Human:

Birth Rate: a
Natural Death Rate: b
become zombie Rate (infectious): α
Zombie kill human Rate: σ
k: carrying capacity

Zombie:

Birth Rate = Infectious Rate = α
Death Rate (Human Killing Zombie) = β
Naturla Death Rate = c

$dH/dt = aH(1-H/k) - bH - \alpha HZ - \sigma HZ$
$dZ/dt = \alpha HZ - \beta HZ - cZ$

```
                 bH                          cZ
                  ^                          ^
                  |                          |
aH(1-H/k) -> H   - αHZ, σHZ  - > Z - > βHZ
```

Assumption:

Human:
Human birth rate is logistic growth.
Zombie does not die naturally.
total population N = H+Z
Initial condition: (Human Population, Zombie Population) = (H_0, Z_0).
k is the carrying capacity for human, $0 \le H \le k$.
Zombie does not have a carrying capacity, it survives in any environmental condition.
Weapon : pending…

Non-dimensionalization (Optional)
Dimensions:

H,	Z,	t,	a,	b,	α,	β,	σ,	k,	c
N,	N,	T,	1/T,	1/T,	1/TN,	1/TN,	1/TN,	N,	1/T

$h = H/k$
$z = Z/k$
$\tau = at$

$dH/dt = d(h*k)/d(\tau/a) = dh/d\tau * (k*a) = a*h*k(1-h) - b*h*k - \alpha * h * z* k^2 - \sigma * h * z* k^2$
$dZ/dt = d(z *k)/d(\tau/a) = dz/d\tau * (k*a) = \alpha * h * z* k^2 - \beta*h*z*k^2 - c*zk$

$dh/d\tau = h(1-h) - b^* h/a - \alpha^*k/a *hz - \sigma^*k/a *hz$
$dz/d\tau = \alpha^*k/a *hz - \beta k/a^*hz - c/a^*z$

$\gamma = b/a.$
$\mu = \alpha^*k/a$
$\nu = \sigma^*k/a$
$\varepsilon = \beta k/a$
$\omega = c/a$

$dh/d\tau = h(1-h) - \gamma h - \mu hz - \nu hz$
$dz/d\tau = \mu hz - \varepsilon^*hz - \omega z$

k: Let k be the unit for the population size of H and Z.
a: Let a^{-1} be the unit for time.
h: Non-dimensionalized population of human
z: Non-dimensionalized population of zombie
τ : Non-dimensionalized time
γ :ratio between human natural death rate and birth rate
μ : infectious rate per capita per unit time
ν : zombie killing human rate per capita per unit time
ε: human killing zombie rate per capita per unit time
ω : ratio between zombie death rate and human natural birth rate.

Fixed Point

Non dimensionalised

Note: γ = s, μ = m, ν = v, ε **= e,** ω **= w**

Results:

$h = 0$ and $z = 0$

$h = 1 - s$ and $z = 0$

$h = 0$ and $w = 0$

$z = -\frac{h + s - 1}{v + e}$ and $w = 0$ and $m = e$ and $v + e \neq 0$

$h = 1 - s$ and $w = 0$ and $v = -e$ and $m = e$

$h = -\frac{w}{e - m}$ and $z = \frac{m\,(s - 1) - e\,s + w + e}{(e - m)\,(m + v)}$ and $m \neq e$ and $m + v \neq 0$

$h = -\frac{w}{v + e}$ and $v + e \neq 0$ and $s = \frac{v + w + e}{v + e}$ and $m = -v$

1. h = 0, z = 0
2. h = $1 - \gamma$, z = 0
3. $h = -\frac{\omega}{\epsilon - \mu}$, $z = \frac{\mu(\gamma - 1) - \epsilon\gamma + \omega + \epsilon}{(\epsilon - \mu)(\mu + \nu)}$

Jacobian

1. h = 0, z = 0

$$J = \begin{bmatrix} -\gamma + 1 & 0 \\ 0 & -\omega \end{bmatrix}$$

λ_1 = $-\gamma + 1$

λ_2 = $-\omega$

If gamma < 1, saddle node, unstable

If gamma > 1, stable node.

2. h = $1 - \gamma$, z = 0

$$J = \begin{bmatrix} \gamma - 1 & (\gamma - 1)(\mu + \nu) \\ 0 & (1 - \gamma)(\mu - \epsilon) - \omega \end{bmatrix}$$

λ_1 = $\gamma - 1$

λ_2 = $(1 - \gamma)(\mu - \epsilon) - \omega$

1) gamma > 1

   Lambda1 >0

      Lambda2 > 0 Unstable Node

      Lambda2 < 0 Saddle Node Unstable.

2) gamma <1

   Lambda1<0

      Lambda2 > 0 Saddle Node Ustable.

