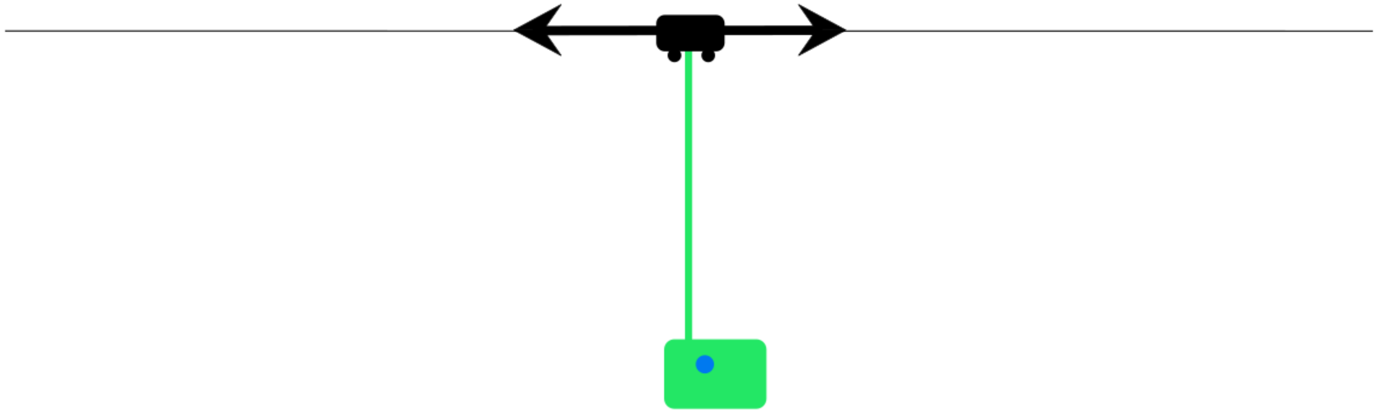
Warm-starting multi-start procedure using penalties instead of constraints to find more optimal trajectories

