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Gruppe 30

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1 Modules and Classes

Path setup for libs:

```
[1]: import sys,os
sys.path.append(os.path.expanduser('./libitmal'))

from libitmal import utils as itmalutils
print(dir(itmalutils))
print(itmalutils.__file__)

['AssertInRange', 'CheckFloat', 'InRange', 'Iterable', 'PrintMatrix',
'ResetRandom', 'TEST', 'TestAll', 'TestCheckFloat', 'TestPrintMatrix',
'TestVarName', 'VarName', '__builtins__', '__cached__', '__doc__', '__file__',
'__loader__', '__name__', '__package__', '__spec__', 'ctxlib', 'inf', 'inspect',
'isFloat', 'isList', 'isNumpyArray', 'nan', 'np', 'random', 're']
/home/lassebp7/code/6.Semester/MAL/libitmal/utils.py
```

1.1 Qa - Load and test libitmal

```
[2]: from libitmal import utils as itmalutils

itmalutils.TestAll()
```

TestPrintMatrix...(no regression testing)

```
X=[[ 1.  2.]
 [ 3. -100.]
 [ 1. -1.]]
X=[[ 1.  2.]
```

```
...
 [ 1. -1.]]
X=[[ 1.
  2.
 [ 3.0001
 -100.
 [ 1.
 -1.
 -1.
 -1.]]
```

```
X=[[ 1.  2.]
 [ 3. -100.]
 [ 1. -1.]]
```

OK

TEST: OK

ALL OK

1.2 Qb Create your own module, with some functions, and test it

Below is two small printer functions placed in malutils and imported:

```
[3]: import malutils

malutils.HelloWorld()
malutils.Greeter("Pokemon!")
```

Hello World!
Hello Pokemon!!

1.3 Qc How do you ‘recompile’ a module?

1.3.1 Answer

Reload of modules can be done in several ways. One simple way is to just restart the kernel. Another is the code below.

```
[4]: import importlib
importlib.reload(malutils)
```

```
[4]: <module 'malutils' from
      '/home/lassebp7/code/6.Semester/MAL/mal_grp30/01/malutils/__init__.py'>
```

If you are using VSCode, it is also possible to add the following code to settings.json, which will make the notebook auto reload the module changes.

```
"jupyter.runStartupCommands": ["%load_ext autoreload", "%autoreload 2"],
```

1.4 Qe Extend the class with some public and private functions and member variables

1.4.1 Answers

As can be seen below, private function and member variables are represented in python by two `__` prefixed to the name.

The meaning of `self` is that it is a reference to the class instance itself. Other languages have ‘this’ as a reference to the class instance itself.

Calling a function without `self` in the parameter list is not allowed in python, as can be seen from the output of the exception catch.

```
[5]: class MyClass:
      def myFun(self):
          self.myvar = "Public function"
          print(f"This is a message inside the class, myvar={self.myvar}.")

      #private function
      def __myfun(self):
          self.myvar = "Private"
          print(f"This is a private message inside the class, myvar={self.myvar}.")
      ↵)
```

```

def callToPrivate(self):
    print(f"Calling private function, myvar={self.myvar}.")
    self.__myfun()
    print("Done with private function")

def myFun2(): # this wont work!
    print("No self")

instance = MyClass()

instance.myFun()
try:
    instance.__myfun()
except:
    print("Exception: can't call private function")

instance.callToPrivate()
try:
    instance.myFun2()
except:
    print("Exception: no self class method!")

```

This is a message inside the class, myvar=Public function.
Exception: can't call private function
Calling private function, myvar=Public function.
This is a private message inside the class, myvar=Private.
Done with private function
Exception: no self class method!

1.5 Qf Extend the class with a Constructor

1.5.1 Answers

As can be seen below, the constructor is named `__init__` and takes the 'self' parameter and an arbitrary number of parameters. There is no real destructor compared to the C++ destructor. Python is a managed language, so objects that are no longer in use are garbage collected.

