Set Theory (D)

Prove:

Prove, using the Laws of Set Operations, for all sets $A : A \cap (\mathcal{U} \oplus A) = \emptyset$.

proof

$$
\begin{aligned}
A \cap (\mathcal{U} \oplus A) &= A \cap ((\mathcal{U} \cap A^c) \cup (\mathcal{U}^c \cap A)) && \text{(Definition of } \oplus) \\
&= A \cap ((A^c \cap \mathcal{U}) \cup (\mathcal{U}^c \cap A)) && \text{(Communitivity of } \cap) \\
&= A \cap (A^c \cup (\mathcal{U}^c \cap A)) && \text{(Identity of } \cap) \\
&= A \cap (A^c \cup (\mathcal{U}^c \cap \mathcal{U} \cap A)) && \text{(Identity of } \cap) \\
&= A \cap (A^c \cup (\mathcal{U} \cap \mathcal{U}^c \cap A)) && \text{(Communitivity of } \cap) \\
&= A \cap (A^c \cup (\emptyset \cap A)) && \text{(Complement with } \cap) \\
&= A \cap (A^c \cup (A \cap \emptyset)) && \text{(Communitivity of } \cap) \\
&= A \cap (A^c \cup (A \cap A \cap A^c)) && \text{(Complement with } \cap) \\
&= A \cap (A^c \cup (A \cap A^c)) && \text{(Idempotence of } \cap) \\
&= A \cap ((A^c \cup A) \cap (A^c \cup A^c)) && \text{(Distributivity of } \textit{cup} \text{ over } \cap) \\
&= A \cap ((A^c \cup A) \cap A^c) && \text{(Idempotence of } \cup) \\
&= A \cap ((A \cup A^c) \cap A^c) && \text{(Communitivity of } \cup) \\
&= A \cap (\mathcal{U} \cap A^c) && \text{(Complement with } \cup) \\
&= (A \cap \mathcal{U}) \cap A^c && \text{(Assosiativity with } \cap) \\
&= A \cap A^c && \text{(Identity of } \cap) \\
&= \emptyset && \text{(Complement with } \cap)
\end{aligned}
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