Question 1: A. Convert the following numbers to their decimal representation. Show your work.

1. $10011011_2 = (1*2^0) + (1*2^1) + (0*2^2) + (1*2^3) + (1*2^4) + (0*2^5) + (0*2^6) + (1*2^7)$
$= (1*1) + (1*2) + (0*4) + (1*8) + (1*16) + (0*32) + (0*64) + (1*128)$
$= 1 + 2 + 0 + 8 + 16 + 0 + 0 + 128$
$= 155_{10}$

2. $456_7 = (6*7^0) + (5*7^1) + (4*7^2)$
$= (6*1) + (5*7) + (4*49)$
$= 6 + 35 + 196$
$= 237_{10}$

3. $38A_{16} = (A*16^0) + (8*16^1) + (3*16^2)$ (A = 10 in decimal)
$= (10*1) + (8*16) + (3*256)$
$= 10 + 128 + 768$
$= 906_{10}$

4. $2214_5 = (4*5^0) + (1*5^1) + (2*5^2) + (2*5^3)$
$= (4*1) + (1*5) + (2*25) + (2*125)$
$= 4 + 5 + 50 + 250$
$= 309_{10}$

B. Convert the following numbers to their binary representation:

1. 69_{10} = 69 / 2 = 34 R 1
34 / 2 = 17 R 0
17 / 2 = 8 R 1
8 / 2 = 4 R 0
4 / 2 = 2 R 0
2 / 2 = 1 R 0
R 1
Collecting the remainders gives us 1000101_2

2. 485_{10}= 485 / 2 = 242 R 1
242 / 2 = 121 R 0
121 / 2 = 60 R 1
60 / 2 = 30 R 0
30 / 2 = 15 R 0
15 / 2 = 7 R 1
7 / 2 = 3 R 1
3 % 2 = 1 R 1
R 1
Collecting the remainders gives us 111100101_2

3. $6D1A_{16} = 0110110100011010_2$
(6 = 0110) (D = 1101) (1 = 0001) (A = 1010)

I used the conversion table. You're going to have to trust me on that because I can't show it.

C. Convert the following numbers to their hexadecimal representation:

1. 1101011_2 = (6 = 0110) (b = 1011) $6b_{16}$ per the table
$= (1*2^0) + (1*2^1) + (0*2^2) + (1*2^3) + (0*2^4) + (1*2^5) + (1*2^6) + (0*2^7)$
$= (1*1 + 1*2 + 0*4 + 1*8)*2^0 + (0*1 + 1*2 + 1*4 + 0*8)*2^4$
$= (1 + 2 + 8)*16^0 + (2 + 4)*16^1$
$= (11)*16^0 + (6)*16^1$ => $b_{16} = 11_{10}$
$= 6b_{16}$

2. 895_{10} = 895 / 16 = 55 R 15
= 55 / 16 = 3 R 7
R 3

15 is f in hex, so
$895_{10} = 37f_{16}$

Question 2: Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8 =$ $^{1}7\ ^{1}5\ ^{1}6\ 6$ Adding 5_8 to 6_8 is equal to $2_8 + 3_8 + 6_8$.
$+\ \ 4\ \ 5\ \ 1\ \ 5$ $(2_8 + 6_8) + 3_8 \Rightarrow (10_8) + 3_8 = 13_8$
$1\ \ 4\ \ 3\ \ 0\ \ 3$ That of course applies to all base 8 numbers.

14303_8

2. $10110011_2 + 1101_2 =$ $1\ ^{1}0\ ^{1}1\ ^{1}1\ ^{1}0\ ^{1}0\ ^{1}1\ 1$
$+\ \ 0\ \ 0\ \ 0\ \ 0\ \ 1\ \ 1\ \ 0\ 1$
$1\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0$

11000000_2

3. $7A66_{16} + 45C5_{16} =$ $^{1}7\ ^{1}a\ 6\ 6$ Adding 5_{16} to 6_{16} is equal to 11_{16}, which uses the
$+\ \ 4\ \ 5\ c\ 5$ symbol b, so no need to carry. Adding c_{16} to 6_{16}
$c\ \ 0\ \ 2\ b$ does require a carry, as the value is 2 over the
'Base'. The same is true for $a_{16} + 1_{16} + 5_{16}$, which
$c02b_{16}$ meets the base.

