EKN-812 Lecture 2

Welfare Measurement; Restrictions on Preferences

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Envelope Theorem

- a direct proof of Shephard's Lemma: consider

$$e(p, u) = p \cdot x^H(p, u)$$

- differentiate with respect to p_i:

$$\frac{\partial e}{\partial p_i} = x_i^H + \sum_{j=1}^{n} p_j \frac{\partial x_j^H}{\partial p_i}$$

- but, we can ignore all the indirect effects:
  - use the utility constraint $u \equiv u(x^H(p, u))$
  - then, notice that by the first-order conditions,

$$p_j = \lambda \frac{\partial u}{\partial x_j}$$

- the envelope theorem says we could have just computed

$$\frac{\partial \mathcal{L}}{\partial p_i} = \frac{\partial e}{\partial p_i}$$

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Comparative Statics

- often we won't be able to solve explicitly for demands
    - but, we can still find the sign of, e.g. price and income effects
    - we do this by implicitly differentiating the system of first-order conditions
- e.g. the first-order conditions give us a system

$$u_x(x, y) - \lambda p_x \equiv 0 \quad (1)$$
$$u_y(x, y) - \lambda p_y \equiv 0 \quad (2)$$
$$m - p_x x - p_y y \equiv 0 \quad (3)$$

- if we differentiate with respect to (say) y we get a linear system for the income effects
    - can solve element-by-element using Cramer's rule
    - can often figure out the sign of the denominator from the second-order conditions

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Elasticity of Substitution

- (Hicks-Allen) partial elasticity of substitution between i and j is

$$\sigma_{ij}(p,u) \quad = \quad \frac{e(p,u)e_{ij}(p,u)}{e_i(p,u)e_j(p,u)} = \left.\frac{\partial \log(x_1/x_2)}{\partial \log(p_1/p_2)}\right|_{u \text{ constant}}$$

- With two goods, a measure of the curvature of indifference curves
  - Does not generalize well to higher dimensions though
  - *Morishima* elasticities are more appropriate then, although non-symmetric
- A different way of defining substitutability: whether the elasticity of substitution σ_{ij} is greater or smaller than 1
  - i.e. do changes in the relative price p_i/p_j lead to more than proportional changes in relative quantities?
  - Can show that σ_{ij} is related to Hicksian elasticities via the formula

$$\varepsilon^H_{ij} \quad = \quad s_j \sigma_{ij}$$

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Cardinal vs. Ordinal Utility

- the demands generated by $u(x)$ are the same as those generated by $f(u(x))$, where $f' > 0$
  - marginal rates of substitution are the same
  - but diminishing marginal utility can be overturned if $f'' > 0$
- certain types of restrictions are preserved by increasing transformations
  - we call those "ordinal" restrictions (depend only on ranking of bundles)
  - e.g. quasiconcavity, elasticity of substitution, price elasticities
  - concavity is *not* preserved by monotone transformations
- this sensitivity to how preferences are represented means we are usually skeptical of conclusions that depend on interpersonal comparisons of utility
  - Pareto efficiency does not have this weakness
  - nevertheless, there are important areas (e.g. optimal taxation) which make use of cardinal utility

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Homothetic Preferences

- first, suppose $u(x)$ is homogenous of degree 1
  - e.g. Cobb-Douglas preferences $u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$
- then all income elasticities are 1:
  - consider $x^M(p, ty)$ for some $t > 0$
  - what is the indirect utility when you scale up income by t?
    - cannot do worse than $tx^M(p, y)$: $u(tx^M(p,y)) = tu(x^M(p,y))$
    - if strictly better, $x^M(p, y)$ wasn't optimal to start with
  - thus, $x^M(p, ty) \equiv tx^M(p, y)$ for all $t > 0$
- to complete the proof: differentiate wrt t and evaluate at $t = 1$.
- now, because utility is ordinal, only need that there is some increasing function f such that $f(u(x))$ is homogenous of degree 1

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  - thus, $x^M(p, ty) \equiv tx^M(p, y)$ for all $t > 0$
- to complete the proof: differentiate wrt t and evaluate at $t = 1$.
- now, because utility is ordinal, only need that there is some increasing function f such that $f(u(x))$ is homogenous of degree 1