Eve Charbonneau*, Francisco Pascoa, Mickaël Begon

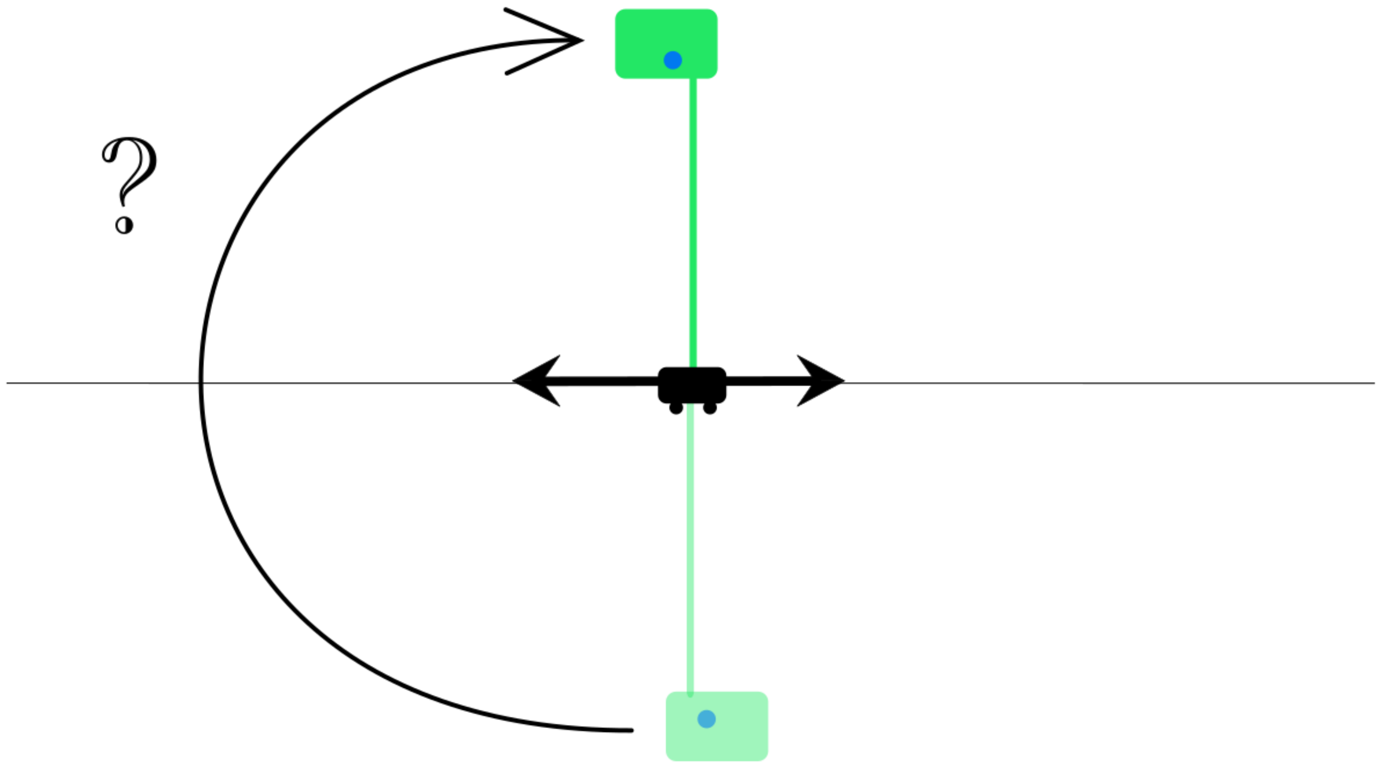
Université de Montréal

*eve.charbonneau.1@umontreal.ca

Example of situation



Example of situation

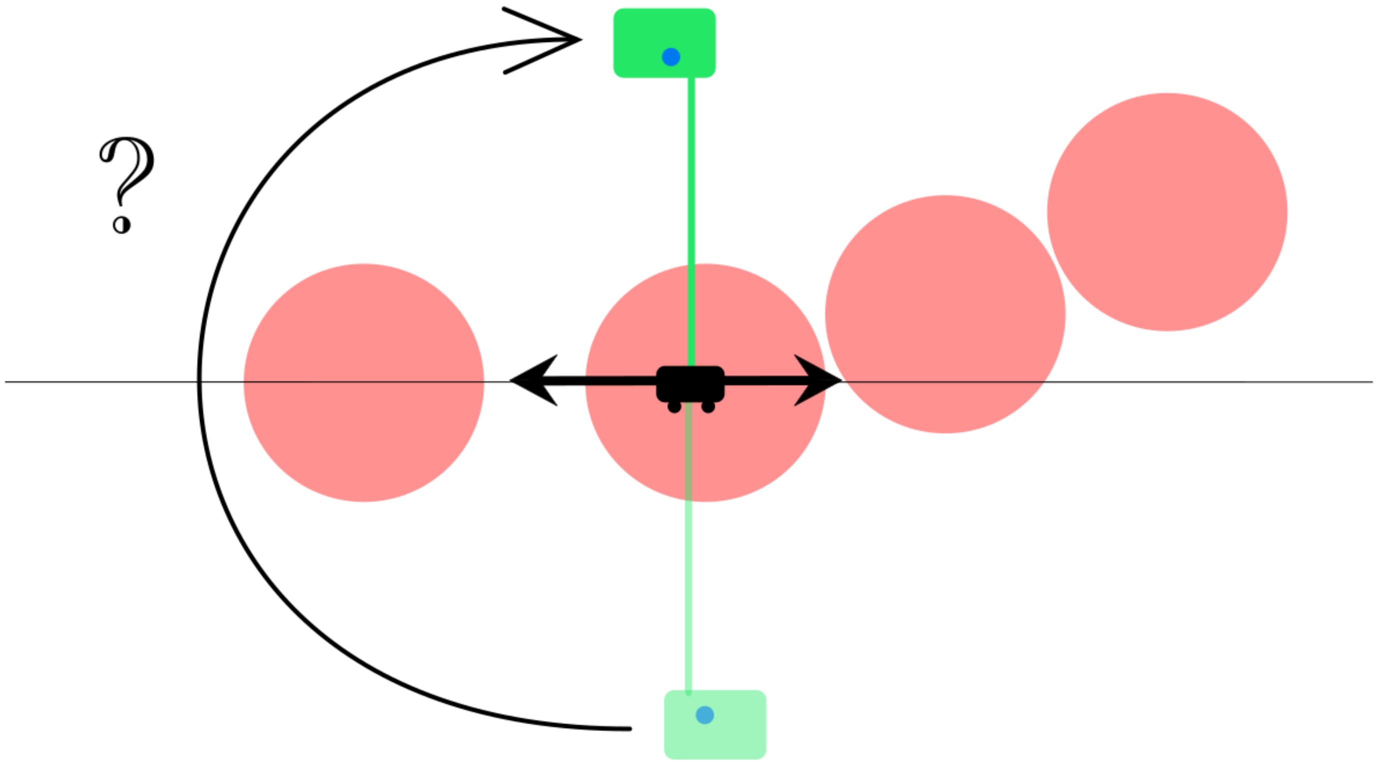


$$\min_{\boldsymbol{x},\boldsymbol{u},T} \phi(\boldsymbol{x},\boldsymbol{u},T)$$

$$s.t. \quad g(x,u,T) > 0$$

optimal control

Example of situation



$$\min_{\boldsymbol{x},\boldsymbol{u},T} \phi(\boldsymbol{x},\boldsymbol{u},T)$$

$$s.t. \quad g(x,u,T) > 0$$

optimal control

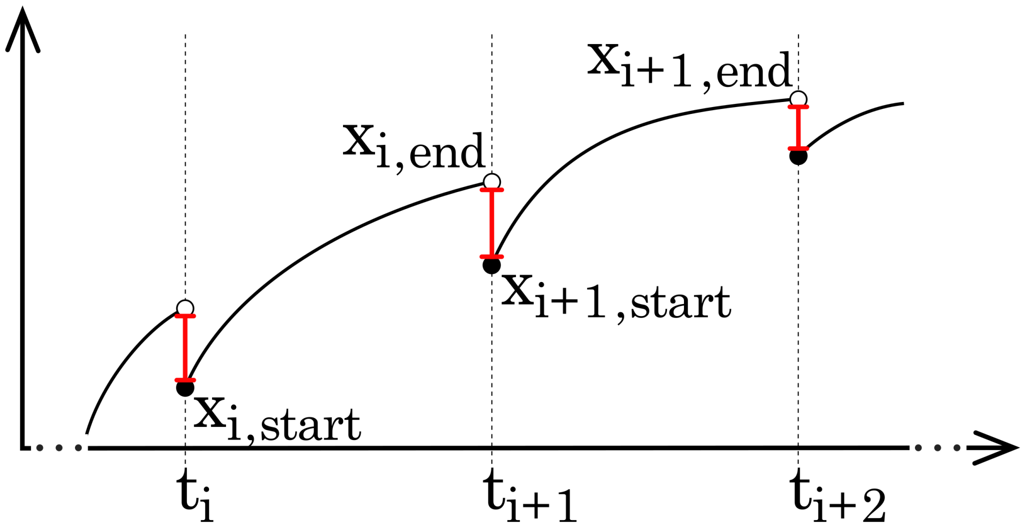
Direct multiple shooting

optimal control

Discretization:

$$time \rightarrow [0, t_1, t_2, \dots, T]$$
$$\boldsymbol{x} \rightarrow [x_0, x_1, \dots, x_N]$$
$$\mathbf{u} \rightarrow [u_0, u_1, \dots, u_N]$$

Continuity constraint:



$$x_{i,\,end} = f(x_i, u_i, T)$$
$$x_{i+1,\,start} = x_{i,\,end}$$

Interior point method

optimal control

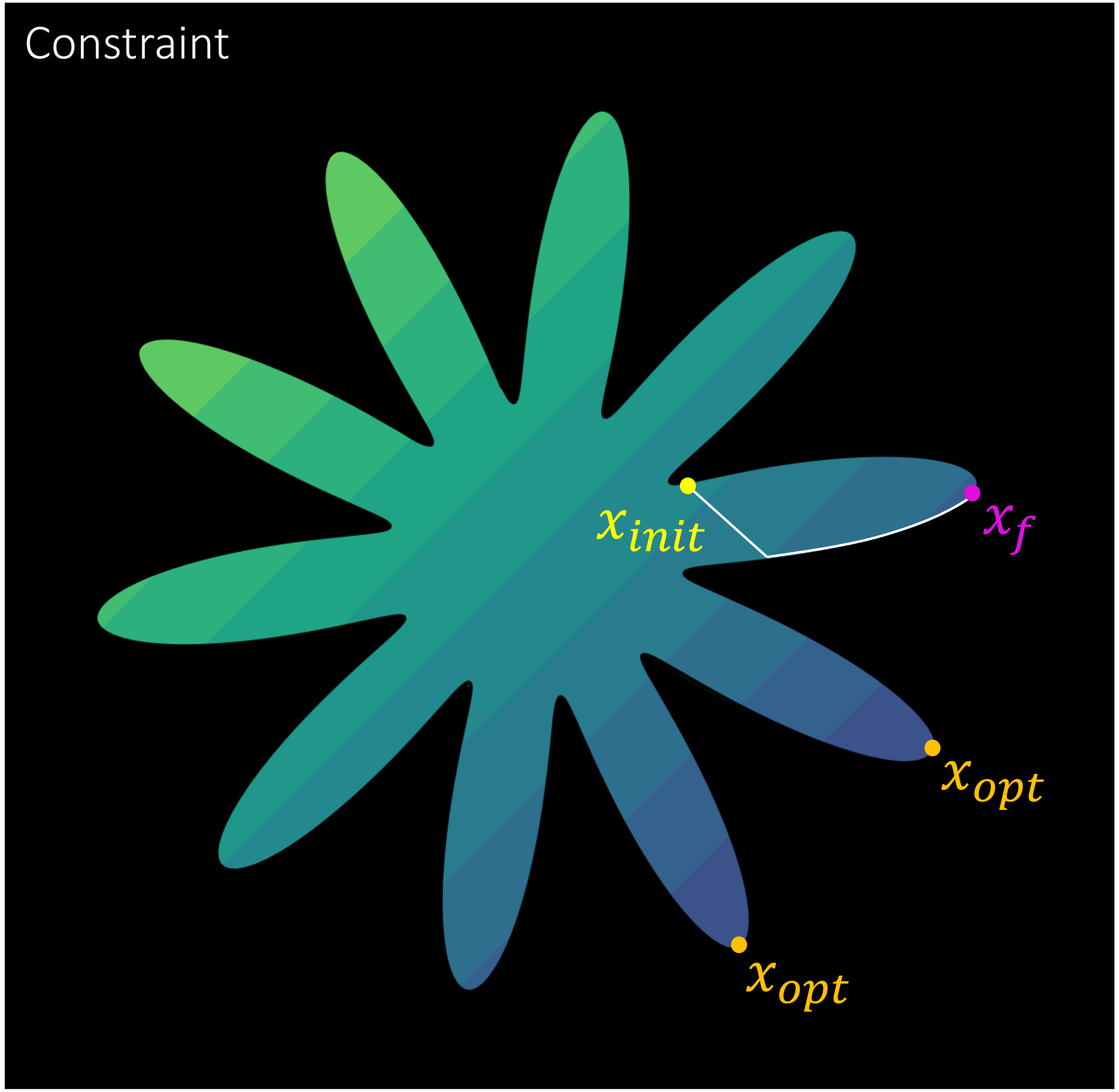
Advantages:

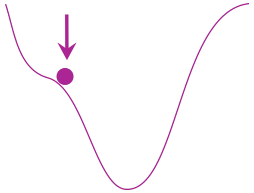
- High convergence rate
- Handles non-linearity and non-convexity
- Robustness to the initial guess

Disadvantages:

- Slower than other algorithms
- Might get stuck in constraint pockets ⟶

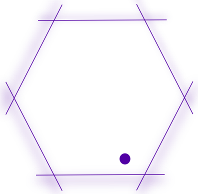
Interior point method





Initialization

×

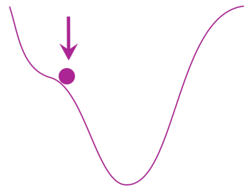


Interior point

×

Constraints

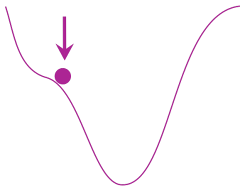
Multi-start

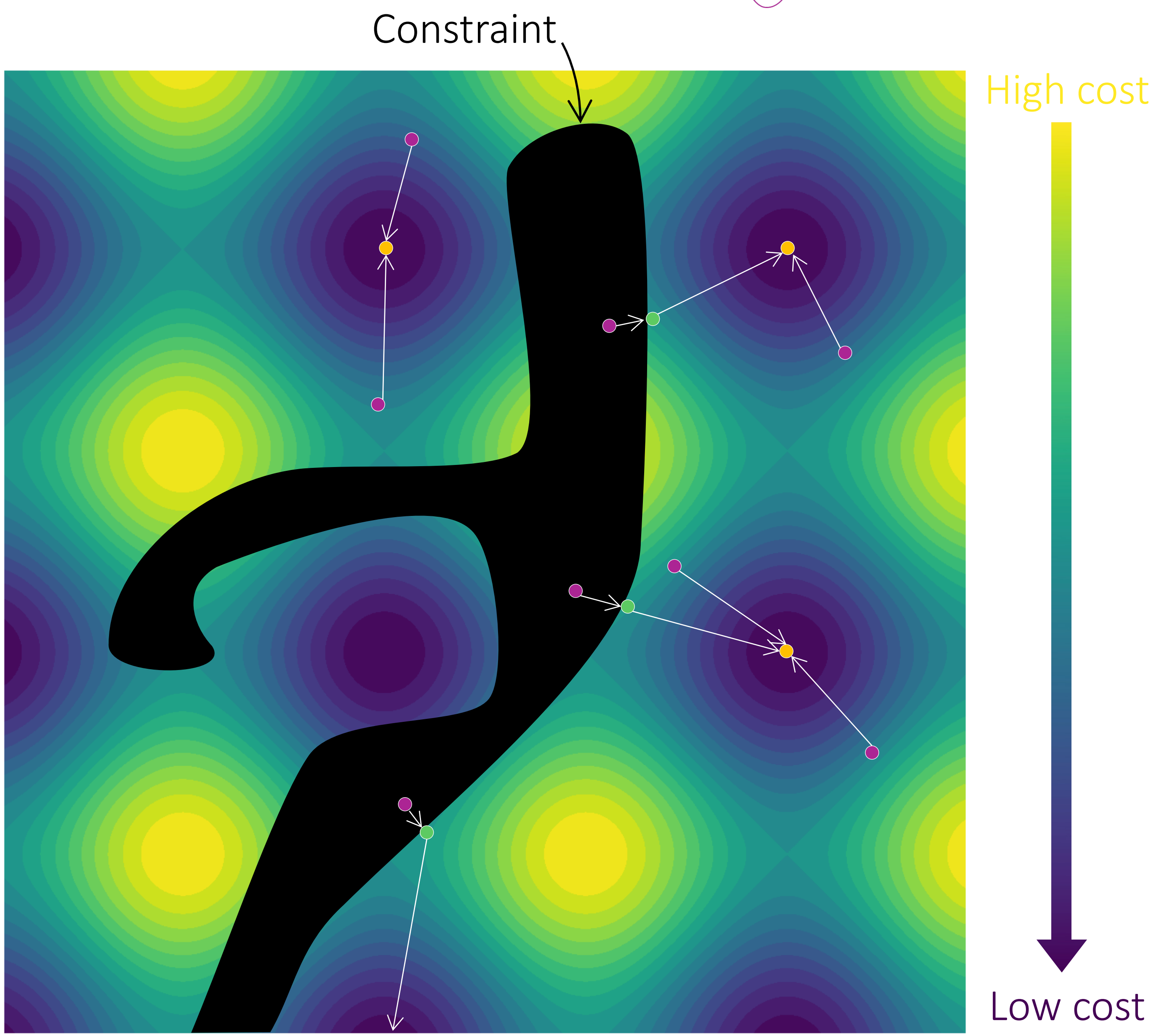


Classical multi-start:

