CSYS 300 PoCS Assignment 7

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Code is located at https://github.com/Evelios/PoCS_Assignment_07

1 Problem 1

Solving for the complamentary cumulative distributions for continuous distributions. The formula used for solving for the complamentary cumulative distribution is,

$$P_{\leq}(A) = \int_{p^{-1}(A^{-\gamma})}^{\infty} p(x)dx = p_{\leq}(p^{-1}(A^{-\gamma})) \tag{1}$$

Here p^{-1} represents the inverse function, so that is the first thing that needs to be found.

(a) Solving for $P_{\leq}(A)$ from $p(x) = cx^{-(q+1)}$

$$p = cx^{-(q+1)} \tag{2}$$

$$x = cp'^{-(q+1)} \tag{3}$$

$$x/c = p'^{-(q+1)} \tag{4}$$

$$p' = (x/c)^{-(\frac{1}{q+1})} \tag{5}$$

$$p'(A^{-\gamma}) = (A^{-\gamma}/c)^{-(\frac{1}{q+1})} \tag{6}$$

$$P_{\leq}(A) = \int_{\frac{A^{-\gamma}}{c}^{-(\frac{1}{q+1})}}^{\infty} cx^{-(q+1)}dx \tag{7}$$

$$= -\frac{c}{q}x^{-q}\Big|_{(A^{-\gamma}/c)^{-(\frac{1}{q+1})}}^{\infty} \tag{8}$$

$$= \left[0 - -\frac{c}{q}\left((A^{-\gamma}/c)^{-(\frac{1}{q+1})}\right)^{-q}\right] \tag{9}$$

$$= \frac{c}{q}\left((A^{-\gamma}/c)^{-(\frac{1}{q+1})}\right)^{-q} \tag{10}$$

$$= \frac{c}{q}\left(\frac{A^{-\gamma}}{c}\right)^{-(\frac{q}{q+1})} \tag{11}$$

$$= \frac{c}{q}\left(\frac{A^{-\gamma}}{c}\right)^{-(1+\frac{1}{q})} \tag{12}$$

(b) Solving for $P_{\leq}(A)$ from $p(x) = ce^{-x}$

$$p = e^{-x} \tag{13}$$

$$x = e^{-p'} \tag{14}$$

$$ln(x) = -p' \tag{15}$$

$$p' = -ln(x) \tag{16}$$

$$p'(A^{-\gamma}) = -ln(A^{-\gamma}) \tag{17}$$

$$P_{\leq}(A) = \int_{-ln(A^{-\gamma})}^{\infty} ce^{-x}dx \tag{18}$$

$$= \left. -ce^{-x} \right|_{-ln(A^{-\gamma})}^{\infty} \tag{19}$$

$$= \left(0 - -ce^{-(-ln(A^{-\gamma}))}\right) \tag{20}$$

$$= ce^{ln(A^{-\gamma})} \tag{21}$$

$$P_{\leq}(A) = cA^{-\gamma} \tag{22}$$

(c) Solving for $P_{\leq}(A)$ from $p(x) = ce^{-x^2}$

$$p = ce^{-x^2} \tag{23}$$

$$x = ce^{-p'^2} \tag{24}$$

$$x = ce^{-p'^2} \tag{25}$$

$$x/c = e^{-p'^2} \tag{26}$$

$$ln(x/c) = -p'^2 \tag{27}$$

$$p' = (-ln(x/c))^{1/2} \tag{28}$$

$$p'(A^{-\gamma}) = (-ln(A^{-\gamma}/c))^{1/2} \tag{29}$$

$$P_{\leq}(A) = \int_{(-ln(A^{-\gamma}/c))^{1/2}}^{\infty} ce^{-x^2} dx \tag{30}$$

$$= c \int_0^{\infty} e^{-x^2} dx \int_0^{(-ln(A^{-\gamma}/c))^{1/2}} e^{-x^2} dx \tag{31}$$

$$= c \int_0^{(-ln(A^{-\gamma}/c))^{1/2}} e^{-x^2} dx \tag{32}$$

$$= c \times erf \left[-ln \left(\frac{A^{-\gamma}}{c} \right)^{1/2} \right] \tag{33}$$