4. $3022_5 - 2433_5 =$ $^{-1}3\ ^{-1}0\ ^{-1}2\ 2$ Subtracting 3_5 from 2_5 will drop the value in the ones
$-\ \ 2\ \ 4\ \ 3\ 3$ place below the base, so we need to borrow leaving
$0\ \ 0\ \ 3\ 4$ a 4 in the ones place and subtracting 3 from 1 in the
tens place.

34_5

Question 3: A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. 124_{10} =	0	1	1	1	1	1	0	0
	128	64	32	16	8	4	2	1

The leading digit indicates whether it is positive or negative, and this is positive, so it is 0. Adding the value of the weighted positions as above results in 124_{10}.

$01111100_{8 \text{ bit 2's comp}}$

2. -124_{10} =>

($\sim 124_{10}$) =	1	0	0	0	0	0	1	1

The leading digit indicates whether it is positive or negative, and this is positive, so it is 1. Per Ratan's instructions, we can hold a temporary value for 124_{10} by inverting all of the numbers. Adding 1 to it should give us the negative 8-bit 2's complement.

	1	0	0	0	0	$^{1}0$	$^{1}1$	1
+								1
	1	0	0	0	0	1	0	0

$10000100_{8 \text{ bit 2's comp}}$

3. 109_{10} =	0	1	1	0	1	1	0	1
	128	64	32	16	8	4	2	1

We use 0 as the leading digit and use the values of the weights to get 109_{10}.

$01101101_{8 \text{ bit 2's comp}}$

4. -79_{10} = I don't have the luxury of knowing the positive value, so I'll work it out.

79_{10} =	0	1	0	0	1	1	1	1
$\sim 79_{10}$ =	0	0	1	1	0	0	0	0
+								1
	1	0	1	1	0	0	0	1

Using Ratan's method, I get the final value from the inverted positive number + 1.

$10110001_{8 \text{ bit 2's comp}}$

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{8\text{ bit 2's comp}}$ = A leading 0 indicates the number is positive, so I will add the values to find the decimal value.

0	0	0	1	1	1	1	0	
128	64	32	16	8	4	2	1	
			16 +	8 +	4 +	2		= 30

30_{10}

2. $11100110_{8\text{ bit 2's comp}}$ = A leading 1 indicates the number is negative. We'll work this out by multiplying the values against their respective weights:

1	1	1	0	0	1	1	0	
$(-1 * 2^7)$	64	32	16	8	4	2	1	
-128 +	64 +	32	+		4 +	2		= - 26

-26_{10}

3. $00101101_{8\text{ bit 2's comp}}$ = A leading 0 indicates the number is positive, so I will add the values to find the decimal value.

0	0	1	0	1	1	0	1	
128	64	32	16	8	4	2	1	
		32	+	8 +	4	+	1	= 45

45_{10}

4. $10011110_{8\text{ bit 2's comp}}$ = A leading 1 indicates the number is negative. We'll do the same as above.

1	0	0	1	1	1	1	0	
-128	64	32	16	8	4	2	1	
-128	+		16 +	8 +	4 +	2		= -98

-98_{10}

Question 4: Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4, sections b, c

(b) $\neg(p \lor q)$

p	q	$\neg(p \lor q)$
T	T	F
T	F	F
F	T	F
F	F	T

(c) $r \lor (p \land \neg q)$

r	p	q	$(p \land \neg q)$	$r \lor (p \land \neg q)$
T	T	T	F	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	T
F	T	T	F	F
F	T	F	T	T
F	F	T	F	F
F	F	F	F	F

2. Exercise 1.3.4, sections b, d Give a truth table for each expression.

(b) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

p	q	$(p \rightarrow q)$	$(q \rightarrow p)$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(d) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$(p \leftrightarrow q)$	$(p \leftrightarrow \neg q)$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

Question 5: Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.7, sections b, c

Consider the following pieces of identification a person might have in order to apply for a credit card:

> B: Applicant presents a birth certificate.
> D: Applicant presents a driver's license.
> M: Applicant presents a marriage license.

(b) The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.

$(B \land D) \lor (B \land M) \lor (D \land M)$

(c) Applicant must present either a birth certificate or both a driver's license and a marriage license.

