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CTII348

Discussão sobre Sistemas Lineares - Sistemas Homogêneos

Discussão sobre Sistemas Lineares

$$\textcircled{1} \begin{cases} ax + 4y = 1 \\ x + 2y = b \end{cases} \quad \left( \begin{array}{cc|c} a & 4 & 1 \\ 1 & 2 & b \end{array} \right)$$

$$\textcircled{2} \xrightarrow{-2} \left( \begin{array}{cc|c} a & 4 & 1 \\ 1 & 2 & 0,5 \end{array} \right) \sim \left( \begin{array}{cc|c} a-2 & 0 & a \end{array} \right) \rightarrow \text{FALSO}$$

$$(a-2)x = 0 \\ x = 0/a-2$$

$$\text{b) } x = \frac{0}{a-2} = \frac{0}{2-2} = \frac{0}{0} \rightarrow \text{Indeterminado} \rightarrow \text{verdadeiro}$$

c) FALSA

d) FALSA

e) FALSA

$$\textcircled{2} \begin{cases} x + Ky = 1 \\ Kx + y = 1-K \end{cases}$$

$$\text{I) } \xrightarrow{-K} \left( \begin{array}{cc|c} 1 & K & 1 \\ K & 1 & 1-K \end{array} \right) \sim \left( \begin{array}{cc|c} & & \\ 0 & 1-2K & 1-K^2 \end{array} \right)$$

$$y = \frac{1-K}{1-2K} = \text{não pode ser indeterminado}$$

$$\text{II) } K=0,5$$

$$y = \frac{1-0,5}{1-2 \cdot 0,5} = \frac{0,5}{0} \rightarrow \text{impossível}$$

III)  $K \neq 0,5 \rightarrow$  S.P. Determinado

(D)  $\rightarrow$  nenhum está correto

$$\textcircled{3} \begin{cases} x + 2y + Cz = 1 \\ y + z = 2 \\ 3x + 2y + 2z = -1 \end{cases}$$

$$2) \begin{array}{ccc|cc} 1 & 2 & C & 1 & 2 \\ 0 & 1 & 1 & 0 & 1 \\ 3 & 2 & 2 & 3 & 2 \end{array} \quad \begin{array}{l} \\ \\ = 8 - 2 \cdot 3C = 6 - 3C \end{array}$$

$2 + 6 = 8$

$$b) \begin{array}{c} 4 \\ 2 \\ -2 \end{array} \left( \begin{array}{ccc|c} 1 & 2 & C & 1 \\ 0 & 1 & 1 & 2 \\ 3 & 2 & 2 & -1 \end{array} \right) \sim \left( \begin{array}{ccc|c} 1 & 0 & C-2 & -3 \\ & 1 & 1 & 2 \\ & 3 & 0 & -5 \end{array} \right) \sim \left( \begin{array}{ccc|c} 1 & 0 & C-2 & -3 \\ & 1 & 1 & 2 \\ 0 & 0 & 3C-6 & -4 \end{array} \right)$$

$$z = 4$$

$$3C - 6 \rightarrow C \cdot 3 \cdot 2 - 6 = 0$$

C tem que ser  $\neq 2$ , para ter uma única solução

$$C \in \mathbb{R} - \{2\}$$

$$\textcircled{4} \begin{cases} x - y = K \\ 12x - Ky + z = -1 \\ 36x + \quad + Kz = 2 \end{cases}$$

$$\begin{array}{l} \rightarrow \left( \begin{array}{ccc|c} 1 & -1 & 0 & K \\ 12 & -K & 1 & 1 \\ 36 & 0 & K & 2 \end{array} \right) \sim \left( \begin{array}{ccc|c} 0 & 12-K & 1 & 12K-1 \\ 0 & 36 & K & 36K-2 \end{array} \right) \end{array}$$

$$0 \quad K^2 - 12K + 36 : 2 - 12K - 37K$$

$$y = \frac{12K^2 - 37K + 2}{K^2 - 12K + 36}$$

$$\begin{cases} 6 + 6 = 12 \\ 6 * 6 = 36 \end{cases}$$

$$K \neq 6$$

(E)

