

Number Theory (P)

Prove:

Prove that if $m =_{(n)} m'$ and $p =_{(n)} p'$ then $m + p =_{(n)} m' + p'$

proof

By the definition of mod, $m =_{(n)} p$ if $n|(m - p)$. So we can get that $n|(m - m')$ and $n|(p - p')$.

Therefore, for some $k_1, k_2 \in Z$,

$$m = k_1n + m',$$

$$p = k_2n + p'.$$

Add two equation, we can get that $m + p = (k_1 + k_2)n + (m' + p')$.

Because $k_1, k_2 \in Z$, then $k_1 + k_2 \in Z$, so $n|((m + p) - (m' + p'))$.

Therefore $m + p =_{(n)} (m' + p')$.