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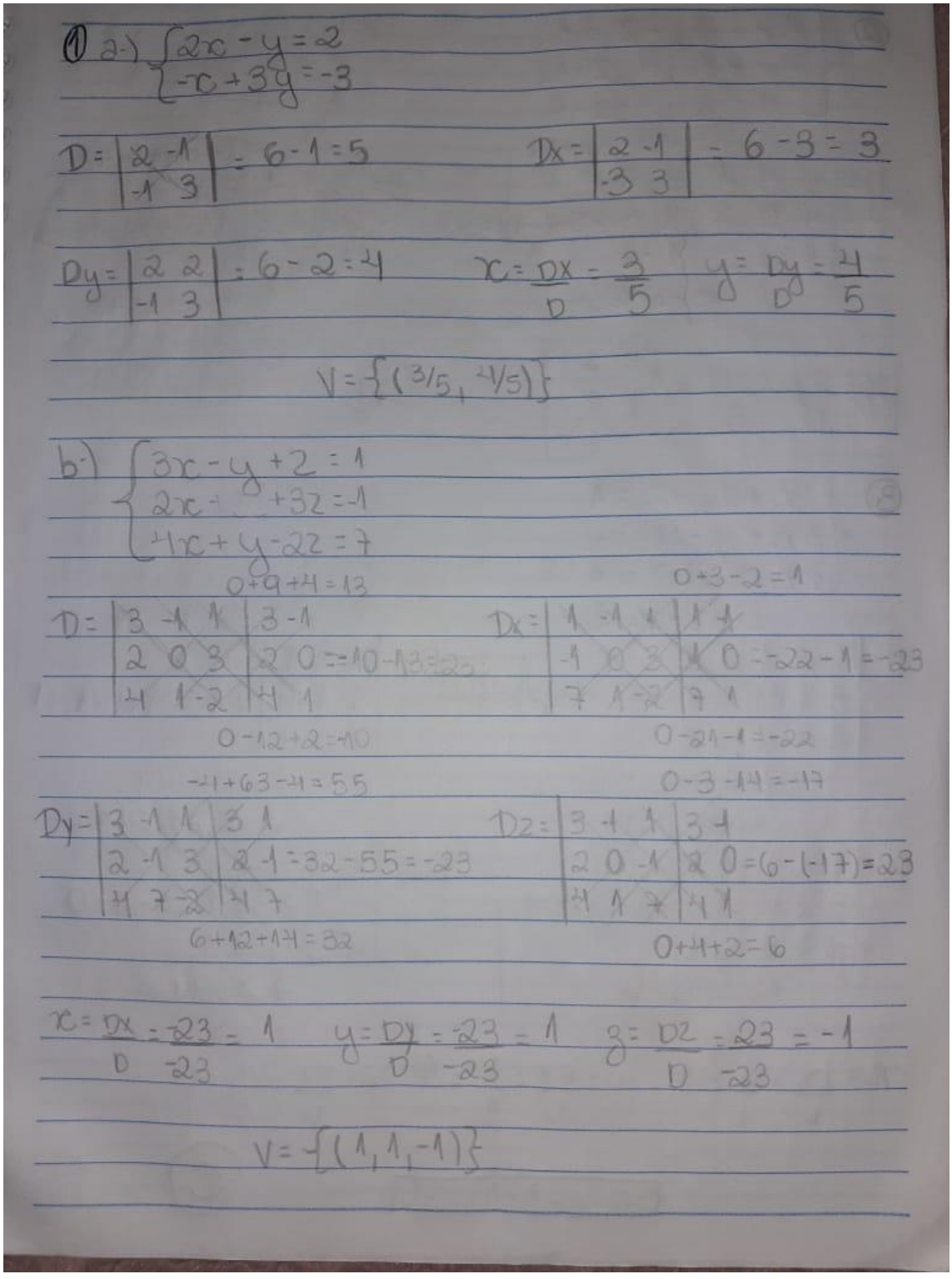
CTII348

Sistemas Lineares - Regra de Cramer e Escalonamento (Gass)

Regra de Cramer



② $\begin{cases} 3x+4y-z=1 \\ 4x+5y+2z=12 \\ x-2y+3z=8 \end{cases}$

$D = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix}$ $\begin{matrix} 3 & 4 \\ 4 & 5 \\ 1 & -2 \end{matrix}$ $4\cdot5 = 59 - 19 = 30$

$-5 - 12 + 36 = 19$

$45 + 6 + 8 = 59$

$D_y = \begin{vmatrix} 3 & 1 & -1 \\ 4 & 12 & 2 \\ 1 & 8 & 3 \end{vmatrix}$ $\begin{matrix} 3 & 1 \\ 4 & 12 \\ 1 & 8 \end{matrix}$ $12\cdot12 = 78 - 48 = 30$

