CS 7641: Problem Set 2
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Problem 1:

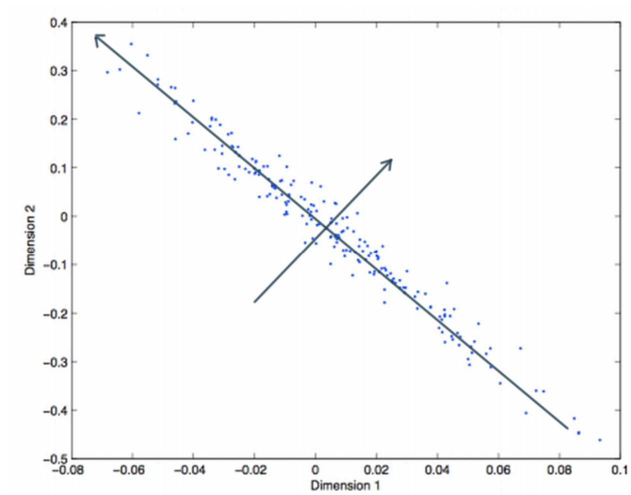
Character	Probability	Encoding (bits)
A	50%	1
B	25%	00
C	25%	01

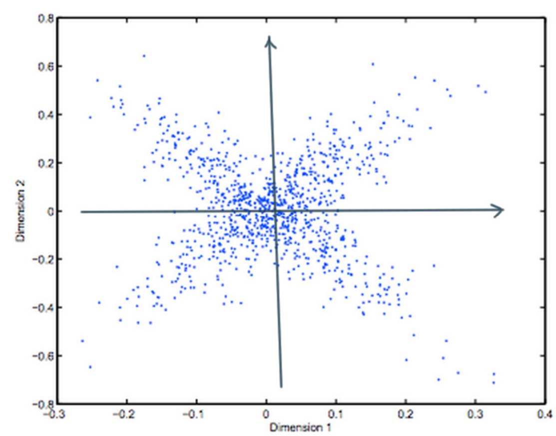
Entropy of the signal = 1*(0.5) + 2*(0.25) + 2*(0.25) = **1.5 bits**

Problem 2:

k-Means using Euclidean distance is a particular case of the EM-Algorithm when we are fitting k-Gaussian distributions with the same variance for each attribute.

Problem 3:





Problem 4:

a) Hierarchical clustering with single link. The clustering cannot be from K-Means or EM because the clustering looks like the nearest points are linked together to form a cluster.
b) K-Means. This looks like two clusters with the mean of the data points as the centroids. The cluster boundaries are well defined and doesn't show any EM like soft clustering.
c) EM. This cluster looks like it was generated by EM because it is able to cluster 2 seemingly overlapping clusters with soft clustering like properties.

Problem 7:

MDP Setup:

States: All states in the grid.
Model: Deterministic transitions
Actions: N, S, E & W

Rewards: +10 for going from 6 to G, +10 for going from 8 to G, 0 for all other transitions.
Need to find the optimal policy.

Value Function: $u(s) = R(s) + \gamma \max_{a} \sum T(s,a,s') * u(s')$

Iteration 0

0.00	0.00	0.00
0.00	0.00	0.00
0.00	0.00	G

Iteration 1

0.00	0.00	0.00
0.00	0.00	10.0
0.00	10.0	G

Iteration 2

0.00	0.00	8.0
0.00	8.0	10.0
8.0	10.0	G

Iteration 3

0.00	6.4	8.0
6.4	8.0	10.0
8.0	10.0	G

Problem 8:

To find the Nash equilibria, we examine each action profile in turn.
(*A*,*B*) Neither player can increase her payoff by choosing an action different from her current one. Thus this action profile is a Nash equilibrium.
(*A*,*B*) By choosing *B* rather than *A*, player 1 obtains a payoff of 1 rather than 0, **given** player 2's action. Thus this action profile is not a Nash equilibrium. [Also, player 2 can increase her payoff by choosing *A* rather than *B*.]
(*B*,*A*) By choosing *A* rather than *B*, player 1 obtains a payoff of 2 rather than 0, **given** player 2's action. Thus this action profile is not a Nash equilibrium. [Also, player 2 can increase her payoff by choosing *B* rather than *A*.]
(*B*,*B*) Neither player can increase her payoff by choosing an action different from her current one. Thus this action profile is a Nash equilibrium.
We conclude that the game has two Nash equilibria, (*A*,*A*) and (*B*,*B*).

To find the Nash equilibria, we examine each action profile in turn.

(A,A) Firm 2 can increase its payoff from 1 to 2 by choosing the action *B* rather than the action *A*. Thus this action profile is not a Nash equilibrium.
(A,B) Firm 1 can increase its payoff from 1 to 2 by choosing the action *B* rather than the action *A*. Thus this action profile is not a Nash equilibrium.
(B,A) Firm 1 can increase its payoff from 1 to 2 by choosing the action *A* rather than the action *B*. Thus this action profile is not a Nash equilibrium.
(B,B) Firm 2 can increase its payoff from 1 to 2 by choosing the action *A* rather than the action *B*. Thus this action profile is not a Nash equilibrium.
We conclude that the game has no Nash equilibrium!

For the last problem,
(T,L) Neither player can increase its payoff by choosing a different action, so this action profile is a Nash equilibrium.
(T,R) Player 1 can increase her payoff from 0 to 1 by choosing the action *B* rather than the action *T*. Thus this action profile is not a Nash equilibrium.
(B,L) Firm 1 can increase its payoff from 0 to 2 by choosing the action *T* rather than the action *B*. Thus this action profile is not a Nash equilibrium.
(B,R) Neither firm can increase its payoff by choosing a different action, so this action profile is a Nash equilibrium.
We conclude that the game has two Nash equilibria, (T,L) and (B,R).