

## THE APPLICATION OF LATIN SQUARE IN AGRONOMIC RESEARCH

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**Abstract:** To plan an experiment means to decide how to observe and measure in order to minimize randomized variations and stress the effects of the factors analyzed. Namely, it is of major importance to minimize experimental errors. Thus, the experimental design is expected to meet some of the principles stated in order to minimize experimental errors.

Local control is one of the principles anticipated to be met by an experimental design. In addition to the variations tested, local control is expected to isolate the significant variations from the experimental error. The choice of the experimental design is one of the means of realization of local control.

The objective of the study was to point to the possibility of increasing reliability of conclusions by changing experimental design. Namely, the Latin square along with some of its characteristics may be anticipated to solve some of the issues with regard to experimental design when agricultural production is concerned.

**Key words:** experimental design, Latin square, treatments, replications.

### I n t r o d u c t i o n

Statistical methods have been applied in experimental design since the 1930s. Fisher, R.A., was the first researcher to publish papers dealing with experimental design. The Latin square design has been often used since then in experimental design and the results have been computed and interpreted using the analysis of variance.

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The experimental design has, been introduced under the direction of R.A. Fischer, into agronomic research at Rothamsted Experimental Station (Great Britain) and later on its use spread to all other scientific branches requiring experimental research.

The mathematical models are identical for the trials conducted according to the same design irrespective of the scientific field. However, an experiment designed and conducted in industry varies from that in agriculture. Trials are far more rapidly conducted in industry compared with agriculture where they may last as long as the vegetation period and the experiment may sometimes take years to be carried out. Trials in industry are known to be expensive but often far less complex. Thus, they may be repeated at even time intervals and the results obtained in one trial may serve to make the necessary corrections in the trial to follow. Also, the main objective of the experiment in industry is to determine the confidence interval within which, along with a certain dose of risk, the set parameter may be anticipated, knowing in advance that the results of the treatment will not be the same. The objective of the experiments in agriculture is to determine potential differences and their intensity between treatments. The use of the Latin square is, among others, one of the ways to solve this issue.

**Types of the Latin square in the function of solving the issues with  
regard to experimental design in agronomic research**

In conventional agronomic research the Latin square represents the factor experiment having a limited randomized treatment on experimental units in relation to the effects of the factors, the influence of which is being controlled with the help of rows and columns. The effects of the treatment are not considered to be mutually related with the factors the effects of which are recorded in rows and columns.

In biological experimental design the Latin square is used mainly as a method for controlling the homogeneity of the experimental units.

The advantage of the square designed plot whose main objective is to eliminate the differences in soil fertility has early been acknowledged. Also, numerous agricultural activities are known to be long running and parallel. For this reason, the Latin square may be regarded convenient for eliminating variations in the experimental material. The use of different Latin square types is of major importance in solving some issues with regard to experimenting in agronomic research.

The issue with regard to residue effects characterizes agronomic research. Namely, a number of successive treatments are often being applied to one experimental unit and for this reason the effect of each treatment in the case of the unit analyzed is affected by both previously applied treatment and earlier applied treatments.

For example, when studying the mode of nutrition in daily intake of animals it is preferable to apply successively different nutrition modes in each of the animals analyzed.

The solution to this problem may be obtained by using the row-complete Latin square. Considering the fact that using this Latin square type each treatment is preceded by other treatments, the result of the treatment may be said to be "balanced" in relation to the residue effects, although the balance may be considered incomplete because none of the treatments have been preceded by themselves.

If the objective of the study is to control residue effects for more than one preceding period, the plan gains in complexity. In that case  $n-1$  balanced squares are needed for  $n$  treatments.

In addition to the direct effects, indirect effects are known to be recorded in agronomic research as well. Quasi-complete Latin squares have successfully been used in studying indirect effects. Also, more than one quasi-complete square can be used in order to study direct balancing.

Biological research is known to be carried out under laboratory conditions using a small number of samples. In that case, a number of measurements can be carried out on one experimental unit in order to get more information and eventually contribute to the reliability of the conclusions. Thus, the Latin square design with replications is preferable. Sometimes manifold measurements are required for a part of the units analyzed because some treatments should be given priority, i.e. their effect is far more variable. Based on the replicated treatments, it is possible to assess the variance between the replications which in some cases may hinder the analysis of the interactions between factors from rows, columns and treatments. On the other hand, considering these interactions to be insignificant, the variance between the replications may be confounded with the residue variance in order to increase the degree of freedom for the variance error in the Latin square. This is of major importance for Latin square designs of smaller dimensions having a small number of degrees of freedom for the variance error.

The Latin squares with partial replications have the objective to increase the number of the degrees of freedom for the error and to indicate whether the effects of the factors from the rows, columns and treatments have been fully separated. Therefore, if there is any indication of interactions going on, a quarter of the replications from the possible  $n^3$  measurements, the interactions of two factors are completely confounded with the main effects. In that case, it is far more recommendable to use other experimental designs instead of the Latin square.

The concept of the F-square has been developed considering that the features of these designs may be used for studying the orthogonal Latin squares and other issues with regard to combinatorics.

