Optimizing Recovered Energy in Rear Axle Regenerative Braking Systems

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***Abstract*—Regenerative braking systems are becoming more common due to the increased popularity of electric and hybrid vehicles. We formulate an expression to optimize the energy regeneration capabilities of a rear axle system and prove that maximizing the regenerated energy is equivalent to a convex optimization of the regenerative acceleration at each time instant. We then propose a model that combines a differential equation solver with a convex optimizer to compute the energy regenerated for any given set of vehicle parameters. Preliminary experiments were also run to examine the effects of different parameters on the overall energy regenerated for a single brake event.**

Keywords - Optimization, regenerative braking, blended braking, FSAE

I. Introduction

As electric vehicles become more popular, it is important to increase their efficiency and range. The operation of a regenerative braking system provides significant benefits to an electric vehicle by allowing it to regain a portion of the energy that would otherwise be lost to friction in a braking event [1]. The Formula Society of Automotive Engineers (FSAE) team at Virginia Tech would like to incorporate such a system into its prototype electric vehicle, which is planned to compete at the annual international competition in May 2017. The goal of this project will be to develop a model that will determine the optimum parameters for the regenerative braking system conditioned on any desired set of braking events. Since the vehicle is still undergoing rapid changes in design, the program should allow for easy alterations of the necessary parameters.

A typical commercial blended braking system uses a brake-by-wire system to dynamically configure the interaction of the frictional and regenerative braking systems to maximize energy returned while maintaining a constant braking force. Such a system is necessary because energy varies with the velocity of the vehicle while the braking force does not. The amount of energy regenerated by the system has a cap. The blended braking system keeps energy at this cap while maintaining constant braking force by varying the percentage of energy each system is handling [1]. This interaction is diagrammed in Figure 1.

The graph depicts the elements present in a braking event. The demanded deceleration ramps up to 0.35g within the first half second and then stays constant for rest of the duration. The frictional and regenerative elements work together to provide the requested braking force, with the regenerative system taking priority until it reaches its energy cap. As the vehicle slows down, the amount of energy decreases and the regenerative portion can form a greater percentage of the deceleration demand while keeping the amount of power regenerated constant. This continues until the regenerative system forms the entirety of the deceleration demand. Afterwards, the energy cap of the system is not being reached and the power regenerated begins to fall. In this region, the system is operating at its maximum regenerative potential. The frictional system reengages when the wheel speed falls below a specified threshold, as the regenerative system is no longer able to provide the power necessary to charge the battery [1].

The system on the Electric Powertrain (EPT) car cannot achieve these levels of efficiency because brake-by-wire systems are forbidden by the FSAE rules for EPT vehicles due to safety concerns [2]. As a result, the force provided by the frictional braking is held constant for a constant deceleration demand. This forces a design choice between the amount of energy regenerated and the smoothness of the braking system; any additional braking force provided by the regenerative system to keep the system at its power cap will contribute additional deceleration.

The system is further complicated by the location of the regenerative braking system. The EPT car only supports regen braking on the rear two wheels. The moment generated due to a braking event shifts the effective center of mass towards the front wheels, reducing the available power that can be regenerated at the rear wheels [3]. A diagram and equations describing the load shift are included in Figure 2.

The energy for each wheel dissipated by the brake system is a function of weight, deceleration, and the effective location of the center of mass. By assuming that the tires do not slip, the effective braking force associated with the power regenerated at each wheel must be less than the force due to friction between each tire and the driving surface. If one assumes further that the coefficient of friction between any one tire and the driving surface is consistent and identical to that associated to any other wheel, then the maximum allowable braking force at any given wheel is proportional to the supported load at that wheel. In other words, the maximum available power for any given wheel depends on the weight distribution of the vehicle. Since

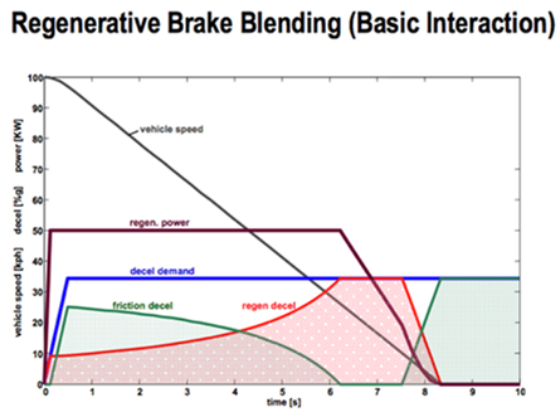


Figure 1: Typical behavior of a commercial blended braking system [1]

