CBSE 10, 2020

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***Abstract*—This manual provides an introduction to vectors and their properties, based on the question papers, year 2020, from Class 10 and 12, CBSE; JEE and JNTU.**

1 Discrete Math

1.1. Show that $5 + 2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number.

1.2. Check whether 12^n can end with the digit 0 for any natural number n.
Solution: If 12^n ends with the digit 0,

$$12^n \equiv 0 \pmod{10} \tag{1.2.1}$$
$$\implies 12^n \equiv 0 \pmod{2} \tag{1.2.2}$$
$$12^n \equiv 0 \pmod{5} \tag{1.2.3}$$

However,

$$12 \not\equiv 0 \pmod{5} \tag{1.2.4}$$
$$12^2 \not\equiv 0 \pmod{5} \tag{1.2.5}$$

Let

$$12^{n-1} \not\equiv 0 \pmod{5} \tag{1.2.6}$$

Then, it is obvious from (1.2.4) and (1.2.6) that

$$12.12^{n-1} \not\equiv 0 \pmod{5} \tag{1.2.7}$$
$$\implies 12^n \not\equiv 0 \pmod{5} \tag{1.2.8}$$

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1.3. Use Euclid Division Lemma to show that the square of any positive integer is either in the form $3q$ or $3q + 1$ for some integer q.
Solution: Let p be any positive integer. Then p can be either $3k, 3k + 1$ or $3k - 1$, for some postive integer k. If

$$p = 3k, \tag{1.3.1}$$
$$p^2 = 9k^2 = 3(q) \tag{1.3.2}$$
$$\text{where } q = 3k^2 \tag{1.3.3}$$

If

$$p = 3k + 1, \tag{1.3.4}$$
$$p^2 = 9k^2 + 6k + 1 = 3(q) + 1, \tag{1.3.5}$$
$$\text{where } q = 3k^2 + 2k \tag{1.3.6}$$

Similarly, if

$$p = 3k - 1, \tag{1.3.7}$$
$$p^2 = 9k^2 - 6k + 1 = 3(q) + 1, \tag{1.3.8}$$
$$\text{where } q = 3k^2 - 2k \tag{1.3.9}$$

1.4. Find the sum of the first 100 natural numbers.
Solution: The sum of the first n natural number is

$$\frac{n(n+1)}{2} \tag{1.4.1}$$

Sustituting $n = 100$ in the above, the desired sum is

$$50 \times 101 = 5050 \tag{1.4.2}$$

1.5. The LCM of 2 numbers is 182 and their HCF is 13. If one of the numbers is 26, find the other.
Solution: The desired number is obtained as

$$\frac{182 \times 13}{26} = 91 \tag{1.5.1}$$

1.6. Which of the following is not an A.P. ?
a) $-1.2, 0.8, 2.8...$
b) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}...$
c) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}...$

d) $\frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}$

Solution: a_0, a_1, a_2 can be terms of an AP only if

$$a_1 - a_0 = a_2 - 2a_1 \tag{1.6.1}$$

Considering each of the above cases,

a)

$$2.8 - 1.2 = 1.6 = 2(0.8) \tag{1.6.2}$$

Hence, the given terms are in A.P.

b)

$$3 + \sqrt{2} - 3 = \sqrt{2} \tag{1.6.3}$$
$$3 + 2\sqrt{2} - \left(3 + \sqrt{2}\right) = \sqrt{2} \tag{1.6.4}$$
$$3 + 3\sqrt{2} - \left(3 + 2\sqrt{2}\right) = \sqrt{2} \tag{1.6.5}$$

Hence, the given terms are in A.P.

c)

$$\frac{7}{3} - \frac{4}{3} = 1 \tag{1.6.6}$$
$$\frac{9}{3} - \frac{7}{3} = \frac{2}{3} \tag{1.6.7}$$
$$\frac{12}{3} - \frac{9}{3} = 1 \tag{1.6.8}$$

Hence, the given terms are not in A.P.

d)

$$-\frac{2}{5} + \frac{1}{5} = -\frac{1}{5} \tag{1.6.9}$$
$$-\frac{3}{5} + \frac{2}{5} = -\frac{1}{5} \tag{1.6.10}$$

Hence, the given terms are in A.P.

1.7. Find the sum

$$\left(-5\right) + \left(-8\right) + \left(-11\right) + ... + \left(-230\right) \tag{1.7.1}$$

Solution: The above series is an A.P. with

$$a_0 = -5, a_n = -230, d = -3 \tag{1.7.2}$$

Since

$$a_n = a_0 + (n-1)\,d, \tag{1.7.3}$$
$$n = \frac{a_n - a_0}{d} + 1 \tag{1.7.4}$$
$$= 76 \tag{1.7.5}$$

Thus the desired sum is given by

$$S_n = \frac{n\left(a_n + a_0\right)}{2} \tag{1.7.6}$$
$$= -235 \times 38 = -8930 \tag{1.7.7}$$

2 Arithmetic

2.1. The radius of a sphere (in cm), whose volume is $12\pi cm^3$, is

a) 3
b) $3\sqrt{2}$
c) $3^{\frac{2}{3}}$
d) $3^{\frac{1}{3}}$

Solution: The volume of a sphere, given the radius r, is given by

$$V = \frac{4}{3}\pi r^3 \tag{2.1.1}$$

Hence the radius is

$$r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}} \tag{2.1.2}$$
$$= 3^{\frac{2}{3}} \tag{2.1.3}$$

2.2. In a cylindrical vessel of radius 10cm, containing some water, 9000 small spherical balls are dropped which are completely immersed in water which raises the water level. If each spherical ball is of radius 0.5 cm then find the rise in the level of water in the vessel.

Solution: The various parameters considered in this problem are listed in Table (2.2.1).

Symbol	Value	Description
R	10	Cylinder Radius
r	0.5	Ball Radius
N	9000	Number of balls
h	?	Rise level

TABLE 2.2.1

From the given information, the volume of all the balls will be equal the increase in the volume of water in the vessel. Thus,

$$N \times \frac{4}{3}\pi r^3 = \pi R^2 h \tag{2.2.1}$$
$$\implies h = \frac{4r^3}{3R^2}N \tag{2.2.2}$$
$$= 15 \tag{2.2.3}$$

2.3. A cone and a cylinder have the same radii, but the height of the cone is 3 times that of the cylinder. Find the ratio of their volumes

Solution: Let $h_i, V_i, i = 1, 2$ be the respective heights and volumes of the cone and the cylinder. Then

$$V_1 = \frac{1}{3}\pi r^2 h_1 \tag{2.3.1}$$
$$V_2 = \pi r^2 h_2 \tag{2.3.2}$$
$$\implies \frac{V_1}{V_2} = \frac{h_1}{3h_2} \tag{2.3.3}$$
$$= 1 \tag{2.3.4}$$

$\because h_1 = 3h_2$.

3 Algebra

1) A motorboat whose speed is 18 kmph in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.
Solution: The input parameters are given in Table (1.1).

Symbol	Value	Description
v	18	Speed of the boat in still water
t	1	Excess time upstream
x	?	Speed of the stream

TABLE 1.1

$$\frac{d}{v-x} = \frac{d}{x+v} + t \tag{1.1}$$

where d is the distance traveled upstream. From the above equation,

$$\frac{2xd}{v^2-x^2} = t \tag{1.2}$$
$$\implies tx^2 + 2xd - tv^2 = 0 \tag{1.3}$$
$$\text{or, } x = -d \pm \sqrt{d^2 + t^2v^2} \tag{1.4}$$
$$= 6 \tag{1.5}$$

2) On dividing a polynomial $p(x)$ by $x^2 - 4$, the quotient and remainder are found to be x and 3 respectively. The polynomial $p(x)$ is
Solution: The input parameters for this problem are available in (2.1).

Symbol	Value	Description
$d(x)$	$x^2 - 4$	Divisor
$q(x)$	x	Quotient
$r(x)$	3	Remainder
$p(x)$	?	Desired Polynomial

TABLE 2.1

In general, the polynomial

$$p(x) = d(x)q(x) + r(x) \tag{2.1}$$
$$= \left(x^2 - 4\right)x + 3 \tag{2.2}$$
$$= x^3 - 4x + 3 \tag{2.3}$$

3) Form a quadratic polynomial, the sum and product of whose zeros are $\left(-3\right)$ and 2 respectively.
Solution: The desired quadratic polynomial is

$$x^2 - (-3)\,x + 2 = 0 \tag{3.1}$$
$$\implies x^2 + 3x + 2 = 0 \tag{3.2}$$

4) Can $\left(x^2 - 1\right)$ be a remainder while dividing $x^4 - 3x^2 + 5x - 9$ by $\left(x^2 + 3\right)$? Justify the reasons.
Solution:

$$\begin{array}{r l}
 & \quad x^2 \qquad\quad -6 \\ \cline{2-2}
x^2+3) & x^4 - 3x^2 + 5x \;\; -9 \\
 & -x^4 - 3x^2 \\ \cline{2-2}
 & \qquad -6x^2 + 5x \;\; -9 \\
 & \qquad\;\; 6x^2 \qquad +18 \\ \cline{2-2}
 & \qquad\qquad\quad 5x \;\; +9
\end{array}$$

which implies that the remainder is $5x + 9$ resulting in a contradiction. Hence, the given statement is not true.