2 Problem 2

The cost: the expected size of the fire on d-dimensional lattice is

$$C_{fire} \propto \sum_{i=1} N_{sites} p_i a_i$$

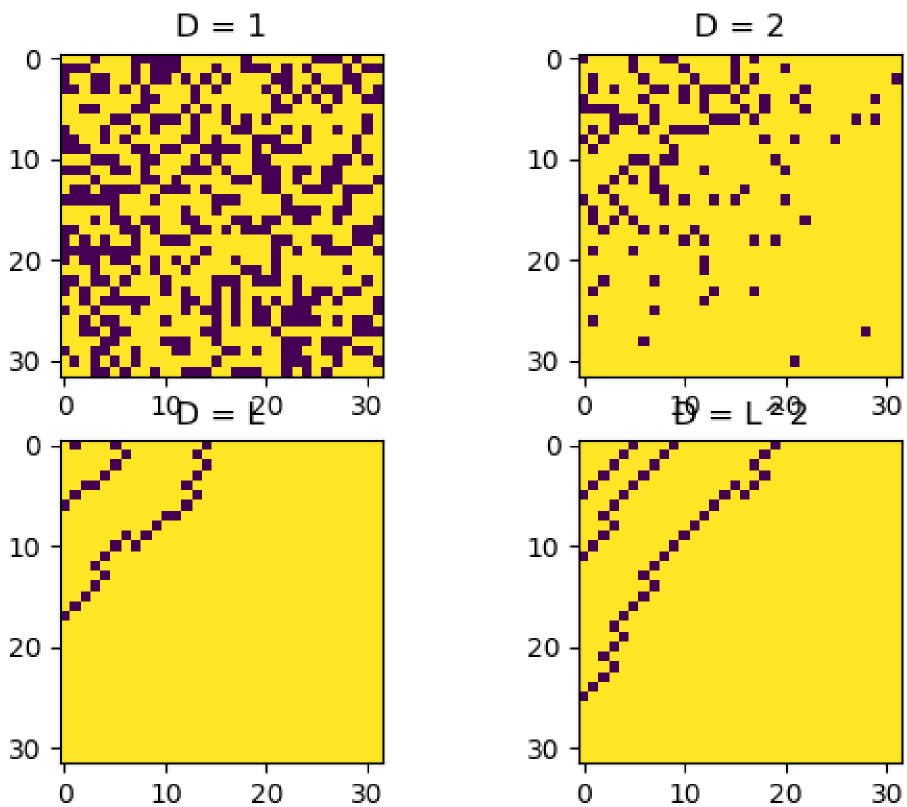
The constraint for building and maintining $(d-1)$-dimensional firewalls in d-dimensions is

