

EKN-812: Problem Set 0

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Suggested Reading

Review these materials:

- David Autor's notes on math tools for microeconomics ([link](#))
- These notes from Royal Holloway's math camp ([link](#))
- Martin Osborne's online tutorials on mathematical methods for economic theory ([link](#)), sections 4 - 7.
- Silberberg and Suen (2000), Ch. 6 (comparative statics)
- Silberberg and Suen (2000), Ch. 7 (envelope theorem)

Sindi has made copies of the chapters from Silberberg and Suen (2000), but you can also find the book on reserve at the library. The appendix to Varian (1992) is also useful for these topics. You may also want to review your undergraduate notes or textbooks on calculus, and on constrained optimization.

The Osborne notes (linked above) are a good review of calculus and the basic theory of convex functions. I'll assume you know all the material in sections 1 - 3 of those notes.

Exercises

We will be using the above techniques often this semester. Try the exercises below and we will discuss the solutions in the first week of classes.

Constrained Optimization

1. Let

$$f(x, y) = (x - 3y + 1)^2 + y^2 + 4.$$

Solve the problem

$$\min_{x \geq 0, y \geq 0} f(x, y)$$

subject to the constraint $x + y = 4$. How do you know you have found a (local) minimum and not a maximum?

2. If $f(x, y) = x^2 - \log x + xy$, solve

$$\max_{x \geq 0, y \geq 0} f(x, y) \quad \text{s.t.} \quad x + y = 10 \tag{1}$$

Is there an interior solution to this problem?

3. Suppose α is a known constant. Solve

$$\begin{aligned} \max_{x,y} 4x + \alpha y \quad \text{s.t.} \quad & x^2 + y^2 \leq 25 \\ & x \geq 0 \\ & 4 \geq y \geq 0 \end{aligned}$$

Which of the constraints bind at the solution? Which are slack? How (if at all) does your answer depend on α ?

4. Minimize $f(x, y, z) = x^2 + y^2 + (z - 3)^2$ subject to the constraints

$$x + 2y \geq 4 \tag{2}$$

$$x + y + z \geq 6. \tag{3}$$

Envelope Theorem

5. Let

$$V(a) = \max_{x,y} 4 - x^2/2 - 4y \quad \text{s.t.} \quad 6x - 4y \leq a$$

Compute the optimal choice of x and y (they may depend on a), and thus find $V(a)$. If λ is the multiplier on the constraint $6x - 4y \leq a$, can you express $V'(a)$ in terms of λ ?

6. Same as question 5 above, but with the objective $2xy + 3x$ and subject to the constraint $x + 2y \leq a$.

References

Silberberg, Eugene, and Wing Suen. 2000. *The Structure of Economics: A Mathematical Analysis*. 3rd ed. Singapore: McGraw-Hill.

Varian, Hal R. 1992. *Microeconomic Analysis*. 3rd ed. New York: WW Norton & Co.