5) The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is
   a) 4
   b) ± 4
   c) -4
   d) 0

Solution: A quadratic equation

$$ax^2 + bx + c = 0 \tag{5.1}$$

has equal roots only if the discriminant

$$b^2 - 4ac = 0 \tag{5.2}$$

Substituting

$$a = 2, b = k, c = 2, \tag{5.3}$$
$$\implies k^2 - 4 = 0 \tag{5.4}$$
$$\text{or, } k = \pm 4 \tag{5.5}$$

6) Obtain other zeros of the polynomial

$$p\left(x\right) = 2x^4 - x^3 - 11x^2 + 5x + 5 \tag{6.1}$$

if the two of its zeros are $\sqrt{5}$ and $-\sqrt{5}$
Solution: The given polynomial can be expressed as

$$p(x) = \left(x - \sqrt{5}\right)\left(x + \sqrt{5}\right)\left(ax^2 + bx + c\right) \tag{6.2}$$

$$= \left(x^2 - 5\right)\left(ax^2 + bx + c\right) \tag{6.3}$$

$$= ax^4 + bx^3 + (c - 5a)\,x^2 - 5bx - 5c \tag{6.4}$$

Comparing the above with (6.1),

$$a = 2, b = c = -1 \tag{6.5}$$

Thus we need to find the zeros of the polynomial

$$2x^2 - x - 1 = 0 \tag{6.6}$$

$$\implies x = 1, -\frac{1}{2} \tag{6.7}$$

7) What minimum must be added to $2x^3 - 3x^2 + 6x + 7$ so that the resulting polynomial will be divisible by $x^2 - 4x + 8$?
Solution: From the following division,

$$\begin{array}{r|rrrr} & & & 2x & +5 \\ \hline x^2 - 4x + 8) & 2x^3 & -3x^2 & +6x & +7 \\ & -2x^3 & +8x^2 & -16x & \\ \hline & & 5x^2 & -10x & +7 \\ & & -5x^2 & +20x & -40 \\ \hline & & & 10x & -33 \end{array}$$

it is obvious that $33 - 10x$ needs to be added.

4 Trigonometry

4.1. Write the value of $\sin^2 30° + \cos^2 60°$
Solution: Since

$$\cos 60° = \sin 30°, \tag{4.1.1}$$

$$\sin^2 30° + \cos^2 60° = 2\sin^2 30° \tag{4.1.2}$$

$$= 1 \tag{4.1.3}$$

4.2. What is the simplest form of $\frac{1+\tan^2 A}{1+\cot^2 A}$
Solution:

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{1 + \tan^2 A}{1 + \frac{1}{\tan^2 A}} \tag{4.2.1}$$

$$= \tan^2 A \tag{4.2.2}$$

4.3. In Fig 4.3.1 the angle of elevation from the top of a tower from a point C on the ground, which is 30m away from the foot of the tower is 30°. Find the height of the tower.
Solution: In general, the height is given by

$$h = d\tan\theta \tag{4.3.1}$$

$$= 30\tan 30° = 30\frac{1}{\sqrt{3}} \tag{4.3.2}$$

$$= 10\sqrt{3} \tag{4.3.3}$$

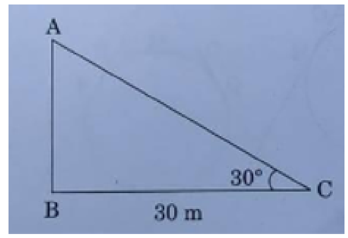

Fig. 4.3.1.

4.4. Prove that

$$\left(\sin^4\theta - \cos^4\theta + 1\right)\csc^2\theta = 2 \tag{4.4.1}$$

Solution: Since

$$\left(\sin^2\theta + \cos^2\theta\right)\left(\sin^2\theta - \cos^2\theta\right) = -\cos 2\theta \tag{4.4.2}$$

the L.H.S in (4.4.1) can be expressed as

$$\left(1 - \cos 2\theta\right)\csc^2\theta = 2\sin^2\theta\csc^2\theta \tag{4.4.3}$$

$$= 2 \tag{4.4.4}$$

4.5. If A, B and C are interior angles of $\triangle ABC$, then show that

$$\cos\left(\tfrac{B+C}{2}\right) = \sin\left(\tfrac{A}{2}\right) \tag{4.5.1}$$

Solution: In a triangle,

$$A + B + C = 180° \tag{4.5.2}$$

$$\implies \left(\tfrac{B+C}{2}\right) = 90° - \frac{A}{2} \tag{4.5.3}$$

$$\text{or, } \cos\left(\tfrac{B+C}{2}\right) = \sin\left(90° - \tfrac{A}{2}\right) \tag{4.5.4}$$

yielding (4.5.1).

4.6. A statue 1.6m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.Use$\left(\sqrt{3} = 1.73\right)$
Solution: Let the height of the pedestal be x and the height of the statue be h. If the given

angles be $\theta_1 = 60°$ and $\theta_2 = 45°$, from the given information,

$$(h + x)\cot\theta_1 = x\cot\theta_2 \tag{4.6.1}$$

$$\implies x = \frac{h\cot\theta_1}{\cot\theta_2 - \cot\theta_1} = \frac{1.6}{\sqrt{3}-1} \tag{4.6.2}$$

5 Linear Algebra

5.1. The distance between the points $(m, -n)$ and $(-m, n)$ is

a) $\sqrt{m^2 + n^2}$
b) $m + n$
c) $2\sqrt{m^2 + n^2}$
d) $\sqrt{2m^2 + 2n^2}$

Solution: Letting

$$\mathbf{A} = \begin{pmatrix} m \\ -n \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -m \\ n \end{pmatrix} \tag{5.1.1}$$

$$\mathbf{A} - \mathbf{B} = 2\begin{pmatrix} m \\ -n \end{pmatrix} \tag{5.1.2}$$

Using the definition of the norm,

$$\|\mathbf{A} - \mathbf{B}\| = 2\left\|\begin{pmatrix} m \\ -n \end{pmatrix}\right\| \tag{5.1.3}$$

$$= 2\sqrt{\begin{pmatrix} m & -n \end{pmatrix}\begin{pmatrix} m \\ -n \end{pmatrix}} \tag{5.1.4}$$

$$= 2\sqrt{m^2 + n^2} \tag{5.1.5}$$

5.2. The point on the x-axis which is equidistant from $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 10 \\ 0 \end{pmatrix}$

a) $\begin{pmatrix} 7, 0 \end{pmatrix}$
b) $\begin{pmatrix} 5, 0 \end{pmatrix}$
c) $\begin{pmatrix} 0, 0 \end{pmatrix}$
d) $\begin{pmatrix} 3, 0 \end{pmatrix}$

Solution: The input parameters for this problem are available in (5.2.1). If $\mathbf{x}$ lies on the x-axis and is equidistant from the points $\mathbf{A}$ and $\mathbf{B}$,

Symbol	Value	Description
A	$\begin{pmatrix} -4 \\ 0 \end{pmatrix}$	First point
B	$\begin{pmatrix} 10 \\ 0 \end{pmatrix}$	Second point
P	?	Desired point

TABLE 5.2.1

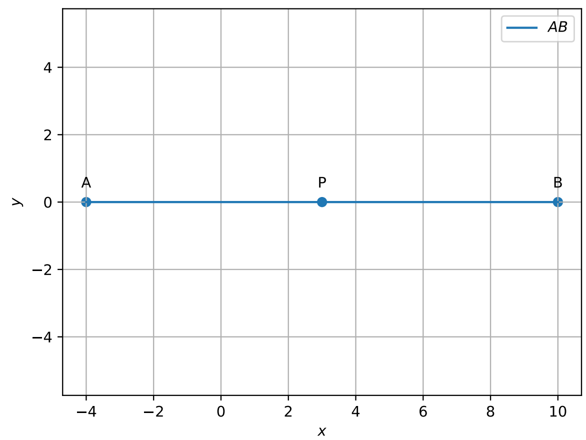

Fig. 5.2.1.

