

Evelyn Santos de Santana

CTII348

Sistemas Lineares - Regra de Cramer e Escalonamento (Gauss)

Regra de Cramer

① a) $\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$

$$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - 1 = 5$$
$$D_x = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6 - 3 = 3$$
$$D_y = \begin{vmatrix} 2 & 2 \\ -1 & 3 \end{vmatrix} = 6 - 2 = 4$$
$$x = \frac{D_x}{D} = \frac{3}{5} \quad y = \frac{D_y}{D} = \frac{4}{5}$$
$$V = \left\{ \left(\frac{3}{5}, \frac{4}{5} \right) \right\}$$

b) $\begin{cases} 3x - y + 2z = 1 \\ 2x - y + 3z = -1 \\ 4x + y - 2z = 7 \end{cases}$

$$D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} = 3(-2) - 1(-4) + 12 = -6 + 4 + 12 = 10$$
$$D_x = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & -2 \end{vmatrix} = 1(-2) - 1(-2) + 7 = -2 + 2 + 7 = 7$$
$$D_y = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 7 & -2 \end{vmatrix} = 3(-2) - 1(-14) + 12 = -6 + 14 + 12 = 20$$
$$D_z = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 7 \end{vmatrix} = 3(-7) - 1(14) + 7 = -21 - 14 + 7 = -28$$
$$x = \frac{D_x}{D} = \frac{7}{10} \quad y = \frac{D_y}{D} = \frac{20}{10} = 2 \quad z = \frac{D_z}{D} = \frac{-28}{10} = -\frac{14}{5}$$
$$V = \left\{ \left(\frac{7}{10}, 2, -\frac{14}{5} \right) \right\}$$

$$\textcircled{2} \begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases}$$

$$-5 \cdot 12 + 36 = 19$$

$$-12 + 48 + 12 = 48$$

$$D = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix}$$

$$4 \cdot 5 = 59 - 19 = 30$$

$$1 \cdot 2 \cdot 3 = 1 \cdot 2$$

$$45 + 6 + 8 = 59$$

$$D_y = \begin{vmatrix} 3 & 1 & -1 \\ 4 & 12 & 2 \\ 1 & 8 & 3 \end{vmatrix}$$

$$4 \cdot 12 \cdot 2 = 78 - 48 = 30$$

$$1 \cdot 8 \cdot 3 = 1 \cdot 8$$

$$108 + 2 - 32 = 78$$

$$y = \frac{D_y}{D} = \frac{30}{30} = 1$$

(A)

$$\textcircled{3} \begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = 2 \\ 2x + 3y - z = 1 \end{cases}$$

$$2 - 33 - 6 = -37$$

$$1 - 33 + 4 = -28$$

$$D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{vmatrix}$$

$$3 \cdot 1 \cdot (-11) = -36 - (-37) = 1$$

$$2 \cdot 3 \cdot (-1) = 2 \cdot 3$$

$$-1 - 44 + 9 = -36$$

$$-4 - 11 - 3 = -18$$

$$D_x = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & -11 \\ 1 & 3 & -1 \end{vmatrix}$$

$$2 \cdot 1 \cdot (-11) = -29 - (-28) = 1$$

$$1 \cdot 3 \cdot (-1) = 2 \cdot 3$$

$$-1 - 22 - 6 = -29$$

$$2 - 6 + 6 = 2$$

$$D_y = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & -11 \\ 2 & 1 & -1 \end{vmatrix}$$

$$3 \cdot 2 \cdot (-11) = -17 - (-18) = 1$$

$$2 \cdot 1 \cdot (-1) = 2 \cdot 1$$

$$12 - 22 + 3 = -17$$

$$D_z = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$3 \cdot 1 \cdot (-2) = -2 - 2 = 0$$

$$2 \cdot 3 \cdot 1 = 2 \cdot 3$$

$$1 - 8 + 9 = 2$$

$$x = \frac{D_x}{D} = \frac{-1}{-1} = 1$$

$$y = \frac{D_y}{D} = \frac{1}{1} = 1$$

$$z = \frac{D_z}{D} = \frac{0}{1} = 0$$

$$SOMA = 1 + 1 + 0 = 2$$

(C)

$$\textcircled{4} \begin{cases} x+2y-3z=29 \\ x+3y+2z=4 \\ x-y-2z=8 \end{cases}$$

$$-9-2-4=-15$$

$$-72-58-16=-146$$

$$D = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix}$$

$$13 = 1 - (-15) = 16$$

$$1-1-2 = -1$$

$$-6+4+3=1$$

$$-12+16-58=-54$$

$$D_x = \begin{vmatrix} 29 & 2 & -3 \\ 4 & 3 & 2 \\ 8 & -1 & -2 \end{vmatrix}$$

$$43 = -130 - (-146) = 16$$

$$8-1-2 = 5$$

$$-174+32+12=-130$$

$$87-4+16=99$$

$$D_y = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix}$$

$$14 = 26 - (-54) = 80$$

$$1-8-2 = -9$$

$$-8+58-24=26$$

$$D_z = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix}$$

$$13 = 3 - 99 = -96$$

$$1-1-8 = -8$$

$$24+8-29=3$$

$$x = \frac{D_x}{D} = \frac{16}{16} = 1 \quad y = \frac{D_y}{D} = \frac{80}{16} = 5 \quad z = \frac{D_z}{D} = \frac{-96}{16} = -6$$