$-12 + 48 + 12 = 48$

$108 + 2 - 32 = 78$

$y = \frac{D_y}{D} = \frac{30}{30} = 1$ (A)

③ $\begin{cases} x+2y+z=1 \\ 3x+y-11z=2 \\ 2x+3y-z=1 \end{cases}$

$D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{vmatrix}$ $\begin{matrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{matrix}$ $3\cdot1 = -36 - (-37) = 1$

$2 - 33 - 6 = -37$

$-1 - 44 + 9 = -36$

$-4 - 11 - 3 = 18$

$D_x = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & -11 \\ 1 & 3 & -1 \end{vmatrix}$ $\begin{matrix} 1 & 2 \\ 2 & 1 \\ 1 & 3 \end{matrix}$ $3\cdot1 = -29 - (-28) = -1$

$1 - 33 + 4 = -28$

$-1 - 22 - 6 = -29$

$2 - 6 + 6 = 2$

$D_y = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & -11 \\ 2 & 1 & -1 \end{vmatrix}$ $\begin{matrix} 1 & 1 \\ 3 & 2 \\ 2 & 1 \end{matrix}$ $3\cdot2 = -17 - (-18) = 1$

$12 - 22 + 3 = -17$

$D_z = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$ $\begin{matrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{matrix}$ $3\cdot1 = 2 - 2 = 0$

$1 - 8 + 9 = 2$

$x = \frac{D_x}{D} = \frac{-1}{1} = -1$ $\quad y = \frac{D_y}{D} = \frac{1}{1} = 1$ $\quad z = \frac{D_z}{D} = \frac{0}{1} = 0$

SOMA $= -1 + 1 + 0 = 0$ (C)

④ $\begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases}$

$-9 - 2 - 4 = -15$

$D = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix} \begin{matrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{matrix}$ $\quad 1 \cdot 3 = 1 - (-15) = 16$

$-6 + 4 + 3 = 1$

$-12 + 16 - 58 = -54$

$D_x = \begin{vmatrix} 29 & 2 & -3 \\ 4 & 3 & 2 \\ 8 & -1 & -2 \end{vmatrix} \begin{matrix} 29 & 2 \\ 4 & 3 \\ 8 & -1 \end{matrix}$ $\quad 4 \cdot 3 = -130 - (-146) = 16$

$-72 - 58 - 16 = -146$

$-174 + 32 + 12 = -130$

$D_y = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix} \begin{matrix} 1 & 29 \\ 1 & 4 \\ 1 & 8 \end{matrix}$ $\quad 1 \cdot 4 = 26 - (-54) = 80$

$-8 + 58 - 24 = 26$

$87 - 4 + 16 = 99$

$D_z = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix} \begin{matrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{matrix}$ $\quad 1 \cdot 3 = 3 - 99 = -96$

$24 + 8 - 29 = 3$

$x = \frac{D_x}{D} = \frac{16}{16} = 1 \qquad y = \frac{D_y}{D} = \frac{80}{16} = 5 \qquad z = \frac{D_z}{D} = \frac{-96}{16} = -6$

$\boxed{SOMA = 1 + 5 - 6 = 0}$ (A)

⑤ $\begin{cases} 2x + y = 5 \\ 2y + z = 3 \\ 3x + 2y + z = 7 \end{cases}$

$0 + 4 + 0 = 4$

$D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} \begin{matrix} 2 & 1 \\ 0 & 2 \\ 3 & 2 \end{matrix}$ $\quad 2 \cdot 2 = 7 - 4 = 3$

$4 + 3 + 0 = 7$

$0 + 10 + 3 = 13$

$D_x = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{vmatrix} \begin{matrix} 5 & 1 \\ 3 & 2 \\ 7 & 2 \end{matrix}$ $\quad 3 \cdot 2 = 17 - 13 = 4$

$10 + 7 + 0 = 17$

0+14+0=14

$D_y = \begin{vmatrix} 2 & 5 & 0 & 2 & 5 \\ 0 & 3 & 1 & 0 & 3 \\ 3 & 7 & 4 & 3 & 7 \end{vmatrix}$ 3 = 21 - 14 = 7

6+15+0=21

30+12+0=42

$D_z = \begin{vmatrix} 2 & 1 & 5 & 2 & 1 \\ 0 & 2 & 3 & 0 & 2 \\ 3 & 2 & 7 & 3 & 2 \end{vmatrix}$ 2 = 37 - 42 = -5