$$\|\mathbf{x} - \mathbf{A}\| = \|\mathbf{A} - \mathbf{B}\| \tag{5.2.1}$$

$$\implies \|\mathbf{x} - \mathbf{A}\|^2 = \|\mathbf{x} - \mathbf{B}\|^2 \tag{5.2.2}$$

which can be expressed as

$$(\mathbf{x} - \mathbf{A})^\top (\mathbf{x} - \mathbf{A}) = (\mathbf{x} - \mathbf{B})^\top (\mathbf{x} - \mathbf{B})$$

$$\implies \|\mathbf{x}\|^2 - 2\mathbf{x}^\top\mathbf{A} + \|\mathbf{A}\|^2 = \|\mathbf{x}\|^2 - 2\mathbf{x}^\top\mathbf{B} + \|\mathbf{B}\|^2 \tag{5.2.3}$$

which can be simplified to obtain

$$\mathbf{x} = x\mathbf{e}_1 \tag{5.2.4}$$

where

$$x = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2(\mathbf{A} - \mathbf{B})^\top \mathbf{e}_1} \tag{5.2.5}$$

$$= 3 \tag{5.2.6}$$

upon substituting numerical values. Hence, the desired point is $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$. See Fig. 5.2.1

5.3. The center of a circle whose end points of a diameter are $\begin{pmatrix} -6, 3 \end{pmatrix}$ and $\begin{pmatrix} 6, 4 \end{pmatrix}$ is

a) $\begin{pmatrix} 8, -1 \end{pmatrix}$
b) $\begin{pmatrix} 4, 7 \end{pmatrix}$
c) $\begin{pmatrix} 0, \frac{7}{2} \end{pmatrix}$
d) $\begin{pmatrix} 4, \frac{7}{2} \end{pmatrix}$

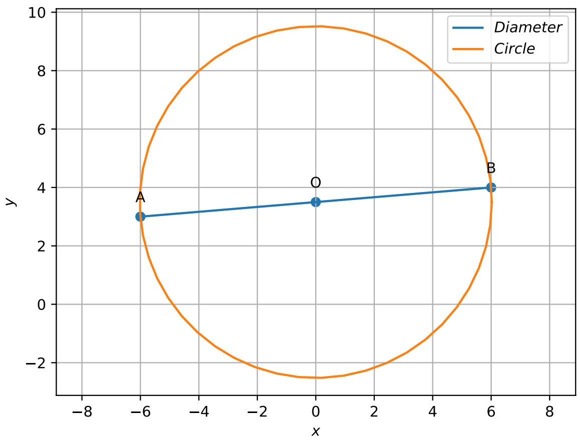

Fig. 5.3.1.

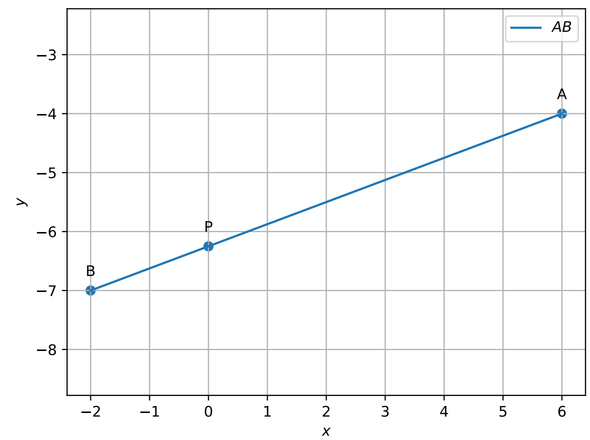

Fig. 5.4.1.

Solution: Using section formula, the desired point is given by

$$\mathbf{O} = \frac{\mathbf{B}+\mathbf{A}}{2} \tag{5.3.1}$$

$$= \frac{1}{2}\left[\begin{pmatrix}-6\\3\end{pmatrix} + \begin{pmatrix}6\\4\end{pmatrix}\right] \tag{5.3.2}$$

$$= \frac{1}{2}\begin{pmatrix}0\\7\end{pmatrix} \tag{5.3.3}$$

See Fig. 5.3.1

5.4. Find the ratio in which the y-axis divides the line segment joining the points $\left(6,-4\right)$ and $\left(-2,-7\right)$. Also find the point of intersection

Solution: In general, letting the given points be $\mathbf{A},\mathbf{B}$,

$$\mathbf{P} = \frac{k\mathbf{B}+\mathbf{A}}{k+1} \tag{5.4.1}$$

Since the point lies on the y-axis,

$$\mathbf{e}_1^\top\mathbf{P} = 0 \tag{5.4.2}$$

$$\implies k\mathbf{e}_1^\top\mathbf{B} + \mathbf{e}_1^\top\mathbf{A} = 0 \tag{5.4.3}$$

$$\text{or, } k = -\frac{\mathbf{e}_1^\top\mathbf{A}}{\mathbf{e}_1^\top\mathbf{B}} \tag{5.4.4}$$

Substituting in (5.4.1) and simplifying,

$$\mathbf{P} = \frac{\left(\mathbf{e}_1^\top\mathbf{B}\right)\mathbf{A} - \left(\mathbf{e}_1^\top\mathbf{A}\right)\mathbf{B}}{\left(\mathbf{e}_1^\top\mathbf{B}\right) - \left(\mathbf{e}_1^\top\mathbf{A}\right)} \tag{5.4.5}$$

See Fig. 5.4.1

5.5. Show that the points $\left(7,10\right)$, $\left(-2,5\right)$ and $\left(3,-4\right)$ and vertices of an isosceles right triangle.

Solution: Let the given points be $\mathbf{A},\mathbf{B},\mathbf{C}$ respectively. Then, the direction vectors of AB, BC and CA are

$$\mathbf{A}-\mathbf{B} = \begin{pmatrix}7\\10\end{pmatrix} - \begin{pmatrix}-2\\5\end{pmatrix} = \begin{pmatrix}9\\5\end{pmatrix} \tag{5.5.1}$$

$$\mathbf{B}-\mathbf{C} = -\begin{pmatrix}-2\\5\end{pmatrix} - \begin{pmatrix}3\\-4\end{pmatrix} = \begin{pmatrix}-5\\9\end{pmatrix} \tag{5.5.2}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix}3\\-4\end{pmatrix} - \begin{pmatrix}7\\10\end{pmatrix} = \begin{pmatrix}-4\\-14\end{pmatrix} \tag{5.5.3}$$

From the above, we find that

$$\left(\mathbf{A}-\mathbf{B}\right)^\top\left(\mathbf{B}-\mathbf{C}\right) = \begin{pmatrix}9 & 5\end{pmatrix}\begin{pmatrix}-5\\9\end{pmatrix} \tag{5.5.4}$$

$$= 0 \tag{5.5.5}$$

$$\left(\mathbf{B}-\mathbf{C}\right)^\top\left(\mathbf{C}-\mathbf{A}\right) = \begin{pmatrix}-5 & 9\end{pmatrix}\begin{pmatrix}-4\\-14\end{pmatrix} \tag{5.5.6}$$

$$= -106 \tag{5.5.7}$$

$$\left(\mathbf{C}-\mathbf{A}\right)^\top\left(\mathbf{A}-\mathbf{B}\right) = \begin{pmatrix}-4 & -14\end{pmatrix}\begin{pmatrix}9\\5\end{pmatrix} \tag{5.5.8}$$

$$= -106 \tag{5.5.9}$$

From the above equations,

$$\left(\mathbf{A}-\mathbf{B}\right) \perp \left(\mathbf{B}-\mathbf{C}\right) \tag{5.5.10}$$

$$\angle BCA = \angle CAB \tag{5.5.11}$$

Thus, the triangle is right angled and isosceles. See Fig. 5.5.1

5.6. $AOBC$ is a rectangle whose 3 vertices are $\mathbf{A} = \left(0,-3\right)$, $\mathbf{O} = \left(0,0\right)$ and $\mathbf{B} = \left(4,0\right)$. The length

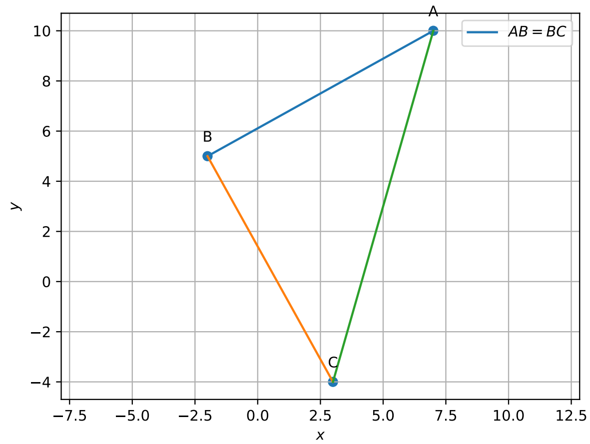

Fig. 5.5.1.

