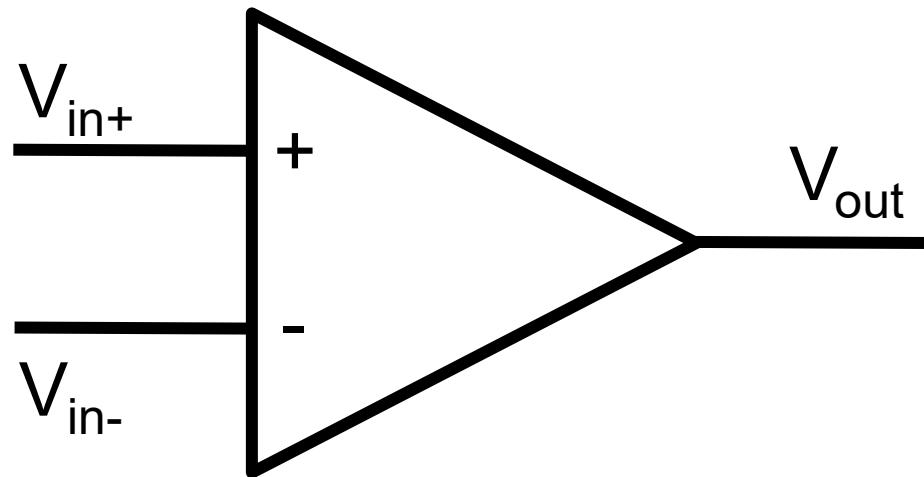


MICROWAVE AMPLIFIER DESIGN - II

EE5303 – Part 2

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Outline

❑ Noise

- Three major types
- Power Spectrum

❑ Noise Figure

- Noise figure circle
- Noise Figure of Multistage Amplifiers

❑ Impedance Matching

- Reasons/objectives
- Smith Chart primer & four sets of principles
- Impedance matching by Smith Chart – stub matching with 3 examples

❑ Nonlinearity & Performance Parameters

- 1-dB suppression point
- Intermodulation Distortion (IMD)
- Two-Tone 3rd Order Intercept Point
- Dynamic range
- Multistage Amplifiers

❑ Design Considerations

- Power Amplifier Operating Modes
- DC Biasing Networks
- Design Considerations

❑ Case Study -- X-band wideband low-noise amplifier

(***Self study &
must study)

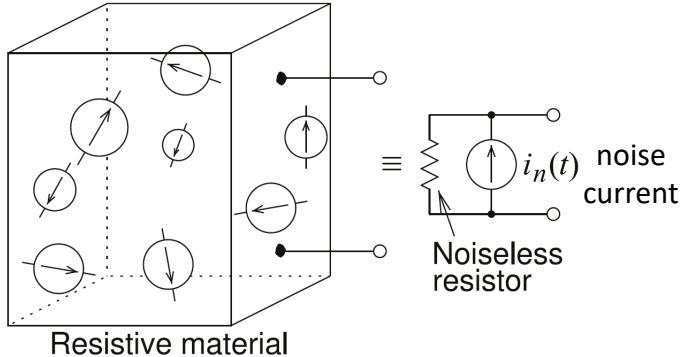
(Random) Noise – the Three Major Types

Noise is ubiquitous in electronic systems:

- Noise (i.e., random fluctuations of voltage and current) defines the minimum detectable signal
- Fluctuations of voltage and current arise from several different physical processes yielding noise with statistical properties
- The three major physical sources of noise affecting electronic circuits are **thermal, shot, and flicker (noise)**

Thermal Noise

(also called Johnson–Nyquist noise)



- It is the best understood noise, caused by the random movement of electrons due to the random vibration of the lattice of a conducting material
- It is explained by the **fluctuation-dissipation Theorem**, which relates thermally-induced fluctuations in a material to the resistance of the material (see the illustration above)

**Thermal noise power (Watts)
in a bandwidth B (Hz)**

$$P_t(f) = kTB$$

- k is the Boltzmann constant
 $= 1.380649 \times 10^{-23}$ joules per kelvin (J/K)
- T is the temperature in kelvin

**Power spectrum density
(per Hz)**

$$S_t(f) = kT \rightarrow$$

S_t is constant throughout the frequency spectrum for a fixed T

Shot noise

(also Poisson noise)

$$S(f) = 2e|I|$$

- e is the electron charge
- I is the average current

- It originates from the fact that current is made of electric charges (electrons/holes) being discrete in nature
- The term “shot” comes from an analogy to pellets (small balls) or bullets being fired randomly - like “a shotgun blast”.
- It is often less significant as compared with the flicker noise and thermal noise
- It varies during an RF cycle as the current flow varies. For active circuit designs, bias current should be minimized.

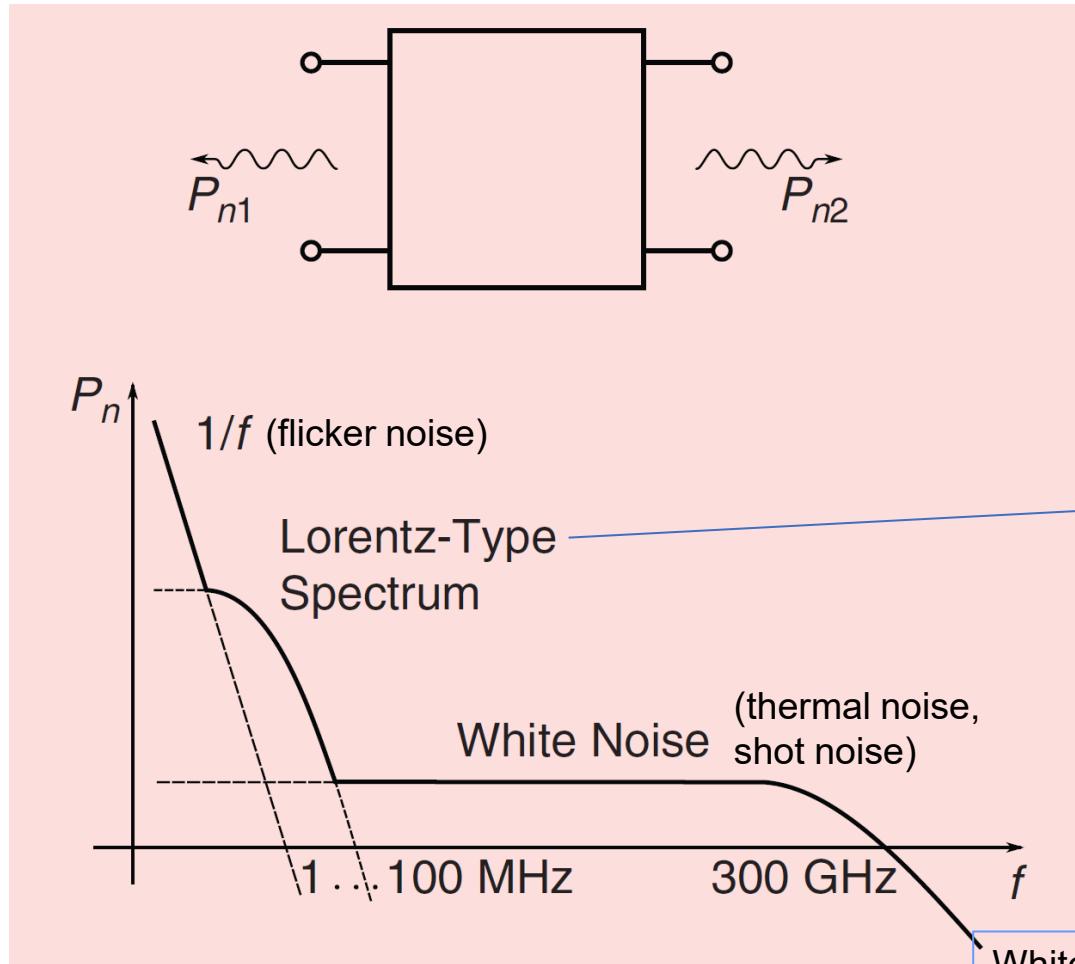
Flicker noise [or 1/f (one-over-f) noise]

- It is due to diffusion, traps in a semiconductor, and surface traps. When several such carriers are trapped, they are not available for conduction and as a result, the resistance of the semiconductor is modulated.
- It dominates at very low frequencies (below a few kHz or MHz)
- A **complete physical understanding** of flicker noise is **not available**, and flicker noise is a major concern with oscillators.

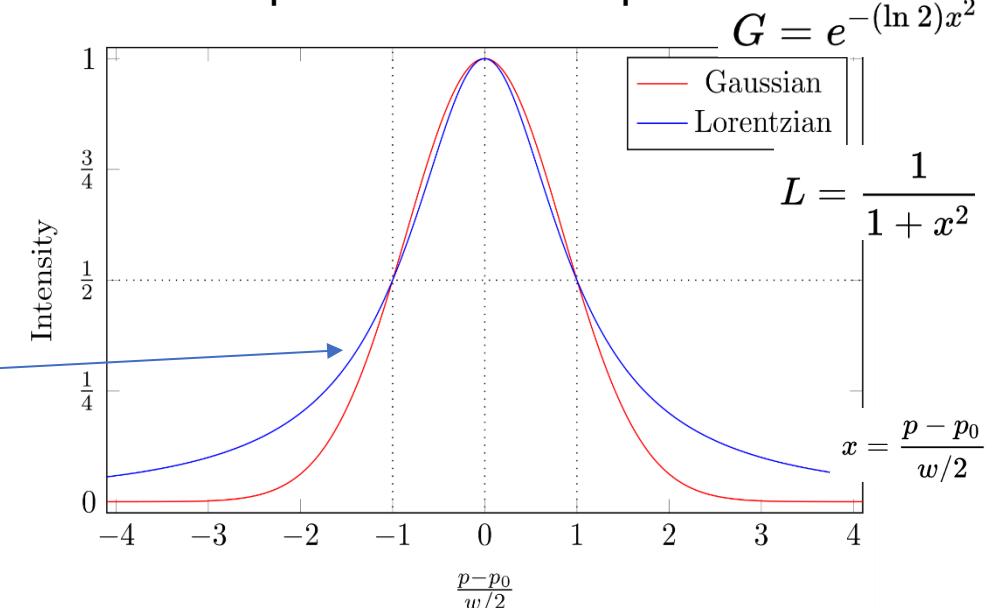
- Flicker noise is temperature & frequency independent
- Thermal noise is proportional to temperature
- Flicker noise has the spectral density decreasing with increasing frequency
- **White Noise:** White noise is the noise that has constant magnitude of power over frequency. Examples of White noise are Thermal noise, and Shot noise.

The noise spectrum

The noise spectrum of three major types of noise: thermal, shot, and flicker (noise)



Spectrum line shape



White noise, if it is of a thermal nature, for example, shows a low-pass characteristic, as predicted by quantum physics, which needs to be accounted for beyond 300 GHz at room temperature.

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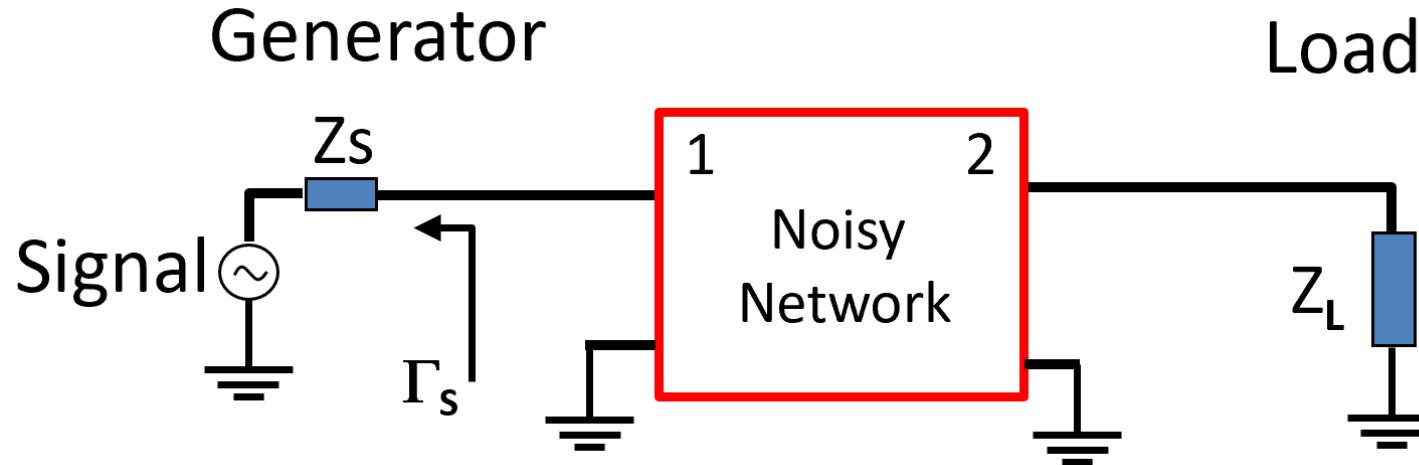
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Noise Figure of Two-Port Amplifier



SNR = signal to noise ratio

$$SNR \doteq \frac{P_s}{P_n} \quad (\text{input signal power over output noise power}) \quad \text{The larger, the better}$$

$$F = \frac{SNR_{input}}{SNR_{output}} \quad \text{The best is 1 (noise free)}$$

- Noise will originate from the generator and noisy network (amplifier).
- Both noise and signals present at the input will be amplified (or attenuated)
- Since, the 2-port network contributes noise, the SNR will deteriorate

Noise Figure is a measure of the deterioration of SNR as the signal passes through the network:

$$F = \frac{SNR_{input}}{SNR_{output}} = \frac{S_i/N_i}{S_o/N_o} = \frac{S_i * N_o}{S_o * N_i}$$

Note that some researchers use the following nomenclature:

- Noise factor -- ratio
- Noise figure --- dB

Here in EE5303, we just use "Noise Figure", i.e., noise figure can be a ratio or in dB.

We will assume that the system bandwidth B is narrow.

Noise Figure of an Amplifier

- The noise figure of a two port network can be shown to be given by

$$F = F_{\min} + \frac{r_n}{g_s} |y_s - y_{opt}|^2 = F_{\min} + \frac{r_n}{g_s} [(g_s - g_{opt})^2 + (b_s - b_{opt})^2]$$

where

F_{\min} = minimum noise figure, which is a function of operating frequency and current.

$r_n = R_n / Z_0$ is the normalized noise resistance

$y_{opt} = g_{opt} + jb_{opt}$ is the normalized optimum source admittance which results in the minimum noise figure.

$y_s = g_s + jb_s$ is the normalized source admittance

- Substituting

$$y_s = \frac{1 - \Gamma_s}{1 + \Gamma_s}$$

$$y_{opt} = \frac{1 - \Gamma_{opt}}{1 + \Gamma_{opt}}$$

the noise figure equation becomes

For a fixed noise figure F_i , the noise figure equation defines a circle in the Γ_s plane. All the Γ_s value/points on this circle give constant noise figure F_i .