- ++ Random initial guesses

Multi-start





1) Random initial guess

2) Constrained optimization
    a) Constraint push
    b) Gradient descent
    c) (Constraint descent)

Two-step optimization workflow

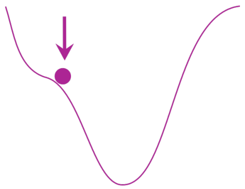
Two-step optimization workflow:

1) Random initial guesses

2) First underconstrained optimization

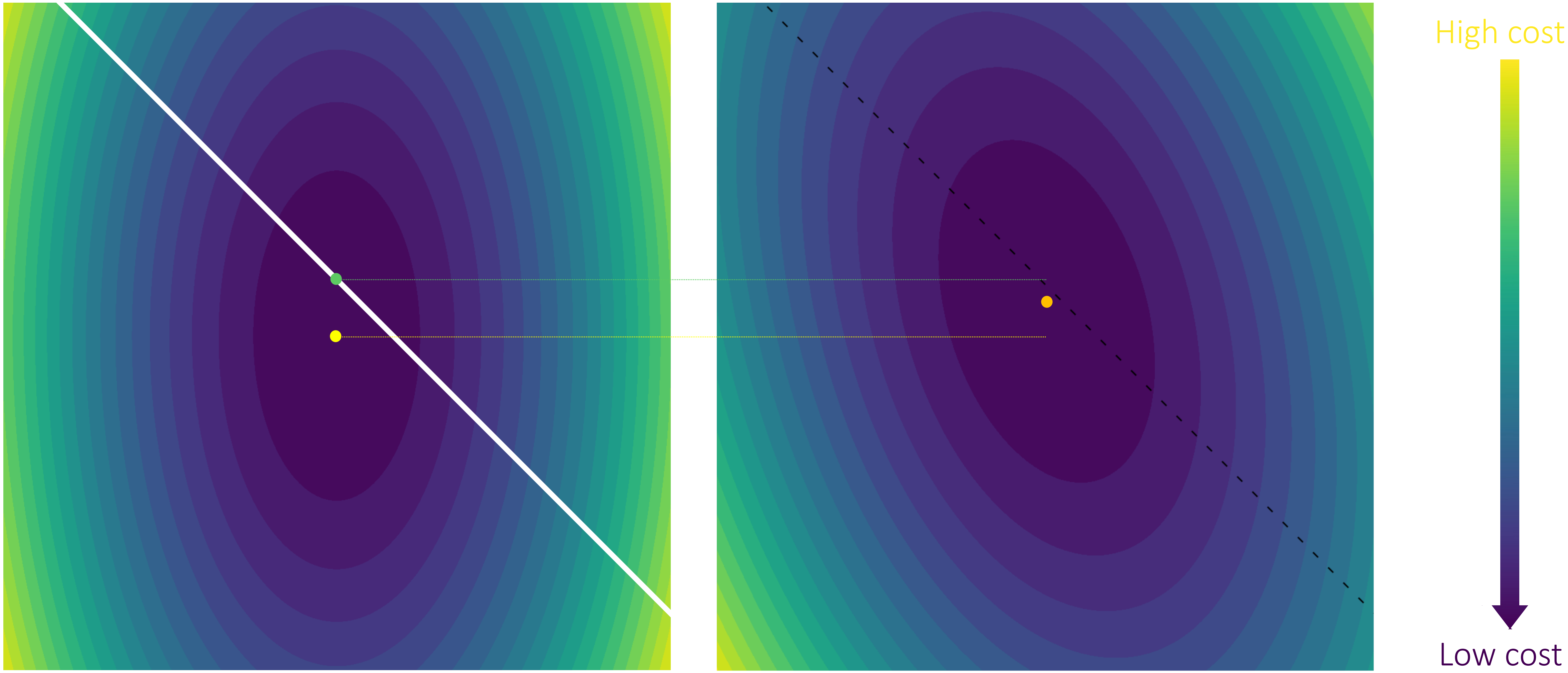
   ↳ Replacing constraints by penalty terms in the cost function

Underconstrained optimization



$$\min_{x,y}\ (0.5x)^2 + y^2 \qquad s.t\ \ x = -(0.6 - y)$$

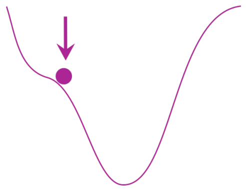
$$\min_{x,y}\ (0.5x)^2 + y^2 + 10(x + (0.6 - y))^2$$



original version

underconstrained version

Two-step optimization workflow



initialization

Two-step optimization workflow:

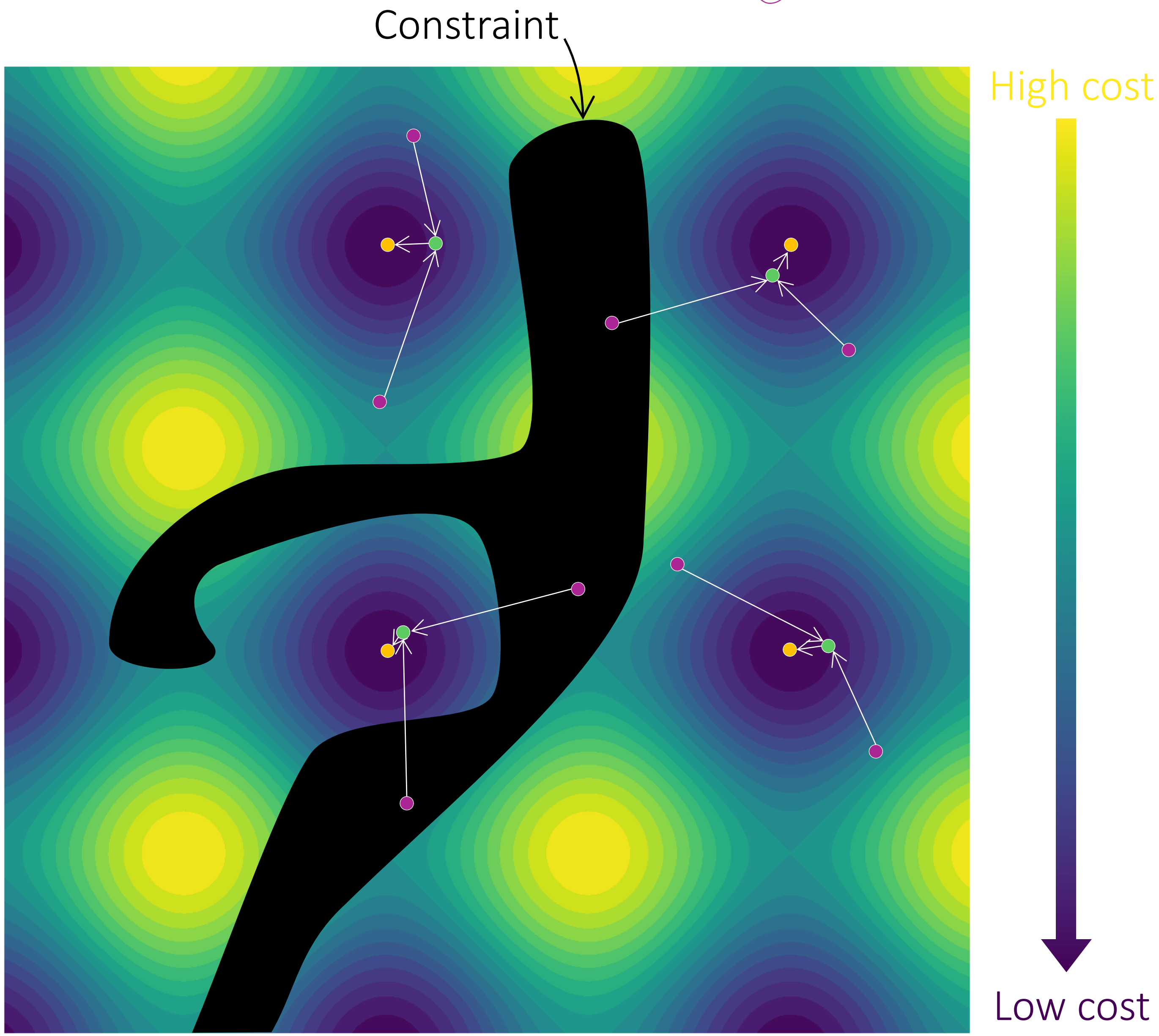
1) Random initial guesses

2) First underconstrained optimization

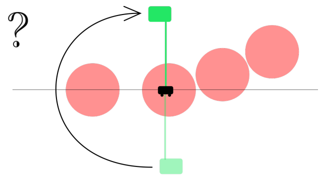
3) Second optimization (original problem)