$B \lor (D \land M)$

2. Exercise 1.3.7, sections b – e

Define the following propositions:

s: a person is a senior
y: a person is at least 17 years of age
p: a person is allowed to park in the school parking lot

Express each of the following English sentences with a logical expression:

(b) A person can park in the school parking lot if they are a senior or at least seventeen years of age.

$(s \lor y) \to p$

(c) Being 17 years of age is a necessary condition for being able to park in the school parking lot.

$p \to y$

(d) A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

$p \leftrightarrow (s \land y)$

(e) Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

$p \to (s \lor y)$

3. Exercise 1.3.9, sections c, d

Use the definitions of the variables below to translate each English statement into an equivalent logical expression.

- y: the applicant is at least eighteen years old
- p: the applicant has parental permission
- c: the applicant can enroll in the course

(c) The applicant can enroll in the course only if the applicant has parental permission.

$p \leftrightarrow c$

(d) Having parental permission is a necessary condition for enrolling in the course.

$p \rightarrow c$

Question 6: Solve the following questions from the Discrete Math zyBook:
1. Exercise 1.3.6, sections b - d

Give an English sentence in the form "If...then..." that is equivalent to each sentence.

(b) Maintaining a B average is necessary for Joe to be eligible for the honors program.

If Joe is to be eligible for the honors program, then he must maintain a B average.

(c) Rajiv can go on the roller coaster only if he is at least four feet tall.

If and only if Rajiv is at least four feet tall, then he can go on the roller coaster.

(d) Rajiv can go on the roller coaster if he is at least four feet tall.

If Rajiv is at least four feet tall, then he can go on the roller coaster.

2. Exercise 1.3.10, sections c – f

The variable p is true, q is false, and the truth value for variable r is unknown. Indicate whether the truth value of each logical expression is true, false, or unknown.

(c) $(p \lor r) \leftrightarrow (q \land r)$

False

(d) $(p \land r) \leftrightarrow (q \land r)$

Unknown

(e) $p \rightarrow (r \lor q)$

Unknown

(f) $(p \land q) \rightarrow r$

True

Question 7: Solve Exercise 1.4.5, sections b – d, from the Discrete Math zyBook:

Define the following propositions:

- j: Sally got the job.
- l: Sally was late for her interview
- r: Sally updated her resume.

Express each pair of sentences using logical expressions. Then prove whether the two expressions are logically equivalent.

(b) If Sally did not get the job, then she was late for her interview or did not update her resume.
If Sally updated her resume and was not late for her interview, then she got the job.

$\neg j \rightarrow (l \lor r)$

$(r \land \neg l) \rightarrow j$

j	l	r	$\neg j \rightarrow (l \lor r)$	$(r \land \neg l) \rightarrow j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	F
F	F	F	F	T

The expressions are not logically equivalent.

(c) If Sally got the job then she was not late for her interview.
If Sally did not get the job, then she was late for her interview.

$j \rightarrow \neg l$ $\quad$ T → F => F

$\neg j \rightarrow l$ $\quad$ F → T => T

The expressions are not logically equivalent.

(d) If Sally updated her resume or she was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.

$(r \lor \neg l) \rightarrow j$ $\quad$ $T \rightarrow T \Rightarrow T$
$j \rightarrow (r \land \neg l)$ $\quad$ $T \rightarrow T \Rightarrow T$

The expressions are logically equivalent.

Question 8: Solve the following questions from the Discrete Math zyBook:
1. Exercise 1.5.2, sections c, f, i

Use the laws of propositional logic to prove the following:

(c) $(p \rightarrow q) \land (p \rightarrow r) \equiv p \rightarrow (q \land r)$

$(\neg p \lor q) \land (\neg p \lor r)$ Conditional Identities

$\neg p \lor (q \land r)$ Distributive Laws

$p \rightarrow (q \land r)$ Conditional Identities

(f) $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$\neg((p \lor \neg p) \land (p \lor q))$ Distributive Laws

