Optical Geometry- BPhO Computational Physics Challenge 2025

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Task 1a)

Using Python, I plotted the refractive index of BK7 glass vs. wavelength over the visible spectrum (400–800 nm) using the Sellmeier equation, with sliders allowing the wavelength range to be adjusted interactively. I entered the Sellmeier coefficients for BK7 crown glass (from `https://refractiveindex.info/?shelf=3d&book=glass&page=BK7`) and defined the Sellmeier equation:

Sellmeier Equation

An empirical formula used for modelling how a transparent material's refractive index n changes with the wavelength λ of light (in µm). It uses the theory that materials bend different wavelengths of light by differing amounts (dispersion):

$$n^2(\lambda) = 1 + \sum_{i=1}^{3} \frac{B_i \lambda^2}{\lambda^2 - C_i}$$

Where:

$n(\lambda)$: refractive index at wavelength λ (µm)

B_i, C_i: material-specific Sellmeier coefficients

λ: Wavelength in micrometres (µm)

BK7 Coefficients

$B_1 = 1.03961212, \quad B_2 = 0.231792344, \quad B_3 = 1.01046945\, C_1 = 0.00600069867, \quad C_2 = 0.0200179144, \quad C_3 = 103.560653$

Example for $\lambda = 550\,\text{nm}$:

Convert to micrometres: $550\,\text{nm} = 0.55\,\mu\text{m}$

Apply the equation:

$$n^2 = 1 + \frac{1.03961212 \cdot (0.55)^2}{(0.55)^2 - 0.00600069867} + \frac{0.231792344 \cdot (0.55)^2}{(0.55)^2 - 0.0200179144} + \frac{1.01046945 \cdot (0.55)^2}{(0.55)^2 - 103.560653}$$

With $(0.55)^2 = 0.3025$, the final result is:

$$n \approx \sqrt{2.3109} \approx 1.5195$$

matching the known value for BK7 at this wavelength.

Task 1b)

Using the empirical formula:

$$\left(n^2 - 1\right)^{-2} = 1.731 - 0.261 \left(\frac{f}{10^{15}\,\mathrm{Hz}}\right)^2$$

where n is the refractive index of water and f is the frequency in Hz, we can compute n over the visible spectrum.

Step 1: Frequency range Visible light wavelengths (400–800 nm) correspond to frequencies via $f = c/\lambda$: 400 nm $\rightarrow$ 750 THz

800 nm $\rightarrow$ 375 THz

In code: "' frequencies$_t hz = np.linspace(375, 750, 1000)$ frequencies$_h z = frequencies_t hz *$ $1e12$"'

Step 2: Compute n Rearranging the equation:

$$n = \sqrt{1 + \frac{1}{\sqrt{1.731 - 0.261 \cdot (f/10^{15})^2}}}$$

Implementation: "'python def refractive$_i ndex_w ater(frequency_h z) : f_s caled = frequency_h z/1e15inv_s q =$ $1.731 - 0.261 * f_s caled * *2n_s quared = 1 + 1/np.sqrt(inv_s q)returnnp.sqrt(n_s quared)$"'

Step 3: Colour mapping Frequencies are converted back to wavelengths via $\lambda = c/f$ and mapped to RGB values using a visible-light colour function.

Step 4: Plotting with Plotly "' fig.add$_t race(go.Scatter(x = frequencies_t hz, y =$ $refractive_i ndices, mode =' markers', marker = dict(color = colors, size = 5), hovertemplate ='$ $Frequency :))$"'

Task 2)

The thin lens equation relates the object distance u, image distance v, and focal length f of a thin lens:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Rearranging into a linear form for analysis:

$$\frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$$

Letting $x = \frac{1}{u}$ and $y = \frac{1}{v}$ gives:

$$y = -x + \frac{1}{f}$$

This is a straight line with slope $m = -1$ and intercept $c = \frac{1}{f}$, allowing f to be found as $f = \frac{1}{c}$.

0.1 Data Processing and Uncertainty Propagation

The inverse distances $\frac{1}{u}$ and $\frac{1}{v}$ were calculated. Uncertainties were propagated via:

$$\sigma_{1/u} = \frac{\sigma_u}{u^2}, \quad \sigma_{1/v} = \frac{\sigma_v}{v^2}$$

Since uncertainties exist in both x and y, Orthogonal Distance Regression (ODR) was used instead of ordinary least squares, minimising perpendicular distances to the regression line. This provides a more accurate estimate of the focal length.