Designers would have to compromise gain for better noise performance or vice versa

Noise Figure Equation

$$F = F_{\min} + \frac{4r_n |\Gamma_s - \Gamma_{opt}|^2}{(1 - |\Gamma_s|^2) |1 + \Gamma_{opt}|^2}$$

- To determine the noise figure circle for a given noise figure F_i , we define an intermediate NF parameter

$$N_i = \frac{F_i - F_{\min}}{4r_n} |1 + \Gamma_{opt}|^2$$

where

F_i = noise figure required (ratio value) **(NOT dB value)**

F_{\min} = device minimum noise figure (ratio value)

$r_n = R_n / Z_0$ (from data sheet or measured)

Γ_{opt} = the optimum source reflection coefficient which results in the minimum noise figure

Noise Figure Circle

- The center of the noise figure circle is

$$C_{Fi} = \frac{\Gamma_{opt}}{1 + N_i}$$

- The radius of the circle is

$$r_{Fi} = \frac{1}{1 + N_i} \sqrt{N_i^2 + N_i (1 - |\Gamma_{opt}|^2)}$$

LNA Example

Design a low noise amplifier using GaAs FET and it has following S-parameters at

4 GHz ($Z_0=50$ Ohms): $S_{11} = 0.6 \angle -60^\circ$ $S_{21} = 1.9 \angle 81^\circ$ $S_{12} = 0$ $S_{22} = 0.5 \angle -60^\circ$

$F_{\min} = 1.6$ dB, $\Gamma_{\text{opt}} = 0.62 \angle 100^\circ$, $R_N = 20$. Design an amplifier with a noise figure of 2.0 dB and with the maximum gain that is compatible with the noise figure.

$\text{dB from/to ratio/amplitude}$
 $x \text{ dB} = 10 \log_{10}(y)$
 $y = 10^{(x/10)}$

Solution:

$$N = \frac{F - F_{\min}}{4R_N/Z_0} |1 + \Gamma_{\text{opt}}|^2 = \frac{1.58 - 1.445}{4(20/50)} |1 + 0.62 \angle 100^\circ|^2 \\ = 0.0986,$$

$$C_F = \frac{\Gamma_{\text{opt}}}{N + 1} = 0.56 \angle 100^\circ \quad \longrightarrow \text{Center of noise figure circle (NFC)}$$

$$R_F = \frac{\sqrt{N(N + 1 - |\Gamma_{\text{opt}}|^2)}}{N + 1} = 0.24 \quad \longrightarrow \text{Radius of NFC}$$

Gain and noise figure are related to Power, so the coefficients for conversion between dB and ratio are 10

(*** if the ratio is a voltage ratio, then the coefficients are 20 --- we do not have such cases in Part 2 of EE5303)

Calculate the radius and center of Input constant gain circle

$G_S(\text{dB})$	g_S	C_S	R_S
1.0	0.805	$0.52 \angle 60^\circ$	0.300
1.5	0.904	$0.56 \angle 60^\circ$	0.205
1.7	0.946	$0.58 \angle 60^\circ$	0.150

Noise figure circle & Gain circles on Smith Chart

2 dB Noise Circle

$$C_F = 0.56 \angle 100^\circ$$

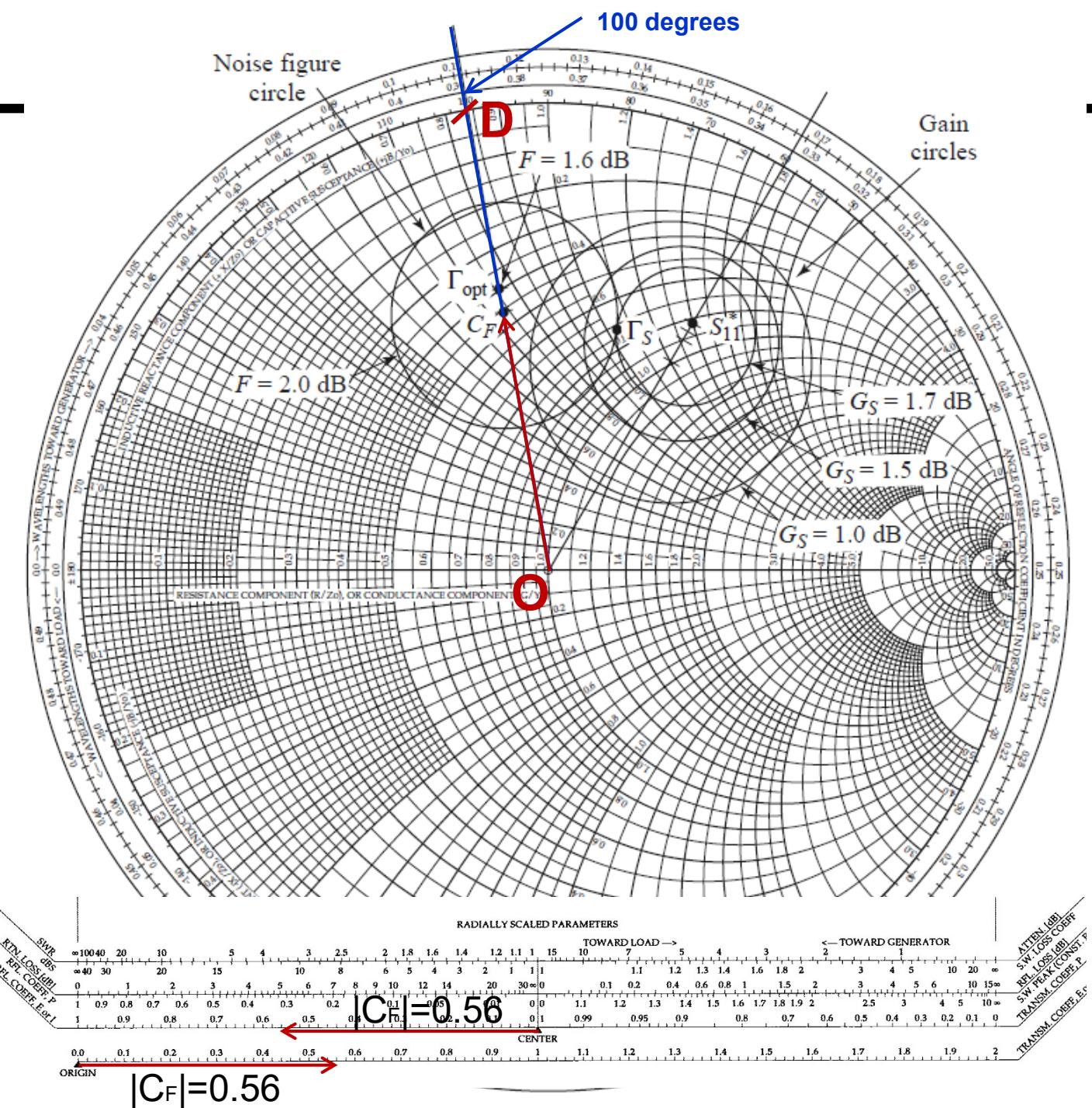
$$R_F = 0.24$$

This noise figure circle is plotted in Figure ($F_{\min} = 1.6$ dB) occurs for $\Gamma_S = \Gamma_{\text{opt}} = 0.62 \angle 100^\circ$.

We can see that the $G_s=1.7$ dB gain circle just intersects the $F=2$ dB noise figure circle, and any higher gain will result in a worse noise figure.

From the Smith Chart, the optimum solution is then

$$\Gamma_S = 0.53 \angle 75^\circ, \text{ yielding } G_S = 1.7 \text{ dB and } F = 2.0 \text{ dB}$$



LNA Example

For the output section we choose $\Gamma_L = S_{22}^* = 0.5 \angle 60^\circ$ for a maximum G_L of 1.33 dB.

$$G_L = \frac{1}{1 - |S_{22}|^2} = 1.33 = 1.25 \text{ dB.}$$

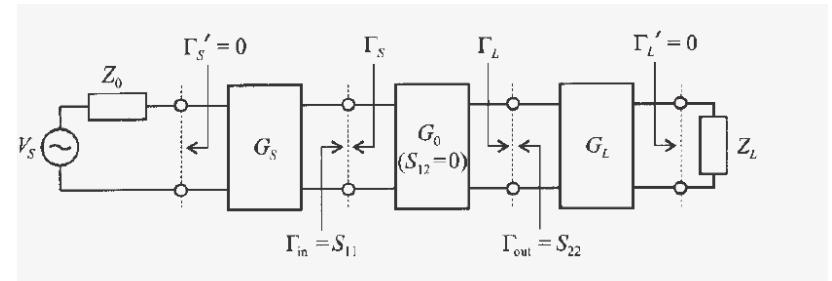
The transistor gain is

$$G_0 = |S_{21}|^2 = 3.61 = 5.58 \text{ dB,}$$

Note that $S_{12}=0$

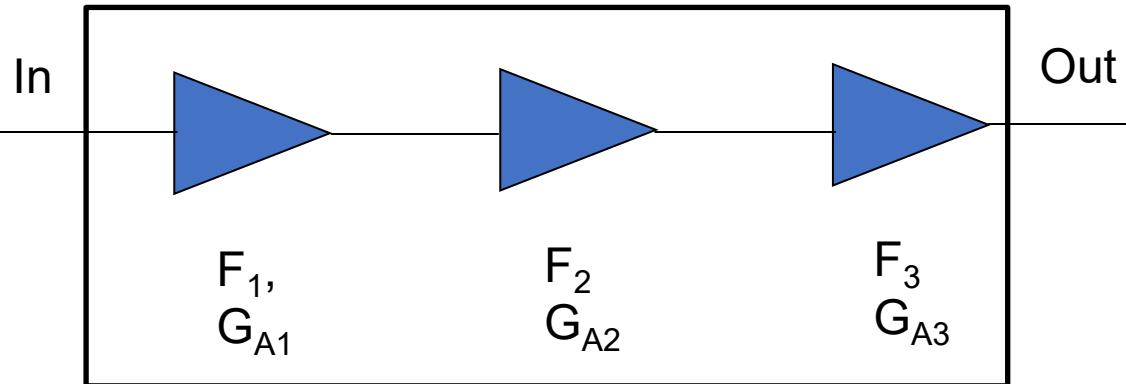
Overall unilateral transducer gain will be

$$G_{TU} = G_s + G_0 + G_L = 8.53 \text{ dB}$$



Noise Figure of a Multistage Amplifier

Block diagram of a multistage amplifier



The overall gain is simply the product of all the individual gains: $G_A = G_{A1}G_{A2}G_{A3}$. (ratio value)

or $G_A = G_{A1} + G_{A2} + G_{A3}$ (dB value)

The overall noise figures is given by:

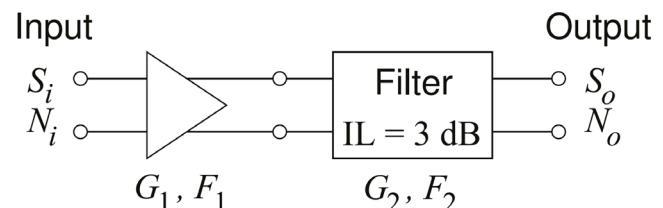
$$\text{(ratio value)} \quad F = F_1 + \frac{F_2 - 1}{G_{A1}} + \frac{F_3 - 1}{G_{A1}G_{A2}}$$

Generalize to m-stage amplifier $F = F_1 + \sum_{n=2}^m \frac{F_n - 1}{\prod_{i=2}^n G_{A(i-1)}}$ Friis's formula

Example

Consider the cascade of an amplifier and a filter (see figure below)

- What is the gain G_2 of the filter in decibels? Note that IL is insertion loss.
- What is the noise figure of the filter?
- The amplifier has a gain $G_1 = 20$ dB and a noise figure of 2 dB. What is the overall gain (in dB) of the cascade system?
- What is the noise figure (in ratio) of the cascade system?
- What is the noise figure (in dB) of the cascade system?



Solution:

- $G_2 = 1/\text{IL}$, thus $G_2 = -3$ dB.
- For a passive element, $\text{NF}_2 = \text{IL} = 3$ dB.
- $G_1 = 20$ dB and $G_2 = -3$ dB, so $G_{\text{TOTAL}} = G_1|_{\text{dB}} + G_2|_{\text{dB}} = 17$ dB.
- $F_1 = 10^{\text{NF}_1/10} = 10^{2/10} = 1.585$, $F_2 = 10^{\text{NF}_2/10} = 10^{-3/10} = 1.995$, $G_1 = 10^{20/10} = 100$, and $G_2 = 10^{-3/10} = 0.5$. Using Friis's formula

$$F_{\text{TOTAL}} = F_1 + \frac{F_2 - 1}{G_1} = 1.585 + \frac{1.995 - 1}{100} = 1.594. \quad (4.38)$$

- $\text{NF}_{\text{TOTAL}} = 10 \log_{10}(F_{\text{TOTAL}}) = 10 \log_{10}(1.594) = 2.03$ dB.

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Impedance Matching

Reasons for Impedance Matching

1. Maximum power transfer

- To minimize loss due to impedance mismatch (ideally, zero reflection)

2. Impedance transformer (transforming networks)

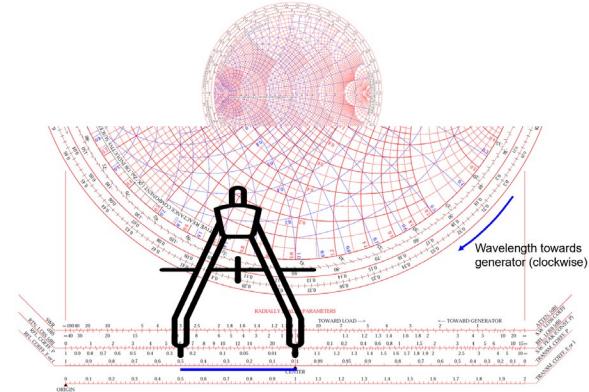
- For receiver LNA, we would need a source resistance to obtain the best noise figure (this source resistance is usually different from that to achieve maximum power transfer).
- To find the best compromise between noise figure (NF) and gain

3. Aid in filtering

- Due to their frequency response, the matching networks can be of help in filtering off unwanted signals



Phillip Hagar Smith
(1905 - 1987)



By the end of the thirties, secret work was afoot in both the USA and the United Kingdom. At Bell Telephone's Radio Research Lab in New Jersey, [Philip Hagar Smith](#), born in Lexington Massachusetts, developed a circular chart form in 1939 that shows the entire universe of complex impedances in one convenient circle.

Wait, that's not entirely correct, as pointed out thanks to Jim... *the Smith Chart only shows one half of the entire universe of complex impedances. The negative impedances (with a negative real part; where gain lives) still reach out to infinity in all directions around the circle.*

The [Smith chart](#) remains in wide use today, and will be around long after we're all gone. Les Besser recalls that Philip Smith submitted [an article on his development to the IRE](#), which was rejected.