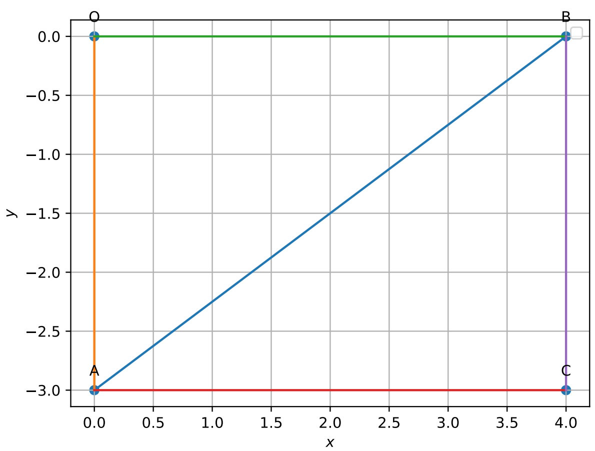

Fig. 5.6.1.

of the diagonal is ________
Solution: The length of the diagonal is

$$\|\mathbf{A}-\mathbf{B}\| = \sqrt{3^2+4^2} = 5 \tag{5.6.1}$$

Also, the the fourth point

$$\mathbf{C} = \mathbf{A}+\mathbf{B}-\mathbf{O} = \begin{pmatrix} 4 \\ -3 \end{pmatrix} \tag{5.6.2}$$

See Fig. 5.6.1

5.7. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 added to its denominator. Find the fraction.

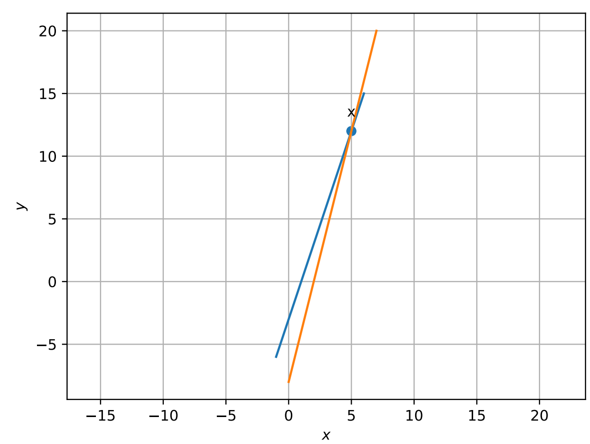

Fig. 5.7.1.

Solution: Let the desired fraction be $\frac{x}{y}$. From the given information,

$$\frac{x-1}{y} = \frac{1}{3} \tag{5.7.1}$$

$$\frac{x}{y+8} = \frac{1}{4} \tag{5.7.2}$$

The above equations result in the system

$$3x - y = 3 \tag{5.7.3}$$

$$4x - y = 8 \tag{5.7.4}$$

which can be expressed as the matrix equation

$$\begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ 8 \end{pmatrix} \tag{5.7.5}$$

The augmented matrix is obtained as

$$\begin{pmatrix} 3 & -1 & 3 \\ 4 & -1 & 8 \end{pmatrix} \tag{5.7.6}$$

Through pivoting, we obtain

$$\begin{pmatrix} 3 & -1 & 3 \\ 0 & 1 & 12 \end{pmatrix} \tag{5.7.7}$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1+R_2}{3}} \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 12 \end{pmatrix} \tag{5.7.8}$$

$$\implies \mathbf{x} = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \tag{5.7.9}$$

See Fig. 5.7.1

5.8. The present age of a father is three years more than three times the age of his son. Three years hence the father's age will be 10 years more

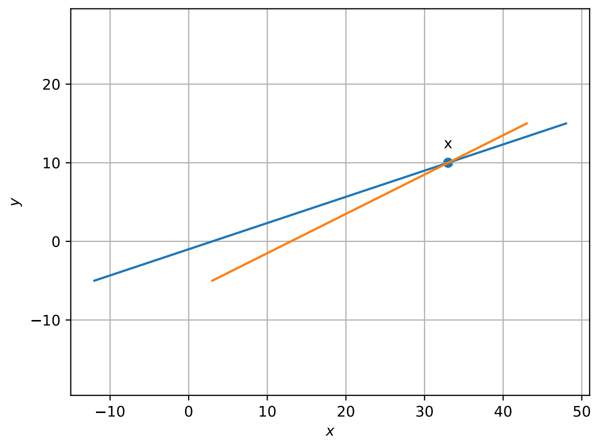

Fig. 5.8.1.

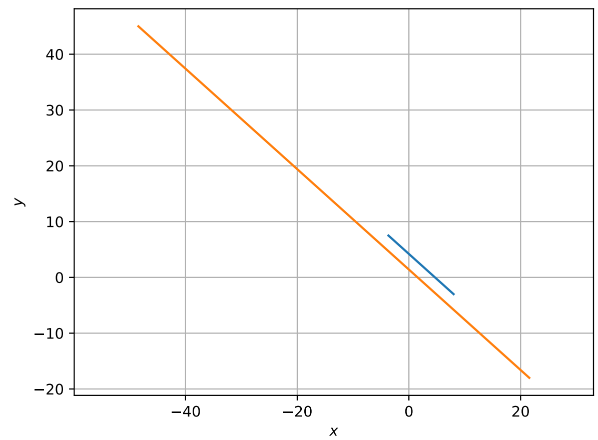

Fig. 5.9.1.

than twice the age of the son. Determine their present ages.

Solution: Let the ages of the father and son be x, y respectively. From the given information,

$$x = 3y + 3 \tag{5.8.1}$$

$$x + 3 = 2(y + 3) + 10 \tag{5.8.2}$$

which can be expressed as

$$x - 3y = 3 \tag{5.8.3}$$

$$x - 2y = 13 \tag{5.8.4}$$

$$\implies \begin{pmatrix} 1 & -3 \\ 1 & -2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ 13 \end{pmatrix} \tag{5.8.5}$$

The augmented matrix for the above matrix equation is

$$\left(\begin{array}{cc|c} 1 & -3 & 3 \\ 1 & -2 & 13 \end{array}\right) \tag{5.8.6}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \left(\begin{array}{cc|c} 1 & -3 & 3 \\ 0 & 1 & 10 \end{array}\right) \tag{5.8.7}$$

$$\xleftrightarrow{R_1 \leftarrow 3R_2 + R_1} \left(\begin{array}{cc|c} 1 & 0 & 33 \\ 0 & 1 & 10 \end{array}\right) \implies \mathbf{x} = \begin{pmatrix} 33 \\ 10 \end{pmatrix} \tag{5.8.8}$$

See Fig. 5.8.1

5.9. The pair of linear equations,

$$\frac{3x}{2} + \frac{5y}{3} = 7 \text{ and} \tag{5.9.1}$$

$$9x + 10y = 14 \tag{5.9.2}$$

is

a) consistent
b) inconsistent
c) consistent with one solution
d) consistent with many solutions

Solution: The given system can be expressed as the matrix equation

$$\begin{pmatrix} \frac{3}{2} & \frac{5}{3} \\ 9 & 10 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 7 \\ 14 \end{pmatrix} \tag{5.9.3}$$

The augmented matrix can be expressed as

$$\begin{pmatrix} \frac{3}{2} & \frac{5}{3} & 7 \\ 9 & 10 & 14 \end{pmatrix} \tag{5.9.4}$$

$$\xleftrightarrow{R_1 \leftarrow 6R_1} \begin{pmatrix} 9 & 10 & 42 \\ 9 & 10 & 14 \end{pmatrix} \tag{5.9.5}$$

$$\xleftrightarrow{R_2 \leftarrow R_1 - R_2} \begin{pmatrix} 9 & 10 & 42 \\ 0 & 0 & 28 \end{pmatrix} \tag{5.9.6}$$

From the above, it is obvious that the rank of the coefficient matrix is not equal to the rank of the augmented matrix. Hence, the system is inconsistent. See Fig. 5.9.1

5.10. If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, prove that the other two sides are divided in the same ratio.

