CBSE MATHEMATICS 2020

G V V Sharma*

1 Matrices

1.1. Show that the plane $x - 5y - 2z = 1$ contains the line $\frac{x-5}{3} = y = 2 - z$.

1.2. Find a vector $\overrightarrow{r}$ equally inclined to the three axes and whose magnitude is $3\sqrt{3}$ units.

1.3. Find the angle between unit vectors $\overrightarrow{a}$ and $\overrightarrow{b}$ so that $\sqrt{3}$ $\overrightarrow{a}$ - $\overrightarrow{b}$ is also a unit vector.

1.4. If $A = \begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, Find scalar k so that $A^2 + I = kA$.

1.5. Find the coordinates of the point where the line

$$\frac{x-1}{3} = \frac{y+4}{7} = \frac{z+4}{2}$$

cuts the xy-plane.

1.6. The angle between the vectors $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$ is

a) $\frac{-\pi}{3}$
b) 0
c) $\frac{\pi}{3}$
d) $\frac{2\pi}{3}$

1.7. If A is a non-singular square matrix of order 3 such that $A^2 = 3A$, then value of $\left|A\right|$ is

a) -3
b) 3
c) 9
d) 27

1.8. If $\left|\overrightarrow{a}\right| = 4$ and $-3 \leq \lambda \leq 2$ then $\left|\lambda\overrightarrow{a}\right|$ lies in

a) $[0, 12]$
b) $[2, 3]$
c) $[8, 12]$
d) $[-12, 8]$

1.9. The area of a triangle formed by vertices O, A and B, where $\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\overrightarrow{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$ is

a) $3\sqrt{5}$ sq.units

b) $5\sqrt{5}$ sq.units

c) $6\sqrt{5}$ sq.units

d) 4 sq.units

1.10. The coordinates of the foot of the perpendicular drawn from the point $(2, -3, 4)$ on the y-axis is

a) $(2, 3, 4)$
b) $(-2, -3, -4)$
c) $(0, -3, 0)$
d) $(2, 0, 4)$

1.11. The distance between parallel planes 2x+y-2z-6=0 and 4x+2y-4z=0 is ———————— units.

1.12. If P(1,0,-3) is the foot of the perpendicular from the origin to the plane, then the Cartesian equation of the plane is ....

1.13. Find the equation of the plane passing through the points $(1, 0, -2)$, $(3, -1, 0)$ and perpendicular to the plane $2x - y + z = 8$. Also find the distance of the plane thus obtained from the origin.

1.14. If

$$\begin{vmatrix} 2x & -9 \\ -2 & x \end{vmatrix} = \begin{vmatrix} -4 & 8 \\ 1 & -2 \end{vmatrix}$$

then value of x is —————-

1.15. Using integration, Find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside the parabola $y^2 = 4x$.

1.16. Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are A(2,0), B(4,5) and C(6,3).

1.17. If $A = \begin{pmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{pmatrix}$, Find A^{-1} and use it to solve the following system of the equations:

$$5x - y + 4z = 5$$
$$2x + 3y + 5z = 2$$
$$5x - 2y + 6z = -1$$

1.18. If x,y,z are different and

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$

then using properties of determinants show that

$$1 + xyz = 0$$

2 Continuous Math

2.1. Evaluate :

$$\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\, dx$$

Solution: Let

$$\tan\theta = \sin x \tag{2.1.1}$$

Then

$$\sec^2\theta\, d\theta = \cos x\, dx \tag{2.1.2}$$

From (2.1.1) and (2.1.2)

$$\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\ dx$$
$$= 2\int_0^{\frac{\pi}{4}} \theta \tan\theta \sec^2\theta\, d\theta \tag{2.1.3}$$

Letting

$$u = \theta, dv = \tan\theta \sec^2\theta\, d\theta,$$
$$v = \int \tan\theta\sec^2\theta\, d\theta$$
$$= \int t\, dt \quad (t = \tan\theta)$$
$$= \frac{t^2}{2} = \frac{\tan^2\theta}{2} \tag{2.1.4}$$

Thus,

$$v\, du = \frac{\tan^2\theta}{2}\, d\theta$$
$$\implies \int v\, du = \int \frac{\tan^2\theta}{2}\, d\theta$$
$$= \int \frac{\sec^2\theta - 1}{2}\, d\theta$$
$$= \frac{\tan\theta - 1}{2} \tag{2.1.5}$$

and

$$\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)\ dx$$
$$= \left[\theta\frac{\tan^2\theta}{2} - \frac{\tan\theta - 1}{2}\right]_0^{\frac{\pi}{4}}$$
$$= \frac{\pi}{8} - \frac{1}{2} \tag{2.1.6}$$

