ASSIGNMENT-1

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Question : Suppose the equations AB, BC and CA are respectively given by

$$\mathbf{n}_i^\top \mathbf{x} = c_i \quad i = 1, 2, 3 \tag{1}$$

The equations of the respective angle bisectors are given by

$$\frac{\mathbf{n}_i^\top \mathbf{x} - c_i}{\|\mathbf{n}_i\|} = \pm \frac{\mathbf{n}_j^\top \mathbf{x} - c_j}{\|\mathbf{n}_j\|} \quad i \neq j \tag{2}$$

Substitute numerical values and find the equations of the angle bisectors of A,B and C.

Solution :

The parametric equations of sides;

$$\begin{aligned} BC, &\quad \mathbf{a} \quad is \quad \begin{pmatrix} 11 & 1 \end{pmatrix} \cdot \mathbf{x} = -38, \\ CA, &\quad \mathbf{b} \quad is \quad \begin{pmatrix} 1 & -1 \end{pmatrix} \cdot \mathbf{x} = 2, \\ AB, &\quad \mathbf{c} \quad is \quad \begin{pmatrix} 7 & 5 \end{pmatrix} \cdot \mathbf{x} = 2, \end{aligned}$$

Using the formula mentioned in the question to find out the angular bisector for sides AB and AC, naming the angular bisector $\mathbf{d}$ we get,

$$\frac{\mathbf{n}_3^\top \mathbf{x} - c_3}{\|\mathbf{n}_3\|} = \pm \frac{\mathbf{n}_2^\top \mathbf{x} - c_2}{\|\mathbf{n}_2\|}$$

As we can see we'll get 2 solutions for $\mathbf{d}$. This is because one of them is internal angular bisector and the other is the external angular bisector. Internal angular bisector can be evaluated if we take "+" in the above formula.

Hence, $\mathbf{d}$ is given by,

$$\begin{aligned} \frac{\mathbf{n}_3^\top \mathbf{x} - c_3}{\|\mathbf{n}_3\|} &= \frac{\mathbf{n}_2^\top \mathbf{x} - c_2}{\|\mathbf{n}_2\|} \\ \left(\frac{\mathbf{n_3}}{\|\mathbf{n_3}\|} - \frac{\mathbf{n_3}}{\|\mathbf{n_3}\|} \right) \cdot \mathbf{x} &= \left(\frac{c_3}{\|\mathbf{n_3}\|} - \frac{c_2}{\|\mathbf{n_2}\|} \right) \\ \left(\frac{\begin{pmatrix} 7 & 5 \end{pmatrix}}{\sqrt{74}} - \frac{\begin{pmatrix} 1 & -1 \end{pmatrix}}{\sqrt{2}} \right) \cdot \mathbf{x} &= \frac{2}{\sqrt{74}} - \frac{2}{\sqrt{2}} \\ \begin{pmatrix} \frac{7-\sqrt{37}}{\sqrt{74}} & \frac{5+\sqrt{37}}{\sqrt{74}} \end{pmatrix} \cdot \mathbf{x} &= \frac{2 - 2\sqrt{37}}{\sqrt{74}} \end{aligned}$$

Hence, the internal angluar bisector of angle A will be,

$$\mathbf{d} = \begin{pmatrix} \frac{7-\sqrt{37}}{\sqrt{74}} & \frac{5+\sqrt{37}}{\sqrt{74}} \end{pmatrix} \cdot \mathbf{x} - \frac{2 - 2\sqrt{37}}{\sqrt{74}}$$

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