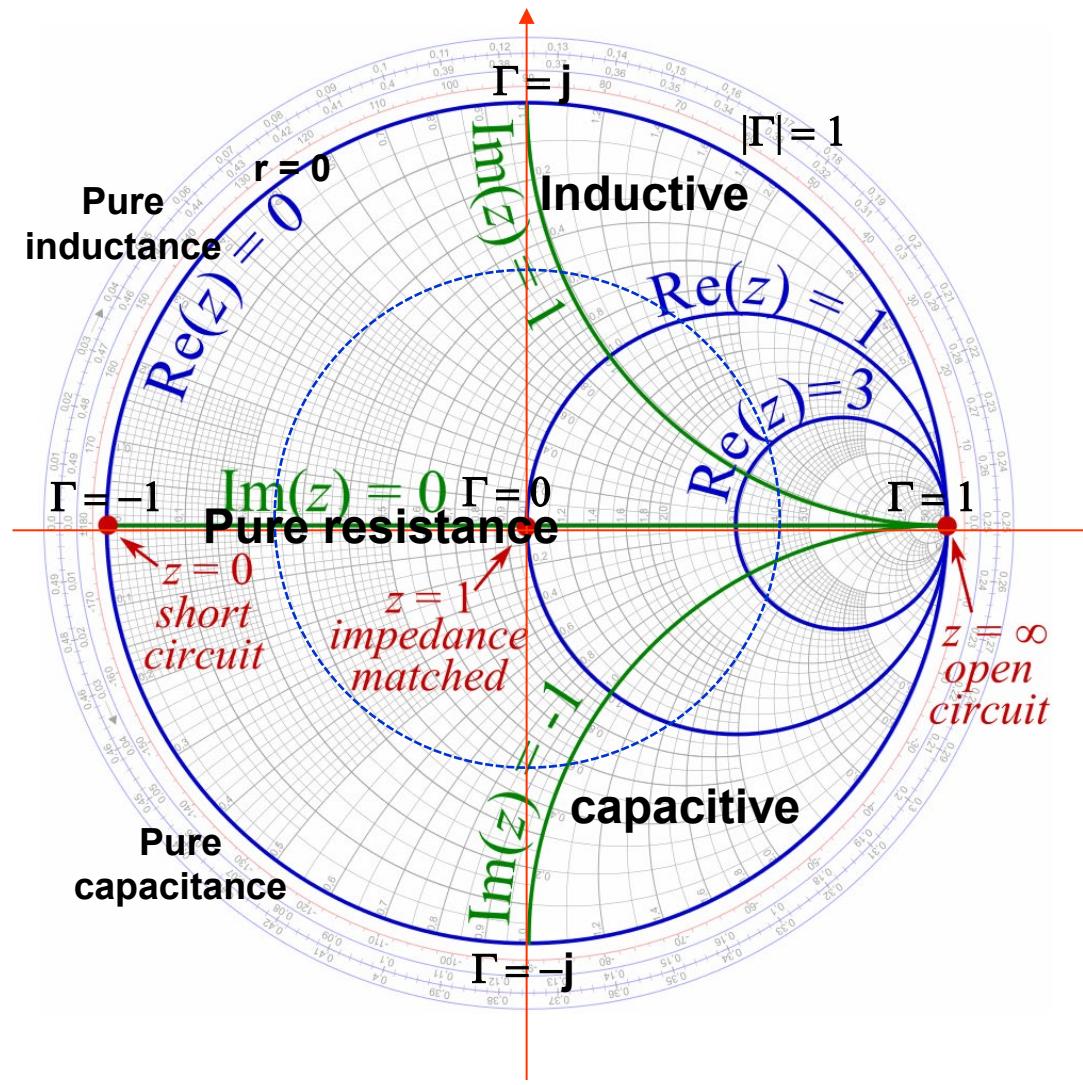
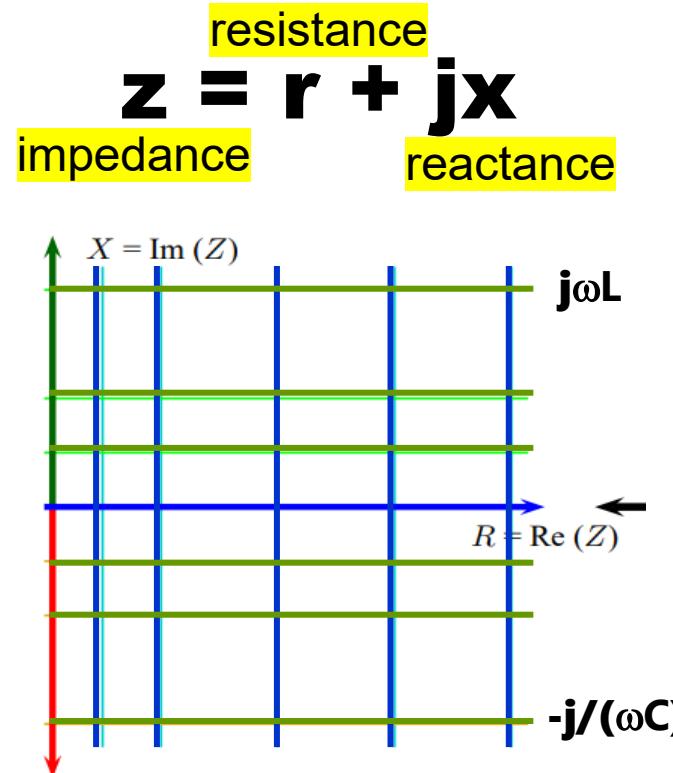
The picture of handsome Phil is courtesy of his wife Anita, and just might be the only picture of him you could find on the entire worldwide web! By the way, Anita's company, Analog Instruments of New Providence, NJ, still supplies the ubiquitous chart in paper form to the microwave industry.

Paul shared this story about following in the footsteps of Phil Smith. As a starting engineer he took over the desk and chair of Phillip Smith when Smith retired from Bell Labs in 1970. To welcome me, my colleagues decorated the desktop with a Plexiglas-covered, poster-sized Smith Chart which served as a humbling reminder that I was occupying a pretty special place.

Smith Chart

Normalized impedance $Z = Z / Z_0$

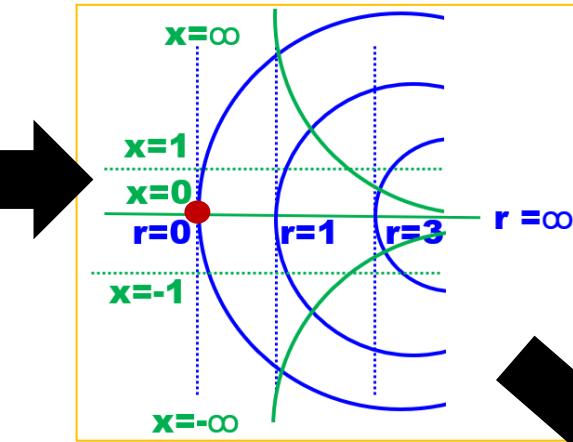
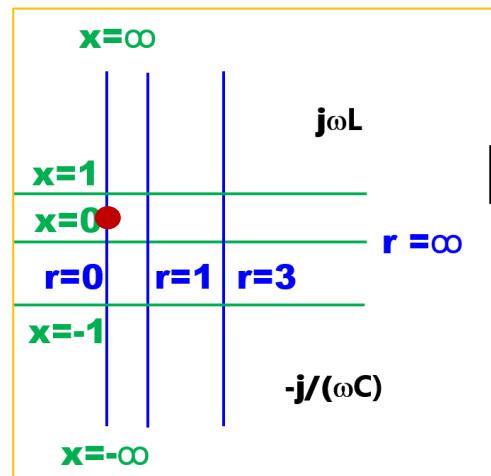
Z Chart = constant r circles + constant X circles + constant $|\Gamma|$ circles



Smith Chart

L & C in series connection

$$z = r + jx = r + j [\omega L - 1/(\omega C)]$$



Bilinear Transform (conformal mapping): Z plane to Γ plane

$$\Gamma = \frac{z_L - 1}{z_L + 1}$$

$$z_L = Z_L/Z_0$$

$$\Gamma = \Gamma_r + j\Gamma_i, \text{ and } z_L = r_L + jx_L$$

$$r_L + jx_L = \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i}$$

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2$$

Center $C_r = \{r/(1+r), 0\}$

Radius $R_r = 1/(1+r)$

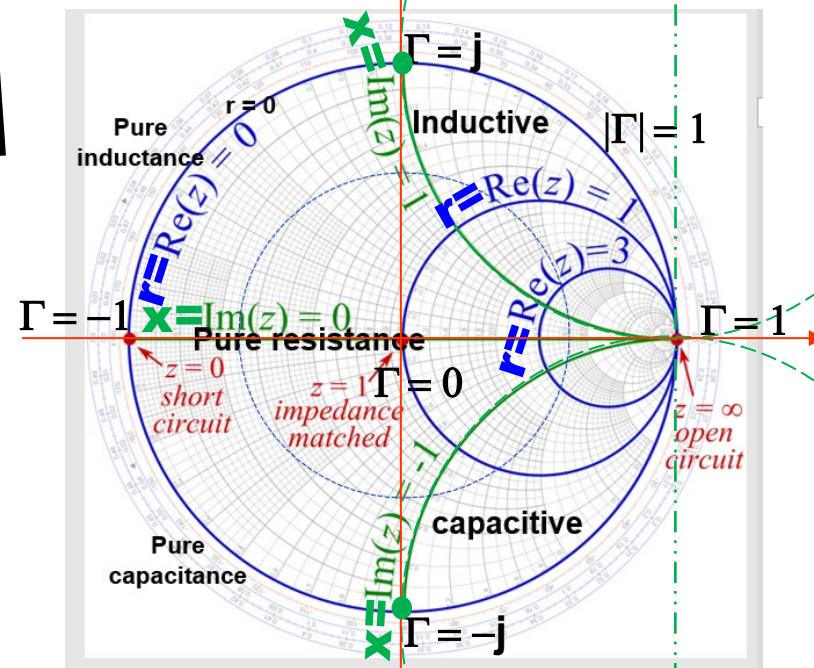
$r=0 \rightarrow C_r = (0,0)$, $R_r=1$ (outermost circle)

$r=\infty \rightarrow C_r = (1,0)$, $R_r=0$ (degenerated to a point)

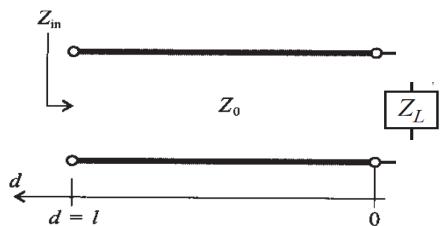
$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

Center $C_x = (1, 1/x_L)$; Radius $R_x = 1/x_L$

**Z Chart = constant r circles
+ constant x circles
+ constant $|\Gamma|$ circles**



Impedance Matching by Smith Chart



Four sets of principles

1

**How to
get y
from z
on
Smith
Chart?**

$$z = \frac{1 + \Gamma}{1 - \Gamma}$$

Impedance
(normalized)

Admittance

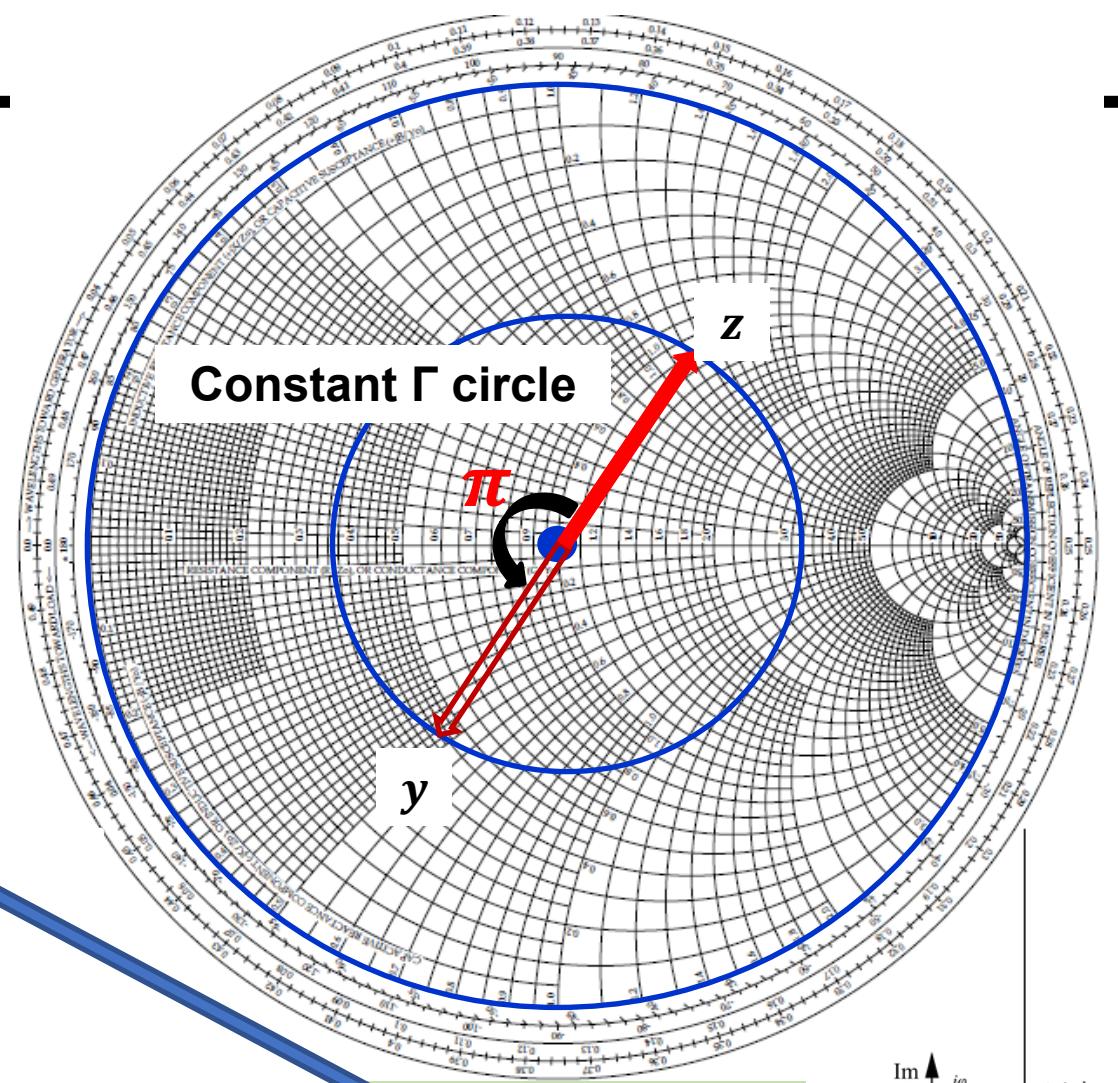
$$y = \frac{1}{z} = \frac{1 - \Gamma}{1 + \Gamma}$$

$$= \frac{1 + (\Gamma e^{j\pi})}{1 - (\Gamma e^{j\pi})}$$

$$\Gamma' = \Gamma e^{j\pi}$$

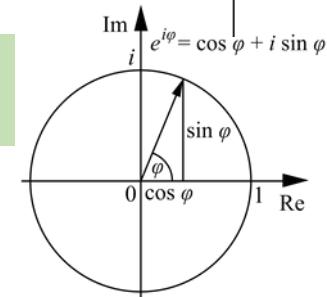
$$y = z' = \frac{1 + \Gamma'}{1 - \Gamma'}$$

$\Gamma e^{j\pi} \rightarrow$ rotate Γ by 180 degrees
 $y (= 1/z)$ can be obtained by rotating z with 180 degrees



$e^{j\pi} = \cos\pi + j\sin\pi = -1$
Euler's Identity/Equation

/œ̄lər/ - eu-lər
Euler
(noun) Swiss mathematician (1707-1783)



