All the terms to subsitute in the code are highlighted in yellow.

$\boxed{\text{Eq. 1}}$

"Dpd" nel codice

$$\mu_d = \frac{\partial P_d}{\partial \eta} \; (*) = (P_s - P_t)(C_2 + 2C_3\eta + 3C_4\eta^2) + (P_o - P_t)(1 - C_2 - 2C_3\eta - 3C_4\eta^2)$$

$$m_y = \frac{\Delta y}{d(x,y)} \quad : \; d(x,y) = \text{distance function} \qquad / \qquad m_x = \frac{\Delta x}{d(x,y)}$$

$$U = \frac{\mu_d}{m_y} u \qquad \frac{\partial m_y}{\partial x} = -\Delta y \frac{1}{d^2(x,y)} \frac{\partial d}{\partial x} \qquad \frac{\partial m_x}{\partial y} = \Delta x \left(-\frac{1}{d^2(x,y)} \cdot \frac{\partial d}{\partial y} \right)$$

"Dxmy" nel codice

"Dymx" nel codice

$$u \cdot U = \frac{\mu_d}{m_y} u^2 = \frac{\partial P_d}{\partial \eta} \frac{d(x,y)}{\Delta y} u^2$$

$$\frac{\partial (u \cdot U)}{\partial x} = \dots = \underbrace{\frac{\partial P_d}{\partial \eta}}_{\text{sostituire } (*)} \frac{u}{\Delta y} \left(u \frac{\partial d}{\partial x} + 2 d(x,y) \frac{\partial u}{\partial x} \right)$$

$$V = \frac{\mu_d}{m_x} v = \frac{\partial P_d}{\partial \eta} \frac{d(x,y)}{\Delta x} v \quad \rightarrow \quad u \cdot V = \frac{\partial P_d}{\partial \eta} \frac{d(x,y)}{\Delta x} u v$$

$$\frac{\partial (u \cdot V)}{\partial y} = \frac{\partial P_d}{\partial \eta} \frac{1}{\Delta x} \left(u v \frac{\partial d}{\partial y} + d(x,y)\, u \frac{\partial v}{\partial y} + d(x,y)\, v \frac{\partial u}{\partial y} \right)$$

$$\Omega = \frac{\mu_d}{m_y} w = \frac{\partial P_d}{\partial \eta} \frac{d(x,y)}{\Delta y} w \quad \rightarrow \quad u \cdot \Omega = \frac{\partial P_d}{\partial \eta} \frac{d(x,y)}{\Delta y} u \cdot w$$

$$\frac{\partial (u \cdot \Omega)}{\partial \eta} = \frac{\partial^2 P_d}{\partial \eta^2} \frac{d(x,y)}{\Delta y} u w + \frac{\partial P_d}{\partial \eta} \frac{d(x,y)}{\Delta y} \frac{\partial u}{\partial \eta} w + \frac{\partial P_d}{\partial \eta} \frac{d(x,y)}{\Delta y} u \frac{\partial w}{\partial \eta}$$

$$= \frac{d(x,y)}{\Delta y} \left(\frac{\partial^2 P_d}{\partial \eta^2} u w + \frac{\partial P_d}{\partial \eta} \frac{\partial u}{\partial \eta} w + \frac{\partial P_d}{\partial \eta} u \frac{\partial w}{\partial \eta} \right)$$

Con $\displaystyle \frac{\partial^2 P_d}{\partial \eta^2} = (P_s - P_t)(2C_3 + 6C_4\eta) + (P_o - P_t)(-2C_3 - 6C_4\eta)$