$$\textcircled{5} \begin{cases} x - y + 2 = 6 \\ 2x + y - 2 = -3 \\ x + 2y - 2 = -5 \end{cases}$$

$$\begin{array}{l} \rightarrow \left( \begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 2 & 1 & -1 & -3 \\ 1 & 2 & -1 & -5 \end{array} \right) \sim \left( \begin{array}{ccc|c} 0 & 3 & -3 & -15 \\ 0 & 3 & -2 & -11 \end{array} \right) \sim \left( \begin{array}{ccc|c} 0 & 0 & -1 & -4 \end{array} \right) \end{array}$$

$$x = 1 + 4$$

$$3y - 3 \cdot 4 = -15$$

$$\cdot 2 = 4 \quad \times (-1)$$

$$x = 6 - 5$$

$$3y - 12 = -15$$

$$/ 2 = 4 /$$

$$x = 1 /$$

$$3y = -15 + 12$$

$$3y = -3$$

$$y = -3/3$$

$$y = -1 /$$

(B)

S.P. Determinado

$$x \cdot y \cdot z =$$

$$1 \cdot (-1) \cdot 4 = -4$$

$$(6) \begin{cases} x + y + z = K \\ Kx + y + z = 1 \\ x + y - z = K \end{cases}$$

$$\left| \begin{array}{ccc|c} 1 & 1 & 1 & K \\ -K & 1 & 1 & 1 \\ 1 & 1 & -1 & K \end{array} \right| \sim \left| \begin{array}{ccc|c} 1 & 1 & 1 & K \\ 0 & 1-K & 1-K & 1-K^2 \\ 0 & 0 & -2 & 0 \end{array} \right|$$

$$(1-K)y + (1-K) \cdot 0 = 1-K^2 \quad 2z=0$$

$$(1-K)y = 1-K^2 \quad z=0/2$$

$$y = \frac{1-K^2}{1-K} \rightarrow 1-K^2=0$$

$$1-K=0 \quad \frac{1-K^2}{1-K} \quad K^2=1$$

$$K=1 \quad K=1$$

$K=1 \rightarrow$  S.P. Indetermined (D)

$$(7) \begin{cases} x + y + z = 1 \\ mx - 2y + 4z = 5 \\ m^2x + 4y + 16z = 25 \end{cases}$$

$$\left| \begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ m & -2 & 4 & 5 \\ m^2 & 4 & 16 & 25 \end{array} \right| \sim \left| \begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ m+2 & 0 & 6 & 7 \\ m^2+4 & 0 & 12 & 21 \end{array} \right| \sim \left| \begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ m+2 & 0 & 6 & 7 \\ m^2-2m-8 & 0 & 0 & 7 \end{array} \right|$$

$$m^2-2m-8=0$$

$$x = \frac{7}{m^2-2m-8} = 0$$

$$\Delta = (-2)^2 - 4 \cdot 1 \cdot (-8)$$

$$m^2-2m-8$$

$$\Delta = 4+32$$

$$\Delta = 36$$

$$m_1 = 8/2 = 4$$

$$m_2 = -4/2 = -2$$

$$m_1 + m_2$$

$$m = \frac{2 \pm 6}{2} =$$

$$4 + (-2) =$$

$$2$$

(B)



## Sistemas Homogêneos

$$\textcircled{1} \begin{bmatrix} 1 & 7 \\ 7 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = K \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{cases} x + 7y = Kx \\ 7x + x = Ky \end{cases}$$

$$\begin{pmatrix} 1 & 7 \\ 7 & 1 \end{pmatrix} \begin{pmatrix} x + 7y \\ 7x + y \end{pmatrix} = \begin{pmatrix} xK \\ yK \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 7 & :K \\ 7 & 1 & :K \end{pmatrix} \sim \begin{pmatrix} 0 & -48 & : -6K \end{pmatrix} \rightarrow -48y = 6K$$

$$y = \frac{-6K}{-48} = \frac{K}{8}$$

$$x + 7y = Kx \quad K = 8 \cdot y$$

$$x + 7y = 8y \quad K = 8 \cdot 1$$

$$\frac{x}{8} = 8y - 7y \quad |K = 8|$$

$$1 = y$$

(E)

