12.13.3.25

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The probability distribution of a random variable X is given below:

X	0	1	2	3
$P(X)$	k	$\frac{k}{2}$	$\frac{k}{4}$	$\frac{k}{8}$

1) Determine the value of k.
2) Determine $P(X \leq 2)$ and $P(X > 2)$.
3) Find $P(X \leq 2) + P(X > 2)$.

Solution:

1) The cumulative distribution function of X is,

$$F_X(k) = \Pr(X \leq k) \tag{1}$$
$$= \sum_{i=0}^{k} p_X(i) \tag{2}$$

As X can only take values to 3, we can say that,

$$F_X(k \geq 3) = 1 \tag{3}$$
$$\sum_{i=0}^{k} p_X(k) = 1 \tag{4}$$
$$\implies p_X(0) + p_X(1) + p_X(2) + p_X(3) = 1 \tag{5}$$
$$\implies k + \frac{k}{2} + \frac{k}{4} + \frac{k}{8} = 1 \tag{6}$$
$$\implies \frac{15k}{8} = 1 \tag{7}$$
$$\implies k = \frac{8}{15} \tag{8}$$

Hence, the value of k is $\frac{8}{15}$. This makes the data given in the question as follows,

$$p_X(k) = \begin{cases} \frac{8}{15} & \text{if } k = 0 \\ \frac{4}{15} & \text{if } k = 1 \\ \frac{2}{15} & \text{if } k = 2 \\ \frac{1}{15} & \text{if } k = 3 \\ 0 & \text{Otherwise} \end{cases} \tag{9}$$

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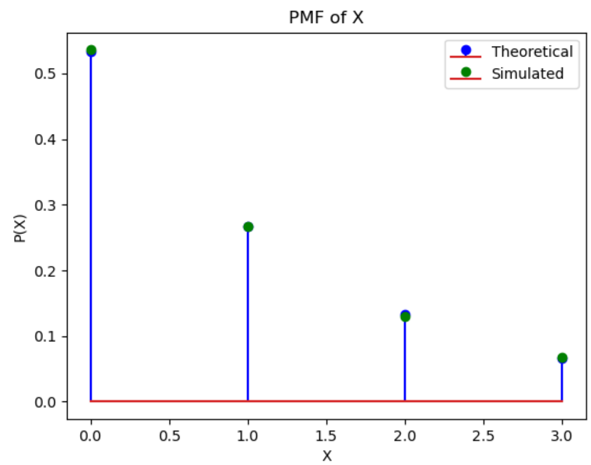

Fig. 3. Generated using (1)

Using the value of k we can also find CDF of X,

$$F_X(k) = \begin{cases} 0 & \text{if } k < 0 \\ \frac{8}{15} & \text{if } k = 0 \\ \frac{4}{5} & \text{if } k = 1 \\ \frac{14}{15} & \text{if } k = 2 \\ 1 & \text{if } k \geq 3 \end{cases} \tag{10}$$

2)

$$\Pr(X \leq 2) = F_X(2) \tag{11}$$
$$= \frac{14}{15} \tag{12}$$

$$\Pr(X > 2) = 1 - F_X(2) \tag{13}$$
$$= 1 - \frac{14}{15} \tag{14}$$
$$= \frac{1}{15} \tag{15}$$

3)

$$\Pr(X \leq 2) + \Pr(X > 2) = F_X(2) + 1 - F_X(2) \tag{16}$$
$$= 1 \tag{17}$$

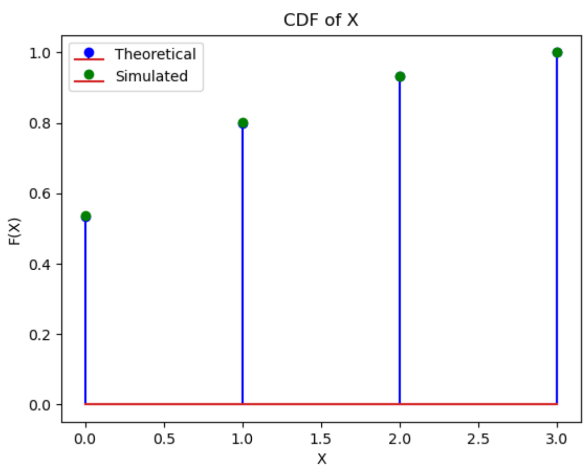

Fig. 3. Generated using (10)