2

z chart vs y chart

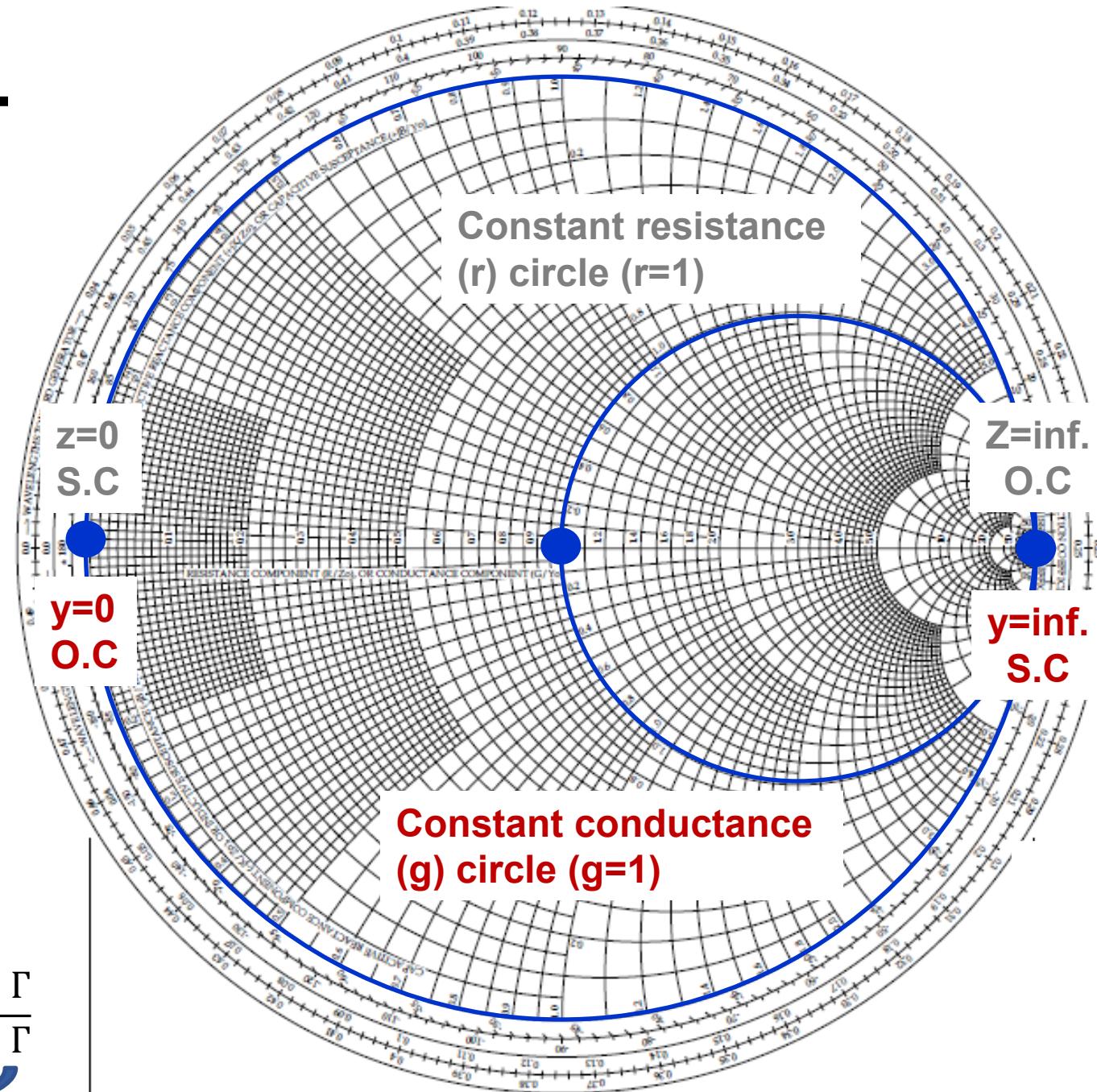
$$z = \frac{1 + \Gamma}{1 - \Gamma}$$

**Impedance
z chart**

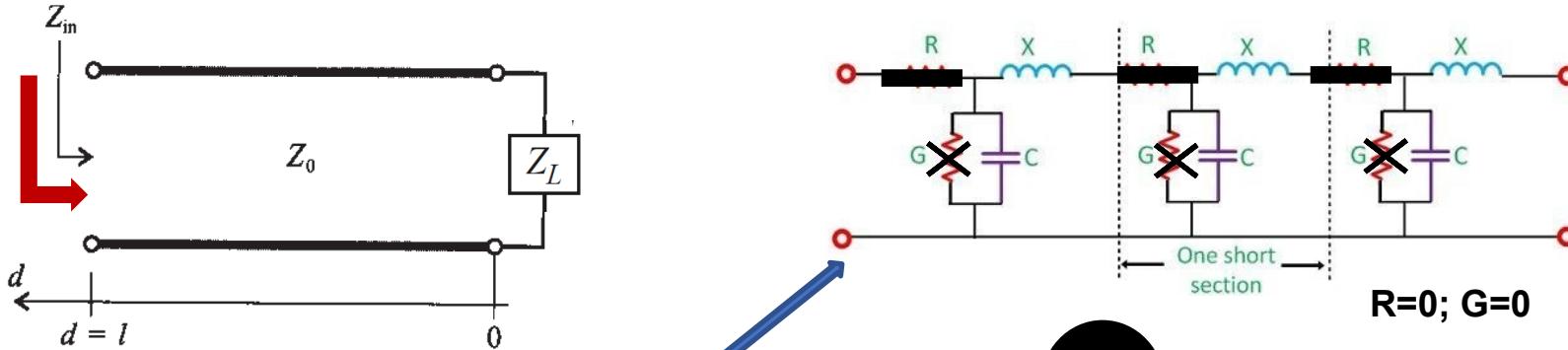
**Admittance
y chart**

$$y = \frac{1}{z} = \frac{1 - \Gamma}{1 + \Gamma}$$

$$y = \frac{1 + (\Gamma e^{j\pi})}{1 - (\Gamma e^{j\pi})} \quad z = \frac{1 + \Gamma}{1 - \Gamma}$$



Transmission line impedance equation



In either case, the knowledge of the circuit parameters ultimately leads to the characteristic impedance and propagation constant of a generic transmission line system:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}, \quad \gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

From this representation, the input impedance of a terminated transmission line is developed. The result is perhaps one of the most important RF equations for lossless transmission lines:

$$Z_{in}(d) = \frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)}$$

The application of this equation for the special cases of open, short, and matched load impedances are investigated in terms of their spatial and frequency domain behaviors. Furthermore, the lambda-quarter or quarter-wave transformer is introduced as a way of matching a load impedance to a desired input impedance.

As an alternative to the input impedance equation, it is often useful to represent the line impedance in terms of the reflection coefficients at load and source end:

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}, \quad \Gamma_s = \frac{Z_G - Z_0}{Z_G + Z_0}$$

It is found that the reflection coefficient is transformed by a lossless transmission line of length d according to

$$\Gamma(d) = \Gamma_0 e^{-j2\beta d}$$

Chapter 2 (TL), Summary, Ludwig book

3

Lossless

Transmission line impedance equation

$$Z_{in}(d) = Z_0 \frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)}$$

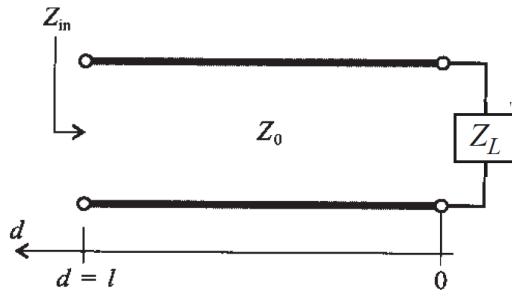
$$Y_{in}(d) = Y_0 \frac{Y_L + jY_0 \tan(\beta d)}{Y_0 + jY_L \tan(\beta d)}$$

$$\begin{aligned} \Gamma(d) &= \Gamma_L e^{-j2\beta d} \\ &= |\Gamma_L| e^{j(\theta_L - 2\beta d)} \end{aligned}$$

$$\beta = \frac{2\pi}{\lambda}$$

Addition of
transmission lines
(in series) does
not change $|\Gamma_L|$,
except for phase

Transmission line impedance equation (Lossless)



$$Z_{in}(d) = Z_0 \frac{Z_L + jZ_0 \tan(\beta d)}{Z_0 + jZ_L \tan(\beta d)}$$

$$\frac{Z_{in}(d)}{Z_0} = \frac{\frac{Z_L}{Z_0} + j \tan(\beta d)}{1 + \frac{Z_L}{Z_0} j \tan(\beta d)}$$

$$z_{in}(d) = \frac{z_L + j \tan(\beta d)}{1 + z_L j \tan(\beta d)}$$

Normalized by characterized impedance Z_0

short-circuited stub



For a **short-circuited stub**, $Z_L = 0$

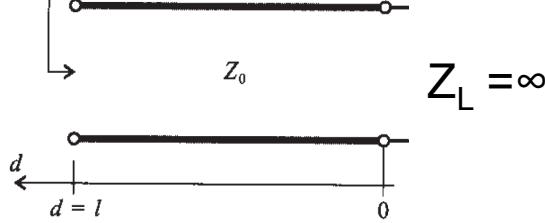
$$Z_{in}(d) = jZ_0 \tan(\beta d)$$

$$z_{in}(d) = j \tan(\beta d)$$

4

Z_{in}
Pure
imaginary
for these two
cases

open stub



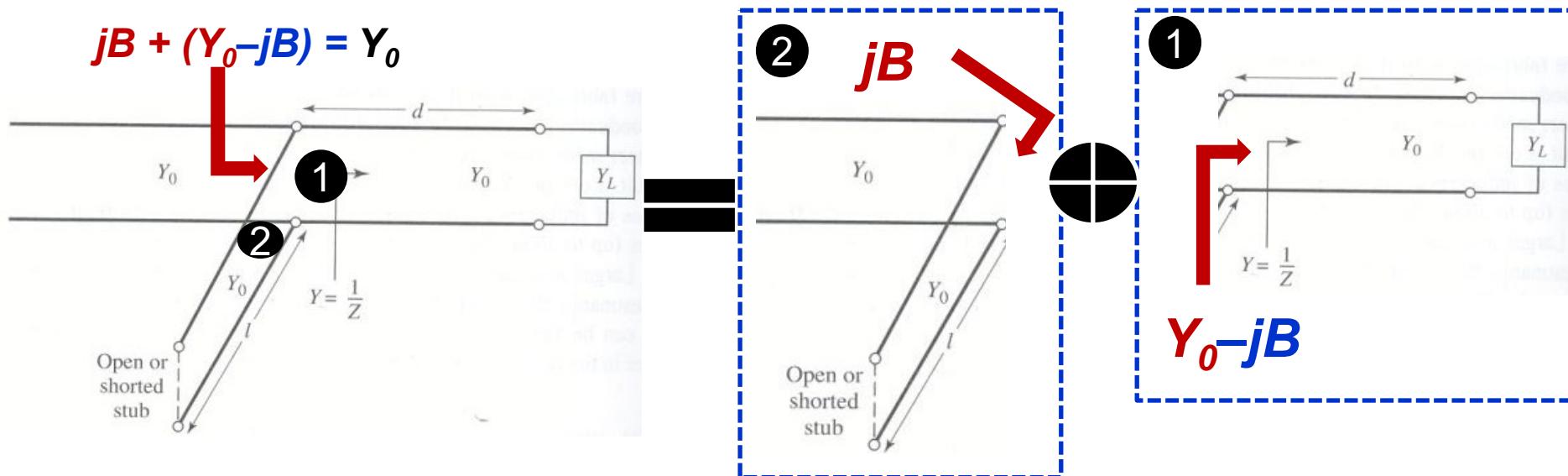
For a **open stub**, $Z_L = \infty$

$$Z_{in}(d) = -jZ_0 / \tan(\beta d)$$

$$z_{in}(d) = -j / \tan(\beta d)$$

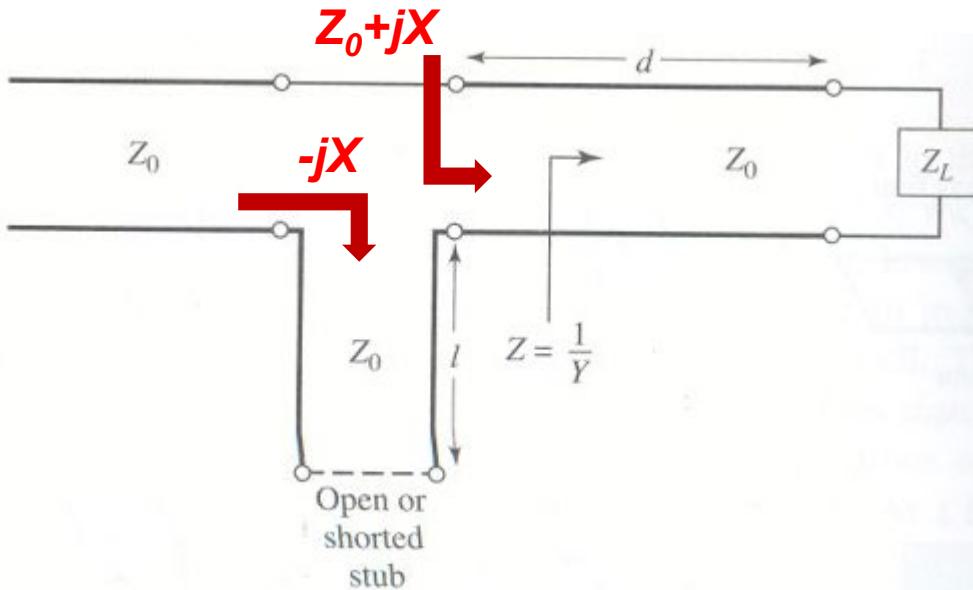
Single Stub Matching – shunt stub

- In single stub tuning, the two adjustable parameters are the distance, d , from the load to the stub position, and the value of susceptance or reactance provided by the shunt or series stub.
- For shunt stub case, the idea is to select d so that admittance, Y looking into the line at distance d from the load is of the form $Y_0 + jB$, then susceptance is chosen as $-jB$, resulting in a matched condition

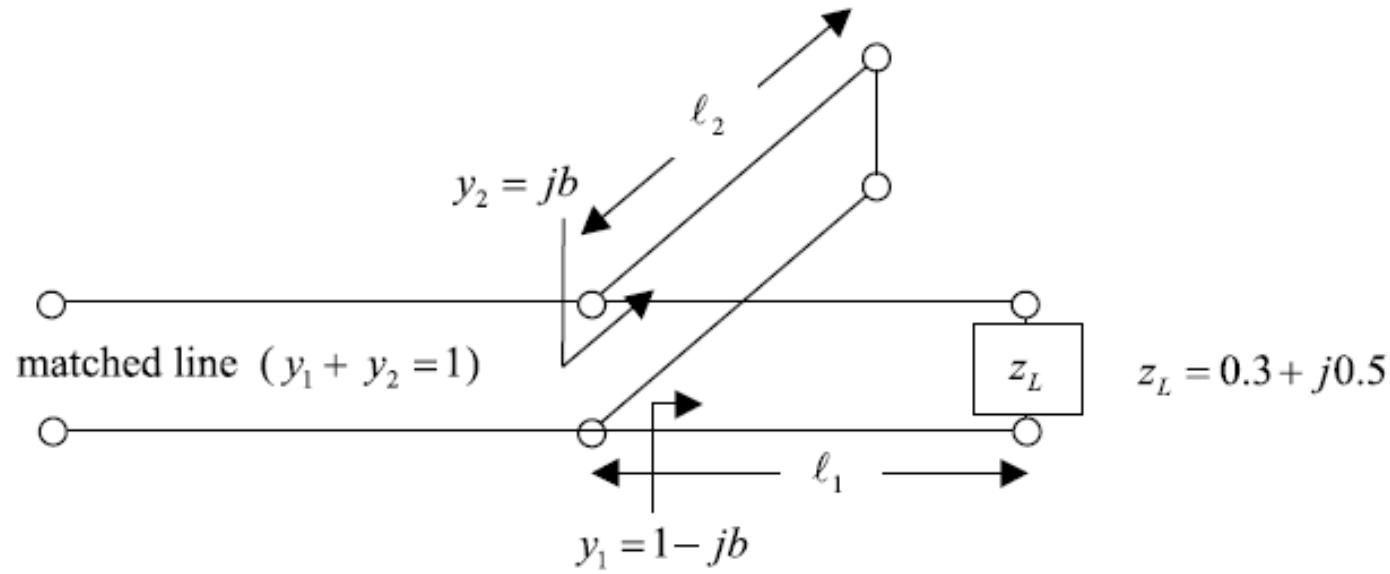


Single Stub Matching – series stub

- For the series stub case, the distance d is selected so that the impedance Z seen looking into the line at a distance d from the load is of the form $Z_0 + jX$. Then the stub reactance is chosen is $-jX$, resulting in a matched condition.