Homothetic Preferences

- first, suppose $u(x)$ is homogenous of degree 1
  - e.g. Cobb-Douglas preferences $u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}$
- then all income elasticities are 1:
  - consider $x^M(p, ty)$ for some $t > 0$
  - what is the indirect utility when you scale up income by t?
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Integrability

- you may be wondering if rational choice has any *other* implications besides the NSD Slutsky matrix
  - the answer is **no**
- the formal result is that if you have a demand function $x^M(p, y)$ with a NSD Slutsky matrix, you can always construct a utility function $u(x)$ that "rationalizes the data"
  - in the sense that maximization of $u(x)$ subject to a budget constraint would give you x^M back again
  - this result is only "local"
  - for the details, see Deaton and Muellbauer (1980), Ch. 2.6
- the implication is that if you want sharper predictions, have to make stronger assumptions
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CV, EV, and Consumer's Surplus

- so far we have been talking about the positive implications of utility theory
- however, consumer theory also has normative uses:
    - tracking changes in the cost of living over time
    - measuring the gains from innovation (new goods)
    - evaluating the distribution of losses (or gains) from government policies
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- the *equivalent variation* is the change in income that is *equivalent* to the price change, starting at the original prices
  - EV implicitly defined by $v(p, y + EV) = v(p', y)$
  - can show this is equivalent to $EV = e(p, u') - y$ where $u' = v(p', y)$
  - the price a consumer is willing to pay to avoid a price increase
- the *compensating variation* is similar but evaluated at the new prices
  - implicitly defined by $v(p', y - CV) = v(p, y)$
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- recall the Hicksian demands are the derivatives of the expenditure function
  - this means we can interpret EV and CV as areas under a Hicksian demand curve
  - but, which ones?
- *consumer surplus* is the area under a Marshallian demand curve
  - in the case of a normal good,

$$EV \leq \Delta CS \leq CV$$

  - for an inferior good the reverse inequalities hold

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Price Indices

- ideal price index for one consumer:

$$\pi_{t+1,t} = \frac{e(p(t+1), u) - e(p(t), u)}{e(p(t), u)}$$

- problems:
  - RHS depends on u, unless preferences are homothetic
  - we don't know preferences (and hence $e(p, u)$)
- obviously, heterogeneity in preferences makes this more complicated
- two common types of price indices:
  - *Laspeyres* indices use initial bundle as weights for later prices
  - *Paasche* indices use current quantities as weights for earlier prices
  - *chained* indices: break up long periods into accumulation of short-term changes
- an important practical issue is how often to update the weights
  - trade off accuracy vs cost of data collection
- dealing with new goods (and exit of old goods) is another important issue
  - one strategy: compare new goods to "appropriate" average of existing ones
  - important to use chained indices here because demand patterns shift over time

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  - important to use chained indices here because demand patterns shift over time

Price Indices

- ideal price index for one consumer:

$$\pi_{t+1,t} = \frac{e(p(t+1),u) - e(p(t),u)}{e(p(t),u)}$$

- problems:
  - RHS depends on u, unless preferences are homothetic
  - we don't know preferences (and hence $e(p,u)$)
- obviously, heterogeneity in preferences makes this more complicated
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  - *Laspeyres* indices use initial bundle as weights for later prices
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Composite Commodity Theorem

- suppose we have three goods, x_1, x_2 and x_3
  - initial prices are p^0
  - however, (p_2, p_3) always move "in parallel"
  - we will see that we can define an artificial "composite commodity" and proceed as if there are only two goods
- let $e^*(p_1, \theta, u) = e(p_1, \theta p_2^0, \theta p_3^0, u)$
  - here $e(p, u)$ is the usual expenditure function
  - we are suppressing (p_2, p_3) as an argument of e^*
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- this means we can define the artificial "good" $z = p_2^0 x_2 + p_3^0 x_3$ with price θ and treat demands as arising from a two-good system in (x_1, z)
  - this is useful because we want to focus on x_1
  - not on the detailed composition of z
- heavily used in macroeconomics
  - aggregate all consumption in a period into c_t
- more generally: if you want to aggregate to broad groups, weight by relative prices

