

12.13.3.25

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The probability distribution of a random variable X is given below:

X	0	1	2	3
P(X)	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

- 1) Determine the value of k.
- 2) Determine $P(X \leq 2)$ and $P(X > 2)$.
- 3) Find $P(X \leq 2) + P(X > 2)$.

Solution:

- 1) **Value of k:** The cumulative distribution function of X is,

$$F_X(k) = \Pr(X \leq k) \quad (1)$$

$$= \sum_{i=0}^k p_X(i) \quad (2)$$

As X can only take values to 3, we can say that,

$$F_X(k \geq 3) = 1 \quad (3)$$

$$\sum_{i=0}^k p_X(k) = 1 \quad (4)$$

$$\Rightarrow p_X(0) + p_X(1) + p_X(2) + p_X(3) = 1 \quad (5)$$

$$\Rightarrow k + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} = 1 \quad (6)$$

$$\Rightarrow \frac{15k}{8} = 1 \quad (7)$$

$$\Rightarrow k = \frac{8}{15} \quad (8)$$

Hence, the value of k is $\frac{8}{15}$. This makes the data given in the question as follows,

$$p_X(k) = \begin{cases} \frac{8}{15} & \text{if } k = 0 \\ \frac{4}{15} & \text{if } k = 1 \\ \frac{2}{15} & \text{if } k = 2 \\ \frac{1}{15} & \text{if } k = 3 \\ 0 & \text{Otherwise} \end{cases} \quad (9)$$

Using the value of k we can also find CDF of X,

$$F_X(k) = \begin{cases} 0 & \text{if } k < 0 \\ \frac{8}{15} & \text{if } k = 0 \\ \frac{4}{5} & \text{if } k = 1 \\ \frac{14}{15} & \text{if } k = 2 \\ 1 & \text{if } k \geq 3 \end{cases} \quad (10)$$

- 2) **Value of $P(X \leq 2)$ and $P(X > 2)$:**

- a) **Value of $P(X \leq 2)$:** $\Pr(X \leq 2)$ translates to sum of probabilities of all states of the random variable that are less than or equal to 2. From (10),

$$\Pr(X \leq 2) = F_X(2) \quad (11)$$

$$= \frac{14}{15} \quad (12)$$

- b) **Value of $P(X > 2)$:** $P(X > 2)$ translates to sum of probabilities of all states of the random variable greater than 2. In other words, it can be obtained by removing the probability of X being less than or equal to 2 from 1. From (10),

$$\Pr(X > 2) = 1 - F_X(2) \quad (13)$$

$$= 1 - \frac{14}{15} \quad (14)$$

$$= \frac{1}{15} \quad (15)$$

- 3) **Value of $P(X \leq 2) + P(X > 2)$:** From (12) and (15) we can easily say that,

$$\Pr(X \leq 2) + \Pr(X > 2) = \frac{14}{15} + \frac{1}{15} \quad (16)$$

$$\Rightarrow = 1 \quad (17)$$

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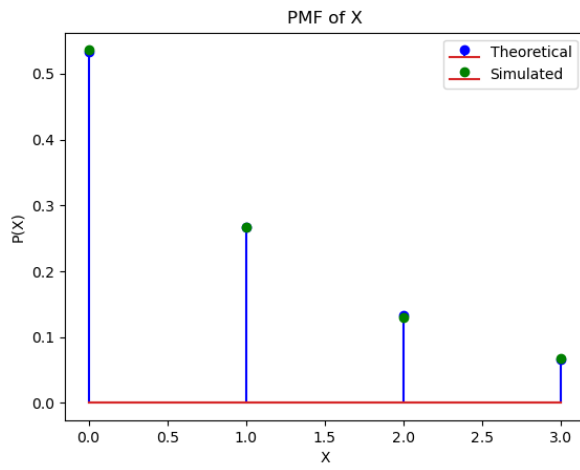


Fig. 3. Generated using (1)

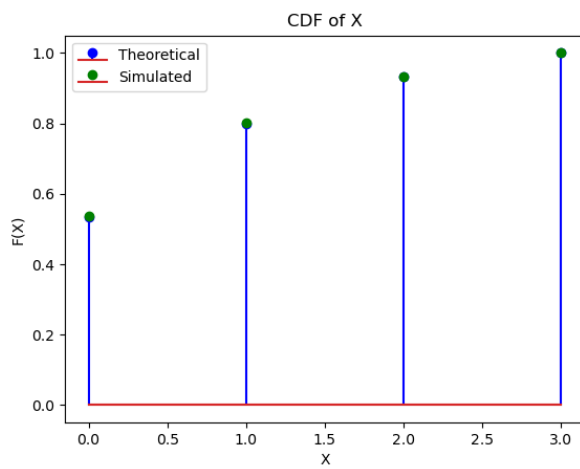


Fig. 3. Generated using (10)