CSYS 300 PoCS Assignment 6

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Code is located at https://github.com/Evelios/PoCS_Assignment_06

1 Problem 1

1-d Theoretical Percolation

On an infinite 1-d lattice forest with a tree present at any site with probability p. The probability of a tree not appearing is then $1 - p$.

The expected distribution of forest sizes L is the chance that a forest will extend for l sites and then be bordered by two non-tree sites. This equation for 1-d is

$$F(p, l) = p^l \tag{1}$$

To find the critical point for a 1-d lattice p_c we need to find when the forest creates one giant component. That is when l become massive (infinte)

$$< l >= \sum_{l=0}^{\infty} lp^l$$

$$p_c = \lim_{l \to \infty} L(p, l) \tag{2}$$

$$p_c = \lim_{l \to \infty} p^l \tag{3}$$

$$p_c = 1 \tag{4}$$

2 Problem 2

Showing analitically that the critical probability for site percolation on a triangular lattice is $p_c = 1/2$.

Using the real space renormalization of a 3-site connection on a triangular lattice we can come up with the percolation properties of the nodes to solve for the critical point p_c. Each of these nodes are made up of a a 3-site triangle connection. There is flow through these nodes if there are at least two of the nodes activated. Since each node has an activation probability of p then the change of flow through a node P' is the sum of the probabilities of each configuration appearing.

There is one configuration state with all the sites active, this happens with probability p^3. There are then three configurations in which two of the sites are activated, each of those configurations have a $p^2(1 - p)$ chance of occuring. Thus the change of a node appearing with an active flow state is,

$$p' = f(p) = p^3 + 3p^2(1-p)$$

There are two important states to check for our node probability. If $p = 0$ then $p' = 0$. If $p = 1$ then $p' = 1$. Our equation satisfies these two requirements

$$f(0) = 0^3 + 3 * 0^2(1-0) = 0$$

$$f(1) = 1^3 + 3 * 1^2(1-1) = 1$$

3 Problem 3

So, unfortunately on my part I read problem 3 and assumed that we were coding up the HOT model for tree placement optimization. I jumped on that gun and coded up the optimization design problem for arbitrary tree sizes and probabilities. Unfortunately I read the problem more and realized I wan't even close. That code is in HOT_trees.R in the repository.

4 Problem 4

5 Problem 5