

EKN-812 Lecture 1

Methodology; Basic Theory of Demand

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The Economic Approach to Human Behavior

- Alfred Marshall (1890): “Economics is a study of mankind in the ordinary business of life.”
- Lionel Robbins (1932): “Economics is the science which studies human behaviour as a relationship between ends and scarce means which have alternative uses.”
- Gary Becker (1976): economics is defined by its method, not its subject matter.
 - rational choice (purposeful behavior)
 - stable preferences
 - equilibrium (in either implicit or explicit “markets”)
- economists study many types of “non-market” behavior:
 - crime
 - education
 - fertility
 - health
 - time use

What Good is (Economic) Theory?

- theories are good for:
 - understanding mechanisms
 - generating predictions for new circumstances
- obviously, theories have to be tested against evidence
 - so, we will avoid theories that are *unfalsifiable* (no *empirical content*)
 - compare: astrology, Tarot reading
- what do we have that journalists don't?
 - coherent body of theory
 - the concept of “equilibrium”: different people's actions have to be mutually consistent
 - a sharp focus on causation
- think of economic theory as
 - a way of building up judgement by studying simplified cases
 - helps us think about how to interpret the evidence

Outline for Today

- budget constraints
- preferences: axiomatic foundations
- Hicksian and Marshallian demand
- implications of rational choice: symmetry, homogeneity, negativity
- substitutes and complements; constrained demand functions and rationing

Choice and Scarcity

- “choice” is meaningless if there is no scarcity
- a simple situation in which resources are limited:
 - two goods x_1, x_2 available in any nonnegative amounts
 - constant prices $p_1, p_2 > 0$ and a given budget y
 - then $p_1x_1 + p_2x_2 \leq y$
- consumer choices give us the functions $x_1^*(p, y), x_2^*(p, y)$
 - here $p = (p_1, p_2)$ is the vector of prices
- the budget constraint alone places some restrictions on behavior

Notation: Shares and Elasticities

- Say we are choosing quantities of n goods with prices p_i ($i = 1, \dots, n$)
- the i -th budget share is

$$s_i = \frac{p_i x_i^*}{y}$$

- the income elasticity of the i -th good is

$$\eta_i = \frac{y}{x_i^*} \frac{\partial x_i^*}{\partial y}$$

- the elasticity of demand for good i with respect to the j th price is

$$\varepsilon_{ij} = \frac{p_j}{x_i^*} \frac{\partial x_i^*}{\partial p_j}$$

- obviously these quantities are all “local”
 - might vary with the particular (p, y) facing the consumer
- shares and elasticities are unitless
 - why is this good?

Cournot and Engel Aggregation

- budget constraint holds as an identity in (p, y) , so we can differentiate it
- differentiate wrt y to get “Engel aggregation”
 - in elasticity form:

$$\sum_{i=1}^n s_i \eta_i \equiv 1$$

- budget-share weighted average of income elasticities is always 1
- differentiate wrt p_j to get “Cournot aggregation”
 - in elasticity form:

$$s_j + \sum_{i=1}^n s_i \varepsilon_{ij} \equiv 0$$

Slutsky Compensation for Price Changes

- price changes have two effects on the budget set
 - change the relative price of goods
 - change aggregate “purchasing power”
- it is often useful to distinguish between these two mechanism for how prices affect choices
 - substitution effects due to changes in relative prices
 - income effects due to changes in overall purchasing power
- as a simple example of why this distinction might matter, consider two policies:
 - levy a general consumption tax to fund highway maintenance
 - collect tolls on drivers
- Slutsky compensation: alter income after a price change such that the original bundle is feasible
 - with two goods, suppose p_1 rises to $p'_1 > p_1$; what is the Slutsky compensation?

Irrational Behavior and the “Law of Demand”

- main result in consumer theory: compensated own-price effects are negative
 - “demand curves slope down”
 - we will see a more precise statement of this later
- however, a lot of the positive content of consumer theory comes from the budget constraint alone
 - not from particular assumptions about rationality
- suppose prices are p_1, p_2 and all consumers have the same income y
- suppose consumers are passive: half buy only x_1 and half buy only x_2
 - what is the market demand?
- now, suppose p_1 rises to p'_1 and we compensate everyone
 - what happens to market demand?
- why does this work?

