

On the Cardinal of the Support of Walsh for Functions of few Variables

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Boolean functions play a crucial role in cryptography and error-correcting codes due to their diverse applications and rich mathematical properties. One such property, the Walsh transform, is a Fourier-Hadamard transform that provides valuable insights into the spectral behavior of Boolean functions. The Walsh support of a Boolean function, defined as the set of points where the Walsh transform is nonzero, offers further structural information. Despite its significance, the Walsh support remains relatively underexplored.

1 Definitions

Definition 1.1. Let $f : \mathbb{F}_2^n \rightarrow \mathbb{F}_2$ be a Boolean function and $a \in \mathbb{F}_2^n$, the Walsh transform in a is defined as :

$$W_f(a) := \sum_{x \in \mathbb{F}_2^n} (-1)^{f(x) + a \cdot x},$$

and the Walsh support is:

$$W_{\text{supp}}(f) := \{a \in \mathbb{F}_2^n, W_f(a) \neq 0\}.$$