

Evelyn Santos de Santana

CTII348

Sistemas Lineares - Regra de Cramer e Escalonamento (Gauss)

Regra de Cramer

① a)
$$\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$$

$$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - 1 = 5$$
$$D_x = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6 - 3 = 3$$
$$D_y = \begin{vmatrix} 2 & 2 \\ -1 & 3 \end{vmatrix} = 6 - 2 = 4$$
$$x = \frac{D_x}{D} = \frac{3}{5} \quad y = \frac{D_y}{D} = \frac{4}{5}$$
$$V = \left\{ \left(\frac{3}{5}, \frac{4}{5} \right) \right\}$$

b)
$$\begin{cases} 3x - y + 2z = 1 \\ 2x - y + 3z = -1 \\ 4x + y - 2z = 7 \end{cases}$$

$$0 + 9 + 4 = 13 \quad 0 + 3 - 2 = 1$$
$$D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} = 3(-1)(-2) - 1(2)(-8) + 1(12 - 4) = 6 + 16 + 8 = 30$$
$$D_x = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & -2 \end{vmatrix} = 1(2 - 21) - 1(2 - 21) + 1(7 - 7) = -19 - 19 + 0 = -38$$
$$D_y = \begin{vmatrix} 3 & 1 & 1 \\ 2 & -1 & 3 \\ 4 & 7 & -2 \end{vmatrix} = 3(-2 - 28) - 1(-4 - 12) + 1(14 - 8) = -90 + 16 + 6 = -68$$
$$D_z = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 7 \end{vmatrix} = 3(-7 - 4) - 1(14 - 4) + 1(14 - 8) = -33 - 10 + 6 = -37$$
$$x = \frac{D_x}{D} = \frac{-38}{30} = -\frac{19}{15}$$
$$y = \frac{D_y}{D} = \frac{-68}{30} = -\frac{34}{15}$$
$$z = \frac{D_z}{D} = \frac{-37}{30}$$
$$V = \left\{ \left(-\frac{19}{15}, -\frac{34}{15}, -\frac{37}{30} \right) \right\}$$

$$\textcircled{2} \begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases}$$

$$-5 \cdot 12 + 36 = 19$$

$$-12 + 48 + 12 = 48$$

$$D = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix}$$

$$4 \cdot 5 = 59 - 19 = 30$$

$$1 \cdot 2 \cdot 3 = 1 \cdot 2$$

$$45 + 6 + 8 = 59$$

$$Dy = \begin{vmatrix} 3 & 1 & -1 \\ 4 & 12 & 2 \\ 1 & 8 & 3 \end{vmatrix}$$

$$4 \cdot 12 \cdot 2 = 78 - 48 = 30$$

$$1 \cdot 8 \cdot 3 = 1 \cdot 8$$

$$108 + 2 - 32 = 78$$

$$y = \frac{Dy}{D} = \frac{30}{30} = 1$$

(A)

$$\textcircled{3} \begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = 2 \\ 2x + 3y - z = 1 \end{cases}$$

$$2 - 33 - 6 = -37$$

$$1 - 33 + 4 = -28$$

$$D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{vmatrix}$$

$$3 \cdot 1 \cdot (-11) = -36 - (-37) = 1$$

$$2 \cdot 3 \cdot (-1) = 2 \cdot 3$$

$$-1 - 44 + 9 = -36$$

$$-4 - 11 - 3 = -18$$

$$Dx = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & -11 \\ 1 & 3 & -1 \end{vmatrix}$$

$$2 \cdot 1 \cdot (-11) = -29 - (-28) = 1$$

$$1 \cdot 3 \cdot (-1) = 2 \cdot 3$$

$$-1 - 22 - 6 = -29$$

$$2 - 6 + 6 = 2$$

$$Dy = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & -11 \\ 2 & 1 & -1 \end{vmatrix}$$

$$3 \cdot 2 \cdot (-11) = -17 - (-18) = 1$$

$$2 \cdot 1 \cdot (-1) = 2 \cdot 1$$

$$12 - 22 + 3 = -17$$

$$Dz = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$3 \cdot 1 \cdot (-2) = -2 - 2 = 0$$

$$2 \cdot 3 \cdot 1 = 2 \cdot 3$$

$$1 - 8 + 9 = 2$$

$$x = \frac{Dx}{D} = \frac{-1}{-1} = 1$$

$$y = \frac{Dy}{D} = \frac{1}{1} = 1$$

$$z = \frac{Dz}{D} = \frac{0}{1} = 0$$

$$SOMA = 1 + 1 + 0 = 0$$

(C)