The use of the F-square in agronomic research has its advantage over other experimental designs in cases when two problems need to be solved in an experiment. Firstly, it is preferable to use the F-square, if based on earlier research, it is known that the difference between some treatments is insignificant and negligible. Secondly, the advantage of the F-square in relation to the Latin square concerns those cases when the number of treatments is smaller than the degree of the square because the use of information on the available experimental units increases the accuracy of the estimated effects of the treatment.

The issue with regard to the small number of replications due to the small number of treatments may be solved by using the experimental design of the Latin square type, whereby the number of replications is greater than the number of treatments. In that case, the incomplete quasi Latin square may be used, whereby the size of the block, which otherwise equals the number of the replications, is by one smaller or greater from the number of treatments multiplied by a number. Thus each treatment in one of the blocks is replicated once less, i.e. more in relation to other blocks.

On the other hand, sometimes there is a need to control the two sources of heterogeneity of the units but with a smaller number of replications than the number of treatments. This problem may be resolved by constructing a modified Latin square.

In big experiments the use of the conventional Latin square design is known to be economically unjustifiable because a greater number of replications is used from the actually required, thus decreasing the accuracy of the trial due to a smaller homogeneity of the units. Thus it is preferable to use the special kind of the Latin square, i.e. the latis square.

In general, the feature of the Latin square design is that the number of replications equals the number of treatments which often poses a limiting factor for planning an agricultural experiment according to this design despite the need for eliminating the heterogeneity of the experimental units in two directions. Therefore, when the available material cannot be used for the equal number of replications and treatments and the need prevails to eliminate the heterogeneity of the experimental material, it is preferable to use the incomplete Latin square.

Accordingly, the issue with regard to the number of treatments and the number of replications may be solved using different types of Latin squares, depending on the cause provoking the problem.

In agricultural research, especially under field conditions, the experimental material is known to be exposed to the effects of both systematic and randomized factors. If the experiment is designed with the aim of determining the relationship between the treatments needing to control three sources of systematic variations, the Greek-Latin square will be required.

The computations need to include a possible trend in cases when experimental units are successively performed. One of the methods for solving the problem is the use of systematic plans which estimate the effects of a number of treatments using orthogonal polynoms irrespective of the trend type. This is one of the ways to estimate a substantially small number of treatments and a comparatively complex trend. A magic square design may be used for estimating the effects of a large number of treatments irrelevant of the type of the trend involved, considering the fact that each trend may be adequately represented as being linear. In that case, the numbers in the fields of the symmetric magic square represent ordinal numerals of the measurings or numbers of certain time parameters, i.e. day or hour of measuring.

### Conclusion

The Latin square design is known to be used in different scientific fields: agronomic, industrial, biological, medical research, psychology, etc.

The basic Latin square model has variously been modified in order to eliminate some of its disadvantages ensuring homogenous experimental conditions. This study stresses the importance of the Latin square in solving the following issues:

- residue effects;
- indirect effects;
- elimination of trend effects;
- monitoring more than two randomized factors;
- increasing the degrees of freedom in squares with a small number of treatments;
- different significance of the treatments;
- smaller number of treatments in relation to the number of replications;
- small number of replications due to a small number of treatments, and
- smaller number of replications than required by a conventional Latin square.

In addition to their use in experimental designs, Latin squares may be applied for other purposes. Namely, the concept of the Latin square construction is being applied in some scientific fields. The concept of the Latin square construction of the degree  $n$  is known to be equivalent to the code concept for detecting errors in telecommunications. In addition, structures of objects distributed according to a scheme may often be seen in mathematics. From the mathematical standpoint, square schemes, i.e. Latin squares are most important. A direct relationship has been recorded between these and quasi-groups. Also, the concept of orthogonality in the case of the Latin square may be linked to the problem of projective planes.

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Received October 16, 2000

Accepted March 27, 2001

PRIMENA LATINSKOG KVADRATA U POLJOPRIVREDNIM  
ISTRAŽIVANJIMA

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## R e z i m e

Pri planiranju eksperimenata javlja se problem izbora tretmana, eksperimentalnih jedinica i tehničke aparature za izvodjenje ogleda s jedne strane, i metoda aplikacije tretmana na eksperimentalne jedinice i broja njihovih ponavljanja, s druge strane. Drugi deo problema u nekim slučajevima može se rešiti postavljanjem ogleda po planu latinskih kvadrata sa određenim svojstvima.

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Pri izvodjenju eksperimenata često je potrebno da se pored ispitivanih varijacija i neke druge značajne varijacije izdvoje iz eksperimentalne greške. U radu je istaknuto da je neophodno poznavanje različitih vrsta latinskih kvadrata, jer se upotrebom latinskih kvadrata sa određenim svojstvima mogu rešiti problemi rezidualnih i indirektnih efekata, eliminisanja uticaja trenda, kontrole većeg broja slučajnih faktora, kao i problemi broja stepeni slobode i odnosa broja tretmana i broja ponavljanja.

Primljeno 16. oktobra 2000.

Odobreno 27. marta 2001.