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Alchian and Allen Effect

- suppose we have two "similar" goods which are substitutes
  - high and low quality wine, apples, etc
  - should be in comparable (physical) units
- now add a constant t to each price (e.g. transportation costs)
- consider $e^*(t, p_3, u) = e(p_1 + t, p_2 + t, p_3, u)$
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  - define the artificial commodity $z = x_1 + x_2$ as above
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  - best South African wines are exported (similar for France, Australia, California)
  - "shipping the good apples out"
- can show that

$$\frac{\partial}{\partial t}\left(\frac{x_1}{x_2}\right) > 0 \quad \text{if} \quad \varepsilon_{23}^H - \varepsilon_{13}^H \geq 0$$

  - possible counterexamples?

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Quasilinear Approximation

- for some $\varepsilon > 0$ and $a > 0$,

$$u(c,q) = \begin{cases} c + \left(\frac{a}{1-\varepsilon^{-1}}\right) q^{1-\varepsilon^{-1}} & \text{if } \varepsilon \neq 1 \\ c + a\log(q) & \text{if } \varepsilon = 1 \end{cases}$$

- Can show that if y is large enough Marshallian elasticity is exactly ε. (If y is too small, spend entire budget on q, so $c^M(p,y) = 0$.)
  - What are the income elasticities of c and q?
  - What is the elasticity of substitution between c and q?
- These preferences are very useful for studying the demand for one good in isolation; c is a "composite commodity" consisting of "all other goods"
- Can think of quasilinear preferences as a limiting case where there are
  - many other goods
  - none of them occupies a large budget share
  - and the elasticity of substitution between any pair of goods is bounded
  - See Vives (1987) for precise sufficient conditions

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$$u(c,q) = \begin{cases} c + \left(\frac{a}{1-\varepsilon^{-1}}\right) q^{1-\varepsilon^{-1}} & \text{if } \varepsilon \neq 1 \\ c + a\log(q) & \text{if } \varepsilon = 1 \end{cases}$$

- Can show that if y is large enough Marshallian elasticity is exactly ε. (If y is too small, spend entire budget on q, so $c^M(p,y) = 0$.)
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- suppose you can group goods e.g.

$$\begin{aligned} u &= u(v, w) \\ v &= f(x_1, x_2) \\ w &= g(y_1, y_2) \end{aligned}$$

- we say (x_1, x_2) is *weakly separable* from (y_1, y_2)
  - immediate implication: can mechanically solve consumer's problem in two stages

$$\begin{aligned} \pi_v^*(p_{x,1}, p_{x,2}, v) &= \min p_{x,1}x_1 + p_{x,2}x_2 \text{ s.t. } f(x_1, x_2) \geq v \\ \pi_w^*(p_{y,1}, p_{y,2}, v) &= \min p_{y,1}y_1 + p_{y,2}y_2 \text{ s.t. } g(y_1, y_2) \geq w \end{aligned}$$

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- more subtly, MRS between x-goods does not depend on y-goods
  - this restricts substitution patterns and cross-price effects
- in particular, the cross-price elasticities for goods in different groups
  - have to be proportional to the income effects within each group
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Strong (Additive) Separability

- suppose preferences can be represented as

$$u(x) = \sum_{i=1}^{n} v_i(x_i)$$

- examples:
  - $u(x_1, x_2) = 1 + x_1 + x_2 + x_1 x_2$ is additively separable
  - $u(x_1, x_2) = x_1 + x_2 + x_1 x_2^2$ is **not** - why?
- if all the v_i are increasing and concave:
  - all goods are normal
  - no (net) complements
- used a lot for
  - intertemporal choice
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- some problems where additivity is “too restrictive”
  - habits, addiction
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- obviously this is restrictive
  - the other side to restrictiveness is that you need less data
  - e.g. Deaton and Muellbauer (1980) shows that the Marshallian price elasticities satisfy

$$\begin{aligned}\varepsilon_{ii}^{M} &= \phi\eta_i - \eta_i s_i(1+\phi\eta_i) \\ \varepsilon_{ij}^{M} &= -\eta_i s_j(1+\phi\eta_j)\end{aligned}$$

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  - here ϕ is some constant that depends on preferences and prices
- important point is that you only need
  - the budget shares (just accounting data)
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  - and then you can identify all cross-price elasticities

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