$\operatorname{div} Vu$

$$\frac{\partial U}{\partial t} = \frac{\mu_d}{m_y}\frac{\partial u}{\partial t} = \frac{\mu_d}{m_y}\frac{\partial u}{\partial t}$$

$$p = p_0\left(\frac{R_d\,\theta_m}{p_0 \cdot \alpha_d}\right)^{\gamma} \longrightarrow \frac{\partial p}{\partial x} = p_0\left(\frac{R_d}{p_0 \cdot \alpha_d}\right)^{\gamma}\gamma\,\theta_m^{\gamma-1}\frac{\partial \theta_m}{\partial x}$$

$$\frac{\partial p}{\partial \eta} = p_0\left(\frac{R_d}{p_0 \cdot \alpha_d}\right)^{\gamma}\gamma\,\theta_m^{\gamma-1}\frac{\partial \theta_m}{\partial \eta}$$

$$\frac{\partial \phi}{\partial x} = g\,\frac{\partial z}{\partial x}$$

$\boxed{\text{Eq. 2}}$

$$\frac{\partial V}{\partial t} = \frac{\mu_d}{m_x}\frac{\partial v}{\partial t}$$

$$U \cdot v = \frac{\mu_d}{m_y} u v \quad , \quad V \cdot v = \frac{\mu_d}{m_x} v^2 \quad , \quad \Omega \cdot v = \frac{\mu_d}{m_y} \omega v$$

$$\left.\begin{aligned}
\frac{\partial (U \cdot v)}{\partial x} &= \frac{\mu_d}{\Delta y}\left(u v \frac{\partial d}{\partial x} + d(x,y)\, u \frac{\partial v}{\partial x} + d(x,y)\, v \frac{\partial u}{\partial x}\right) \\
\frac{\partial (V \cdot v)}{\partial y} &= \frac{\mu_d}{\Delta x}\left(v^2 \frac{\partial d}{\partial y} + d(x,y)\, 2 v \frac{\partial v}{\partial y}\right) \\
\frac{\partial (\Omega \cdot v)}{\partial \eta} &= \frac{1}{m_y}\left(\frac{\partial^2 p_d}{\partial \eta^2}\omega v + \mu_d \frac{\partial \omega}{\partial \eta} v + \mu_d\, \omega \frac{\partial v}{\partial \eta}\right)
\end{aligned}\right\} \underline{\mathrm{div}Vv}$$

$$\frac{\partial p}{\partial y} = p_0\left(\frac{R_d}{p_0 \cdot \alpha_d}\right)^{\gamma}\gamma\,\theta_m^{\gamma-1}\frac{\partial \theta_m}{\partial y}$$

$\frac{\partial p}{\partial \eta}$ as above , $\frac{\partial \phi}{\partial y} = g\,\frac{\partial z}{\partial y}$

Eq. 3

$$\frac{\partial W}{\partial t} = \frac{\mu_d}{m_y}\frac{\partial w}{\partial t}$$

$$U \cdot w = \frac{\mu_d}{m_y} u \cdot w = \frac{\mu_d}{\Delta y} d(x,y) \cdot u \cdot w \quad , \quad V \cdot w = \frac{\mu_d}{\Delta x} d(x,y)\, v \cdot w \quad , \quad \Omega \cdot w = \frac{\mu_d}{m_y} \cdot \omega \cdot w$$

$$\left.\begin{aligned}
\frac{\partial (U \cdot w)}{\partial x} &= \frac{\mu_d}{\Delta y}\left(\frac{\partial d}{\partial x} u w + d(x,y)\frac{\partial u}{\partial x} w + d(x,y)\, u \frac{\partial w}{\partial x}\right) \\
\frac{\partial (V \cdot w)}{\partial y} &= \frac{\mu_d}{\Delta x}\left(\frac{\partial d}{\partial y} v \cdot w + d(x,y)\frac{\partial v}{\partial y} w + d(x,y)\, v \frac{\partial w}{\partial y}\right) \\
\frac{\partial (\Omega \cdot w)}{\partial \eta} &= \frac{1}{m_y}\left(\frac{\partial^2 p_d}{\partial \eta^2} \omega w + \mu_d \frac{\partial \omega}{\partial \eta} w + \mu_d\, \omega \frac{\partial w}{\partial \eta}\right)
\end{aligned}\right\} \underline{\text{div } V w}$$

