Set Theory (HD)

Prove:

Prove that for all sets $A \cup (B \cap A) = A$.

proof

$$\begin{aligned}
A \cup (B \cap A) &= (A \cup B) \cap (A \cup A) && \text{(Distributivity of } \cup \text{ over } \cap) \\
&= (A \cup B) \cap A && \text{(Idempotence of } \cup) \\
&= (A \cup B) \cap (A \cup \emptyset) && \text{(Identity of } \cup) \\
&= A \cup (B \cap \emptyset) && \text{(Distributivity of } \cup \text{ over } \cap) \\
&= A \cup (B \cap (B \cap B^c)) && \text{(Complement with } \cap) \\
&= A \cup ((B \cap B) \cap B^c) && \text{(Associativity of } \cap) \\
&= A \cup (B \cap B^c) && \text{(Idempotence of } \cap) \\
&= A \cup \emptyset && \text{(Complement with } \cap) \\
&= A && \text{(Identity of } \cup)
\end{aligned}$$