Evelyn Santos de Santana

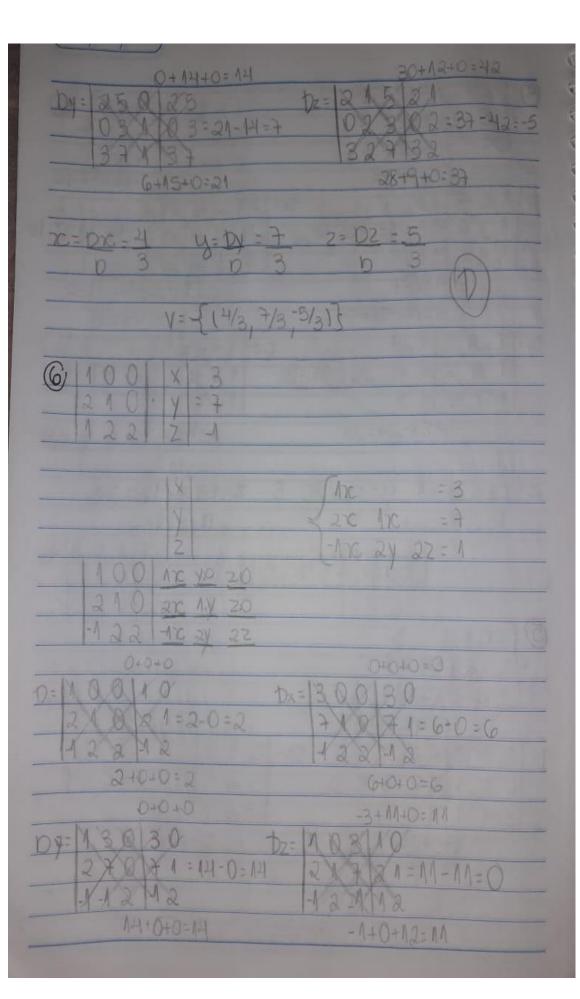**CTII348**

Sistemas Lineares - Regra de Cramer e Escalonamento (Gass)

Regra de Cramer

① a) $\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases}$

$D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - 1 = 5$ $Dx = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6 - 3 = 3$

$Dy = \begin{vmatrix} 2 & 2 \\ -1 & 3 \end{vmatrix} = 6 - 2 = 4$ $x = \dfrac{Dx}{D} = \dfrac{3}{5}$ $y = \dfrac{Dy}{D} = \dfrac{4}{5}$

$V = \left\{ (3/5, 4/5) \right\}$

b) $\begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 7 \end{cases}$

$0 + 9 + 4 = 13$ $0 + 3 - 2 = 1$

$D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} \begin{matrix} 3 & -1 \\ 2 & 0 \\ 4 & 1 \end{matrix} = -10 - 13 = -23$ $Dx = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & -2 \end{vmatrix} \begin{matrix} 1 & -1 \\ -1 & 0 \\ 7 & 1 \end{matrix} = -22 - 1 = -23$

$0 - 12 + 2 = -10$ $0 - 21 - 1 = -22$

$-21 + 63 - 4 = 55$ $0 - 3 - 14 = -17$

$Dy = \begin{vmatrix} 3 & -1 & 1 \\ 2 & -1 & 3 \\ 4 & 7 & -2 \end{vmatrix} \begin{matrix} 3 & 1 \\ 2 & -1 \\ 4 & 7 \end{matrix} = 32 - 55 = -23$ $Dz = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & -1 \\ 4 & 1 & 7 \end{vmatrix} \begin{matrix} 3 & -1 \\ 2 & 0 \\ 4 & 1 \end{matrix} = 6 - (-17) = 23$

$6 + 12 + 14 = 32$ $0 + 4 + 2 = 6$

$x = \dfrac{Dx}{D} = \dfrac{-23}{-23} = 1$ $y = \dfrac{Dy}{D} = \dfrac{-23}{-23} = 1$ $z = \dfrac{Dz}{D} = \dfrac{23}{-23} = -1$

$V = \left\{ (1, 1, -1) \right\}$

② $\begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases}$

$-5 - 12 + 36 = 19$ $-12 + 48 + 12 = 48$

$D = \begin{vmatrix} 3 & 4 & -1 \\ 4 & 5 & 2 \\ 1 & -2 & 3 \end{vmatrix} \begin{matrix} 3 & 4 \\ 4 & 5 \\ 1 & -2 \end{matrix} = 59 - 19 = 30$ $Dy = \begin{vmatrix} 3 & 1 & -1 \\ 4 & 12 & 2 \\ 1 & 8 & 3 \end{vmatrix} \begin{matrix} 3 & 1 \\ 4 & 12 \\ 1 & 8 \end{matrix} = 78 - 48 = 30$