Preferences

- we've just seen that the budget constraint alone implies restrictions on behavior (at least in aggregate)
- even so it's often useful to introduce more structure into a model of how people make choices
 - get sharper predictions
 - a coherent language for thinking about *why* people do what they do
 - closely related: derive normative implications
- you have already worked with utility functions, say $u(x)$
 - it's possible to give an axiomatic foundation to the existence of such an object
 - we won't use these too much
 - but, it's good to know what exactly we are assuming when we work with utility functions

Preferences: Axiomatic Foundations

- suppose the set of bundles you can choose from is a subset of \mathbb{R}_+^n (n-tuples of nonnegative reals)
- what do we mean by “preferences”?
 - a “relation” \succeq describing a person’s evaluation of the statement “is at least as good as”
 - a “binary relation” in the sense that you need two bundles x, y in order to evaluate “ $x \succeq y$ ”
- if this relation satisfies certain properties then:
 - we can construct a utility function $u(x)$ that “represents” this person’s preferences
 - in the sense that: $u(x) \geq u(y)$ if and only if $x \succeq y$

Preferences: Axiomatic Foundations

- We can construct a continuous utility function u that represents the preference relation \succeq if the preferences relation is
 - reflexive: for all x , $x \succeq x$
 - complete: for all x, y either $x \succeq y$ or $y \succeq x$
 - transitive: if $x' \succeq x$ and $x'' \succeq x'$, then also $x'' \succeq x$
 - continuous: for any x , the sets $\{y : y \succeq x\}$ and $\{y : y \preceq x\}$ are closed
- often it's convenient to add:
 - convexity: if $x' \succeq x$, then for any $\alpha \in [0, 1]$, $\alpha x' + (1 - \alpha)x \succeq x$
 - if we assume this, we get that u is quasiconcave

Duality: Hicksian and Marshallian Demand

- two ways to think of the consumer's problem
- maximize utility subject to a budget constraint
 - $x^M(p, y) = \arg \max u(x)$ s.t. $px \leq y$ are the *Marshallian demands*
 - $v(p, y) = \max u(x)$ s.t. $px \leq y$ is the *indirect utility function*
- minimize the cost of obtaining a target level of utility
 - $x^H(p, u) = \arg \min px$ s.t. $u(x) \geq u$ are the *Hicksian demands*
 - $e(p, u) = \min px$ s.t. $u(x) \geq u$ is the *expenditure function* (or *cost function*)
- “obviously”:
 - $x^M(p, e(p, u)) \equiv x^H(p, u)$ for all (p, u)
 - and, $x^H(p, v(p, y)) \equiv x^M(p, y)$ for all (p, y)
- what are the analogous relationships between the expenditure and indirect utility functions?

Properties of Hicksian Demand

- Hicksian demand is:
 - easier to work with for welfare analysis (why?)
 - harder to use, because we don't get to observe utility levels
- let's notice some useful properties of $e(p, u)$:
 - homogenous of degree one in p : for all (p, u) and $t > 0$, $e(tp, u) \equiv te(p, u)$
 - increasing in u and in p
 - concave in p (why?)
 - continuously differentiable in p
 - Shephard's Lemma:

$$\frac{\partial}{\partial p_i} e(p, u) \equiv x_i^H(p, u).$$

Properties of Hicksian Demand

- these special properties of the expenditure function imply restrictions on Hicksian demands

- adding-up:

$$\sum_{i=1}^n p_i x_i^H(p, u) \equiv e(p, u)$$

- homogeneity of degree zero: for all $t > 0$ and all (p, u) , $x^H(tp, u) \equiv x^H(p, u)$
- symmetry: for all i, j ,

$$\frac{\partial^2 e}{\partial p_j \partial p_i} = \frac{\partial x_i^H}{\partial p_j} = \frac{\partial x_j^H}{\partial p_i} = \frac{\partial^2 e}{\partial p_i \partial p_j}.$$

- negativity: the matrix σ with i, j -th entry

$$\sigma_{ij} = \frac{\partial x_i^H}{\partial p_j}$$

is negative semidefinite.