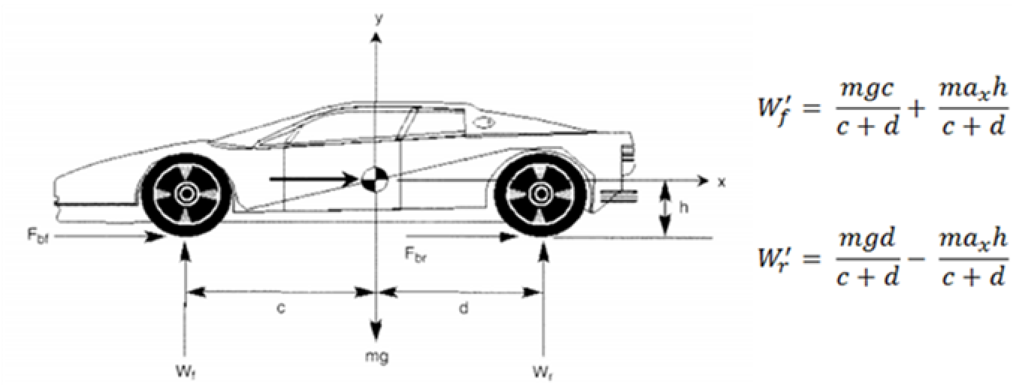


Figure 2: Load transfer of a two axle vehicle [3]

the vehicle weight supported by the rear axle is reduced during a braking event, the available power that can be regenerated at the rear axle is reduced proportionately [3]. This means that a good portion of the energy dissipated is unavailable to this regenerative system.

II. Derivation

Two wheel regen systems are uncommon, and no literature detailing their operation was found. There was also no prior documentation available in our team, as this is the first time a regenerative braking system has been used. Thus, a derivation of how the system worked from basic physics was necessary. Equations for physics as relating to brakes were taken from [4].

The model was implemented using a series of difference equations to track changes in four major variables: velocity, power regenerated, regenerative contribution to deceleration, and frictional contribution to deceleration. The majority of it was fairly straightforward to implement; however, the addition of the load shift on the axles during a brake event made the calculations for the regenerative portion of the system more complicated.

Some brake models begin with the full desired deceleration applied. This simplifies the model significantly, as the ramp-up time and associated weight transfer (assuming a constant brake force system) can be ignored. This is not suitable for this model, however, as it significantly skews the amount of power regenerated early in the brake event. In addition, the presence of a non-constant brake force caused by the regenerative system necessitates that the load transfer be taken into account anyway. Thus, the brake force ramps up from zero to its maximum value over 0.6 seconds. This is a typical system response time used in braking models. The model assumes that the kinetic energy of the vehicle is converted to other forms, such as heat, by means of the braking system. As such, the maximum power available to the regenerative braking system is equal to the instantaneous rate of change of the kinetic energy of the vehicle. This model simplifies the expression for available power by restricting the available kinetic energy to only the translational kinetic energy:

$$P = \frac{\partial}{\partial t}\left(\frac{1}{2}\right) mv^2 = mav \tag{1}$$

where P is the total power available, m is the mass of the vehicle, a is the vehicle's acceleration, and v is the vehicle's velocity. It is clear from equation 1 that the available power may be expressed as an equivalent force multiplied by a velocity. When viewed from this perspective, the link between the 'no slip' condition, relating the force of friction to the braking force, and the power regenerated becomes apparent.