Two-step optimization workflow

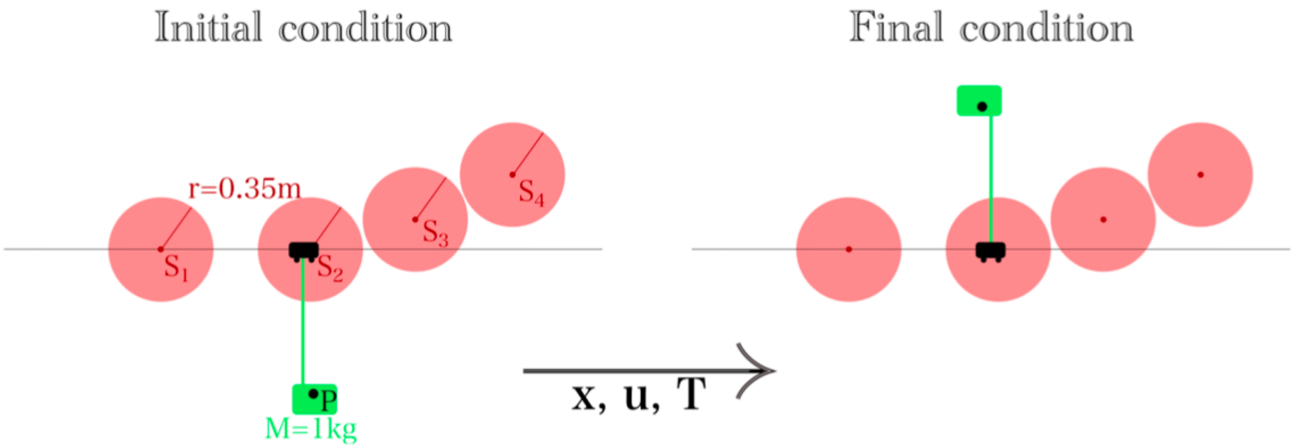
1) Random initial guess
2) Underconstrained optimization
3) Constrained optimization



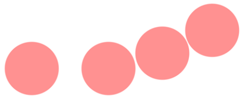
Application to a constrained problem



application

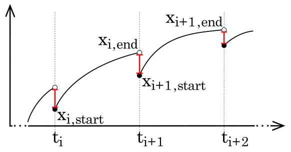


Constraints:



Not hit the obstacles $\mathbf{P}_i(\mathbf{x}_{i,start}) - \mathbf{S}_j \geq r$

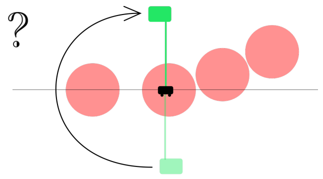
$$\sum_{i=0}^{N_i+1} \sum_{j=1}^{4} \left(\begin{cases} \|\mathbf{P}_i(\mathbf{x}_{i,start}) - \mathbf{S}_j\| - r, & \text{if negative,} \\ 0, & \text{otherwise} \end{cases} \right)^2$$



States continuity $x^k_{i+1,start} - x^k_{i,end}(\mathbf{x}_{i,start},\ \mathbf{u}_i,\ T) = 0$

$$\sum_{i=1}^{N_i-1} \sum_{k=1}^{N_k} (x^k_{i+1,start} - x^k_{i,end}(\mathbf{x}_{i,start},\ \mathbf{u}_i,\ T))^2$$

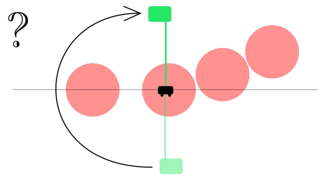
Problem definition



application

Constrained Penalized continuity Penalized obstacles

Problem definition



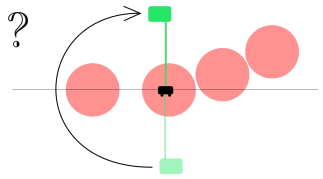
application

$$\min_{\mathbf{x},\mathbf{u},T} \quad \omega_1 T + \omega_2 \sum_{i=0}^{N_i} \mathbf{u}_i^2$$

$$\begin{aligned} s.t. \quad & x_{i+1,start}^k - x_{i,end}^k(\mathbf{x}_{i,start},\ \mathbf{u}_i,\ T) = 0 \ \Big\} \ \textit{Continuity constraint} \\ & \mathbf{P}_i(\mathbf{x}_{i,start}) - \mathbf{S}_j \geq r \ \Big\} \ \textit{Obstacles constraint} \\ & \mathbf{x} \in \mathscr{X}, \mathbf{u} \in \mathscr{U}, T \in \mathscr{T} \end{aligned}$$

Constrained Penalized continuity Penalized obstacles

Problem definition



application

$$\min_{\mathbf{x},\mathbf{u},T} \quad \omega_1 T + \omega_2 \sum_{i=0}^{N_i} \mathbf{u}_i^2 + \overbrace{\omega_3 \sum_{i=1}^{N_i-1} \sum_{k=1}^{N_k} \left(x^k_{i+1,start} - x^k_{i,end}(\mathbf{x}_{i,start},\ \mathbf{u}_i,\ T)\right)^2}^{\textit{Continuity penalty}}$$

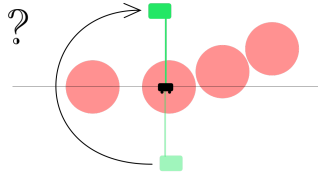
$$s.t.$$

$$\left. \mathbf{P}_i(\mathbf{x}_{i,start}) - \mathbf{S}_j \geq r \ \right\} \textit{Obstacles constraint}$$

$$\mathbf{x} \in \mathscr{X}, \mathbf{u} \in \mathscr{U}, T \in \mathscr{T}$$

Constrained *Penalized continuity* Penalized obstacles

Problem definition



application

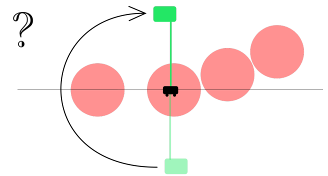
$$\min_{\mathbf{x},\mathbf{u},T} \quad \omega_1 T + \omega_2 \sum_{i=0}^{N_i} \mathbf{u}_i^2 + \underbrace{\omega_4 \sum_{i=0}^{N_i+1} \sum_{j=1}^{4} \left(\begin{cases} \|\mathbf{P}_i(\mathbf{x}_{i,start}) - \mathbf{S}_j\| - r, & \text{if negative,} \\ 0, & \text{otherwise} \end{cases} \right)^2}_{Obstacles\ penalty}$$

$$s.t. \quad \left. x^k_{i+1,start} - x^k_{i,end}(\mathbf{x}_{i,start},\, \mathbf{u}_i,\, T) = 0 \right\} \textit{Continuity constraint}$$

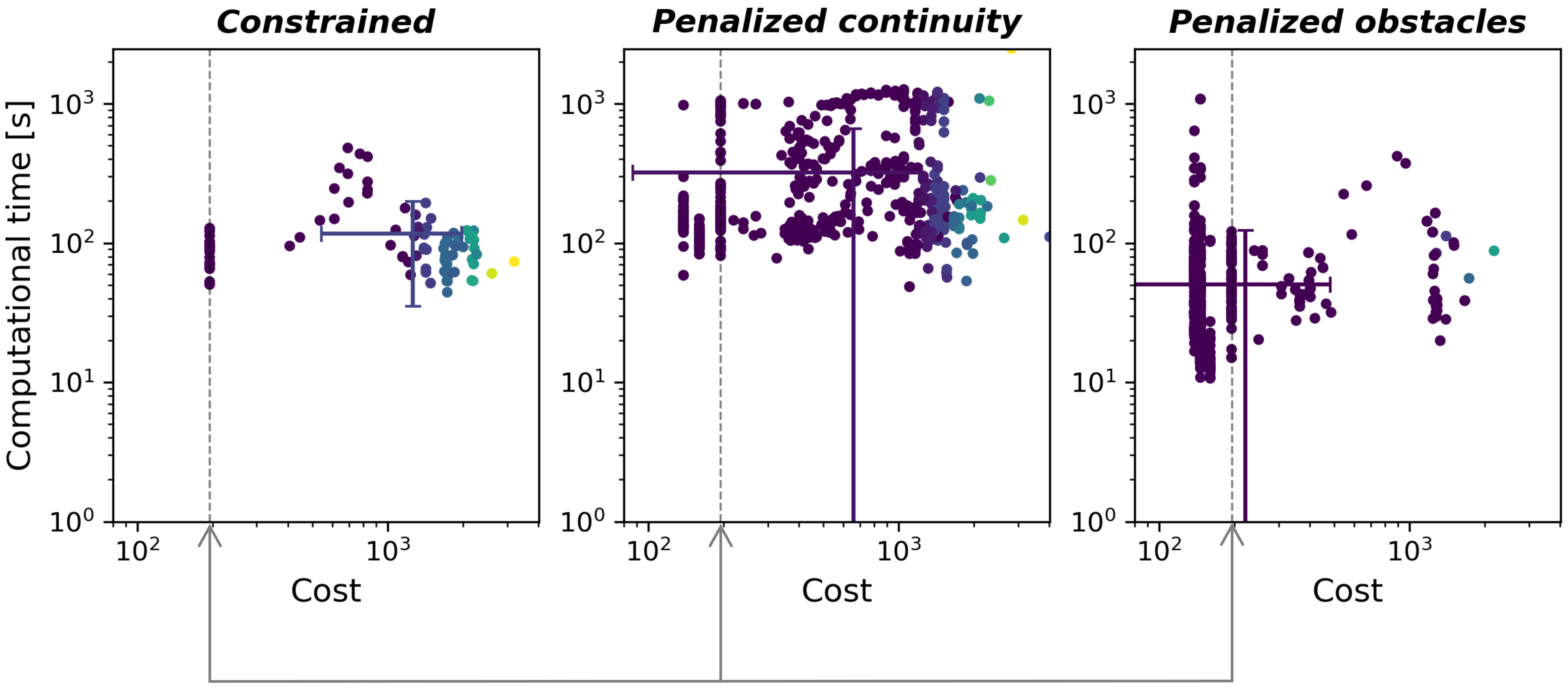
$$\mathbf{x} \in \mathscr{X}, \mathbf{u} \in \mathscr{U}, T \in \mathscr{T}$$