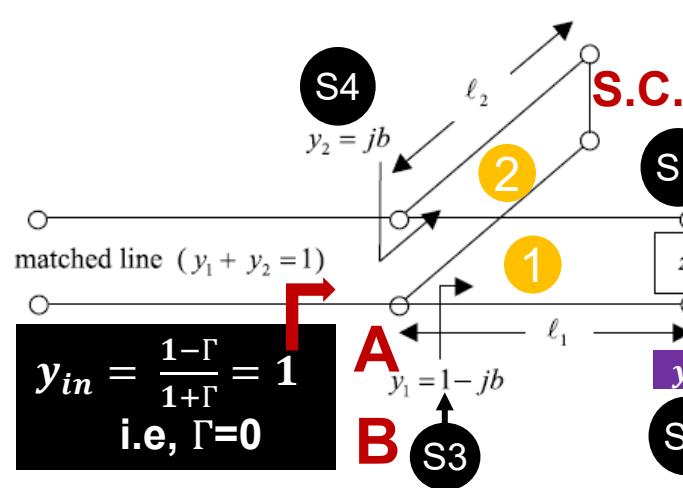


Example (Single stub matching)



Use a single parallel stub tuner to match the above line to its normalized load. Use a shorted stub and find its distance from the load, ℓ_1 , and its length ℓ_2 .

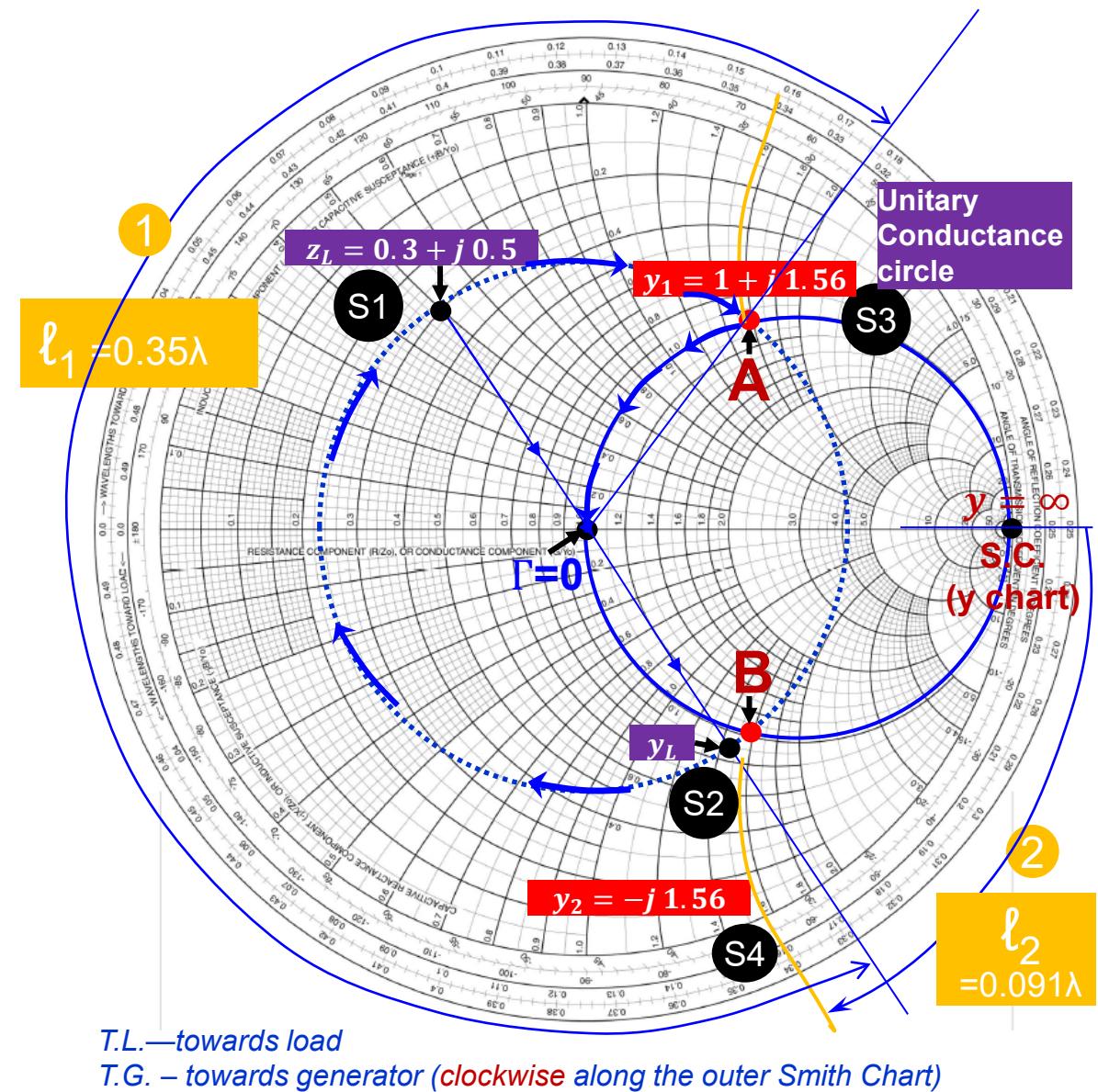
Example 1



- S1 Step-1: Locate z_L on Smith chart,
- S2 Step-2: Draw the $|\Gamma|$ circle passing through z_L point to get y_L
- S3 Step-3: The two intersection points of the $|\Gamma|$ circle and the unit conductance circle ($y=1$) are two solutions with $y_1=1+jb$ (or $1-jb$)
- S4 Step-4: locate $y_2=-jb$ (or $+jb$)
- S5 Read the length of ℓ_1 (matching from y_L towards generator to y_1) & ℓ_2

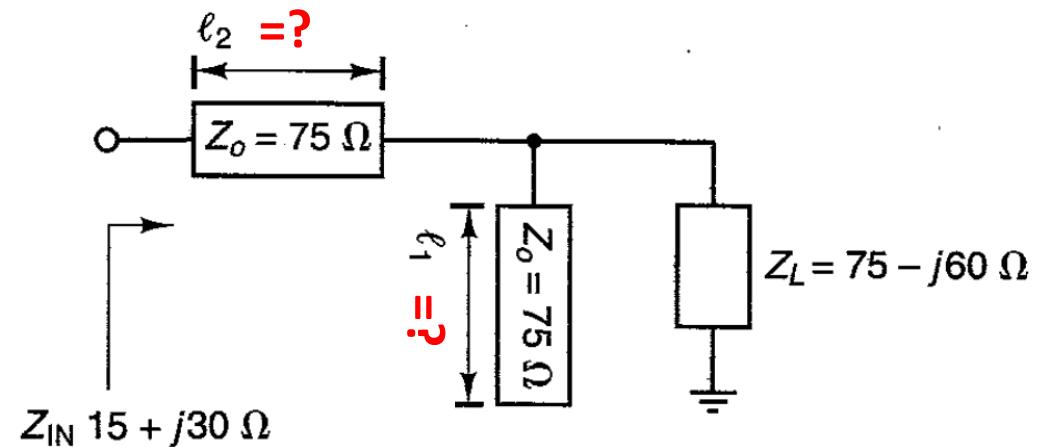
Note that

- 1) the first solution shown in the chart gives shorter length of ℓ_1 & ℓ_2
- 2) This question is about using shorted stub, so the reading of ℓ_2 is from $y=\infty$ towards generator



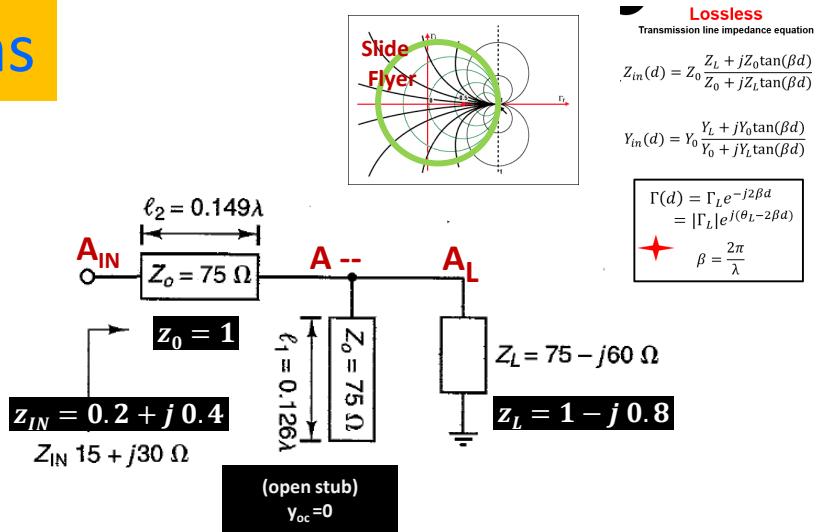
Example 2

Design a microstrip matching network to transform the load $Z_L = 75 - j60 \Omega$ to an input impedance of value $Z_{IN} = 15 + j30 \Omega$. The characteristic impedance is $Z_o = 75 \Omega$



Example 2

Ans



S0: normalization (by Z_0)

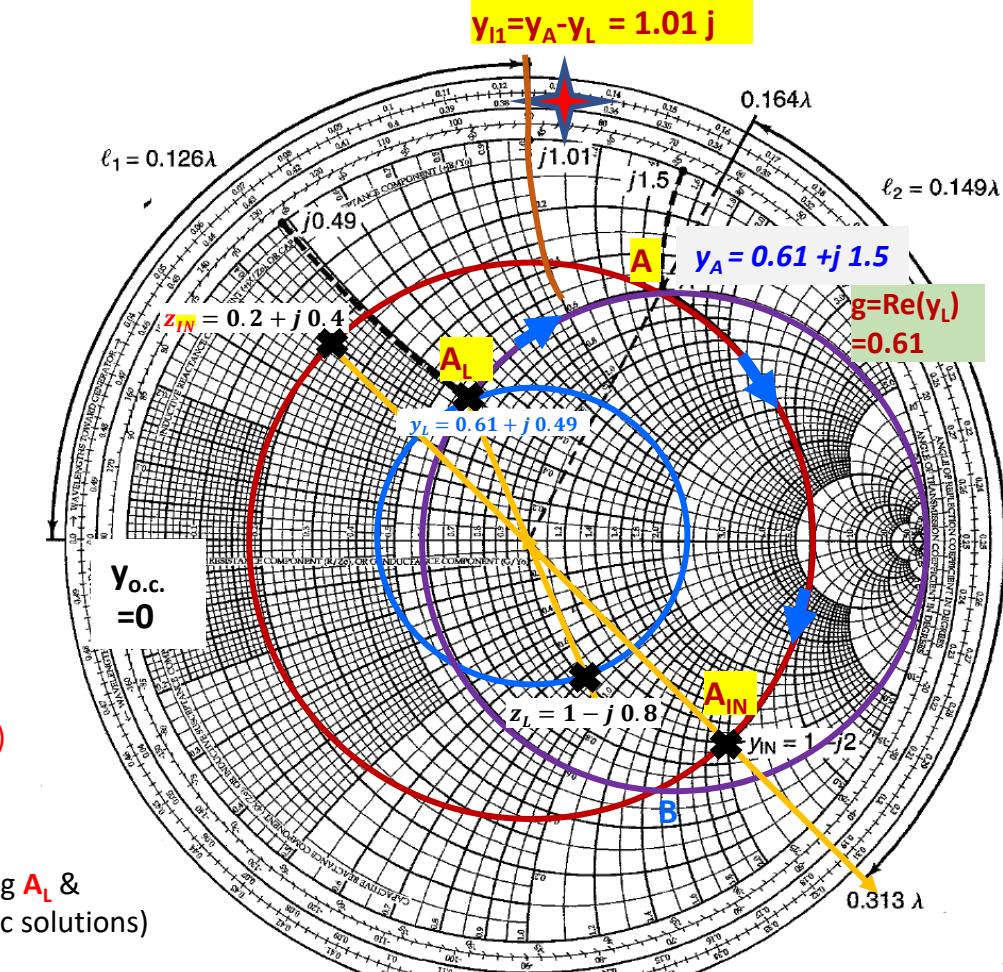
S1: {ride on the $|\Gamma|$ circle}

(i) identify z_L on the Smith chart, draw the constant $|\Gamma|$ circle, passing through z_L , then read y_L at the other end of the diameter

$$y_L = 0.61 + j0.49 \text{ (at } A_L)$$

$$y_{IN} = 1.0 - j2.0 \text{ (at } A_{IN})$$

(ii) repeat (i) for z_{IN} and we get



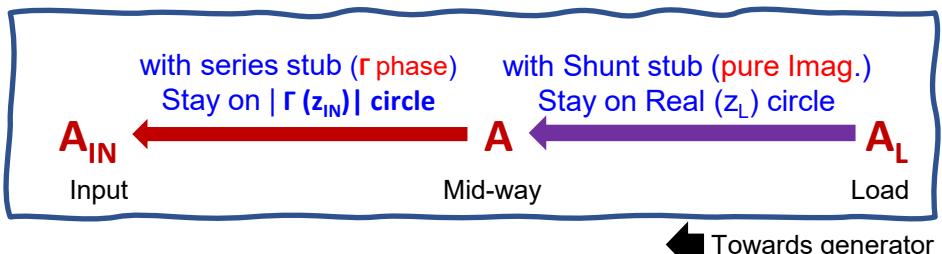
S2: {build a bridge to travel from the load to A}

(i) draw the constant conductance $g = \text{Re}(y_L) = 0.61$ circle crossing A_L & intersecting with $|\Gamma(y_{IN})|$ circle at two points A & B (two basic solutions)

Read $A \rightarrow y_A = 0.61 + j1.5$

(ii) from $A_L \rightarrow y_L = 0.61 + j0.49$ to $A \rightarrow$ the shunt open stub L_1 , whose normalized admittance $y_{11} = y_A - y_L = j(1.5 - 0.49) = j1.01$

S3: Read, towards the generator, the length of the Transmission lines l_2 (from A clockwise to A_{IN}) and l_1 .



Example 3

Q4

[Gonzalez, p.288]

3.18 A microwave amplifier is to be designed for $G_{TU,\max}$ using a transistor with

$$S_{11} = 0.5 \angle 140^\circ \quad S_{12} = 0$$

$$S_{21} = 5 \angle 45^\circ \quad S_{22} = 0.6 \angle -95^\circ$$

The S parameters were measured in a 50Ω system at $f = 900$ MHz, $V_{CE} = 15$ V, and $I_C = 15$ mA.

