Proof of Product of Even and Odd Numbers

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1 Proof that the product of two even numbers are even

Definition: An even number is any number that is divisible by 2. This means that any even number can be written in the form $x = 2n$ where $n \in \mathbb{Z}$ (The set of integers is denoted by $\mathbb{Z} = \{..., -2, -1, 0, 1, 2, ...\}$

Let $a, b \in \mathbb{Z}$, where a and b are even numbers. That means they can be written as follows:

$$a = 2m$$
$$b = 2n$$

where $m, n \in \mathbb{Z}$. Then:

$$\begin{aligned} ab &= (2m)(2n) \\ &= 4mn \\ &= 2(2mn) \end{aligned}$$

Let $l = 2mn$. Since $\mathbb{Z}$ is closed under multiplication, then $2mn \in \mathbb{Z}$, and therefore, $l \in \mathbb{Z}$. Then ab is, by definition, an even number, since $ab = 2l$, $l \in \mathbb{Z}$. Therefore, any even numbers times an even number is even itself. ■