EKN-812: Problem Set 0

Jesse Naidoo

Suggested Reading

Review these materials:

- David Autor's notes on math tools for microeconomics (link)
- These notes from Royal Holloway's math camp (link)
- Martin Osborne's online tutorials on mathematical methods for economic theory (link), sections 4 - 7.
- Varian (1992), Ch. 27 (optimization)

For a deeper treatment of why the above "cookbook" techniques work, you should read:

- Silberberg and Suen (2000), Ch. 6 (comparative statics)
- Silberberg and Suen (2000), Ch. 7 (envelope theorem)
- Silberberg and Suen (2000), Ch. 14 (problems with inequality constraints; Kuhn-Tucker conditions)

Sonja has made copies of the chapters from Silberberg and Suen (2000), but you can also find the book on reserve at the library. I have also placed Varian (1992) on reserve.

You may also want to review your undergraduate notes or textbooks on calculus, and on constrained optimization. Chapter 26 of Varian (1992) contains most of what you should already know (i.e. if it looks unfamiliar, go revise it!). The Osborne notes (linked above) are a good review of calculus and the basic theory of convex functions. I'll assume you know all the material in sections 1 - 3 of those notes.

Exercises

We will be using the above techniques often this semester. Try the exercises below and we will discuss the solutions in the first week of classes.

Constrained Optimization

1. Let
$$f(x, y) = (x - 3y + 1)^2 + y^2 + 4.$$

We want to solve the problem

$$\min_{x,y} f(x, y)$$

subject to the constraints $x + y \geq 4$, $x \geq 0$ and $y \geq 0$.

(a) Write down the Lagrangian for this problem. Use λ as the multiplier on the constraint that $x + y \geq 4$, and use δ_x and δ_y for the nonnegativity constraints on x and $y. Write out the system of inequalities that comes from the first-order conditions.
(b) How many different cases are there to consider when finding a solution? (i.e. $x^* > 0$ vs. $x^* = 0$, etc) Can you eliminate any of them? (Hint: use the complementary slackness conditions!)
(c) Find the solution(s) $(x^*, y^*, \lambda^*, \delta_x^*, \delta_y^*)$.
(d) How do you know you have found a (local) minimum and not a maximum?

2. Suppose α is a known constant. Solve

$$\max_{x,y} 4x + \alpha y \quad \text{s.t.} \quad \begin{aligned} &x^2 + y^2 \leq 25 \\ &x \geq 0 \\ &4 \geq y \geq 0 \end{aligned}$$

Which of the constraints bind at the solution? Which are slack? How (if at all) does your answer depend on α? You may want to use the notation $x^*(\alpha)$, $y^*(\alpha)$ etc. to emphasize this dependence.

3. Minimize $f(x, y, z) = x^2 + y^2 + (z - 3)^2$ subject to the constraints

$$\begin{aligned} x + 2y &\geq 4 & (1) \\ x + y + z &\geq 6. & (2) \end{aligned}$$

4. If $f(x, y) = x^2 - \log x + xy$, solve

$$\max_{x \geq 0, y \geq 0} f(x, y) \quad \text{s.t.} \quad x + y = 10 \tag{3}$$

using the same steps (a) - (d) as above. Is there an interior solution to this problem?

Envelope Theorem

5. Let

$$V(a) = \max_{x,y} 4 - x^2/2 - 4y \text{ s.t. } 6x - 4y \leq a$$

Compute the optimal choice of x and y (they may depend on a), and thus find $V(a)$. If λ is the multiplier on the constraint $6x - 4y \leq a$, can you express $V'(a)$ in terms of λ?

6. Same as question 5 above, but with the objective $2xy + 3x$ and subject to the constraint $x + 2y \leq a$.

References

Silberberg, Eugene, and Wing Suen. 2000. *The Structure of Economics: A Mathematical Analysis*. 3rd ed. Singapore: McGraw-Hill.

Varian, Hal R. 1992. *Microeconomic Analysis*. 3rd ed. New York: WW Norton & Co.