28+9+0=37

$x = \frac{Dx}{D} = \frac{4}{3}$ $y = \frac{Dy}{D} = \frac{7}{3}$ $z = \frac{Dz}{D} = \frac{5}{3}$ (D)

$V = \{(4/3, 7/3, -5/3)\}$

(6) $\begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 2 \end{vmatrix} \cdot \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 3 \\ 7 \\ -1 \end{vmatrix}$

$\begin{cases} 1x & = 3 \\ 2x & 1x & = 7 \\ -1x & 2y & 2z = 1 \end{cases}$

$\begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{vmatrix}$ $\begin{matrix} 1x & 0y & 0z \\ 2x & 1y & 0z \\ -1x & 2y & 2z \end{matrix}$

0+0+0

$D = \begin{vmatrix} 1 & 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 & 1 \\ -1 & 2 & 2 & -1 & 2 \end{vmatrix}$ 2 = 2 - 0 = 2

2+0+0=2

0+0+0=0

$D_x = \begin{vmatrix} 3 & 0 & 0 & 3 & 0 \\ 7 & 1 & 0 & 7 & 1 \\ 1 & 2 & 2 & -1 & 2 \end{vmatrix}$ 6 + 0 = 6

6+0+0=6

0+0+0

$D_y = \begin{vmatrix} 1 & 3 & 0 & 1 & 3 \\ 2 & 7 & 0 & 2 & 7 \\ -1 & -1 & 2 & -1 & -1 \end{vmatrix}$ 14 - 0 = 14

14+0+0=14

-3+14+0=11

$D_z = \begin{vmatrix} 1 & 0 & 3 & 1 & 0 \\ 2 & 1 & 7 & 2 & 1 \\ -1 & 2 & -1 & -1 & 2 \end{vmatrix}$ 11 - 11 = 0

