

Exemplar - 10.13.3.39

EE22BTECH11043 - Rambha Satvik*

A die has its face marked 0,1,1,1,6,6. Two such dice are thrown together and their score is recorded.

- 1) How many different scores are possible ?
- 2) What is the probability of getting a total 7 ?

Solution: Let the random variables be defined as:

Random Variable	Values	Description
X	X = {0,1,6}	First Dice Roll
Y	Y = {0,1,6}	Second Dice Roll

- 1) **Possible outcomes:** The following data can be interpreted from the data given in the question,

$$p_X(k) = \begin{cases} \frac{1}{6} & \text{if } k = 0 \\ \frac{1}{2} & \text{if } k = 1 \\ \frac{1}{3} & \text{if } k = 6 \\ 0 & \text{Otherwise} \end{cases} \quad (1)$$

$$p_Y(k) = \begin{cases} \frac{1}{6} & \text{if } k = 0 \\ \frac{1}{2} & \text{if } k = 1 \\ \frac{1}{3} & \text{if } k = 6 \\ 0 & \text{Otherwise} \end{cases} \quad (2)$$

(3)

The Z-transform of a die is defined as

$$M_X(z) = z^{-X} = \sum_{k=-\infty}^{\infty} p_X(k)z^{-k} \quad (4)$$

The Z-transform of the first die X_1 is given by

$$M_{X_1}(z) = \frac{1}{6}z^0 + \frac{1}{2}z^{-1} + \frac{1}{3}z^{-6} \quad (5)$$

$$= \frac{z^0 + 3z^{-1} + 2z^{-6}}{6} \quad (6)$$

(7)

The Z-transform of the second die X_2 is given by

$$M_{X_2}(z) = \frac{1}{6}z^0 + \frac{1}{2}z^{-1} + \frac{1}{3}z^{-6} \quad (8)$$

$$= \frac{z^0 + 3z^{-1} + 2z^{-6}}{6} \quad (9)$$

(10)

$$M_{X_1+X_2}(z) = \frac{z^0 + 3z^{-1} + 2z^{-6}}{6} \times \frac{z^0 + 3z^{-1} + 2z^{-6}}{6} \quad (11)$$

$$M_{X_1+X_2}(z) = \frac{(1 + 3z^{-1} + 2z^{-6})^2 (1 - z^{-1})^2}{36(1 - z^{-1})^2} \quad (12)$$

We also know that,

$$p_{X_1+X_2}(n-k) \xleftrightarrow{Z} M_{X_1+X_2}(z)z^{-k}; \quad (13)$$

$$nu(n) \xleftrightarrow{Z} \frac{z^{-1}}{(1 - z^{-1})^2} \quad (14)$$

Hence, after some algebra, it can be shown that,

$$\begin{aligned} & \frac{1}{36}[(n+1)u(n+1) + 4nu(n) - 2n - 1u(n-1) \\ & -12(n-2)u(n-2) + 9(n-3)u(n-3) + 4(n-5)u(n-5) \\ & -20(n-7)u(n-7) + 12(n-8)u(n-8) \\ & +4(n-11)u(n-11) - 8(n-12)u(n-12) \\ & +4(n-13)u(n-13)] \xleftrightarrow{Z} \\ & \frac{(1 + 3z^{-1} + 2z^{-6})^2 (1 - z^{-1})^2}{36(1 - z^{-1})^2} \quad (15) \end{aligned}$$

The probability function for the sum of the numbers appearing on both the dice,

$$p_X(n) = \begin{cases} \frac{1}{36} & n = 0 \\ \frac{1}{6} & n = 1 \\ \frac{1}{4} & n = 2 \\ \frac{1}{9} & n = 6, 12 \\ \frac{1}{3} & n = 7 \\ 0 & \text{else} \end{cases} \quad (16)$$

The possible outcomes: 0,1,2,6,7&12

- 2) **Probability of getting a 7 :** From (16), the probability of getting the sum of 2 numbers appearing on the dice to be 7, would be $\frac{1}{3}$.

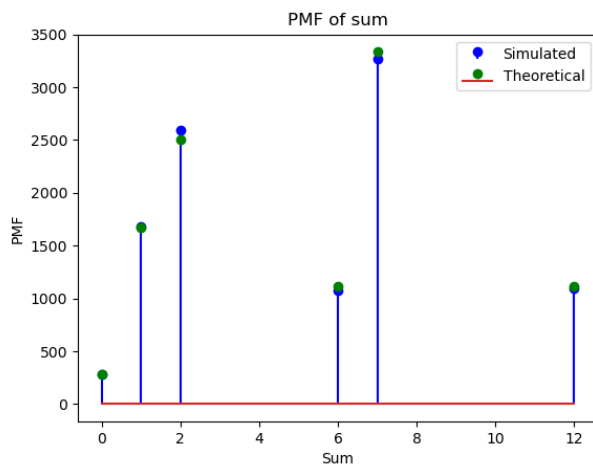


Fig. 1. Sketch of Probability Mass Function for Sum generated using (16)