ASSIGNMENT-1

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Question : Suppose the equations *AB, BC* and *CA* are respectively given by

$$\mathbf{n}_i^\top \mathbf{x} = c_i \qquad i = 1, 2, 3 \tag{1}$$

The equations of the respective angle bisectors are given by

$$\frac{\mathbf{n}_i^\top \mathbf{x} - c_i}{\|\mathbf{n}_i\|} = \pm \frac{\mathbf{n}_j^\top \mathbf{x} - c_j}{\left\|\mathbf{n}_j\right\|} \qquad i \neq j \tag{2}$$

Substitute numerical values and find the equations of the angle bisectors of *A*,*B* and *C*.

Solution :

The parametric equations of sides;

$$\text{BC}, \quad is \quad \begin{pmatrix} 11 & 1 \end{pmatrix} \mathbf{x} = -38, \tag{3}$$
$$\text{CA}, \quad is \quad \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 2, \tag{4}$$
$$\text{AB}, \quad is \quad \begin{pmatrix} 7 & 5 \end{pmatrix} \mathbf{x} = 2 \tag{5}$$
$$\tag{6}$$

Using the formula mentioned in the question to find out the angular bisector for sides AB and AC, naming the angular bisector **d** we get,

$$\frac{\mathbf{n}_3^\top \mathbf{x} - c_3}{\|\mathbf{n}_3\|} = \pm \frac{\mathbf{n}_2^\top \mathbf{x} - c_2}{\|\mathbf{n}_2\|} \tag{7}$$

As we can see we'll get 2 solutions for **d**. This is because one of them is internal angular bisector and the other is the external angular bisector. Internal angular bisector can be evaluated if we take "+" in the above formula.

Hence, **d** is given by,

$$\frac{\mathbf{n}_3^\top \mathbf{x} - c_3}{\|\mathbf{n}_3\|} = \frac{\mathbf{n}_2^\top \mathbf{x} - c_2}{\|\mathbf{n}_2\|} \tag{8}$$
$$\left(\frac{\mathbf{n_3}}{\|\mathbf{n_3}\|} - \frac{\mathbf{n_3}}{\|\mathbf{n_3}\|} \right) \mathbf{x} = \left(\frac{c_3}{\|\mathbf{n_3}\|} - \frac{c_2}{\|\mathbf{n_2}\|} \right) \tag{9}$$
$$\left(\frac{\begin{pmatrix} 7 & 5 \end{pmatrix}}{\sqrt{74}} - \frac{\begin{pmatrix} 1 & -1 \end{pmatrix}}{\sqrt{2}} \right) \mathbf{x} = \frac{2}{\sqrt{74}} - \frac{2}{\sqrt{2}} \tag{10}$$
$$\begin{pmatrix} \frac{7-\sqrt{37}}{\sqrt{74}} & \frac{5+\sqrt{37}}{\sqrt{74}} \end{pmatrix} \mathbf{x} = \frac{2 - 2\sqrt{37}}{\sqrt{74}} \tag{11}$$

Hence, the internal angluar bisector of angle A, **d** will be,

$$\begin{pmatrix} \frac{7-\sqrt{37}}{\sqrt{74}} & \frac{5+\sqrt{37}}{\sqrt{74}} \end{pmatrix} \mathbf{x} = \frac{2 - 2\sqrt{37}}{\sqrt{74}} \tag{12}$$

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