$$\textcircled{4} \begin{cases} x+2y-3z=29 \\ x+3y+2z=4 \\ x-y-2z=8 \end{cases}$$

$$-9-2-4=-15$$

$$-72-58-16=-146$$

$$D = \begin{vmatrix} 1 & 2 & -3 & | & 12 \\ 1 & 3 & 2 & | & 13 \\ 1 & -1 & -2 & | & 11 \end{vmatrix}$$

$$-6+4+3=1$$

$$-12+16-58=-54$$

$$D_x = \begin{vmatrix} 29 & 2 & -3 & | & 29 & 2 \\ 4 & 3 & 2 & | & 4 & 3 \\ 8 & -1 & -2 & | & 8 & -1 \end{vmatrix}$$

$$-174+32+12=-130$$

$$87-4+16=99$$

$$D_y = \begin{vmatrix} 1 & 29 & -3 & | & 12 & 29 \\ 1 & 4 & 2 & | & 1 & 4 \\ 1 & 8 & -2 & | & 1 & 8 \end{vmatrix}$$

$$-8+58-24=26$$

$$D_z = \begin{vmatrix} 1 & 2 & 29 & | & 12 \\ 1 & 3 & 4 & | & 13 \\ 1 & -1 & 8 & | & 11 \end{vmatrix}$$

$$24+8-29=3$$

$$x = \frac{D_x}{D} = \frac{16}{16} = 1 \quad y = \frac{D_y}{D} = \frac{80}{16} = 5 \quad z = \frac{D_z}{D} = \frac{-96}{16} = -6$$

$$SOMA = 1+5-6=0$$

(A)

$$\textcircled{5} \begin{cases} 2x+y=5 \\ 2y+z=3 \\ 3x+2y+z=7 \end{cases}$$

$$0+4+0=4$$

$$0+10+3=13$$

$$D = \begin{vmatrix} 2 & 1 & 0 & | & 2 & 1 \\ 0 & 2 & 1 & | & 0 & 2 \\ 3 & 2 & 1 & | & 3 & 2 \end{vmatrix}$$

$$4+3+0=7$$

$$D_x = \begin{vmatrix} 5 & 1 & 0 & | & 5 & 1 \\ 3 & 2 & 1 & | & 3 & 2 \\ 7 & 2 & 1 & | & 7 & 2 \end{vmatrix}$$

$$10+7+0=17$$

$$0+14+0=14 \quad 30+12+0=42$$

$$D_1 = \begin{vmatrix} 2 & 5 & 0 \\ 0 & 3 & 1 \\ 3 & 7 & 1 \end{vmatrix} \begin{vmatrix} 2 & 5 \\ 0 & 3 \\ 3 & 7 \end{vmatrix}$$

$$0 \cdot 3 \cdot 1 - 0 \cdot 3 = 21 - 14 = 7$$

$$6+15+0=21$$

$$D_2 = \begin{vmatrix} 2 & 1 & 5 \\ 0 & 2 & 3 \\ 3 & 2 & 7 \end{vmatrix} \begin{vmatrix} 2 & 1 \\ 0 & 2 \\ 3 & 2 \end{vmatrix}$$

$$0 \cdot 2 \cdot 3 - 0 \cdot 2 = 37 - 42 = -5$$

$$28+9+0=37$$

$$x = \frac{D_1}{D} = \frac{7}{3} \quad y = \frac{D_2}{D} = \frac{-5}{3} \quad z = \frac{D_3}{D} = \frac{5}{3}$$

$$V = \left\{ \frac{14}{3}, \frac{7}{3}, \frac{5}{3} \right\}$$

⑥

$$\begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 2 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 3 \\ 7 \\ -1 \end{vmatrix}$$

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

$$\begin{cases} 1x = 3 \\ 2x + 1y = 7 \\ -1x + 2y + 2z = 1 \end{cases}$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{vmatrix} \begin{vmatrix} 1x & y & z \\ 2x & 1y & z \\ -1x & 2y & 2z \end{vmatrix}$$

$$0+0+0=0$$

$$0+0+0=0$$

$$D_1 = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ 2 & 1 \\ -1 & 2 \end{vmatrix}$$

$$2+0+0=2$$

$$0+0+0=0$$

$$D_2 = \begin{vmatrix} 3 & 0 & 0 \\ 7 & 1 & 0 \\ -1 & 2 & 2 \end{vmatrix} \begin{vmatrix} 3 & 0 \\ 7 & 1 \\ -1 & 2 \end{vmatrix}$$

$$6+0+0=6$$

$$-3+11+0=8$$

$$D_3 = \begin{vmatrix} 1 & 3 & 0 \\ 2 & 7 & 0 \\ -1 & -1 & 2 \end{vmatrix} \begin{vmatrix} 3 & 0 \\ 7 & 1 \\ -1 & 2 \end{vmatrix}$$

$$14+0+0=14$$

$$D_4 = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & 7 \\ -1 & 2 & -1 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ 2 & 1 \\ -1 & 2 \end{vmatrix}$$

$$-1+0+12=11$$

$$x = \frac{dx}{dt} = 6 = 3 \quad y = \frac{dy}{dt} = 14 = 7 \quad z = \frac{dz}{dt} = 0 = 0$$

$$0 \quad 2$$

$$0 \quad 2$$

$$0 \quad 2$$

(E)