$\neg(p \lor \neg p) \lor \neg(p \lor q)$ De Morgan's Laws

$\neg(T) \lor \neg(p \lor q)$ Complement Laws

F v $\neg(p \lor q)$ Complement Laws

$\neg(p \lor q)$ Identity Laws

$\neg p \land \neg q$ De Morgan's Laws

(i) $(p \land q) \rightarrow r \equiv (p \land \neg r) \rightarrow \neg q$

$\neg(p \land q) \lor r$ Conditional Identities

$(\neg p \lor \neg q) \lor r$ De Morgan's Laws

$(\neg p \lor r) \lor \neg q$ Associative Laws

$\neg(p \land \neg r) \lor \neg q$ De Morgan's Laws

$(p \land \neg r) \rightarrow \neg q$ Conditional Identities

2. Exercise 1.5.3, sections c, d

Use the laws of propositional logic to prove that each statement is a tautology.

(c) $\neg r \lor (\neg r \to p)$

$\neg r \lor (\neg\neg r \lor p)$	Conditional Identities
$\neg r \lor (r \lor p)$	Double Negation Law
$(\neg r \lor r) \lor p$	Associative Laws
$T \lor p$	Complement Laws
T	Final value

(d) $\neg(p \to q) \to \neg q$

$\neg(\neg p \lor q) \to \neg q$	Conditional Identities
$(\neg\neg p \land \neg q) \to \neg q$	De Morgan's Laws
$(p \land \neg q) \to \neg q$	Double Negation Law
$\neg(p \land \neg q) \lor \neg q$	Conditional Identities again
$(\neg p \lor \neg\neg q) \lor \neg q$	De Morgan's Laws
$(\neg p \lor q) \lor \neg q$	Double Negation Law
$\neg p \lor (q \lor \neg q)$	Associative Laws
$\neg p \lor T$	Complement Laws
T	Final value

Question 9: Solve the following questions from the Discrete Math zyBook:
1. Exercise 1.6.3, sections c, d

Consider the following statements in English. Write a logical expression with the same meaning. The domain is the set of all real numbers.

(c) There is a number that is equal to its square.

$\exists x\ (x = x^2)$

(d) Every number is less than or equal to its square plus 1.

$\forall x\ (x \leq x^2 + 1)$

2. Exercise 1.7.4, sections b - d

In the following question, the domain is a set of employees who work at a company. Ingrid is one of the employees at the company. Define the following predicates:

S(x): x was sick yesterday
W(x): x went to work yesterday
V(x): x was on vacation yesterday

Translate the following English statements into a logical expression with the same meaning.

(b) Everyone was well and went to work yesterday.

$\forall x\ (\neg S(x) \land W(x))$

(c) Everyone who was sick yesterday did not go to work.

$\forall x\ (S(x) \rightarrow \neg W(x))$

(d) Yesterday someone was sick and went to work.

$\exists x\ (S(x) \land W(x))$

Question 10: Solve the following questions from the Discrete Math zyBook:
1. Exercise 1.7.9, sections c - i

The domain for this question is the set {a, b, c, d, e}. The following table gives the value of predicates P, Q, and R for each element in the domain. For example, Q(c) = T because the truth value in the row labeled c and the column Q is T. Using these values, determine whether each quantified expression evaluates to true or false.

	P(x)	**Q(x)**	**R(x)**
a	T	T	F
b	T	F	F
c	F	T	F
d	T	T	F
e	T	T	T

(c) $\exists x((x = c) \rightarrow P(x))$ False

(d) $\exists x(Q(x) \land R(x))$ True, row e

(e) $Q(a) \land P(d)$ True

(f) $\forall x ((x \neq b) \rightarrow Q(x))$ True

(g) $\forall x (P(x) \lor R(x))$ False, row C is False on both.

(h) $\forall x (R(x) \rightarrow P(x))$ True

(i) $\exists x(Q(x) \lor R(x))$ True

2. Exercise 1.9.2, sections b - i

The tables below show the values of predicates P(x, y), Q(x, y), and S(x, y) for every possible combination of values of the variables x and y. The row number indicates the value for x and the column number indicates the value for y. The domain for x and y is {1, 2, 3}.

P	**1**	**2**	**3**
1	T	F	T
2	T	F	T
3	T	T	F

Q	**1**	**2**	**3**
1	F	F	F
2	T	T	T
3	T	F	F

S	**1**	**2**	**3**
1	F	F	F
2	F	F	F
3	F	F	F

Indicate whether each of the quantified statements is true or false.

