EKN-812 Lecture 1

Methodology; Basic Theory of Demand

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The Economic Approach to Human Behavior

What Good is (Economic) Theory?

Outline for Today

- budget constraints
- preferences: axiomatic foundations
- Hicksian and Marshallian demand
- implications of rational choice: symmetry, homogeneity, negativity
- substitutes and complements; constrained demand functions and rationing

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Choice and Scarcity

- "choice" is meaningless if there is no scarcity
- a simple situation in which resources are limited:
  - two goods x_1, x_2 available in any nonnegative amounts
  - constant prices $p_1, p_2 > 0$ and a given budget y
  - then $p_1 x_1 + p_2 x_2 \leq y$
- consumer choices give us the functions $x_1^*(p, y), x_2^*(p, y)$
  - here $p = (p_1, p_2)$ is the vector of prices
- the budget constraint alone places some restrictions on behavior

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Notation: Shares and Elasticities

- Say we are choosing quantities of n goods with prices p_i $(i = 1, \dots n)$
- the i-th budget share is

$$s_i = \frac{p_i x_i^*}{y}$$

- the income elasticity of the i-th good is

$$\eta_i = \frac{y}{x_i^*} \frac{\partial x_i^*}{\partial y}$$

- the elasticity of demand for good i with respect to the $j$þprice is

$$\varepsilon_{ij} = \frac{p_j}{x_i^*} \frac{\partial x_i^*}{\partial p_j}$$

- obviously these quantities are all "local"
    - might vary with the particular (p, y) facing the consumer
- shares and elasticities are unitless
    - why is this good?

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Cournot and Engel Aggregation

- budget constraint holds as an identity in (p, y), so we can differentiate it
- differentiate wrt y to get "Engel aggregation"
  - in elasticity form:

$$\sum_{i=1}^{n} s_i \eta_i = 1$$

  - budget-share weighted average of income elasticities is always 1
- differentiate wrt p_j to get "Cournot aggregation"
  - in elasticity form:

$$s_j + \sum_{i=1}^{n} s_i \varepsilon_{ij} = 0$$

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Slutsky Compensation for Price Changes

- price changes have two effects on the budget set
  - change the relative price of goods
  - change aggregate "purchasing power"
- it is often useful to distinguish between these two mechanism for how prices affect choices
  - substitution effects due to relative prices
  - income effects due to changes in overall purchasing power
- as a simple example of why this distinction might matter, consider two policies:
  - levy a general consumption tax to fund highway maintainence
  - collect tolls on drivers
- Slutsky compensation: alter income after a price change such that the original bundle is feasible
  - with two goods, suppose p_1 rises to $p_1' > p_1$; what is the Slutsky compensation?

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Irrational Behavior and the "Law of Demand"

- main result in consumer theory: compensated own-price effects are negative
  - "demand curves slope down"
  - we will see a more precise statement of this later
- however, a lot of the positive content of consumer theory comes from the budget constraint alone
  - not from particular assumptions about rationality
- suppose prices are p_1, p_2 and all consumers have the same income y
- suppose consumers are passive: half buy only x_1 and half buy only x_2
  - what is the market demand?
- now, suppose p_1 rises to p_1' and we compensate everyone
  - what happens to market demand?
- why does this work?

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Preferences

- we've just seen that the budget constraint alone implies restrictions on behavior (at least in aggregate)
- even so it's often useful to introduce more structure into a model of how people make choices
  - get sharper predictions
  - a coherent language for thinking about *why* people do what they do
  - closely related: derive normative implications
- you have already worked with utility functions, say $u(x)$
  - it's possible to give an axiomatic foundation to the existence of such an object
  - we won't use these too much
  - but, it's good to know what exactly we are assuming when we work with utility functions

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Preferences: Axiomatic Foundations

- suppose the set of bundles you can choose from is a subset of $\mathbb{R}^n_+$ (n-tuples of nonnegative reals)
- what do we mean by "preferences"?
  - a "relation" $\succeq$ describing a person's evaluation of the statement "is at least as good as"
  - a "binary relation" in the sense that you need two bundles x, y in order to evaluate "$x \succeq y$"
- if this relation satisfies certain properties then:
  - we can construct a utility function $u(x)$ that "represents" this person's preferences
  - in the sense that: $u(x) \geq u(y)$ if and only if $x \succeq y$

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  - transative: if $x' \succeq x$ and $x'' \succeq x'$, then also $x'' \succeq x$
  - continuous: for any x, the sets $\{y : y \succeq x\}$ and $\{y : y \preceq x\}$ are closed
- often it's convenient to add:
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- two ways to think of the consumer's problem
- maximize utility subject to a budget constraint
  - $x^M(p, y) = \arg\max u(x)$ s.t. $px \leq y$ are the *Marshallian demands*
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  - $x^H(p, u) = \arg\min px$ s.t. $u(x) \geq u$ are the *Hicksian demands*
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Properties of Hicksian Demand