$$SOMA = 1+5-6=0$$

(A)

$$\textcircled{5} \begin{cases} 2x+y=5 \\ 2y+z=3 \\ 3x+2y+z=7 \end{cases}$$

$$0+4+0=4$$

$$0+10+3=13$$

$$D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$02 = 7-4=3$$

$$32-1 = 5$$

$$4+3+0=7$$

$$D_x = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{vmatrix}$$

$$32 = 17-13=4$$

$$72-1 = 5$$

$$10+7+0=17$$

$$\begin{array}{l}
 0+14+0=14 \\
 D_1 = \begin{array}{|ccc|c} 2 & 5 & 0 & 25 \\ \hline 0 & 3 & 1 & 3 \\ 3 & 7 & 1 & 37 \end{array} \\
 3=21-14=7 \\
 6+15+0=21 \\
 \end{array}
 \qquad
 \begin{array}{l}
 30+12+0=42 \\
 D_2 = \begin{array}{|ccc|c} 2 & 1 & 5 & 21 \\ \hline 0 & 2 & 3 & 2 \\ 3 & 2 & 7 & 32 \end{array} \\
 2=37-42=-5 \\
 28+9+0=37
 \end{array}$$

$$x = \frac{D_1}{D} = \frac{7}{3} \qquad y = \frac{D_2}{D} = \frac{7}{3} \qquad z = \frac{D_3}{D} = \frac{5}{3}$$

$$V = \left\{ \frac{14}{3}, \frac{7}{3}, \frac{5}{3} \right\}$$

$$\textcircled{6} \quad \begin{array}{|ccc|c} 1 & 0 & 0 & x \\ \hline 2 & 1 & 0 & y \\ 1 & 2 & 2 & z \end{array} \quad \begin{array}{l} x = 3 \\ y = 7 \\ z = -1 \end{array}$$

$$\begin{array}{c} x \\ y \\ z \end{array}$$

$$\begin{cases} 1x = 3 \\ 2x + 1y = 7 \\ -1x + 2y + 2z = 1 \end{cases}$$

$$\begin{array}{|ccc|ccc} 1 & 0 & 0 & 1x & y & z \\ \hline 2 & 1 & 0 & 2x & 1y & z \\ -1 & 2 & 2 & -1x & 2y & 2z \end{array}$$

$$\begin{array}{l}
 0+0+0 \\
 D_1 = \begin{array}{|ccc|c} 1 & 0 & 0 & 10 \\ \hline 2 & 1 & 0 & 21 \\ 1 & 2 & 2 & 12 \end{array} \\
 2=20-10=10 \\
 2+0+0=2 \\
 0+0+0
 \end{array}
 \qquad
 \begin{array}{l}
 0+0+0=0 \\
 D_2 = \begin{array}{|ccc|c} 3 & 0 & 0 & 30 \\ \hline 7 & 1 & 0 & 71 \\ 1 & 2 & 2 & 12 \end{array} \\
 7=60-50=10 \\
 6+0+0=6 \\
 -3+11+0=8
 \end{array}$$

$$\begin{array}{l}
 D_3 = \begin{array}{|ccc|c} 1 & 3 & 0 & 30 \\ \hline 2 & 7 & 0 & 71 \\ 1 & 2 & 2 & 12 \end{array} \\
 7=14-0=14 \\
 14+0+0=14
 \end{array}
 \qquad
 \begin{array}{l}
 D_4 = \begin{array}{|ccc|c} 1 & 0 & 3 & 10 \\ \hline 2 & 1 & 7 & 21 \\ 1 & 2 & 1 & 12 \end{array} \\
 2=11-11=0 \\
 -1+0+12=11
 \end{array}$$

$$x = \frac{dx}{dt} = 6 = 3 \quad y = \frac{dy}{dt} = 14 = 7 \quad z = \frac{dz}{dt} = 0 = 0$$

0 2

0 2

0 2

(E)

