2D Geometry

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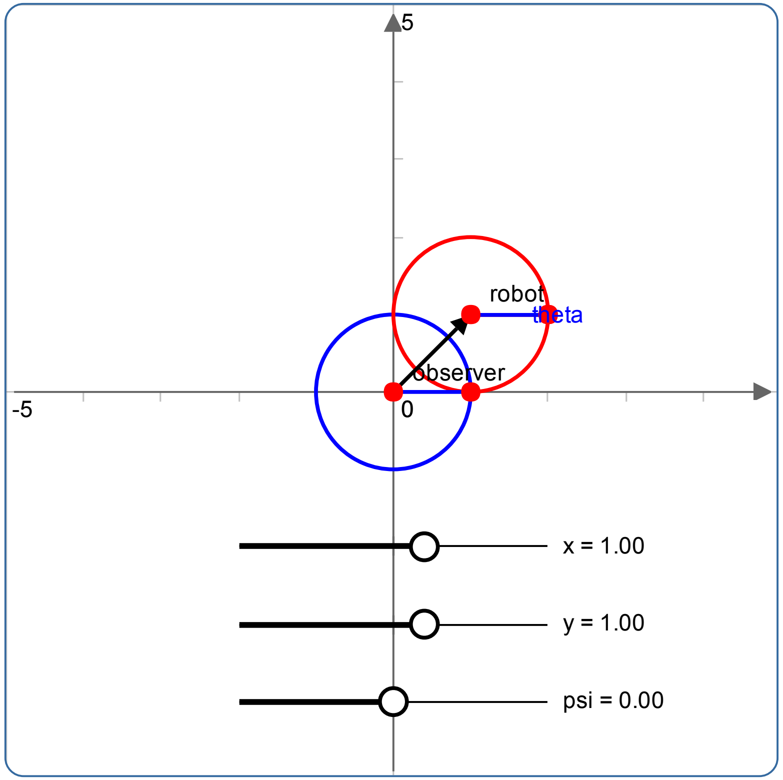
2D Geometry is used to express the robot's position and heading as well as to translate between the different coordinate frames. In 2D the pose of a robot can be uniquely defined by a position vector

$$\mathbf{t} = \begin{pmatrix} x \\ y \end{pmatrix}$$

and the orientation angle ψ (yaw) relative to a reference frame. At the start, the local coordinate system may be coincident with the observer's (global) coordinate system but this changes quickly as the robot moves around and rotates. Set some transforms with the sliders and get a feeling for the robot moving around:



Translation & Rotation

0/2 points (graded)

Assume your robot flew 2 m in x direction, 3 m in y direction and turned 90 degrees (counterclockwise) around its yaw axis. Please enter the translation vector:

$$\mathbf{t} = \begin{pmatrix} x \\ y \end{pmatrix}$$

1; 2

✖ **Answer:** 2, 3

Please enter the rotation matrix:

$$\mathbf{R} = \begin{pmatrix} cos(\psi) & -sin(\psi) \\ sin(\psi) & cos(\psi) \end{pmatrix}$$

-0.45, -0.89; 0.89, -0.45

✖ **Answer:** 0, -1; 1, 0

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Transformation

1/1 point (graded)

Together, $\mathbf{t}$ and $\mathbf{R}$ form a homogeneous transformation matrix $\mathbf{T}_1^0$ transforming local into global coordinates. Please enter the homogeneous transformation matrix:

$$\mathbf{T}_1^0 = \begin{pmatrix} \mathbf{R} & \mathbf{t} \\ 0 & 1 \end{pmatrix}$$

0,-1,2;1,0,3;0,0,1

✔ **Answer:** 0, -1, 2; 1, 0, 3; 0, 0 , 1

Explanation

$$\mathbf{T}_1^0 = \begin{pmatrix} cos(90) & -sin(90) & 2 \\ sin(90) & cos(90) & 3 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