From here, the weight distribution is calculated to determine the effective mass at each wheel. Equations 2-5 are derived by assuming that only two forces are present during braking on a flat, level surface: a forward force during deceleration, and a lateral force while the vehicle is turning. Each of these forces shift the weight distribution and change the power that can be regenerated at each wheel.

$$m_{front\_R} = M\left(\frac{d}{c+d}\right)\left(\frac{d^{'}}{c^{'}+d^{'}}\right)+\frac{Ma_c h}{2g\left(c^{'}+d^{'}\right)}+\frac{Ma_c h}{2g\left(c+d\right)} \tag{2}$$

$$m_{front\_R} = M\left(\frac{d}{c+d}\right)\left(\frac{d^{'}}{c^{'}+d^{'}}\right)+\frac{Ma_c h}{2g\left(c^{'}+d^{'}\right)}-\frac{Ma_c h}{2g\left(c+d\right)} \tag{3}$$

$$m_{front\_R} = M\left(\frac{d}{c+d}\right)\left(\frac{d^{'}}{c^{'}+d^{'}}\right)-\frac{Ma_c h}{2g\left(c^{'}+d^{'}\right)}-\frac{Ma_c h}{2g\left(c+d\right)} \tag{4}$$

$$m_{front\_R} = M\left(\frac{d}{c+d}\right)\left(\frac{d^{'}}{c^{'}+d^{'}}\right)-\frac{Ma_c h}{2g\left(c^{'}+d^{'}\right)}+\frac{Ma_c h}{2g\left(c+d\right)} \tag{5}$$

where a_c is the centrifugal acceleration, a_T is the total forward deceleration, M is the total vehicle mass, g is the acceleration due to gravity, h is the height of the center of gravity, and the constants c, d identify the location of the center of mass relative to the wheels, as depicted in Figure 2. The primed counterparts, $c^{'}$ and $d^{'}$, identify the horizontal position of the center of mass relative to the lateral position of the center of mass. The centrifugal acceleration is assumed to be positive for left turns, and the forward deceleration is assumed to always be a positive value. To simplify the problem and construct it to fit the task at hand, only the rear axle is considered, and the power generated is assumed to be the total power obtained from both rear wheels. Using this situation, the equivalent mass becomes the sum of equations 3 and 4. It is also assumed that the total forward deceleration may be linearly separated into components which separately contribute to either frictional braking or regenerative braking. Equation 6 expresses the total equivalent mass on the rear axle.

$$m_{rear} = \frac{Mh}{g\left(c+d\right)} + \left(\frac{gc}{h} - a_F - a_R\right) \tag{6}$$

where a_F is the deceleration due to friction and a_R is the deceleration due to power regeneration. Assuming that no slipping occurs such that the forward velocity at each wheel is equal to the vehicle speed, equations 1 and 6 can be combined to write an expression for the power regenerated, in terms of the portion of deceleration which is due to the regeneration.

$$P_R = \frac{\epsilon_R M h v}{g\left(c+d\right)}\left(\left(\frac{gc}{h} - a_F\right)a_R - a_R^2\right) \tag{7}$$

Where a_R is the deceleration due to the power regeneration, a_F is the deceleration due to frictional braking, v is the vehicle's forward speed, and ϵ_R is the efficiency of the power regeneration.

Using equation 7, one can now determine the total energy regenerated for a given braking event, defined by an initial velocity, a final velocity, and deceleration curves. Equation 8 expresses the total energy regenerated in a given braking event.

$$E_R = \frac{\epsilon_R M h}{g\left(c+d\right)}\int_{t_i}^{t_f} v\left(\left(\frac{gc}{h} - a_F\right)a_R - a_R^2\right)dt \tag{8}$$

where t_i is the time at the initial velocity, t_f is the time a the final velocity, a_F is some known deceleration due to friction, a_R is the deceleration due to the regenerative braking. The quantity a_R is a function which depends on time that is to be determined such that equation 8 is maximized for a given braking event. Only braking events are considered in the model and, as such, all acceleration variables are assumed to have positive values which correspond to decelerations. Considerations will also be made for drivability, to ensure that the braking system does not become too unbalanced. The major design tradeoff will be between drivability and regenerative efficiency; since this is not a brake-by-wire system, a tradeoff will have to be made between using the system to recapture the most power or maintain a linear braking curve.