$$C_{firewalls} \propto \sum_{i=1}^{N_{sites}} a_i^{(d-1)/d} a_i^{-1}$$

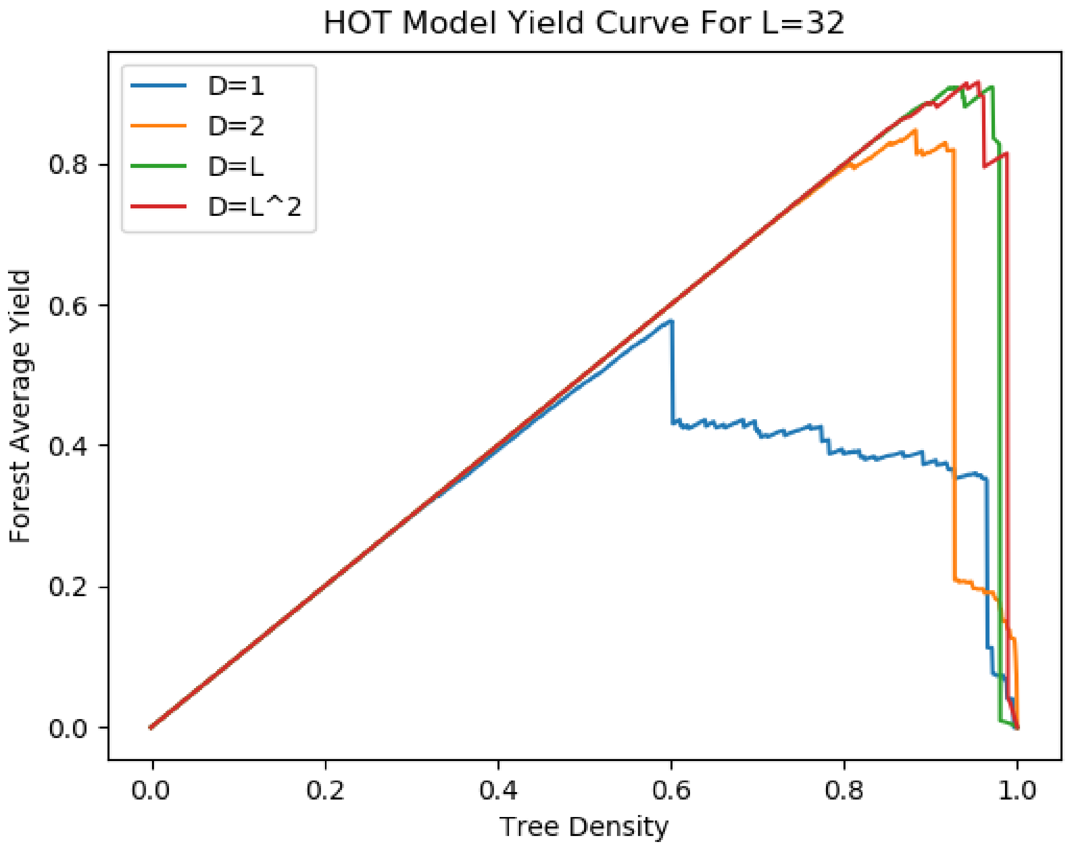
3 Problem 3

I was able to code up the HOT model of the lighning prone square forest farm stead. I have the model running with a grid size of $L = 32$ and with the design parameters of $D = 1, 2, L, L^2$.

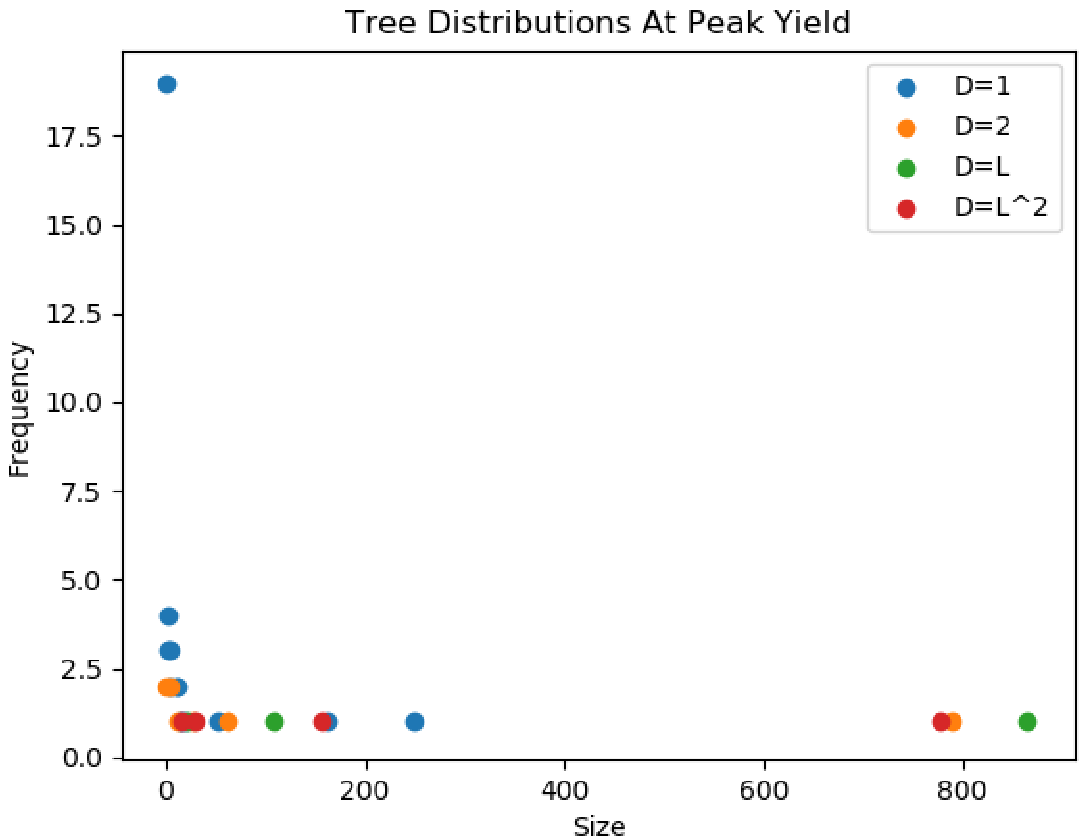
(a) The plot of the forests at approximate peak yield for the simulation



(b) The yield curves for all of the design parameters that were run



(c) The distribution of forest sizes for the different design parameters that were run


Tree Distributions At Peak Yield
D=1
D=2
D=L
D=L^2
Frequency
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
0
200
400
600
800
Size