(b) $\exists x\ \forall y\ Q(x, y)$ True, row 2 is full of T

(c) $\exists y\ \forall x\ P(x, y)$ True, the entire column in y = 1 is true

(d) $\exists x\ \exists y\ S(x, y)$ False, there are no possible selections for True

(e) $\forall x\ \exists y\ Q(x, y)$ False, the top column is all false

(f) $\forall x\ \exists y\ P(x, y)$ True, there is a true value in every column in every row

(g) $\forall x\ \forall y\ P(x, y)$ False, there are false values on the board

(h) $\exists x\ \exists y\ Q(x, y)$ True, at least one square is true

(i) $\forall x\ \forall y\ \neg S(x, y)$ True, ¬S would make the whole board true

Question 11: Solve the following questions from the Discrete Math zyBook:
1. Exercise 1.10.4, sections c - g

Translate each of the following English statements into logical expressions. The domain is the set of all real numbers

(c) There are two numbers whose sum is equal to their product.
$\exists x \ \exists y \ ((x + y) = (x * y))$

(d) The ratio of every two positive numbers is also positive.
$\forall x \ \forall y \ (((x > 0) \land (y > 0)) \rightarrow (x/y > 0) \land (y/x > 0))$

(e) The reciprocal of every positive number less than one is greater than one.
$\forall x \ ((x > 0) \land (x < 1) \rightarrow (1/x > 1))$

(f) There is no smallest number.
$\forall x \ \exists y \ (x > y)$

(g) Every number other than 0 has a multiplicative inverse.
$\forall x \ ((x \neq 0)) \rightarrow (x^{-1}))$

2. Exercise 1.10.7, sections c - f

The domain is a group working on a project at a company. One of the members of the group is named Sam. Define the following predicates.

- P(x, y): x knows y's phone number. (A person may or may not know their own phone number.)
- D(x): x missed the deadline.
- N(x): x is a new employee.

Give a logical expression for each of the following sentences.

(c) There is at least one new employee who missed the deadline.
$\exists x \ (N(x) \land D(x))$

(d) Sam knows the phone number of everyone who missed the deadline.
$\forall y \ (D(y) \rightarrow P(Sam, y))$

(e) There is a new employee who knows everyone's phone number.
$\exists x \ \forall y \ (N(x) \land P(x, y))$

(f) Exactly one new employee missed the deadline.

$\exists x\ ((N(x) \land D(x)) \land \forall y((x \neq y) \rightarrow \neg D(y)))$

3. Exercise 1.10.10, sections c – f

The domain for the first input variable to predicate T is a set of students at a university. The domain for the second input variable to predicate T is the set of Math classes offered at that university. The predicate T(x, y) indicates that student x has taken class y. Sam is a student at the university and Math 101 is one of the courses offered at the university. Give a logical expression for each sentence.

(c) Every student has taken at least one class other than Math 101.

$\forall x\ \exists y\ ((y \neq \text{Math } 101) \rightarrow T(x, y)))$

(d) There is a student who has taken every math class other than Math 101.

$\exists x\ \forall y\ ((y = \text{a math class} \land y \neq \text{Math } 101) \rightarrow T(x, y)))$

(e) Everyone other than Sam has taken at least two different math classes.

$\forall x\ \exists y\ \exists z\ (((x \neq \text{Sam}) \land (y \neq z)) \rightarrow (T(x, y) \land T(x, z)))$

(f) Sam has taken exactly two math classes

$\exists x\ \exists y\ \exists z\ ((x = \text{Sam}) \land (y \neq z) \land T(x, y) \land T(x, z))$

Question 12: Solve the following questions from the Discrete Math zyBook:
1. Exercise 1.8.2, sections b – e

In the following question, the domain is a set of male patients in a clinical study. Define the following predicates:

P(x): x was given the placebo
D(x): x was given the medication
M(x): x had migraines

Translate each statement into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English.

Sample question: Some patient was given the placebo and the medication.

- $\exists x\ (P(x) \land D(x))$
- Negation: $\neg\exists x\ (P(x) \land D(x))$
- Applying De Morgan's law: $\forall x\ (\neg P(x) \lor \neg D(x))$
- English: Every patient was either not given the placebo or not given the medication (or both).

