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If A and B are two events and A $\neq \phi$, B $\neq \phi$, then

1) $\Pr(A|B) = \Pr(A) \,.\, \Pr(B)$
2) $\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$
3) $\Pr(A|B) \,.\, \Pr(B|A) = 1$
4) $\Pr(A|B) = \frac{\Pr(A)}{\Pr(B)}$

Solution:

Let us take an example. The following table describes events when a die is rolled.

Event	Description	Probability
A	Even number shows up	$\frac{1}{2}$
B	Perfect number shows up	$\frac{1}{3}$
AB	Both Event A and B happen	$\frac{1}{6}$

1) As $\Pr(A|B)$ represents the probability of occurence of A given that B has occured. Hence,

$$\Pr(A|B) = \frac{1/6}{1/3} \tag{1}$$

$$= \frac{1}{2} \tag{2}$$

$$\Pr(A) \,.\, \Pr(B) = \frac{1}{2} \times \frac{1}{3} \tag{3}$$

$$= \frac{1}{6} \tag{4}$$

As RHS of (2) and (4) are not equal, option 1 is not correct.

2)

$$\frac{\Pr(AB)}{\Pr(B)} = \frac{\frac{1}{6}}{\frac{1}{3}} \tag{5}$$

$$= \frac{1}{2} \tag{6}$$

As (2) and (6) are equal, option 2 is correct.

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \tag{7}$$

Equation (7) is one of the axioms of probability.

3) From (7),

$$\Pr(A|B) \,.\, \Pr(B|A) = \frac{\Pr(AB)}{\Pr(B)} \times \frac{\Pr(AB)}{\Pr(A)} \tag{8}$$

$$= \frac{\Pr(AB)^2}{\Pr(A) \,.\, \Pr(B)} \tag{9}$$

$$\neq 1 \tag{10}$$

Hence, option 3 is incorrect. Let us verify it using the example.

$$\frac{\Pr(AB)^2}{\Pr(A) \,.\, \Pr(B)} = \frac{\frac{1}{6}^2}{\frac{1}{2} \times \frac{1}{3}} \tag{11}$$

$$= \frac{1}{6} \tag{12}$$

Hence, option 3 is incorrect.

4)

$$\frac{\Pr(A)}{\Pr(B)} = \frac{\frac{1}{2}}{\frac{1}{3}} \tag{13}$$

$$= \frac{3}{2} \tag{14}$$

As (2) and (14) are not equal, option 4 is incorrect.

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