0.2 Results

Slope: -1.0172 ± 0.0288 Intercept ($\frac{1}{f}$): $0.0662 \pm 0.0009\,\mathrm{cm}^{-1}$ Focal length: $f = 14.95 \pm 0.20\,\mathrm{cm}$

The slope was close to the theoretical value of -1, confirming the linearity predicted by the thin lens equation.

0.3 Potential Sources of Error

Measurement inaccuracies in u and v

Lens thickness and aberrations (thin lens approximation limitations)

Parallax errors in image position measurements

Task 3/4)

0.4 Fermat's Principle and the Law of Reflection

Consider a mirror, a ray originating from a point on the left L can reach a point on the right R in two ways: either travelling directly from L to R, or by reflecting off the mirror first. These two paths have different lengths and thus take different amounts of time.

Each corresponds to a local minimum in travel time, meaning the actual path taken by light is a minimum compared to other paths in its immediate vicinity, even if it's not the absolute shortest time path; this arises from the wave nature of light, although we usually model it as a ray.

Define the x-axis as the line where the mirror intersects the plane containing points L and R, perpendicular to the mirror surface. The y-axis is normal to the mirror. The coordinates of points L and R are (x_L, y_L) and (x_R, y_R), respectively.

Suppose the ray hits the mirror at $(x, 0)$. The total travel time for the ray going from L to R via this point on the mirror is:

$$\text{travel time} = ncd_1(x) + ncd_2(x) = nc\sqrt{(x - x_L)^2 + y_L^2} + nc\sqrt{(x_R - x)^2 + y_R^2}$$

Where n is the refractive index of the medium above the mirror ($y > 0$) and c is the speed of light in vacuum.

According to Fermat's Principle, the actual path corresponds to the minimum travel time. Thus, the value of x at the reflection point satisfies:

$$\frac{d}{dx}[d_1(x) + d_2(x)] = \frac{x - x_L}{d_1(x)} - \frac{x_R - x}{d_2(x)} = 0$$

From this, it follows that the angle of incidence θ_i equals the angle of reflection θ_r:

$$\sin\theta_i = \sin\theta_r \quad \Rightarrow \quad \theta_r = \theta_i$$

Where θ_i and θ_r are the angles the incoming and reflected rays make with the normal to the mirror surface.

0.5 Problem Setup

A source $A = (x_A, y_A)$ and target $B = (x_B, y_B)$, both with $y > 0$, are positioned above a flat reflecting surface at $y = 0$. For a candidate reflection point $R = (x, 0)$ on the mirror, the total path length is:

$$L(x) = \sqrt{(x - x_A)^2 + y_A^2} + \sqrt{(x_B - x)^2 + y_B^2}.$$

Fermat's principle predicts that the physical reflection point minimises $L(x)$. As it states that light travels between two points along the path that makes the travel time stationary (usually a minimum). In a uniform medium of constant speed v, travel time is proportional to path length, so minimising time is equivalent to minimising path length.

0.6 Derivation

Differentiating $L(x)$ with respect to x:

$$\frac{dL}{dx} = \frac{x - x_A}{\sqrt{(x - x_A)^2 + y_A^2}} - \frac{x_B - x}{\sqrt{(x_B - x)^2 + y_B^2}}.$$

Setting $dL/dx = 0$ for an extremum gives:

$$\frac{x - x_A}{|AR|} = \frac{x_B - x}{|RB|}.$$

Geometrically,

$$\frac{x - x_A}{|AR|} = \sin\theta_i, \quad \frac{x_B - x}{|RB|} = \sin\theta_r,$$

where θ_i and θ_r are the incident and reflected angles concerning the normal. The extremum condition becomes:

$$\sin\theta_i = \sin\theta_r \quad \Rightarrow \quad \theta_i = \theta_r$$

(for $0° \leq \theta < 90°$), which is the law of reflection.

