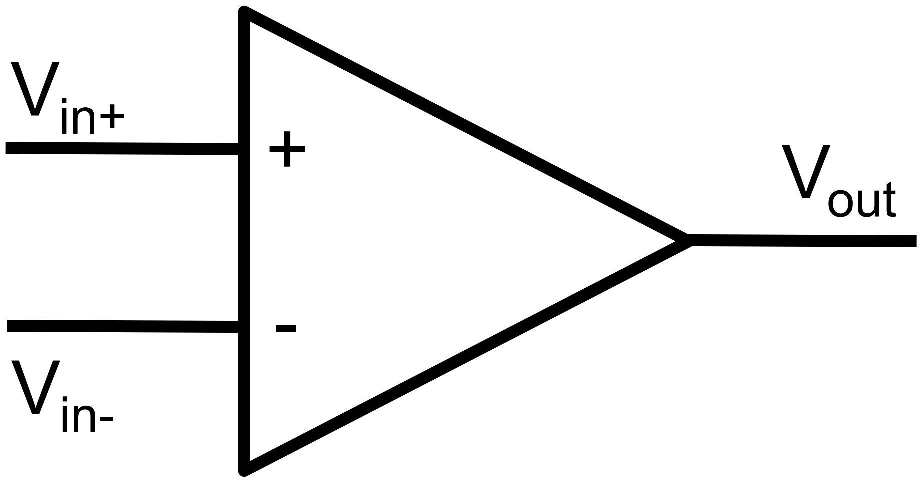
MICROWAVE AMPLIFIER DESIGN - I



EE5303 – Part 2

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EE5303 Part 2 -- Study Scopes & Schedule

Week #	Date (6pm-9pm, Friday @ LT 6)	Topics	Remark
8	10-Oct	**Microwave Amplifier Design - I**	**LT6**
9	17-Oct	**Microwave Amplifier Design - II** --- ****Release of Mini-Project #2: Amplifier Design by ADS* *(*Same grouping as Part 1) [Report submission deadline: 14 Nov]*	**LT6**
10	24-Oct	**Microwave Oscillators**	**LT6**
11	31-Oct	**Microwave Mixers** --- **** Week 11: Computer Lab sessions (ADS) for Mini-Project #2*	**LT6** **computer lab**
12	7-Nov	**Quiz #2 (1 hour, 6:15pm-7:15pm)** --- **** Week 12: Computer Lab sessions (ADS) for Mini-Project #2*	**LT6** **computer lab**
13	14-Nov	****Course Revision (Parts I and II)** --- ***** Project 2 -- Report submission deadline: 14 Nov***	**LT6**
Reading week	15-Nov	*(Reading Week)*	
Exam weeks	22 Nov onwards	01 Dec @ 5pm (for 2 hours) -- to be confirmed	

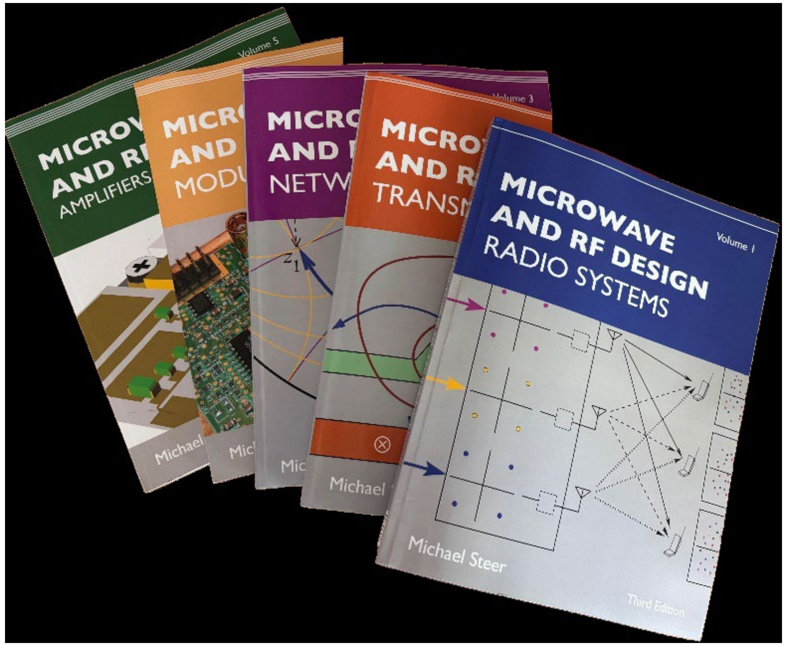
Four lectures

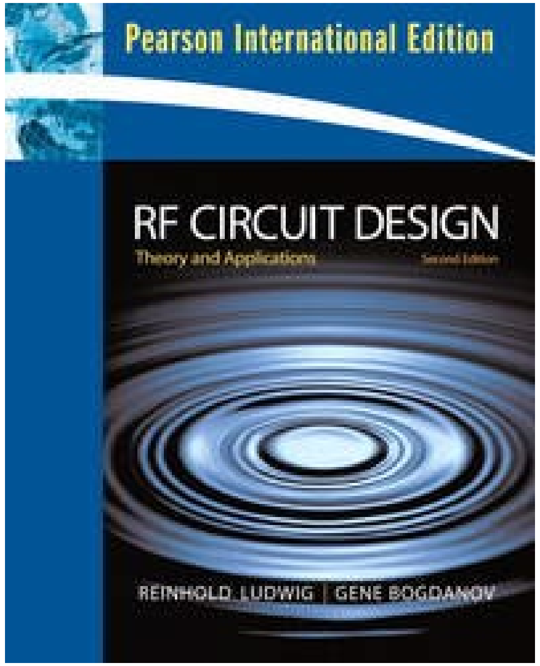
One Quiz

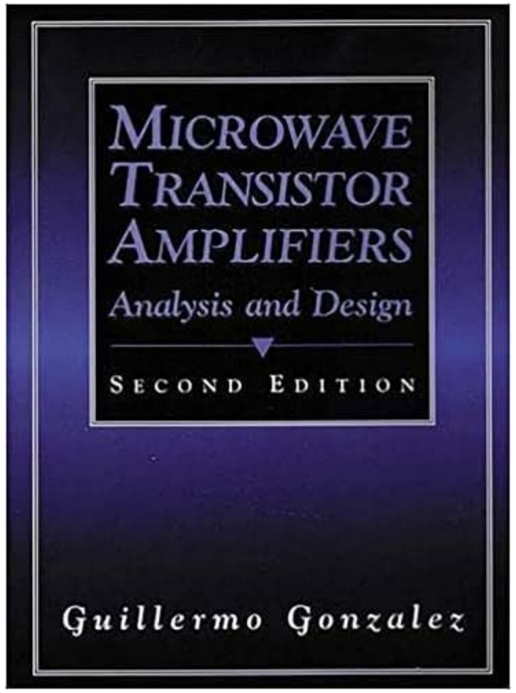
One mini-Project (4-GA guided ADS Sessions)

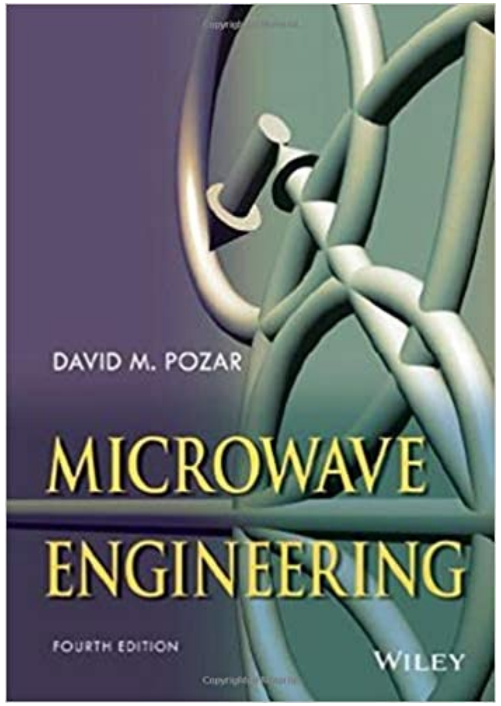
The unexamined life is not worth living.
-- Socrates

Reference Textbooks









Microwave and RF Design, vol.1-5 (3rd ed.) by Michael Steer

- Hard copies available in Amazon
- Softcopies, free: https://repository.lib.ncsu.edu/items/c920fb47-8458-433e-9cd8-b5cacf70e859

RF Circuit Design –Theory & Applications (2nd ed.) by Reinhold Ludwig and Gene Bogdanov

Microwave Transistor Amplifiers: Analysis and Design (2nd ed.) by G Gonzalez

Microwave Engineering (4th ed.) by David M. Pozar

Outline

Introduction
- Block diagram of RF Transceiver
- Transistor Technologies for Amplifiers
- Block Diagram of Amplifiers & Key Parameters

Stability
- Stability Conditions & Stability Tests
- Stability Circles & Interpretation

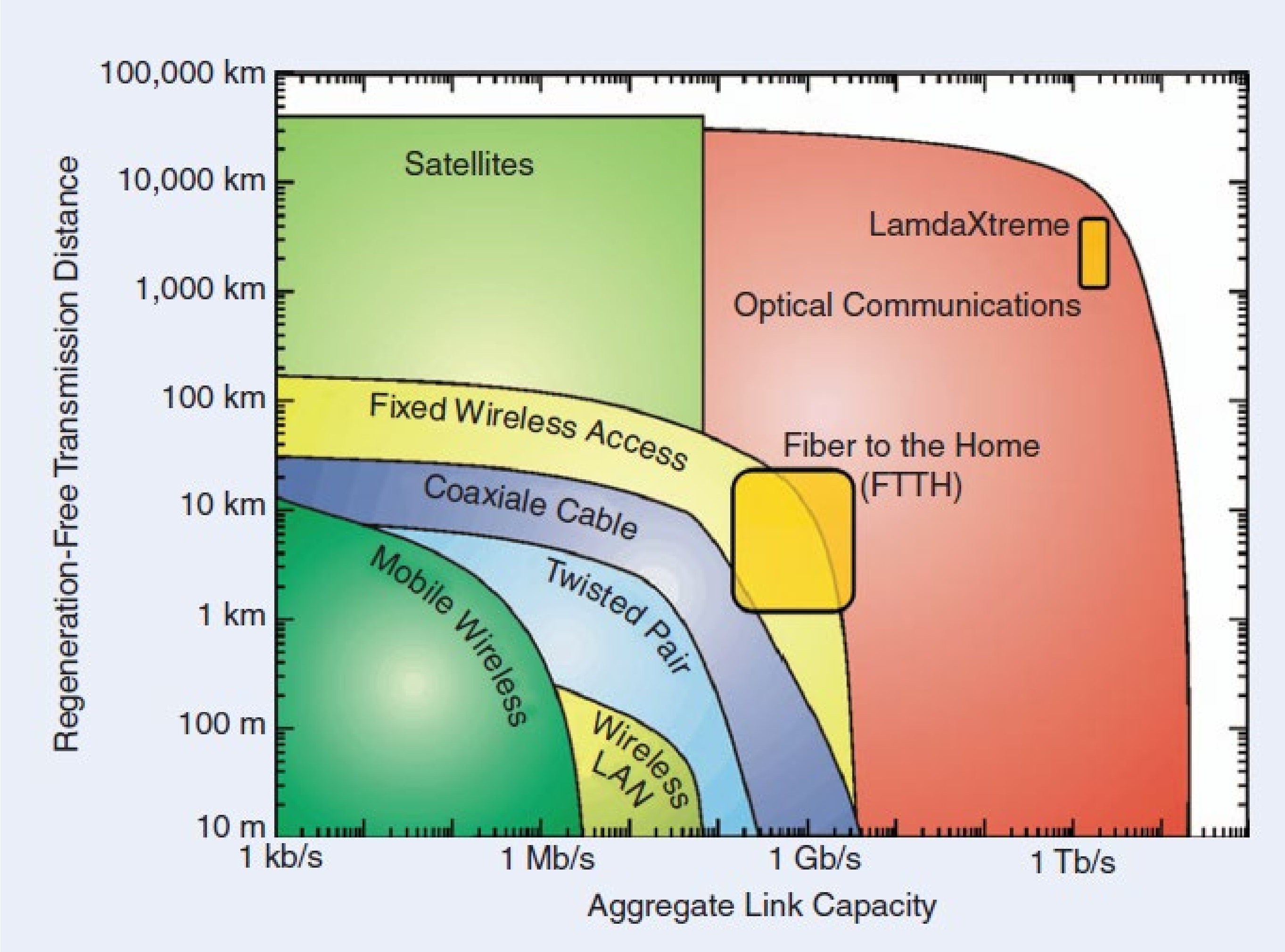
Power Gain
- Definitions of Power Gains
- Unilateral Design & Constant Gain Circles

Other short topics
- Bilateral Design (a brief intro)
- Bias Tee
- Power Efficiency
- Amplifier Design Considerations

Reach versus *data rate*

for radio, wireline, and optical communications technologies

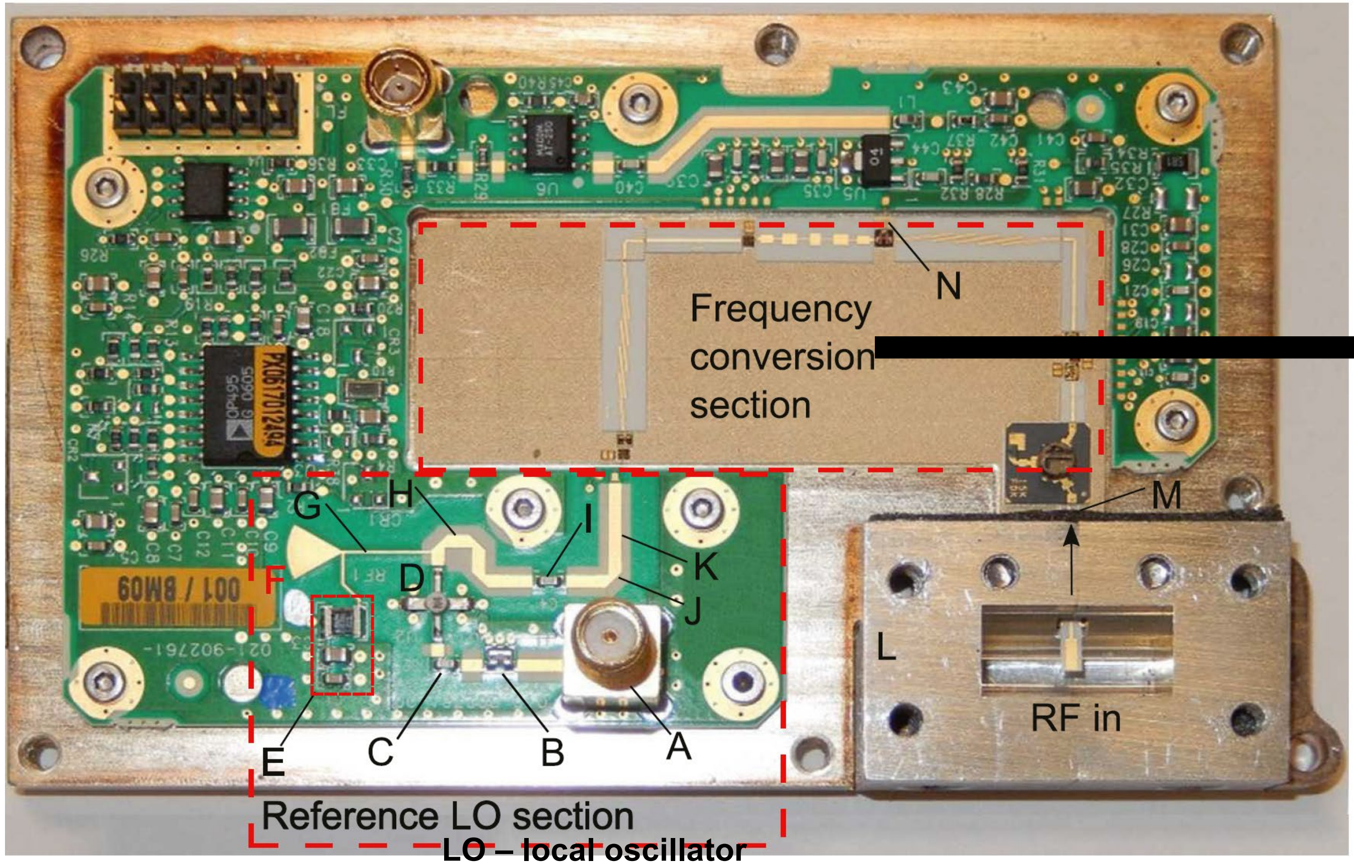
Can you see the Microwave/RF electronics?



Communication Link Capacity vs Transmission Distance

P. J. Winzer and R.-J. Essiambre, "Advanced optical modulation formats," Proc. IEEE, vol. 94, no. 5, pp. 952–985, 2006.

A 15-GHz Receiver Subsystem



A SMA connector, Reference LO in
B Attenuator
C DC blocking capacitor
D Reference LO amplifier
E Bias line
F Radial stub
G High-impedance transmission line
H 50-Ohm transmission line
I DC blocking capacitor
J Mitered bend
K 50 transmission line
L Waveguide-to-microstrip adaptor, RF in
M Interface to frequency conversion section
N Interface to IF section

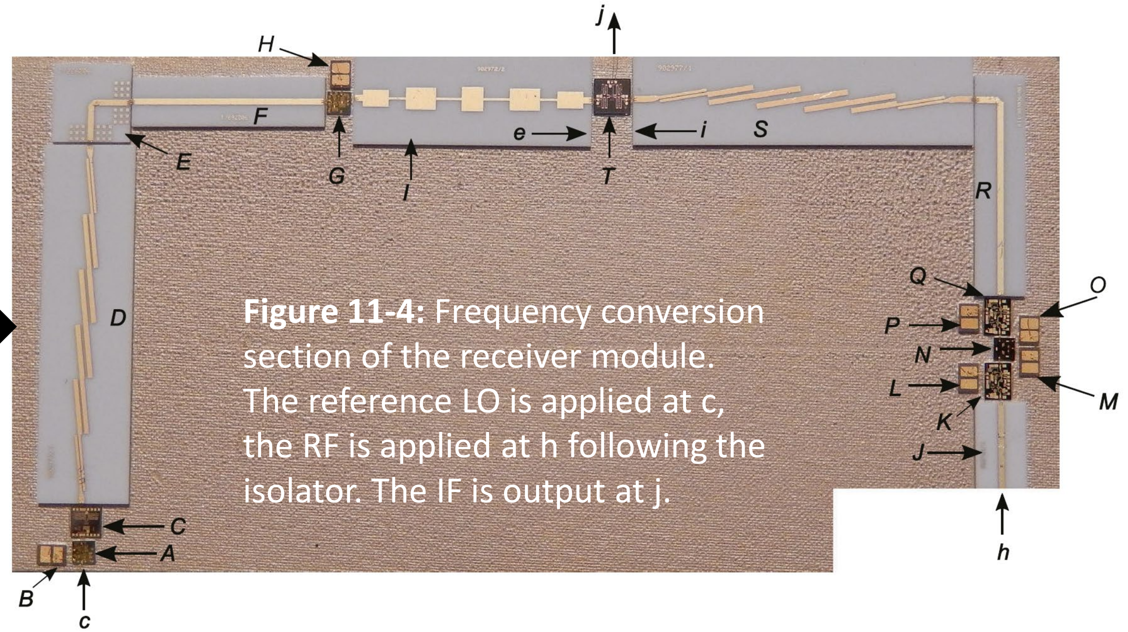


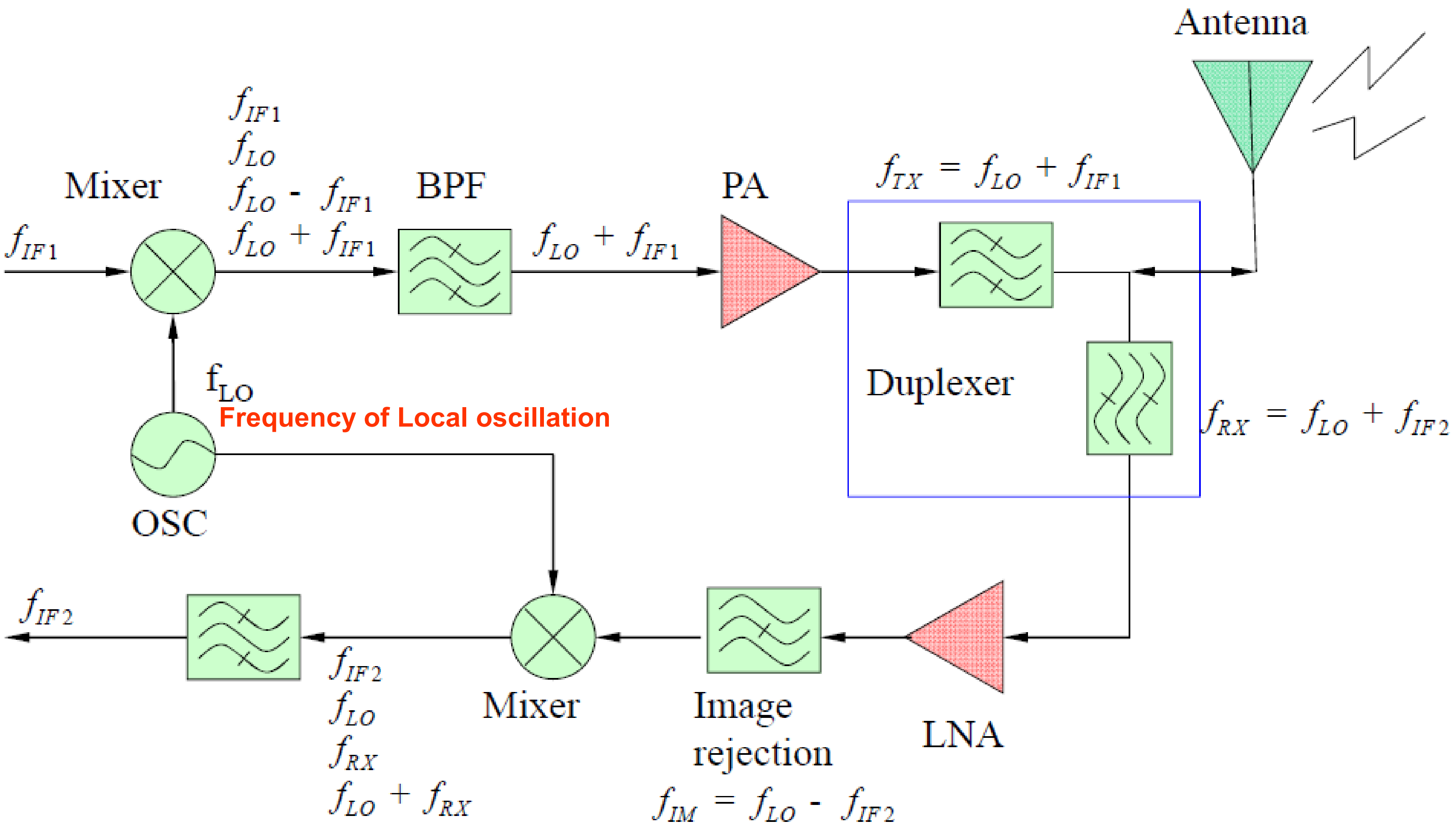
Figure 11-4: Frequency conversion section of the receiver module. The reference LO is applied at c, the RF is applied at h following the isolator. The IF is output at j.

