

12.13.3.25

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The probability distribution of a random variable X is given below:

X	0	1	2	3
P(X)	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

- 1) Determine the value of k.
- 2) Determine $P(X \leq 2)$ and $P(X > 2)$.
- 3) Find $P(X \leq 2) + P(X > 2)$.

Solution:

- 1) **Value of k:** We know that sum of probability of all the random variables is 1, i.e.,

$$\sum_{i=-\infty}^{\infty} P(i) = 1 \quad (1)$$

$$\Rightarrow P(0) + P(1) + P(2) + P(3) = 1 \quad (2)$$

$$\Rightarrow k + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} = 1 \quad (3)$$

$$\Rightarrow \frac{15k}{8} = 1 \quad (4)$$

$$\Rightarrow k = \frac{8}{15} \quad (5)$$

Hence, the value of k is $\frac{8}{15}$. This makes the data given in the question as follows,

$$P(k) = \begin{cases} \frac{8}{15} & \text{if } k = 0 \\ \frac{4}{15} & \text{if } k = 1 \\ \frac{2}{15} & \text{if } k = 2 \\ \frac{1}{15} & \text{if } k = 3 \\ 0 & \text{Otherwise} \end{cases} \quad (6)$$

2) Value of $P(X \leq 2)$ and $P(X > 2)$:

- a) **Value of $P(X \leq 2)$:** $P(X \leq 2)$ translates to sum of probabilities of all random variables less than or equal to 2, i.e.,

$$P(X \leq 2) = \sum_{i=-\infty}^2 P(i) \quad (7)$$

$$\Rightarrow P(X \leq 2) = P(0) + P(1) + P(2) \quad (8)$$

$$\Rightarrow P(X \leq 2) = \frac{8}{15} + \frac{4}{15} + \frac{2}{15} \quad (9)$$

$$\Rightarrow P(X \leq 2) = \frac{14}{15} \quad (10)$$

- b) **Value of $P(X > 2)$:** $P(X > 2)$ translates to sum of probabilities of all random variables greater than 2, i.e.,

$$P(X > 2) = \sum_{i=3}^{\infty} P(i) \quad (11)$$

$$\Rightarrow P(X > 2) = P(3) \quad (12)$$

$$\Rightarrow P(X > 2) = \frac{1}{15} \quad (13)$$

- 3) **Value of $P(X \leq 2) + P(X > 2)$:** From (10) and (13) we can easily say that,

$$P(X \leq 2) + P(X > 2) = \frac{14}{15} + \frac{1}{15} \quad (14)$$

$$\Rightarrow P(X \leq 2) + P(X > 2) = 1 \quad (15)$$

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