Snell's Law of Refraction

Next, consider refraction at the boundary between two media. Let the interface lie along $y = 0$, separating a medium with refractive index n_i in the upper half-plane ($y > 0$) and n_t in the lower half-plane ($y < 0$). Points L and R lie in these two media with coordinates (x_L, y_L) and (x_R, y_R), where $y_L > 0$ and $y_R < 0$, meaning a ray traveling from L to R crosses the interface at $(x, 0)$. The total travel time is:

$$n_i c d_1(x) + n_t c d_2(x) = n_i c\sqrt{(x - x_L)^2 + y_L^2} + n_t c\sqrt{(x_R - x)^2 + y_R^2}$$

Fermat's Principle requires the travel time to be minimised, leading to:

$$\frac{d}{dx}[n_i d_1(x) + n_t d_2(x)] = n_i \frac{x - x_L}{d_1(x)} - n_t \frac{x_R - x}{d_2(x)} = 0$$

This implies Snell's Law:

$$n_i \sin\theta_i = n_t \sin\theta_t$$

Where θ_i and θ_t are the angles between the ray and the normal in the upper and lower media, respectively. Thus, Fermat's Principle can be used to derive both the law of reflection and Snell's law of refraction.

Model setup

- Source $A = (x_A, y_A)$, target $B = (x_B, y_B)$, both above the flat reflecting surface at $y = 0$.
- Reflection point $R = (x, 0)$.
- Total travel time (proportional to length):

  $$t(x) \propto L(x) = |AR| + |RB|$$

  with

  $$|AR| = \sqrt{(x - x_A)^2 + y_A^2}, \quad |RB| = \sqrt{(x_B - x)^2 + y_B^2}$$

  Differentiate $L(x)$ with respect to x:

  $$\frac{dL}{dx} = \frac{x - x_A}{|AR|} - \frac{x_B - x}{|RB|}$$

  Set $\frac{dL}{dx} = 0$ for an extremum:

  $$\frac{x - x_A}{|AR|} = \frac{x_B - x}{|RB|}$$

  Geometrically, this means

  $$\sin\theta_i = \sin\theta_r$$

which implies

$$\theta_i = \theta_r$$

for angles between 0° and 90°, confirming the law of reflection.

0.7 Output

Left graph: Travel time t vs. horizontal position x: total travel time of the ray for every possible point S where the ray could meet the interface (mirror for reflection, boundary for refraction).

Horizontal axis (x): The position along the interface where the ray meets it.

Vertical axis (Travel Time): The total time taken for light to travel from point $A \to S \to B$, calculated using:

Reflection:

$$t(x) = \frac{\sqrt{x^2 + y^2} + \sqrt{(L - x)^2 + y^2}}{c}$$

Where c is the wave speed in the medium.

Refraction:

$$t(x) = \frac{\sqrt{x^2 + y^2}}{c_1} + \frac{\sqrt{(L - x)^2 + Y^2}}{c_2}$$

Where c_1 and c_2 are the wave speeds in the two media. The curve always has a minimum. According to Fermat's theorem, this is the path that light will take.

The red dot marks the minimum point.

In **Reflection mode**, the minimum occurs exactly at $x = L/2$ when the setup is symmetric, which corresponds to equal angles of incidence and reflection.

In **Refraction mode**, the minimum occurs at the x where Snell's Law holds.

Right graph: Ray path: The actual geometry of the light path from $A \to S \to B$ using the x value from the minimum time on the left graph.

Reflection mode:

A is above the mirror, and B is also above the mirror on the opposite side. The point S is the contact point on the mirror.

Calculation:

$$\theta = \text{angle of incidence}, \quad \phi = \text{angle of reflection}$$

And show in the title that they're equal (law of reflection).

Refraction mode:

A is above the boundary (medium 1), B is below it (medium 2). The point S is where the ray crosses the interface.

Calculation:

$$\frac{\sin\theta}{c_1} \quad \text{and} \quad \frac{\sin\phi}{c_2}$$

This shows that they match (Snell's Law).

The left graph is the mathematics of Fermat's principle; the right graph is the geometry that results from that x:

At the minimum travel time point:

- In reflection, $\theta = \phi$ (law of reflection).
- In refraction, $\frac{\sin\theta}{c_1} = \frac{\sin\phi}{c_2}$ (Snell's Law).

Task 5

A horizontal plane mirror positioned at $y = y_m$ reflects a point (x, y) to (x', y') according to:

$$x' = x, \quad y' = 2y_m - y \tag{1}$$

Derivation

For a perfect mirror, the law of reflection states that the angle of incidence equals the angle of reflection. A horizontal mirror at $y = y_m$ reverses the perpendicular (vertical) component of the position relative to the mirror plane, while keeping the horizontal (parallel) coordinate unchanged. Therefore, the reflected point is equidistant from the mirror on the opposite side, hence:

$$\Delta y = y - y_m \tag{2}$$

$$y' = y_m - \Delta y = 2y_m - y \tag{3}$$

The resulting image is **virtual** because the reflected rays diverge after reflection. To the observer, these rays appear to originate from a point behind the mirror, although no actual light passes through that location.