Constrained Penalized continuity *Penalized obstacles*

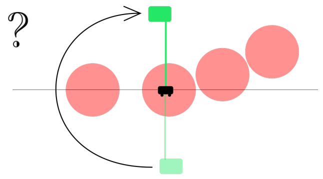
Results

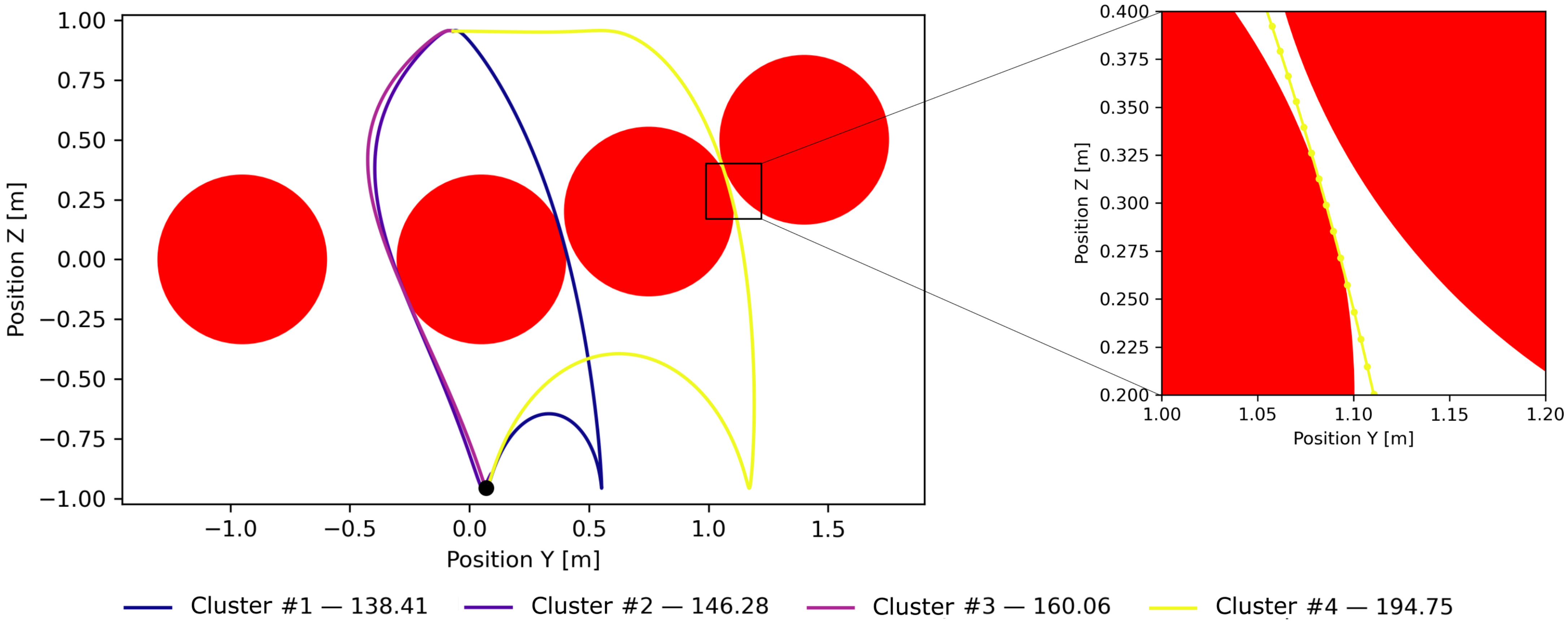


application

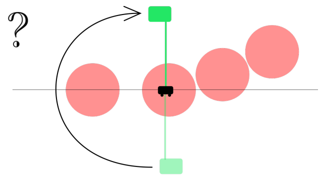


Results

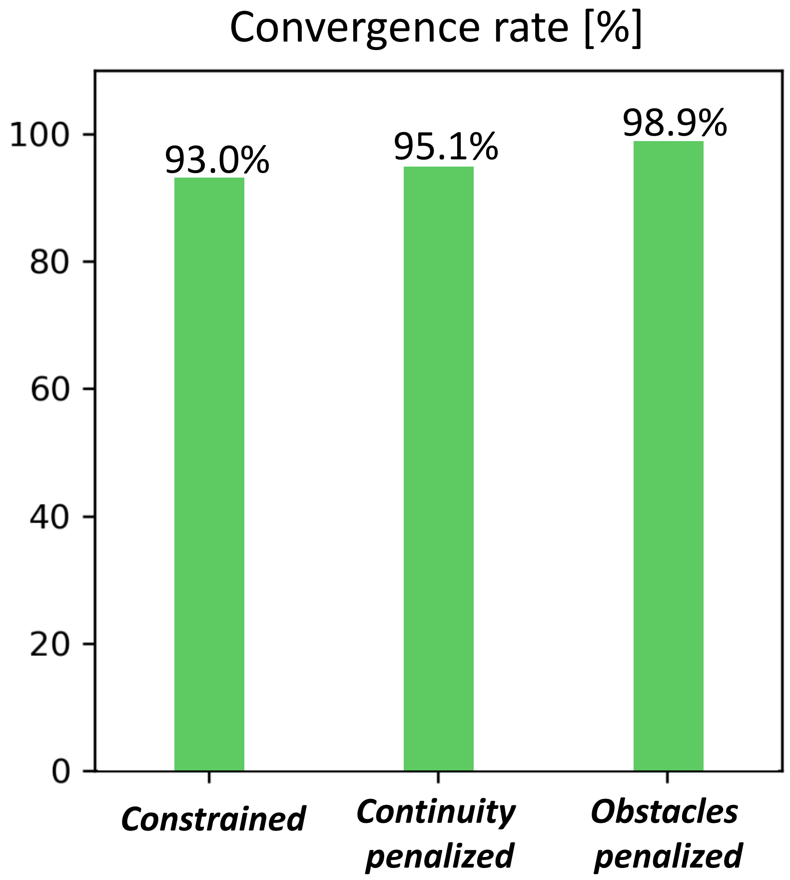

?
application

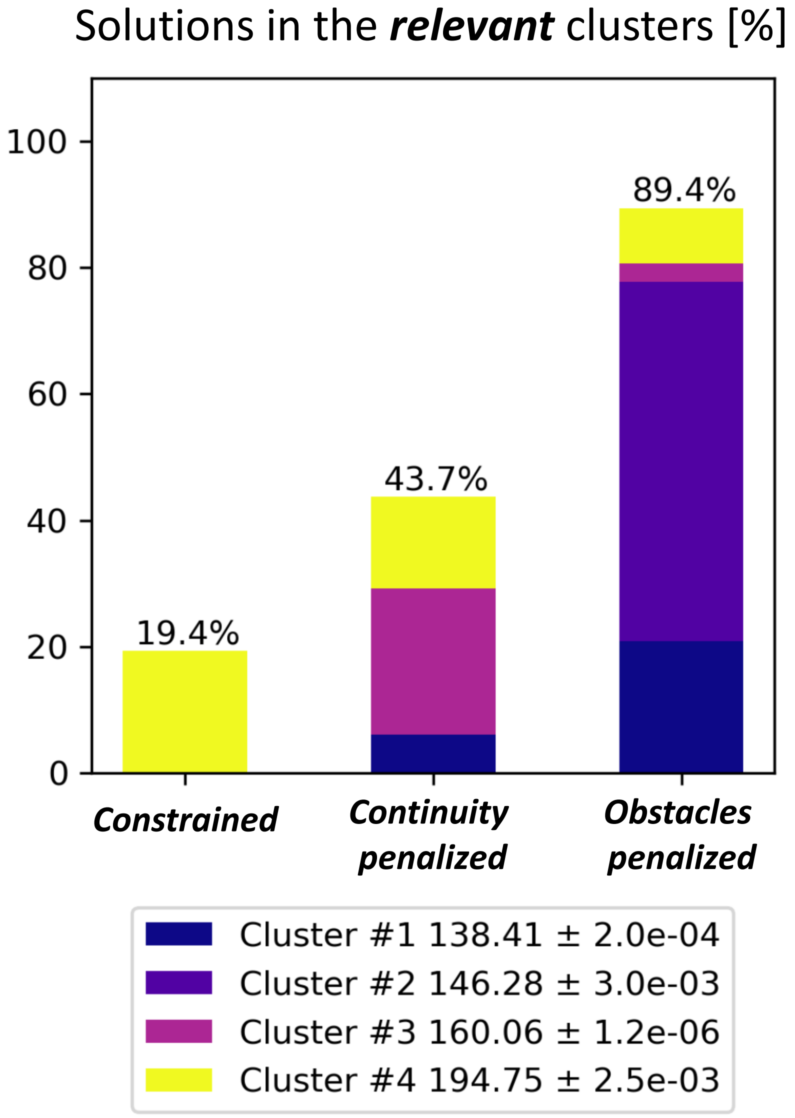

1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Position Z [m]
−1.0
−0.5
0.0
0.5
1.0
1.5
Position Y [m]
0.400
0.375
0.350
0.325
0.300
0.275
0.250
0.225
0.200
Position Z [m]
1.00
1.05
1.10
1.15
1.20
Position Y [m]
Cluster #1 — 138.41
Cluster #2 — 146.28
Cluster #3 — 160.06
Cluster #4 — 194.75

