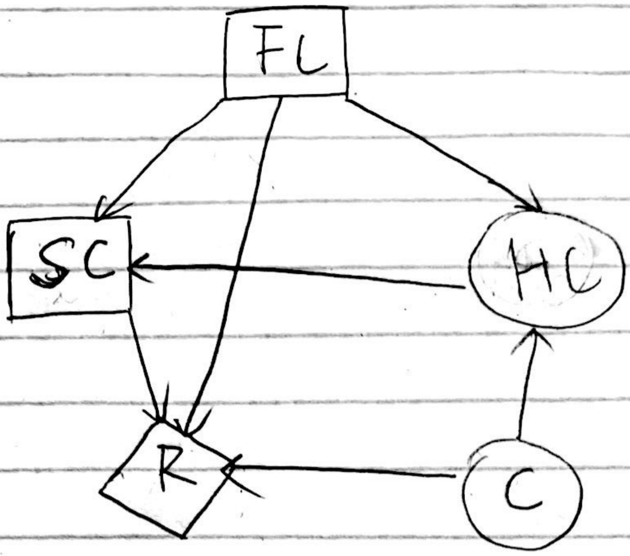
2. (a)



C	P(C)
d_1	1/3
d_2	1/3
d_3	1/3

For doors 1, 2, 3.

variable $FC \in \{d_1, d_2, d_3\}$: First choice of door.

$SC \in \{0, 1\}$: Second choice whether choose to stay(0) or switch (1) a door

$C \in \{d_1, d_2, d_3\}$: the car is behind which door.

$HC \in \{s, l\}$: for the remaining two doors left by my choice, whether the smaller or larger one the host choose.

$R \in \{0, 1\}$: reward

Probability table for FC:

FC	C	$P(HC=s \mid FC, C)$	$P(HC=l \mid FC, C)$
d_1	d_1	0.5	0.5
d_1	d_2	0	1
d_1	d_3	1	0
d_2	d_1	0	1
d_2	d_2	0.5	0.5
d_2	d_3	1	0
d_3	d_1	0	1
d_3	d_2	1	0
d_3	d_3	0.5	0.5

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Utility table for R

C	FC	SC	R(C, FC, SC)
d1	d1	0	1
d1	d1	1	0
d1	d2	0	0
d1	d2	1	1
d1	d3	0	0
d1	d3	1	1
d2	d1	0	0
d2	d1	1	1
d2	d2	0	1
d2	d2	1	0
d2	d3	0	0
d2	d3	1	1
d3	d1	0	0
d3	d1	1	1
d3	d2	0	0
d3	d2	1	1
d3	d3	0	1
d3	d3	1	0

Factors are.

$$f_0(C, FC, SC) = R(C, FC, SC)$$

$$f_1(C) = P(C)$$

$$f_2(FC, C, HC) = P(HC \mid FC, C)$$

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(b) Using the Variable Elimination alg, we first sum out C:

$$f_3(FL, SL, HL) = \sum_C f_0(C, FL, SL) f_2(FL, C, HL) f_3(C)$$

=

FL	SL	HL=s	HL=l
d_1	0	$\frac{1}{3}\times 0.5\times 1 + \frac{1}{3}\times 0\times 0 + \frac{1}{3}\times 1\times 0 = 1/6$	1/6
d_1	1	$\frac{1}{3}\times 0.5\times 0 + \frac{1}{3}\times 0\times 1 + \frac{1}{3}\times 1\times 1 = 1/3$	1/3
d_2	0	$\frac{1}{3}\times 0\times 0 + \frac{1}{3}\times 0.5\times 1 + \frac{1}{3}\times 1\times 0 = 1/6$	1/6
d_2	1	$\frac{1}{3}\times 0\times 1 + \frac{1}{3}\times 0.5\times 0 + \frac{1}{3}\times 1\times 1 = 1/3$	1/3
d_3	0	$\frac{1}{3}\times 0\times 0 + \frac{1}{3}\times 1\times 0 + \frac{1}{3}\times 0.5\times 1 = 1/6$	1/6
d_3	1	$\frac{1}{3}\times 0\times 1 + \frac{1}{3}\times 1\times 1 + \frac{1}{3}\times 0.5\times 0 = 1/3$	1/3

Now, we max out SC

$$f_4(FL, HL) = \max_{SL} f_3(FL, SL, HL)$$

=

FL	HL	$f_4(FL, HL)$	policy(FL)
d_1	s	1/3	1
d_1	l	1/3	1
d_2	s	1/3	1
d_2	l	1/3	1
d_3	s	1/3	1
d_3	l	1/3	1

then, we sum out HL

$$f_5(FL) = \sum_{HL} f_4(FL, HL)$$

=

FL	$f_5(FL)$
d_1	2/3
d_2	2/3
d_3	2/3

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then, max out FL

$$f_6() = \max_{FL} f_5(FL) = 2/3, \text{ policy is random}$$

Therefore, the expected probability of winning a car is 66.67%, and the policy is to pick the first door randomly, and always switch door next.

(c) $p_1 = 0.8$, $p_2 = 0.1$, $p_3 = 1 - p_1 - p_2 = 0.1$

thus, $f_1(C) = P(C) =$

C	P(C)
d_1	0.8
d_2	0.1
d_3	0.1

Same as (b), sum out C, we get

$$f_3(FL, SC, HL) = \sum_C f_0(C, FL, SC) f_2(FL, C, HL) f_1(C)$$

=

FL	SC	HL=S	HL=L
d_1	0	0.4	0.4
d_1	1	0.1	0.1
d_2	0	0.05	0.05
d_2	1	0.1	0.8
d_3	0	0.05	0.05
d_3	1	0.1	0.8

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Then max out SC

$$f_4(FC, HC) = \max_{SC} f_3(FC, SC, HC)$$

FC	HC	$f_4(FC, HC)$	policy(FC)
d_1	s	0.4	0
d_1	t	0.4	0
d_2	s	0.1	1
d_2	t	0.8	1
d_3	s	0.1	1
d_3	t	0.8	1

then Sum out HC

$$f_5(FC) = \sum_{HC} f_4(FC, HC)$$

FC	$f_5(FC)$
d_1	0.8
d_2	0.9
d_3	0.9

then max out FC

$$f_6() = \max_{FC} f_5(FC) = 0.9$$

The ~~ex~~ expected probability of winning a car is 90%, and the policy is to pick door 2 (or door 3) first, and then switch.

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