- (a) Determine $G_{TU,\max}$.
- (b) Design two different microstrip matching networks.
- (c) Draw the constant gain circle for $G_L = 1$ dB.
- (d) If the S parameters at 1 GHz are

$$S_{11} = 0.48 \angle 137^\circ \quad S_{12} = 0$$

$$S_{21} = 4.6 \angle 48^\circ \quad S_{22} = 0.57 \angle -99^\circ$$

calculate the gain G_T at 1 GHz for the designs in part (b).

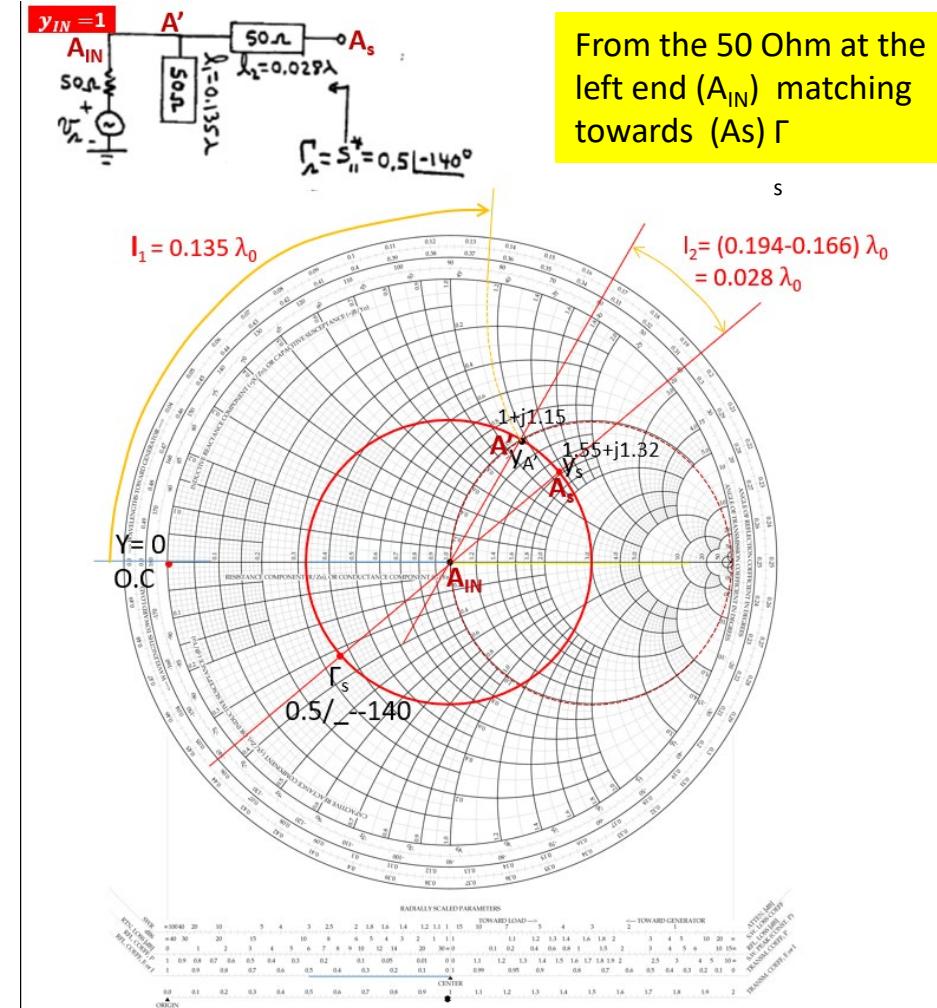
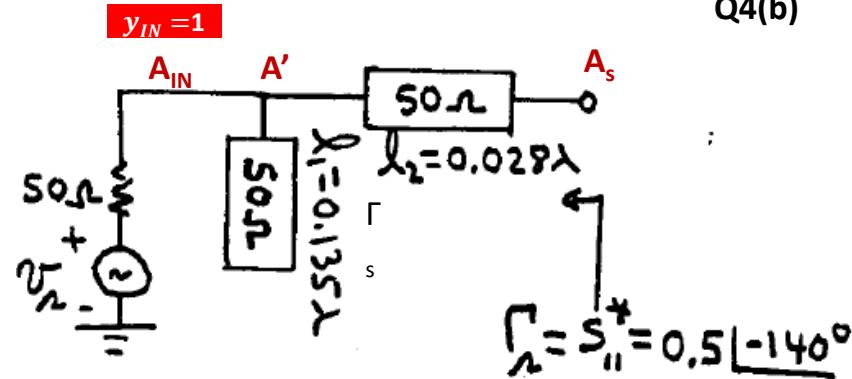
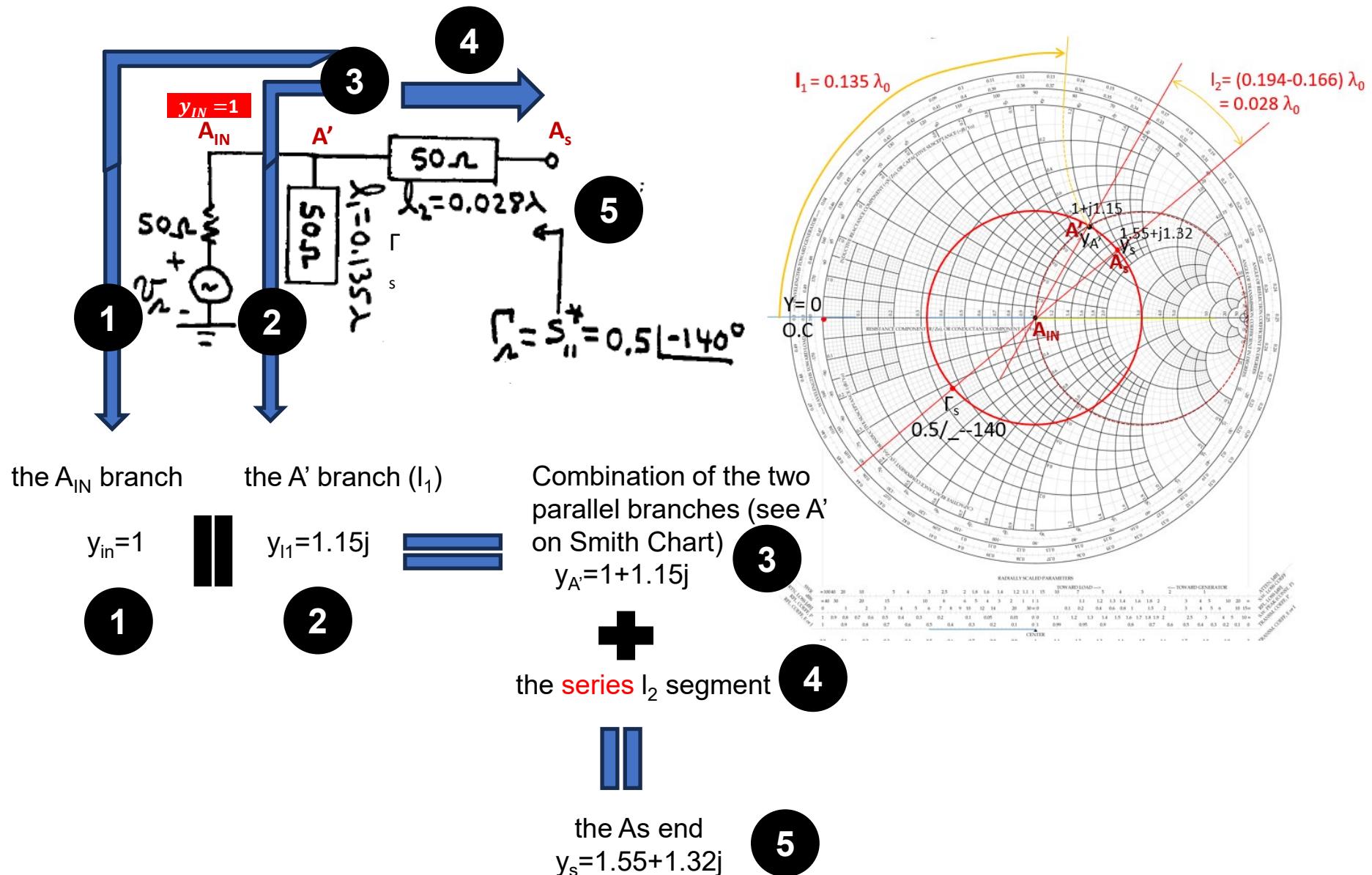


Illustration of the impedance matching process



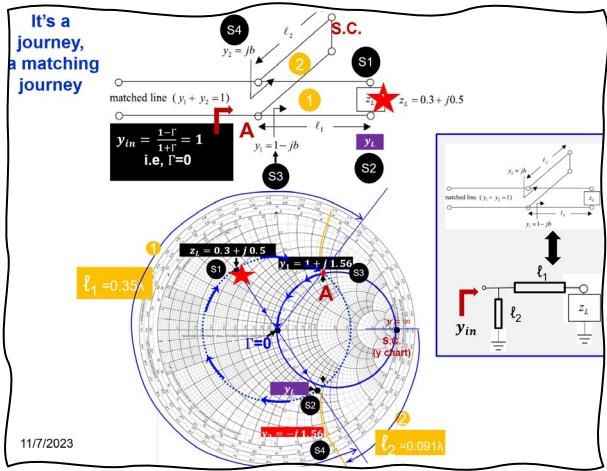
Compare the 3 Cases of Impedance Matching

“flip”

$$y_1 = 1 + jb \text{ (or } 1 - jb\text{)}$$

$$y_2 = -jb \text{ (or } +jb\text{)}$$

So that $y_1 + y_2 = 1$
 (start: y_1 ; end: $y=1$)



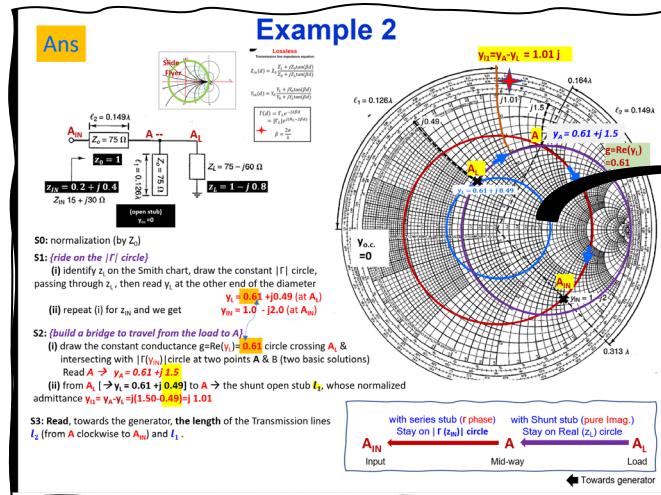
This is a common question in Part 1

“jump/bridge”

Start: the blue circle

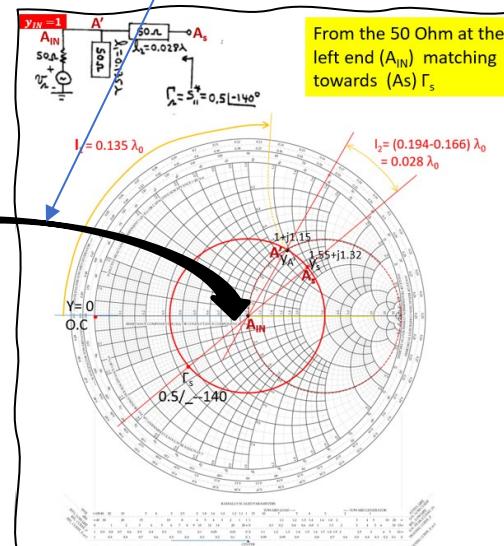
End: the red circle

Jump via the purple circle



Special case of “jump/bridge”

The blue circle (in the middle case) now degenerates into a point (the center of the Smith Chart for this case)



Outline

❑ Noise

- Three major types
- Power Spectrum

❑ Noise Figure

- Noise figure circle
- Noise Figure of Multistage Amplifiers

❑ Impedance Matching

- Reasons/objectives
- Smith Chart primer & four sets of principles
- Impedance matching by Smith Chart – stub matching with 3 examples

❑ Nonlinearity & Performance Parameters

- 1-dB suppression point
- Intermodulation Distortion (IMD)
- Two-Tone 3rd Order Intercept Point
- Dynamic range
- Multistage Amplifiers

❑ Design Considerations

- Power Amplifier Operating Modes
- DC Biasing Networks
- Design Considerations

❑ Case Study -- X-band wideband low-noise amplifier

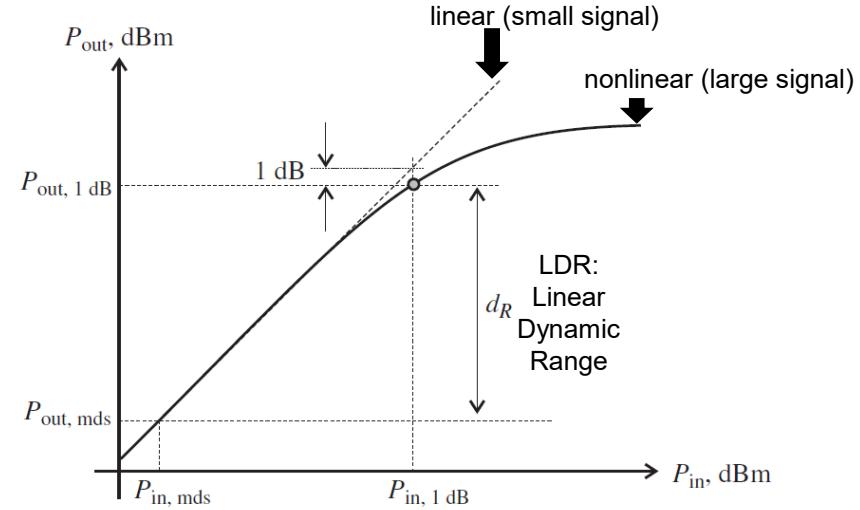
Nonlinear Performance Parameters of Amplifier

- So far, our discussion of amplifier design is based on the assumption of “linear & small signal S-parameters”
- For high-power amplifiers, they usually operate in the non-linear region, so large-signal S-parameters are used for their design.

1- dB Compression Point

dBm or **dB_{mW}** (decibel-milliwatts) is a unit of level used to indicate that a power level is expressed in decibels (dB) with reference to one milliwatt (mW):

$$P(\text{dBm}) = 10 \cdot \log(P/1\text{mW})$$



Output power vs input power of an amplifier

- The output power of RF amplifier can be written as

$$P_{\text{out}} (\text{dBm}) = P_{\text{in}} (\text{dBm}) + G_p (\text{dB})$$
 where G_p is power gain
- As P_{in} is increased, P_{out} increases up to the saturation point of the RF amplifier.
- Towards the saturation point, P_{out} no longer follows the rate of increase in P_{in} . This results in an effective decrease in the gain of the amplifier.
- The decrease in effective gain is termed **gain compression**.