2.2. Prove that

$$\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\sin^{-1}\left(\frac{4}{5}\right)$$

Solution:

$$\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \tan^{-1}\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}} \tag{2.2.1}$$
$$= \tan^{-1}\frac{1}{2} \tag{2.2.2}$$
$$= \frac{1}{2}\tan^{-1}\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2} \tag{2.2.3}$$
$$= \frac{1}{2}\tan^{-1}\frac{4}{3} \tag{2.2.4}$$
$$= R.H.S \tag{2.2.5}$$

2.3. Differentiate $\sec^2(x^2)$ with respect to x^2.
Solution:

$$\frac{\sec^2(x^2)}{d\left(x^2\right)} = 22\sec(x^2)\tan(x^2) \tag{2.3.1}$$

2.4. If $y = f(x^2)$ and $f'(x) = e^{\left(\sqrt{x}\right)}$, then find $\frac{dy}{dx}$.
Solution:

$$\frac{dy}{dx} = 2xf'(x) = 2xe^{\sqrt{x}} \tag{2.4.1}$$

2.5. If

$$\tan^{-1}\left(\frac{y}{x}\right) = \log\sqrt{x^2+y^2}$$

prove that

$$\frac{dy}{dx} = \frac{x+y}{x-y} \tag{2.5.1}$$

. **Solution:** Let

$$y = x\tan\theta \tag{2.5.2}$$

$$\implies \frac{dy}{dx} = \tan\theta + x\sec^2\theta\frac{d\theta}{dx} \tag{2.5.3}$$

Then, (2.5.1) can be expressed as

$$\theta = \log\left(x\sec\theta\right) \tag{2.5.4}$$

$$\implies \frac{d\theta}{dx} = \frac{1}{x\sec\theta}\left(\sec\theta + x\sec\theta\tan\theta\frac{d\theta}{dx}\right) \tag{2.5.5}$$

$$= \frac{1}{x}\left(1 + x\tan\theta\frac{d\theta}{dx}\right) \tag{2.5.6}$$

$$\text{or, } \frac{d\theta}{dx} = \frac{1}{x\left(1-\tan\theta\right)} \tag{2.5.7}$$

From (2.5.3) and (**??**)

$$\frac{dy}{dx} = \tan\theta + \frac{\sec^2\theta}{\left(1-\tan\theta\right)} \tag{2.5.8}$$

$$= \frac{1+\tan\theta}{1-\tan\theta} \tag{2.5.9}$$

$$= \frac{x+y}{x-y} \tag{2.5.10}$$

upon substituting from (2.5.1) and simplifying.

2.6. If

$$y = e^{\left(a\cos^{-1}x\right)}, -1 < x < 1$$

then show that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$$

Solution: From the given information,

$$y_1 = -\frac{ae^{\left(a\cos^{-1}x\right)}}{\sqrt{1-x^2}} \tag{2.6.1}$$

$$= -\frac{ay}{\sqrt{1-x^2}} \implies \sqrt{1-x^2}y_1 + ay \quad = 0 \tag{2.6.2}$$

Hence, differentiating the above equation,

$$\sqrt{1-x^2}y_2 - \frac{2xy_1}{\sqrt{1-x^2}} + ay_1 = 0 \tag{2.6.3}$$

$$\implies \left(1-x^2\right)y_2 - 2xy_1 - a^2y = 0 \tag{2.6.4}$$

2.7. If $\cos(\sin^{-1}\frac{2}{\sqrt{5}} + \cos^{-1}x) = 0$, then x is

a) $\frac{1}{\sqrt{5}}$

b) $\frac{-2}{\sqrt{5}}$

c) $\frac{2}{\sqrt{5}}$

d) 1

Solution: Using a simplistic approach, $\because \cos\frac{\pi}{2} = 0$,

$$\sin^{-1}\frac{2}{\sqrt{5}} + \cos^{-1}x = \frac{\pi}{2} \tag{2.7.1}$$

$$\implies \frac{\pi}{2} - \cos^{-1}x = -\sin^{-1}\frac{2}{\sqrt{5}} \tag{2.7.2}$$

$$\text{or, } \sin^{-1}x = \sin^{-1}\left(-\frac{2}{\sqrt{5}}\right) \tag{2.7.3}$$