Solution: Let the vertices of the triangle be $\mathbf{A}, \mathbf{B}, \mathbf{C}$ such that

$$\mathbf{B} = \mathbf{0} \tag{5.10.1}$$

and $\mathbf{A}, \mathbf{C}$ are known. Let $\mathbf{P}$ be a known point on AB such that PQ is parallel to BC. Let

$$\mathbf{P} = \lambda(\mathbf{A} - \mathbf{B}) \tag{5.10.2}$$
$$= \lambda\mathbf{A} \tag{5.10.3}$$
$$\text{and }, \frac{\|\mathbf{P}\|}{\|\mathbf{A}\|} = \frac{BP}{AB} = |\lambda| \tag{5.10.4}$$

Since

$$PQ \parallel BC, \tag{5.10.5}$$
$$\mathbf{Q} = P + \mu\mathbf{B} - \mathbf{C} \tag{5.10.6}$$
$$= \lambda\mathbf{A} - \mu\mathbf{C} \tag{5.10.7}$$

using the equation of the line PQ and substituting from (5.10.3) Also, since $\mathbf{Q}$ lies on the line AC,

$$\mathbf{Q} = \mathbf{A} + k(\mathbf{A} - \mathbf{C}) \tag{5.10.8}$$
$$= (1+k)\mathbf{A} - k\mathbf{C} \tag{5.10.9}$$
$$\text{and } \frac{\|\mathbf{A} - \mathbf{Q}\|}{\|\mathbf{A} - \mathbf{C}\|} = \frac{AQ}{AC} = |k| \tag{5.10.10}$$

From (5.10.7) and (5.10.9)

$$\lambda\mathbf{A} - \mu\mathbf{C} = (1+k)\mathbf{A} - k\mathbf{C} \tag{5.10.11}$$
$$\implies (1+k+\lambda)\mathbf{A}(k-\mu)\mathbf{C} = 0 \tag{5.10.12}$$
$$\implies k = \mu, \lambda = -1 - \mu \tag{5.10.13}$$
$$\text{or, } |\lambda| = 1 + k \tag{5.10.14}$$

From (5.10.4), (5.10.10) and (5.10.14),

$$\frac{AQ}{AC} = \frac{AP}{AB} \tag{5.10.15}$$

See Fig. 5.10.1 for the input parameters in Table 5.10.1.

Symbol	Value	Description
a	4	BC
c	5	AB
θ	$\frac{\pi}{3}$	$\angle B$
k	$\frac{3}{2}$	$\frac{BP}{AP} = \frac{CQ}{AQ}$

TABLE 5.10.1

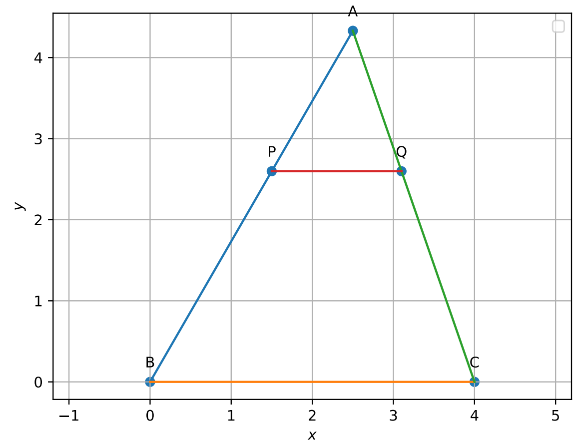

Fig. 5.10.1.

5.11. In Fig. 5.11.1, if $\mathbf{PQ} \parallel \mathbf{BC}$ and $\mathbf{PR} \parallel \mathbf{CD}$, prove that $\frac{QB}{AQ} = \frac{DR}{AR}$.
Solution: From the previous problem, it is obvious that

$$\frac{AP}{PC} = \frac{AQ}{QB} = \frac{AR}{RD} \tag{5.11.1}$$

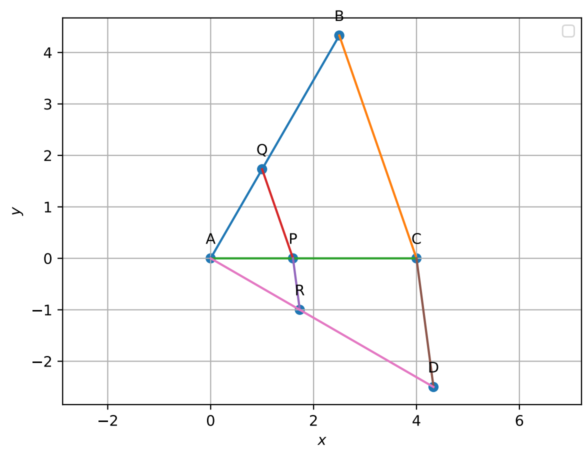

Fig. 5.11.1.

For construction, let $\mathbf{A} = \mathbf{0}, \mathbf{e}_2^\top\mathbf{C} = 0, \mathbf{B}, \mathbf{C}, \mathbf{D}$ be the input vectors. Choose $0 < k < 1$ and define

$$\mathbf{P} = \frac{k\mathbf{C}}{k+1} \tag{5.11.2}$$
$$\mathbf{Q} = \frac{k\mathbf{B}}{k+1} \tag{5.11.3}$$
$$\mathbf{R} = \frac{k\mathbf{D}}{k+1} \tag{5.11.4}$$

5.12. Construct a $\triangle ABC$ with sides $\mathbf{BC} = 6cm$, $\mathbf{AB} = 5cm$ and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of $\triangle ABC$.
Solution: The input parameters for drawing $\triangle ABC$ are given in Table (5.12.1). The second

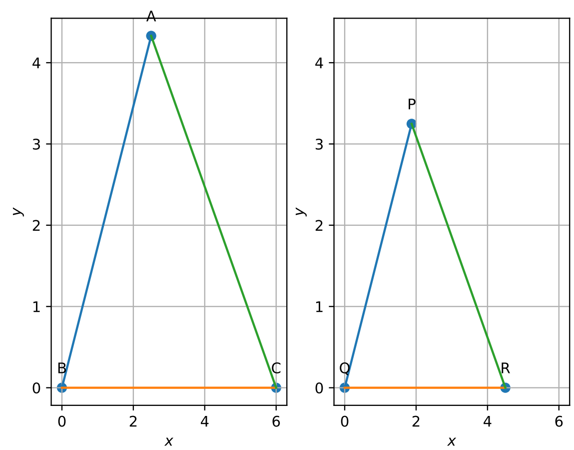

Fig. 5.12.1.

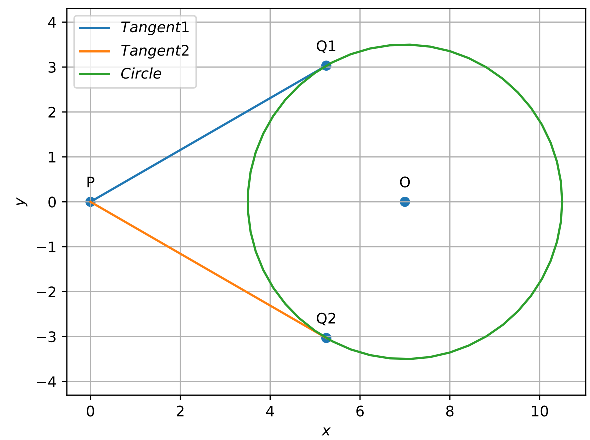

Fig. 5.13.1.

triangle can be drawn by taking the sides to be $\frac{3}{4}c, \frac{3}{4}a$. See Fig. 5.12.1.

Symbol	Value	Description
c	5	AB
a	6	BC
θ	$60°$	$\angle ABC$
$\mathbf{B}$	$\mathbf{0}$	Origin
$\mathbf{C}$	$\begin{pmatrix} a \\ 0 \end{pmatrix}$	Vertex on the x-axis
$\mathbf{A}$	$c\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$	Vertex

TABLE 5.12.1

5.13. Draw a circle of radius $3.5cm$. Take a point P outside the circle at a distance of $7cm$ from the centre of the circle and construct a pair of tangents to the circle from that point.
Solution: The input parameters for this construction are available in Table (5.13.1) See

Symbol	Value	Description
r	3.5	*Radius*
d	7	Distance of $\mathbf{P}$ from the origin
$sin\theta$	$\frac{r}{d}$	Angle between the tangent from $\mathbf{P}$ and d
$\mathbf{P}$	$\mathbf{0}$	Origin
$\mathbf{O}$	$\begin{pmatrix} d \\ 0 \end{pmatrix}$	Centre of the circle
$\mathbf{Q}_i$	$r\cot\theta\begin{pmatrix} \cos\theta \\ \pm\sin\theta \end{pmatrix}$	Points of Contact