(b) Every patient was given the medication or the placebo or both.

$\forall x\ (P(x) \lor D(x)) \lor (P(x) \land D(x))$
$\neg\forall x\ (P(x) \lor D(x)) \lor (P(x) \land D(x))$
$\exists x\ \neg((P(x) \lor D(x)) \land (P(x) \land D(x)))$
$\exists x\ \neg(P(x) \lor D(x)) \lor \neg(P(x) \land D(x))$

There exists a patient who was not given the medication or placebo or was not given both.

(c) There is a patient who took the medication and had migraines.

$\exists x\ (D(x) \land M(x))$
$\neg\exists x\ (D(x) \land M(x))$
$\forall x\ \neg(D(x) \land M(x))$
$\forall x\ \ (\neg D(x) \lor \neg M(x))$

Every patient who didn't take the medication didn't have migraines.

(d) Every patient who took the placebo had migraines. (Hint: you will need to apply the conditional identity, $p \to q \equiv \neg p \lor q$.)

$$\forall x\, (P(x) \to M(x))$$
$$\neg\forall x\, (P(x) \to M(x))$$
$$\exists x\, \neg(P(x) \to M(x))$$
$$\exists x\, \neg(\neg P(x) \lor M(x))$$
$$\exists x\ \ (\neg\neg P(x) \land \neg M(x))$$
$$\exists x\ \ (P(x) \land \neg M(x))$$

There was a patient who took the placebo and didn't have a migraine.

(e) There is a patient who had migraines and was given the placebo.

$$\exists x\, (M(x) \land P(x))$$
$$\neg\exists x\, (M(x) \land P(x))$$
$$\forall x\, \neg(M(x) \land P(x))$$
$$\forall x\, (\neg M(x) \lor \neg P(x))$$

Every patient didn't have a migraine or take the placebo.

2. Exercise 1.9.4, sections c - e

Write the negation of each of the following logical expressions so that all negations immediately precede predicates. In some cases, it may be necessary to apply one or more laws of propositional logic.

(c) $\exists x\ \forall y\, (P(x, y) \to Q(x, y))$

$$\forall x\, \neg\forall y\, (P(x, y) \to Q(x, y))$$
$$\forall x\ \exists y\, \neg(P(x, y) \to Q(x, y))$$
$$\forall x\ \exists y\, \neg(\neg P(x, y) \lor Q(x, y))$$
$$\forall x\ \exists y\ \ (\neg\neg P(x, y) \land \neg Q(x, y))$$
$$\forall x\ \exists y\ \ (P(x, y) \land \neg Q(x, y))$$

(d) $\exists x\ \forall y\, (P(x, y) \leftrightarrow P(y, x))$

$$\forall x\, \neg\forall y\, (P(x, y) \leftrightarrow P(y, x))$$
$$\forall x\ \exists y\, \neg(P(x, y) \leftrightarrow P(y, x))$$
$$\forall x\ \exists y\, \neg(P(x, y) \leftrightarrow P(y, x))$$
$$\forall x\ \exists y\, \neg((P(x, y) \to P(y, x)) \land (P(y, x) \to P(x, y))$$
$$\forall x\ \exists y\, \neg((\neg P(x, y) \lor P(y, x)) \land (\neg P(y, x) \lor P(x, y))$$
$$\forall x\ \exists y\ \ (\neg(\neg P(x, y) \lor P(y, x)) \lor \neg(\neg P(y, x) \lor P(x, y))$$
$$\forall x\ \exists y\ \ ((\neg\neg P(x, y) \land \neg P(y, x)) \lor (\neg\neg P(y, x) \land \neg P(x, y))$$
$$\forall x\ \exists y\ \ ((P(x, y) \land \neg P(y, x)) \lor (P(y, x) \land \neg P(x, y))$$

(e) $\exists x\ \exists y\, P(x, y) \land\ \forall x\ \forall y\, Q(x, y)$

$$\forall x\, \neg\exists y\, P(x, y) \land\ \exists x\, \neg\forall y\, Q(x, y)$$
$$\forall x\ \forall y\, \neg P(x, y) \land\ \exists x\ \exists y\, \neg Q(x, y)$$