A Reference LO amplifier
B Power supply decoupling capacitor
C ×2 frequency multiplier
D Edge-coupled PCL bandpass filter
E Microstrip bend
F Microstrip transmission line
G ×2 frequency multiplier
H Power supply decoupling capacitor
I stepped impedance lowpass filter
J Microstrip transmission line
K RF amplifier
L Power supply decoupling capacitor
M Power supply decoupling capacitor
N Voltage variable attenuator
O Power supply decoupling capacitor
P Power supply decoupling capacitor
Q RF amplifier
R Microstrip transmission line and bend
S Edge-coupled PCL bandpass filter
T Subharmonic mixer

[source] The Fundamentals of Microwave and RF Design, *(3rd ed.) by Michael Steer*

Figure 11-3: A 14.4–15.35 GHz receiver consisting of cascaded modules interconnected by microstrip lines. Surrounding the microwave circuit are DC conditioning and control circuitry. RF in is 14.4 GHz to 15.35 GHz, LO in is 1600.625 MHz to 1741.875 MHz. The frequency of the IF is 70–1595MHz.

Block Diagram of RF Transceiver (TRX)

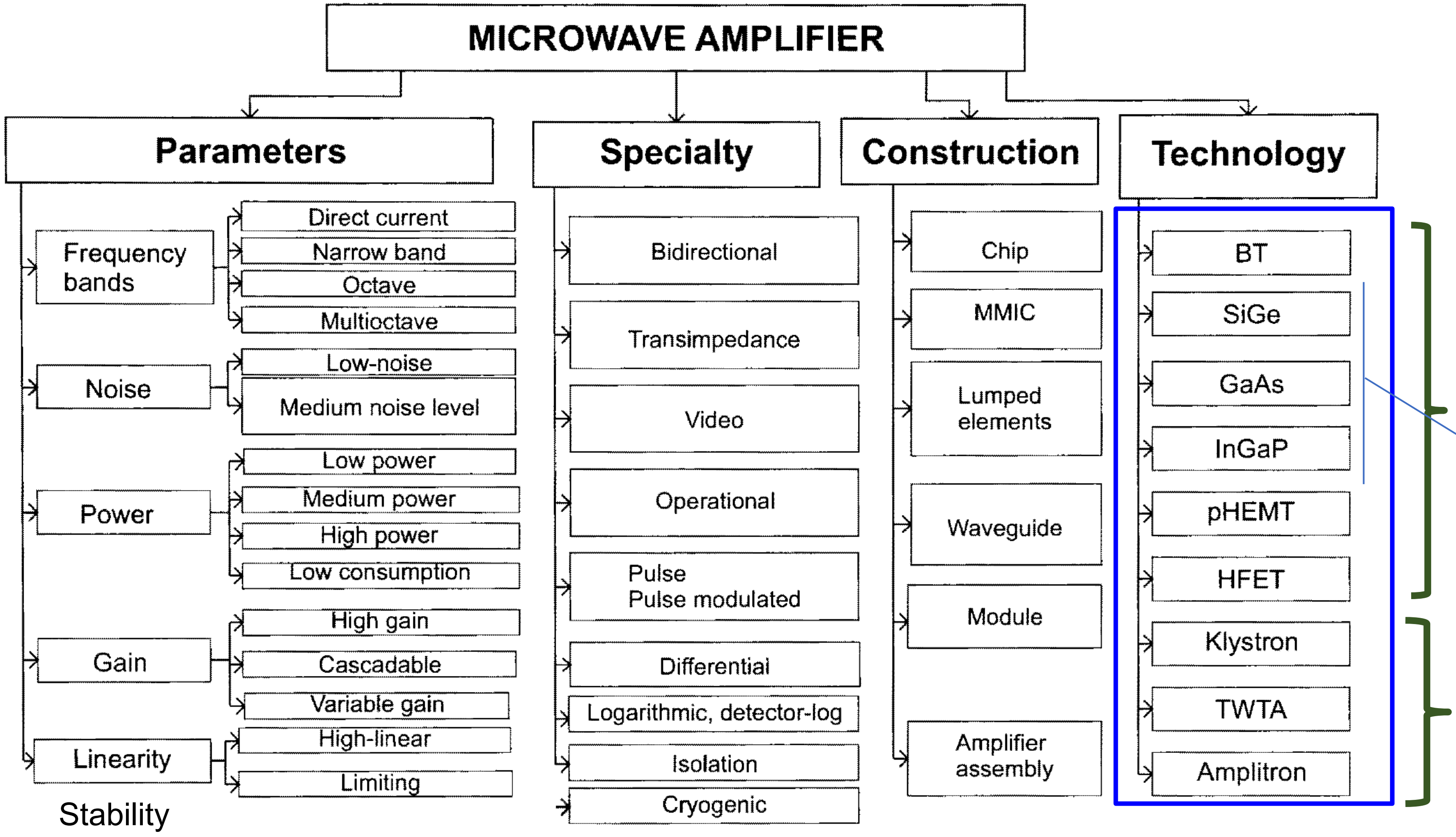


Filter
Oscillator
Amplifier
(**A**ntenna)
Mixer

FOA^{2}M

Amplifier: definition & Classification

Microwave/RF amplifiers increase the power of a Microwave/RF signal by converting DC power to AC power.



Classification of RF/Microwave Amplifier

Handbook of Rf, Microwave, and Millimeter-wave Components, editors: Sergey M_ Smolski et al., Artech House.

- *BT - Bipolar transistor*
- *FET - field-effect transistor*
  - HFET - heterostructure FET
- HEMT - *High Electron Mobility Transistor*
  - *p*HEMT - pseudomorphic HEMT
- *MOS - metal-oxide-semiconductor*

Solid state amplifiers

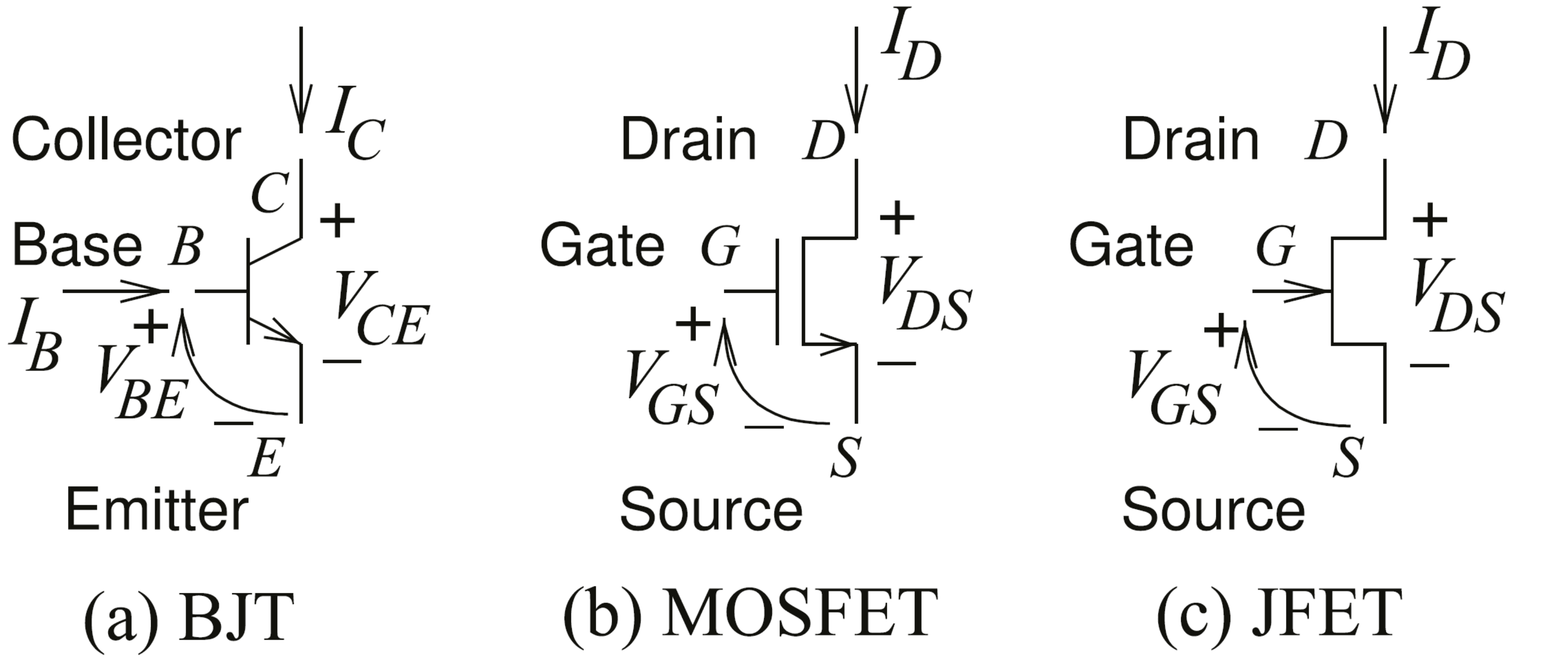
Material
- Silicon Germanium (SiGe)
- Gallium Arsenide (GaAs)
- Indium Gallium Phosphide (InGaP)
- gallium-nitride (GaN)

Microwave tube amplifiers
- Traveling Wave Tube amplifiers (TWTA)
- Gyrotrons
- Klystrons

Vacuum Tube Amplifiers

Transistor Technologies: BJT vs FET

BJT　　　　FET



(a) npn bipolar transistor

B for the base terminal, C for the collector terminal, and E for the emitter terminal

(b) n-type MOSFET (nMOS); (c) n-type JFET (nJFET)

G for the gate terminal, D for the drain terminal, and S for the source terminal.

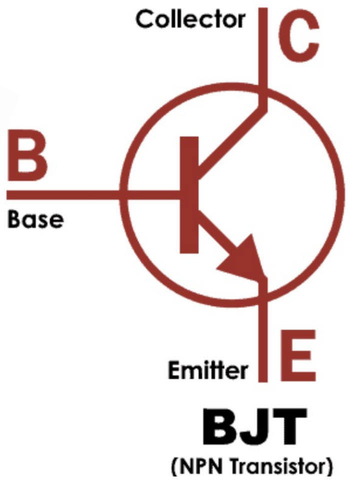
- **Bipolar junction transistors, (BJTs)**;
- **Junction field effect transistors, (JFETs)**;
- **Insulated gate FETs, (IGFETs)** (mainly MOSFTE = **metal-oxide-semiconductor FETs**)

Figure 1-2: Transistor schematics
The schematic symbol for a BJT is used for HBTs;
and the schematic symbol for a JFET is used for MESFETs, HEMTS, and pHEMTs.

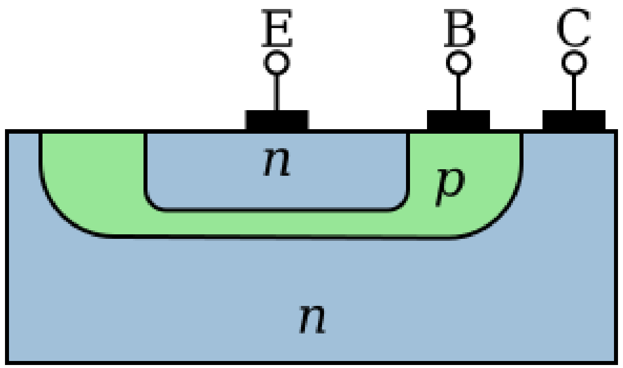
[source] Microwave and RF Design, Vol. 5, *(3rd ed.) by Michael Steer*

Transistor Technologies: BJT vs FET

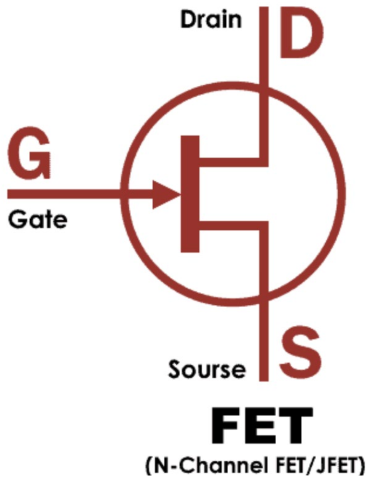
Bipolar junction transistor (BJT) uses two charge carries i.e. electrons and holes.
while unipolar transistor like FETs (Field Effect Transistors) uses only one charge carrier.

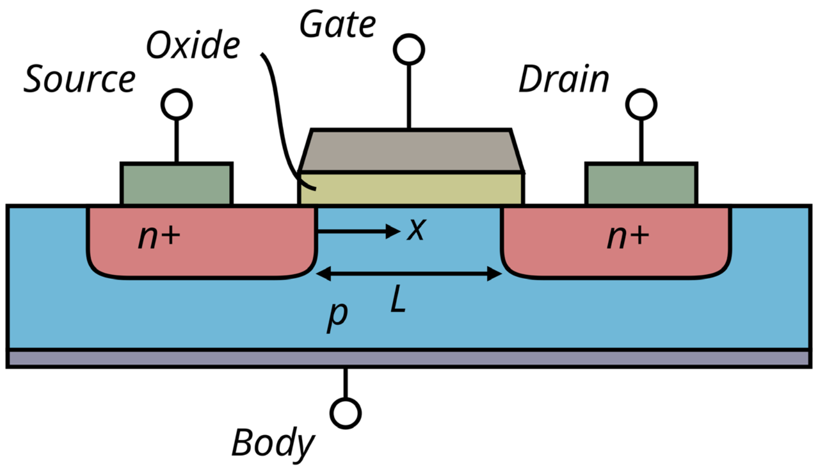


Introduced in 1948 by Shockley, **BJT** is an electronic component mainly used for switching and amplification purpose.



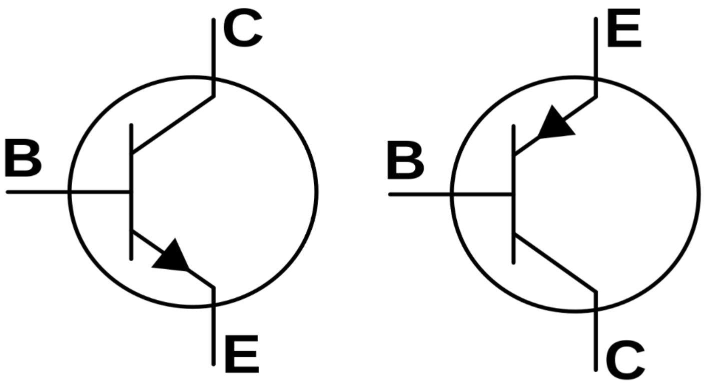
Simplified cross section of a planar **npn BJT**





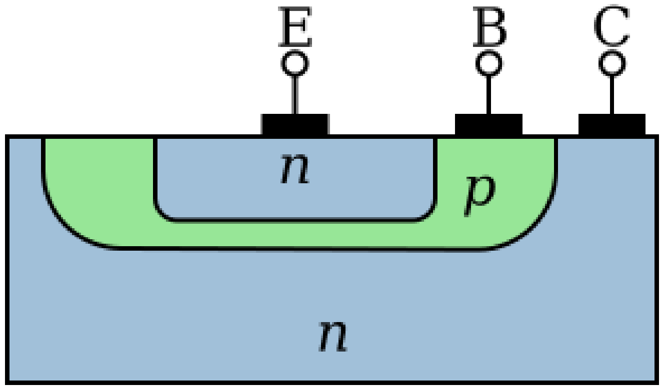
Cross-sectional view of an n-type MOSFET type field-effect transistor, showing *source*, *gate* and *drain* terminals, and insulating oxide layer.

BJT (Bipolar Junction Transistors)

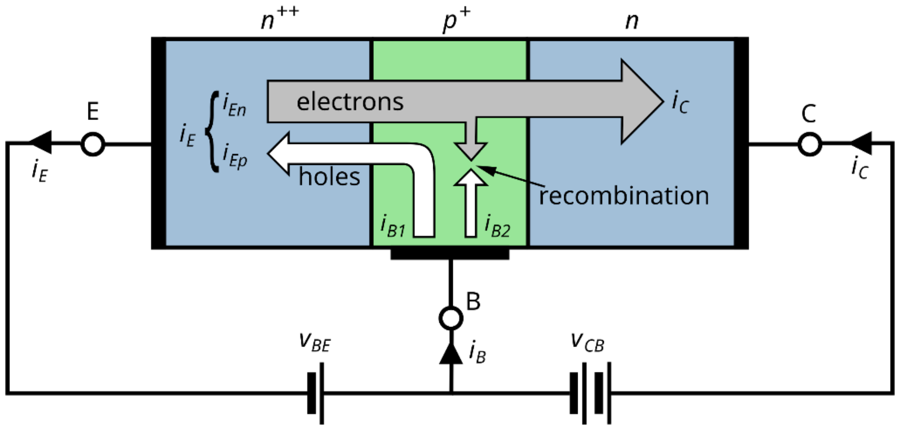


NPN BJT PNP BJT

Two types of BJTs (npn vs pnp)



Simplified cross section of a planar **npn BJT**

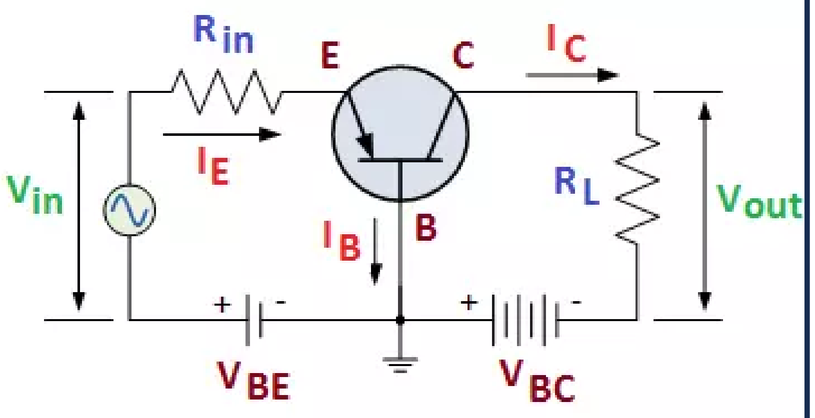


npn BJT with forward-biased B–E junction and reverse-biased B–C junction

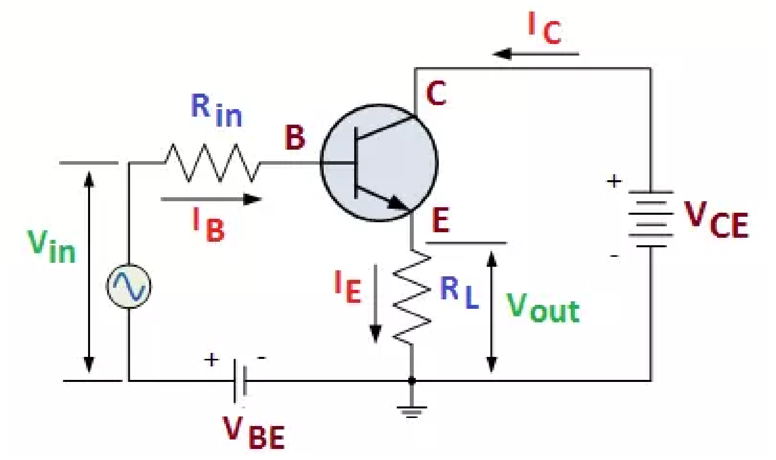
- BJTs exist as PNP and NPN types, based on the doping types of the three main terminal regions.
- An NPN transistor comprises two semiconductor junctions that share a thin p-doped region,
- While a PNP transistor comprises two semiconductor junctions that share a thin n-doped region.
- N-type means doped with impurities (such as phosphorus or arsenic) that provide mobile electrons,
- while p-type means doped with impurities (such as boron) that provide holes that readily accept electrons.

