Assignment 2 (Due: May 29, 2022)

1. (**Math**) In the augmented Euclidean plane, there is a line $x-3y+4=0$, what is the homogeneous coordinate of the infinity point of this line?

2. (**Math**) On the normalized retinal plane, suppose that $\mathbf{p}_n$ is an ideal point of projection without considering distortion. If distortion is considered, $\mathbf{p}_n=(x, y)^T$ is mapped to $\mathbf{p}_d=(x_d, y_d)^T$ which is also on the normalized retinal plane. Their relationship is,

   $$\begin{cases} x_d = x\left(1+k_1r^2+k_2r^4\right)+2\rho_1xy+\rho_2\left(r^2+2x^2\right)+xk_3r^6 \\ y_d = y\left(1+k_1r^2+k_2r^4\right)+2\rho_2xy+\rho_1\left(r^2+2y^2\right)+yk_3r^6 \end{cases}$$

   where $r^2=x^2+y^2$

   For performing nonlinear optimization in the pipeline of camera calibration, we need to compute the Jacobian matrix of $\mathbf{p}_d$ w.r.t $\mathbf{p}_n$, i.e.,

   $$\frac{d\mathbf{p}_d}{d\mathbf{p}_n^T}$$

   It should be noted that in this question $\mathbf{p}_d$ is the function of $\mathbf{p}_n$ and all the other parameters can be regarded as constants.

3. (**Math**) In our lecture, we mentioned that for performing nonlinear optimization in the pipeline of camera calibration, we need to compute the Jacobian of the rotation matrix (represented in a vector) w.r.t its axis-angle representation. In this question, your task is to derive the concrete formula of this Jacobian matrix. Suppose that

   $\mathbf{r}=\theta\mathbf{n}\in\mathbb{R}^{3\times1}$, where $\mathbf{n}=\begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$ is a 3D unit vector and θ is a real number denoting the rotation angle.

   With Rodrigues formula, $\mathbf{r}$ can be converted to its rotation matrix form,

   $$\mathbf{R}=\cos\theta\mathbf{I}+(1-\cos\theta)\mathbf{n}\mathbf{n}^T+\sin\theta\mathbf{n}^\wedge$$

   and obviously $\mathbf{R}\triangleq\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$ is a 3×3 matrix.

   Denote $\mathbf{u}$ by the vectorized form of $\mathbf{R}$, i.e.,

   $$\mathbf{u}\triangleq\left(R_{11},R_{12},R_{13},R_{21},R_{22},R_{23},R_{31},R_{32},R_{33}\right)^T$$

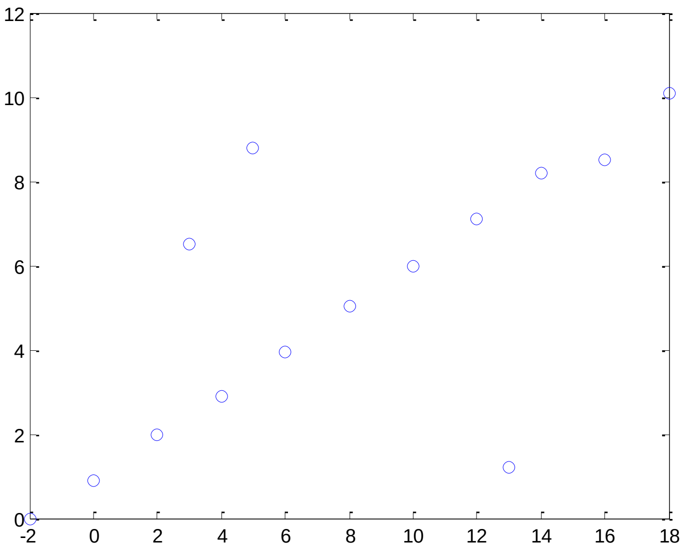
   Please give the concrete form of Jacobian matrix of $\mathbf{u}$ w.r.t $\mathbf{r}$, i.e., $\frac{d\mathbf{u}}{d\mathbf{r}^T}\in\mathbb{R}^{9\times3}$.

   In order to make it easy to check your result, please follow the following notation requirements,

   $$\alpha\triangleq\sin\theta,\beta\triangleq\cos\theta,\gamma\triangleq1-\cos\theta$$

   In other words, the ingredients appearing in your formula are restricted to $\alpha,\beta,\gamma,\theta,n_1,n_2,n_3$.

4. (**Programming**) RANSAC is widely used in fitting models from sample points with outliers. Please implement a program to fit a straight 2D line using RANSAC from the following sample points:
   (-2, 0), (0, 0.9), (2, 2.0), (3, 6.5), (4, 2.9), (5, 8.8), (6, 3.95), (8, 5.03), (10, 5.97), (12, 7.1), (13, 1.2), (14, 8.2), (16, 8.5) (18, 10.1). Please show your result graphically.



5. (**Programming**) Bird's-eye-view generation. The geometric transform between the physical plane and its bird's-eye-view image can be simply described by a **similarity transformation** matrix. Bird's-eye-view is very useful in autonomous industrial inspection, ADAS, etc. In this question, your task is to create the bird's-eye-view image of a physical plane, e.g., the wall of your room. For this purpose, you may need to,
   1) make a calibration board with chessboard patterns;
   2) calibrate your camera (the camera mounted on your laptop or the camera of your mobile phone with fixed focal length) to get its intrinsics;
   3) attach regular patterns (e.g., chessboard patterns) to the wall, determine the 2D coordinate system C_W of the wall, and determine the coordinates $\{\mathbf{x}_{Wi}\}_{i=1}^{N}$ of the feature points of the regular patterns with respect to C_W;
   4) take the image I_d of the wall with regular patterns;
   5) undistort image I_d with the camera's intrinsics to get the undistorted image I;
   6) For each $\mathbf{x}_{Wi}$, determine its image $\mathbf{x}_{Ii}$ on I;
   7) solve the homography matrix $P_{W \to I}$ between the wall and the image I wall using $\{\mathbf{x}_{Wi} \leftrightarrow \mathbf{x}_{Ii}\}_{i=1}^{N}$;
   8) generate the final bird's-eye-view image of the wall using the technique introduced in our lecture.

   For submission, you **only** need to submit the following items to TA:
   1) the intrinsic parameters of your camera;
   2) the original image of the wall (or other physical planes) taken by your camera; make sure that your name is painted or attached on the wall (or the plane); (maybe similar to following image I provide to you)
   3) the generated bird's-eye-view image of the wall (or other physical planes).