$45 + 6 + 8 = 59$ $108 + 2 - 32 = 78$

$y = \dfrac{Dy}{D} = \dfrac{30}{30} = 1$ Ⓐ

③ $\begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = 2 \\ 2x + 3y - z = 1 \end{cases}$

$2 - 33 - 6 = -37$ $-1 - 33 + 4 = -28$

$D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{matrix} = -36 - (-37) = 1$ $Dx = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & -11 \\ 1 & 3 & -1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 2 & 1 \\ 1 & 3 \end{matrix} = -29 - (-28) = 1$

$-1 - 44 + 9 = -36$ $-1 - 22 - 6 = -29$

$-4 - 11 - 3 = -18$ $2 - 6 + 6 = 2$

$Dy = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & -11 \\ 2 & 1 & -1 \end{vmatrix} \begin{matrix} 1 & 1 \\ 3 & 2 \\ 2 & 1 \end{matrix} = -17 - (-18) = 1$ $Dz = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 3 & 1 \\ 2 & 3 \end{matrix} = 2 - 2 = 0$

$12 - 22 + 3 = -17$ $1 - 8 + 9 = 2$

$x = \dfrac{Dx}{D} = \dfrac{-1}{1} = -1$ $y = \dfrac{Dy}{D} = \dfrac{1}{1} = 1$ $z = \dfrac{Dz}{D} = \dfrac{0}{1} = 0$

$\boxed{SOMA = -1 + 1 + 0 = 0}$ Ⓒ

④ $\begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases}$

$-9-2-4 = -15$ $-72-58-16 = -146$

$D = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix} \begin{matrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{matrix} = 1 - (-15) = 16$

$Dx = \begin{vmatrix} 29 & 2 & -3 \\ 4 & 3 & 2 \\ 8 & -1 & -2 \end{vmatrix} \begin{matrix} 29 & 2 \\ 4 & 3 \\ 8 & -1 \end{matrix} = -130 - (-146) = 16$

$-6+4+3 = 1$ $-174+32+12 = -130$

$-12+16-58 = -54$ $87-4+16 = 99$

$Dy = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & 2 \end{vmatrix} \begin{matrix} 1 & 29 \\ 1 & 4 \\ 1 & 8 \end{matrix} = 26 - (-54) = 80$

$Dz = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix} \begin{matrix} 1 & 2 \\ 1 & 3 \\ 1 & -1 \end{matrix} = 3 - 99 = -96$

$-8+58-24 = 26$ $24+8-29 = 3$

$x = \dfrac{Dx}{D} = \dfrac{16}{16} = 1$ $y = \dfrac{Dy}{D} = \dfrac{80}{16} = 5$ $z = \dfrac{Dz}{D} = \dfrac{-96}{16} = -6$

$\boxed{SOMA = 1 + 5 - 6 = 0}$ Ⓐ

⑤ $\begin{cases} 2x + y = 5 \\ 2y + z = 3 \\ 3x + 2y + z = 7 \end{cases}$

$0+4+0 = 4$ $0+10+3 = 13$

$D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} \begin{matrix} 2 & 1 \\ 0 & 2 \\ 3 & 2 \end{matrix} = 7 - 4 = 3$

$Dx = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{vmatrix} \begin{matrix} 5 & 1 \\ 3 & 2 \\ 7 & 2 \end{matrix} = 17 - 13 = 4$

$4+3+0 = 7$ $10+7+0 = 17$

$$0 + 14 + 0 = 14 \qquad\qquad 30 + 12 + 0 = 42$$

$$Dy = \begin{vmatrix} 2 & 5 & 0 \\ 0 & 3 & 1 \\ 3 & 7 & 1 \end{vmatrix} \begin{matrix} 2 & 5 \\ 0 & 3 \\ 3 & 7 \end{matrix} = 21 - 14 = 7 \qquad Dz = \begin{vmatrix} 2 & 1 & 5 \\ 0 & 2 & 3 \\ 3 & 2 & 7 \end{vmatrix} \begin{matrix} 2 & 1 \\ 0 & 2 \\ 3 & 2 \end{matrix} = 37 - 42 = -5$$

$$6 + 15 + 0 = 21 \qquad\qquad 28 + 9 + 0 = 37$$

$$x = \frac{Dx}{D} = \frac{4}{3} \qquad y = \frac{Dy}{D} = \frac{7}{3} \qquad z = \frac{Dz}{D} = \frac{5}{3}$$