- this last one implies that compensated own-price effects are nonpositive:
 $\partial x_i^H / \partial p_i \leq 0$

Slutsky Equation

- we can relate the observable Marshallian demands to the (unobservable) Hicksian ones
 - differentiate the identity $x^M(p, e(p, u)) \equiv x_i^H(p, u)$ wrt p_j
- the result is called the “Slutsky equation”:

$$\varepsilon_{ij}^M = \varepsilon_{ij}^H - s_j \eta_i$$

- so, we can form the matrix σ , assuming we had enough data with exogenous variation in prices and incomes
- then, the properties we mentioned are testable implications of rational choice
 - adding-up
 - homogeneity
 - symmetry
 - negativity

Hicksian Elasticities

- we can translate the properties of Hicksian demand into elasticity form:
 - homogeneity: for all goods i ,

$$\sum_{j=1}^n \epsilon_{ij}^H = 0$$

- adding-up (or use the utility constraint): for all j ,

$$\sum_{i=1}^n s_i \epsilon_{ij}^H = 0$$

- symmetry: for all i, j

$$s_i \epsilon_{ij}^H = s_j \epsilon_{ji}^H.$$

- two of these imply the third
 - we don't have as many independent restrictions as it appears
 - with a two-good demand system, how much information do we need to fully characterize demand?

Marshallian Elasticities

- we've already seen some restrictions on Marshallian demand
 - Cournot aggregation:

$$s_j + \sum_{i=1}^n s_i \varepsilon_{ij}^M = 0$$

- Engel aggregation:

$$\sum_{i=1}^n s_i \eta_i = 1.$$

- of course, we can exploit the relationship to the compensated elasticities to derive other implications

Substitutes and Complements

- we could define i and j to be *substitutes* if an increase in the price of j leads to an increase in the (compensated) demand for i
 - i.e. if $\varepsilon_{ij}^H > 0$
- similarly, let's call i and j *complements* if an increase in the price of j leads to a *decrease* in the compensated demand for i
 - i.e. if $\varepsilon_{ij}^H < 0$
- what might go wrong if we defined things in terms of Marshallian demand?
- with a two-good demand system, you can only have substitutes (prove this!)

Constrained Demand

- so far we have only imposed the budget constraint on consumers
- suppose the quantities of some goods were fixed, e.g. because of rationing
 - say the quantity of good 1 is fixed at z , and x_{-1} is the vector of all the other goods
 - the consumer's expenditure-minimization problem becomes:

$$\bar{e}(p, u, z) = \min_{x_{-1}} p_1 z + p_{-1} x_{-1} \text{ s.t. } u(z, x_{-1}) \geq u$$

- let $\bar{x}^H(p_{-1}, u, z)$ be the constrained Hicksian demands
- notice how the price of the rationed good does *not* directly affect the constrained demands

Constrained Demand

- let $\Delta(p, u, z) = \bar{e}(p, u, z) - e(p, u)$ be the “excess” expenditure due to rationing
 - clearly $\Delta(p, u, z) \geq 0$ with a minimum where $z = x_1^H(p, u)$ - why?
 - and, the constrained Hicksian demands satisfy

$$\bar{x}_i^H(p_{-1}, u, x_1^H(p_1, p_{-1}, u)) \equiv x_i^H(p_1, p_{-1}, u)$$

- if we differentiate wrt the price of the rationed good (p_1),

$$\frac{\partial \bar{x}_i}{\partial z} = \frac{\sigma_{i1}}{\sigma_{11}}$$

- what does this tell us about the effects of rationing on the demand for other goods?
- and, if we differentiate with respect to p_i , we get

$$\frac{\partial \bar{x}_i^H}{\partial p_i} = \sigma_{ii} - \frac{\sigma_{i1}^2}{\sigma_{11}}$$

- interpretation?

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