

EKN-812 Lecture 3

Intertemporal Choice; Demand for Durable Goods

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CV, EV, and Consumer Surplus

- consider the case of a price increase
- $EV = e(p_0, u_1) - e(p_0, u_0)$
 - old prices; new level of utility (*equivalent* variation)
- $CV = e(p_1, u_0) - e(p_0, u_0)$
 - new prices, old level of utility (*compensating* variation)
- remember:
 - $y = e(p_0, u_0) = e(p_1, u_1)$
 - these points lie on the Marshallian demand curve too
- write the differences in $e(p, u)$ as an integral under a Hicksian demand curve

Proof That $\eta_i = 1$ for Homothetic Demands

- first, suppose that $u(x)$ is homogeneous of degree 1
 - this is stronger than we need: recall utility is ordinal
 - so we only need that there is some increasing f such that $f(u(x))$ is h.d. 1
- suppose $x^M(p, y_0) = x_0^* = \arg \max u(x) \text{ s.t. } px \leq y_0$
 - let $t > 0$ be given (say $t = 2$ for concreteness)
 - and let $x^M(p, ty_0) = x_1^* = \arg \max u(x) \text{ s.t. } px \leq ty_0$
- we know $v(p, ty_0) \geq tv(p, y_0)$ by feasibility
 - could always choose tx_0^*
- suppose the inequality is strict, so $v(p, ty_0) = u(x_1^*) > tu(x_0^*)$
 - then $t^{-1}x_1^*$ is feasible when $y = y_0$ (check this)
 - and $u(t^{-1}x_1^*) > v(p, y_0) = u(x_0^*)$ by assumption
 - then x_0^* was not optimal to begin with - a contradiction

Proof That $\eta_i = 1$ for Homothetic Demands

- interpretation: with u homogeneous,
 - if you can do better when rich (x_1^*) than just a scaled-up version of what you did when you were poor (x_0^*)
 - then, you should have chosen a scaled-down version of your choices when rich in the first place
- thus, we have shown that for all t and all y_0 ,

$$x^M(p, ty_0) \equiv x^M(p, y_0)$$

- final step: differentiate wrt t and evaluate at $t = 1$

Implications of Weak Separability

- say we have $u = u(x, v(y, z))$
 - (y, z) are weakly separable from x
- suppose p_y changes
 - conditional on v , there is a within-group substitution effect
 - the cost of v changes, inducing a substitution effect between (x, v)
 - this in turn gives us an income effect on v and thus on y and z
- example: suppose
 - $u(x, v) = x + \log(v)$
 - $v(y, z) = \min\{y, z\}$
 - normalize the price of x to unity
- first, compute the Hicksian demands
 - $y^H(p_y, p_z, v)$ and similarly for z^H
- what is the objective for the cost-minimization problem over (x, v) ?

Implications of Weak Separability

- compute ε_{yz}^H directly:

$$\varepsilon_{yz}^H = -\frac{p_z}{(p_y + p_z)}$$

- is this what we would have expected?
- what about across-group substitution, say ε_{xy}^H ?
 - would expect ε_{xy}^H to be proportional to

$$\varepsilon_{xy}^H \propto \frac{\partial \log c_v}{\partial \log p_y}$$

- notice the latter is the cost share of y in v
- this is not specific to these preferences (use the envelope theorem!)

Implications of Weak Separability

- suppose $y = \text{cars}$ and $z = \text{fuel}$
 - would a tax on fuel and a subsidy to cars result in more or less traffic?
 - would people benefit from such a policy?
 - will spending on transport rise or fall?
- notice that this structure allows us to define and measure the “cost of driving”
 - does not depend on the consumption of other goods
 - other examples of price subindices?

Consumption and Savings

- suppose we have a two-period model
 - income in each period $t = 0, 1$ is y_t
 - consumption in each period is c_t
 - there is a capital market offering gross returns of $(1 + r)$
 - saving (possibly negative) is s_0
- what are the constraints?
 - what if there were a limit to borrowing?
- when does the borrowing constraint bind?
 - implications for consumption?
 - if an increase in y_0 causes an increase in c_0 , is this evidence of “credit constraints”?
 - are “credit constraints” the same as poverty?