①

$$V = \left\{ \left(\tfrac{4}{3}, \tfrac{7}{3}, \tfrac{-5}{3} \right) \right\}$$

⑥ $\begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 2 \end{vmatrix} \cdot \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 3 \\ 7 \\ 1 \end{vmatrix}$

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} \qquad \begin{cases} 1x & = 3 \\ 2x & 1x = 7 \\ -1x & 2y & 2z = 1 \end{cases}$$

$$\begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{vmatrix} \begin{matrix} 1x & y0 & 20 \\ 2x & 1y & 20 \\ -1x & 2y & 22 \end{matrix}$$

$$0 + 0 + 0 \qquad\qquad 0 + 0 + 0 = 0$$

$$D = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{vmatrix} \begin{matrix} 1 & 0 \\ 2 & 1 \\ -1 & 2 \end{matrix} = 2 - 0 = 2 \qquad Dx = \begin{vmatrix} 3 & 0 & 0 \\ 7 & 1 & 0 \\ 1 & 2 & 2 \end{vmatrix} \begin{matrix} 3 & 0 \\ 7 & 1 \\ -1 & 2 \end{matrix} = 6 + 0 = 6$$

$$2 + 0 + 0 = 2 \qquad\qquad 6 + 0 + 0 = 6$$

$$0 + 0 + 0 \qquad\qquad -3 + 14 + 0 = 11$$

$$Dy = \begin{vmatrix} 1 & 3 & 0 \\ 2 & 7 & 0 \\ -1 & 1 & 2 \end{vmatrix} \begin{matrix} 3 & 0 \\ 7 & 0 \\ 1 & 2 \end{matrix} = 14 - 0 = 14 \qquad Dz = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & 7 \\ -1 & 2 & 1 \end{vmatrix} \begin{matrix} 1 & 0 \\ 2 & 1 \\ -1 & 2 \end{matrix} = 11 - 11 = 0$$

$$14 + 0 + 0 = 14 \qquad\qquad -1 + 0 + 12 = 11$$

$$x = Dx = \frac{6}{D} = \frac{3}{2} \qquad y = Dy = \frac{14}{D} = \frac{7}{2} \qquad z = Dz = \frac{0}{D} = \frac{0}{2}$$

(E)

Escalonamento (Gass)

① $\begin{cases} 2x - y - 3z = -5 \\ x + 3y - z = 11 \\ x \quad\quad - 5z = 3 \end{cases}$

$\overset{3}{\underset{\llcorner}{}}\begin{pmatrix} 2 & 1 & -3 & \vdots & 5 \\ 1 & 3 & -1 & \vdots & 11 \\ 1 & 0 & -5 & \vdots & 3 \end{pmatrix} \sim_1 \begin{array}{c} \\ 7 \\ -7 \end{array}\begin{pmatrix} 7 & 0 & -10 & \vdots & -4 \\ 1 & 0 & -5 & \vdots & 3 \end{pmatrix} \sim \begin{pmatrix} & & & \vdots \\ & & & \vdots \\ 0 & 0 & 25 & \vdots & -25 \end{pmatrix}$

$2x - y - 3z = -5$
$2 \cdot (-2) - y - 3 \cdot (-1) = -5$
$-4 - y + 3 = -5$
$-4 + 3 + 5 = y$
$\boxed{y = 4}$

$7x - 10z = -4$
$7x - 10 \cdot (-1) = -4$
$7x + 10 = -4$
$7x = -4 - 10$
$7x = -14$
$x = -14/7$
$\boxed{x = -2}$

$25z = -25$
$z = -25/25$
$\boxed{z = -1}$

$V = \{(-2, 4, -1)\}$

② $\begin{cases} x \quad\quad = 2y \\ \quad 2y \quad = 3z \\ x + y + z = 11 \end{cases}$ $\rightarrow \begin{array}{c} 2y = 3z \\ \boxed{z = 2y/3} \end{array}$

$\rightarrow \boxed{x = 2y}$

$x + y + z = 11$
$2y + y + 2y/3 = 11 \;(\cdot 3)$
$6y + 3y + 6y = 33$
$11y = 33$
$y = 33/11$
$y = 3$