Escalonamento (Gauss)

$$\textcircled{1} \begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{cases}$$

$$\begin{array}{c} 3 \\ \downarrow \end{array} \left(\begin{array}{ccc|c} 2 & 1 & -3 & -5 \\ 1 & 3 & -1 & 11 \\ 1 & 0 & -5 & 3 \end{array} \right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & 3 & -1 & 11 \\ 2 & 1 & -3 & -5 \\ 1 & 0 & -5 & 3 \end{array} \right) \xrightarrow{R_1 - R_3} \left(\begin{array}{ccc|c} 0 & 3 & 4 & 8 \\ 2 & 1 & -3 & -5 \\ 1 & 0 & -5 & 3 \end{array} \right) \xrightarrow{R_2 - 2R_3} \left(\begin{array}{ccc|c} 0 & 3 & 4 & 8 \\ 0 & 1 & 7 & -1 \\ 1 & 0 & -5 & 3 \end{array} \right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 0 & 1 & 7 & -1 \\ 0 & 3 & 4 & 8 \\ 1 & 0 & -5 & 3 \end{array} \right) \xrightarrow{R_2 - 3R_1} \left(\begin{array}{ccc|c} 0 & 1 & 7 & -1 \\ 0 & 0 & -17 & 11 \\ 1 & 0 & -5 & 3 \end{array} \right) \xrightarrow{R_2 \cdot (-1/17)} \left(\begin{array}{ccc|c} 0 & 1 & 7 & -1 \\ 0 & 0 & 1 & -11/17 \\ 1 & 0 & -5 & 3 \end{array} \right) \xrightarrow{R_1 - 7R_2} \left(\begin{array}{ccc|c} 0 & 1 & 0 & 76/17 \\ 0 & 0 & 1 & -11/17 \\ 1 & 0 & -5 & 3 \end{array} \right) \xrightarrow{R_3 + 5R_2} \left(\begin{array}{ccc|c} 0 & 1 & 0 & 76/17 \\ 0 & 0 & 1 & -11/17 \\ 1 & 0 & 0 & -25/17 \end{array} \right)$$

$$2x - y - 3z = -5 \quad 7x - 10z = -4 \quad 25z = -25$$

$$2(-2) - y - 3(-1) = -5 \quad 7x - 10(-1) = -4 \quad z = -25/25$$

$$-4 - y + 3 = -5 \quad 7x + 10 = -4 \quad \underline{z = -1}$$

$$-4 + 3 + 5 = y \quad 7x = -4 - 10$$

$$\underline{y = 4} \quad 7x = -14$$

$$\underline{x = -2} \quad V = \{-2, 4, -1\}$$

$$\underline{x = -2}$$

$$\textcircled{2} \begin{cases} x = 2y \\ 2y = 3z \\ x + y + z = 11 \end{cases} \xrightarrow{x = 2y} \begin{cases} 2y = 3z \\ 2y + y + z = 11 \end{cases} \xrightarrow{2y = 3z} \begin{cases} 3z + z = 11 \\ 2y = 3z \end{cases} \xrightarrow{3z + z = 11} \begin{cases} 4z = 11 \\ 2y = 3z \end{cases} \xrightarrow{4z = 11} \begin{cases} z = 11/4 \\ 2y = 3(11/4) \end{cases} \xrightarrow{2y = 3(11/4)} \begin{cases} z = 11/4 \\ y = 33/8 \end{cases} \xrightarrow{z = 11/4} \begin{cases} y = 33/8 \\ x = 2y \end{cases} \xrightarrow{y = 33/8} \begin{cases} x = 33/4 \\ y = 33/8 \\ z = 11/4 \end{cases}$$

