Number Theory (HD)

Proof

prove

Prove that if $m =_{(n)} m'$, $p =_{(n)} p'$ and $q =_{(n)} q'$ then $m + p * q =_{(n)} m' + p' * q'$.

proof

Let $m = k_1 * n + m'$, $p = k_2 * n + p'$ and $q = k_3 * n + q'$, Therefore,

$$m + p * q = (k_1 * n + m') + (k_2 * n + p') * (k_3 * n + q')$$

$$= k_1 * n + m' + k_2 * k_3 * n^2 + k_2 * q' * n + k_3 * p' * n + p' * q'$$

$$= (k_1 + k_2 * k_3 * n + k_2 * q' + k_3 * p') * n + m' + p' * q'$$

Let $k_1 + k_2 * k_3 * n + k_2 * q' + k_3 * p' = k$, so m + p * q = k * n + m' + p' * q'. Therefore, $m + p * q =_{(n)} m' + p' * q'$.