$$\implies x = -\frac{2}{\sqrt{5}} \tag{2.7.4}$$

2.8. The interval in which the function f given by $f(x) = x^2e^{-x}$ is strictly increasing, is

a) $(\infty, -\infty)$
b) $(\infty, 0)$
c) $(2, \infty)$
d) $(0, 2)$

Solution: Taking the derivative

$$f'(x) = 2xe^{-x} - x^2e^{-x} \tag{2.8.1}$$

$$= \left(2x - x^2\right)e^{-x} \tag{2.8.2}$$

and

$$f'(x) > 0 \tag{2.8.3}$$

$$\implies x\left(2-x\right) > 0 \tag{2.8.4}$$

$$\text{or, } x \in (0, 2) \tag{2.8.5}$$

2.9. The function $f\left(x\right) = \frac{x-1}{x(x^2-1)}$ is discontinuous at

a) Exactly one point
b) Exactly two points
c) Exactly three points
d) No point

Solution: The given function can be expressed as

$$f\left(x\right) = \frac{x-1}{x\left(x-1\right)\left(x+1\right)} \tag{2.9.1}$$

Hence, the denominator of the given function vanishes at $x = 0, 1, -1$, which are the points of discontinuity. However, there is a removable discontinuity at $x = 1$.

2.10. The function $f : R \to [-1, 1]$ defined by $f\left(x\right) = \cos x$ is

a) Both one-one and onto
b) Not one-one, but onto
c) one-one, but Not onto
d) Neither one-one, nor onto

Solution:

$$\cos(-2\pi) = \cos(2\pi) = 1 \tag{2.10.1}$$

Hence, $f(x)$ is not one to one. However, it is onto.

2.11. If the radius of the circle is increasing at the rate of 0.5cm/s, then the rate of increase of its circumference is ———
Solution: Let the p be the circumference and r the radius. Then

$$p = 2\pi r \tag{2.11.1}$$
$$\implies \frac{dp}{dt} = 2\pi\frac{dr}{dt} = \pi \tag{2.11.2}$$

2.12. a) The range of the principle value branch of the function $y = \sec^{-1} x$ is ———-
Solution: The range is $[0,\pi] - \left\{\frac{\pi}{2}\right\}$.
b) The principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is ———
Solution: Since

$$\cos\frac{\pi}{3} = \frac{1}{2}, \tag{2.12.1}$$
$$\cos\left(\pi - \frac{\pi}{3}\right) = -\frac{1}{2} \tag{2.12.2}$$

Thus the desired principal value is $\frac{2\pi}{3}$

2.13. Evaluate :

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x\cos^2 x dx$$

2.14. Find the value of k, so that the function

$$f(x) = \begin{cases} \text{k}x^2\text{+ 5 ,} & \text{if x} \le 1 \\ 2, & \text{if x} > 1 \end{cases}$$

is continuous at x=1.

2.15. Find the integrating factor of the differential equation

$$x\frac{dy}{dx} = 2x^2 + y$$

.

2.16. If

$$f(x) = \sqrt{\frac{secx - 1}{secx + 1}}$$

Find

$$f'\left(\frac{\pi}{3}\right)$$

.

2.17. Find $f'(x)$ if

$$f(x) = (\tan x)^{(\tan x)}$$

2.18. Find :

$$\int \frac{\tan^3 x}{\cos^3 x} dx$$

2.19. Solve the following differential equation:

$$(1 + e^{\frac{y}{x}})dy + e^{\frac{y}{x}(1-\frac{y}{x})}dx = 0; (x \neq 0)$$

3 Discrete Math

1) The relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2)(2, 1)(1, 1)\}$ is
   a) Symmetric and transitive, but not reflexive
   b) reflexive and symmetric, but not transitive
   c) Symmetric, but neither reflexive nor transitive
   d) An equivalence relation
2) Check whether the relation R in the set N set of natural numbers given by R= {(a,b):a is divisor of b} is reflexive, symmetric or transitive. Also determine whether R is an equivalence relation.

4 Probability

1) A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn randomly one-by-one without replacement and are found to to be both kings. Find the probability of the lost card being a king.
2) A fair dice is thrown two times. Find the probability distribution of the number of sixes. Also determine the mean of the number of sixes.

5 Linear Programming

1) The corner points of the feasible region of an LPP are (0,0),(0,8),(2,7),(5,4) and (6,0). The maximum profit P=3x+2y occurs at the point ....
2) A cottage industry manufactures pedestal lamps and wooden shades. Both the products

require machine time as well as craftsman time in the making. The number of hours required for producing 1 unit of each and the corresponding profit is given in the following table : In a day, the factory has availability of

Item	Machine Time	Craftsman Time	Profit(in INR)
Pedestal Lamp	1.5 hours	3 hours	30
Wooden shades	3 hours	1 hour	20

TABLE 2

not more than 42 hours of machine time and 24 hours of craftsman time.
Assuming that all items manufactured are sold, how should the manufacturer schedule his daily production in order to maximise the profit? Formulate it as an LPP and solve it graphically.

3) Amongst all open (from the top) right circular cylindrical boxes of volume $125\pi \ cm^3$, find the dimensions of the box which has the least surface area.