The `__del__` function is not a destructor. It's just a function that gets called when the garbage collector destroys the instance.

```

[6]: class MyCtorClass:
    def __init__(self,x):
        self.x = x
        print(f"Constructor called with x={x}")

    def GetX(self):
        return self.x

```

```
ctorInstance = MyCtorClass(42)
num = ctorInstance.GetX()
print(f"numn from instance = {num}")
```

Constructor called with x=42
numn from instance = 42

1.6 Qg Extend the class with a to-string function

Below is a small class with a “to string” method:

```
[7]: class MyToStringClass:
      def __init__(self,x):
          self.x = x

      def __str__(self):
          return f"MyToStringClass (x={self.x})"

strClass = MyToStringClass(420)
print(strClass)
```

MyToStringClass (x=420)

2 Intro

```
[8]: import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import sklearn.linear_model
import os

def prepare_country_stats(oecd_bli, gdp_per_capita):
    oecd_bli = oecd_bli[oecd_bli["INEQUALITY"]=="TOT"]
    oecd_bli = oecd_bli.pivot(index="Country", columns="Indicator",
        ↪ values="Value")
    gdp_per_capita.rename(columns={"2015": "GDP per capita"}, inplace=True)
    gdp_per_capita.set_index("Country", inplace=True)
    full_country_stats = pd.merge(left=oecd_bli, right=gdp_per_capita,
        ↪ left_index=True, right_index=True)
    full_country_stats.sort_values(by="GDP per capita", inplace=True)
    remove_indices = [0, 1, 6, 8, 33, 34, 35]
    keep_indices = list(set(range(36)) - set(remove_indices))
    ↪ return full_country_stats[["GDP per capita", 'Life satisfaction']].
    ↪ iloc[keep_indices]

datapath = os.path.join("./datasets", "lifesat", "")

# Load the data
```

```

oecd_bli = pd.read_csv(datapath + "oecd_bli_2015.csv", thousands=',')
gdp_per_capita = pd.read_csv(datapath + "gdp_per_capita.
    ↪csv", thousands=',', delimiter='\t',
                                encoding='latin1', na_values="n/a")

# Prepare the data
country_stats = prepare_country_stats(oecd_bli, gdp_per_capita)
X = np.c_[country_stats["GDP per capita"]]
y = np.c_[country_stats["Life satisfaction"]]

# Visualize the data
#country_stats.plot(kind='scatter', x="GDP per capita", y='Life satisfaction')
#plt.show()

# Select a linear model
model = sklearn.linear_model.LinearRegression()
# Train the model
model.fit(X, y)

print("OK")

```

OK

2.1 Qa) The θ parameters and the R^2 Score

2.1.1 Answers

Maximum for R^2 is 1.

Minimum for R^2 is negative infinity. This is if the model makes predictions worse than just guessing the average, and can be a result of overfitting, bad test data etc.

It's better to have a higher R^2 score. This measures the fitness of the model.

```

[9]: # skæring ved x-aksen
theta_0 = model.intercept_
# koefficienten
theta_1 = model.coef_[0]
print(f"h(x) = {theta_0[0]:.4f} + {theta_1[0]}x")

u = np.sum((y - model.predict(X))**2)
v = np.sum((y - np.mean(y))**2)

R2 = 1 - u/v
R2_skl = model.score(X, y)
print(f"R2 = {R2}")
print(f"R2_skl = {R2_skl}")

```

$h(x) = 4.8531 + 4.911544589158484e-05x$

$R2 = 0.7344414355437031$

```
R2_sk1 = 0.7344414355437031
```

2.2 Qb) Using k-Nearest Neighbors

2.2.1 Answers

KNN regressor also uses R^2 as a score, so in that regard they can be compared to each other. However, the knn model might overfit to the data, since $k=3$ allows for the model to fluctuate a bit.

This information about the score function was found at the following locations in the documentation.

Linear Reg https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LinearRegression.html#sklearn.linear_model.LinearRegression