TABLE 5.13.1

Fig. 5.13.1. The desired circle can be expressed as a conic with parameters

$$\mathbf{V} = \mathbf{I}, \mathbf{u} = 0, f = -r^2. \tag{5.13.1}$$

The slopes of the tangents from $\mathbf{P}$ to the circle are given by

$$L_P: \quad \mathbf{x} = \mathbf{P} + \mu\mathbf{m} \quad \mu \in \mathbb{R} \tag{5.13.2}$$

where

$$\mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{5.13.3}$$

$$\mathbf{m}^\top \mathbf{\Sigma} \mathbf{m} = 0 \tag{5.13.4}$$

$$\mathbf{\Sigma} = (\mathbf{VP} + \mathbf{u})(\mathbf{VP} + \mathbf{u})^\top - \mathbf{V}\left(\mathbf{P}^\top \mathbf{VP} + 2\mathbf{u}^\top \mathbf{P} + f\right) \tag{5.13.5}$$

and μ given by

$$\mu_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{Vq} + \mathbf{u}) \pm \sqrt{\left[\mathbf{m}^\top (\mathbf{Vq} + \mathbf{u})\right]^2 - \left(\mathbf{q}^\top \mathbf{Vq} + 2\mathbf{u}^\top \mathbf{q} + f\right)\left(\mathbf{m}^\top \mathbf{Vm}\right)}\right) \tag{5.13.6}$$

Substituting from (5.13.1) in (5.13.7),

$$\mathbf{\Sigma} = \mathbf{PP}^\top - \left(\|\mathbf{P}\|^2 - r^2\right)\mathbf{I} \tag{5.13.7}$$

5.14. In Fig. 5.14.1, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that $\mathbf{AB} + \mathbf{CD} = \mathbf{BC} + \mathbf{AD}$.
Solution: The input parameters for drawing the figure are available in Table (5.14.1). The steps for construcing the figure are

a) $\theta_1 = \angle ADC = 2\tan^{-1}\frac{r}{d}$.

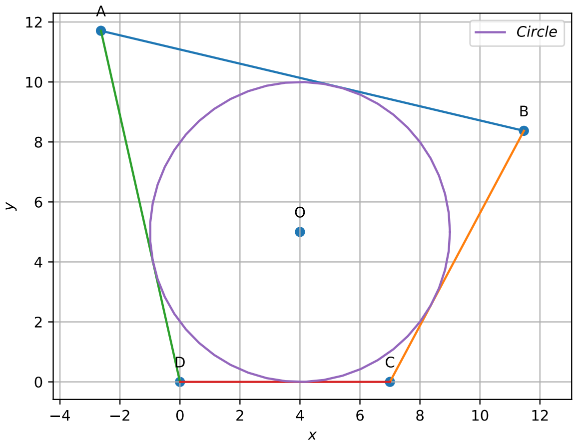

Fig. 5.14.1.

Symbol	Value	Description
$\mathbf{D}$	$\mathbf{0}$	Origin
r		Circle Radius
$\mathbf{O}$	$\begin{pmatrix} d \\ r \end{pmatrix}$	Circle Centre
$\mathbf{C}$	$\begin{pmatrix} c+d \\ 0 \end{pmatrix}$	Vertex on the x-axis
AD	$d+a$	Vertex

TABLE 5.14.1

b) $\mathbf{A} = (a+d)\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$
c) $\theta_2 = \angle BAD = 2\tan^{-1}\frac{r}{a}$
d) The slope of AB is $m_1 = \tan(\angle ADC + \angle BAD)$. This is obtained by extending AB and CD so that they meet.
e) $\theta_3 = \angle BCD = 2\tan^{-1}\frac{r}{c}$
f) The slope of BC is $m_2 = \tan\angle BCD$
g) The direction vectors of AB and BC are

$$\mathbf{m}_1 = \begin{pmatrix} 1 \\ m_1 \end{pmatrix}, \mathbf{m}_2 = \begin{pmatrix} 1 \\ m_2 \end{pmatrix} \tag{5.14.1}$$

respectively.
h) $\mathbf{B}$ is obtained as the point of intersection of the lines AB and BC

The equation of AB and BC are respectively

$$\begin{aligned} \mathbf{x} &= \mathbf{A} + \lambda_1\mathbf{m}_1 \\ \mathbf{x} &= \mathbf{C} + \lambda_2\mathbf{m}_2 \end{aligned} \tag{5.14.2}$$

and their intersection is given by

$$\mathbf{A} + \lambda_1\mathbf{m}_1 = \mathbf{C} + \lambda_2\mathbf{m}_2 \tag{5.14.3}$$

$$\implies \begin{pmatrix} \mathbf{m}_1 & \mathbf{m}_2 \end{pmatrix}\begin{pmatrix} \lambda_1 \\ -\lambda_2 \end{pmatrix} = \mathbf{C} - \mathbf{A} \tag{5.14.4}$$

which can be used to obtained λ_1, λ_2 and consequently $\mathbf{B}$, using (5.14.2)

5.15. In Fig 5.15.1 find the perimeter of $\triangle ABC$ if $\mathbf{AP} = 12cm$

Solution: The steps for constructing Fig. 5.15.1 are

a) Draw a circle of radius r.
b) From any point $\mathbf{A}$ outside the circle, draw two tangents AP and AQ using Table 5.13.1

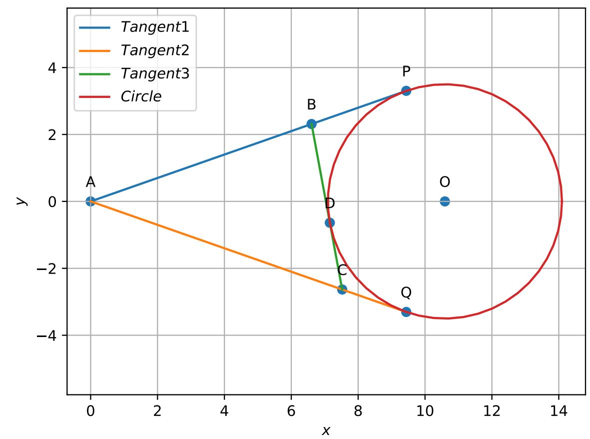

Fig. 5.15.1.

c) Choose any point $\mathbf{B}$ on AP. Find its reflection $\mathbf{R}$ about the line BO using the formula

$$\mathbf{R} = \mathbf{B} + 2\frac{c - \mathbf{n}^\top\mathbf{B}}{\|\mathbf{n}\|^2} \tag{5.15.1}$$

where

$$\mathbf{m} = \mathbf{B} - \mathbf{O}, \mathbf{m}^\top\mathbf{n} = 0, c = \mathbf{n}^\top\mathbf{O}, \tag{5.15.2}$$

d) Find the equations of the lines AQ and BR and use them to find the intersection $\mathbf{C}$.

5.16. Sum of the areas of two squares is $544m^2$. If the difference of their perimeters is $32m$, find the sides of the two squares.

Solution: Let the sides be x, y. From the given information,

$$x^2 + y^2 = 544 \tag{5.16.1}$$

$$4x - 4y = 32 \tag{5.16.2}$$

From the given information, the above equations can be expressed in vector form as

$$\mathbf{x}^\top\mathbf{x} = r^2 \tag{5.16.3}$$

$$\mathbf{n}^\top\mathbf{x} = c \tag{5.16.4}$$

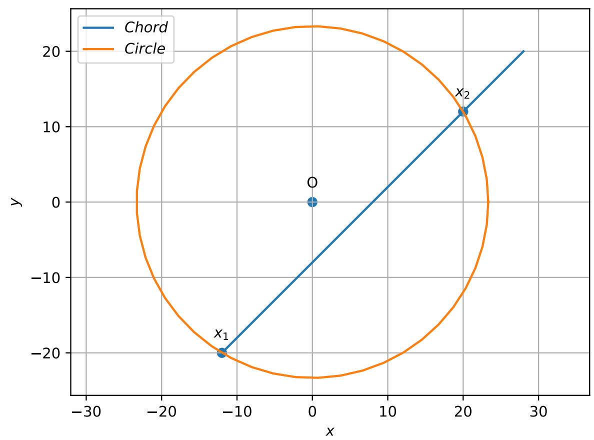

Fig. 5.16.1.

with parameters listed in Table (5.16.1) Thus,

Symbol	Value	Description
r	$\sqrt{544}$	Radius of the Circle
c	8	Line Parameter
$\mathbf{n}$	$\begin{pmatrix}1\\-1\end{pmatrix}$	Normal to the Line
$\mathbf{m}$	$\begin{pmatrix}1\\1\end{pmatrix}$	Direction Vector of the Line
$\mathbf{A}$	$\begin{pmatrix}8\\0\end{pmatrix}$	x-intercept of the Line

TABLE 5.16.1

the desired solution is the point of intersection of the line with the circle in the first quadrant as shown in Fig. 5.16.1