Results

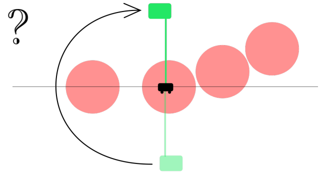


application





Penalty weightings



application

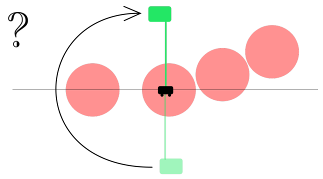
$$\min_{\mathbf{x},\mathbf{u},T} \quad \omega_1 T + \omega_2 \sum_{i=0}^{N_i} \mathbf{u}_i^2 + \overbrace{\boxed{\omega_3} \sum_{i=1}^{N_i-1} \sum_{k=1}^{N_k} \left(x_{i+1,start}^k - x_{i,end}^k(\mathbf{x}_{i,start},\, \mathbf{u}_i,\, T)\right)^2}^{Continuity\ penalty} +$$

$$\underbrace{\boxed{\omega_4} \sum_{i=0}^{N_i+1} \sum_{j=1}^{4} \left(\begin{cases} \|\mathbf{P}_i(\mathbf{x}_{i,start}) - \mathbf{S}_j\| - r, & \text{if negative,} \\ 0, & \text{otherwise} \end{cases} \right)^2}_{Obstacles\ penalty}$$

Sensitivity analysis

	Maximum iterations	Continuity penalty weight (ω_3)	Obstacles penalty weight (ω_4)
Constrained	NA	constrained	constrained
Penalized continuity	100, 1000, 10000	10000, 100000, 1000000	constrained
Penalized obstacles	100, 1000, 10000	constrained	10000, 100000, 1000000

Results



application

Penalized continuity:

- Higher penalty weight ⟶ increased cost value
- Larger number of iterations ⟶ decreased cost, more relevant solutions

Conclusion: maximum number of iterations ≥ 1000

penalty weight of 1000

Penalized obstacles:

- Every combination of weight and number of iteration gave good results

Useful in real optimization context

Advantages:

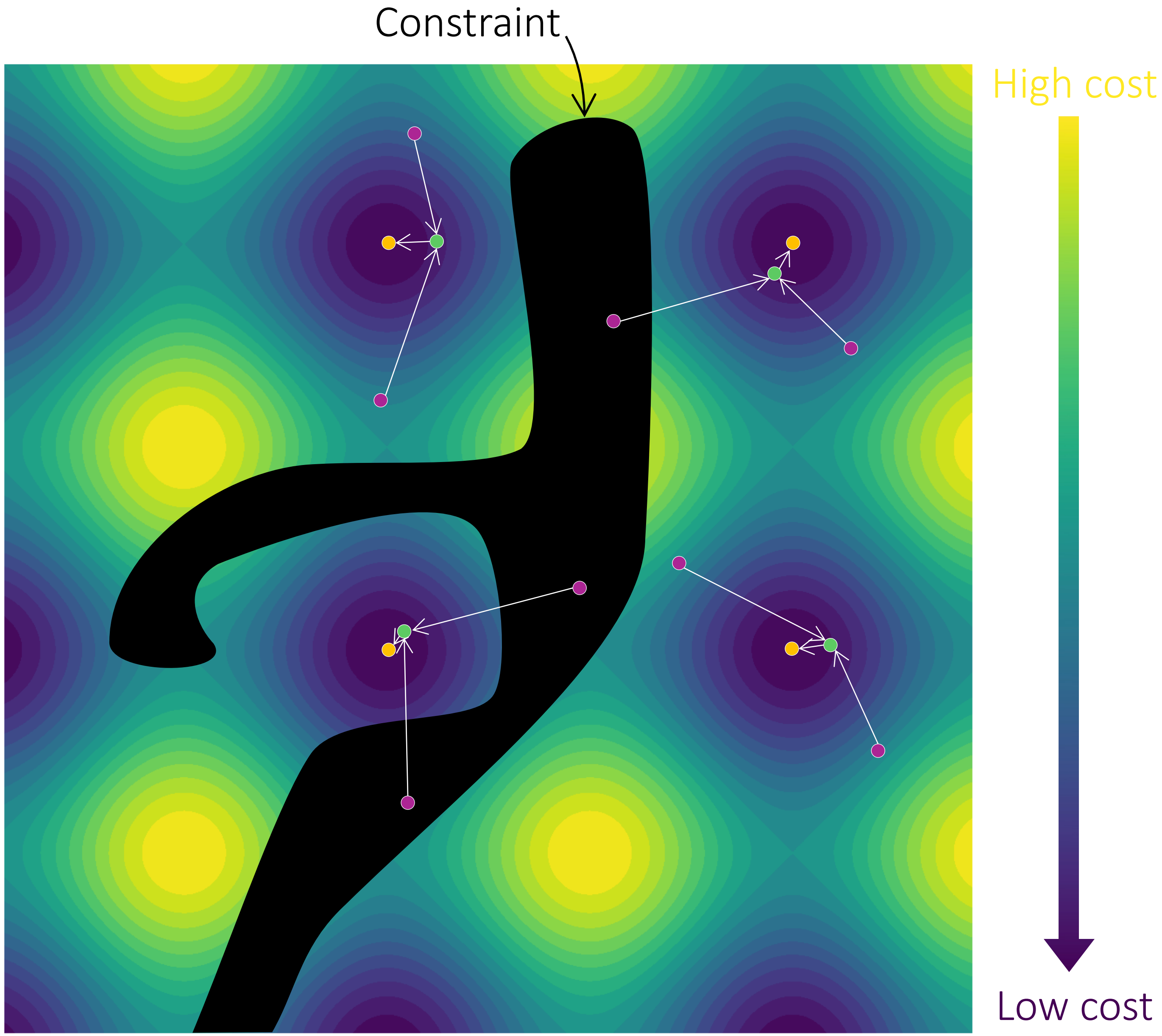
- Increased convergence rate
- Finding better solutions
- Finding relevant solutions more often
- Decrease computational time

Useful in real optimization context

Limitations:

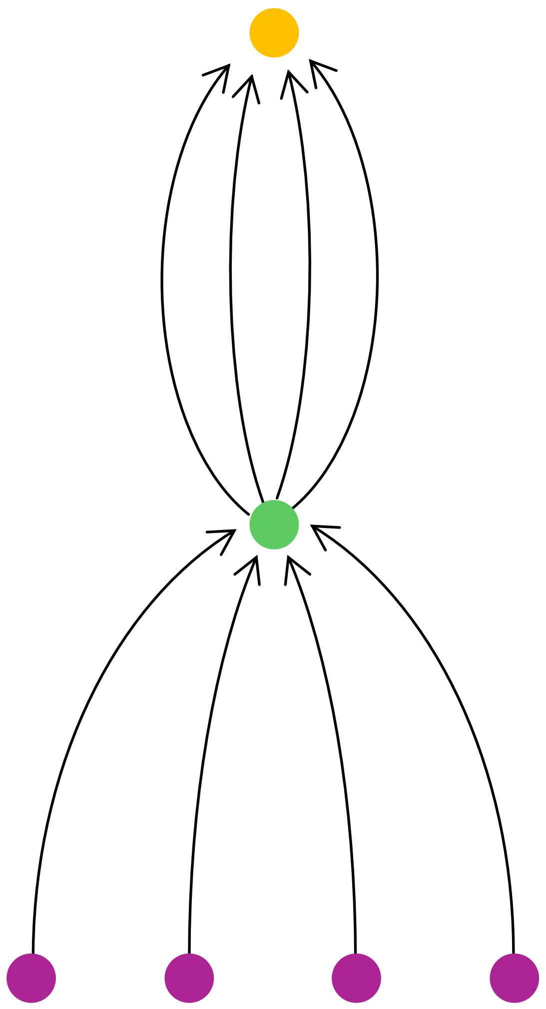
- Longer than solving the problem once

1) Random initial guess
2) Underconstrained optimization
3) Constrained optimization



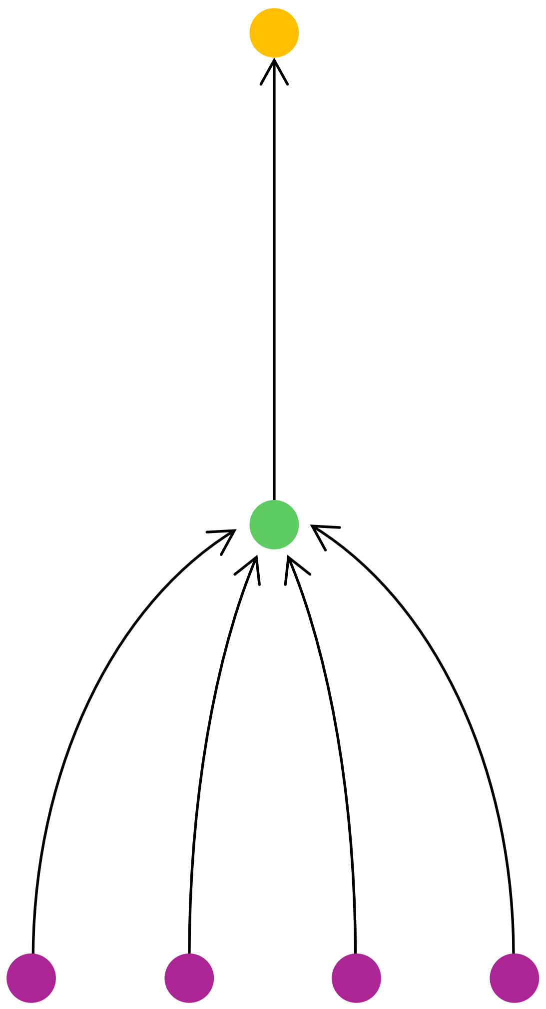
Second optimization
(original version)

