CSYS 300 PoCS Assignment 7

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Code is located at https://github.com/Evelios/PoCS_Assignment_07

1 Problem 1

Solving for the complamentary cumulative distributions for continuous distributions. The formula used for solving for the complamentary cumulative distribution is,

$$P_{\leq}(A) = \int_{p^{-1}(A^{-\gamma})}^{\infty} p(x)dx = p_{\leq}(p^{-1}(A^{-\gamma})) \tag{1}$$

Here p^{-1} represents the inverse function, so that is the first thing that needs to be found.

(a) Solving for $P_{\leq}(A)$ from $p(x) = cx^{-(q+1)}$

$$p = cx^{-(q+1)} \tag{2}$$

$$x = cp'^{-(q+1)} \tag{3}$$

$$x/c = p'^{-(q+1)} \tag{4}$$

$$p' = (x/c)^{-(\frac{1}{q+1})} \tag{5}$$

$$p'(A^{-\gamma}) = (A^{-\gamma}/c)^{-(\frac{1}{q+1})} \tag{6}$$

$$P_{\leq}(A) = \int_{\frac{A^{-\gamma}}{c}^{-(\frac{1}{q+1})}}^{\infty} cx^{-(q+1)}dx \tag{7}$$

$$= -\frac{c}{q}x^{-q}\Big|_{(A^{-\gamma}/c)^{-(\frac{1}{q+1})}}^{\infty} \tag{8}$$

$$= \left[0 - -\frac{c}{q}\left((A^{-\gamma}/c)^{-(\frac{1}{q+1})}\right)^{-q}\right] \tag{9}$$

$$= \frac{c}{q}\left((A^{-\gamma}/c)^{-(\frac{1}{q+1})}\right)^{-q} \tag{10}$$

$$= \frac{c}{q}\left(\frac{A^{-\gamma}}{c}\right)^{-(\frac{q}{q+1})} \tag{11}$$

$$= \frac{c}{q}\left(\frac{A^{-\gamma}}{c}\right)^{-(1+\frac{1}{q})} \tag{12}$$

(b) Solving for $P_{\leq}(A)$ from $p(x) = ce^{-x}$

$$p = e^{-x} \tag{13}$$

$$x = e^{-p'} \tag{14}$$

$$ln(x) = -p' \tag{15}$$

$$p' = -ln(x) \tag{16}$$

$$p'(A^{-\gamma}) = -ln(A^{-\gamma}) \tag{17}$$

$$P_{\leq}(A) = \int_{-ln(A^{-\gamma})}^{\infty} ce^{-x}dx \tag{18}$$

$$= -ce^{-x}\Big|_{-ln(A^{-\gamma})}^{\infty} \tag{19}$$

$$= \left(0 - -ce^{-(-ln(A^{-\gamma}))}\right) \tag{20}$$

$$= ce^{ln(A^{-\gamma})} \tag{21}$$

$$P_{\leq}(A) = cA^{-\gamma} \tag{22}$$

(c) Solving for $P_{\leq}(A)$ from $p(x) = ce^{-x^2}$

$$p = ce^{-x^2} \tag{23}$$

$$x = ce^{-p'^2} \tag{24}$$

$$x = ce^{-p'^2} \tag{25}$$

$$x/c = e^{-p'^2} \tag{26}$$

$$ln(x/c) = -p'^2 \tag{27}$$

$$p' = (-ln(x/c))^{1/2} \tag{28}$$

$$p'(A^{-\gamma}) = (-ln(A^{-\gamma}/c))^{1/2} \tag{29}$$

$$P_{\leq}(A) = \int_{(-ln(A^{-\gamma}/c))^{1/2}}^{\infty} ce^{-x^2} dx \tag{30}$$

$$= c \int_{0}^{\infty} e^{-x^2} dx \int_{0}^{(-ln(A^{-\gamma}/c))^{1/2}} e^{-x^2} dx \tag{31}$$

$$= c \int_{0}^{(-ln(A^{-\gamma}/c))^{1/2}} e^{-x^2} dx \tag{32}$$

$$= c \times erf \left[-ln \left(\frac{A^{-\gamma}}{c} \right)^{1/2} \right] \tag{33}$$

2 Problem 2

The cost: the expected size of the fire on d-dimensional lattice is

$$C_{fire} \propto \sum_{i=1} N_{sites} p_i a_i$$

The constraint for building and maintining $(d-1)$-dimensional firewalls in d-dimensions is

$$C_{firewalls} \propto \sum_{i=1}^{N_{sites}} a_i^{(d-1)/d} a_i^{-1}$$

3 Problem 3