III. System Constraints

The subsystem used to generate power during a braking event will have limitations. The total instantaneous power generation will be limited by either the power generation capabilities of the selected components or the power handling capabilities of the battery charging system. Either way, a maximum power restriction is imposed on the system. Equation 9 uses equation 7 to express the restriction on the maximum instantaneous power generation allowed by the system.

$$\frac{\epsilon_R M h v}{g\left(c+d\right)}\left(\left(\frac{gc}{h} - a_F\right)a_R - a_R^2\right) \leq P_{Rmax} \tag{9}$$

The vehicle is assumed to be driven in a safe manner. As such, power generation is limited to velocities below some predetermined, safe speed. Similarly, power generation does not occur efficiently at very low speeds, and the system will define some minimum velocity, below which power regeneration will not be engaged. Additionally, slow velocities measured at each wheel may indicate vehicle skidding or wheel lock-up, both of which are undesired events. Although, this work will not consider either of these events. Equation 10 expresses the range of vehicle velocity during which the power regeneration system may be engaged.

$$v_{max} \geq v = v\left(t_i\right) - \int_{t_i}^{t_f} a_T dt \geq v_{min} \tag{10}$$

The vehicle is also assumed to maintain contact between all four tires and the driving surface. This constraint requires that the equivalent mass at any given wheel is non-negative, leading to equation 11.

$$a_R \leq \frac{gc}{h} - a_F \tag{11}$$

Similarly, the drivability factor, or non-linear tolerance, which imposes an upper limit on the deceleration available to the driver is expressed in equation 12 as a percentage of the desired deceleration. Additionally, upon examination of equation 11, it is clear that the non-linear tolerance constraint is only active for a limited range, specified by equation 13. For values outside this range, the constraint in equation 11 is more restrictive and the nonlinear tolerance does not alter the system.

$$a_R \leq \eta_{nonlinear} * a_{desired} - a_F \tag{12}$$

$$1 \leq \eta_{nonlinear} \leq \frac{gc}{ha_{desired}} \tag{13}$$

IV. System Analysis

The objective function, equation 8, is used to maximize the energy regenerated by the system during a given braking event. The primary work will be focused on braking from some nominal speed until the vehicle comes to a complete stop. Due to the fact that the deceleration attributed to regeneration is independent of previous values, the problem may be broken down and viewed from different instances in time. This allows the objective function to be reduced to only consider the integrand, and the integrand is a concave function with respect to the variable a_R.

Lemma 1. *The integrand of equation 8 is a concave function with respect to a_R*

Proof. Let each of the variables, M, ϵ_R, a_F, a_R, and v be non-negative. Let h, g, c and d be positive. The integrand of equation 8 is given by equation 7. Taking the second derivative of equation 7 with respect to a_R,

$$\frac{\partial^2}{\partial a_R^2} P_R = \frac{\epsilon_R M h v}{g\left(c+d\right)} \left(\frac{\partial}{\partial a_R}\right) \left(\left(\frac{gc}{h} - a_F\right) - 2a_R\right)$$

$$= \frac{-2\epsilon_R M h v}{g\left(c+d\right)} \leq 0$$

Thus, the objective function is a concave function since the second derivative is non-positive for all possible values of a_R. □

The quadratic constraint may be further simplified to convert the problem into a convex optimization problem. By examining the boundary of the constraint, it becomes apparent that two possible cases apply. The boundary condition is given in equation 14.

$$-\frac{P_{Rmax} g\left(c+d\right)}{Mhv} + \left(\frac{gc}{h} - a_F\right) a_R - a_R^2 = 0 \tag{14}$$

The solution to equation 14 can be divided into two cases.

Case 1. *The roots of equation 14 are real. For this case, the system is generating the maximum allowable power and the constraint is active. Physically, the feasible set of valid aR values must be continuous and non-negative. Also, both roots of equation 14 will have the same sign. Therefore, the maximum value of a_R is limited to the lower root, if both are real-valued.*

Proof. Let

$$4P_{Rmax} g\left(c+d\right) \leq Mhv \left(\frac{cg}{h} - a_F\right)^2$$

Then the roots of equation 14 are,

$$\frac{1}{2}\left(\frac{gc}{h} - a_F\right) \pm \left(\frac{gc}{h} - a_F\right) \sqrt{1 - \frac{4P_{Rmax} g\left(c+d\right)}{Mhv\left(\frac{gc}{h} - a_F\right)^2})}$$

$$= \frac{1}{2}\left(\frac{gc}{h} - a_F\right) \left(1 \pm \sqrt{1 - \frac{4P_{Rmax} g\left(c+d\right)}{Mhv\left(\frac{gc}{h} - a_F\right)^2}}\right)$$

By our assumption we have,

$$0 \leq 1 \pm \sqrt{1 - \frac{4P_{Rmax} g\left(c+d\right)}{Mhv\left(\frac{gc}{h} - a_F\right)^2}} \leq 2$$