First optimization
(underconstrained)



Second optimization
(original version)

First optimization
(underconstrained)



Useful in real optimization context

Limitations:

- Longer than solving the problem once
- Only tested on a simple example

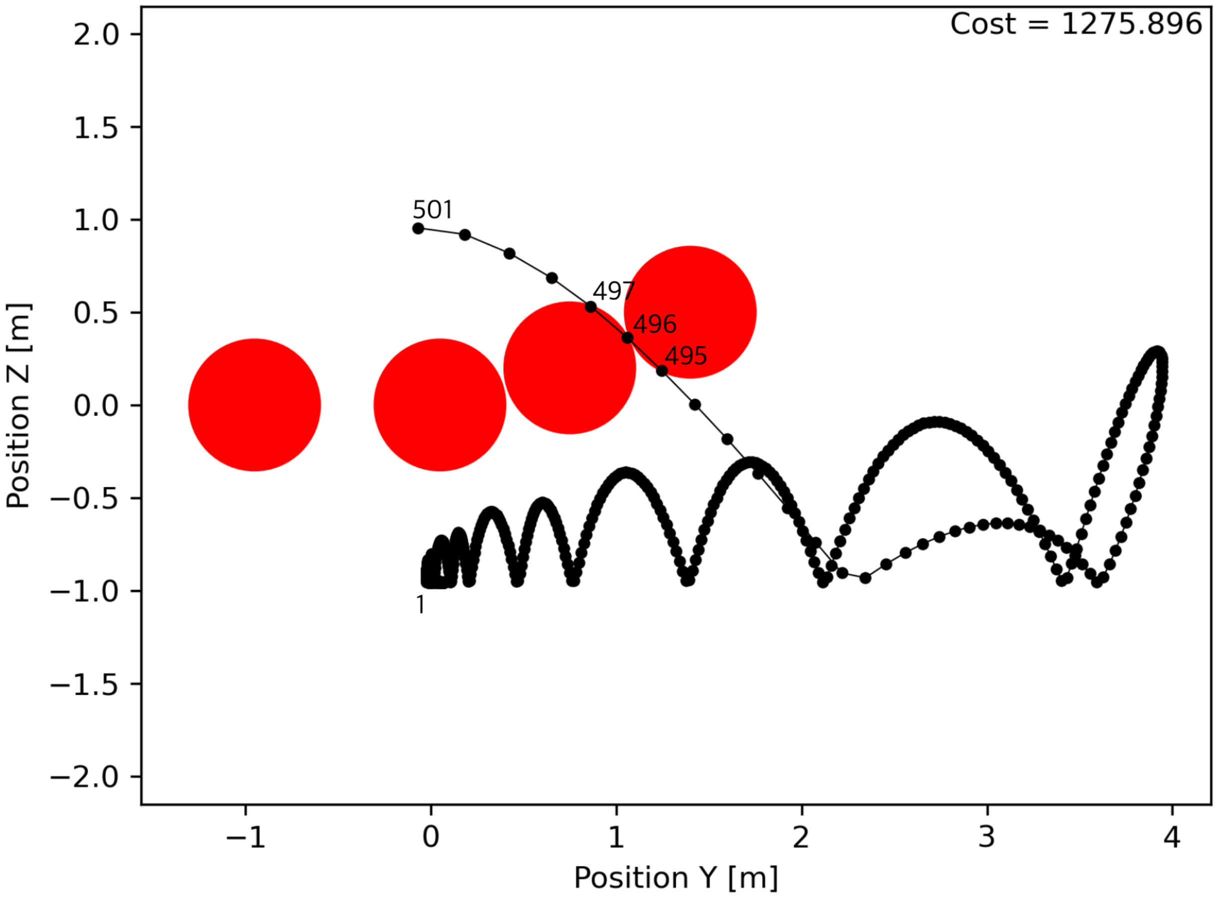
Highlights:

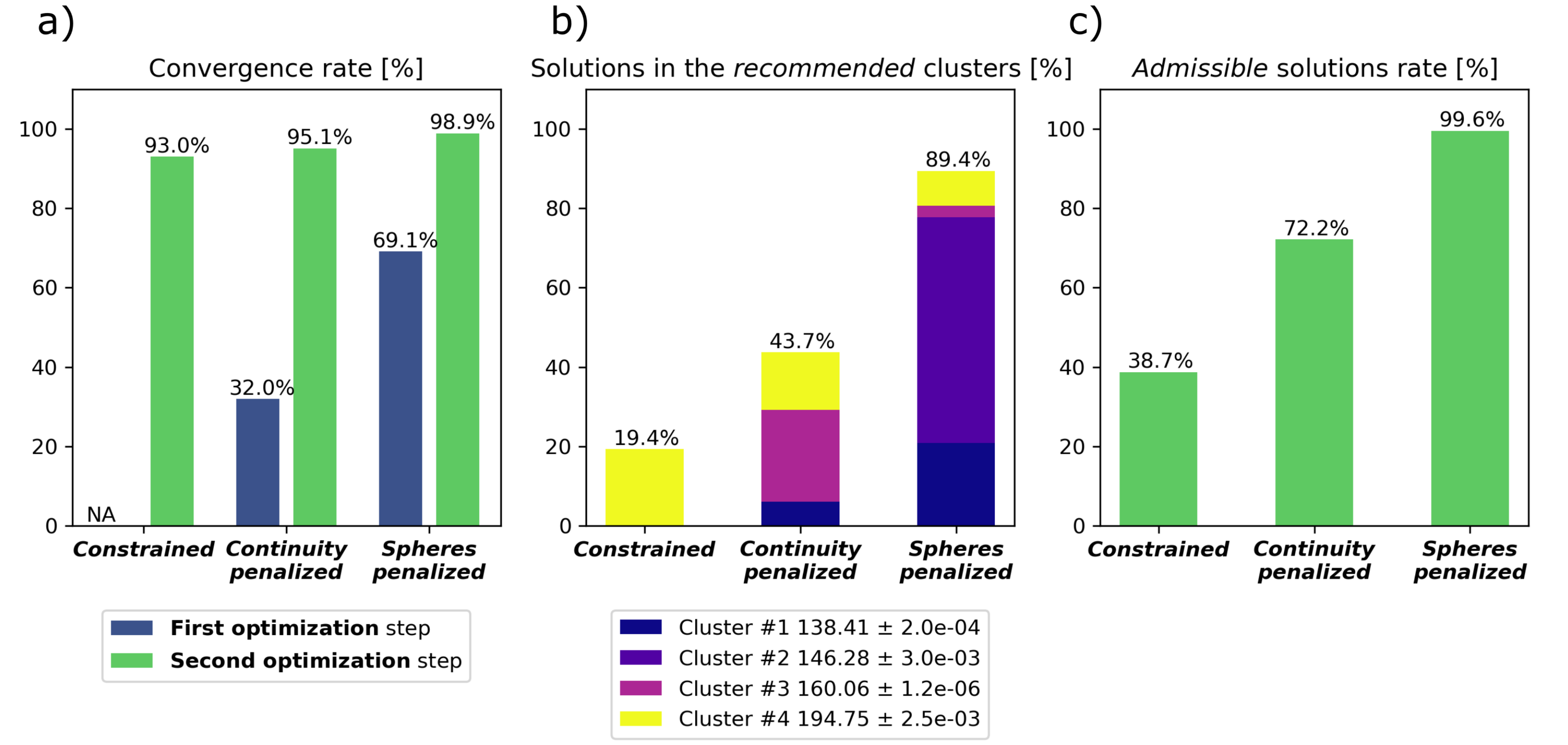
- Initial guesses independent from the researcher’s insights
- Not specific to the problem or this transcription

Questions ?