$x = 2y$
$x = 2 \cdot 3$
$\boxed{x = 6}$

$2y = 3z$
$2 \cdot 3 = 3z$
$6 = 3z$
$z = 6/3$
$\boxed{z = 2}$

$x + 2y + 3z = ?$
$6 + 6 + 6 = \boxed{18}$

$\textcircled{B}$

③ $\begin{cases} x+y+z=0 \\ 2x-y-2z=1 \\ 6y+3z=-12 \end{cases}$

$\overset{-2}{}\begin{pmatrix} 1 & 1 & 1 & \vdots & 0 \\ 2 & -1 & -2 & \vdots & 1 \\ 0 & 6 & 3 & \vdots & -12 \end{pmatrix} \sim \begin{pmatrix} 0 & -3 & -4 & \vdots & 1 \\ 0 & 6 & 3 & \vdots & -12 \end{pmatrix} \sim \begin{pmatrix} & & & \vdots & \\ & & & \vdots & \\ 0 & 0 & -5 & \vdots & -10 \end{pmatrix}\downarrow$

Ⓓ

$-5 \cdot 2 = -10$

$2 = -10/-5$

$\boxed{2 = 2}$

④ A = Ali

B = Bia $A+B+C = 68$

C = Caco

$B + \dfrac{20}{100} \cdot C = A \to B + 0{,}2C = A$

$C + \dfrac{20}{100} \cdot A = 3B \to C + 0{,}2A = 3B$

$A+B+C = 68$ (I)

$B + 0{,}2 = A$ (II)

$0{,}2 + AC = 3B$ (III)

$I \to II \to (B+0{,}2C) + B + C = 68$

$2B + 1{,}2C = 68$

$B = \dfrac{68 - 1{,}2C}{2}$

$\boxed{B = 34 - 0{,}6C}\downarrow$

III → $0,2A+C=3.(34-0,6C)$ → $0,2(B+0,2C)+2,8C=102$

$0,2A+C=102-1,8C$ → $0,2B+0,4C+2,8C=102$

$0,2A+2,8C=102 ← II$ → $0,2B+2,84=102$

$0,2(34-0,6C)+2,84C=102$ $B=34-0,6C$

$6,8-0,12C+2,84C=102$ $B=34-0,6\cdot35$

$6,8+2,72=102$ $B=34-21$

$2,72C=102-6,8$ $|B=13|$

$2,72C=95,2$

$C=\dfrac{95,2}{2,72}$ $A+B+C=68$

        $A+13+35=68$

$|C=35|$ $A+48=68$

        $A=68-48$

Ali tem R$ 20,00 $|A=20|$

Bia tem R$ 13,00

Caco tem R$ 35,00   Caco - Ali ?

        $35-20=|15|$

Ali tem R$ 15,00 a   (A)
menos que Caco

⑤ x = Alfeu = 134

y = Bento = 115

z = Cínthia = 48

$A=\begin{vmatrix} 0 & 3 & 4 \\ 1 & 0 & 5 \\ 2 & 1 & 0 \end{vmatrix}$ $x=\begin{vmatrix} x & -134 \\ y & -115 \\ z & -48 \end{vmatrix}$ $\begin{cases} 3y+4z=134 \\ x \quad +5z=115 \\ 2x+y \quad =48 \end{cases}$

$$\begin{array}{c} \\ 2.\\ 4 \end{array}\left(\begin{array}{ccc:c} 0 & 3 & 4 & 134\\ 1 & 0 & 5 & 115\\ 2 & 1 & 0 & 48 \end{array}\right) \underset{\sim}{\overset{1.}{}} \left(\begin{array}{ccc:c} 0 & 3 & 4 & 134\\ & & : & \\ 0 & 1 & -10 & -182 \end{array}\right) \underset{\sim}{} \left(\begin{array}{ccc:c} & & : & \\ & & : & \\ 0 & 0 & 34 & 680 \end{array}\right)$$

$3y + 4z = 134$

$3y + 4 \cdot 20 = 134$

$3y + 80 = 134$

$3y = 134 - 80$

$3y = 54$

$y = 54/3$

$\underline{y = 18}$

$x - 5z = 115$

$x - 5 \cdot 20 = 115$

$x + 100 = 115$

$x = 115 - 100$

$\underline{\underline{x = 15}}$

$34z = 680$

$z = 680/34$

$\underline{z = 20}$

$x + y + z = ?$

$15 + 18 + 20 = 53$

R$ 53,00

(A)