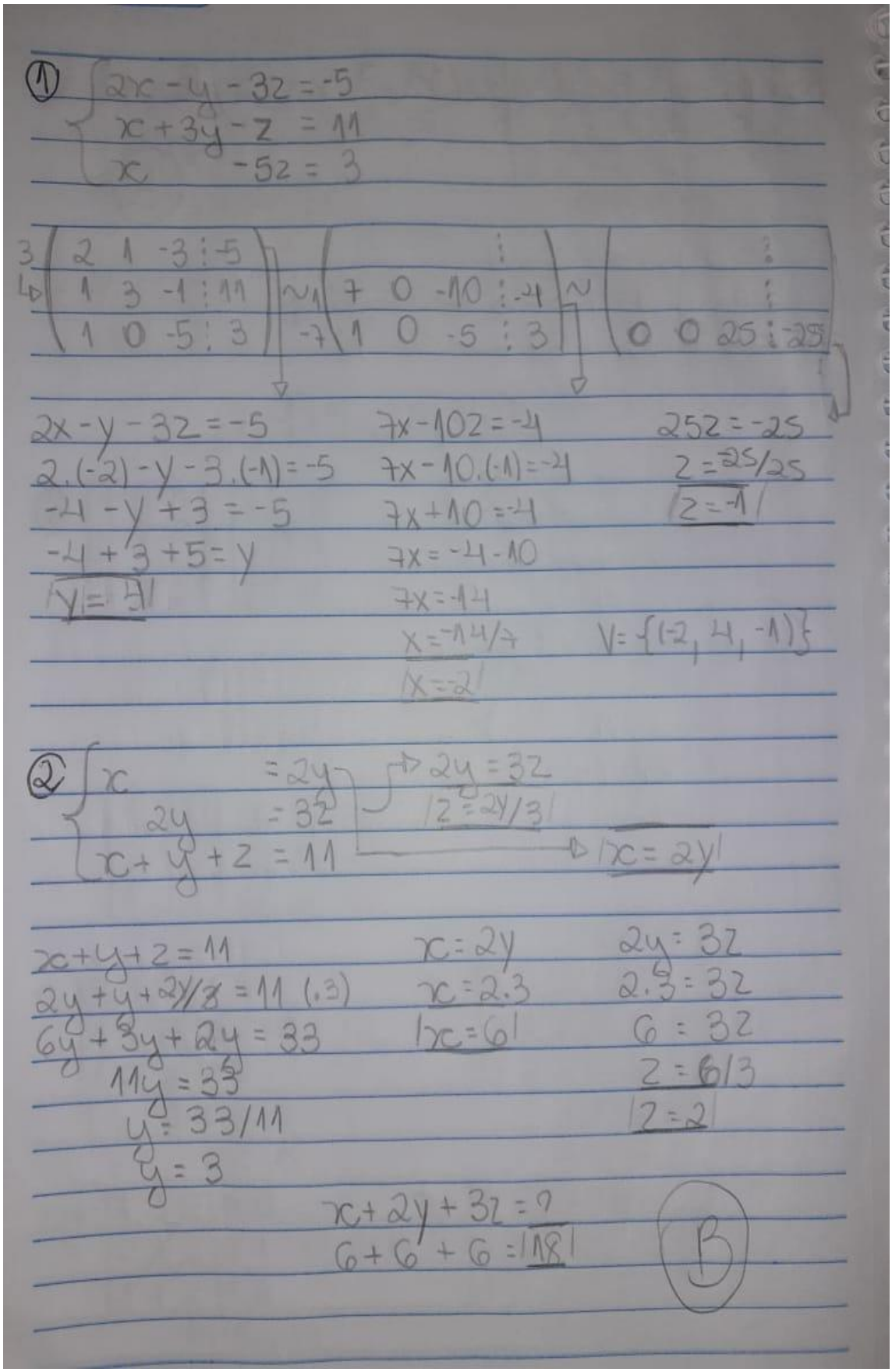
-1+0+12=11

11

$x = \frac{Dx}{D} = \frac{6}{2} = 3 \quad y = \frac{Dy}{D} = \frac{14}{2} = 7 \quad z = \frac{Dz}{D} = \frac{0}{2} = 0$

(E)

Escalonamento (Gass)



③ $\begin{cases} x+y+z=0 \\ 2x-y-2z=1 \\ 6y+3z=-12 \end{cases}$

$$\begin{pmatrix} 1 & 1 & 1 & : & 0 \\ 2 & -1 & -2 & : & 1 \\ 0 & 6 & 3 & : & -12 \end{pmatrix} \xrightarrow{-2} \sim \begin{pmatrix} & & & : & \\ 0 & -3 & -4 & : & 1 \\ 0 & 6 & 3 & : & -12 \end{pmatrix} \sim \begin{pmatrix} & & & : & \\ & & & : & \\ 0 & 0 & -5 & : & -10 \end{pmatrix}$$

$-5.z = -10$

$z = -10 / -5$

$z = 2$

(D)

④ A = Al

B = Bia

C = Caco

$A+B+C=68$

$B + 20/100 . C = A \rightarrow B + 0,2C = A$

$C + 20/100 . A = 3B \rightarrow C + 0,2A = 3B$

$A+B+C=68$ (I)

$B+0,2=A$ (II)

$0,2+AC=3B$ (III)

$I \rightarrow II \rightarrow (B+0,2C)+B+C=68$

$2B+1,2C=68$

$B=\frac{68-1,2C}{2}$

$B=34-0,6C$

/ /

III → $0,2A+C=3.(34-0,6C)$

$0,2A+C=102-1,8C$

$0,2A+2,8C=102$ ← II

→ $0,2(B+0,2C)+2,8C=102$

$0,2B+0,4C+2,8C=102$

$0,2B+2,84=102$

$0,2(34-0,6C)+2,84C=102$

$6,8-0,12C+2,84C=102$

$6,8+2,72=102$

$2,72C=102-6,8$

$2,72C=95,2$

$C=\frac{95,2}{2,72}$

$\boxed{C=35}$

$B=34-0,6C$

$B=34-0,6.35$

$B=34-21$

$\boxed{B=13}$

$A+B+C=68$

$A+13+35=68$

$A+48=68$

$A=68-48$

$\boxed{A=20}$ (A)

Ali tem R$ 20,00

Bia tem R$ 13,00

Caco tem R$ 35,00

Caco - Ali?

$35-20=\boxed{15}$

Ali tem R$ 15,00 a menos que Caco (A)

(5) x = Alpeu = 134

y = Bento = 115

z = Cinthia = 48

$$A=\begin{vmatrix}0&3&4\\1&0&5\\2&1&0\end{vmatrix} \qquad X=\begin{vmatrix}x&-134\\y&-115\\z&-48\end{vmatrix}$$

$$\begin{cases}3y+4z=134\\x+5z=115\\2x+y=48\end{cases}$$

	0	3	4	:	134	1.	0	3	21	:	134			:
2.	1	0	5	:	115	~				:		~		:
4.	2	1	0	:	48	3.	0	1	10	:	182		0	0 34 : 680

$3y + 42 = 134$

$3y + 4.20 = 134$

$3y + 80 = 134$

$3y = 134 - 80$

$3y = 54$

$y = 54/3$

$y = 18$

$x + 52 = 115$

$x - 5.20 = 115$

$x + 100 = 115$

$x = 115 - 100$

$x = 15$

$342 = 680$

$z = 680/34$

$z = 20$

$x + y + z = ?$

$15 + 18 + 20 = 53$

R$ 53,00

(A)