$\frac{\partial P}{\partial \eta}$ as above

Eq. 4

$$\frac{\partial \Theta_m}{\partial t} = \mu_d \frac{\partial \theta_m}{\partial t}$$

$$U \cdot \theta_m = \frac{\mu_d}{\Delta y} d(x,y)\, u\, \theta_m \quad , \quad V \cdot \theta_m = \frac{\mu_d}{\Delta x} d(x,y)\, v\, \theta_m \quad , \quad \Omega \cdot \theta_m = \frac{\mu_d}{m_y} \omega\, \theta_m$$

$$\left.\begin{aligned}
\frac{\partial(U\cdot\theta_m)}{\partial x} &= \frac{\mu_d}{\Delta y}\left(\frac{\partial d}{\partial x}u\theta_m + d(x,y)\frac{\partial u}{\partial x}\theta_m + d(x,y)u\frac{\partial\theta_m}{\partial x}\right)\\
\frac{\partial(V\cdot\theta_m)}{\partial y} &= \frac{\mu_d}{\Delta x}\left(\frac{\partial d}{\partial y}v\theta_m + d(x,y)\frac{\partial v}{\partial y}\theta_m + d(x,y)v\frac{\partial\theta_m}{\partial y}\right)\\
\frac{\partial(\Omega\cdot\theta_m)}{\partial\eta} &= \frac{1}{m_y}\left(\frac{\partial^2 p_d}{\partial\eta^2}w\theta_m + \mu_d\frac{\partial w}{\partial\eta}\theta_m + \mu_d w\frac{\partial\theta_m}{\partial\eta}\right)
\end{aligned}\right\}\ \underline{\operatorname{div} V\theta_m}$$

$\boxed{\text{Eq. 5}}$

$\frac{\partial\mu_d}{\partial t} = 0$!! (since μ is an independent variable)

$$V = \frac{\mu_d}{m_x}v = \frac{\mu_d}{\Delta x}d(x,y)v$$

$$\left.\begin{aligned}
\frac{\partial V}{\partial x} &= \frac{\mu_d}{\Delta x}\left(\frac{\partial d}{\partial x}v + d(x,y)\frac{\partial v}{\partial x}\right)\\
\frac{\partial V}{\partial y} &= \frac{\mu_d}{\Delta x}\left(\frac{\partial d}{\partial y}v + d(x,y)\frac{\partial v}{\partial y}\right)\\
\frac{\partial V}{\partial\eta} &= \frac{1}{m_x}\left(\frac{\partial^2 p_d}{\partial\eta^2}v + \mu_d\frac{\partial v}{\partial\eta}\right)
\end{aligned}\right\}\ \underline{\operatorname{div} V}$$

$\boxed{\text{Eq. 6}}$

$$\frac{\partial\phi}{\partial t} = g\frac{\partial z}{\partial t}$$

$\boxed{\text{Eq. 7}}$

$$\frac{\partial Q_m}{\partial t} = \mu_d\frac{\partial q_m}{\partial t}$$

$$U\cdot q_m = \frac{\mu_d}{\Delta y} d(x,y)\, u\, q_m \quad , \quad V\cdot q_m = \frac{\mu_d}{\Delta x} d(x,y)\, v\, q_m \quad , \quad \Omega\cdot q_m = \frac{\mu_d}{m_y} w\, q_m$$

$$\left.\begin{aligned}
\frac{\partial(U\cdot q_m)}{\partial x} &= \frac{\mu_d}{\Delta y}\left(\frac{\partial d}{\partial x} u\, q_m + d(x,y)\frac{\partial u}{\partial x} q_m + d(x,y)\, u\, \frac{\partial q_m}{\partial x}\right) \\
\frac{\partial(V\cdot q_m)}{\partial y} &= \frac{\mu_d}{\Delta x}\left(\frac{\partial d}{\partial y} v\, q_m + d(x,y)\frac{\partial v}{\partial y} q_m + d(x,y)\, v\, \frac{\partial q_m}{\partial y}\right) \\
\frac{\partial(\Omega\cdot q_m)}{\partial \eta} &= \frac{1}{m_y}\left(\frac{\partial^2 p_d}{\partial \eta^2} w\, q_m + \mu_d \frac{\partial w}{\partial \eta} q_m + \mu_d\, w\, \frac{\partial q_m}{\partial \eta}\right)
\end{aligned}\right\} \underline{\operatorname{div} V q_m}$$