(Source: Wikipedia)

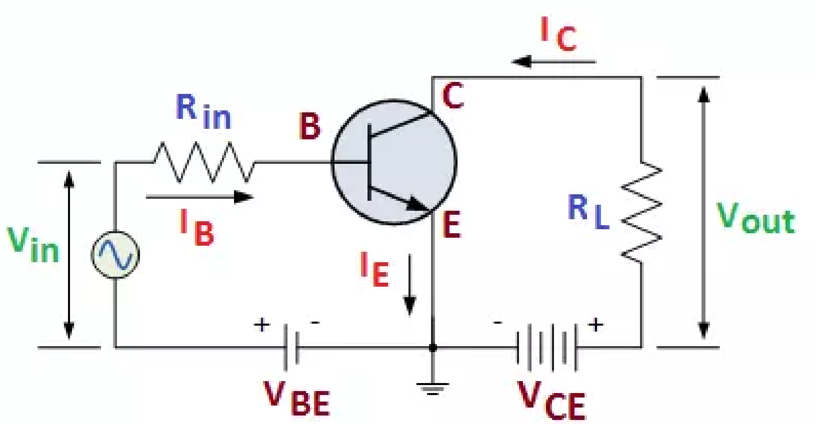
Three Basic Configurations of BJT with External Circuits



Common **base** configuration



Common **collector** configuration

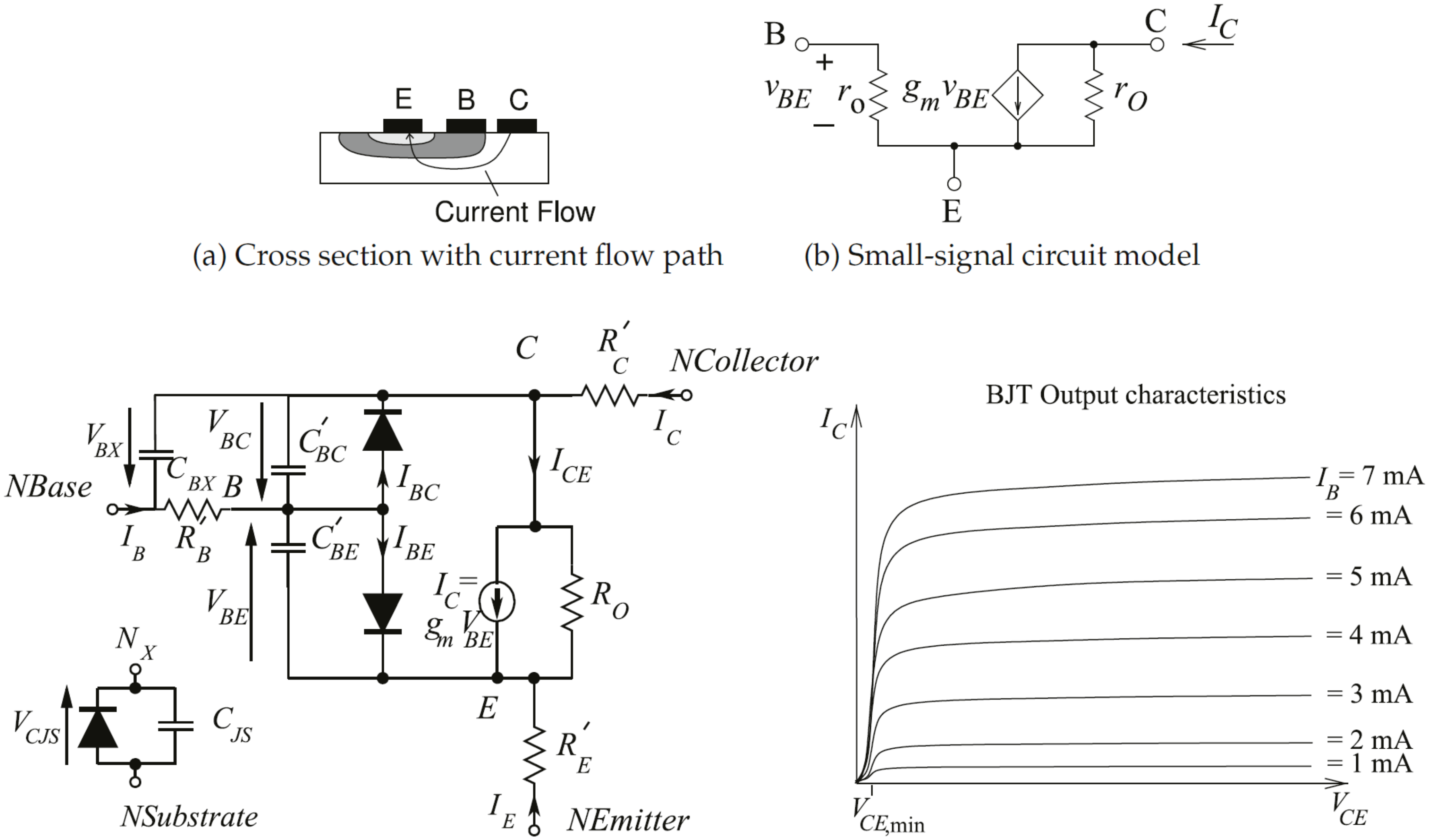


Common **emitter** configuration

- BJT is a current controlled device, mainly used for amplification and switching purposes.
- There are three ways to connect a BJT with external electronic circuits -- Common Base / collector / emitter Configurations. The nature of the current being controlled at the output is different for different configurations.
- A BJT is composed of two junctions, called emitter-base (E-B) junction and collector-base (C-B) junction.
- A BJT uses two charge carries i.e. electrons and holes.

https://www.theengineeringprojects.com/2018/05/introduction-to-bjt-bipolar-junction-transistor.html

BJT Schematics, Circuit Models & I-V Curve



(a) Cross section with current flow path

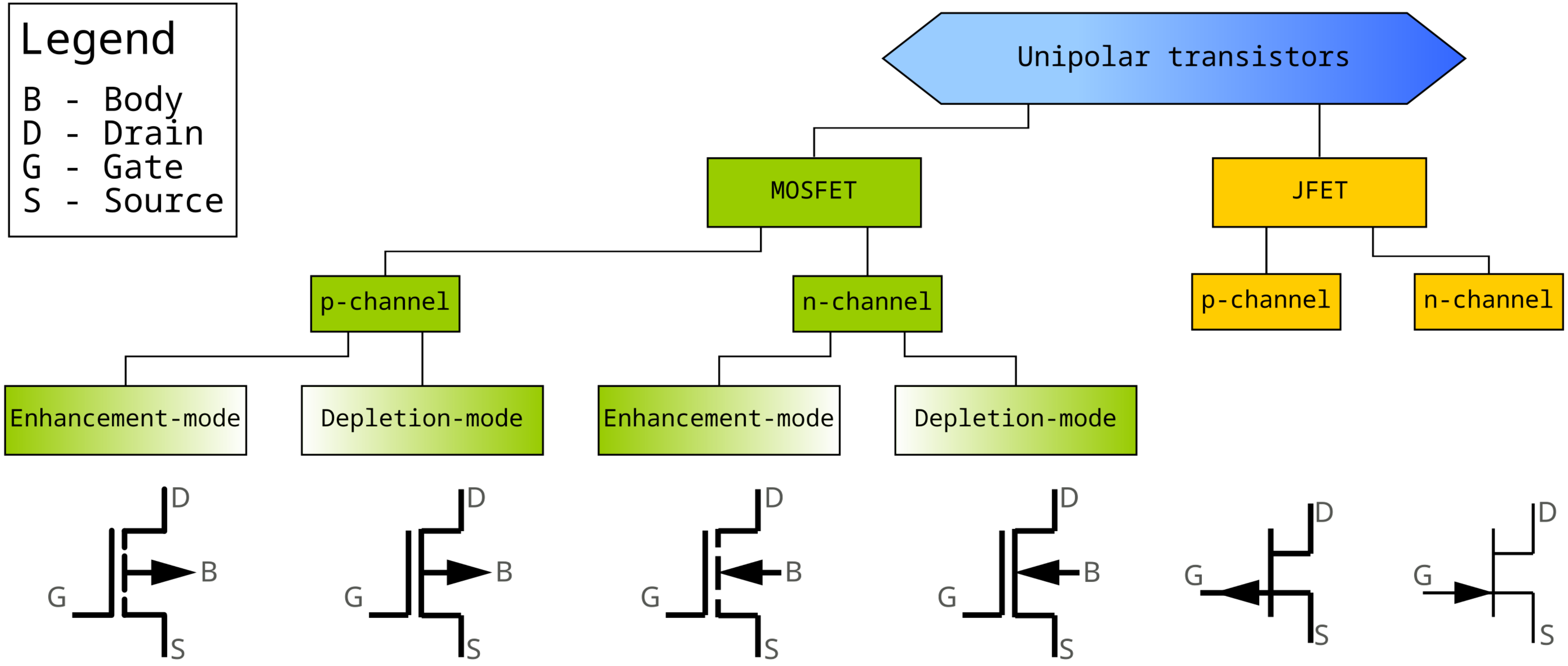
(b) Small-signal circuit model

(c) Gummel–Poon model schematic

(d) Output i-v characteristic

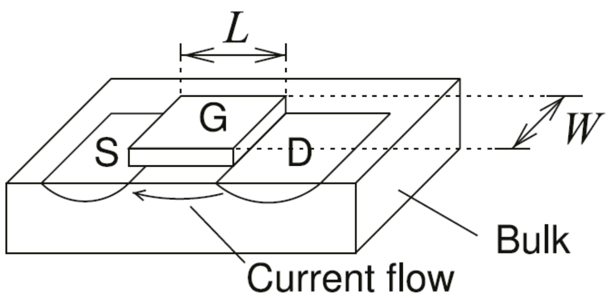
Figure 1-3: BJT details.

FET (field effect transistor)

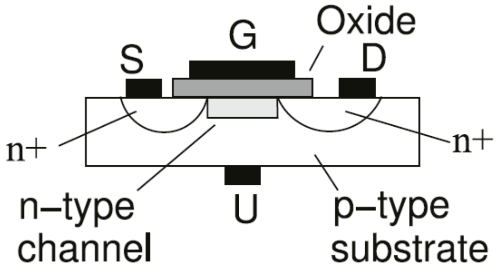


- The field-effect transistor (FET) uses an electric field to control the current through a semiconductor.
- It comes in two types: junction FET (JFET) and metal–oxide–semiconductor FET (MOSFET).
- FETs have three terminals: source, gate, and drain.
- FETs control the current by applying a voltage to the gate, which in turn alters the conductivity between the drain and source.
- **The gate voltage is used to reduce current conduction --> a depletion-mode FET**
- **A gate field is applied to pull carriers from the bulk into the channel --> an enhancement-mode FET**

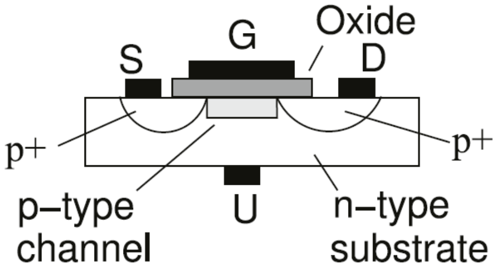
MOSFET



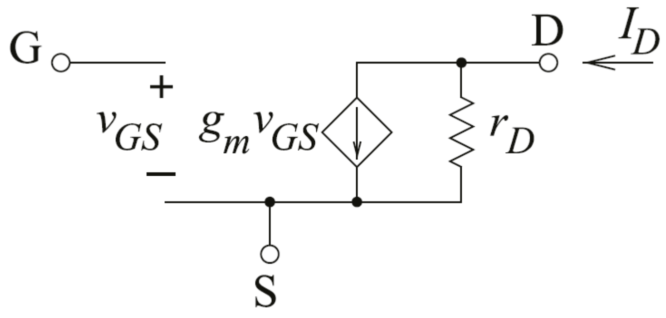
(a) MOS



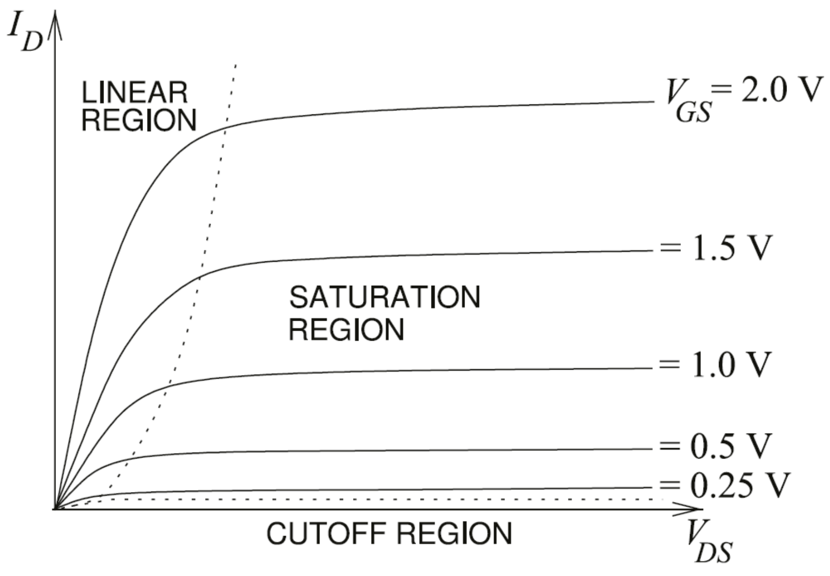
(b) nMOS



(c) pMOS



(e) Small-signal model

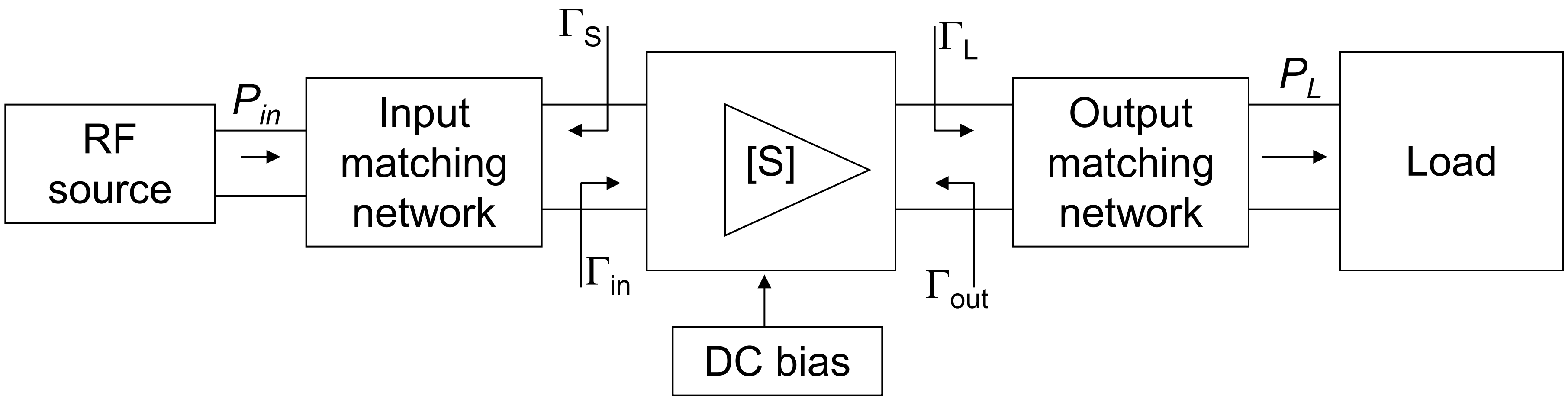


(d) nMOS

- *The gate voltage is used to reduce current conduction --> a depletion-mode FET*
- *A gate field is applied to pull carriers from the bulk into the channel --> an enhancement-mode FET*

Figure 1-5: MOSFET details:
(a) three-dimensional view of a MOSFET;
(b) cross section of an nMOS transistor with metal or **polysilicon** contacts indicated by the black blocks;
(c) the corresponding cross section of a pMOS transistor;
(d) current-voltage characteristics of an enhancement-mode MOSFET;
(e) circuit model of fundamental operation

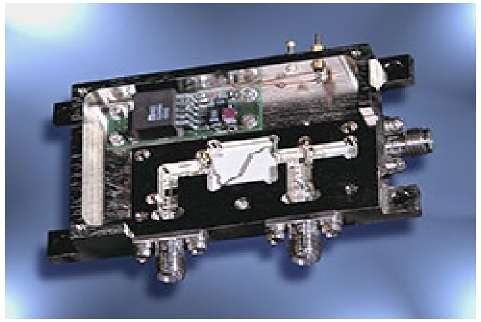
Generic Diagram of Amplifier



- Key amplifier parameters
  - Gain (G) and gain flatness
  - Operating frequency (f) and bandwidth (BW)
  - Noise figure (F)
  - VSWR (in/out)
  - Intermodulation distortion (IMD)
  - Harmonics
  - Output Power
  - Heating/thermal effects

An amplifier design will require some definition of power relations and tools for analysis of stability, gain noise and VSWR performance

Examples of Amplifiers & Specification



Amplifier Model Number	Frequency Range	Saturated Power	Linear Power	Gain	Flatness	IP_3
5302015	2.4 - 2.5 GHz	10 Watts	8 Watts	41 dB	± 0.75 dB	49 dB
5303075	0.8 - 4.2 GHz	1.5 Watts	1.2 Watts	31 dB	± 2.0 dB	41 dB
5303011A	0.8 - 2 GHz	25 Watts	20 Watts	17 dB	± 1.5 dB	53 dB
5303009	2 - 4 GHz	25 Watts	20 Watts	12 dB	± 1.5 dB	53 dB
5303006A	20 - 500 MHz	50 Watts	30 Watts	48 dB	± 1.5 dB	51 dB
5302024	1.2 - 1.6 GHz	10 Watts	8 Watts	43 dB	± 1.0 dB	49 dB
5162	0.8 - 4.2 GHz	28 Watts	22 Watts	46 dB	± 2.0 dB	54 dB
5140	0.7 - 3 GHz	10 Watts	8 Watts	41 dB	± 2.0 dB	49 dB
4061	1.85 - 2.17 GHz	200 Watts	160 Watts	53 dB	± 1.0 dB	62 dB
4043	2 - 30 MHz	1000 Watts	600 Watts	60 dB	± 2.0 dB	64 dB
5127	20 - 1000 MHz	200 Watts	120 Watts	54 dB	± 2.0 dB	57 dB
5086	0.01 - 200 MHz	200 Watts	125 Watts	52 dB	± 2.0 dB	57 dB

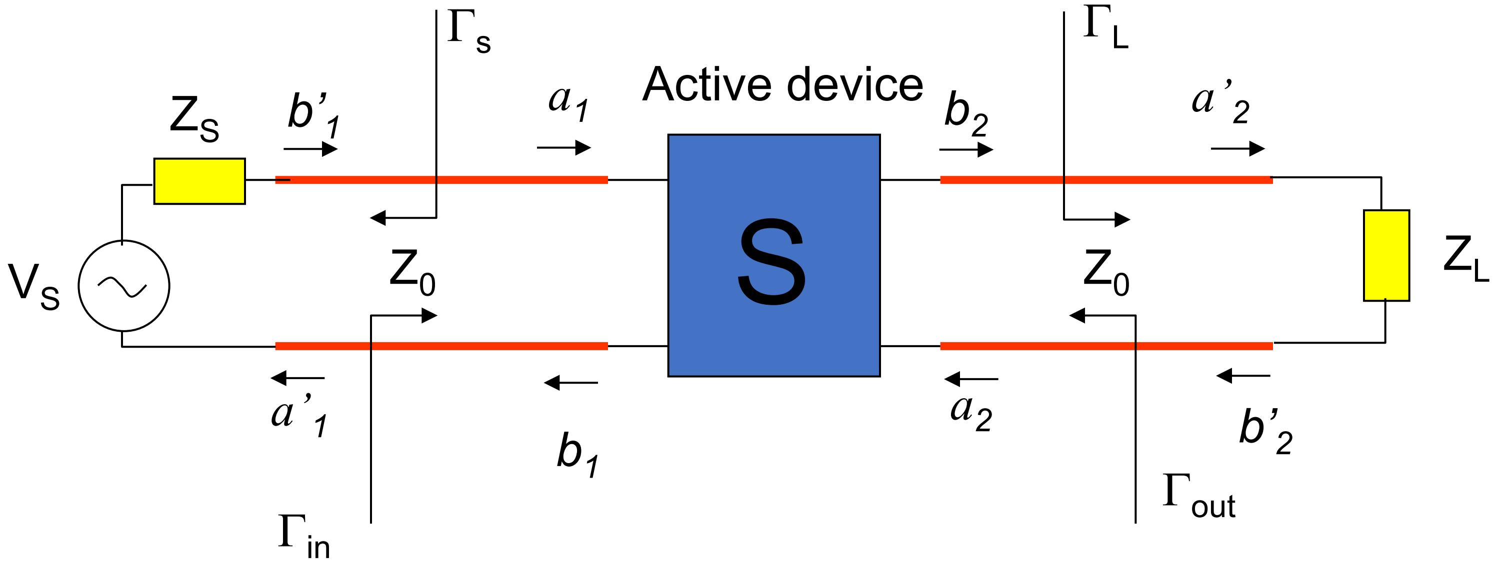
Third Order Intercept Point (IP3)

Basic Tools in Amplifier Design

- Before the actual design, we need to determine some basic parameters of an amplifier:
  - Stability (the amplifier must be stable, not oscillating)
  - Achievable power gain
  - Noise figure (how much signal to noise ratio at the amplifier output will differ from the signal to noise ratio at the amplifier input (noise figure is always more than one)
- Tools for analysis of these parameters:
  - Stability analysis
  - Gain analysis
  - Noise figure analysis and optimization
- Since in practice we measure S parameters of the device by connecting it to the measurement equipment with transmission lines, the tools we are going to develop must take in account this measurement conditions

Simplified Block Diagram of Amplifier

- To facilitate and simplify the analysis, we will include the input and output matching networks in the source and load impedances
- We will also assume that the source and the load are connected to the amplifier by means of transmission lines with characteristic impedance Z_0 and having negligible lengths.

