

# CSYS 300 PoCS Assignment 7

Thomas Waters

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**Code is located at** [https://github.com/Evelios/PoCS\\_Assignment\\_07](https://github.com/Evelios/PoCS_Assignment_07)

# 1 Problem 1

Solving for the complementary cumulative distributions for continuous distributions. The formula used for solving for the complementary cumulative distribution is,

$$P_{\leq}(A) = \int_{p^{-1}(A^{-\gamma})}^{\infty} p(x) dx = p_{\leq}(p^{-1}(A^{-\gamma})) \quad (1)$$

Here  $p^{-1}$  represents the inverse function, so that is the first thing that needs to be found.

(a) Solving for  $P_{\leq}(A)$  from  $p(x) = cx^{-(q+1)}$

$$p = cx^{-(q+1)} \quad (2)$$

$$x = cp'^{-(q+1)} \quad (3)$$

$$x/c = p'^{-(q+1)} \quad (4)$$

$$p' = (x/c)^{-\left(\frac{1}{q+1}\right)} \quad (5)$$

$$p'(A^{-\gamma}) = (A^{-\gamma}/c)^{-\left(\frac{1}{q+1}\right)} \quad (6)$$

$$P_{\leq}(A) = \int_{\frac{A^{-\gamma}}{c}^{-\left(\frac{1}{q+1}\right)}}^{\infty} cx^{-(q+1)} dx \quad (7)$$

$$= -\frac{c}{q} x^{-q} \Big|_{(A^{-\gamma}/c)^{-\left(\frac{1}{q+1}\right)}}^{\infty} \quad (8)$$

$$= \left[ 0 - -\frac{c}{q} \left( (A^{-\gamma}/c)^{-\left(\frac{1}{q+1}\right)} \right)^{-q} \right] \quad (9)$$

$$= \frac{c}{q} \left( (A^{-\gamma}/c)^{-\left(\frac{1}{q+1}\right)} \right)^{-q} \quad (10)$$

$$= \frac{c}{q} \left( \frac{A^{-\gamma}}{c} \right)^{-\left(\frac{q}{q+1}\right)} \quad (11)$$

$$= \frac{c}{q} \left( \frac{A^{-\gamma}}{c} \right)^{-(1+\frac{1}{q})} \quad (12)$$

(b) Solving for  $P_{\leq}(A)$  from  $p(x) = ce^{-x}$

$$p = e^{-x} \quad (13)$$

$$x = e^{-p'} \quad (14)$$

$$\ln(x) = -p' \quad (15)$$

$$p' = -\ln(x) \quad (16)$$

$$p'(A^{-\gamma}) = -\ln(A^{-\gamma}) \quad (17)$$

$$P_{\leq}(A) = \int_{-\ln(A^{-\gamma})}^{\infty} ce^{-x} dx \quad (18)$$

$$= -ce^{-x} \Big|_{-\ln(A^{-\gamma})}^{\infty} \quad (19)$$

$$= \left( 0 - -ce^{-(-\ln(A^{-\gamma}))} \right) \quad (20)$$

$$= ce^{\ln(A^{-\gamma})} \quad (21)$$

$$P_{\leq}(A) = cA^{-\gamma} \quad (22)$$

(c) Solving for  $P_{\leq}(A)$  from  $p(x) = ce^{-x^2}$

$$p = ce^{-x^2} \quad (23)$$

$$x = ce^{-p'^2} \quad (24)$$

$$x = ce^{-p'^2} \quad (25)$$

$$x/c = e^{-p'^2} \quad (26)$$

$$\ln(x/c) = -p'^2 \quad (27)$$

$$p' = (-\ln(x/c))^{1/2} \quad (28)$$

$$p'(A^{-\gamma}) = (-\ln(A^{-\gamma}/c))^{1/2} \quad (29)$$

$$P_{\leq}(A) = \int_{(-\ln(A^{-\gamma}/c))^{1/2}}^{\infty} c e^{-x^2} dx \quad (30)$$

## 2 Problem 2

The cost: the expected size of the fire on  $d$ -dimensional lattice is

$$C_{fire} \propto \sum_{i=1} N_{sites} p_i a_i$$

The constraint for building and maintaining  $(d-1)$ -dimensional firewalls in  $d$ -dimensions is

$$C_{firewalls} \propto \sum_{i=1}^{N_{sites}} a_i^{(d-1)/d} a_i^{-1}$$

## 3 Problem 3