

EKN-812 Lecture 4

Uncertainty; Risk-Sharing

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Contingent Commodities

- suppose there are S “states of nature”
 - probability that s occurs is π_s
 - e.g. $s = 0$ is “sunny”, $s = 1$ is “rainy”
- more generally:
 - there is a vector of n “physical” commodities $x = (x_1, \dots, x_n)$
 - we could consider the set of all nS *contingent commodities*
 - and, define preferences over this set of goods in the usual way
- however, we mainly use a special form of preferences where

$$u(x_{1,1}, \dots, x_{n,1}, \dots, x_{n,S}) = \sum_{s=1}^S \pi_s v(x_s)$$

- in this formulation, it is useful to think of x_s as a random vector

Arrow-Debreu Securities

- trade in contingent markets may seem farfetched
 - but, there is an equivalent way to support such trades (“Arrow-Debreu securities”)
- suppose you have S assets, each of which pays off one unit in state s
 - then, you could implement any consumption lottery by trading in these assets
- in fact, this is stronger than we need
 - we do need that there are “enough” different assets to replicate any contingent allocation
 - if not, we say we have “incomplete markets” and equilibria will not necessarily be efficient
 - see e.g. Sargent and Ljungqvist (2018), Ch. 8
- a larger point is that trade in asset markets
 - provide opportunities for consumption smoothing
 - not just for investment over time

Expected Utility

- we can justify the construction of expected utility preferences with some axioms
 - see e.g. Varian (1992), Ch. 11
- note that expected utility preferences are a special type of additively separable preferences

$$u(x_{1,1}, \dots, x_{n,1}, \dots, x_{n,S}) = \sum_{s=1}^S \pi_s v(x_s)$$

- this does impose restrictions on substitution across states, if feasible

Risk Aversion

- let's consider the special case where there is only one physical good
 - if relative prices don't vary with the state s
 - could justify with the composite commodity theorem
- Jensen's inequality: if $v(x)$ is concave, for any $\lambda \in [0, 1]$
 - $\lambda v(x_0) + (1 - \lambda)v(x_1) \leq v(\lambda x_0 + (1 - \lambda)x_1)$
 - concavity means v always lies below its tangent
 - or, any chord connecting two points on the graph of v lies below v
- the economics of this are that, if you have a risky "lottery", $E[v(c)] \leq v(E[c])$

Certainty Equivalents

- consider \tilde{c} such that

$$v(\tilde{c}) = E[v(c)]$$

- this is called the *certainty equivalent* of the risky consumption lottery c
- certainty equivalents are really just a monotone transformation of expected utility
 - ▶ as such, we can rank lotteries equally well by their certainty equivalents
 - ▶ this is because they represent the same preferences
- e.g. if $c = 1$ with probability p and $c = 2$ with probability $1 - p$ and $v(c) = \log(c)$
 - show that $\tilde{c} = 2^{1-p}$
 - what if $c \sim U(1, \bar{c})$?

Certainty Equivalents

- a useful fact: if $\log X \sim N(\mu, \sigma^2)$, then
 - $E[X] = \exp[\mu + \sigma^2/2]$
 - what about $E[X^\delta]$?
 - this distribution is called the “lognormal” distribution
- if we have $v(c) = -\exp[-rc]$, and c is normal,
 - what is $E[v(c)]$?
 - these are called *constant absolute risk aversion* (CARA) preferences
 - the name refers to the fact that $-u''(c)/u'(c)$ is constant, for all levels of wealth

Certainty Equivalents

- with CARA utility, the certainty equivalent of $c \sim N(\mu, \sigma^2)$ is

$$\tilde{c} = \mu - r\sigma^2/2$$

- the difference $E[c] - \tilde{c}$ is called the *risk premium*
- i.e. the extra expected return you'd need to accept the risk
- if the (vNM) utility function is $v(c) = c^{1-\gamma}/(1-\gamma)$
 - first, confirm v is concave
 - next, note that the *coefficient of relative risk aversion* is constant:

$$\frac{cv''(c)}{v'(c)} = \gamma$$

- these preferences are called “CRRA preferences” or “power utility”
- with *lognormal* consumption risk, the risk premium is $\gamma\sigma^2/2$
 - note, this distribution of c is different to the CARA case above!
 - in particular we have $c > 0$ always
 - unfortunately the distribution of the sum of lognormals is hard to express
 - ▶ we mostly ignore the possibility of $c < 0$ for convenience in the CARA case

Insurance

- with “actuarially fair” prices, would demand full insurance
- what is the “fair” price of R1 in state s ?
 - must be $\pi_s < 1$ - why?
- thus, the budget constraint is

$$\sum_{s=1}^S \pi_s c_s = y$$

- with expected-utility preferences:

$$\max_{(c_s)_s} \sum_{s=1}^S \pi_s v(c_s) \text{ s.t. } \sum_{s=1}^S \pi_s c_s = y$$

- the FOC are that $\pi_s v'(c_s) = \lambda \pi_s$
- thus, $c_s = c_{s'}$ for all states s, s'

Insurance

- key to the full-insurance result is that $v(c)$ is *state-independent*
 - what if the marginal utility of consumption is affected by e.g. poor health?
 - leads to a theory that allows us to value improvements in health or disability insurance
 - ▶ e.g. Murphy and Topel (2006) or Low and Pistaferri (2015)

Risk Sharing

- if people are risk-averse, and their incomes are not perfectly correlated
 - there will be gains from sharing risk
 - i.e. providing insurance to each other
- this is one of the key elements in the economic theory of the family
 - other institutions provide informal risk-sharing:
 - ▶ neighbors can provide help (often in kind)
 - ▶ merchants can extend credit
 - ▶ temporary migration (e.g. Morten (2019))
 - storage of goods or sale of assets can also help to smooth consumption
- links between risk-sharing and investment are central in development economics

Risk Sharing

- suppose there are $s = 1, \dots, S$ states with probabilities π_s
- there are two agents with expected-utility preferences
 - vNM utility for a is $v_a(c_{a,s})$ and similarly for b
- each has a random endowment $y_{a,s}$ and $y_{b,s}$
- what would be the *optimal* way to share their joint income?
 - for now let's take "efficiency" to mean "maximizes a weighted average of utilities"
 - this leads to the objective function - for some $\theta \in [0, 1]$ -

$$\theta E[v_a(c_{a,s})] + (1 - \theta) E[v_b(c_{b,s})] = \sum_{s=1}^S \pi_s [\theta v_a(c_{a,s}) + (1 - \theta) v_b(c_{b,s})]$$

- what are the constraints?
 - a and b cannot move resources across states (absent some external market)
 - so, for each state s , we have to have $c_{a,s} + c_{b,s} \leq y_{a,s} + y_{b,s}$
- there are typically also "participation constraints" on these allocations
 - e.g. would a accept a contract that sets $c_{a,s} = 0$ and $c_{b,s} = y_{a,s} + y_{b,s}$?
 - what is the worst contract that a would accept?

References

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