EveCharbie/ContinuityObjectiveTests

Bad solution



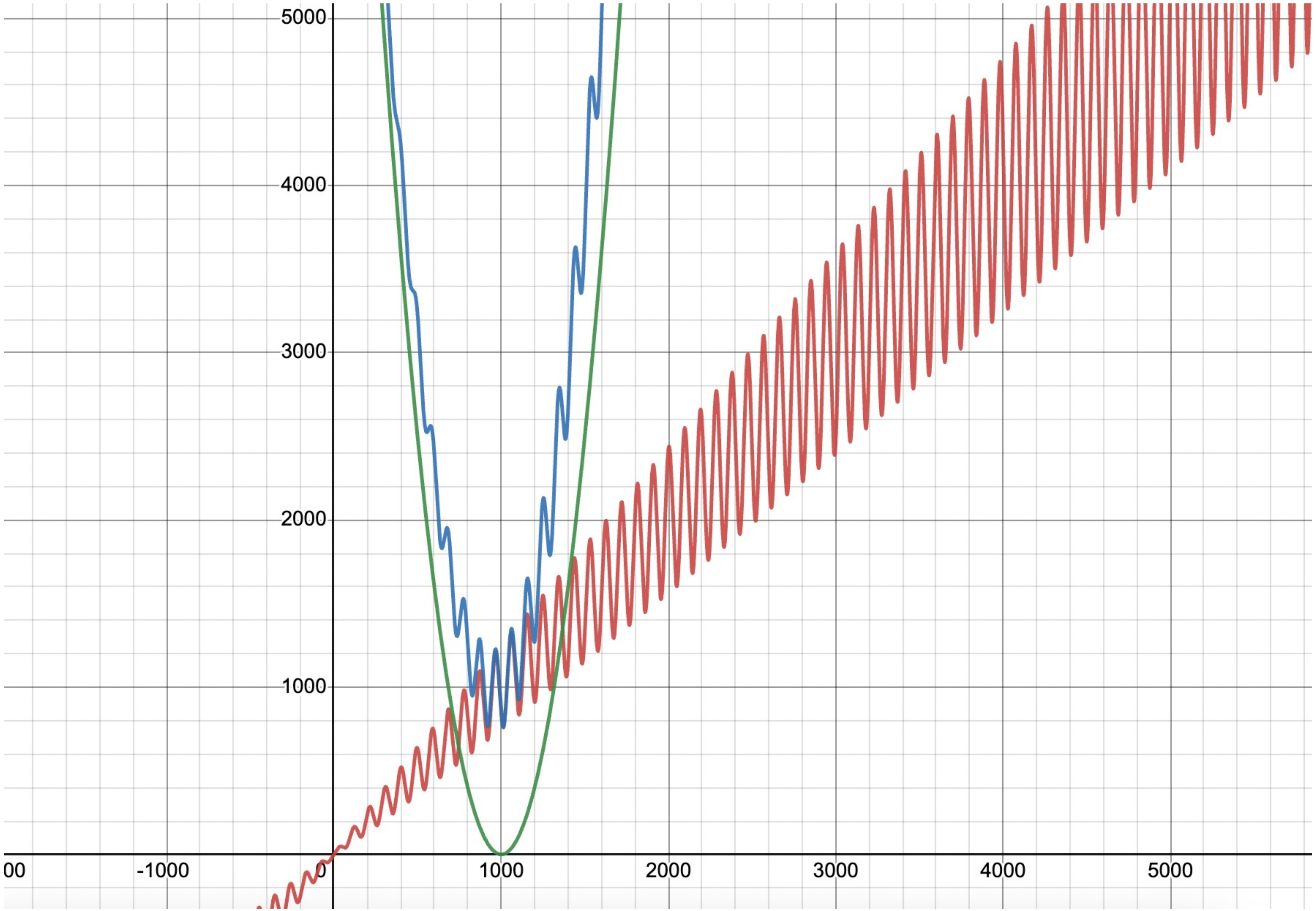


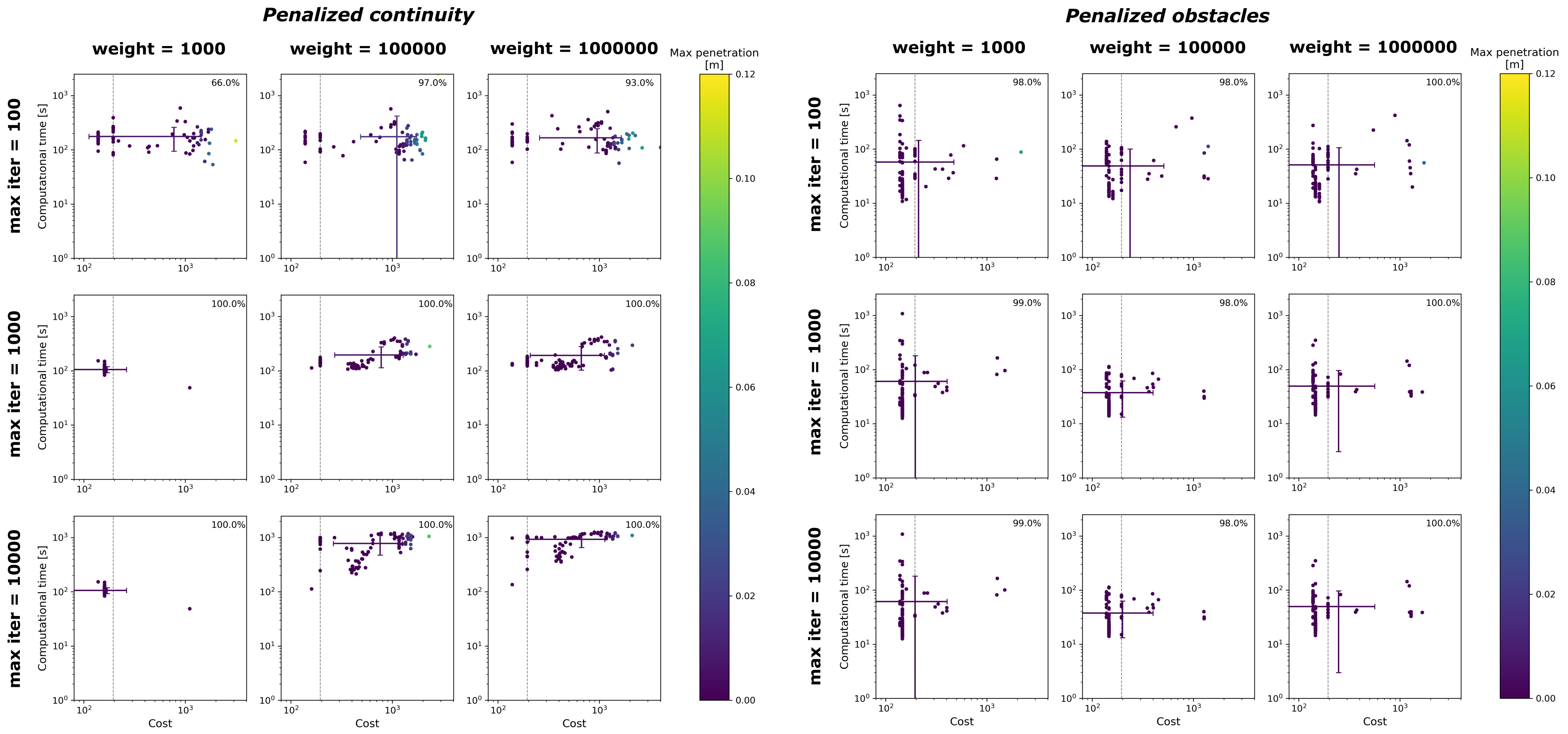
a)
Convergence rate [%]
93.0%
95.1%
98.9%
69.1%
32.0%
NA
Constrained
Continuity penalized
Spheres penalized
First optimization step
Second optimization step
b)
Solutions in the recommended clusters [%]
19.4%
43.7%
89.4%
Cluster #1 138.41 ± 2.0e-04
Cluster #2 146.28 ± 3.0e-03
Cluster #3 160.06 ± 1.2e-06
Cluster #4 194.75 ± 2.5e-03
c)
Admissible solutions rate [%]
38.7%
72.2%
99.6%

1 $\sin\left(\frac{x}{15}\right)\cdot x^{0.8}+x$

2 $\left(\frac{x}{10}-100\right)^{2}$

3 $\left(\frac{x}{10}-101\right)^{2}+\sin\left(\frac{x}{15}\right)\cdot x^{0.8}+x$




Penalized continuity
Penalized obstacles
weight = 1000
weight = 100000
weight = 1000000
max iter = 100
max iter = 1000
max iter = 10000
Computational time [s]
Cost
Max penetration [m]
66.0%
97.0%
93.0%
100.0%
98.0%
99.0%

$$
\min_{\mathbf{x},\mathbf{u},T} \quad \omega_1 T + \omega_2 \sum_{i=0}^{N_i} \mathbf{u}_i^2 + \overbrace{\omega_3 \sum_{i=1}^{N_i-1} \sum_{k=1}^{N_k} (x_{i+1,start}^k - x_{i,end}^k(\mathbf{x}_{i,start},\, \mathbf{u}_i,\, T))^2}^{\textit{Continuity penalty}} +
$$

$$
\underbrace{\omega_4 \sum_{i=0}^{N_i+1} \sum_{j=1}^{4} \left(\begin{cases} \|\mathbf{P}_i(\mathbf{x}_{i,start}) - \mathbf{S}_j\| - r, & \text{if negative,} \\ 0, & \text{otherwise} \end{cases} \right)^2}_{\textit{Obstacles penalty}} \tag{3a}
$$

$$
s.t. \quad x_{i+1,start}^k - x_{i,end}^k(\mathbf{x}_{i,start},\, \mathbf{u}_i,\, T) = 0 \tag{3b}
$$

$$
\mathbf{P}_i(\mathbf{x}_{i,start}) - \mathbf{S}_j \geq r \tag{3c}
$$

$$
\mathbf{x} \in \mathscr{X}, \mathbf{u} \in \mathscr{U}, T \in \mathscr{T} \tag{3d}
$$

where r was the spheres' radius, $\mathbf{P}_i$ was the position of the point on the pendulum at the i^{th} node, $\mathbf{S}_j$ was the j^{th} sphere's center position ω_i were the weightings $\boxed{(\omega_1 = 100,\ \omega_2 = 1,}$ $\omega_3 \in \{10000, 100000, 1000000\}$ and $\omega_4 \in \{1000, 100000, 10000000\}$), and $\mathscr{X}$, $\mathscr{U}$ $\mathscr{T}$ were the decision variable domains (see Tab. 1 and Tab. 2 for numerical values). The first two objective terms (i.e., minimizing the movement duration and the horizontal forces applied on the cart) were included in all implementations of the OCP. Their sum was used to compare the cost between implementations. The continuity term was included only to the ***penalized continuity*** OCP, while, for the other two OCPs, it was enforced by the continuity condition constraints (Eq. 3b). The sphere avoidance term was included only for the ***penalized obstacles*** OCP, while, for the other two OCPs, it was enforced by obstacle avoidance constraints (Eq. 3c). The tolerance on all constraints was set to 10^{-6} when solving the nonlinear program with IPOPT [20].