Task 6–9: Modelling Image Formation by Lenses and Mirrors

Key Equations

$$\text{Thin Lens Formula:} \quad \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \tag{4}$$

$$\text{Mirror Equation:} \quad \text{Same as the lens formula for spherical mirrors} \tag{5}$$

$$\text{Magnification:} \quad m = \frac{v}{u} \tag{6}$$

Transforming Equations for Tasks

Task	Setup	Transform Equations
6	Real image by a convex lens	$X = -\frac{fx}{x-f}, \quad Y = X \cdot \frac{y}{x}$
7	Virtual image (inside focal length)	Same, but with $0 < x < f$
8	Concave mirror (real image)	Use the mirror formula and derive the transformat
9	Convex mirror (virtual image)	Modify for negative focal length

section*Task 6 The Gaussian thin lens equation is:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \tag{7}$$

Where:

- u = object distance (measured from the lens to the object)
- v = image distance (from the lens to the image)
- f = focal length of the lens (positive for convex lenses)

Derivation

From geometric optics, a convex lens refracts light so that parallel light rays from a point on the object converge to the focal point on the image plane. Using a ray diagram with two rays—one parallel to the axis (which refracts through the focal point) and one passing through the centre of the lens (which continues in a straight line)—these form similar triangles along with the object and image, which, when the focal length is known, lead to the thin lens equation.

Magnification

$$m = -\frac{v}{u} \tag{8}$$

The minus sign denotes image inversion. If $|m| > 1$, the image is magnified; if $|m| < 1$, it is reduced. A real image is formed when the refracted rays physically converge to a point. This happens when $u > f$, so v is positive and on the opposite side of the lens from the object.

Task 7

The same Gaussian lens equation applies, but for $u < f$:

- $v < 0$, meaning the image is on the same side as the object.
- Magnification becomes:

$$m = -\frac{v}{u} \quad \Rightarrow \quad m > 0 \tag{9}$$

  which indicates the image is upright.

When the object is inside the focal length, refracted rays diverge, but to an observer, they appear to come from a point on the same side as the object. This is the principle of a magnifying glass.

Task 8

For spherical mirrors, the mirror equation is:

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R} \tag{10}$$

where R is the radius of curvature. Since the focal length is $f = \frac{R}{2}$, this becomes:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \tag{11}$$

Concave mirrors reflect parallel rays inward toward the focal point. A real image is formed when $u > f$, with rays physically converging in front of the mirror.

Task 9

The same mirror equation applies, but for convex mirrors:

- $f < 0$, which always produces $v < 0$ for real object distances.
- The image is therefore virtual, upright, and smaller than the object.

Physics: Convex mirrors cause parallel rays to diverge. The reflected rays appear to originate from a point behind the mirror. This is why convex mirrors are used for wide-angle viewing in vehicles.

Task 10

Anamorphic imaging distorts spatial geometry and can be used to map coordinates (x, y) in a unit circle to polar coordinates:

$$r = \sqrt{x^2 + y^2}, \tag{12}$$

$$\theta = \arctan 2(y, x) \tag{13}$$

By then scaling the radius ($r \mapsto R_f(r)$), distances can be stretched or compressed through non-linear mapping, and the angles ($\theta \mapsto \theta' = s_\theta \cdot \theta$), the angular spread can be modified. To get the final image, the new positions of the pixels are mapped back to Cartesian:

$$x' = r' \cos\theta', \tag{14}$$

$$y' = r' \sin\theta' \tag{15}$$

Therefore, when a distorted image is placed around a cylindrical mirror, the reflected rays reconstruct the intended picture. This requires a transformation that pre-distorts the original so that the mirror's geometry undoes the distortion.