Using the parametric equation of the line

$$\mathbf{x} = \mathbf{A} + \lambda\mathbf{m} \tag{5.16.5}$$

Substituting the above in (5.16.3),

$$(\mathbf{A} + \lambda\mathbf{m})^\top (\mathbf{A} + \lambda\mathbf{m}) = r^2$$
$$\implies \lambda^2 \|\mathbf{m}\|^2 + 2\lambda\mathbf{m}^\top\mathbf{A} + \|\mathbf{A}\|^2 - r^2 = 0 \tag{5.16.6}$$

yielding

$$\lambda = \frac{-\mathbf{m}^\top\mathbf{A} \pm \sqrt{(\mathbf{m}^\top\mathbf{A})^2 - \|\mathbf{m}\|^2\left(\|\mathbf{A}\|^2 - r^2\right)}}{\|\mathbf{m}\|^2} \tag{5.16.7}$$

For this problem, the numerical values are

$$\mathbf{n} = \begin{pmatrix}1\\-1\end{pmatrix}, c = 8, \mathbf{m} = \begin{pmatrix}1\\1\end{pmatrix}, \tag{5.16.8}$$

$$\mathbf{A} = \begin{pmatrix}8\\0\end{pmatrix}, r^2 = 544 \tag{5.16.9}$$

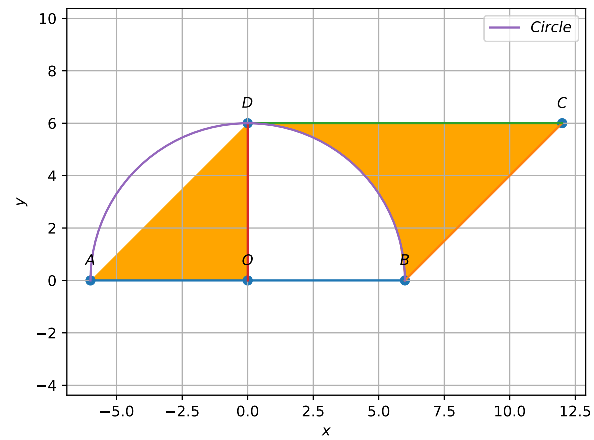

Fig. 5.17.1.

Substituting the above in (5.16.7),

$$\lambda = 12 \tag{5.16.10}$$

Thus, substituting from (5.16.10) and (5.16.8) in (5.16.5) the desired point of intersection is

$$\mathbf{x} = \begin{pmatrix}8\\0\end{pmatrix} + 12\begin{pmatrix}1\\1\end{pmatrix} \tag{5.16.11}$$
$$= \begin{pmatrix}20\\12\end{pmatrix} \tag{5.16.12}$$

Thus, the sides are $20m$ and $12m$.

5.17. In Fig. 5.17.1 ABCD is a parallelogram. A semicircle with centre O and the diameter AB has been drawn and it passes through D. If $\mathbf{AB} = 12$ and $\mathbf{OD} \perp \mathbf{AB}$, then find the area of the shaded region. Use $\left(\pi = 3.14\right)$

Solution: From the figure, the radius of the circle is

$$r = \frac{AB}{2} = OD = 6 \tag{5.17.1}$$

The area of the parallelogram is

$$AB \times OD = 2r^2 \tag{5.17.2}$$

The area of the sector DOB is

$$\frac{1}{4}\pi r^2 \tag{5.17.3}$$

Thus, the desired area is

$$\left(2 - \frac{\pi}{4}\right) r^2 = 9\left(8 - \pi\right) \tag{5.17.4}$$

For construction, the input parameters are given in Table 5.17.1. The various coordinates

Symbol	Value	Description
O	0	Circle Centre
r	6	Circle Radius

TABLE 5.17.1

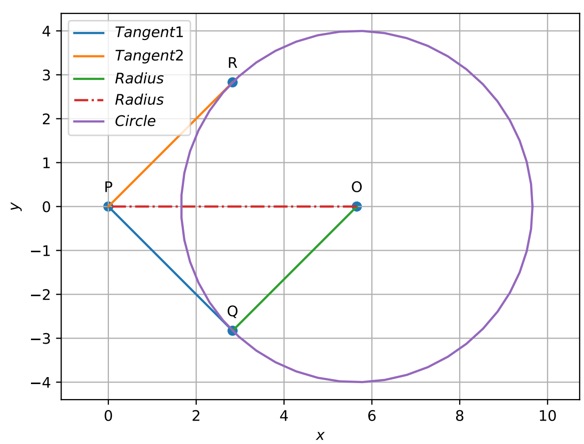

Fig. 5.18.1.

are

$$\mathbf{A} = \begin{pmatrix} -r \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} r \\ 0 \end{pmatrix}, \tag{5.17.5}$$

$$\mathbf{D} = \begin{pmatrix} 0 \\ r \end{pmatrix}, \tag{5.17.6}$$

$$\mathbf{C} = \mathbf{B} - \mathbf{A} + \mathbf{D} - \mathbf{A} \tag{5.17.7}$$

$$= \mathbf{B} + \mathbf{D} - \mathbf{A} \tag{5.17.8}$$

5.18. In Fig. 5.18.1, from an external point **P**, two tangents PQ and PR are drawn to a circle of radius 4cm with center **O**. If $\angle QPR = 90°$, then length of PQ is

a) 3
b) 4
c) 2
d) $2\sqrt{2}$

Solution: In general, for a circle with radus r and $\angle QPR = \theta$,

$$PQ = r \cot \frac{\theta}{2} \tag{5.18.1}$$

$$= 4 \cot 45° = 4 \tag{5.18.2}$$

upon substituting numerical values.

5.19. In Fig. 5.19.1, $DE \parallel BC$. If $\frac{AD}{DB} = \frac{3}{2}$ and $AE = 2.7cm$, then EC is equal to

a) $2.0cm$

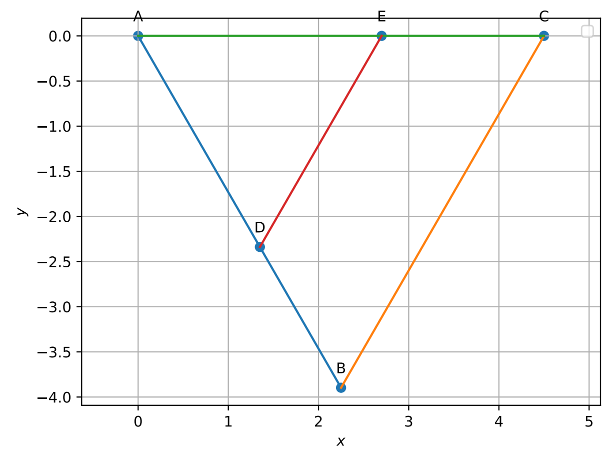

Fig. 5.19.1.

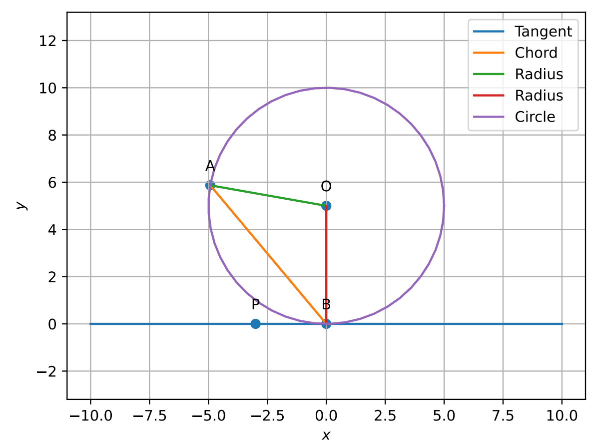

Fig. 5.20.1.

b) $1.8cm$
c) $4.0cm$
d) $2.7cm$

Solution: Since

$$\frac{AD}{DB} = \frac{AE}{EC} \tag{5.19.1}$$

$$\implies \frac{3}{2} = \frac{2.7}{EC} \tag{5.19.2}$$

$$\text{or, } EC = 1.8 \tag{5.19.3}$$

5.20. In Fig. 5.20.1 PQ is tangent to the circle with center at **O**, at the point **B**. If $\angle AOB = 100°$, then $\angle ABP$ is equal to

a) 50°
b) 40°
c) 60°

d) 80°

Solution: In general,

$$\angle ABP = \frac{1}{2}\angle AOB \tag{5.20.1}$$
$$= 50° \tag{5.20.2}$$

The input parameters for drawing the figure are available in Table 5.20.1

Symbol	Value	Description
r	5	Radius of the Circle
$\mathbf{B}$	$\mathbf{0}$	Origin
$\mathbf{O}$	$\begin{pmatrix}0\\ r\end{pmatrix}$	Centre of the circle
θ	100°	Angle subtended by chord AB
AB	$2r\sin\frac{\theta}{2}$	Lenght of AB
$\mathbf{A}$	$2r\sin\frac{\theta}{2}\begin{pmatrix}-\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}\end{pmatrix}$	

TABLE 5.20.1

5.21. All concentric circles are ________ to each other

6 Probability and Statistics

6.1. A game in a booth at a Diwali Fair involves using a spinner first. Then, if the spinner stops on an even number, the player is allowed to pick a marble from a bag. The spinner and the marbles in the bag are represented in Fig 6.1.1. Prizes are given, when a black marble is picked. Shweta plays the game once.