      Lambda2 < 0 Node Stable **STABLE**

      [gamma < 1

         mu < epsilon

         or

         (1-gamma)(mu-epsilon) < omega]

3. $h = -\frac{\omega}{\epsilon - \mu}$, $z = \frac{\mu(\gamma - 1) - \epsilon\gamma + \omega + \epsilon}{(\epsilon - \mu)(\mu + \nu)}$

$$J = \begin{bmatrix} \frac{\omega}{\epsilon-\mu} & \frac{\omega}{\epsilon-\mu}(\mu+\nu) \\ -\frac{\mu(\gamma-1)-\epsilon\gamma+\omega+\epsilon}{\mu+\nu} & 0 \end{bmatrix}$$

$$\lambda_1 = \frac{\omega - \sqrt{\omega^2 - 4(\mu - \epsilon)(-\gamma\mu\omega + \gamma\epsilon\omega + \mu\omega - \epsilon\omega - \omega^2)}}{2(\mu - \nu)}$$

$$\lambda_1 = \frac{\omega + \sqrt{\omega^2 - 4(\mu - \epsilon)(-\gamma\mu\omega + \gamma\epsilon\omega + \mu\omega - \epsilon\omega - \omega^2)}}{2(\mu - \nu)}$$

Imaginary, depend on omega/(mu-epsilon)

omega/(mu-epsilon) > 0, unstable spiral

omega/(mu-epsilon) < 0, stable spiral. **STABLE**

Real, only possible combination for stable:

epsilon < mu:

Stable when Det > 0:

gamma > 1:

(gamma -1)(mu + epsilon) < omega

gamma < 1

epsilon > mu: Unstable

$$Trace = \frac{\omega}{\epsilon - \mu}$$

$$Determinant = \frac{\omega((\gamma - 1)(\mu + \epsilon) - \omega)}{\epsilon - \mu}$$

Stable:

fixed point one (0, 0) is stable when:

gamma > 1

fixed point two (1-gamma, 0) is stable when:

gamma < 1 & mu < epsilon

or

gamma < 1 & mu > epsilon and

(1-gamma)(mu-epsilon) < omega

fixed point three ($h = -\frac{\omega}{\epsilon - \mu}$, $z = \frac{\mu(\gamma - 1) - \epsilon\gamma + \omega + \epsilon}{(\epsilon - \mu)(\mu + \nu)}$) is **stable** when:

epsilon < mu

gamma > 1 (but z < 0) non-existent

or

gamma < 1

only when (gamma-1)(mu-epsilon) < -omega

1. h = 0, z = 0
2. h = 1-gamma, z = 0

$$h = -\frac{\omega}{\epsilon - \mu} \quad z = \frac{\mu(\gamma - 1) - \epsilon\gamma + \omega + \epsilon}{(\epsilon - \mu)(\mu + \nu)}$$

3.

k: Let k be the unit for the population size of H and Z.
a: Let a^{-1} be the unit for time.
h: Non-dimensionalized population of human
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ε : human killing zombie rate per capita per unit time
ω : ratio between zombie death rate and human natural birth rate.

Gamma > 1 Human natural death rate > Birth rate

Fixed point 1 is always stable;
Fixed point 2 is always unstable;
fixed point 3 is non-existent;

[If epsilon < mu Human killing zombie rate < Infectious rate
and (gamma -1)(mu + epsilon) < omega,
and (u(r-1) - er + w + e) < 0 (so that z > 0)
fixed point 3 is stable;
otherwise, fixed point 3 is unstable. Or
Fixed point 3 could be non-existent because z may be < 0]

Gamma < 1 Human natural death rate < Birth rate

Fixed point 1 is always unstable;
If epsilon > mu, Human killing zombie rate > Infectious rate
fixed point 2 is stable;
fixed point 3 is non-existent because h < 0 and z < 0;
If epsilon < mu, Human killing zombie rate < Infectious rate
fixed point 3 is always stable;
fixed point 3 exists when (gamma-1)(mu-epsilon) < -omega
which means (1-gamma)(mu-epsilon) > omega
so fixed point 2 is unstable;
fixed point 3 does not exist when (gamma-1)(mu-epsilon) > -omega
which means (1-gamma)(mu-epsilon) < omega
fixed point 2 is stable.

ε μ

Just fixed point 1:

gamma > 1 and

epsilon > mu

or

epsilon < mu and

(gamma -1)(mu + epsilon) > omega

Just fixed point 2:

gamma < 1

mu < epsilon

Just fixed point 3:

epsilon < mu

gamma < 1 and

(1-gamma)(mu-epsilon) > omega

Fixed point 1 and Fixed point 3:

epsilon < mu

gamma > 1 and

(gamma -1)(mu + epsilon) < omega

Fixed point 2 and Fixed point 3:

gamma < 1

mu > epsilon and

(1-gamma)(mu - epsilon) < omega

gamma

http://valeriecoffman.com/walking-dead-mathematics/

h=0.597315436242, z=0.140939597315

tau = 27.4274274274