# CSYS 300 PoCS Assignment 6

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**Code is located at**   https://github.com/Evelios/PoCS_Assignment_06

# 1 Problem 1

1-d Theoretical Percolation

On an infinite 1-d lattice forest with a tree present at any site with probability $p$. The probability of a tree not appearing is then $1 - p$.

The expected distribution of forest sizes $L$ is the chance that a forest will extend for $l$ sites and then be bordered by two non-tree sites. This equation for 1-d is

$$F(p, l) = p^l \tag{1}$$

To find the critical point for a 1-d lattice $p_c$ we need to find when the forest creates one giant component. That is when $l$ become massive (infinte)

$$< l >= \sum_{l=0}^{\infty} l p^l$$

$$p_c = \lim_{l \to \infty} L(p, l) \tag{2}$$

$$p_c = \lim_{l \to \infty} p^l \tag{3}$$

$$p_c = 1 \tag{4}$$

# 2 Problem 2

Showing analitically that the critical probability for site percolation on a triangular lattice is $p_c = 1/2$.

Using the real space renormalization of a 3-site connection on a triangular lattice we can come up with the percolation properties of the nodes to solve for the critical point $p_c$. Each of these nodes are made up of a a 3-site triangle connection. There is flow through these nodes if there are at least two of the nodes activated. Since each node has an activation probability of $p$ then the change of flow through a node $P'$ is the sum of the probabilities of each configuration appearing.

There is one configuration state with all the sites active, this happens with probability $p^3$. There are then three configurations in which two of the sites are activated, each of those configurations have a $p^2(1 - p)$ chance of occuring. Thus the change of a node appearing with an active flow state is,

$$p' = f(p) = p^3 + 3p^2(1 - p)$$

There are two important states to check for our node probability. If $p = 0$ then $p' = 0$. If $p = 1$ then $p' = 1$. Our equation satisfies these two requirements

$$f(0) = 0^3 + 3 * 0^2(1 - 0) = 0$$

$$f(1) = 1^3 + 3 * 1^2(1 - 1) = 1$$

# 3 Problem 3

# 4 Problem 4

# 5 Problem 5