Question 3: Solve the following questions from the Discrete Math zyBook:

a) Exercise 4.1.3, sections b, c

Which of the following are functions from **R** to **R**? If f is a function, give its range.

(b) $f(x) = 1/(x^2 - 4)$ Not a function. If x = 2 or x = -2, the result is 1/0.

(c) $f(x) = \sqrt{x^2}$ It is a function. The range is **R**.

b) Exercise 4.1.5, sections b, d, h, i, l

Express the range of each function using roster notation.

(b) Let A = {2, 3, 4, 5}.

f: A → **Z** such that f(x) = x^2.

f(x) = {4,9,16,25}

(d) f: $\{0,1\}^5$ → **Z**. For x ∈ $\{0,1\}^5$, f(x) is the number of 1's that occur in x. For example f(01101) = 3, because there are three 1's in the string "01101".

f(x) = {0,1,2,3,4,5}

(h) Let A = {1, 2, 3}.

f: A × A → **Z**×**Z**, where f(x,y) = (y, x).

f(x,y) = {(3,3),(3,2),(3,1),(2,3),(2,2),(2,1),(1,3),(1,2),(1,1)}

(i) Let A = {1, 2, 3}.

f: A × A → **Z**×**Z**, where f(x,y) = (x,y+1).

f(x,y) = {(4,3),(4,2),(4,1),(3,3),(3,2),(3,1),(2,3),(2,2),(2,1)}

(l) Let A = {1, 2, 3}.

f: P(A) → P(A). For X ⊆ A, f(X) = X - {1}.

f(X) = {∅, {2}, {3}, {1, 2}, {1,3} {2, 3}, {1, 2, 3}}

Question 4: I. Solve the following questions from the Discrete Math zyBook:

a. Exercise 4.2.2, sections c, g, k

For each of the functions below, indicate whether the function is onto, one-to-one, neither or both. If the function is not onto or not one-to-one, give an example showing why.

(c) h: **Z** → **Z**. h(x) = x^3	One-to-one. $1^3 = 1$, but $2^3 = 8$. There's not a way to get to any value in between. Negative numbers will map to negative numbers as well
(g) f: **Z** × **Z** → **Z** × **Z**, f(x, y) = (x+1, 2y)	One-to-one. For y, there is no odd integer that would allow for an odd number. $2y \neq 3$, for example.
(k) f: $\mathbf{Z}^+ \times \mathbf{Z}^+ \to \mathbf{Z}^+$, f(x, y) = $2^x + y$.	Neither. $2^2 + 1 = 2^1 + 3 = 5$, so it's not one-to-one. $2^2 + 2 = 2^1 + 4 = 6$, so it's not onto.

b. Exercise 4.2.4, sections b, c, d, g

For each of the functions below, indicate whether the function is onto, one-to-one, neither or both. If the function is not onto or not one-to-one, give an example showing why.

(b) f: $\{0, 1\}^3 \to \{0, 1\}^3$. The output of f is obtained by taking the input string and replacing the first bit by 1, regardless of whether the first bit is a 0 or 1. For example, f(001) = 101 and f(110) = 110.

Neither. It is not possible to map onto 000, 001, 010, or 011. Furthermore those numbers map onto the same target as 100, 101, 110, and 111. For example, $000_{domain} = 100_{domain} = 100_{target}$.

(c) f: $\{0, 1\}^3 \to \{0, 1\}^3$. The output of f is obtained by taking the input string and reversing the bits. For example f(011) = 110.

Both. Since $x_1 \neq x_2$, and the target is the same as the domain, it is impossible for any two inputs to have the same output.

(d) f: $\{0, 1\}^3 \to \{0, 1\}^4$. The output of f is obtained by taking the input string and adding an extra copy of the first bit to the end of the string. For example, f(100) = 1001.

One-to-one, but not onto. f(000) = 0000, f(001) = 0010. An exhaustive list would show that each input maps onto a different output, but it's impossible to map onto 0001, so not onto.

(g) Let A be defined to be the set {1, 2, 3, 4, 5, 6, 7, 8} and let B = {1}. f: P(A) → P(A). For $X \subseteq A$, f(X) = X - B. Recall that for a finite set A, P(A) denotes the power set of A which is the set of all subsets of A.

Neither. X - B would map to the respective element for {2} or {3}, but {1} - {1} and ∅ - {1} would both result in ∅.