Nonlinear Performance Parameters of Amplifier

- The 1 dB compression point is defined as the point in the nonlinear region where power gain is reduced by 1 dB over the small-signal linear power gain:

linear small-signal power gain

$$G_{1dB} (\text{dB}) = G_o (\text{dB}) - 1$$

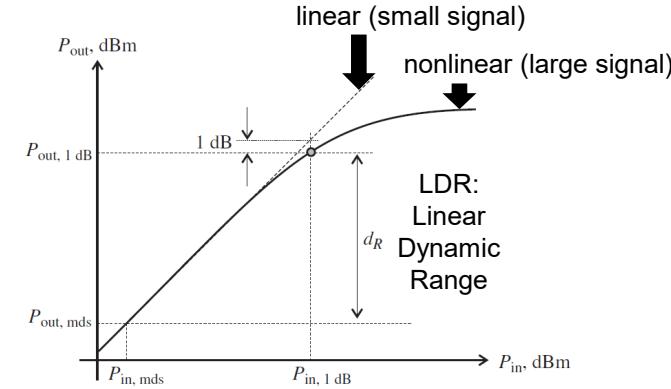
$$P_{1dB} (\text{dBm}) = P_{out,1dB} (\text{dBm}) = P_{in, 1dB} (\text{dBm}) + G_{1dB} (\text{dB})$$

$$= P_{in, 1dB} (\text{dBm}) + [G_o (\text{dB}) - 1]$$

[Example] An amplifier operating at 4 GHz has an output power of $P_{1dB} = 30 \text{ dBm}$, and its corresponding linear power gain is 36 dB. What is the $P_{in,1dB}$ (in dBm) ?

$$P_{1dB} (\text{dBm}) = P_{out,1dB} (\text{dBm}) = P_{in, 1dB} (\text{dBm}) + [G_o (\text{dB}) - 1]$$

$$P_{in, 1dB} (\text{dBm}) = P_{1dB} (\text{dBm}) - [G_o (\text{dB}) - 1] = 30 - (36-1) = -5 \text{ dBm}$$



- Gain compression is a measure of the dynamic range of the RF amplifier and 1 dB compression point is commonly used as a standard of measurement.
- The linear dynamic range (LDR) specifies the range over which a RF amplifier provides linear power gain.
- The upper limit of the dynamic range is the 1 dB compression point while the lower limit is dependent on the noise power level. (An input signal is detectable only if its output power level is above the noise power level). The linear dynamic range (LDR) can be written as

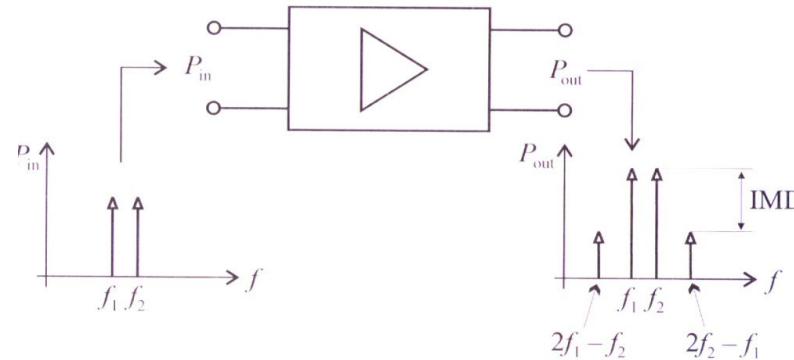
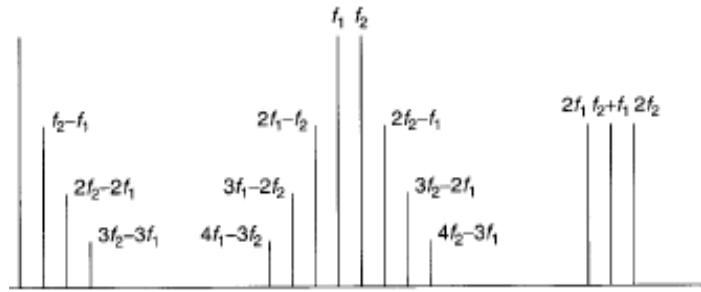
$$\text{LDR} = P_{1dB} - P_{o, mds} \quad \text{Where, } P_{o, mds} = \text{minimum detectable signal output power}$$

- Obviously, the higher the 1 dB compression point, the better the linear dynamic range.

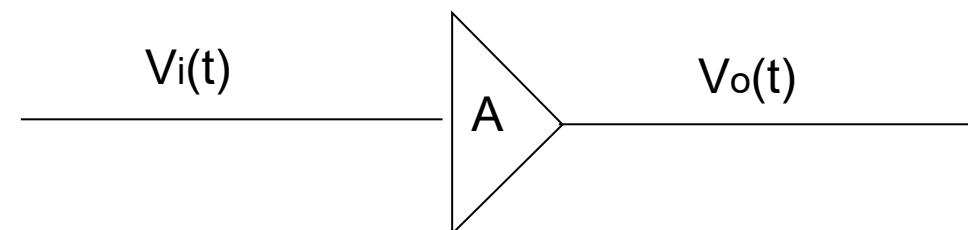
Two Tone 3rd Order Intercept Point

- When two or more sinusoidal frequencies are applied simultaneously to a RF amplifier that is not perfectly linear, the output contains additional frequency components called “intermodulation products”, which are desirable for mixers but undesirable for amplifiers. The difference between the desired and undesired power level (in dBm) at the output port is typically defined as “intermodulation distortion” (IMD) in dB.

The third-order IMD in dBm is written as $\text{IMD3 (dBm)} = P_{f_1} - P_{2f_1-f_2}$ (dBm)



- If f_1 and f_2 are the frequencies of the two signals arriving at the input of the RF amplifier, the amplifier generates intermodulation products at its output due to inherent nonlinearity, in the form of $[\pm mf_1 \pm nf_2]$ where m, n are positive integers which may assume any value from 1 to infinity.



Two Tone 3rd Order Intercept Point

- $V_o(t) = AV_i(t) + BV_i^2(t) + CV_i^3(t)$
 - Second harmonics: $2f_1, 2f_2$ (caused by V_i^2 term)
 - Third harmonics: $3f_1, 3f_2$ (caused by V_i^3 term)
 - Second order intermodulation products: $f_1 \pm f_2$ (caused by V_i^2 term)
 - Third-order intermodulation products: $2f_1 \pm f_2, 2f_2 \pm f_1$ (caused by V_i^3 term)
- (**we'll study again in the Lecture notes on Mixer)
- For example, if 100 and 101 MHz are the frequencies of the applied signals,
Then 99 and 102 MHz are the two-tone 3rd order products closer to the original frequencies.
- The two-tone 3rd order products are a major concern because of their
($2f_1-f_2, 2f_2-f_1$) proximity to the original / desired frequencies ($f_1 & f_2$) and they cannot be filtered out effectively. Hence their considerations are of great importance in system design.

Two-Tone 3rd Order Intercept Point

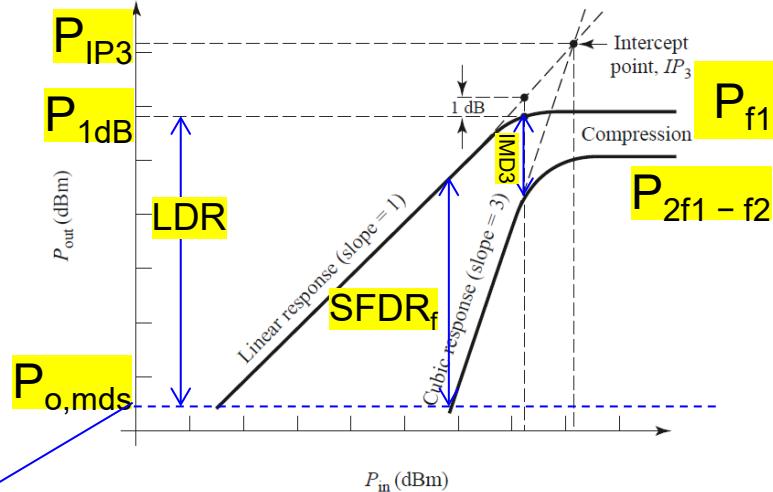
- In the linear region, the 3rd order products decrease / increase by 3 dB for every 1 dB decrease / increase of the input power, while the desired output signal power decrease / increase by 1 dB for every 1 dB decrease / increase of the input power.
- By extending the linear portions of the responses, they intercept at a point called the 3rd order intercept point (IP₃, or TOI, or P_{IP}). This P_{IP} is a measure of the linearity of the RF amplifier and should be as high as possible.
- It is also possible to predict P_{IP} without plotting the curves as

$$P_{IP3} \text{ (dBm)} = P_{1\text{dB}} \text{ (dBm)} + 10 \text{ (dB)}$$

(Note that this 10 dB is a general rule of thumb, which may not be true for all kinds of amplifiers)
- Difference b/w two curves i.e. P_{f1} – P_{2f1-f2}

$$\text{IMD3} = P_{f1} - P_{2f1-f2} = (P_{IP3} - P_{2f1-f2}) \times 2/3 \text{ (dBm)}$$

(Note that P_{xx} are output power in dBm)



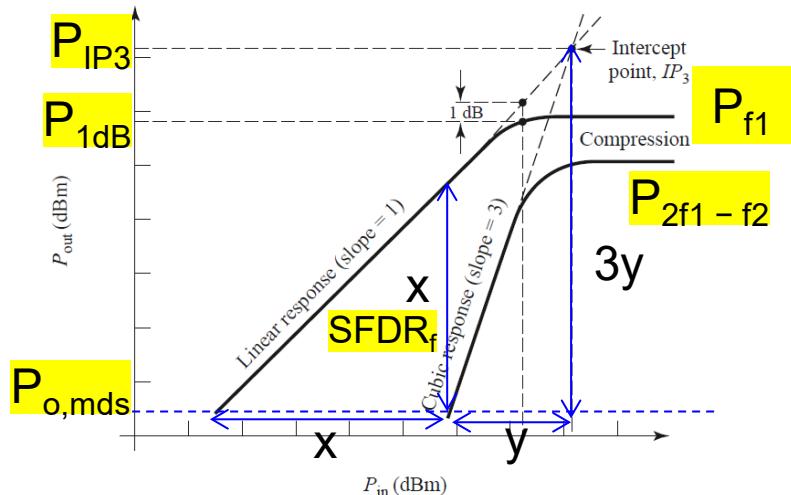
- The spurious free dynamic range (SFDR_f) is the range (P_{f1} – P_{2f1-f2}) when P_{2f1-f2} is equal to the **minimum detectable** output **signal** i.e. P_{0,mds}.

$$\text{SFDR}_f \text{ (dB)} = (P_{IP3} - P_{o, mds}) \times 2 / 3$$

Derivation

- The spurious free dynamic range ($SFDR_f$) is the range ($P_{f1} - P_{2f1-f2}$) when P_{2f1-f2} is equal to the **minimum detectable** output signal i.e. $P_{o, mds}$.

$$SFDR_f \text{ (dB)} = (P_{IP3} - P_{o, mds}) \times 2 / 3$$



$$\begin{aligned}\frac{x}{3y} &= \frac{x}{(x+y)} \\ x(x+y) &= 3xy \\ x(x+y-3y) &= 0 \\ x &= 0 \\ \text{or} \\ x &= 2y = (3y) * 2/3 \\ SFDR_f \text{ (dB)} &= (P_{IP3} - P_{o, mds}) \times 2 / 3\end{aligned}$$

Example

(Adapted from Pozar's book (4th Ed., p.512)) EXAMPLE 10.5 DYNAMIC RANGES)

An amplifier has a 1-dB compression point of 25 dBm (referenced to output), a gain of 40 dB, and a third-order intercept point of 35 dBm (referenced to output). If the thermal noise at the input port of the amplifier is $N_{in} = -87.4$ dBm and a designer intends to achieve a desired output SNR of 10 dB, find the linear dynamic range (LDR) and the spurious free dynamic range (SFDR_f).

Solution

The noise power at the amplifier output can be calculated as

$$N_o = N_{in} (\text{dBm}) + G (\text{dB}) = -87.4 + 40 = -47.4 \text{ dBm}$$

The linear dynamic range in dB is,

$$\text{LDR} = P_{1\text{dB}} (\text{dBm}) - N_o (\text{dBm}) = 25 + 47.4 = 72.4 \text{ dB}$$

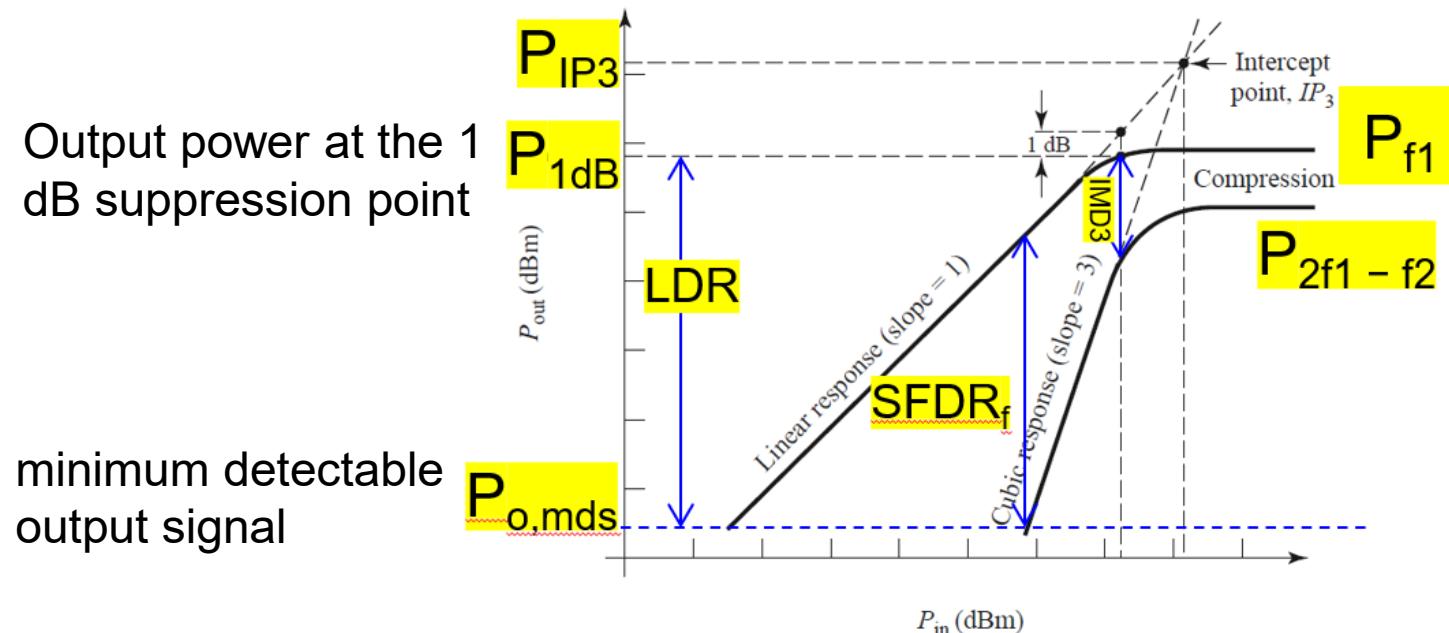
LDR, SFDR ---dB (ratio)

The spurious free dynamic range in dB is

$$\text{SFDR}_f = 2/3 * (P_{IP3} - N_o) - \text{SNR} = 2/3 * (35 + 47.4) - 10 = 44.9 \text{ dB}$$

Observe that SFDR << LDR.