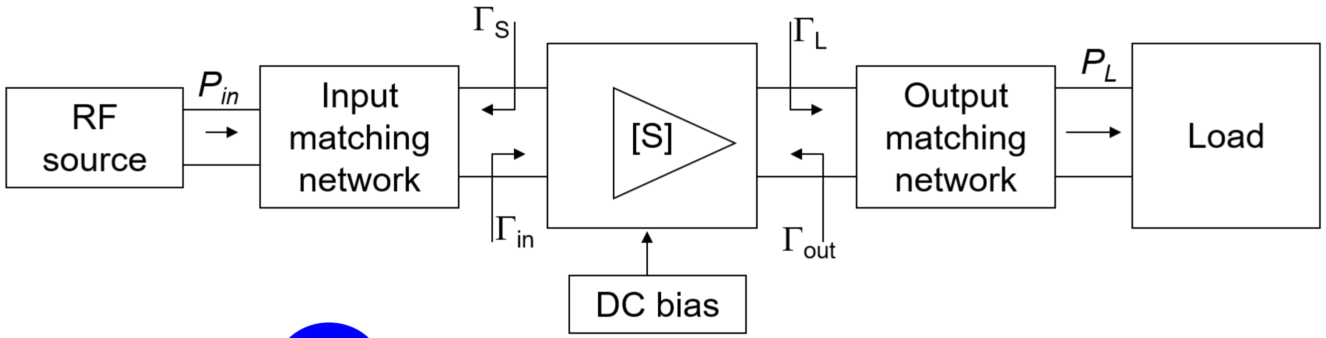


a_n, b_n are normalized incident and reflected waves

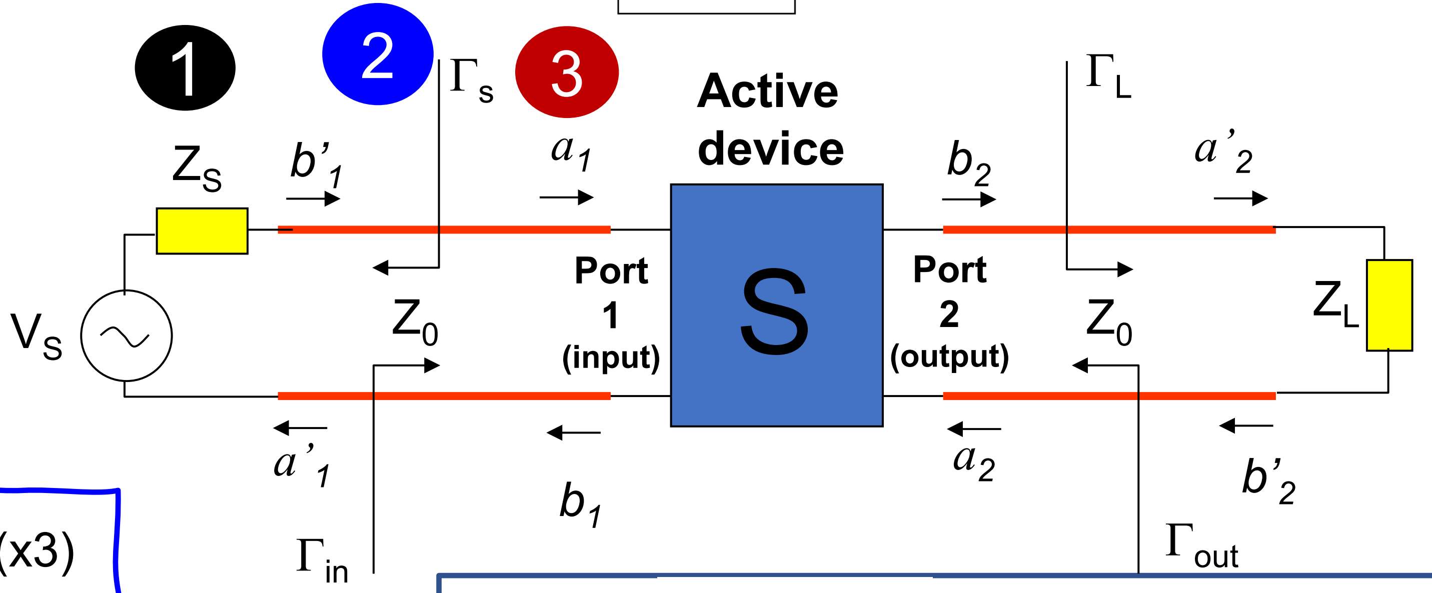
$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}; \quad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Block Diagram of Amplifier & Definition of Key Parameters



a_n , b_n are normalized incident & reflected waves



1 Z (x3)
2 Γ (x4)
3 a, b

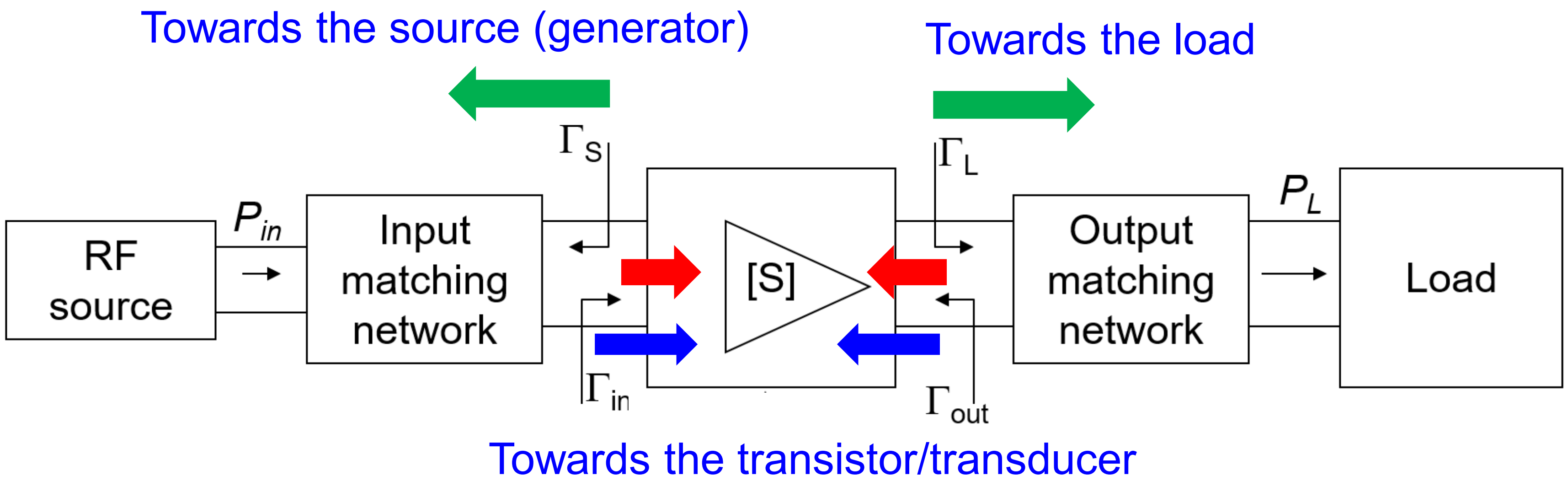
Reference point (observation point)

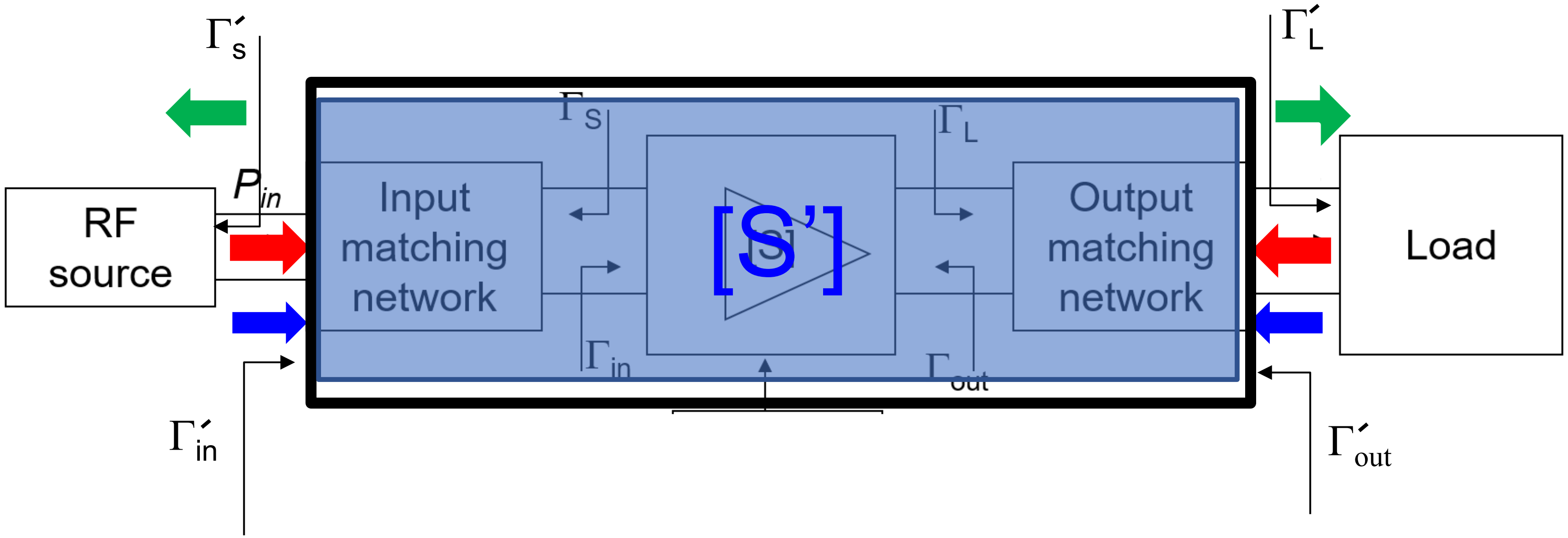
$$\Gamma_x = \frac{Z_x - Z_0}{Z_x + Z_0}$$

Gamma = Difference / Sum

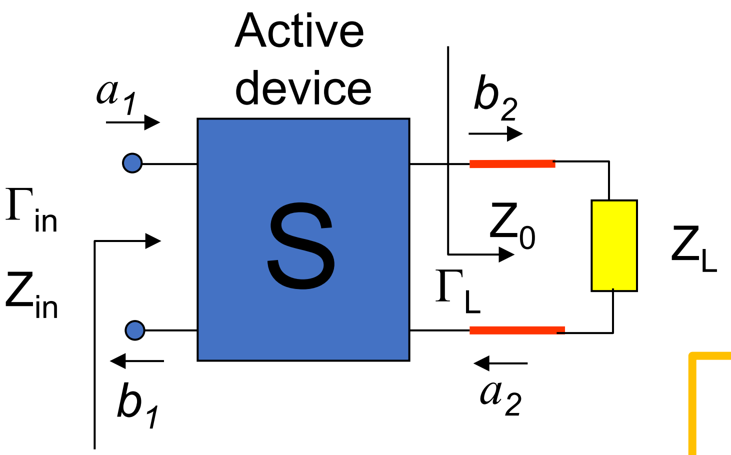
$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0}; \quad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Reference (observation) point





Calculation of Γ_{in} and Γ_{out}

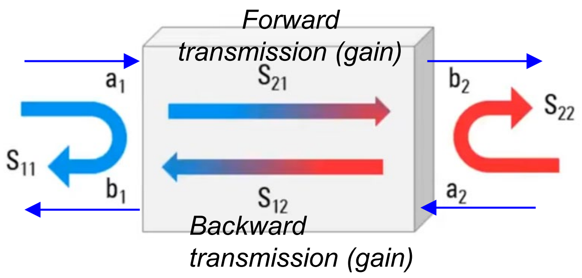


$$\Gamma_{in} = \frac{b_1}{a_1}; \quad \Gamma_L = \frac{a_2}{b_2}$$

$$\Gamma_{in} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L}$$

When will this happen?

$$\Gamma_{in} = S_{11}$$



- *Diagonal elements == reflection coefficients (at the same port)*
- *Off-diagonal elements == transmission coefficients (across two ports)*

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

$\Rightarrow$ [b]=[S][a]

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

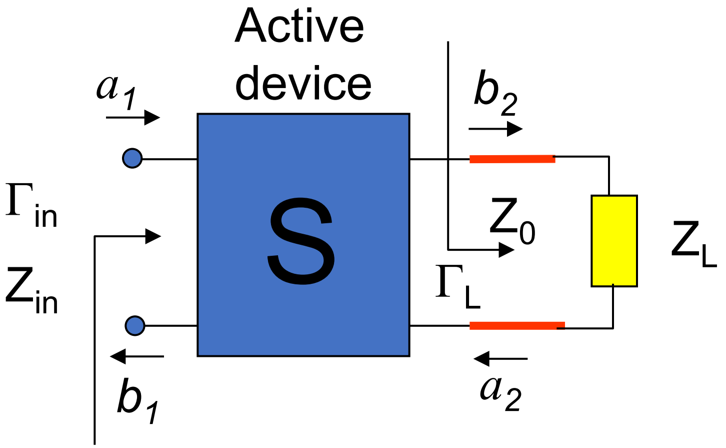
$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

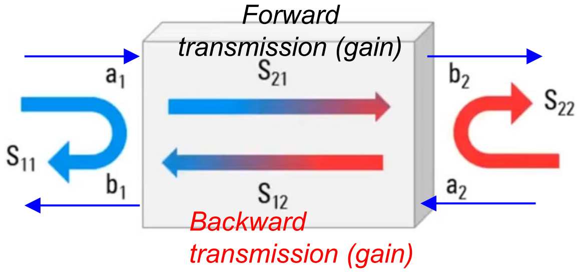
$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \qquad S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$

For a passive network/system

- Sij is usually a complex number
- |Sij| ≤ 1

Explanation of cases for S21=0 or S12=0





- Diagonal elements == reflection coefficients (at the same port)
- Off-diagonal elements == transmission coefficients (across two ports)

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$\Gamma_{in} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$\Gamma_{in} = S_{11}$

- if $\Gamma_L = 0$, i.e., $Z_0 = Z_L$ (matched)
- if $S_{21} = 0$
- if $S_{12} = 0$, The transistor's backward transmission (or feedback) is neglected ---- it's called **Unilateral case (design)**

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$\boxed{b_2} = \underbrace{\cancel{S_{21}}}_{=0}a_1 + \boxed{S_{22}a_2}$$

$$\boxed{b_1 = S_{11}a_1} + \underbrace{\cancel{S_{12}}}_{=0}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

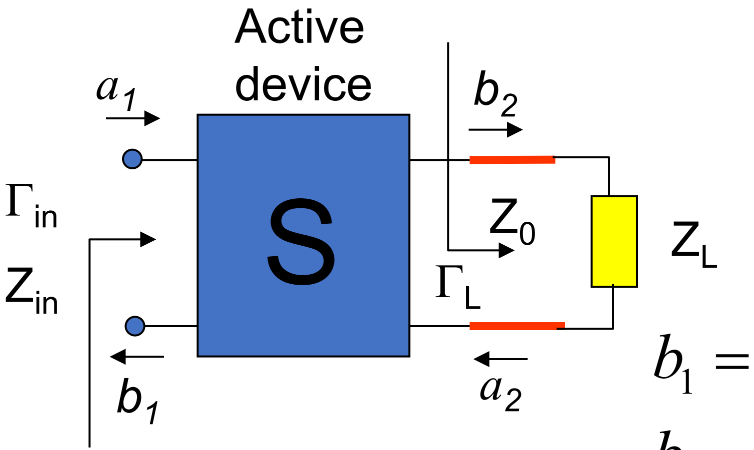
In other words,

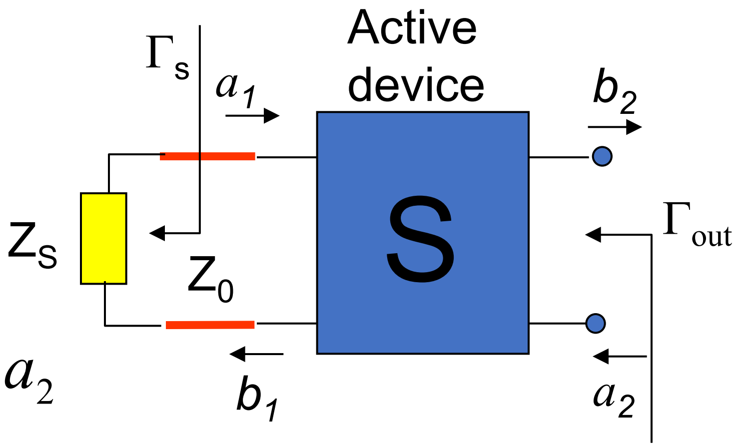
S11 is a special case of Γ_{in}

Γ_{in} degenerates to S11 under the above special cases.

