Number Theory (D)

Proof

prove:
   Prove that $m =_{(n)} p$ if and only if $m\%n = p\%n$.

proof

I will prove that $m =_{(n)} p$ if and only if $m\%n = p\%n$, by showing that

(a) if $m =_{(n)} p$, then $m\%n = p\%n$,

(b) if $m\%n = p\%n$, then $m =_{(n)} p$.

proof of (a)

Because $m =_{(n)} p$, therefore $p < n$.
Assume $p\%n \neq m\%n$, so $p\%n \neq p$, therefore $p \geq n$.
contradiction!
Therefore, if $m =_{(n)} p$, then $m\%n = p\%n$.

proof of (b)

Let $m\%n = p\%n = p'$,
so $m = k_1 * n + p'$ and $p = k_2 * n + p'$.
so $m - p = (k_1 - k_2) * n$
so $m = (k_1 - k_2) * n + p$
Let $k_1 - k_2 = k$,so $m = k * n + p$
Therefore, $m =_{(n)} p$.

Therefore,$m =_{(n)} p$ if and only if $m\%n = p\%n$.