II. Give an example of a function from the set of integers to the set of positive integers that is:

a. one-to-one, but not onto. $f(x) = |x| * 2$ if $x \leq 0$, $f(x) = 2x + 3$ if $x > 0$

b. onto, but not one-to-one. $f(x) = |x| + 1$

c. one-to-one and onto. $f(x) = -2x$ if $x \leq 0$, $2x - 1$ if $x > 0$.

d. neither one-to-one nor onto $f(x) = |x| * 2$ if > 0 or < 0, or $x + 1$ if $x = 0$

Question 5: Solve the following questions from the Discrete Math zyBook:

a) Exercise 4.3.2, sections c, d, g, i

For each of the following functions, indicate whether the function has a well-defined inverse. If the inverse is well-defined, give the input/output relationship of f^{-1}.

(c) $f : R \rightarrow R.\ f(x) = 2x + 3$

Well-defined. f^{-1}(x) = $\frac{x-3}{2}$

(d) Let A be defined to be the set {1,2,3,4,5,6,7,8}
$f : P(A) \rightarrow \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$
For X ⊆ A, $f(x) = |x|$. Recall that for a finite set A, P(A) denotes the power set of which is the set of all subsets of A.

Not well-defined. The power set results in the set of A + 0, which results in the cardinality of the power set. Reversing the cardinality could result in {1}, {2}, {3}, etc., which is certainly not one-to-one.

(g) $f : \{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of f is obtained by taking the input string and reversing the bits. For example, $f : \{0, 1\}^3 = 110$

Well-defined. f^{-1}: $\{0, 1\}^3$ → $\{0, 1\}^3$ will take the reversed string and revert it to the original string.

(i) $f: Z\,x\,Z \rightarrow Z\,x\,Z,\ f(x, y) = (x + 5,\ y - 2)$

Well-defined. $f^{-1}(x, y)$ = (x - 5, y + 2)

b) Exercise 4.4.8, sections c, d

The domain and target set of functions f, g, and h are **Z**. The functions are defined as:

- $f(x) = 2x + 3$
- $g(x) = 5x + 7$
- $h(x) = x^2 + 1$

Give an explicit formula for each function given below.

(c) f o h $f(h(x)) = 2(x^2 + 1) + 3$

(d) h o f $h(f(x)) = (2x + 3)^2 + 1$

c) Exercise 4.4.2, sections b-d

Consider three functions f, g, and h, whose domain and target are **Z**. Let

$$f(x) = x^2 \qquad g(x) = 2^x \qquad h(x) = \frac{x}{5}\,(ceiling\ round)$$

(b) Evaluate (f o h)(52) $f(h(52)) = (\frac{52}{5})^2 = (11)^2 = 121$

(c) Evaluate (g o h o f)(4) $g(h(f(4))) = 2$^$(\frac{4^2}{5}) = 2^4 = 16$

(d) Give a mathematical expression for h o f. $h(f(x)) = \frac{x^2}{5}$

d) Exercise 4.4.6, sections c-e

Define the following functions f, g, and h:

- f: $\{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of f is obtained by taking the input string and replacing the first bit by 1, regardless of whether the first bit is a 0 or 1. For example, f(001) = 101 and f(110) = 110.
- g: $\{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of g is obtained by taking the input string and reversing the bits. For example, g(011) = 110.
- h: $\{0, 1\}^3 \rightarrow \{0, 1\}^3$. The output of h is obtained by taking the input string x, and replacing the last bit with a copy of the first bit. For example, h(011) = 010.

(c) What is (h o f)(010)? (h(f(010)) = h(110) = 111

(d) What is the range of h o f? 101, 111

(e) What is the range of g o f? 001, 101, 111

e) Extra Credit: Exercise 4.4.4, sections c, d

Let f: X → Y and g: Y → Z be two functions.

(c) Is it possible that f is not one-to-one and g o f is one-to-one? Justify your answer. If the answer is "yes", give a specific example for f and g.

The function f maps onto Y, which is the input of function g. If $f(x) = x^2$, it is not one-to-one. However, if $g(x) = \sqrt{x} + 1$, the $g(f(x)) = \sqrt{x^2} = x + 1$.

(d) Is it possible that g is not one-to-one and g o f is one-to-one? Justify your answer. If the answer is "yes", give a specific example for f and g.

The function g maps onto Z, which ostensibly isn't part of function f. I do not think it is possible. If it is, then a similar approach as my last answer should work.