$$K = 8y \rightarrow y = \frac{K}{8}$$

$$\textcircled{2} \begin{cases} 3x + 4y - z = 0 \\ 2x - y + 3z = 0 \\ x + y = 0 \end{cases}$$

$$\begin{pmatrix} 3 & 4 & -1 & :0 \\ 2 & -1 & 3 & :0 \\ 1 & 1 & 0 & :0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & :0 \\ 2 & -1 & 3 & :0 \\ 3 & 4 & -1 & :0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & :0 \\ 0 & -3 & 3 & :0 \\ 0 & 1 & -1 & :0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & :0 \\ 0 & 1 & -1 & :0 \\ 0 & -3 & 3 & :0 \end{pmatrix}$$

Indeterminado  
(VARIAS soluções)

filas 1  
iguais

$$x+y=0 \quad y=\alpha \quad 2x-y+3z=0$$

$$x+\alpha=0 \quad -2\alpha-\alpha+3z=0$$

$$x=-\alpha \quad -3\alpha+3z=0$$

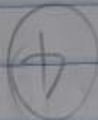
$$3z=3\alpha$$

$$z=\alpha$$

$$z=\alpha$$

$$z=\alpha$$

$$V=\{(-\alpha, \alpha, \alpha)\}$$



$$\textcircled{B} \begin{cases} x+y+z=0 \\ Kx+3y+4z=0 \\ x+Ky+3z=0 \end{cases}$$

$$Kx+3y+4z=0$$

$$x+Ky+3z=0$$

$$3+4K+3K=3+7K$$

$$\begin{vmatrix} 1 & 1 & 1 & 1 & 1 \\ K & 3 & 4 & K & 3 \\ 1 & K & 3 & 1 & K \end{vmatrix}$$

$$K^2-7K+10=0$$

$$9+4+K^2=13+K^2$$

$$13+K^2-(3+7K)$$

$$K^2-7K+10=0$$

$$9+4+K^2=13+K^2$$

$$K^2-7K+10=0$$

$$\Delta=(-7)^2-4 \cdot 1 \cdot 10$$

$$\Delta=49-40$$

$$\Delta=9$$

$$K=\frac{7 \pm \sqrt{9}}{2}$$

$$2.1$$

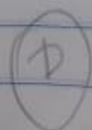
$$K_1+K_2$$

$$5+2=7$$

$$K=\frac{7 \pm 3}{2}$$

$$2$$

$$|K=7|$$



$$K_1=10/2=5$$

$$K_2=4/2=2$$

$$\textcircled{4} \begin{cases} x + K = 0 \\ Kx + y = 0 \\ x + Ky = 0 \end{cases}$$

$$\xrightarrow{-K} \begin{pmatrix} 1 & 0 & K & : & 0 \\ K & 1 & 0 & : & 0 \\ 1 & K & 0 & : & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & K & : & 0 \\ 1-K & 0 & 0 & : & 0 \end{pmatrix}$$

$$x = \frac{0}{1-K^2} \quad \begin{cases} 1-K^2 \neq 0 \\ 1 \neq K^2 \\ K \neq \pm 1 \end{cases} \quad K \neq 0 \rightarrow \text{Solução Única}$$

$$V = \{ (K \in \mathbb{R} / K \neq 0 / K \neq 1 / K \neq -1) \}$$

$$\textcircled{5} \begin{cases} -x + 2y - 3 = 0 \\ 3x - y - 3 = 0 \\ 2x - 2y - 6 = 0 \end{cases}$$

$$\xrightarrow{2} \begin{pmatrix} -1 & 2 & : & 3 \\ 3 & -1 & : & -3 \\ 2 & -2 & : & 6 \end{pmatrix} \sim \begin{pmatrix} 0 & 5 & : & 6 \\ 0 & 0 & : & 0 \end{pmatrix} \rightarrow \begin{cases} 5y = 6 \\ y = 6/5 \end{cases}$$

$$-x + 2 \cdot \frac{6}{5} = 3$$

$$-x + \frac{12}{5} = 3$$

$$-x = 3 - \frac{12}{5}$$

$$-x = \frac{3}{5} \cdot (-1) \rightarrow x = -\frac{3}{5}$$

→ determinante

(B)