Since the quantity $1/2(gc/h - a_F)$ must be non-negative, per equation 11, and using the non-negativity requirement of a_R, both roots must be non-negative. Thus, the lower root,

$$\frac{1}{2}\left(\frac{gc}{h} - a_F\right) \left(1 - \sqrt{1 - \frac{4P_{Rmax} g\left(c+d\right)}{Mhv\left(\frac{gc}{h} - a_F\right)^2}}\right) \tag{15}$$

is the maximum valid value for a_R, since it is the only root which allows a continuous feasible set of a_R.

For a given vehicle design and frictional deceleration setting, the minimum velocity threshold for determining when case 1 applies is,

$$v \leq v_{thres} = \frac{4P_{Rmax} g\left(c+d\right)}{Mhv\left(\frac{gc}{h} - a_F\right)^2}$$

□

Case 2. *The roots of equation 14 are complex. For this case, the system is not able to generate the maximum power, and the constraint of equation 9 is not active. The quadratic constraint from equation 9 may be dropped in this case. The maximum value is also readily calculated, subject to the other constraints of the system.*

Proof. Consider the derivative of equation 9 with respect to a_R,

$$0 = \frac{\partial}{\partial a_R} P_R = \frac{\epsilon_R M h v}{g\left(c+d\right)} \left(\left(\frac{gc}{h} - a_F\right) - 2a_R\right)$$

Then,

$$0 = \frac{gc}{h} - a_F - 2a_R$$

Resulting in the maximum value of,

$$a_{Rmax} = \frac{1}{2}\left(\frac{gc}{h} - a_F\right) \tag{16}$$

Since a_{Rmax} is both non-negative and the maximum value for case 2, values of a_R which exceed a_{Rmax} do not need to be considered. Therefore, the constraint for case 2 is,

$$a_R \leq \frac{1}{2}\left(\frac{gc}{h} - a_F\right) \tag{17}$$

□

Upon examination of both cases, it is apparent that they may be expressed as a single constraint. Consider the expression for the lower root in equation 15. Suppose the term under the radical is non-positive. Then the resulting constraint from case 1 becomes,

$$a_R \leq \frac{1}{2}\left(\frac{gc}{h} - a_F\right)\left(1 - i\sqrt{1 - \frac{4P_{Rmax}g\left(c+d\right)}{Mhv\left(\frac{gc}{h} - a_F\right)^2}}\right) \tag{18}$$

Clearly, equation 18 is equivalent to equation 17 if one considers only the real part of equation 18. Therefore, the restriction imposed for case 1, requiring that the roots be real, may be relaxed by expressing the linear constraint as the real portion of the lower root of equation 14. The constraint for both cases becomes,

$$a_R \leq \mathbb{R}\{a_{R-}\} \tag{19}$$

Additionally, the constraint of equation 19 has a maximum value when the term under that radical is non-positive, resulting in a tighter bound than that provided by the constraint in equation 11. Comparing equations 11 and 19,

$$a_R \leq \mathbb{R}\{a_{R-}\} \leq \frac{1}{2}\left(\frac{gc}{h} - a_F\right)$$

Since the maximum value of the linear constraint of equation 19 is always less than the constraint of equation 11, the constraint found in equation 11 may be dropped.

V. Objective Function

After analyzing the system and its constraints, it has been determined that only three constraints on the acceleration due to power regeneration need to be considered. The velocity constraints are implicitly tied to both the frictional and regenerative accelerations and they determine the valid range of velocities within which the regenerative system may be operated.

Given the above system constraints, the objective function for this project is then:

$$E_R^* = \int_{t_i}^{t_f} P_R^* dt = \frac{\epsilon_R Mhv}{g\left(c+d\right)} \int_{t_i}^{t_f} \left(\left(\frac{gc}{h} - a_F\right) a_R - a_R^2\right) dt$$

Subject to the constraints,

$$a_R \leq \mathbb{R}\left\{\frac{1}{2}\left(\frac{gc}{h} - a_F\right)\left(1 - \sqrt{1 - \frac{4P_{Rmax}g\left(c+d\right)}{Mhv\left(\frac{gc}{h} - a_F\right)^2}}\right)\right\}$$

$$a_R \leq \eta_{nonlinear} * a_{desired} - a_F$$

$$v_{min} \leq v = v\left(t_i\right) - \int_{t_i}^{t_f} a_T dt \leq v_{max}$$

$$-a_R \leq 0$$

While the maximum power constraint may not be significant for a single brake event, it will affect the optimization when considering maximizing the power regenerated over a wide range of braking events.