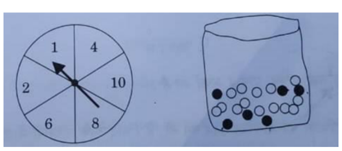

Fig. 6.1.1.

a) What is the probability that she will be allowed to pick a marble from the bag?

Solution: Let $X \in \{0, 1\}$ represent the odd and even outcomes of the spinner and $Y \in \{0, 1\}$ represent the black and white marbles respectively. Shweta can pick a marble only if she gets an even number in the spinner. Hence, the desired probability is

$$\Pr(X = 1) = \frac{5}{6} \tag{6.1.1}$$

b) Suppose she is allowed to pick a marble from the bag, what is the probability of getting a prize, when it is given that the bag contains 20 balls out of which 6 are black?

Solution: Shweta will get a prize only if she picks a black marble. The desired probability can be expressed as

$$\Pr(Y = 0|X = 1) = \frac{6}{20} = \frac{3}{10} \tag{6.1.2}$$

6.2. For the following data, in Table 6.2.1, draw a 'less than' ogive and hence find the median of the distribution.

Age	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of people	5	15	20	25	15	11	9

TABLE 6.2.1

Solution: The less than ogive graph is available in Fig. 6.2.1. Let the age data be represented as

$$\mathbf{a} = \begin{pmatrix}0\\ 10\\ \vdots\\ 70\end{pmatrix} \tag{6.2.1}$$

and the people data be

$$\mathbf{b} = \begin{pmatrix}5\\ 15\\ \vdots\\ 9\end{pmatrix} \tag{6.2.2}$$

The cumulative frequency data is then obtained as

$$\mathbf{c} = \mathbf{P}\mathbf{b} \tag{6.2.3}$$

where

$$\mathbf{P} = \begin{pmatrix}1 & 0 & 0 & \cdots & 0\\ 1 & 1 & 0 & \cdots & 0\\ 1 & 1 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 1 & 1 & 1 & \cdots & 1\end{pmatrix} \tag{6.2.4}$$

If n be the data size, the median point is obtained as

$$\mathbf{M} = \begin{pmatrix}M_1\\ M_2\end{pmatrix} \tag{6.2.5}$$
$$= \left[\mathbf{A}_i + \frac{\frac{A_{2n}}{2} - \mathbf{e}_2^\top \mathbf{A}_i}{\mathbf{e}_2^\top\left(\mathbf{A}_{i+1} - \mathbf{A}_i\right)}\left(\mathbf{A}_{i+1} - \mathbf{A}_i\right)\right] \tag{6.2.6}$$

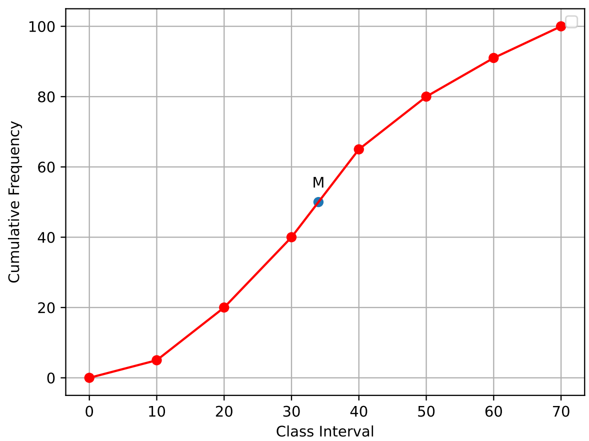

Fig. 6.2.1.

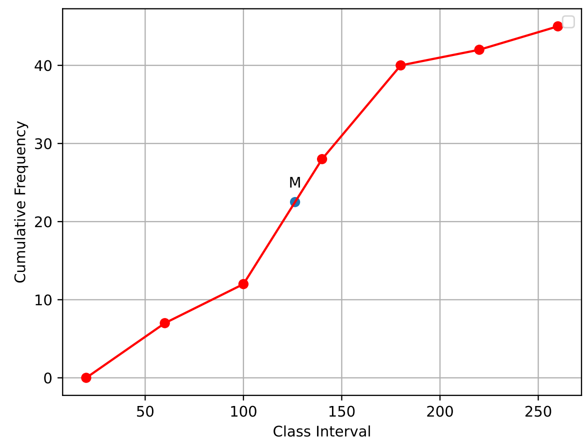

Fig. 6.3.1.

and the median is given by M_1 and the matrix

$$\mathbf{A} = \begin{pmatrix} \mathbf{a} \\ \mathbf{c} \end{pmatrix}, A_{2i} < \frac{A_{2n}}{2} < A_{2,i+1} \tag{6.2.7}$$

From Fig. 6.2.1,

$$n = 8, A_{2n} = 100, i = 4, \tag{6.2.8}$$

$$\mathbf{A}_i = \begin{pmatrix} 30 \\ 40 \end{pmatrix}, \mathbf{A}_{i+1} = \begin{pmatrix} 40 \\ 65 \end{pmatrix} \tag{6.2.9}$$

Thus,

$$\mathbf{M} = \begin{pmatrix} 34 \\ 50 \end{pmatrix}, M_1 = 34 \tag{6.2.10}$$

6.3. The distribution given below in Table 6.3.1 shows the number of wickets taken by bowlers in one-day cricket matches. Find the mean and the median of the number of wickets. Find the mean and the median of the number of wickets taken.

Solution: Defining $\mathbf{a}, \mathbf{b}$ as before in the previous problem, we use a matrix

Number of Wickets	20-60	60-100	100-140	140-180	180-220	220-260
Number of Bowlers	7	5	16	12	2	3

TABLE 6.3.1

$$\mathbf{Q} = \begin{pmatrix} 1 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 1 \end{pmatrix} \tag{6.3.1}$$

to obtain

$$\mathbf{d} = \frac{\mathbf{Q}}{2}\mathbf{a} \tag{6.3.2}$$

The mean is then obtained as

$$\frac{\mathbf{b}^\top \mathbf{d}}{\mathbf{1}^\top \mathbf{b}} \tag{6.3.3}$$

The less than ogive graph is available in Fig. 6.3.1

6.4. Find the mode of the following distribution in Table 6.4.1

Marks	0-10	10-20'	20-30	30-40	40-50	50-60
Number of Students	4	6	7	12	5	6

TABLE 6.4.1

Solution:
Fig. 6.4.1 outlines the approach for calculating the mode for the data in Table 6.4.1. The mode class is first obtained by identifying the interval corresponding to the maximum marks. The mode point is then obtained as the intersection of the lines PQ and RS. The $x-$coordinate of the mode point is the desired mode. For the given problem,

$$\mathbf{P} = \begin{pmatrix} 40 \\ 12 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 30 \\ 7 \end{pmatrix}, \tag{6.4.1}$$

$$\mathbf{R} = \begin{pmatrix} 30 \\ 12 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 40 \\ 5 \end{pmatrix} \tag{6.4.2}$$

and the desired mode point is

$$\mathbf{M} = \begin{pmatrix} 34.167 \\ 9.083 \end{pmatrix} \tag{6.4.3}$$

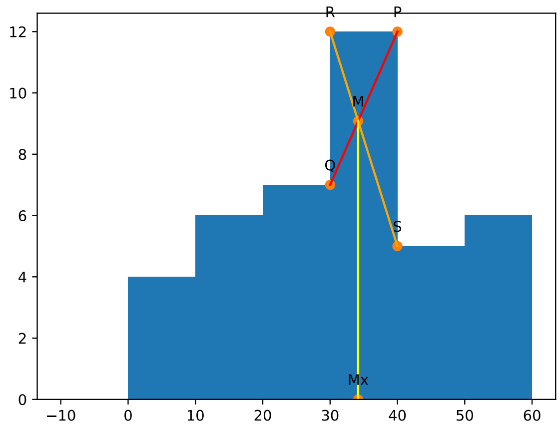

Fig. 6.4.1.

and the mode is 34.167

6.5. If the probablity of an event E happening is 0.023, then $P\left(\bar{E}\right) =$

6.6. The probability of an event that is sure to happen is, ________