EKN-812 Lecture 3

Intertemporal Choice; Demand for Durable Goods

Jesse Naidoo

University of Pretoria

CV, EV, and Consumer Surplus

- consider the case of a price increase
- $EV = e(p_0, u_1) - e(p_0, u_0)$
  - old prices; new level of utility (*equivalent* variation)
- $CV = e(p_1, u_0) - e(p_0, u_0)$
  - new prices, old level of utility (*compensating* variation)
- remember:
  - $y = e(p_0, u_0) = e(p_1, u_1)$
  - these points lie on the Marshallian demand curve too
- write the differences in $e(p, u)$ as an integral under a Hicksian demand curve

Proof That $\eta_i = 1$ for Homothetic Demands

Implications of Weak Separability

- say we have $u = u(x, v(y, z))$
  - (y, z) are weakly separable from x
- suppose p_y changes
  - conditional on v, there is a within-group substitution effect
  - the cost of v changes, inducing a substitution effect between (x, v)
  - this in turn gives us an income effect on v and thus on y and z
- example: suppose
  - $u(x, v) = x + \log(v)$
  - $v(y, z) = \min\{y, z\}$
  - normalize the price of x to unity
- first, compute the Hicksian demands
  - $y^H(p_y, p_z, v)$ and similarly for z^H
- what is the objective for the cost-minimization problem over (x, v)?

Implications of Weak Separability

- compute ε^H_{yz} directly:

$$\varepsilon^H_{yz} = -\frac{p_z}{(p_y + p_z)}$$

  - is this what we would have expected?
- what about across-group substitution, say ε^H_{xy}?
  - would expect ε^H_{xy} to be proportional to

$$\varepsilon^H_{xy} \propto \frac{\partial \log c_v}{\partial \log p_y}$$

  - notice the latter is the cost share of y in v
  - this is not specific to these preferences (use the envelope theorem!)

Implications of Weak Separability

- suppose y = cars and z = fuel
  - would a tax on fuel and a subsidy to cars result in more or less traffic?
  - would people benefit from such a policy?
  - will spending on transport rise or fall?
- notice that this structure allows us to define and measure the "cost of driving"
  - does not depend on the consumption of other goods
  - other examples of price subindices?

Consumption and Savings

- suppose we have a two-period model
  - income in each period $t = 0, 1$ is y_t
  - consumption in each period is c_t
  - there is a capital market offering gross returns of $(1 + r)$
  - saving (possibly negative) is s_0
- what are the constraints?
  - what if there were a limit to borrowing?
- when does the borrowing constraint bind?
  - implications for consumption?
  - if an increase in y_0 causes an increase in c_0, is this evidence of "credit constraints"?
  - are "credit constraints" the same as poverty?

Consumption and Savings

- let $p = (1+r)^{-1}$ be the "market discount factor" (relative price of c_1)
- notice that, by the envelope theorem,

$$\frac{\partial u^*}{\partial p} = \lambda^* \cdot (y_1 - c_1^*)$$

- net borrowers are worse off when interest rates rise
  - conversely, borrowers benefit from lower interest rates
- net savers are better off when interest rates rise

Consumption and Savings

- suppose we have time-separable CRRA utility:

$$v(c_t) = \frac{c_t^{1-\gamma^{-1}}}{1-\gamma^{-1}}$$

- what is the MRS $=$ MRT condition?
  - this is often called the "Euler equation"
- what can we say about the effect of interest rates on consumption over time?
  - here, γ is called the "elasticity of intertemporal substitution"
- what happens when $(1+r)$ rises?
  - higher interest rate $\longrightarrow$ present consumption more expensive relative to future
  - substitution effect: save more

Consumption and Savings

- find the Marshallian demands:
  - for convenience, assume $\beta(1 + r) = 1$
  - can show that
$$\frac{\partial \log c_0}{\partial \log p} = \frac{py_1}{y_0 + py_1} - (\gamma - 1)\frac{p}{1 + p}$$
  - what if y_1 is large relative to y_0?
- notice also that
$$\frac{\partial \log c_0}{\partial \log y_0} = \frac{y_0}{y_0 + py_1} < 1$$
- this is a version of the "permanent income hypothesis"
  - temporary increases in income have small effects on current consumption
  - increases in *permanent* income have much larger effects

Life-Cycle Labor Supply

- suppose period-specific utility is

$$u(c_t, h_t) = \frac{c_t^{1-\eta}}{1-\eta} - \alpha \frac{h_t^{1+\gamma}}{1+\gamma}$$

  - here $\alpha > 0$ is some known constant
- suppose you live for T periods and face a given sequence of wages $w_0, w_1, \ldots$
  - if you can borrow and save freely at gross rate $1 + r$, what is the budget constraint?
- from the first-order conditions
  - elasticity of substitution for labor supply in different periods is $1/\gamma$
  - people often call this the "Frisch elasticity of labor supply"
- these preferences are useful for studying "dynamic" or life-cycle labor supply
  - $\eta \geq 0$ governs the strength of income effects
  - γ governs the strength of substitution effects
  - see Keane (2011)

Life-Cycle Labor Supply

- consider some different policies:
  - tax labor earnings and use the revenue to pay for unconditional transfers
  - tax labor earnings and use the revenue to pay for public goods (or a war)
  - a temporary "tax holiday" that expires after a year
- Frisch elasticity is useful for predicting the effects of a small increase in w_{t+1}/w_t
  - if T is large, and $w_{t+1} + w_t$ is constant, income effects will be negligible
  - a small, temporary demand shock
- you can show that the Frisch elasticity is an upper bound for the Hicksian and Marshallian elasticities
  - estimates of $1/\gamma$ are very close to zero, at least for prime-age men who work full-time
  - taking this at face value, what are the implications?
- macroeconomists tend to believe $1/\gamma$ is large, say 2-4
  - this is because aggregate wages don't fluctuate much at business cycle frequencies
  - however, aggregate hours do move a lot over the business cycle

Rental vs. Capital Prices

- suppose we have some durable good S_t and a nondurable c_t
  - durables depreciate at rate δ and sell for v_t
  - and, some financial asset A_t with gross returns $1 + r$
- what is the one-period ahead budget constraint ("law of motion" for wealth)?
- suppose we have two periods and impose that $A_2 = 0$
  - this can be justified by $A_2 \geq 0$ (a no-Ponzi condition) + optimality
- what is the present-value form of the intertemporal budget constraint?
  - notice that

$$v_t^* = v_t - v_{t+1}\left(\frac{1-\delta}{1+r}\right)$$

  - this is called the *user cost* of durables: the implied rental rate

Demand for Durables

- so far we have just described the budget constraint
- now, suppose preferences are

$$u = \sum_{t=0}^{T} \beta^t v(c_t, S_t)$$

  - assumption is that the *service flow* is proportional to stock of durables
- how would we introduce borrowing constraints into this model?

References

Keane, Michael P. 2011. "Labor Supply and Taxes: A Survey." *Journal of Economic Literature* 49 (4): 961–1075. https://doi.org/10.1257/jel.49.4.961.

Table of Contents