(For print)

Calculation of Γ_{in} and Γ_{out}





$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

$$\Gamma_{in} = \frac{b_1}{a_1}; \quad \Gamma_L = \frac{a_2}{b_2}$$

$$\Gamma_S = \frac{a_1}{b_1}; \quad \Gamma_{out} = \frac{b_2}{a_2}$$

$$\Gamma_x \sim \frac{\text{out}}{\text{in}}$$

$$\Gamma_{in} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{out} = S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1 - S_{11}\Gamma_S}$$

$$Z_{in} = Z_0\frac{1+\Gamma_{in}}{1-\Gamma_{in}} \quad Z_L = Z_0\frac{1+\Gamma_L}{1-\Gamma_L} \quad Z_S = Z_0\frac{1+\Gamma_S}{1-\Gamma_S} \Rightarrow Z_x = Z_0\frac{1+\Gamma_x}{1-\Gamma_x}$$

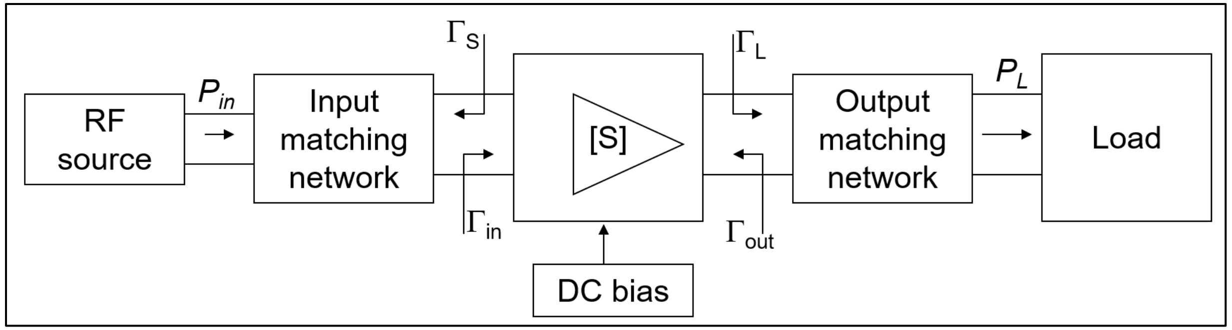
X = { in, L, S}

Outline

- ❑ **Introduction**
  - Block diagram of RF Transceiver
  - Transistor Technologies for Amplifiers
  - Block Diagram of Amplifiers & Key Parameters
- ❑ **Stability**
  - Stability Conditions & Stability Tests
  - Stability Circles & Interpretation
- ❑ **Power Gain**
  - Definitions of Power Gains
  - Unilateral Design & Constant Gain Circles
- ❑ **Other short topics**
  - Bilateral Design (a brief intro)
  - Bias Tee
  - Power Efficiency
  - Amplifier Design Considerations

Observations

1. Γ_{in} is equal to S_{11} only when Γ_L and/ or S_{12} are zero.
2. Γ_{out} is equal to S_{22} only when Γ_S and/ or S_{12} are zero.
3. **CONJUGATE MATCHING** at input and output is: $\Gamma_S = \Gamma^*_{in}$ and $\Gamma_L = \Gamma^*_{out}$



$$\Gamma_x = \frac{Z_x - Z_0}{Z_x + Z_0}$$

But the most important observations are:

1. There are some values of Γ_L that give rise to $|\Gamma_{in}| > 1$
2. There are some values of Γ_S that give rise to $|\Gamma_{out}| > 1$

Recall that:

- When $|\Gamma|$ is greater than unity, the corresponding impedance has a negative real part.
- A negative resistance will result in oscillation if the net loop resistance is negative.

Stability Conditions & Stability Tests

- In a two-port network, oscillations may occur when either the input or output ports presents a negative resistance, i.e. when $|\Gamma_{in}|>1$ or $|\Gamma_{out}|>1$.
- The tendency of a transistor toward oscillation can be determined by its S parameters.
- Such determination can be made even before an amplifier is built and is useful to find the suitable transistor for a particular application.
- A two-port network is said to be unconditionally stable if for all passive load and source terminations, the input and output immitances remain to be passive. Mathematically, $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ for all $|\Gamma_L| < 1$ and $|\Gamma_s| < 1$.

IMMITTANCE is electrical admittance (Y) or impedance (Z)

- To test the unconditional stability condition, we first calculate the intermediate quantity, delta factor $\Delta = S_{11}S_{22} - S_{12}S_{21}$
- The Rollett stability factor is then calculated as

$$K = \frac{1+|\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}S_{21}|}$$

Stability Test-1

k-Δ test

- The stability criteria read:
  - Unconditionally stable $K > 1$ and $|\Delta| < 1$
  - Potentially unstable $K > 1$ and $|\Delta| > 1$; $K < 1$ and $|\Delta| < 1$
- There are also other stability criteria involving different necessary and sufficient condition(s) for testing the unconditional stability.

Stability Test-2

μ test

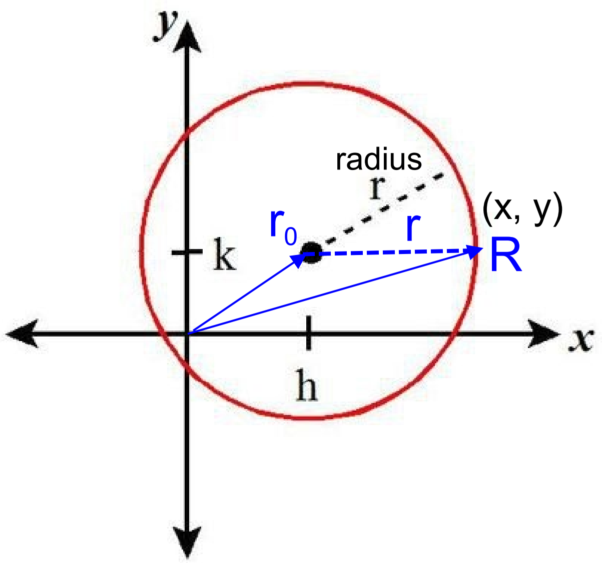
- The two parameter test, K- Δ provides a set of mathematical conditions on two parameters test for unconditional stability.
- A new criterion has been derived that combines K- Δ parameters into single parameter test, this is referred to as "**μ-parameter test**"

μ-factor : the transistor is absolutely stable if:

$$\mu = \frac{1-|S_{11}|^2}{|S_{22} - S_{11}^*\Delta| + |S_{21}S_{12}|} > 1$$

Circle

& its Essential Elements



$$(x-h)^2+(y-k)^2 = r^2$$

$$|R-r_0|= r$$

Equation of a circle

$$(x-h)^2+(y-k)^2=r^2$$

center = (h, k)

radius = r

Stability Circles

- If $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$, it implies that the device is stable.
- If $|\Gamma_{in}| > 1$ or/and $|\Gamma_{out}| > 1$, the device is unstable and may oscillate with certain combination of terminations.
- So $|\Gamma_{in}| = 1$ or/and $|\Gamma_{out}| = 1$ are the boundary conditions, i.e. the maximum limits of input and output reflection coefficients before the amplifier becomes unstable.
- $|\Gamma_{in}|$ and $|\Gamma_{out}|$ are functions of Γ_L and Γ_s respectively, and if we try to solve e.g.

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| = 1$$

it is found that the Γ_L solutions are not unique and they form a circle.

- The stability circle is simply a circle on the Smith chart which represents the boundary between those values of source or load impedances (reflection coefficients) that cause instability and those that do not.
- The perimeter of the circle represents the locus of points satisfying

$$|\Gamma_{in}(\Gamma_L)| = 1 \quad \text{or} \quad |\Gamma_{out}(\Gamma_s)| = 1$$

which can be written as

$$|\Gamma_L - c_L| = |r_L| \quad \text{or} \quad |\Gamma_s - c_s| = |r_s|$$

- Computation of the centers and radii of the source (input) and load (output) stability circles follows:

$$|\Gamma_L| < 1, \; |\Gamma_S| < 1$$

$$|\Gamma_{in}| = \left| \frac{S_{11} - \Gamma_L \Delta}{1 - S_{22}\Gamma_L} \right| < 1$$

$$|\Gamma_{out}| = \left| \frac{S_{22} - \Gamma_S \Delta}{1 - S_{11}\Gamma_S} \right| < 1$$

Source (input) stability circle, $|\Gamma_{out}(\Gamma_s)| = 1$:

$$c_s = \frac{C_1^*}{|S_{11}|^2 - |\Delta|^2}$$

$$r_s = \frac{|S_{12}S_{21}|}{\left||S_{11}|^2 - |\Delta|^2\right|}$$

Load (output) stability circle, $|\Gamma_{in}(\Gamma_L)| = 1$:

$$c_L = \frac{C_2^*}{|S_{22}|^2 - |\Delta|^2}$$

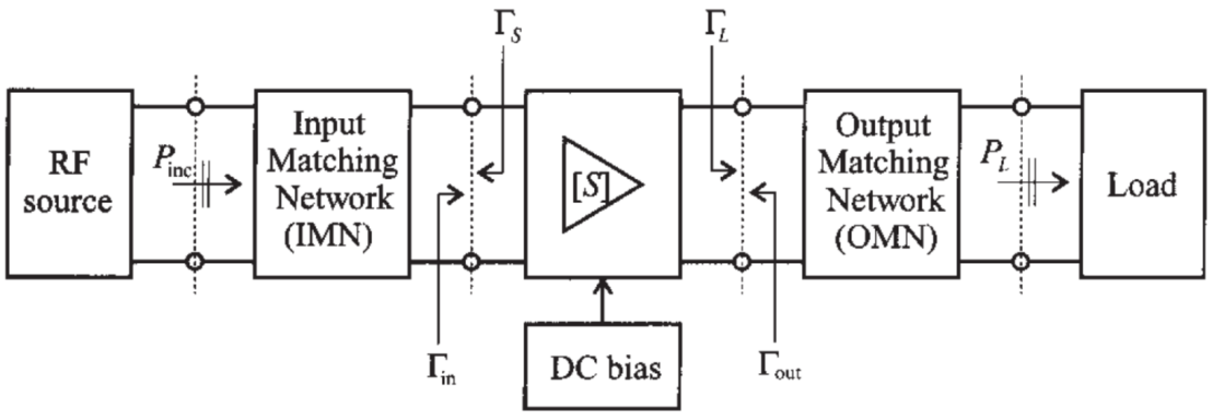
$$r_L = \frac{|S_{12}S_{21}|}{\left||S_{22}|^2 - |\Delta|^2\right|}$$

where
c_s = center of so
r_s = radius of source stability circle
c_L = center of load stability circle
r_L = radius of load stability circle
$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}|$
$C_1 = S_{11} - \Delta S_{22}^*$
$C_2 = S_{22} - \Delta S_{11}^*$

- If the transistor is unconditionally stable, stability circles need not be plotted.
- However, if the transistor is conditionally stable or potentially unstable, we should plot the stability circles.
- Either the inside or the outside of the stability circle may represent the unstable region.

Amplifier: Interpretation of Stability Circles

Stability Circles of Amplifiers



Stability condition

$$|\Gamma_L| < 1, \ |\Gamma_S| < 1$$

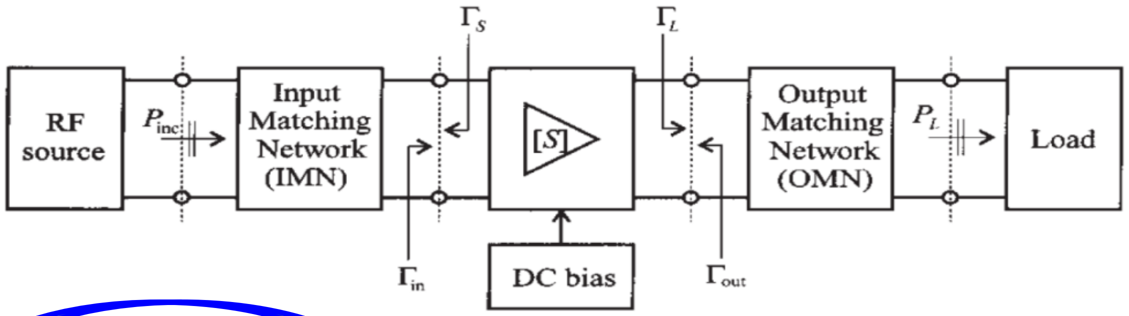
$$|\Gamma_{in}| = \left|\frac{S_{11} - \Gamma_L \Delta}{1 - S_{22}\Gamma_L}\right| < 1$$

$$|\Gamma_{out}| = \left|\frac{S_{22} - \Gamma_S \Delta}{1 - S_{11}\Gamma_S}\right| < 1$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

For a given frequency f_i, the S parameters are fixed,
So Γ_L and Γ_S are the variables to be considered

Stability Circles of Amplifiers



Output Stability circles

This circle is about "output port"

$$|\Gamma_{in}| = \left|\frac{S_{11} - \Gamma_L \Delta}{1 - S_{22}\Gamma_L}\right| < 1$$

$$(\Gamma_L^R - C_{out}^R)^2 + (\Gamma_L^I - C_{out}^I)^2 = r_{out}^2$$

the circle radius is given by

$$r_{out} = \frac{|S_{12}S_{21}|}{\left||S_{22}|^2 - |\Delta|^2\right|}$$

e center of this circle is located at

$$C_{out} = C_{out}^R + jC_{out}^I = \frac{(S_{22} - S_{11}^*\Delta)^*}{|S_{22}|^2 - |\Delta|^2}$$

Γ_{in}

In Γ_L plane

Input Stability circles

This circle is about "input port"

$$|\Gamma_{out}| = \left|\frac{S_{22} - \Gamma_S \Delta}{1 - S_{11}\Gamma_S}\right| < 1$$

$$(\Gamma_S^R - C_{in}^R)^2 + (\Gamma_S^I - C_{in}^I)^2 = r_{in}^2$$

$$r_{in} = \frac{|S_{12}S_{21}|}{\left||S_{11}|^2 - |\Delta|^2\right|}$$

$$C_{in} = C_{in}^R + jC_{in}^I = \frac{(S_{11} - S_{22}^*\Delta)^*}{|S_{11}|^2 - |\Delta|^2}$$

Γ_{out}

In Γ_S plane

(continued)

$$|\Gamma_{in}| = \left|\frac{S_{11} - \Gamma_L \Delta}{1 - S_{22}\Gamma_L}\right| < 1 \quad = 1$$

$$|S_{11} - \Delta\Gamma_L| = |1 - S_{22}\Gamma_L|.$$

Now square both sides and simplify to obtain

$$|S_{11}|^2 + |\Delta|^2|\Gamma_L|^2 - (\Delta\Gamma_L S_{11}^* + \Delta^*\Gamma_L^* S_{11}) = 1 + |S_{22}|^2|\Gamma_L|^2 - (S_{22}^*\Gamma_L^* + S_{22}\Gamma_L)$$

$$(|S_{22}|^2 - |\Delta|^2)\Gamma_L\Gamma_L^* - (S_{22} - \Delta S_{11}^*)\,\Gamma_L - (S_{22}^* - \Delta^* S_{11})\,\Gamma_L^* = |S_{11}|^2 - 1$$

$$\Gamma_L\Gamma_L^* - \frac{(S_{22} - \Delta S_{11}^*)\Gamma_L + (S_{22}^* - \Delta^* S_{11})\,\Gamma_L^*}{|S_{22}|^2 - |\Delta|^2} = \frac{|S_{11}|^2 - 1}{|S_{22}|^2 - |\Delta|^2}. \qquad (12.23)$$

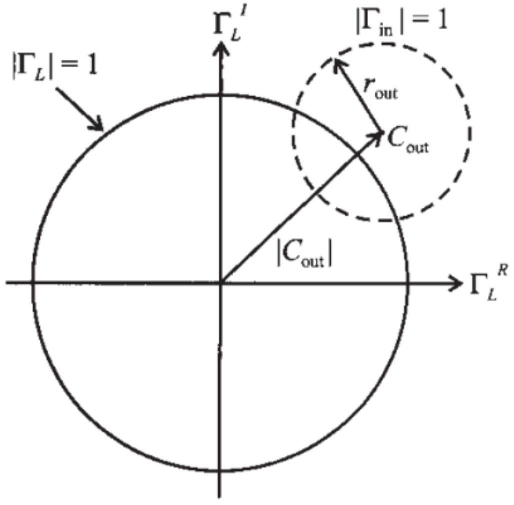
Next, complete the square by adding $\left|S_{22} - \Delta S_{11}^*\right|^2 / \left(|S_{22}|^2 - |\Delta|^2\right)^2$ to both sides:

$$\left|\Gamma_L - \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}\right|^2 = \frac{|S_{11}^2| - 1}{|S_{22}|^2 - |\Delta|^2} + \frac{\left|S_{22} - \Delta S_{11}^*\right|^2}{\left(|S_{22}|^2 - |\Delta|^2\right)^2},$$

or

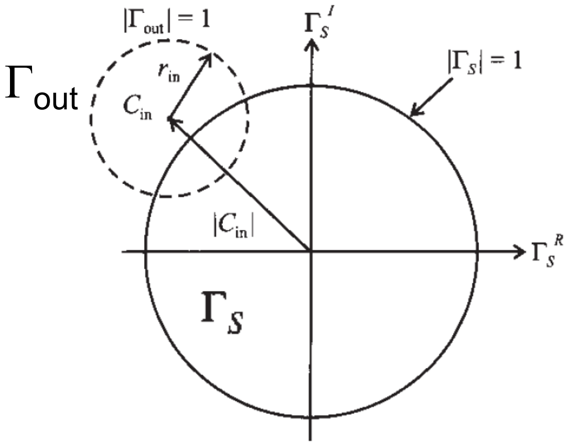
$$\left|\Gamma_L - \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}\right| = \left|\frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2}\right|. \qquad (12.24)$$

C_{out} (center) $\qquad$ R_{out} (radius)

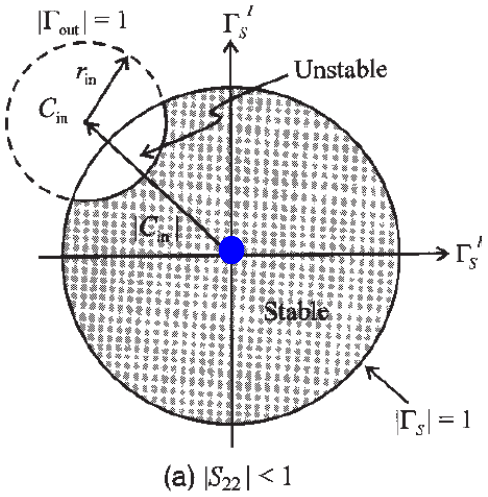


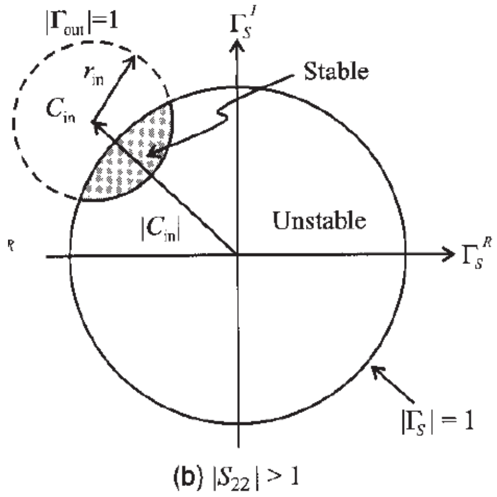
(a) Output stability circle

Input stability circle: interpretation "the origin testing"



Γ_{out} in Γ_S plane





"the origin testing"

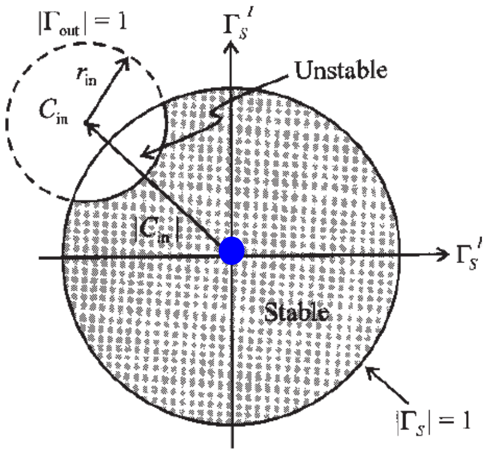
Note: Inside the $|\Gamma_s| = 1$ circle, it is possible that $|\Gamma_{out}| < 1$ or $|\Gamma_{out}| > 1$

$$|\Gamma_{out}| = \left|\frac{S_{22} - \Gamma_S \Delta}{1 - S_{11}\Gamma_S}\right| < 1$$

- $\Gamma_S = 0$ ➔ (a) $|\Gamma_{out}| = |S_{22}| < 1$, so **the origin** $\Gamma_S = 0$ is inside the stable region

  (b) $|\Gamma_{out}| = |S_{22}| > 1$, so the origin $\Gamma_S = 0$ is outside the stable region