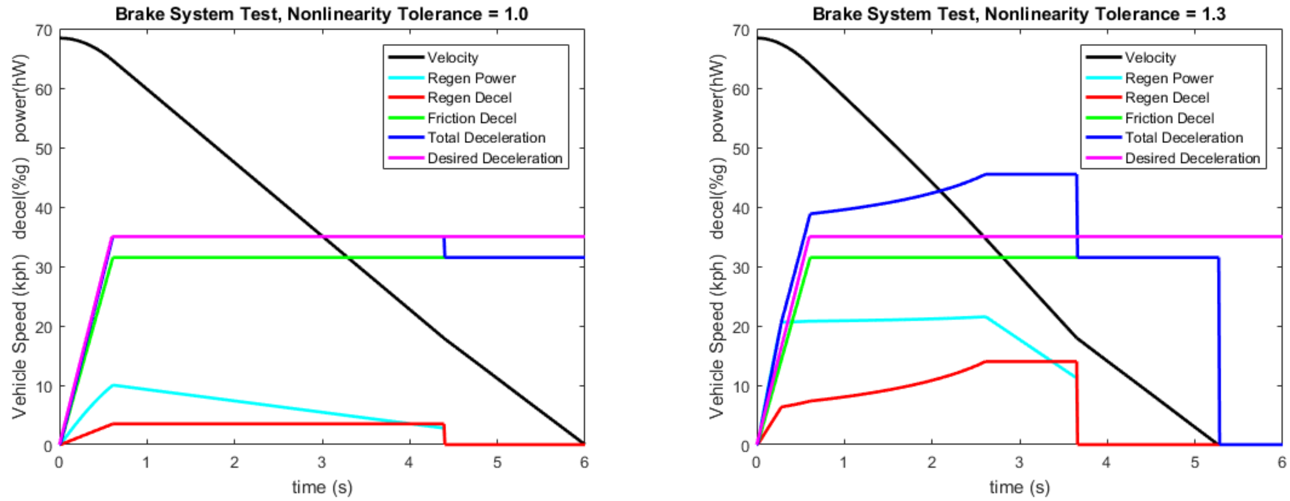
VI. Simulations and Results

Modeling the above objective function is difficult because there are constraints on both the acceleration and velocity. However, as we proved above, the acceleration at a given point in time is convex. Furthermore, the system memory only applies to the velocity constraints; maximizing acceleration at each point it time is equivalent to maximizing the energy regenerated. This allows us to split our model into two components: a convex solver for the acceleration constraints and a differential equation simulation for the velocity constraints. The differential equation portion checks whether the velocity constraint is satisfied before deciding whether to call the convex solver to compute the regenerative acceleration for a given time block and keeps track of the change in velocity throughout the braking event. We use Matlab's quadprog function (an interior point convex solver) to calculate the regenerative acceleration and a simple forward Euler routine to compute the velocity.

Most of the values placed in the model were taken from known information about the EPT car. However, certain aspects of the regenerative system depend on the motors being used. Since the motors have not come in yet, placeholder values are used for preliminary testing of the system. Maximum regenerative power is set to 2 kW, minimum speed for power regeneration to occur is 5 m/s, and regenerative efficiency is assumed to be 1. These values will be replaced when the motors are available for testing.

Two sample runs of the model are included in Figures 3a and 3b. The only factor that changes between the two is the nonlinearity tolerance factor.