Consumption and Savings

- let $p = (1 + r)^{-1}$ be the “market discount factor” (relative price of c_1)
- notice that, by the envelope theorem,

$$\frac{\partial u^*}{\partial p} = \lambda^* \cdot (y_1 - c_1^*)$$

- net borrowers are worse off when interest rates rise
 - conversely, borrowers benefit from lower interest rates
- net savers are better off when interest rates rise

Consumption and Savings

- suppose we have time-separable CRRA utility:

$$v(c_t) = \frac{c_t^{1-\gamma^{-1}}}{1-\gamma^{-1}}$$

- what is the $MRS = MRT$ condition?
 - this is often called the “Euler equation”
- what can we say about the effect of interest rates on consumption over time?
 - here, γ is called the “elasticity of intertemporal substitution”
- what happens when $(1 + r)$ rises?
 - higher interest rate \rightarrow present consumption more expensive relative to future
 - substitution effect: save more

Consumption and Savings

- find the Marshallian demands:
 - for convenience, assume $\beta(1+r) = 1$
 - can show that

$$\frac{\partial \log c_0}{\partial \log p} = \frac{py_1}{y_0 + py_1} - (\gamma - 1) \frac{p}{1 + p}$$

- what if y_1 is large relative to y_0 ?
- notice also that
- this is a version of the “permanent income hypothesis”

$$\frac{\partial \log c_0}{\partial \log y_0} = \frac{y_0}{y_0 + py_1} < 1$$

- temporary increases in income have small effects on current consumption
 - increases in *permanent* income have much larger effects

Life-Cycle Labor Supply

- suppose period-specific utility is

$$u(c_t, h_t) = \frac{c_t^{1-\eta}}{1-\eta} - \alpha \frac{h_t^{1+\gamma}}{1+\gamma}$$

- here $\alpha > 0$ is some known constant
- suppose you live for T periods and face a given sequence of wages w_0, w_1, \dots
 - if you can borrow and save freely at gross rate $1 + r$, what is the budget constraint?
- from the first-order conditions
 - elasticity of substitution for labor supply in different periods is $1/\gamma$
 - people often call this the “Frisch elasticity of labor supply”
- these preferences are useful for studying “dynamic” or life-cycle labor supply
 - $\eta \geq 0$ governs the strength of income effects
 - γ governs the strength of substitution effects
 - see Keane (2011)

Life-Cycle Labor Supply

- consider some different policies:
 - tax labor earnings and use the revenue to pay for unconditional transfers
 - tax labor earnings and use the revenue to pay for public goods (or a war)
 - a temporary “tax holiday” that expires after a year
- Frisch elasticity is useful for predicting the effects of a small increase in w_{t+1}/w_t
 - if T is large, and $w_{t+1} + w_t$ is constant, income effects will be negligible
 - a small, temporary demand shock
- you can show that the Frisch elasticity is an upper bound for the Hicksian and Marshallian elasticities
 - estimates of $1/\gamma$ are very close to zero, at least for prime-age men who work full-time
 - taking this at face value, what are the implications?
- macroeconomists tend to believe $1/\gamma$ is large, say 2-4
 - this is because aggregate wages don't fluctuate much at business cycle frequencies
 - however, aggregate hours do move a lot over the business cycle

Rental vs. Capital Prices

- suppose we have some durable good S_t and a nondurable c_t
 - durables depreciate at rate δ and sell for v_t
 - and, some financial asset A_t with gross returns $1 + r$
- what is the one-period ahead budget constraint (“law of motion” for wealth)?
- suppose we have two periods and impose that $A_2 = 0$
 - this can be justified by $A_2 \geq 0$ (a no-Ponzi condition) + optimality
- what is the present-value form of the intertemporal budget constraint?
 - notice that

$$v_t^* = v_t - v_{t+1} \left(\frac{1 - \delta}{1 + r} \right)$$

- this is called the *user cost* of durables: the implied rental rate
 - ▶ exactly analogous to nondurables
 - ▶ useful for e.g. construction of price indices

Demand for Durables

- so far we have just described the budget constraint
- now, suppose preferences are

$$u = \sum_{t=0}^T \beta^t v(c_t, S_t)$$

- assumption is that the *service flow* is proportional to stock of durables
- how would we introduce borrowing constraints into this model?
- what about a “wedge” between the buying and selling price?

References

Keane, Michael P. 2011. "Labor Supply and Taxes: A Survey." *Journal of Economic Literature* 49 (4): 961–1075.
<https://doi.org/10.1257/jel.49.4.961>.

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