Question 7: Solve the following questions from the Discrete Math zyBook:
a) Exercise 3.1.1, sections a-g

Use the definitions for the sets given below to determine whether each statement is true or false:

$A = \{ x \in \mathbf{Z}$: x is an integer multiple of 3 $\}$
$B = \{ x \in \mathbf{Z}$: x is a perfect square $\}$
$C = \{ 4, 5, 9, 10 \}$
$D = \{ 2, 4, 11, 14 \}$
$E = \{ 3, 6, 9 \}$
$F = \{ 4, 6, 16 \}$

An integer x is a perfect square if there is an integer y such that $x = y^2$.

(a) $27 \in A$ — True. 27/3 = 9

(b) $27 \in B$ — False. $5^2 = 25$ and $6^2 = 36$

(c) $100 \in B$. — True. $10^2 = 100$

(d) $E \subseteq C$ or $C \subseteq E$. — False. Only one element of E is in C

(e) $E \subseteq A$ — True. All elements of E are integer multiples of 3.

(f) $A \subset E$ — False. A contains elements not in E.

(g) $E \in A$ — False. The set of elements in E are not a subset of A.

b) Exercise 3.1.2, sections a-e

Use the definitions for the sets given below to determine whether each statement is true or false:

$A = \{ x \in \mathbf{Z}: x \text{ is an integer multiple of } 3 \}$
$B = \{ x \in \mathbf{Z}: x \text{ is a perfect square} \}$
$C = \{ 4, 5, 9, 10 \}$
$D = \{ 2, 4, 11, 14 \}$
$E = \{ 3, 6, 9 \}$
$F = \{ 4, 6, 16 \}$

An integer x is a perfect square if there is an integer y such that $x = y^2$.

(a) $15 \subset A$ False.

(b) $\{15\} \subset A$ True

(c) $\varnothing \subset C$ True

(d) $D \subseteq D$ True

(e) $\varnothing \in B$ False

c) Exercise 3.1.5, sections b, d

Express each set using set builder notation. Then if the set is finite, give its cardinality. Otherwise, indicate that the set is infinite.

(b) $\{ 3, 6, 9, 12, \ldots. \}$ $B = \{ x \in B : x \text{ is an integer multiple of } 3 \}$

The ellipses indicate the set is infinite.

(d) $\{ 0, 10, 20, 30, \ldots., 1000 \}$ $D = \{ x \in D : x \text{ is an integer multiple of } 10 \}$

The set terminates at 1000.

d) Exercise 3.2.1, sections a-k

Let X = {1, {1}, {1, 2}, 2, {3}, 4 }. Which statements are true?

(a) $2 \in X$ True

(b) $\{2\} \subseteq X$ True

(c) $\{2\} \in X$ False

(d) $3 \in X$ False

(e) $\{1, 2\} \in X$ True

(f) $\{1, 2\} \subseteq X$ True

(g) $\{2, 4\} \subseteq X$ True

(h) $\{2, 4\} \in X$ False

(i) $\{2, 3\} \subseteq X$ False

(j) $\{2, 3\} \in X$ False

(k) $|X| = 7$ False

Question 8: Solve Exercise 3.2.4, section b from the Discrete Math zyBook.

(b) Let A = {1, 2, 3}. What is {X ∈ P(A): 2 ∈ X}?

P(A): = { ∅, {2}, {1, 2}, {2, 3}, {1, 2, 3}}

Question 9: Solve the following questions from the Discrete Math zyBook:

a) Exercise 3.3.1, sections c-e

Define the sets A, B, C, and D as follows:

A = {-3, 0, 1, 4, 17}
B = {-12, -5, 1, 4, 6}
C = {x ∈ **Z**: x is odd}
D = {x ∈ **Z**: x is positive}

For each of the following set expressions, if the corresponding set is finite, express the set using roster notation. Otherwise, indicate that the set is infinite.

(c) A ∩ C {-3, 1, 17}

(d) A ∪ (B ∩ C) {-5, -3, 0, 1, 4, 17}

(e) A ∩ B ∩ C {1}

b) Exercise 3.3.3, sections a, b, e, f

Use the following definitions to express each union or intersection given. You can use roster or set builder notation in your responses, but no set operations. For each definition, i ∈ $\mathbf{Z}^+$.

$A_i = \{ i^0, i^1, i^2 \}$ (Recall that for any number x, $x^0 = 1$.)

$B_i = \{ x \in R : -i \leq x \leq \frac{1}{i} \}$

$C_i = \{ x \in R : \frac{-1}{i} \leq x \leq \frac{1}{i} \}$

(a) $\bigcap_{i=2}^{5} A_i$ $\{ i^0, i^1, i^2 \} = \{1\}$

(b) $\bigcup_{i=2}^{5} A_i$ $\{ i^0, i^1, i^2 \} = \{ 1, 2, 4, 3, 9, 16, 5, 25 \}$

(e) $\bigcap_{i=1}^{100} C_i$ $\{ x \in R : \frac{-1}{i} \leq x \leq \frac{1}{i} \}$

$= \{ x \in R : \frac{-1}{100} \leq x \leq \frac{1}{100} \}$

(f) $\bigcup_{i=1}^{100} C_i$ $\{ x \in R : \frac{-1}{i} \leq x \leq \frac{1}{i} \}$

$= \{ x \in R : -1 \leq x \leq 1 \}$

c) Exercise 3.3.4, sections b, d

Use the set definitions A = {a, b} and B = {b, c} to express each set below. Use roster notation in your solutions.