$$x + y + z = 11$$

$$2y + y + 2y/3 = 11 \quad (.3)$$

$$6y + 3y + 2y = 33$$

$$11y = 33$$

$$y = 33/11$$

$$y = 3$$

$$x = 2y$$

$$x = 2 \cdot 3$$

$$\underline{x = 6}$$

$$2y = 3z$$

$$2 \cdot 3 = 3z$$

$$6 = 3z$$

$$z = 6/3$$

$$\underline{z = 2}$$

$$x + 2y + 3z = 9$$

$$6 + 6 + 6 = 18$$

(B)

$$\textcircled{3} \begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6x + 3z = -12 \end{cases}$$

$$\begin{pmatrix} 1 & 1 & 1 & : & 0 \\ 2 & -1 & -2 & : & 1 \\ 0 & 6 & 3 & : & -12 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & : & 0 \\ 0 & -3 & -4 & : & 1 \\ 0 & 6 & 3 & : & -12 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & : & 0 \\ 0 & -3 & -4 & : & 1 \\ 0 & 0 & -5 & : & -10 \end{pmatrix} \downarrow$$

\textcircled{D}

$$\begin{aligned} -5 \cdot 2 &= -10 \\ 2 &= -10 / -5 \\ 2 &= 2 \end{aligned}$$

$$\textcircled{4} A = \text{Alu}$$

$$B = \text{Bua}$$

$$C = \text{Caco}$$

$$A + B + C = 68$$

$$B + 20\% \cdot C = A \rightarrow B + 0,2C = A$$

$$C + 20\% \cdot A = 3B \rightarrow C + 0,2A = 3B$$

$$I \quad A + B + C = 68 \quad (I)$$

$$B + 0,2C = A \quad (II)$$

$$0,2A + C = 3B \quad (III)$$

$$I + II \rightarrow (B + 0,2C) + B + C = 68$$

$$2B + 1,2C = 68$$

$$B = 68 - 1,2C$$

2

$$B = 34 - 0,6C \quad \downarrow$$

$$\text{III} \rightarrow 0,2A + C = 3(34 - 0,6C) \quad \rightarrow 0,2(B + 0,2C) + 2,8C = 102$$

$$0,2A + C = 102 - 1,8C$$

$$0,2B + 0,4C + 2,8C = 102$$

$$0,2A + 2,8C = 102 + I$$

$$0,2B + 2,8C = 102$$

$$0,2(34 - 0,6C) + 2,8C = 102$$

$$B = 34 - 0,6C$$

$$6,8 - 0,12C + 2,8C = 102$$

$$B = 34 - 0,6 \cdot 35$$

$$6,8 + 2,72 = 102$$

$$B = 34 - 21$$

$$2,72C = 102 - 6,8$$

$$|B = 13|$$

$$2,72C = 95,2$$

$$C = 95,2$$

$$A + B + C = 68$$

$$2,72$$

$$A + 13 + 35 = 68$$

$$|C = 35|$$

$$A + 48 = 68$$

$$A = 68 - 48$$

Ali tem R\$ 20,00

$$|A = 20|$$

Bia tem R\$ 13,00

Caco tem R\$ 35,00

Caco - Ali ?

$$35 - 20 = 15$$

Ali tem R\$ 15,00 a menos que Caco

(A)

$$⑤ \quad x = \text{Alfeu} = 134$$

$$y = \text{Bento} = 115$$

$$z = \text{Cíntia} = 48$$

$$A = \begin{vmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{vmatrix}$$

$$X = \begin{vmatrix} x & -134 \\ y & -115 \\ z & -48 \end{vmatrix}$$

$$\begin{cases} 3y + 4z = 134 \\ x + 5z = 115 \\ 2x + y = 48 \end{cases}$$

$$\begin{array}{c|c|c} 0 & 3 & 4 : 134 \\ \hline 2. & 1 & 0 & 5 : 115 \\ \hline 4. & 2 & 1 & 0 : 48 \end{array} \quad \begin{array}{c|c|c} 1. & 0 & 3 & 4 : 134 \\ \hline \sim & & & : \\ \hline 3. & 0 & 1 & 10 : 182 \end{array} \quad \begin{array}{c|c|c} & & : \\ \hline \sim & & : \\ \hline & 0 & 0 & 3 & 4 : 680 \end{array}$$

$$3y + 42 = 134$$

$$3y + 4 \cdot 20 = 134$$

$$3y + 80 = 134$$

$$3y = 134 - 80$$

$$3y = 54$$

$$y = 54/3$$

$$y = 18$$

$$x + 5z = 115$$

$$x + 5 \cdot 20 = 115$$

$$x + 100 = 115$$

$$x = 115 - 100$$

$$x = 15$$

$$34z = 680$$

$$z = 680/34$$

$$z = 20$$

$$x + y + z = ?$$

$$15 + 18 + 20 = 53$$

(A)

R\$ 53,00