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CTII348

Matriz Inversa

1) $A = \begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix}$

$\begin{bmatrix} x & 1 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ y & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$15 + 3y = 0$ $-x + 2 = 0$ $3x + y = 1$

$3y = -15$ $-x = -2$ $-x + 2 = 0$

$y = -15/3$ $\boxed{x = 2}$ $15 + 3y = 0$

$\boxed{y = -5}$ $-5 + 6 = 1$

C

2) $1 + 3k + 0$

$\begin{vmatrix} 1 & 0 & 1 \\ k & 1 & 3 \\ 1 & k & 3 \end{vmatrix} \begin{matrix} 1 & 0 \\ k & 1 \\ 1 & k \end{matrix} = 3 + K^2 - (1 + 3K) = 0$

$3 + k^2$

$K^2 - 3K + 2 = 0$

$\Delta = (-3)^2 - 4 \cdot 1 \cdot 2$

$\Delta = 9 - 8$

$\Delta = 1$

$K = \dfrac{3 \pm \sqrt{1}}{2 \cdot 1} = \dfrac{3 \pm 1}{2} = K_1 = 4/2 = 2$; $K_2 = 2/2 = 1$

C

Os valores são: 1 e 2.

3) $A = \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} \xrightarrow{\text{INVERSA}} B = ?$

$\begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix} = 12 - 10 = 2$

$det = 2$

$$\text{ORDEM } 2: \begin{bmatrix} 4 & -5 \\ -2 & 3 \end{bmatrix} \div 2 \longrightarrow B = \begin{bmatrix} 2 & -5/2 \\ -1 & 3/2 \end{bmatrix}$$ C

4

$$20+2x+3x = 20+5x$$

$$\left[\begin{array}{ccc|cc} x & 1 & 2 & x & 1 \\ 3 & 1 & 2 & 3 & 1 \\ 10 & 1 & x & 10 & 1 \end{array}\right] \neq 10$$

$$x^2+20+6 = x^2+26$$

$$x^2+26-(20+5x) \neq 0$$

$$x^2-5x+6 \neq 0$$

$$x^2-5x+6=0$$

$$\Delta = (-5) - 4.1.6$$

$$\Delta = 25-24$$

$$\Delta = 1$$

$$x \neq \frac{5 \pm \sqrt{1}}{2.1} \neq \frac{5 \pm 1}{2}$$

$$x_1 \neq 6/2 = 3$$

$$x_2 \neq 4/2 = 2$$

A

5

$$\begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} \to A' = ?$$

SOMA = $A + A^{-1}$

$$2+2+2=6$$

$$\left[\begin{array}{ccc|cc} -1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & 2 & 1 \\ 1 & 1 & -1 & 1 & 1 \end{array}\right] = 7-6 = 1$$

$$1+2+4=7$$

$$A' = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} \div 1$$

$$A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

MATRIZ SOMA = $\begin{bmatrix} -1 & -1 & 2 \\ 2 & 1 & -2 \\ 1 & 1 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$

(B)

(6) A e B → inversíveis de mesma ORDEM

X é $(X.A)^t = B$ $\qquad x = ?$

$((X.A)^T)^T = B$

$X.A = B^T$

$X.\underbrace{A.A^{-1}}_{In} = B^T.A^{-1}$

$\boxed{X = B^T.A^{-1}}$

(B)

(7) $B = \begin{bmatrix} x \\ y \end{bmatrix}$ $\quad C = \begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix}$ → A^{-1} → $A.B = C$

$A.B = C$

$A = C/B$ → $\begin{bmatrix} 4x + 5y \\ 5x + 6y \end{bmatrix} \div \begin{bmatrix} x \\ y \end{bmatrix} = A$

$A = \begin{bmatrix} 4 & 5 \\ 5 & 6 \end{bmatrix} = 24 - 25$

DET = -1

ORDEM 2 = $\begin{bmatrix} 6 & -5 \\ -5 & 4 \end{bmatrix} \div |-1|$

$A^{-1} = \begin{bmatrix} -6 & 5 \\ 5 & -4 \end{bmatrix}$

(D)

⑧ $A = \begin{vmatrix} 2 & K \\ -2 & 1 \end{vmatrix}$ det A = det A^{-1}
soma de K

$\text{Det } A = 2 - (-2K) \Rightarrow 2+2K$
$\text{Det } A = \text{Det } A^{-1}$
$2+2K = \dfrac{1}{2+2K}$

$(2+2K).(2+2K)$
$4+4K+4K+4K^2$
$4K^2+8K+4$
$\Delta = 8^2 - 4.4.4$
$\Delta = 64 - 64$
$\Delta = 0$

$K = \dfrac{-8 \pm 0}{2.4} = \dfrac{-8 \pm 0}{8}$

$K_1 = -8/8 = -1$
$K_2 = -8/8 = -1$

$\begin{cases} K_1 + K_2 \\ -1 + (-1) = \end{cases}$
$\boxed{-2}$

B

$\boxed{\text{SOMA de } K = -2}$

⑨ A e B → ORDEM 2, Det A ≠ 0, Det B ≠ 0

ⓐ $(A+B).(A-B)$
$\boxed{A^2 - AB + BA - B^2}$

ⓑ $(A+B)^2 = (A^2 + 2A.B + B^2)$
$(A+B).(A+B) = A^2 + 2AB + B^2$
$A^2 + AB + BA + B^2 = A^2 + 2AB + B^2$
$AB + BA = 2AB$
$AB + BA = AB + AB$
$AB - AB + BA = AB$
$\boxed{AB = BA}$ ← CONDIÇÃO

(c) $\dfrac{\det A}{\det \cdot A}$

$\det(-A) = (-1)^2 \cdot \det A = \det A \neq 0$

$\dfrac{\det A}{\det(A)} = 1$

(d) $\det(AB) = 1$

$\det(A) \cdot \det(B) = 1$

$\det B = \dfrac{1}{\det A}$