Set Theory (C)

Prove:

Prove that for all sets $A, B : A = B$ if and only if $A \cap B = A \cup B$.

proof

I will prove that for all sets $A, B : A = B$ if and only if $A \cap B = A \cup B$, by showing that

(a) if $A = B$, then $A \cap B = A \cup B$,

(b) if $A \cap B = A \cup B$, then $A = B$.

Proof of (a)

$$
\begin{aligned}
A \cap B &= A \cap A && (A = B) \\
&= A && (\text{Idempotence of } \cap) \\
&= A \cup A && (\text{Idempotence of } \cup) \\
&= A \cup B && (A = B)
\end{aligned}
$$

Proof of (b)

By the result of set theory(HD), we know that $A = A \cup (B \cap A)$.

$$
\begin{aligned}
A &= A \cup (B \cap A) && (\text{Result of (HD)}) \\
&= A \cup (A \cap B) && (\text{Communitativity of } \cap) \\
&= A \cup (A \cup B) && (A \cap B = A \cup B) \\
&= (A \cup A) \cup B && (\text{Communitativity of } \cap) \\
&= A \cup B && (\text{Idempotence of } \cup) \\
&= A \cup (B \cup B) && (\text{Idempotence of } \cup) \\
&= (B \cup B) \cup A && (\text{Communitativity of } \cup) \\
&= B \cup (B \cup A) && (\text{Associativity of } \cup) \\
&= B \cup (A \cup B) && (\text{Communitativity of } \cup) \\
&= B \cup (A \cup B) && (A \cap B = A \cup B) \\
&= B && (\text{Result of (HD)})
\end{aligned}
$$