- Hicksian demand is:
  - easier to work with for welfare analysis (why?)
  - harder to use, because we don't get to observe utility levels
- let's notice some useful properties of $e(p, u)$:
  - homogenous of degree one in p: for all (p, u) and $t > 0$, $e(tp, u) = te(p, u)$
  - increasing in u and in p
  - concave in p (why?)
  - continuously differentiable in p
  - Shephard's Lemma:

$$\frac{\partial}{\partial p_i} e(p, u) = x_i^H(p, u).$$

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- these special properties of the expenditure function imply restrictions on Hicksian demands
  - adding-up:
  $$\sum_{i=1}^{n} p_i x_i^H(p,u) \equiv e(p,u)$$
  - homogeneity of degree zero: for all $t > 0$ and all (p,u), $x^H(tp,u) \equiv x^H(p,u)$
  - symmetry: for all i, j,
  $$\frac{\partial^2 e}{\partial p_j p_i} = \frac{\partial x_i^H}{\partial p_j} = \frac{\partial x_j^H}{\partial p_i} = \frac{\partial^2 e}{\partial p_i p_j}.$$
  - negativity: the matrix σ with i, j-th entry
  $$\sigma_{ij} = \frac{\partial x_i^H}{\partial p_j}$$
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- this last one implies that compensated own-price effects are nonpositive: $\partial x_i^H / \partial p_i \leq 0$

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Slutsky Equation

- we can relate the observable Marshallian demands to the (unobservable) Hicksian ones
    - differentiate the identity $x^M(p, e(p, u)) \equiv x_i^H(p, u)$ wrt p_j
- the result is called the "Slutsky equation":

$$\varepsilon_{ij}^M = \varepsilon_{ij}^H - s_j \eta_j$$

- so, we can form the matrix σ, assuming we had enough data with exogenous variation in prices and incomes
- then, the properties we mentioned are testable implications of rational choice
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Hicksian Elasticities

- we can translate the properties of Hicksian demand into elasticity form:
  - homogeneity: for all goods i,

$$\sum_{j=1}^{n} \varepsilon_{ij}^{H} = 0$$

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- two of these imply the third
  - we don't have as many independent restrictions as it appears
  - with a two-good demand system, how much information do we need to fully characterize demand?

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Substitutes and Complements

- we could define i and j to be *substitutes* if an increase in the price of j leads to an increase in the (compensated) demand for i
  - i.e. if $\varepsilon_{ij}^H > 0$
- similarly, let's call i and j *complements* if an increase in the price of j leads to a *decrease* in the compensated demand for i
  - i.e. if $\varepsilon_{ij}^H < 0$
- what might go wrong if we defined things in terms of Marshallian demand?
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Constrained Demand

- so far we have only imposed the budget constraint on consumers
- suppose the quantities of some goods were fixed, e.g. because of rationing
  - say the quantity of good 1 is fixed at z, and x_{-1} is the vector of all the other goods
  - the consumer's expenditure-minimization problem becomes:

$$\overline{e}(p,u,z) = \min_{x_{-1}} p_1 z + p_{-1}x_{-1} \text{ s.t. } u(z,x_{-1}) \geq u$$

  - let $\overline{x}^H(p_{-1},u,z)$ be the constrained Hicksian demands
  - notice how the price of the rationed good does *not* directly affect the constrained demands

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- let $\Delta(p, u, z) = \overline{e}(p, u, z) - e(p, u)$ be the "excess" expenditure due to rationing
  - clearly $\Delta(p, u, z) \geq 0$ with a minumum where $z = x_1^H(p, u)$ - why?
  - and, the constrained Hicksian demands satisfy

$$\overline{x}_i^H(p_{-1}, u, x_1^H(p_1, p_{-1}, u)) \equiv x_i^H(p_1, p_{-1}, u)$$

- if we differentiate wrt the price of the rationed good (p_1),

$$\frac{\partial \overline{x}_i}{\partial z} = \frac{\sigma_{i1}}{\sigma_{11}}$$

  - what does this tell us about the effects of rationing on the demand for other goods?
- and, if we differentiate with respect to p_i, we get

$$\frac{\partial \overline{x}_i^H}{\partial p_i} = \sigma_{ii} - \frac{\sigma_{i1}^2}{\sigma_{11}}$$

  - interpretation?

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- if we differentiate wrt the price of the rationed good (p_1),

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