Quick Summary



linear dynamic range

$$LDR = P_{1dB} - P_{o, mds}$$

Output power at the 3rd order intercept point (IP3)

$$P_{IP3} (\text{dBm}) = P_{1dB} (\text{dBm}) + 10 (\text{dB})$$

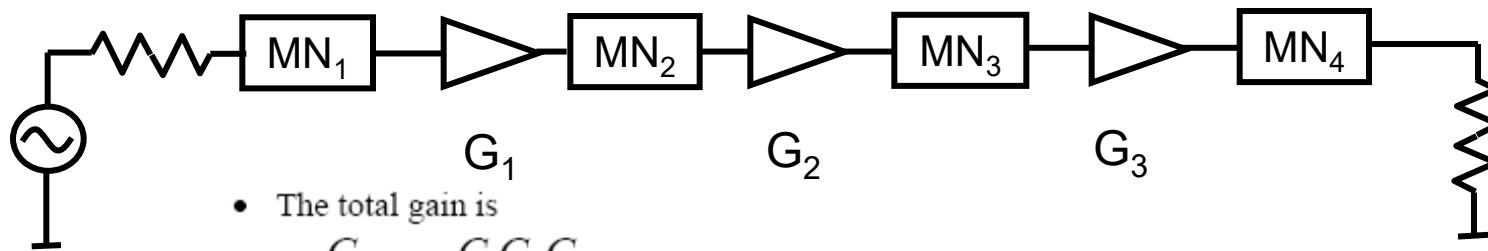
third-order intermodulation distortion (IMD3)

$$IMD3 = P_{f1} - P_{2f1-f2} = (P_{IP3} - P_{2f1-f2}) \times 2/3 (\text{dBm})$$

spurious free dynamic range

$$SFDR_f (\text{dB}) = (P_{IP3} - P_{o, mds}) \times 2 / 3$$

Multistage Amplifiers



- The total gain is

$$G_{\text{total}} = G_1 G_2 G_3$$

- The total noise figure is

$$F_{\text{total}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

- The total third-order (output) intercept point is (assuming worst-case in-phase addition of distortions)

$$\frac{1}{P_{IP3,\text{total}}} = \frac{1}{P_{IP3,1} G_2 G_3} + \frac{1}{P_{IP3,2} G_3} + \frac{1}{P_{IP3,3}}$$

- The total 1-dB gain compression point is (assuming 10 dB below IP3)

$$\frac{1}{P_{1\text{dB},\text{total}}} = \frac{1}{P_{1\text{dB},1} G_2 G_3} + \frac{1}{P_{1\text{dB},2} G_3} + \frac{1}{P_{1\text{dB},3}}$$

- If the minimum detectable input signal is 3 dB above thermal noise.

$$P_{i,\text{mds}} \text{ dBm} = kTB \text{ dBm} + 3 \text{ dB} + F_1 \text{ dB}$$

the total minimum detectable output signal is

$$P_{o,\text{mds}} \text{ dBm} = kTB \text{ dBm} + 3 \text{ dB} + F_{\text{total}} \text{ dB} + G_{\text{total}} \text{ dB}$$

Outline

❑ Noise

- Three major types
- Power Spectrum

❑ Noise Figure

- Noise figure circle
- Noise Figure of Multistage Amplifiers

❑ Impedance Matching

- Reasons/objectives
- Smith Chart primer & four sets of principles
- Impedance matching by Smith Chart – stub matching with 3 examples

❑ Nonlinearity & Performance Parameters

- 1-dB suppression point
- Intermodulation Distortion (IMD)
- Two-Tone 3rd Order Intercept Point
- Dynamic range
- Multistage Amplifiers

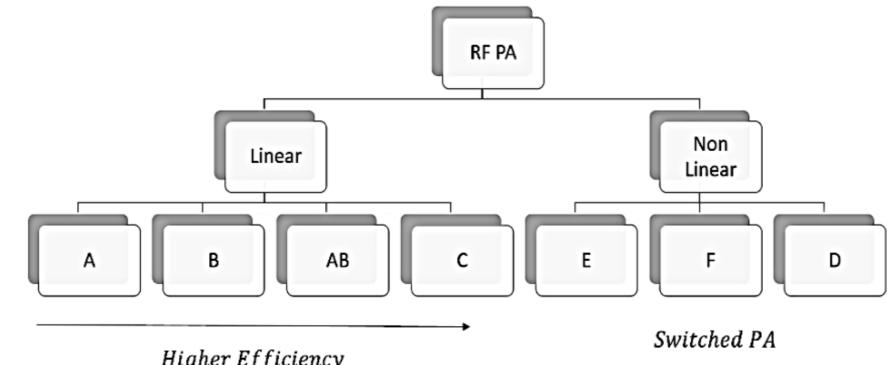
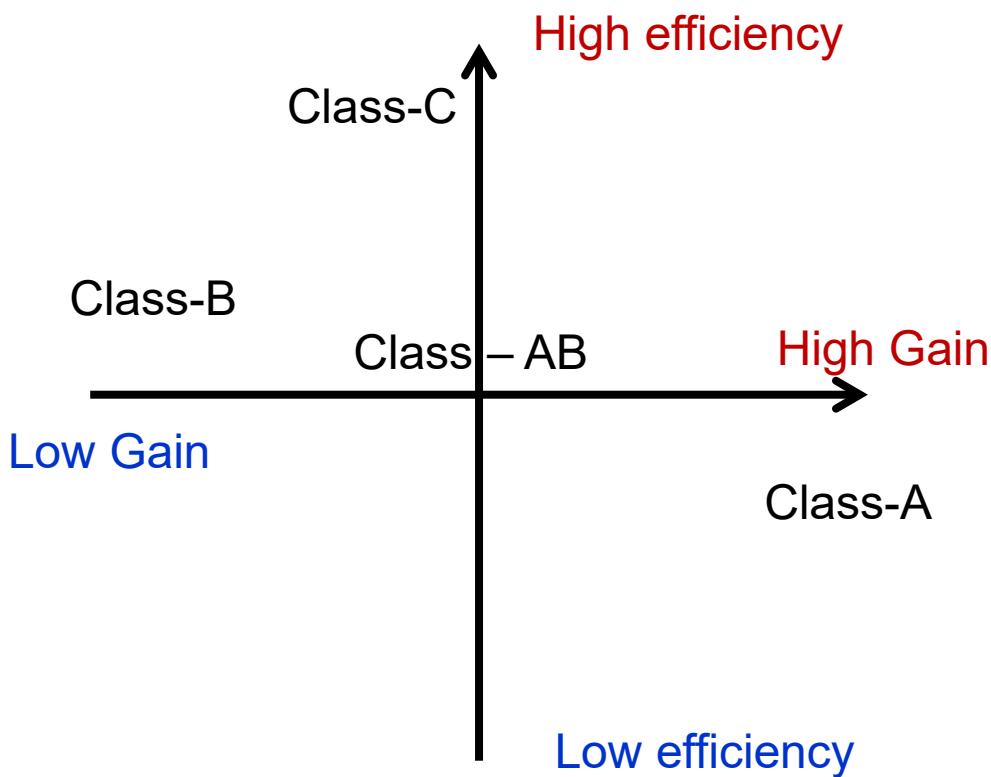
❑ Other topics

- Power Amplifier Operating Modes
- DC Biasing Networks
- Design Considerations

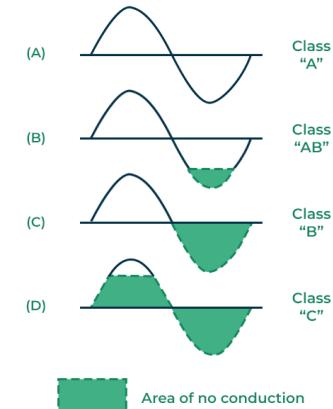
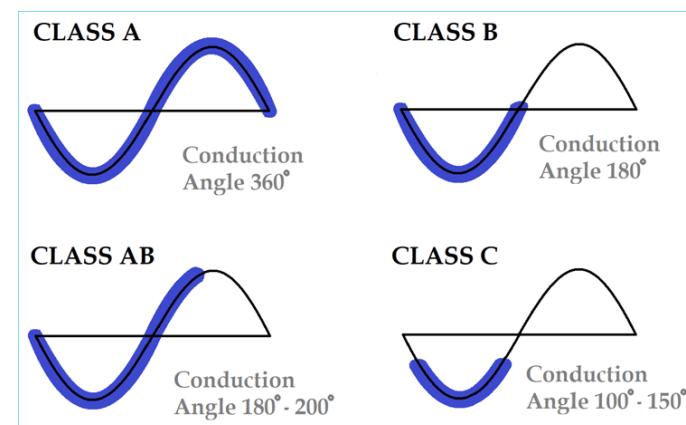
❑ Case Study -- X-band wideband low-noise amplifier

Power Amplifier Operating Modes

- Class-A operation provides high gain but low efficiency
- Class-B operation provides improved efficiency but low gain.
- Class – AB microwave amplifiers operate between class A & class B.
- Class-C has highest possible efficiency. Class C amplifier operates at the essentially non-linear region.



Amplifier class	Maximum efficiency
Class A (resistive bias)	25%
Class A (inductive bias)	50%
Class B	78.53%
Class C	100%
Class D	100%, but typically 75%
Class E	96%
Class F	88.36%

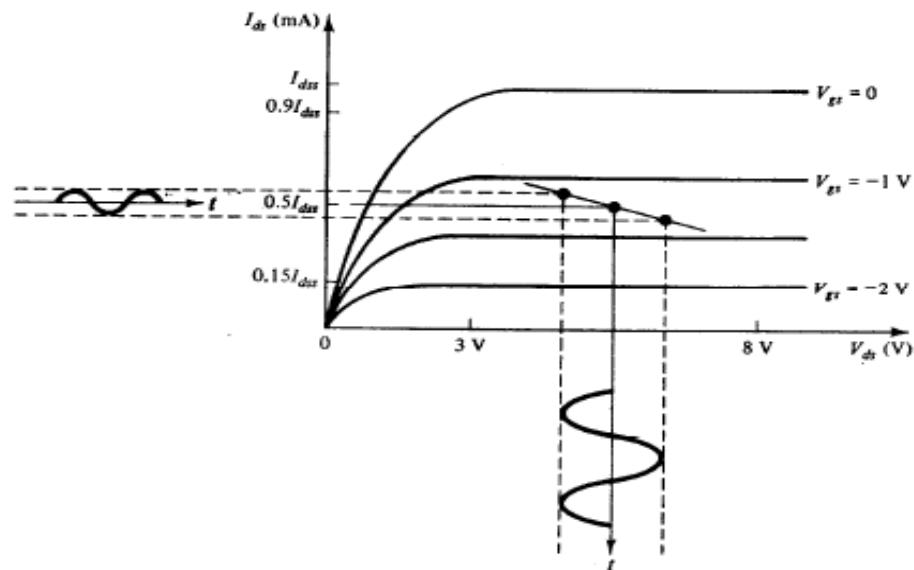


DC Biasing Networks

The design of DC biasing circuits is as important as the design of matching networks for amplifiers, because the amplifiers' high gain, high power, high efficiency, low noise, etc., also depend on the DC bias network.

➤ **The main objectives of DC biasing are:**

- - To establish proper quiescent point and hence the RF amplifier performance.
- - To hold Q point constant over temperature fluctuations and device parameter variations.
- - To make transparent to RF operation.



Q point or the operating point of a device, also known as a bias point, or quiescent point is **the steady-state DC voltage or current at a specified terminal of an active device such as a diode or transistor with no input signal applied**.

Design Considerations

1. TRANSISTORS

- Si BJTs, Si MOSFET - for use below 5 GHz
- GaAs FETs - for use above 2 GHz
- GaAlAs HEMTs for low-noise applications up to 100 GHz
- SiGe Heterojunction Bipolar Transistors (up to 60 GHz)

2. TECHNOLOGY

- Hybrid Microwave Integrated Circuit (HMIC):
- Monolithic Microwave Integrated Circuit (MMIC):

3. IMPEDANCE TRANSFORMER DESIGN

- transformation of an arbitrary impedance to another arbitrary impedance within a specified bandwidth

4. BIAS NETWORKS DESIGN

- Must provide the necessary DC bias voltages without disturbance of the in-band performance and stability at low frequencies.

5. OVERALL STABILITY

Outline

❑ Noise

- Three major types
- Power Spectrum

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- Noise figure circle
- Noise Figure of Multistage Amplifiers

❑ Impedance Matching

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- Intermodulation Distortion (IMD)
- Two-Tone 3rd Order Intercept Point
- Dynamic range
- Multistage Amplifiers

❑ Design Considerations

- Power Amplifier Operating Modes
- DC Biasing Networks
- Design Considerations

❑ Case Study -- X-band wideband low-noise amplifier

(***Self study &
must study)

Case Study -- X-band wideband low-noise amplifier

Case Study: Wideband Amplifiers (CS_AMP2), by Michael Steer

Access the notes and videos in the CANVAS Files folder

The design specification:

To design a LNA (low-noise amplifier) to achieve a maximum noise figure (NF) of 1 dB and a gain of **14±1 dB** from **8 GHz to 12 GHz**.

Notes: Notes about Case Study_Wideband Amplifier Design_Pages from RFDesign_vol5.pdf

VIDEOS:

- CS_Amp2a.mp4: Part A: Wideband Amplifier Design Using the Negative Image Model
- CS_Amp2b.mp4: Part B: Wideband Amplifiers Design, Image Model
- CS_Amp2c.mp4: Part C: Wideband Amplifiers Design, Gain
- CS_Amp2d.mp4: Part D: Wideband Amplifiers Design, Noise
- CS_Amp2e.mp4: Part E: Wideband Amplifiers Design, Stability
- CS_Amp2f.mp4: Part F: Wideband Amplifier Design Image Model-Based Design
- CS_Amp2g.mp4: Part G: Wideband Amplifiers Design, Completing the Design

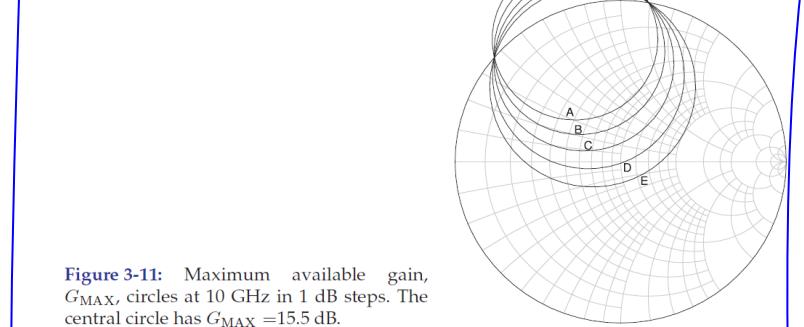
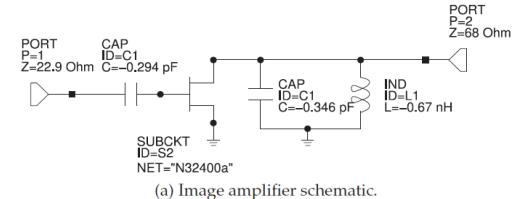


Figure 3-11: Maximum available gain, G_{MAX} , circles at 10 GHz in 1 dB steps. The central circle has $G_{MAX} = 15.5 \text{ dB}$.



(a) Image amplifier schematic.

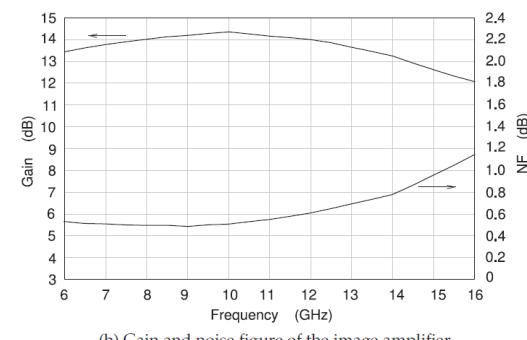


Figure 3-12: Amplifier using negative image model.
(b) Gain and noise figure of the image amplifier.

