12.13.3.62

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If A and B are two events and A $\neq \phi$, B $\neq \phi$, then

1) $\Pr(A|B) = \Pr(A) \,.\, \Pr(B)$
2) $\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$
3) $\Pr(A|B) \,.\, \Pr(B|A) = 1$
4) $\Pr(A|B) = \frac{\Pr(A)}{\Pr(B)}$

Solution:

Let us take an example. The following table describes events when a die is rolled.

Event	Description	Probability
A	Even number shows up	$\frac{1}{2}$
B	Perfect number shows up	$\frac{1}{3}$
A.B	Both Event A and B happen	$\frac{1}{6}$

1)

$$\Pr\left(\frac{A}{B}\right) = \frac{1}{2} \tag{1}$$

$$\Pr(A) \,.\, \Pr(B) = \frac{1}{2} \times \frac{1}{3} \tag{2}$$

$$= \frac{1}{6} \tag{3}$$

As (1) and (3) are not equal, option 1 is not correct.

2)

$$\frac{\Pr(A.B)}{\Pr(B)} = \frac{\frac{1}{6}}{\frac{1}{3}} \tag{4}$$

$$= \frac{1}{2} \tag{5}$$

As (1) and (5) are equal, option 2 is correct.

$$\Pr\left(\frac{A}{B}\right) = \frac{\Pr(A.B)}{\Pr(B)} \tag{6}$$

3) From (5),

$$\Pr\left(\frac{A}{B}\right) \,.\, \Pr\left(\frac{B}{A}\right) = \frac{\Pr(A.B)}{\Pr(B)} \times \frac{\Pr(A.B)}{\Pr(A)} \tag{7}$$

$$= \frac{\Pr(A.B)^2}{\Pr(A) \,.\, \Pr(B)} \tag{8}$$

$$\neq 1 \tag{9}$$

Hence, option 3 is incorrect. Let us verify it using the example. From (5),

$$\frac{\Pr(A.B)^2}{\Pr(A) \,.\, \Pr(B)} = \frac{\frac{1}{6}^2}{\frac{1}{2} \times \frac{1}{3}} \tag{10}$$

$$= \frac{1}{6} \tag{11}$$

Hence, option 3 is incorrect.

4)

$$\frac{\Pr(A)}{\Pr(B)} = \frac{\frac{1}{2}}{\frac{1}{3}} \tag{12}$$

$$= \frac{3}{2} \tag{13}$$

As (1) and (13) are not equal, option 4 is incorrect.

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