Input stability circle: interpretation



(a) $|S_{22}| < 1$

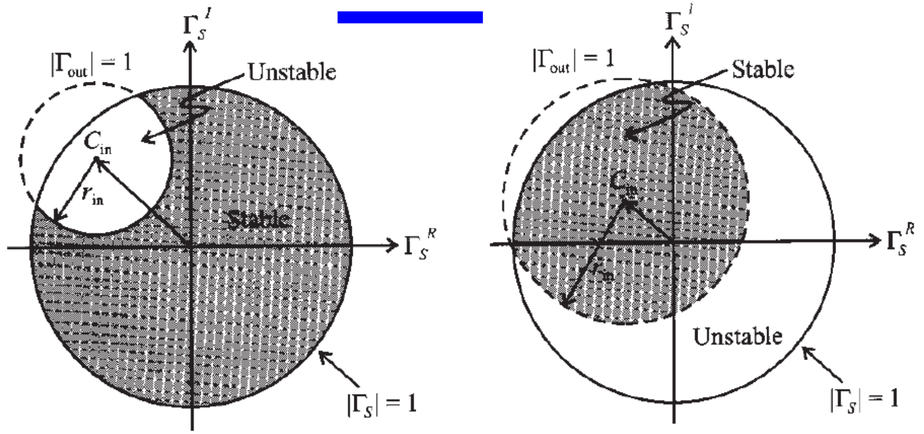
Note: Inside the $|\Gamma_s| = 1$ circle, it is possible that $|\Gamma_{out}| < 1$ **or** $|\Gamma_{out}| > 1$

$$|\Gamma_{out}| = \left|\frac{S_{22} - \Gamma_S \Delta}{1 - S_{11}\Gamma_S}\right| < 1$$

- $\Gamma_S = 0$ ➔ (a) $|\Gamma_{out}| = |S_{22}| < 1$, so the origin $\Gamma_S = 0$ is inside the stable region

  (b) $|\Gamma_{out}| = |S_{22}| > 1$, so the origin $\Gamma_S = 0$ is outside the stable region

|S22|<1 , more scenarios



(a) $r_{in} < |C_{in}|$ (b) $r_{in} > |C_{in}|$

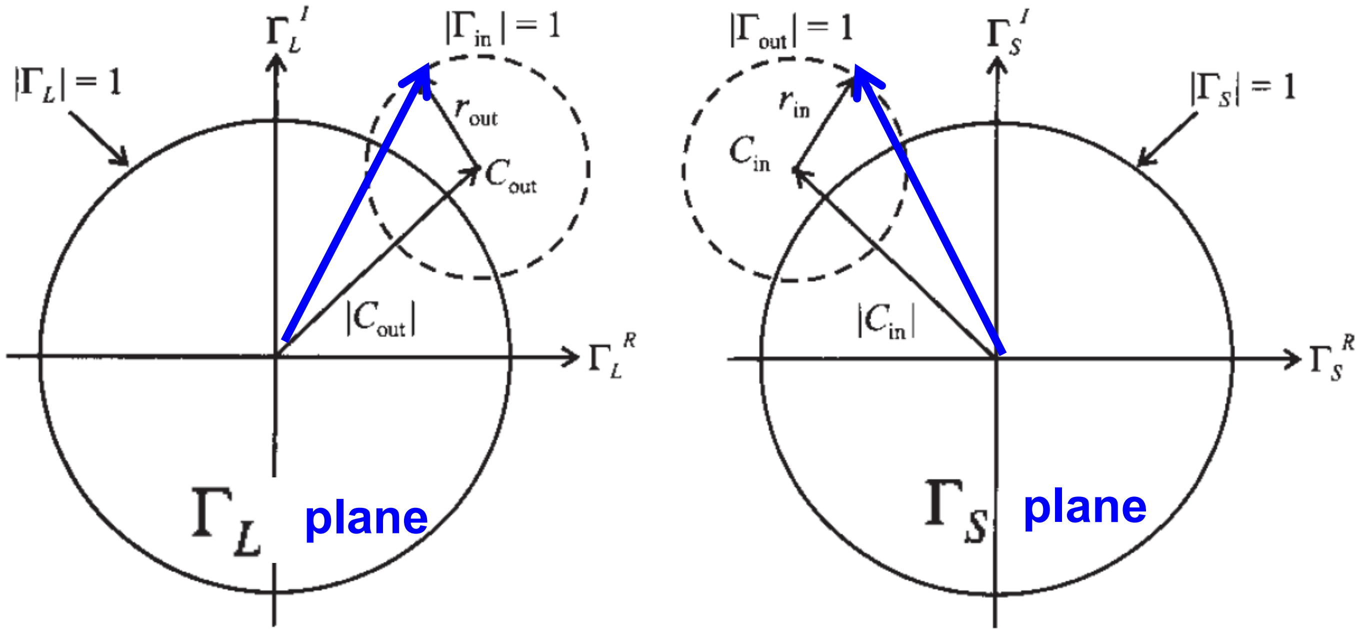
Figure 9-6 Different input stability regions for $|S_{22}| < 1$ depending on ratio between r_S and $|C_{in}|$.

- Amplifier: Interpretation of Stability Circles

Additional Essential Reading/study material

(from Ludwig’s book)

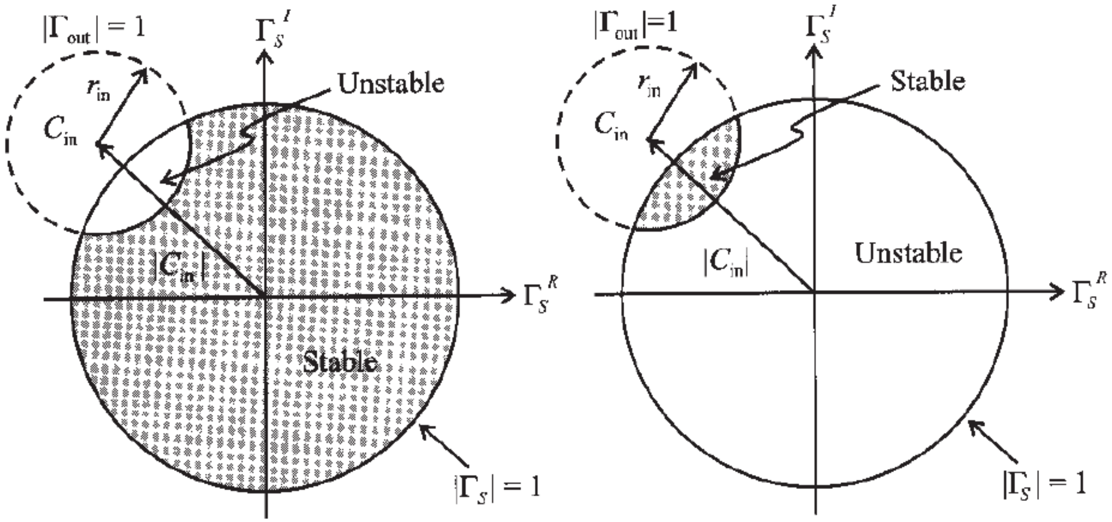
General cases



(a) Output stability circle

(b) Input stability circle

Figure 9-3 Stability circle $|\Gamma_{in}| = 1$ in the complex Γ_L plane and stability circle $|\Gamma_{out}| = 1$ in the complex Γ_S plane.



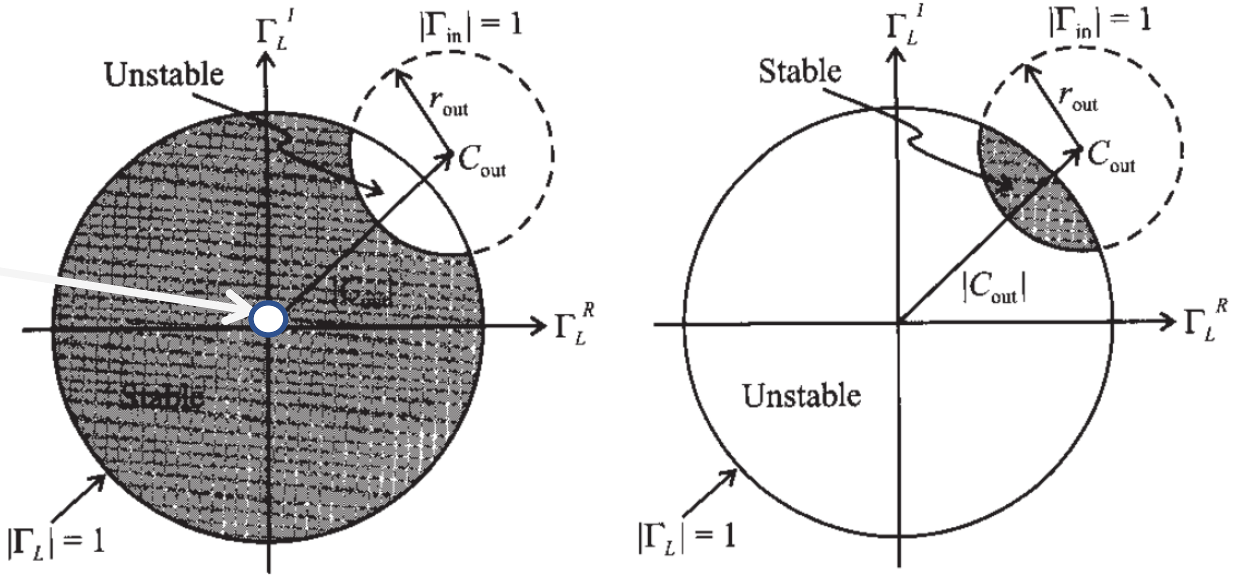
(a) $|S_{22}| < 1$

(b) $|S_{22}| > 1$

Figure 9-5 Input stability circles denoting stable and unstable regions.

For completeness, Figure 9-5 shows the two stability domains for the input stability circle. The rule-of-thumb is the inspection if $|S_{22}| < 1$, which leads to the conclusion that the center ($\Gamma_S = 0$) must be stable; otherwise the center becomes unstable for $|S_{22}| > 1$.

$\Gamma_L = 0$



(a) Shaded region is stable, since $|S_{11}| < 1$

(a) Stable region excludes the origin, $\Gamma_L = 0$, since $|S_{11}| > 1$

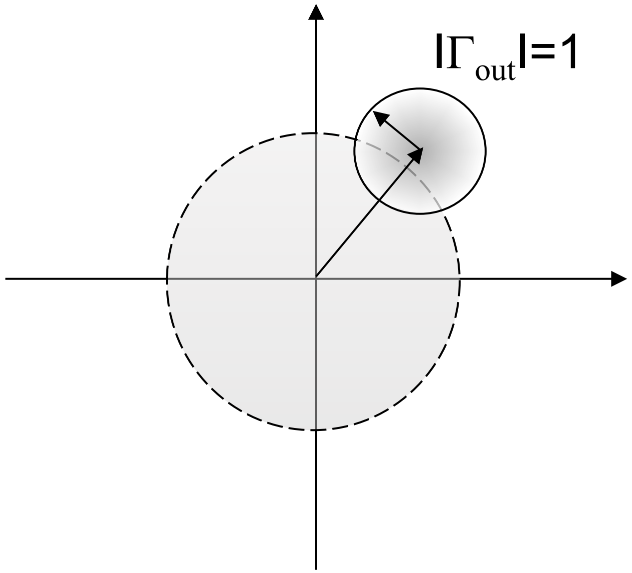
Figure 9-4 Output stability circles denoting stable and unstable regions.

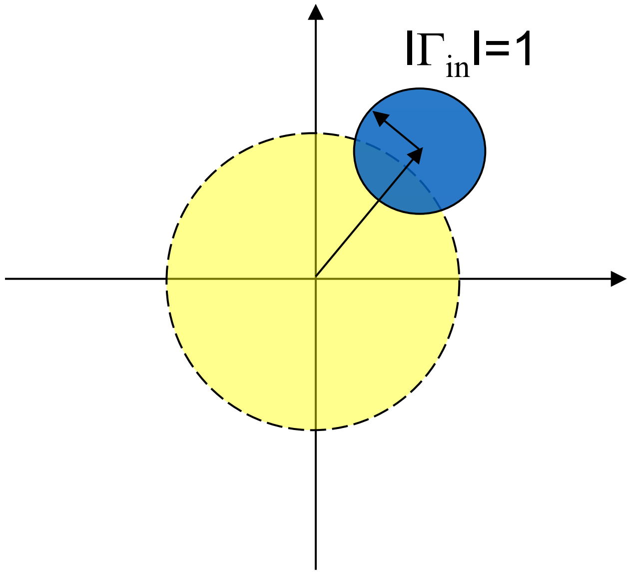
$$|\Gamma_{in}| = \left|\frac{S_{11} - \Gamma_L \Delta}{1 - S_{22}\Gamma_L}\right| < 1$$

Output matching

To interpret the meaning of Figure 9-3 correctly, a critical issue arises that is investigated for the output circle [Figure 9-3(a)], although the same argument holds for the input circle. If $\Gamma_L = 0$, then $|\Gamma_{in}| = |S_{11}|$ and two cases have to be differentiated depending on $|S_{11}| < 1$ or $|S_{11}| > 1$. For $|S_{11}| < 1$, the origin (the point $\Gamma_L = 0$) is part of the stable region, see Figure 9-4(a). However, for $|S_{11}| > 1$ the matching condition $\Gamma_L = 0$ results in $|\Gamma_{in}| = |S_{11}| > 1$, i.e. the origin is part of the unstable region. In this case the only stable region is the shaded domain between the output stability circle $|\Gamma_{in}| = 1$ and the $|\Gamma_L| = 1$ circle, see Figure 9-4(b).

Interpretation of the Stability Circles





$$|\Gamma_L| < 1, \; |\Gamma_S| < 1$$

$$|\Gamma_{in}| = \left| \frac{S_{11} - \Gamma_L \Delta}{1 - S_{22}\Gamma_L} \right| < 1$$

$$|\Gamma_{out}| = \left| \frac{S_{22} - \Gamma_S \Delta}{1 - S_{11}\Gamma_S} \right| < 1$$

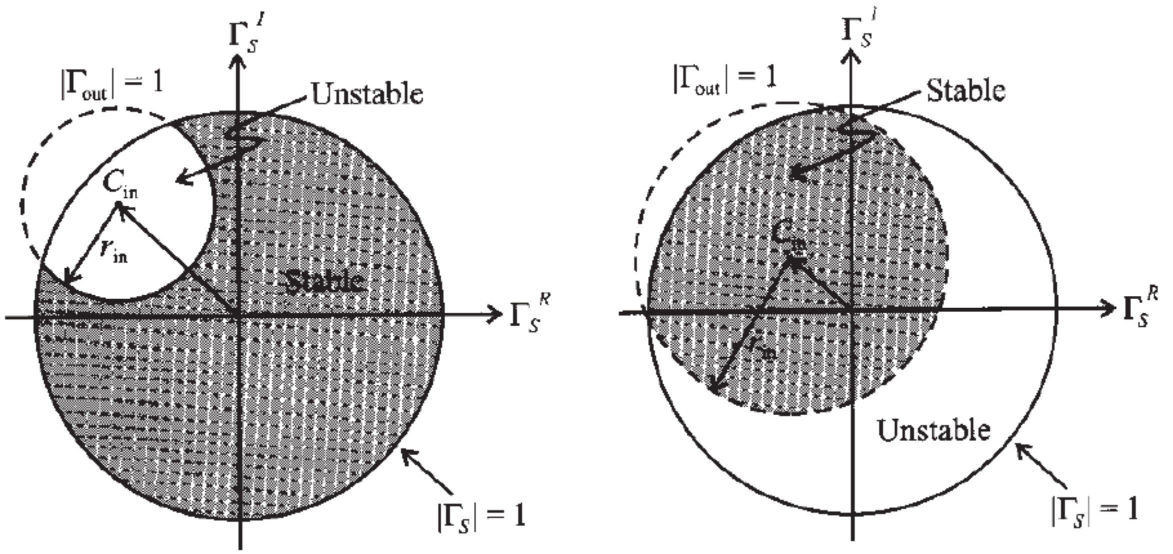
The values of $|\Gamma_{in,\, out}|<1$ can be either inside or outside the circle

We can identify the regions of stability by calculating the value of reflection coefficient only at one point inside or outside the circle of stability. It is convenient to calculate this value at the center of the diagram

$$\Gamma_S = 0; \quad \Gamma_{out} = S_{22} \qquad\qquad \Gamma_L = 0; \quad \Gamma_{in} = S_{11}$$

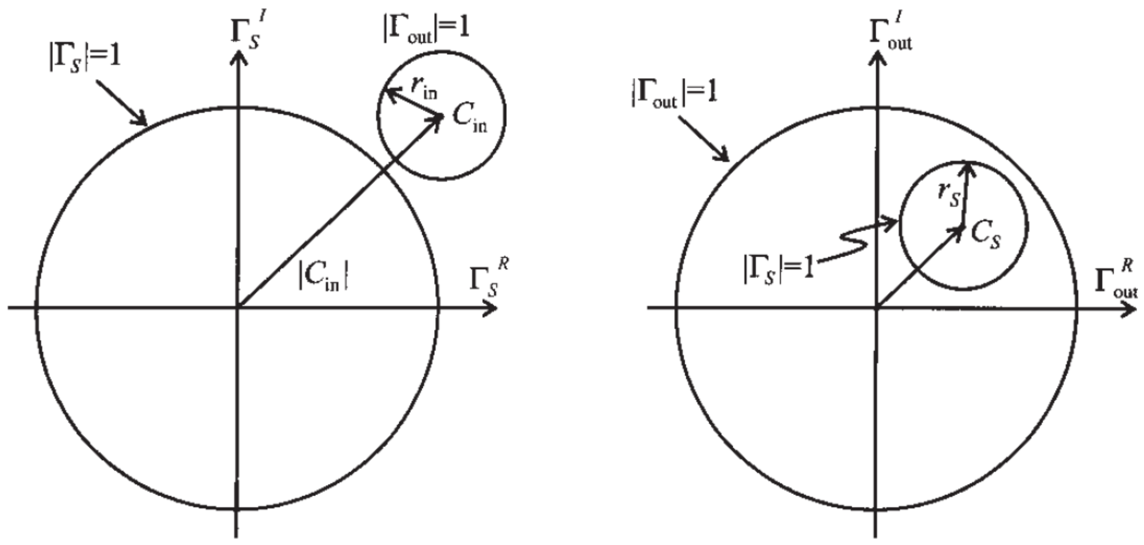
|S22|<1

$$|\Gamma_{out}| = \left|\frac{S_{22} - \Gamma_S\Delta}{1 - S_{11}\Gamma_S}\right| < 1$$



(a) $r_{in} < |C_{in}|$ (b) $r_{in} > |C_{in}|$

Figure 9-6 Different input stability regions for $|S_{22}| < 1$ depending on ratio between r_S and $|C_{in}|$.



(a) $|\Gamma_{out}| = 1$ circle must reside outside (b) $|\Gamma_S| = 1$ circle must reside inside

Figure 9-7 Unconditional stability in the Γ_S and Γ_{out} planes for $|S_{11}| < 1$.

Care has to be exercised in correctly interpreting the stability circles if the circle radius is larger than $|C_{in}|$ or $|C_{out}|$. Figure 9-6 depicts the input stability circles for $|S_{22}| < 1$ and the two possible stability domains depending on $r_{in} < |C_{in}|$ or $r_{in} > |C_{in}|$.

Input Stability circles

$$(\Gamma_S^R - C_{in}^R)^2 + (\Gamma_S^I - C_{in}^I)^2 = r_{in}^2$$

$$|\Gamma_{out}| = \left|\frac{S_{22} - \Gamma_S \Delta}{1 - S_{11}\Gamma_S}\right| < 1 \qquad r_{in} = \frac{|S_{12}S_{21}|}{\left||S_{11}|^2 - |\Delta|^2\right|} \qquad C_{in} = C_{in}^R + jC_{in}^I = \frac{(S_{11} - S_{22}^*\Delta)^*}{|S_{11}|^2 - |\Delta|^2}$$

Γ_{ou}

In tΓ_S plane

The caption of Fig. 9-7 where showing |s11|<1 is not a typo, but causing a bit confusion. (it should be |S11|<1 & |S22|<1.

Pls see further explanation below.

Unconditionally stable

9.23a&b are still in consistent with our previous discussion where we use |S22| as a special checking point to identify the stable region. See Fig. 9-7a.