Conventions

Coordinate Conventions (Pixel Coordinates Used for Plotting & Mapping)

- **Image array indices:** $(\text{row}, \text{col}) = (y_{\text{pixel}}, x_{\text{pixel}})$
- **Pixel coordinates:** x increases rightwards, y increases upwards for plotting convenience (matplotlib `origin=lower` used).
- The lens (or mirror vertex) lies at axial position $x = 0$.
- **Object distance** u: distance from lens to object along optical axis; by our chosen sign convention, $u > 0$ when the object is to the left of the lens (incoming light side).
- **Image distance** v: distance from lens to image along optical axis; $v > 0$ when the image is formed to the right of the lens (transmitted side). For mirrors, the same algebraic form should be adopted, but treat the focal length accordingly ($f = R/2$, sign depends on concave/convex).

Task11)

Rainbows form when sunlight interacts with spherical water droplets via refraction, reflection, and dispersion. The angular position and intensity of the resulting colored arcs depend on wavelength-dependent optical properties of water and the geometric path of light inside the droplet. We simulate both the primary rainbow (one internal reflection) and the secondary rainbow (two internal reflections). Optical Path in a Raindrop Light undergoes the following stages in a spherical droplet: Refraction (Air $\rightarrow$ Water): Light bends according to Snell's law. Internal Reflection(s): Light reflects from the back surface. Refraction (Water $\rightarrow$ Air): Light exits the droplet. Dispersion: Different wavelengths bend differently due to the refractive index variation with frequency.

0.8 Speed of Light and Frequency Conversion

The speed of light in a vacuum is:

$$c = 299,792,458 \text{ m/s}$$

Wavelength λ (in meters) corresponds to frequency:

$$f = \frac{c}{\lambda}$$

Example: For $\lambda = 550$ nm:

$$f \approx 5.45 \times 10^{14} \text{ Hz}$$

0.9 Dispersion Model for Water

Using the same formula in 1b), we calculate the refractive index, $n(\lambda)$, which acts a a base to work off of for the entirety of the task

$$\left[(n^2-1)^{-2}\right] = 1.731 - 0.261\left(\frac{f}{10^{15}}\right)^2$$

Steps: Compute $x = \left(\frac{f}{10^{15}}\right)^2$ Compute $A = 1.731 - 0.261 \cdot x$ Then $(n^2-1)^{-2} = A \Rightarrow (n^2-1)^{-1} = \sqrt{A}$ Rearrange to get n. This ensures shorter wavelengths (blue) have larger n and thus refract more strongly than longer wavelengths (red).

0.10 Snell's Law for Entry Refraction

When light enters the droplet:

$$n_1 \sin(i) = n_2 \sin(r)$$

$n_1 \sin(i) = n_2 \sin(r)$ $n_1 \sin(i) = n_2 \sin(r)$

i = incident angle (measured from normal to surface) r = refracted angle inside the drop n_1 = refractive index of air (1.0) n_2 = refractive index of water (1.33)

For incident angle i and refracted angle r:

$$n_1 \sin i = n_2 \sin r$$

With $n_1 \approx 1.0$ (air) and $n_2 = n(\lambda)$ (water).

In Snell's law, the three angles θ_i, θ_r, and θ_t are not independent, linking to the law of reflection, the reflected angle matches the incident angle, and the transmitted angle obeys Snell's law. Refraction, therefore, refers to the fact that θ_i and θ_t are different. That is, light 'bends' as it transmits through an interface.

0.11 Ray Deviation Angles

After bending in and reflecting inside the drop, the total deviation angle D_k (difference between incoming and outgoing direction) is:

Primary (k=1 reflection):

$$D_1 = \pi + 2i - 4r$$

Secondary (k=2 reflections):

$$D_2 = 2\pi + 2i - 6r$$

π = turnaround due to back reflection $2i$ = two encounters with the drop surface from outside $4r$ or $6r$ = multiple passes inside the drop

The stationary point (minimum deviation) produces the bright rainbow arc as the rainbow radius corresponds to the minimum deviation for a given wavelength.

0.12 Rainbow Angular Position

The observed angular radius θ_{rainbow} is:

Primary:

$$\theta_{\text{rainbow}} = 180^\circ - D_1$$

Secondary:

$$\theta_{\text{rainbow}} = D_2 - 180^\circ$$

0.13 Fresnel Reflection Transmission Losses

Fresnel Equations mathematically describe the behaviour of light at a dielectric interface, determining the reflection and transmission coefficients for both amplitude and intensity, meaning that when creating models of rainbows, using these equations helps to determine their brightness, particularly affecting the secondary bow. As some light reflects at each interface, not all of it makes it out, meaning for unpolarized light, the Fresnel coefficients are:

Reflection (s-polarisation):

$$R_s = \left(\frac{n_1 \cos i - n_2 \cos r}{n_1 \cos i + n_2 \cos r} \right)^2$$

Reflection (p-polarisation):

$$R_p = \left(\frac{n_1 \cos r - n_2 \cos i}{n_1 \cos r + n_2 \cos i}\right)^2$$

For unpolarized light:

$$R = \frac{R_s + R_p}{2}$$

Transmission:

$$T = 1 - R$$

The throughput for the primary rainbow:

$$T_{\text{primary}} = T_{\text{in}} \cdot R_{\text{int}} \cdot (1 - R_{\text{int}})$$

And for the secondary:

$$T_{\text{secondary}} = T_{\text{in}} \cdot R_{\text{int}}^2 \cdot (1 - R_{\text{int}})$$

0.14 Sky Projection Geometry

Rainbow arcs are mapped in altitude–azimuth space relative to the antisolar point: Altitude–azimuth coordinates: Antisolar point altitude = −sun altitude A rainbow is a small circle of radius = rainbow angle, centred at the antisolar point. To map the ring of points where the rainbow is seen:

$$\text{alt} = \arcsin(\sin \phi_c \cos r + \cos \phi_c \sin r \cos \phi)$$

$$\text{az} = \text{az}_c + \arctan 2(\sin \phi \sin r \cos \phi_c,\ \cos r - \sin \phi_c \sin \text{alt})$$

0.15 Methodology

Define Wavelength Range: $\lambda \in [380, 780]$ nm Convert to Frequency: $f = c/\lambda$ Compute $n(\lambda)$ from empirical dispersion formula Loop Over Incident Angles i: Compute r using Snell's law Compute D_k for $k = 1, 2$ Identify the minimum deviation angle Calculate Fresnel losses for each wavelength Map deviation minima to rainbow angular radii Project arcs into sky coordinates for given solar altitude Convert wavelength to RGB colour using the human eye sensitivity model (Blue: 380–490 nm, Green: 490–580 nm, Red: 580–780 nm) The code adjusts brightness near the extremes (deep violet far red are dimmer). Render visualisations of deviation curves, Fresnel brightness, dispersion, and rainbow arcs.

Primary rainbow appears at $\sim 42°$ for green light; secondary at $\sim 51°$. Dispersion causes angular separation of colours, producing visible spectrum order. (n vs λ – dispersion curve for water) Fresnel analysis shows the relative brightness of both types of rainbow, of which the secondary rainbow is significantly dimmer, as each reflection and refraction causes some light to be lost. Sky projection reproduces realistic arcs centred on the antisolar point.

1 Task 12)

The app uses matplotlib to display an interactive ray-tracing diagram of white light entering a triangular prism. This allows for in-code drawing and, in turn,n results in easier and more flexible models. The simulation is split into three key stages of the light's journey: Air $\rightarrow$ Prism refraction (entry face) Internal propagation (inside the prism) Prism $\rightarrow$ Air refraction (exit face)

Because the primary beam is white light, the app calculates separate paths for different wavelengths (e.g., 400 nm violet to 700 nm red) using the Sellmeier equation for BK7 glass (Task 1a) to get accurate refractive indices.

1.1 A. White Beam Entry

At the far left, a white ray is drawn pointing towards the first prism face.

Although it is observed as a single ray of white light, in reality, it is broken down into several rays of different colours, each corresponding to a wavelength. This is because dispersion means different wavelengths refract differently (Shorter wavelengths disperse more).

1.2 B. Dispersion Inside the Prism

The Sellmeier equation:

$$n(\lambda) = \sqrt{1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3}}$$

It is used to calculate the refractive index n for each wavelength in the visible spectrum.

Shorter wavelengths (blue/violet) have slightly higher refractive indices than longer wavelengths (red), causing them to bend more.

Inside the prism, each coloured ray follows a different angle determined by Snell's Law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

This is why the coloured rays appear to fan out inside the prism.

1.3 C. Exit Face Refraction

At the second face, each ray refracts again — but now from a higher n (prism) to a lower n (air).

The exit angles are different for each wavelength, making the rainbow spread more.

Mathematically, Snell's Law is applied again, but n_1 is now the prism's refractive index for that wavelength, and n_2 is 1.00029 (air).

D. The Output Spectrum After exiting, the rays form a spectral spread — the angular separation between red and violet is visible.