EKN-812 Lecture 4

Uncertainty; Risk-Sharing

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Contingent Commodities

- suppose there are S "states of nature"
  - probability that s occurs is π_s
  - e.g. $s = 0$ is "sunny", $s = 1$ is "rainy"
- more generally:
  - there is a vector of n "physical" commodities $x = (x_1, \ldots x_n)$
  - we could consider the set of all nS *contingent commodities*
  - and, define preferences over this set of goods in the usual way
- however, we mainly use a special form of preferences where

$$u(x_{1,1}, \ldots x_{n,1}, \ldots x_{n,S}) = \sum_{s=1}^{S} \pi_s v(x_s)$$

  - in this formulation, it is useful to think of x_s as a random vector

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Arrow-Debreu Securities

- trade in contingent markets may seem farfetched
  - but, there is an equivalent way to support such trades ("Arrow-Debreu securities")
- suppose you have S assets, each of which pays off one unit in state s
  - then, you could implement any consumption lottery by trading in these assets
- in fact, this is stronger than we need
  - we do need that there are "enough" different assets to replicate any contingent allocation
  - if not, we say we have "incomplete markets" and equilibria will not necessarily be efficient
  - see e.g. Sargent and Ljungqvist (2018), Ch. 8
- a larger point is that trade in asset markets
  - provide opportunities for consumption smoothing
  - not just for investment over time

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Expected Utility

- we can justify the construction of expected utility preferences with some axioms
  - see e.g. Varian (1992), Ch. 11
- note that expected utility preferences are a special type of additively separable preferences

$$u(x_{1,1}, \ldots x_{n,1}, \ldots x_{n,S}) = \sum_{s=1}^{S} \pi_s v(x_s)$$

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Risk Aversion

- let's consider the special case where there is only one physical good
  - if relative prices don't vary with the state s
  - could justify with the composite commodity theorem
- Jensen's inequality: if $v(x)$ is concave, for any $\lambda \in [0, 1]$
  - $\lambda v(x_0) + (1-\lambda)v(x_1) \leq v(\lambda x_0 + (1-\lambda)x_1)$
  - concavity means v always lies below its tangent
  - or, any chord connecting two points on the graph of v lies below v
- the economics of this are that, if you have a risky "lottery", $E[v(c)] \leq v(E[c])$

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Certainty Equivalents

- consider $\widetilde{c}$ such that

$$v(\widetilde{c}) = E[v(c)]$$

  - this is called the *certainty equivalent* of the risky consumption lottery c
  - certainty equivalents are really just a monotone transformation of expected utility
    - as such, we can rank lotteries equally well by their certainty equivalents
    - this is because they represent the same preferences
- e.g. if $c = 1$ with probability p and $c = 2$ with probability $1 - p$ and $v(c) = \log(c)$
  - show that $\widetilde{c} = 2^{1-p}$
  - what if $c \sim U(1, \overline{c})$?

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- a useful fact: if $\log X \sim N(\mu, \sigma^2)$, then
  - $E[X] = \exp[\mu + \sigma^2/2]$
  - what about $E[X^\delta]$?
  - this distribution is called the "lognormal" distribution
- if we have $v(c) = -\exp[-rc]$, and c is normal,
  - what is $E[v(c)]$?
  - these are called *constant absolute risk aversion* (CARA) preferences
  - the name refers to the fact that $-u''(c)/u'(c)$ is constant, for all levels of wealth

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  - first, confirm v is concave
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$$\frac{cv''(c)}{v'(c)} = \gamma$$

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Insurance

- with "actuarially fair" prices, would demand full insurance
- what is the "fair" price of R1 in state s?
    - must be $\pi_s < 1$ - why?
- thus, the budget constraint is

$$\sum_{s=1}^{S} \pi_s c_s = y$$

- with expected-utility preferences:

$$\max_{(c_s)_s} \sum_{s=1}^{S} \pi_s v(c_s) \text{ s.t. } \sum_{s=1}^{S} \pi_s c_s = y$$

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    - e.g. Murphy and Topel (2006) or Low and Pistaferri (2015)

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Risk Sharing

- if people are risk-averse, and their incomes are not perfectly correlated
  - there will be gains from sharing risk
  - i.e. providing insurance to each other
- this is one of the key elements in the economic theory of the family
  - other institutions provide informal risk-sharing:
    - neighbors can provide help (often in kind)
    - merchants can extend credit
    - temporary migration (e.g. Morten (2019))
  - storage of goods or sale of assets can also help to smooth consumption
- links between risk-sharing and investment are central in development economics

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- suppose there are $s = 1, \dots S$ states with probabilities π_s
- there are two agents with expected-utility preferences
  - vNM utility for a is $v_a(c_{a,s})$ and similarly for b
- each has a random endowment $y_{a,s}$ and $y_{b,s}$
- what would be the *optimal* way to share their joint income?
  - for now let's take "efficiency" to mean "maximizes a weighted average of utilities"
  - this leads to the objective function - for some $\theta \in [0,1]$ -

$$\theta E[v_a(c_{a,s})] + (1-\theta)E[v_b(c_{b,s})] = \sum_{s=1}^{S} \pi_s[\theta v_a(c_{a,s}) + (1-\theta)v_b(c_{b,s})]$$

- what are the constraints?
  - a and b cannot move resources across states (absent some external market)
  - so, for each state s, we have to have $c_{a,s} + c_{b,s} \leq y_{a,s} + y_{b,s}$
- there are typically also "participation constraints" on these allocations
  - e.g. would a accept a contract that sets $c_{a,s} = 0$ and $c_{b,s} = y_{a,s} + y_{b,s}$?
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References

Low, Hamish, and Luigi Pistaferri. 2015. "Disability Insurance and the Dynamics of the Incentive Insurance Trade-Off." *American Economic Review* 105 (10): 2986–3029.

Morten, Melanie. 2019. "Temporary Migration and Endogenous Risk-Sharing in Village India." *Journal of Political Economy (Forthcoming)*.

Murphy, Kevin M, and Robert H Topel. 2006. "The Value of Health and Longevity." *Journal of Political Economy* 114 (5). The University of Chicago Press: 871–904.

Sargent, Thomas J, and Lars Ljungqvist. 2018. *Recursive Macroeconomic Theory*. 4th ed. MIT Press.

Varian, Hal R. 1992. *Microeconomic Analysis*. 3rd ed. New York: WW Norton & Co.

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