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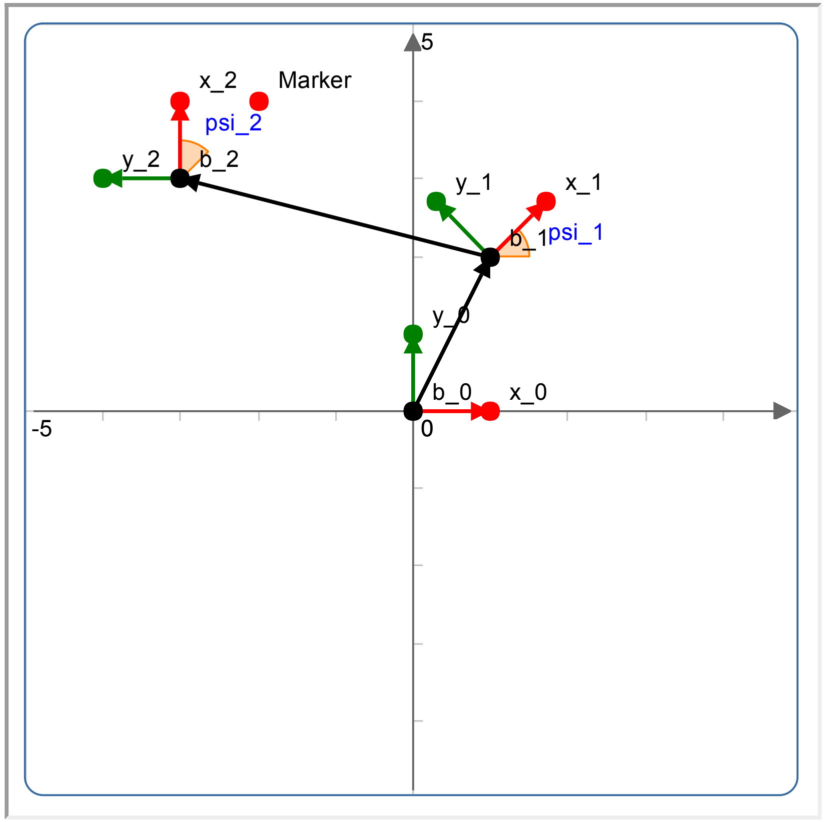
Transformation Concatenation

1/1 point (graded)

During the mission, the robot's pose is continuously reestimated relative to its last known pose.



The transformations describing the poses can simply be concatenated by multiplication to yield the transformation from local to global frame.

$$\mathbf{T}_2^0 = \mathbf{T}_1^0 \cdot \mathbf{T}_2^1$$

$\mathbf{T}_1^0$ with: $\mathbf{t}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\psi_1 = 45^\circ$

$\mathbf{T}_2^1$ with: $\mathbf{t}_2 = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$ and $\psi_2 = 45^\circ$

What is the resulting homogeneous transformation matrix for the example above ?

0,-0.999698,-3.242;0.999698,0,0.586;0,0,1

Answer: 0, -1, -3.24264068712; 1, 0, 0.58578643762; 0, 0 , 1

Explanation

$$\mathbf{T}_1^0 = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{T}_2^1 = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -4 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{T}_2^0 = \mathbf{T}_1^0 \cdot \mathbf{T}_2^1 =$$

$$\begin{pmatrix} 0 & -1 & -3.24264068712 \\ 1 & 0 & 0.58578643762 \\ 0 & 0 & 1 \end{pmatrix}$$

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Local to Global Transformation

1/1 point (graded)

Often the robot has detected a marker in its local coordinate system and we want to know where the marker is on our global map. This can be achieved by finding the translation and rotation that transforms the global into the local coordinate system and applying it to our point of interest.

$$\tilde{p}_{global} = \mathbf{T}_{local}^{global} \cdot \tilde{p}_{local}$$

The robot has detected a marker at position $\tilde{p}_{local} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ of its local coordinate system.

The transform is given as:

$$\mathbf{T}_{local}^{global} = \begin{pmatrix} 0 & -1 & -3 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

Where is the marker located in the global coordinate system?

-2,4,1

✔ **Answer:** -2, 4, 1

Solution

$$\tilde{p}_{global} = \begin{pmatrix} 0 & -1 & -3 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 1 \end{pmatrix}.$$

Submit You have used 3 of 3 attempts

Global to Local Transformation

0/1 point (graded)

Moreover, we are often interested in reaching a goal position in the world coordinate frame but can only give local commands to the robot i.e. we need to translate the global to local commands in order to execute them. This can be achieved by applying the inverse rotation

$$v_{local} = \mathbf{R}^T \cdot v_{global}$$

to the commands.

Now assume you want to fly in y direction of your global coordinate frame with speed 1.5 m/s.

Given $\mathbf{R} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$,

what command v_{local} has to be issued to the robot?

0,1.5;-1.5,0

✖ **Answer:** 1.5, 0

Solution

$$\tilde{v}_{local} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1.5 \end{pmatrix} = \begin{pmatrix} 1.5 \\ 0 \end{pmatrix}.$$