(b) $P(A \cup B) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\} \}$

(d) $P(A) \cup P(B) = \{ \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\} \}$

Question 10: Solve the following questions from the Discrete Math zyBook:
a) Exercise 3.5.1, sections b, c

The sets A, B, and C are defined as follows:

A = {tall, grande, venti}
B = {foam, no-foam}
C = {non-fat, whole}

Use the definitions for A, B, and C to answer the questions. Express the elements using n-tuple notation, not string notation.

(b) Write an element from the set B × A × C.

(foam, tall, non-fat)

(c) Write the set B × C using roster notation.

{ (foam, non-fat), (foam, whole), (no-foam, non-fat), (no-foam, whole) }

b) Exercise 3.5.3, sections b, c, e

Indicate which of the following statements are true.

(b) $\mathbf{Z}^2 \subseteq \mathbf{R}^2$

True. R is the set of real numbers, which definitionally includes all integers, Z.

(c) $\mathbf{Z}^2 \cap \mathbf{Z}^3 = \varnothing$

True. $\mathbf{Z}^2$ contains binary pairs, and $\mathbf{Z}^3$ contains triplets. The sets wouldn't intersect.

(e) For any three sets, A, B, and C, if A ⊆ B, then A × C ⊆ B × C.

True. If A = {1}, B = {1, 2}, and C = {4}, then A x C = {(1, 4)} and B x C = {(1, 4), (2, 4)}

c) Exercise 3.5.6, sections d, e

Express the following sets using the roster method. Express the elements as strings, not n-tuples.

(d) {xy: where x ∈ {0} ∪ $\{0\}^2$ and y ∈ {1} ∪ $\{1\}^2$}

{01, 011, 001, 0011}

(e) {xy: x ∈ {aa, ab} and y ∈ {a} ∪ $\{a\}^2$}

{aaa, aaaa, aba, abaa}

d) Exercise 3.5.7, sections c, f, g

Use the following set definitions to specify each set in roster notation. Except where noted, express elements of Cartesian products as strings.

- A = {a}
- B = {b, c}
- C = {a, b, d}

(c) (A × B) ∪ (A × C)

{ab, ac, aa, ad}

(f) P(A × B)

{∅, {ab}, {ac}, {ab,ac}}

(g) P(A) × P(B). Use ordered pair notation for elements of the Cartesian product.

{ (∅,∅), (∅,{b}), (∅,{c}), (∅,{b,c}), ({a},∅), ({a},{b}), ({a},{c}), ({a},{b,c}) }

Question 11: Solve the following questions from the Discrete Math zyBook:
a) Exercise 3.6.2, sections b, c

Use the set identities given in the table to prove the following new identities. Label each step in your proof with the set identity used to establish that step.

(b) (B ∪ A) ∩ ($\overline{B}$ ∪ A) = A

Line	Statement	Set Identity
1	(B ∪ A) ∩ ($\overline{B}$ ∪ A)	Theorem
2	(B ∪ A) ∩ (A ∪ $\overline{B}$)	Commutative Law, 1
3	A ∪ (B ∩ $\overline{B}$)	Distributive Law, 2
4	A ∪ ∅	Complement Law, 3
5	A	Identity Law, 4

(c) $\overline{A \cap \overline{B}}$ = $\overline{A}$ ∪ B

Line	Statement	Set Identity
1	$\overline{A \cap \overline{B}}$	Theorem
2	¬(A ∩ $\overline{B}$)	Definition of a Complement
3	¬(A ∧ $\overline{B}$)	Definition of Intersection
4	¬(A) ∨ ¬($\overline{B}$)	De Morgan's Law
5	$\overline{A}$ ∨ B	Definition of a Complement
6	$\overline{A}$ ∪ B	Definition of a Union

b) Exercise 3.6.3, sections b, d

A set equation is not an identity if there are examples for the variables denoting the sets that cause the equation to be false. For example $A \cup B = A \cap B$ is not an identity because if $A = \{1, 2\}$ and $B = \{1\}$, then $A \cup B = \{1, 2\}$ and $A \cap B = \{1\}$, which means that $A \cup B \neq A \cap B$.

Show that each set equation given below is not a set identity.

(b) $A - (B \cap A) = A$ If $A = \{4, 5\}$ and $B = \{3, 4, 5\}$, $B \cap A = \{4, 5\}$. $\{4, 5\} - \{4, 5\} = \varnothing \neq A$

(d) $(B - A) \cup A = A$ If $B = \{1, 6\}$ and $A = \{1\}$, $(B - A) = \{6\}$. $(B - A) \cup A = \{6\} \cup \{1\} \neq A$

c) Exercise 3.6.4, sections b, c

The set subtraction law states that A - B = A ∩ $\overline{B}$. Use the set subtraction law as well as the other set identities given in the table to prove each of the following new identities. Label each step in your proof with the set identity used to establish that step.

(b) A ∩ (B - A) = ∅

Line	Statement	Set Identity
1	A ∩ (B - A)	Theorem
2	A ∩ (B ∩ $\overline{A}$)	Definition of Intersection
3	A ∩ ($\overline{A}$ ∩ B)	Commutative Law
4	(A ∩ $\overline{A}$) ∩ B	Associative Law
5	∅ ∩ B	Complement Law
6	∅	Domination Law

(c) A ∪ (B - A) = A ∪ B

Line	Statement	Set Identity
1	A ∪ (B - A)	Theorem
2	A ∪ (B ∩ $\overline{A}$)	Definition of Intersection
3	(A ∪ B) ∩ (A ∪ $\overline{A}$)	Distributive Law
4	(A ∪ B) ∩ *U*	Complement Law
5	A ∪ B	Identity Law