9.3.2 Unconditional Stability

As the name implies, unconditional stability refers to the situation where the amplifier remains stable throughout the entire domain of the Smith Chart at the selected frequency and bias conditions. This applies to both the input and output ports. For $|S_{11}| < 1$ and $|S_{22}| < 1$ it is stated as

$$\left||C_{in}| - r_{in}\right| > 1 \tag{9.23a}$$

$$\left||C_{out}| - r_{out}\right| > 1 \tag{9.23b}$$

In other words, the stability circles have to reside completely outside the $|\Gamma_S| = 1$ and $|\Gamma_L| = 1$ circles. In the following discussion we concentrate on the $|\Gamma_S| = 1$ circle shown in Figure 9-7(a). It is shown in Example 9-2 that condition (9.23a) can be reexpressed in terms of the stability or Rollett factor k:

$$k = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}||S_{21}|} > 1 \tag{9.24}$$

Alternatively, unconditional stability can also be viewed in terms of the Γ_S behavior in the complex $\Gamma_{out} = \Gamma_{out}^R + j\Gamma_{out}^I$ plane. Here, the $|\Gamma_S| \le 1$ domain must reside completely within the $|\Gamma_{out}| = 1$ circle, as depicted in Figure 9-7(b). Plotting $|\Gamma_S| = 1$ in the Γ_{out} plane produces a circle whose center is located at

$$C_S = S_{22} + \frac{S_{12}S_{21}S_{11}^*}{1 - |S_{11}|^2} \tag{9.25}$$

and which possesses a radius c

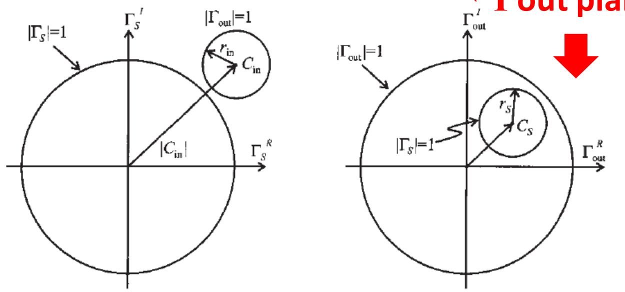
$$r_S = \frac{|S_{12}S_{21}|}{1 - |S_{11}|^2} \tag{9.26}$$

For unconditionally stable case, not only the four |Γ| <1, but also |S11|<1 & |S22|<1.

Fig. 9-7b is to interpret the instability by drawing Γs in the Γout plane.

From 9.23 & 9.27b, we can see that it is also required that |S11|<1, otherwise, 9-7b does not hold.

Plot Γs in the Γout plane



(a) $|\Gamma_{out}| = 1$ circle must reside outside (b) $|\Gamma_S| = 1$ circle must reside inside

Figure 9-7 Unconditional stability in the Γ_S and Γ_{out} planes for $|S_{11}| < 1$.

where the condition $|C_S| + r_S < 1$ must hold. We note that (9.25) can be rewritten as $C_S = (S_{22} - \Delta S_{11}^*)/(1 - |S_{11}|^2)$. Employing $|C_S| + r_S < 1$ and (9.26) it is seen that

$$|S_{22} - \Delta S_{11}^*| + |S_{12}S_{21}| < 1 - |S_{11}|^2 \tag{9.27a}$$

and since $|S_{12}S_{21}| \le |S_{22} - \Delta S_{11}^*| + |S_{12}S_{21}|$ we conclude

$$|S_{12}S_{21}| < 1 - |S_{11}|^2 \tag{9.27b}$$

--------------------End of the **Additional Essential Reading/study** material ------------------------

Example

A certain MESFET has the following S parameter measured at 9 GHz with a 50 Ω reference.

$S_{11} = 0.894\angle -60.6°$
$S_{12} = 0.02\angle 62.4°$
$S_{21} = 3.122\angle 123.6°$
$S_{22} = 0.781\angle -27.6°$

Compute: (a) the delta factor Δ and the Rollett stability factor K, (b) the geometrically derived parameter μ and (c) plot the input and output stability circles.

Solution:

a) The delta factor is

$$\Delta = 0.894\angle -60.6° \times 0.781\angle -27.6° - 0.02\angle 62.4° \times 3.122\angle 123.6°$$
$$= 0.6964\angle -83.1°$$
$$|\Delta| = 0.6964 < 1$$

The Rollett stability factor is

$$K = \frac{1 + |0.6964|^2 - |0.894|^2 - |0.781|^2}{2 \times |0.02 \times 3.122|} = 0.607 < 1$$

b) For the geometrically derived parameter,

$$C_2 = 0.781\angle -27.6° - 0.6964\angle -83.1° \times 0.894\angle 60.6° = 0.170\angle -46.7°$$

$$\mu = \frac{1 - |0.894|^2}{|0.170| + |0.02 \times 3.122|} = 0.86 < 1$$

$$C_1 = 0.894\angle -60.6° - 0.6964\angle -83.1° \times 0.781\angle 27.6° = 0.356\angle -68.5°$$

c) For the input stability circle, $C_1^* = 0.356\angle 68.5°$, and the center and radius are

$$c_s = \frac{0.356\angle 68.5°}{|0.894|^2 - |0.6964|^2} = 1.13\angle 68.5°$$

$$r_s = \frac{|0.02 \times 3.122|}{\left||0.894|^2 - |0.6964|^2\right|} = 0.20$$

Source (input) stability circle, $|\Gamma_{out}(\Gamma_s)| = 1$:

$$c_s = \frac{C_1^*}{|S_{11}|^2 - |\Delta|^2}$$

$$r_s = \frac{|S_{12}S_{21}|}{\left||S_{11}|^2 - |\Delta|^2\right|}$$

Example

For the output stability circle, $C_2^* = 0.170\angle 46.7°$, and the center and radius are

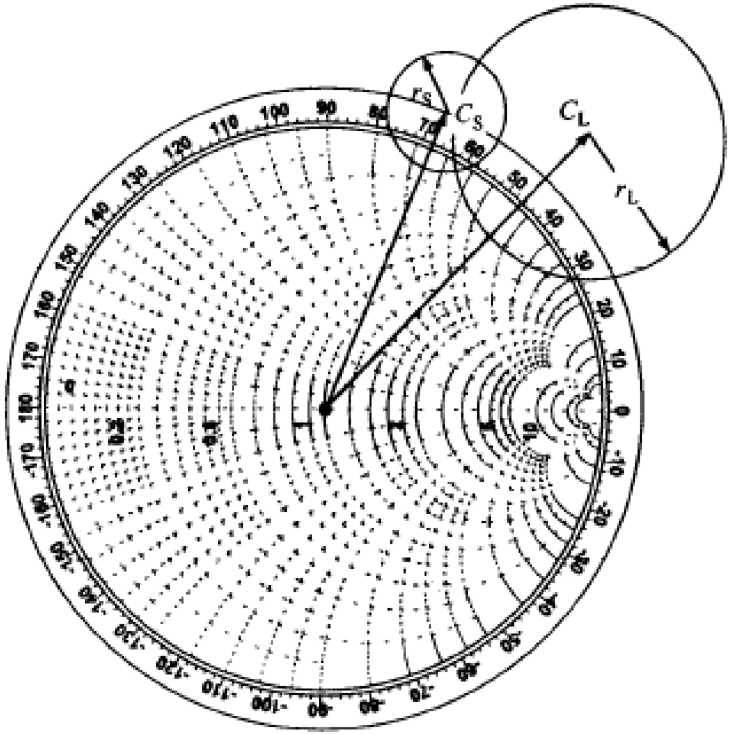
$$c_L = \frac{0.170\angle 46.7°}{|0.781|^2 - |0.6964|^2} = 1.36\angle 46.7°$$

$$r_L = \frac{|0.02\times 3.122|}{\left||0.781|^2 - |0.6964|^2\right|} = 0.50$$

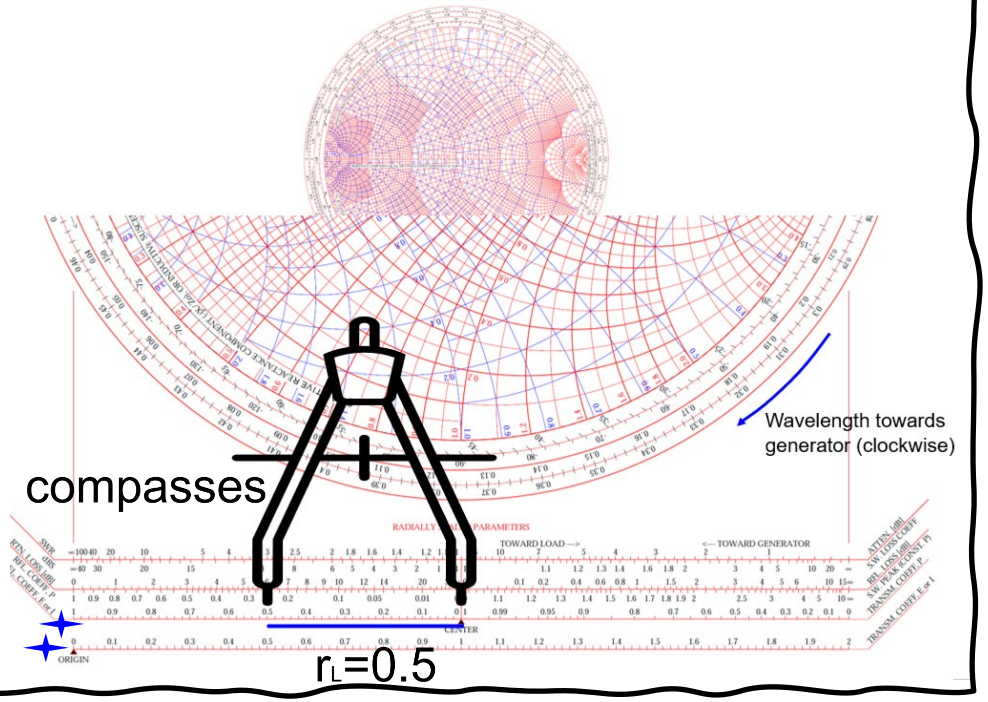
Load (output) stability circle, $|\Gamma_{in}(\Gamma_L)| = 1$:

$$c_L = \frac{C_2^*}{|S_{22}|^2 - |\Delta|^2}$$

$$r_L = \frac{|S_{12}S_{21}|}{\left||S_{22}|^2 - |\Delta|^2\right|}$$



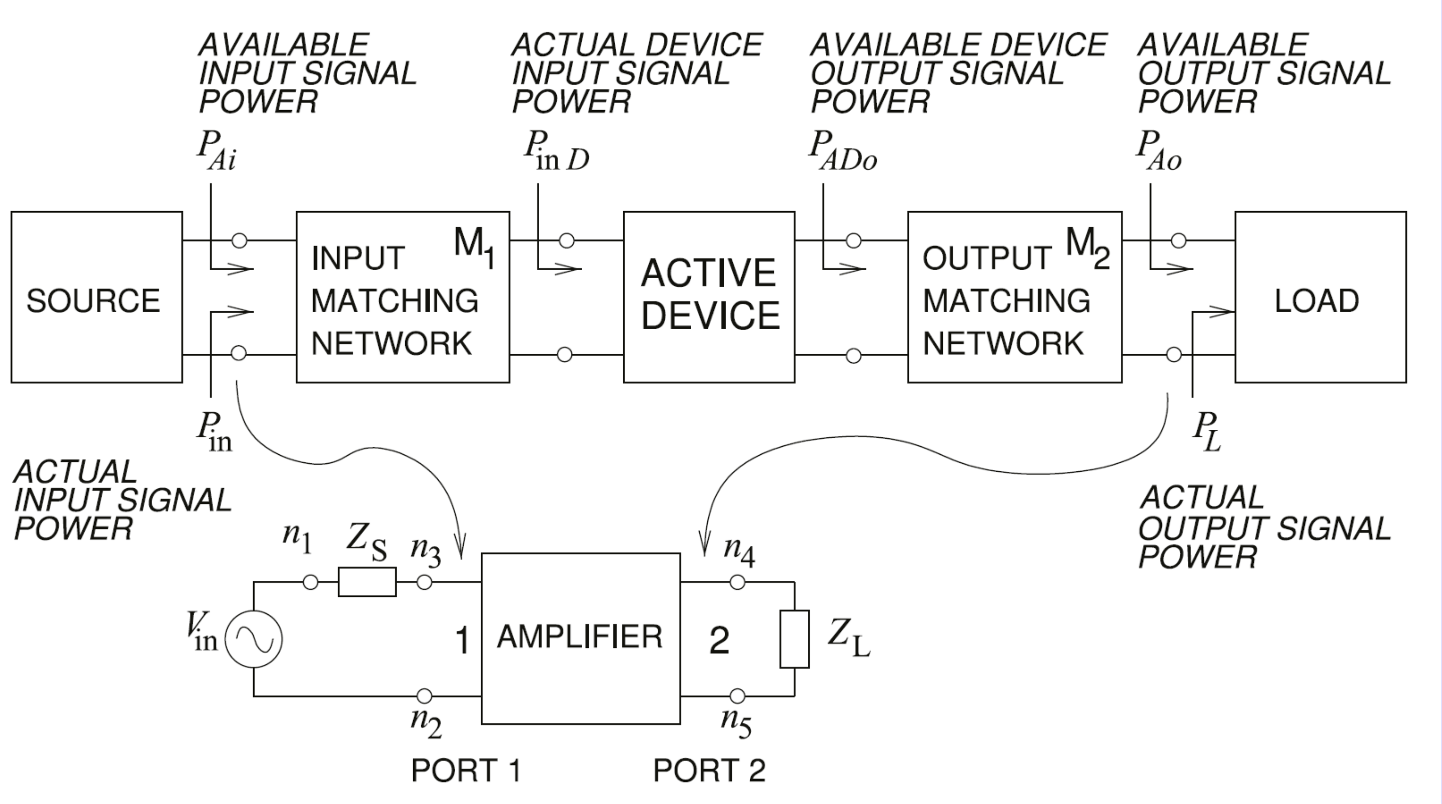
How to measure $C_{L,S}$ & $r_{L,s}$ on a Smith Chart?



Outline

- Introduction
  - Block diagram of RF Transceiver
  - Transistor Technologies for Amplifiers
  - Block Diagram of Amplifiers & Key Parameters
- Stability
  - Stability Conditions & Stability Tests
  - Stability Circles & Interpretation
- **Power Gain**
  - Definitions of Power Gains
  - Unilateral Design & Constant Gain Circles
- Other short topics
  - Bilateral Design (a brief intro)
  - Bias Tee
  - Power Efficiency
  - Amplifier Design Considerations

Amplifier Power Gain



Power	Description
P_{in}	Actual input power delivered to the amplifier.
P_{Ai}	Available input power from the source. $P_{\text{in}} \le P_{Ai}$. If M_1 provides conjugate matching as seen from the source, then $P_{\text{in}} = P_{Ai}$.
$P_{\text{in}D}$	Actual device input power delivered to the active device. $P_{\text{in}D} \le P_{\text{in}}$. If M_1 is lossless, $P_{\text{in}D} = P_{\text{in}}$.
P_{ADo}	Available device output power of the active device.
P_{Ao}	Available amplifier output power. $P_{Ao} \le P_{ADo}$. If M_2 is lossless, $P_{Ao} = P_{ADo}$.
P_L	Actual output power delivered to load. Amplifier output power. $P_L \le P_{Ao}$. If M_2 is lossless and provides conjugate matching, $P_L = P_{Ao} = P_{ADo}$.

$$\begin{aligned}
P_{\text{in}} &= P_{Ai}, && \text{if the generator is conjugately matched} \\
P_{\text{in}} &= P_{\text{in}D}, && \text{if } M_1 \text{ is lossless} \\
P_o &= P_{Ao}, && \text{if the output of active device is conjugately matched} \\
P_L &= P_o, && \text{if } M_2 \text{ is lossless.}
\end{aligned}$$

Figure 2-3: Parameters used in defining gain measures. The input and output matching networks are lossless so that the actual device input signal power, PinD, is the power delivered by the source. Similarly the actual output signal power delivered to the load, PL, is the power delivered by the active device (the transistor including biasing network).

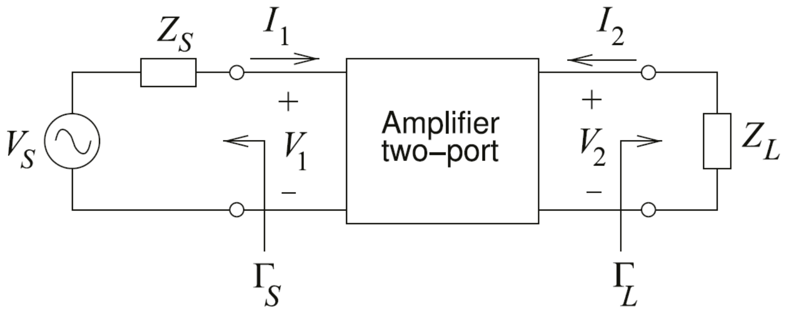
System gain (the actual power gain): $G = \dfrac{P_L}{P_{\text{in}}}$

Transducer gain: $G_T = \dfrac{P_L}{P_{Ai}}$

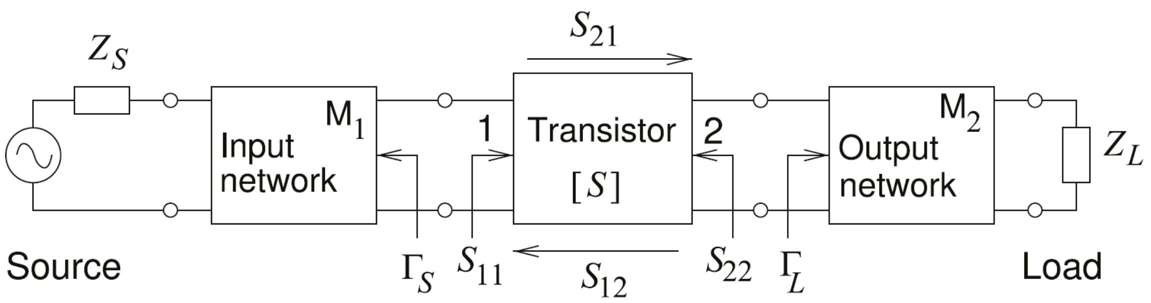
Power gain: $G_P = \dfrac{P_L}{P_{\text{in},D}}$

Available gain: $G_A = \dfrac{P_{Ao}}{P_{Ai}}$

Gain in terms of Scattering Parameters



(a) An amplifier



(b) Active device with input and output matching networks

Figure 2-4: Two-port network with source and load

Let **S** be the normalized scattering matrix of the two-port, with Z_0 being the normalizing real characteristic impedance:

$$\mathbf{S} = [S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

Transducer gain in terms of S parameters:

$$\boxed{G_T = |S_{21}|^2 \frac{(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|(1-\Gamma_S S_{11})(1-\Gamma_L S_{22}) - \Gamma_S \Gamma_L S_{12} S_{21}|^2}}$$

If S12=0 (no feedback from the output to the input

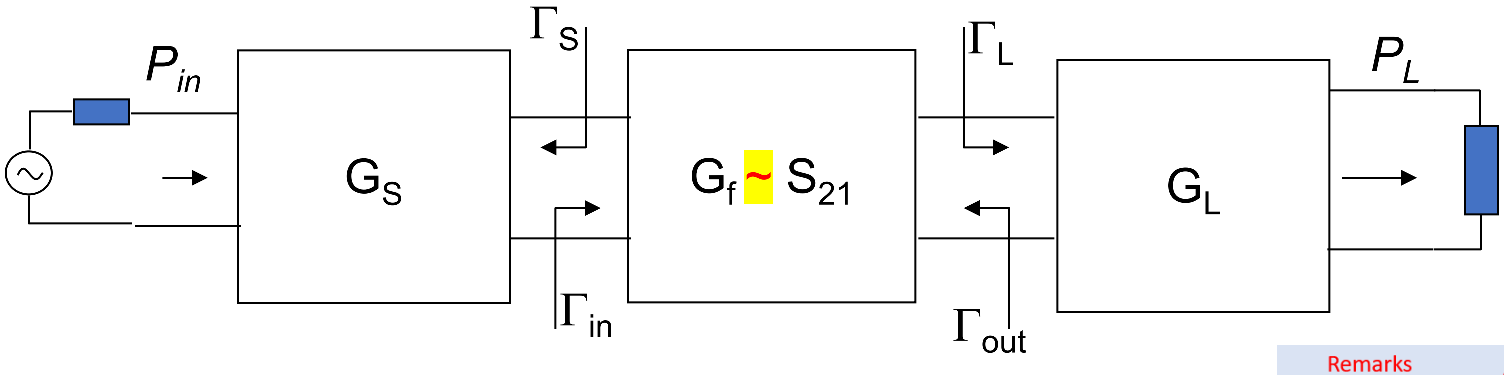
Unilaterial two port

unilateral (transducer) gain

$$G_{TU} = |S_{21}|^2 \left(\frac{1-|\Gamma_S|^2}{|1-\Gamma_S S_{11}|^2} \right) \left(\frac{1-|\Gamma_L|^2}{|1-\Gamma_L S_{22}|^2} \right)$$

Unilateral Case: Constant Gain Circle

- **Unilateral case, (S_{12}= 0)** Assumed that there is no feedback from the output of the amplifier



- Substituting $S_{12} = 0$, we obtain the unilateral transducer power gain

$$G_{tu} = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

- We can think of G_{tu} as comprising three independent gain terms:

$$G_s = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

$$G_f = |S_{21}|^2$$

where

G_s = gain contribution from input matching network
G_L= gain contribution from output matching network
G_f= forward transistor gain