Escalonamento (Gass)

$$\textcircled{1} \begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{cases}$$

$$2x - y - 3z = -5$$

$$7x - 102 = -4$$

$$252 = -25$$

$$2, (-2) - y - 3, (-1) = -5$$

$$7x - 10(-1) = -24$$

$$Z = 25/25$$

$$-4 - y + 3 = -5$$

$$7x + 10 = -4$$

$$Z = -1$$

$$-4 + 3 + 5 = y$$

$$7x = -4 - 10$$

$$Y = A$$

$$7x = -14$$

$$V = \{(-2, 4, -1)\}$$

② $\begin{cases} x = 2y \\ 2y = 3z \\ x + y + z = 11 \end{cases} \rightarrow \begin{cases} 2y = 3z \\ |z = 2y/3| \end{cases} \rightarrow |x = 2y|$

$$x + y + z = 11$$

$$x = 2y$$

$$2y = 3z$$

$$24 + 4 + 2y/3 = 11 \quad (3)$$

$$x = 2.3$$

$$2.3 = 32$$

$$64 + 34 + 24 = 33$$

$$17\pi = 61$$

$G = 32$

$$114^{\circ} = 33^{\circ}$$

$$\underline{Z = 6/3}$$

$$1A = 33/11$$

12 = 2

$$2 = 3$$

$$x + 2y + 3z = 9$$

$$6 + 6 + 6 = 18$$

(B)

$$\textcircled{3} \begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6x + 3z = -12 \end{cases}$$

$$\begin{array}{c} -2 \\ \downarrow \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -1 & -2 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & -4 & 1 \\ 0 & 6 & 3 & -12 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & -4 & 1 \\ 0 & 0 & -5 & -10 \end{array} \right) \downarrow$$

\textcircled{D}

$$\begin{aligned} -5 \cdot 2 &= -10 \\ z &= 10 / -5 \\ z &= -2 \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad A &= \text{Ali} \\ B &= \text{Bua} \\ C &= \text{Caca} \end{aligned} \quad A + B + C = 68$$

$$B + 20\% \cdot C = A \rightarrow B + 0,2C = A$$

$$C + 20\% \cdot A = 3B \rightarrow C + 0,2A = 3B$$

$$A + B + C = 68 \quad (\text{I})$$

$$B + 0,2C = A \quad (\text{II})$$

$$0,2A + C = 3B \quad (\text{III})$$

$$\text{I} \rightarrow \text{II} \rightarrow (B + 0,2C) + B + C = 68$$

$$2B + 1,2C = 68$$

$$B = 68 - 1,2C$$

2

$$B = 34 - 0,6C \quad \downarrow$$

$$\begin{aligned} \text{III} \rightarrow 0,2A+C &= 3(34-0,6C) \\ 0,2A+C &= 102-1,8C \\ 0,2A+2,8C &= 102 + \text{II} \end{aligned} \quad \begin{aligned} 0,2(B+0,2C)+2,8C &= 102 \\ 0,2B+0,4C+2,8C &= 102 \\ 0,2B+2,84C &= 102 \end{aligned}$$

$$0,2(34-0,6C)+2,84C=102$$

$$6,8-0,12C+2,84C=102$$

$$6,8+2,72C=102$$

$$2,72C=102-6,8$$

$$2,72C=95,2$$

$$C=95,2$$

$$2,72$$

$$C=35$$

$$B=34-0,6C$$

$$B=34-0,6 \cdot 35$$

$$B=34-21$$

$$B=13$$

$$A+B+C=68$$

$$A+13+35=68$$

$$A+48=68$$

$$A=68-48$$

$$A=20$$

Ali tem R\$ 20,00

Bia tem R\$ 13,00

Caco tem R\$ 35,00

Caco - Ali ?

$$35-20=15$$

Ali tem R\$ 15,00 a

menos que Caco

$$5) \quad x = \text{Ali} = 134$$

$$y = \text{Bento} = 115$$

$$z = \text{Cynthia} = 48$$

$$A = \begin{bmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} x & -134 \\ y & -115 \\ z & -48 \end{bmatrix}$$

$$\begin{cases} 3y+4z=134 \\ x+5z=115 \\ 2x+y=48 \end{cases}$$

$$\begin{array}{c|c|c} 0 & 3 & 4 : 134 \\ \hline 2. & 1 & 0 & 5 : 115 \\ \hline 4. & 2 & 1 & 0 : 48 \end{array} \quad \begin{array}{c|c|c} 1. & 0 & 3 & 4 : 134 \\ \hline \sim & & & : \\ \hline 3. & 0 & 1 & 10 : 182 \end{array} \quad \begin{array}{c|c|c} & & : \\ \hline \sim & & : \\ \hline & 0 & 0 & 3 & 4 : 680 \end{array}$$

$$3y + 42 = 134$$

$$3y + 4 \cdot 20 = 134$$

$$3y + 80 = 134$$

$$3y = 134 - 80$$

$$3y = 54$$

$$y = 54/3$$

$$y = 18$$

$$x + 5z = 115$$

$$x + 5 \cdot 20 = 115$$

$$x + 100 = 115$$

$$x = 115 - 100$$

$$x = 15$$

$$34z = 680$$

$$z = 680/34$$

$$z = 20$$

$$x + y + z = ?$$

$$15 + 18 + 20 = 53$$

(A)

R\$ 53,00