Assignment 1 (Due: Apr. 10, 2022)

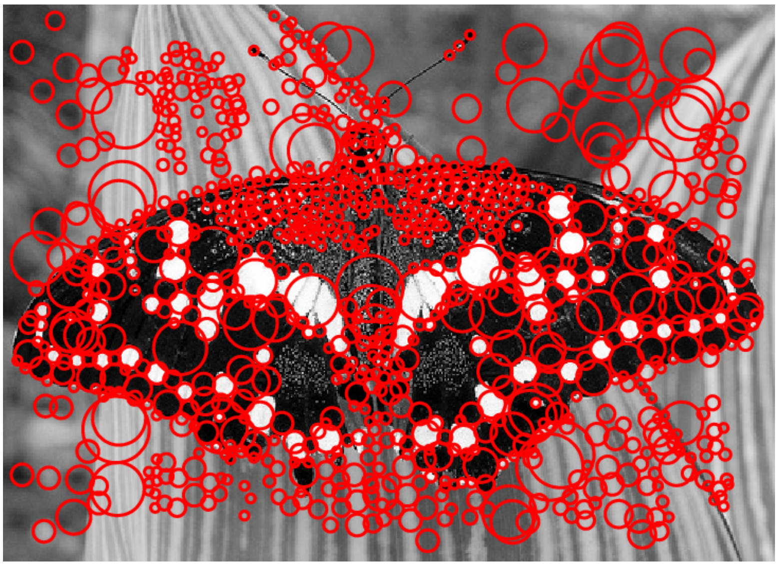
1. (Math) In our lectures, we mentioned that matrices that can represent Euclidean transformations can form a group. Specifically, in 3D space, the set comprising matrices $\{M_i\}$ is actually a group, where $M_i = \begin{bmatrix} R_i & \mathbf{t}_i \\ \mathbf{0}^T & 1 \end{bmatrix} \in \mathbb{R}^{4\times 4}$, $R_i \in \mathbb{R}^{3\times 3}$ is an orthonormal matrix, det($\mathbf{R}_i$)=1, and $\mathbf{t}_i \in \mathbb{R}^{3\times 1}$ is a vector.
Please prove that the set $\{M_i\}$ forms a group.
Hint: You need to prove that $\{M_i\}$ satisfies the four properties of a group, i.e., the closure, the associativity, the existence of an identity element, and the existence of an inverse element for each group element.

2. (Math) When deriving the Harris corner detector, we get the following matrix M composed of first-order partial derivatives in a local image patch w,
$$M = \begin{bmatrix} \sum\limits_{(x_i,y_i)\in w} (I_x)^2 & \sum\limits_{(x_i,y_i)\in w} (I_x I_y) \\ \sum\limits_{(x_i,y_i)\in w} (I_x I_y) & \sum\limits_{(x_i,y_i)\in w} (I_y)^2 \end{bmatrix}$$
   a) Please prove that M is positive semi-definite.
   b) In practice, M is usually positive definite. If M is positive definite, prove that in the Cartesian coordinate system, $[x, y]M\begin{bmatrix} x \\ y \end{bmatrix} = 1$ represents an ellipse.
   c) Suppose that M is positive definite and its two eigen-values are λ_1 and λ_2 and $\lambda_1 > \lambda_2 > 0$. For the ellipse defined by $[x, y]M\begin{bmatrix} x \\ y \end{bmatrix} = 1$, prove that the length of its semi-major axis is $\frac{1}{\sqrt{\lambda_2}}$ while the length of its semi-minor axis is $\frac{1}{\sqrt{\lambda_1}}$.

3. (Math) In the lecture, we talked about the least square method to solve an over-determined linear system $A\mathbf{x} = b, A \in \mathbb{R}^{m\times n}, \mathbf{x} \in \mathbb{R}^{n\times 1}, m > n, rank(A) = n$. The closed form solution is $\mathbf{x} = \left(A^T A\right)^{-1} A^T b$. Try to prove that $A^T A$ is non-singular (or in other words, it is invertible).

4. (Programming) Get two images, taken from the same scene but with scale transformations. Detect the scale invariant points on the two images. You can use the center of the circle to indicate the spatial position of the point and use the

radius of the circle to indicate the characteristic scale of the point, just like the following example.



5. (Programming) Get two images I_1 and I_2 of our campus and make sure that the major parts of I_1 and I_2 are from the same physical plane. Stitch I_1 and I_2 together to get a panorama view using scale-normalized LoG (or DoG) based interest point detector and SIFT descriptor. You can use OpenCV or VLFeat.