Remarks

$$\Gamma_{in} = S_{11} + \frac{S_{21}\overset{0}{\cancel{S_{12}}}\Gamma_L}{1-S_{22}\Gamma_L}$$

$$\Gamma_{in} = S_{11}$$

$$\Gamma_{out} = S_{22} + \frac{S_{21}\overset{0}{\cancel{S_{12}}}\Gamma_S}{1-S_{11}\Gamma_S}$$

$$\Gamma_{out} = S_{22}$$

(For unilateral case)

Unilateral Case: Constant Gain Circle

- Input gain circle:

- Consider

$$G_s = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}$$

To deliver maximum power, it requires CONJUGATE MATCHING at input and output : $\Gamma_S = \Gamma^*_{in}$ and $\Gamma_L = \Gamma^*_{out}$

Because of Unilateral case, we have $\Gamma_S = \Gamma^*_{in} = S^*_{11}$ (For unilateral case only)

$|S11|^2$
$=|S^*11|^2$
$=|S11\, S^*11|$

G_s is maximum when $\Gamma_s = S_{11}^*$

$$G_{s,\max} = \frac{1}{1-|S_{11}|^2}$$

and let g_s be the normalized gain value where

$$g_s = \frac{G_s}{G_{s,\max}} = G_s\left(1-|S_{11}|^2\right)$$

- For a certain G_s between 0 and $G_{s,\max}$, the solutions for Γ_s lie on the input constant gain circle. The center of this circle is along the S_{11}^* vector given by

$$C_{gs} = \frac{g_s S_{11}^*}{1-|S_{11}|^2(1-g_s)}$$

- The radius of the input constant gain circle is

$$r_{gs} = \frac{\sqrt{1-g_s}\left(1-|S_{11}|^2\right)}{1-|S_{11}|^2(1-g_s)}$$

- Output gain circle:

- Consider

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

G_L is maximum when $\Gamma_L = S_{22}^*$

$$G_{L,\max} = \frac{1}{1-|S_{22}|^2}$$

and let g_L be the normalized gain value where

$$g_L = \frac{G_L}{G_{L,\max}} = G_L\left(1-|S_{22}|^2\right)$$

- For a certain G_L between 0 and $G_{L,\max}$, the solutions for Γ_L lie on the output constant gain circle. The center of this circle is along the S_{22}^* vector given by

$$C_{gL} = \frac{g_L S_{22}^*}{1-|S_{22}|^2(1-g_L)}$$

- The radius of the output constant gain circle is

$$r_{gL} = \frac{\sqrt{1-g_L}\left(1-|S_{22}|^2\right)}{1-|S_{22}|^2(1-g_L)}$$

Unilateral Transducer Gain

In the above constant gain circle derivation, we can see when $\Gamma_S = (S_{11})^*$ and $\Gamma_L = (S_{22})^*$ (i.e., the complex conjugates) $\mathbf{G_{TU}}$ achieves its maximum value --- the **maximum unilateral transducer gain**:

$$G_{TU\max} = |S_{21}|^2 \left(\frac{1}{1 - |S_{11}|^2} \right) \left(\frac{1}{1 - |S_{22}|^2} \right)$$

EXAMPLE (Amplifier Gain)

A source that drives an amplifier has an available output power of 1 mW. However, the load of the amplifier is mismatched so that the load reflection coefficient is 1 dB. The power actually delivered to the load of the amplifier is 1 W. Is it possible to determine the system gain? If so, what is it in decibels?

Solution:

The power delivered to the load is PL = 1 W, and the available input power PAi = 1 mW. Examining the gains defined in Equations (2.1)–(2.4), Equation (2.3) yields

$$G_T = \frac{P_L}{P_{Ai}} = \frac{1\text{w}}{1\text{ mW}} = 1000 = 30\text{ dB}.$$

No other gain can be determined, i.e., the system gain cannot be determined as the actual input power is unknown.

EXAMPLE: Fig 2-2 shows the S parameters of a pHEMT transistor.

Frequency (GHz)	$\lvert S_{11}\rvert$	$\angle S_{11}$ degrees	$\lvert S_{21}\rvert$	$\angle S_{21}$ degrees	$\lvert S_{12}\rvert$	$\angle S_{12}$ degrees	$\lvert S_{22}\rvert$	$\angle S_{22}$ degrees
0.500	0.976	−20.9	11.395	161.5	0.011	78.3	0.635	−11.5
1.000	0.925	−41.3	10.729	145.1	0.021	67.8	0.614	−22.2
2.000	0.796	−78.2	8.842	116.7	0.034	51.4	0.553	−37.9
3.000	0.694	−106.8	7.180	94.5	0.041	40.4	0.506	−48.9
4.000	0.614	−127.3	6.002	76.7	0.044	33.9	0.475	−57.7
5.000	0.555	−147.0	5.249	60.3	0.048	28.4	0.453	−66.4
6.000	0.511	−170.2	4.729	43.7	0.052	23.3	0.438	−76.0
7.000	0.493	163.9	4.261	26.8	0.057	14.0	0.391	−87.6
8.000	0.486	140.4	3.784	11.2	0.057	6.4	0.340	−99.1
9.000	0.473	122.5	3.448	−2.4	0.059	5.2	0.332	−109.6
10.000	0.488	103.4	3.339	−17.3	0.073	0.9	0.355	−124.8
11.000	0.539	79.8	3.166	−35.0	0.086	−10.1	0.349	−145.6
12.000	0.626	60.8	2.877	−51.9	0.095	−21.4	0.307	−169.6
13.000	0.685	47.6	2.604	−68.2	0.100	−32.5	0.295	165.3
14.000	0.724	36.2	2.392	−83.8	0.106	−43.3	0.312	142.7
15.000	0.787	20.9	2.225	−99.7	0.109	−55.1	0.320	125.4
16.000	0.818	5.2	2.067	−116.6	0.112	−68.4	0.340	103.9
17.000	0.831	−9.6	1.855	−134.4	0.108	−83.5	0.373	76.1
18.000	0.852	−19.5	1.603	−148.6	0.103	−94.2	0.406	54.7
19.000	0.815	−20.5	1.440	−159.3	0.102	−103.0	0.449	43.1
20.000	0.780	−26.8	1.382	−171.2	0.106	−113.5	0.460	37.9
21.000	0.779	−46.8	1.333	171.2	0.109	−130.7	0.438	31.4
22.000	0.786	−62.1	1.195	152.0	0.110	−148.4	0.417	6.0
23.000	0.774	−70.1	1.073	137.2	0.108	−162.4	0.428	−16.5
24.000	0.744	−81.7	1.025	123.5	0.112	−175.2	0.433	−29.0
25.000	0.704	−90.9	1.061	107.3	0.132	170.0	0.396	−46.5
26.000	0.677	−111.1	1.065	85.8	0.148	147.8	0.298	−71.0

Figure 2-2: Scattering parameters of an enhancement mode pHEMT transistor biased at V_{DS} = 5 V, I_D = 55 mA, V_{GS} = −0.42 V. Extract from the data sheet of the FPD6836P70 discrete transistor [1].

Under the assumption of the unilateral 2-port, what are the maximum unilateral transducer gain $\mathbf{G_{TUmax}}$ in dB?

Frequency (GHz)	$G_{TU\max}$ (dB)
0.5	36.62
1	31.07
2	24.88
3	21.26
4	18.73
5	16.00
6	15.74
7	14.52
8	13.26
9	12.36
10	12.24
11	12.07
12	11.77
13	11.46

Frequency (GHz)	$G_{TU\max}$ (dB)
14	11.25
15	11.61
16	11.64
17	11.11
18	10.50
19	8.89
20	7.91
21	7.48
22	6.55
23	5.46
24	4.62
25	4.22
26	3.61

Example

A certain RF transistor has the following S parameters measured at 500 MHz with a 50 Ω reference:

$S_{11} = 0.8\angle -80°$; $S_{22} = 0.8\angle -80°$; $S_{12} \approx 0$; $S_{21} \approx 2$

Determine the maximum unilateral transducer power gain and plot the input constant gain circles for 3, 2, 1, 0, -1 dB.

Solution:

The maximum unilateral transducer power gain is obtained when input and output are conjugately matched ($\Gamma_s = S_{11}^*$ and $\Gamma_L = S_{22}^*$)

$G_f = |S_{21}|^2 = 4 = 6$ dB

$G_{s,max} = 1 / (1 - |S_{11}|^2) = 1 / (1 - 0.64) = 2.78 = 4.4$ dB

$G_{L,max} = 1 / (1 - |S_{22}|^2) = 1 / (1 - 0.64) = 2.78 = 4.4$ dB

Then the maximum unilateral transducer power gain is

$G_{tu,max} = 4 \times 2.78 \times 2.78 = 30.91 = (6 + 4.4 + 4.4)$ dB $= 14.8$ dB

For a 3 dB (ratio of 2) input constant gain circle

3 dB = 10 log (2)

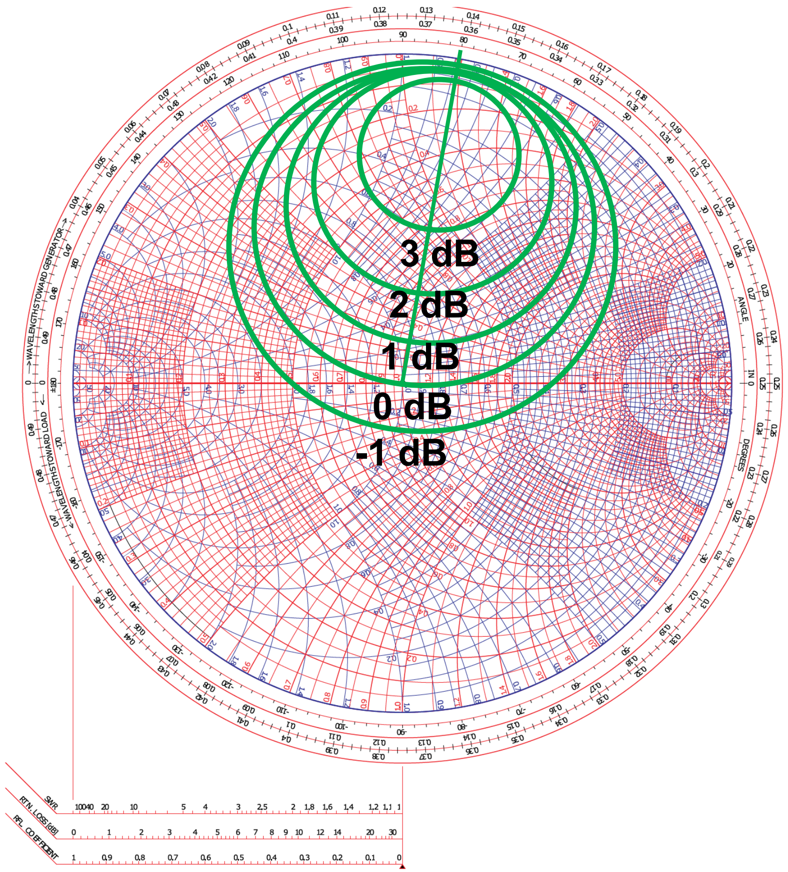
$$g_s = \frac{G_s}{G_{s,\max}} = \frac{2}{2.78} = 0.72$$

$$d_s = \frac{g_s|S_{11}|}{1-|S_{11}|^2(1-g_s)} = \frac{0.72\times 0.8}{1-|0.8|^2(1-0.72)} = 0.70$$

$$r_{gs} = \frac{\sqrt{1-g_s}\left(1-|S_{11}|^2\right)}{1-|S_{11}|^2(1-g_s)} = \frac{\sqrt{1-0.72}\left(1-|0.8|^2\right)}{1-|0.8|^2(1-0.72)} = 0.23$$

Repeat for different power gain levels and the results are tabulated as follows:

G_s dB	-1	0	1	2	3
G_s value	0.79	1	1.26	1.59	2
g_s	0.29	0.36	0.45	0.57	0.72
d_s	0.43	0.49	0.55	0.63	0.70
r_{gs}	0.55	0.49	0.41	0.33	0.23



Additional Questions:

By examininng the plot, what are the two observations/conclusions that you can make?
(1) about the centers of the five gain circles
(2) about the 0-dB gain circle

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  - Amplifier Design Considerations

Bilateral Design (a brief intro)

When S_{12} cannot be assumed as 0 we need to take in account the feedback in calculations of the input and output reflection coefficients. The condition of maximum power gain will be:

$$\Gamma_{IN} = \Gamma_S^* = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} \qquad \Gamma_{OUT} = \Gamma_L^* = S_{22} + \frac{S_{21}S_{12}\Gamma_S}{1 - S_{11}\Gamma_S} \qquad G_t = \frac{1}{1 - |\Gamma_s|^2}|S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

For a bilateral case, Γ_s depends on Γ_L and vice versa, this means this need to be solved simultaneously to obtain conjugate values & referred as Γ_{Ms} and Γ_{ML}

$$\Gamma_{Ms} = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$

$$\Gamma_{ML} = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

where

$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$
$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$

- For unconditional stable case, simultaneous conjugate match can always be achieved and the solutions with minus sign above must be used, i.e. $\Gamma_s = \Gamma_{Ms-}$ and $\Gamma_L = \Gamma_{ML-}$.

$$G_{t\max} = \frac{1}{1 - |\Gamma_{MS}|^2}|S_{21}|^2 \frac{1 - |\Gamma_{ML}|^2}{|1 - S_{22}\Gamma_{ML}|^2}$$

Bias Tee

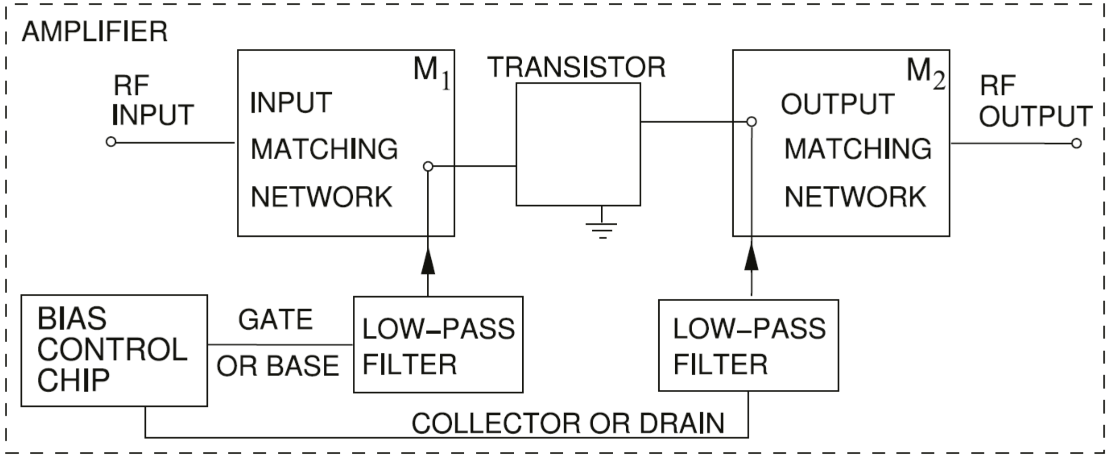
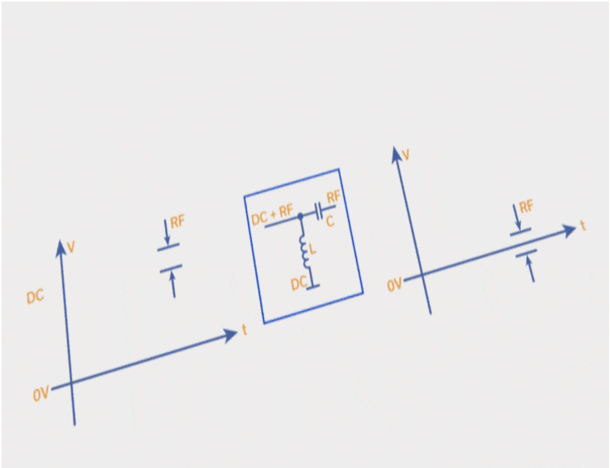


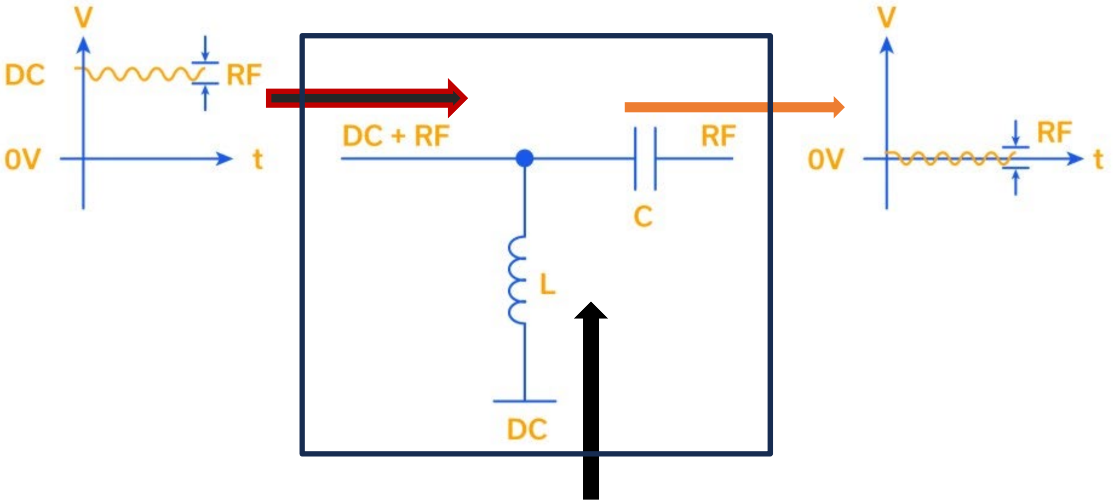
Figure 2-1: Block diagram of a RF amplifier including biasing networks.



animation

The bias tee is an essential component for applying DC voltage to any component that must also pass RF/microwave signals, most commonly an RF amplifier that requires a DC supply.

- For narrowband applications, bias tee design and construction are relatively straightforward, except for component self-resonant frequencies (SRFs).
- For broadband applications, however, bias tee design and construction are nontrivial, dependent heavily on component characteristics.



Fundamental bias tee arrangement of an inductor and capacitor showing the signals at each of the three ports.

https://blog.minicircuits.com/rf-microwave-bias-tee-basics/

Amplifier Efficiency

power-added efficiency (PAE):

$$\eta_{\mathrm{PAE}} = \frac{P_{\mathrm{RF,out}} - P_{\mathrm{RF,in}}}{P_{\mathrm{DC}}}$$

total power-added efficiency:

overall amplifier efficiency:

$$\eta_{\mathrm{overall}} = \eta_{\mathrm{total}} = \frac{P_{\mathrm{RF,out}}}{P_{\mathrm{DC}} + P_{\mathrm{RF,in}}}$$

average amplifier efficiency:

$$\eta_{\mathrm{avg}} = \frac{P_{\mathrm{RF,out,avg}}}{P_{\mathrm{DC,avg}}}$$

It takes into account the time-varying level of a modulated communications signal

For high-gain amplifiers, $P_{\mathrm{RF,in}} \ll P_{\mathrm{DC}}$, and all of the efficiencies become approximately equivalent and is simply called the **efficiency**, η, of the amplifier:

$$\eta = \frac{P_{\mathrm{RF,out}}}{P_{\mathrm{DC}}} \approx \eta_{\mathrm{PAE}} \approx \eta_{\mathrm{total}} \approx \eta_{\mathrm{avg}} \approx \eta_{\mathrm{overall}} \quad \text{(high gain).} \tag{2.38}$$

When the primary input DC power is fed to the drain of a FET, the term **drain efficiency**, η_D, is used:

$$\eta_D = \frac{P_{\mathrm{RF,out}}}{P_{\mathrm{DC}}}. \tag{2.39}$$

This term is also used when the device is a BJT or HBT, although the term **collector efficiency** would be more appropriate.

Amplifier Design Considerations

1. TRANSISTORS
   - Si BJTs, Si MOSFET - for use below 5 GHz
   - GaAs FETs - for use above 2 GHz
   - AlGaAs HEMTs for low-noise applications up to 100 GHz
   - SiGe Heterojunction Bipolar Transistors (up to 60 GHz)
2. TECHNOLOGY
   - Hybrid Microwave Integrated Circuit (HMIC):
   - Monolithic Microwave Integrated Circuit (MMIC):
3. IMPEDANCE TRANSFORMER DESIGN
   - transformation of an arbitrary impedance to another arbitrary impedance within a specified bandwidth
4. BIAS NETWORKS DESIGN
   - Must provide the necessary DC bias voltages without disturbance of the in-band performance and stability at low frequencies.
5. OVERALL STABILITY

Acknowledgment

- Part 2 of the EE5303 Module, consisting of four lectures, was originally designed and taught before 2017 by

Dr. Muhammad Faeyz Karim

Remark