

# 12.13.3.62

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If A and B are two events and  $A \neq \phi$ ,  $B \neq \phi$ , then

$$1) \Pr(A|B) = \Pr(A) \cdot \Pr(B)$$

$$2) \Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

$$3) \Pr(A|B) \cdot \Pr(B|A) = 1$$

$$4) \Pr(A|B) = \frac{\Pr(A)}{\Pr(B)}$$

**Solution:**

Let us take an example. The following table describes events when a die is rolled.

Event	Description	Probability
A	Even number shows up	$\frac{1}{2}$
B	Perfect number shows up	$\frac{1}{3}$
AB	Both Event A and B happen	$\frac{1}{6}$

1) As  $\Pr\left(\frac{A}{B}\right)$  represents the probability of occurrence of A given that B has occurred. Hence,

$$\Pr\left(\frac{A}{B}\right) = \frac{1/6}{1/3} \quad (1)$$

$$= \frac{1}{2} \quad (2)$$

$$\Pr(A) \cdot \Pr(B) = \frac{1}{2} \times \frac{1}{3} \quad (3)$$

$$= \frac{1}{6} \quad (4)$$

As RHS of (2) and (4) are not equal, option 1 is not correct.

2)

$$\frac{\Pr(AB)}{\Pr(B)} = \frac{\frac{1}{6}}{\frac{1}{3}} \quad (5)$$

$$= \frac{1}{2} \quad (6)$$

As (2) and (6) are equal, option 2 is correct.

$$\Pr\left(\frac{A}{B}\right) = \frac{\Pr(AB)}{\Pr(B)} \quad (7)$$

Equation (7) is one of the axioms of probability.

3) From (7),

$$\Pr\left(\frac{A}{B}\right) \cdot \Pr\left(\frac{B}{A}\right) = \frac{\Pr(AB)}{\Pr(B)} \times \frac{\Pr(AB)}{\Pr(A)} \quad (8)$$

$$= \frac{\Pr(AB)^2}{\Pr(A) \cdot \Pr(B)} \quad (9)$$

$$\neq 1 \quad (10)$$

Hence, option 3 is incorrect. Let us verify it using the example.

$$\frac{\Pr(AB)^2}{\Pr(A) \cdot \Pr(B)} = \frac{\frac{1}{6}^2}{\frac{1}{2} \times \frac{1}{3}} \quad (11)$$

$$= \frac{1}{6} \quad (12)$$

Hence, option 3 is incorrect.

4)

$$\frac{\Pr(A)}{\Pr(B)} = \frac{\frac{1}{2}}{\frac{1}{3}} \quad (13)$$

$$= \frac{3}{2} \quad (14)$$

As (2) and (14) are not equal, option 4 is incorrect.

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