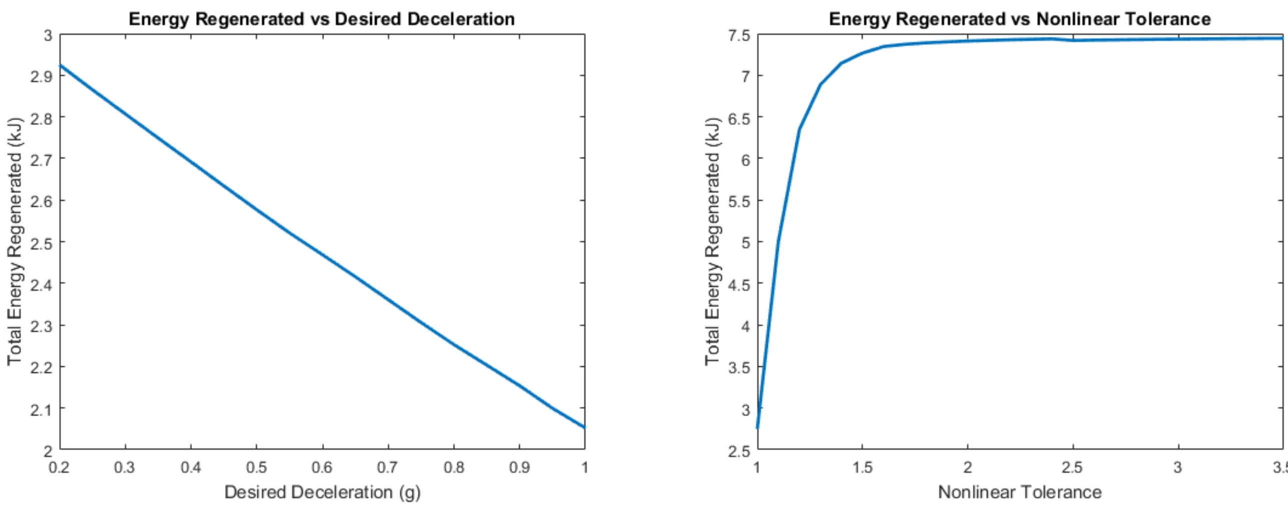
The total amount of energy regenerated by the first run was 2.7 kJ, while the second run regenerated 6.9 kJ – a 4.2 kJ difference from one brake event. The tradeoff is the effect on the brake curve. While the first run has a steady brake output



(a) Model output of system where the nonlinearity tolerance = 1.

(b) Model output of system where the nonlinearity tolerance = 1.3.

Figure 3: Two sample outputs from our model. Only the nonlinearity tolerance was changed between the two simulations.



(a) Model simulation sweeping the desired deceleration parameter.

(b) Model simulation sweeping the nonlinear tolerance parameter.

Figure 4: Two simulated sweeps over tunable parameters using our model.

until the regenerative system disengages, the second run has a significant amount of acceleration throughout the regenerative portion of the curve.

We also did two experiments to examine the effects that nonlinearity tolerance and desired deceleration had on the system. These are tunable parameters, so learning how they effect the system will aid in design decisions for the final system. The desired deceleration sweep had a nonlinear tolerance fixed at 1.3 and swept the deceleration demand from 0.2g to 1g. The nonlinear sweep had the deceleration demand fixed at 1.3 and swept the nonlinear tolerance from 1.0 to 3.5. The initial velocity was set at 19 m/s and the ending velocity at 0 m/s, as in the model simulations in Figures 3a and 3b. The results for the deceleration sweep are included in Figure 4a. The results for the nonlinear tolerance sweep are included in Figure 4b.

The desired deceleration sweep is effectively linear, indicating that energy regenerated increases with less braking force applied. This is likely due to the shift of mass in the vehicle towards the front. While the total amount of energy being dissipated stays the same, a higher braking force throws more mass of the vehicle towards the front resulting in less power for the regenerative system to capture on the back wheels.

The nonlinear sweep has a logarithmic curve. Early on, small increases in the nonlinear factor give significant returns on the amount of energy regenerated. This is because it allows the regenerative system to use more of its capacity by loosening the total acceleration constraint. Later on the regenerative system saturates and further increases in nonlinearity tolerance lead to less returns on energy. The cap for nonlinear tolerance was derived in equation 13, but seeing the actual shape provides greater insight into the expected energy returns.

VII. Conclusion

We have proved that maximizing the regenerated energy for a rear-axle regenerative braking system can be formulated as a convex optimization problem with respect to the regenerative component of acceleration. Furthermore, we have shown that the memory constraint in the system is orthogonal to the optimal value at each time step. We also present a model capable of simulating the total amount of energy regenerated in a brake event give model parameters, and have used it to explore expected returns for tweaking design changes to the vehicle.

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