KNN <https://scikit-learn.org/stable/modules/generated/sklearn.neighbors.KNeighborsRegressor.html#sklearn.neighbors.KNeighborsRegressor>

```
[10]: # Prepare the data
X = np.c_[country_stats["GDP per capita"]]
y = np.c_[country_stats["Life satisfaction"]]

print("X.shape=", X.shape)
print("y.shape=", y.shape)

# Visualize the data
country_stats.plot(kind='scatter', x="GDP per capita", y='Life satisfaction')
plt.show()

# Select and train a model
k = 3
knn = sklearn.neighbors.KNeighborsRegressor(k)
knn.fit(X, y)

# Plot knn
m = np.linspace(0, 60000, 1000)
M = np.empty([m.shape[0], 1])
M[:, 0] = m

y_pred_lin = model.predict(M) # Linear regression predictions
y_pred_knn = knn.predict(M)

# Create the plot
country_stats.plot(kind='scatter', x="GDP per capita", y='Life satisfaction',
    figsize=(8, 6))
plt.axis([0, 60000, 0, 10])

# Plot both model predictions
plt.plot(m, y_pred_lin, "r-", label="Linear Regression", linewidth=2)
plt.plot(m, y_pred_knn, "b--", label=f"KNN (k={k})", linewidth=2)
```

```

# Add labels and legend
plt.xlabel("GDP per capita (USD)")
plt.ylabel("Life satisfaction")
plt.legend()
plt.title("Comparison of Linear Regression vs KNN Regression")
plt.show()

# Print model performance
print(f"Linear Regression Score ( $R^2$ ): {model.score(X, y):.3f}")
print(f"KNN Score ( $R^2$ ): {knn.score(X, y):.3f}")

# Make prediction for Cyprus
X_cyprus = [[22587]]
lin_pred = model.predict(X_cyprus)
knn_pred = knn.predict(X_cyprus)

print(f"\nPredictions for Cyprus (GDP = 22587 USD):")
print(f"Linear Regression: {lin_pred[0][0]:.2f}")
print(f"KNN (k={k}): {knn_pred[0]}")

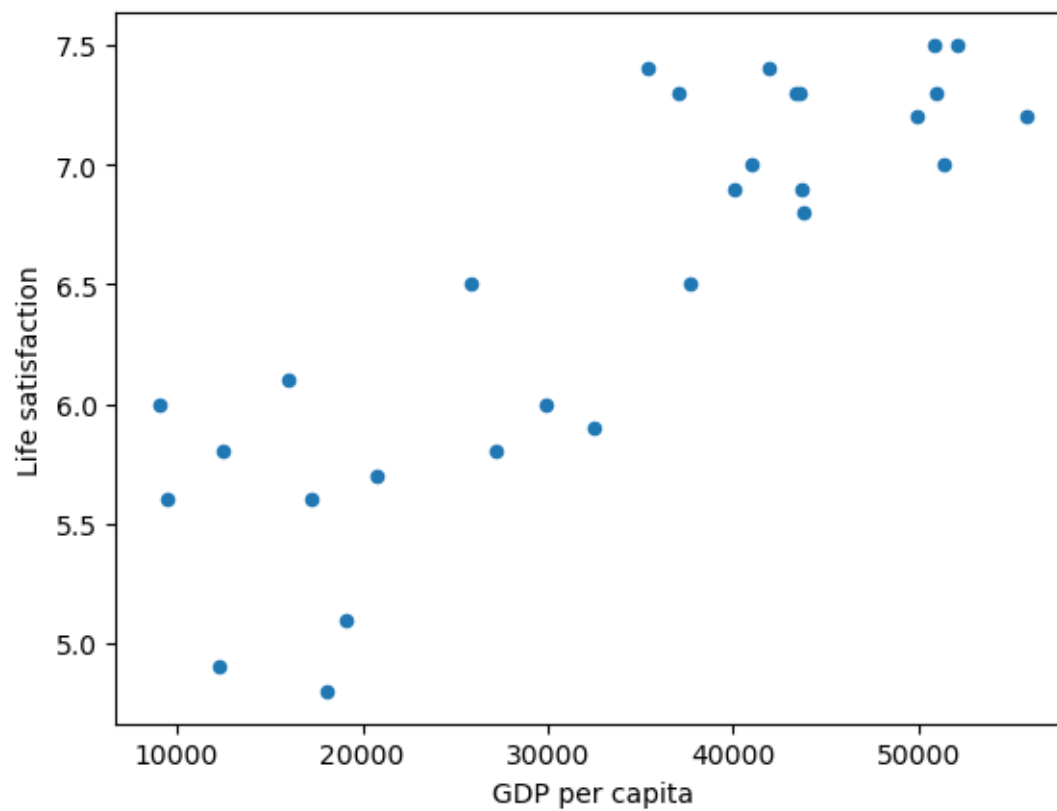
knn_score = knn.score(X, y)
print(f"KNN score : {knn_score}")
print(f"Linear Regression score: {R2_sk1}")

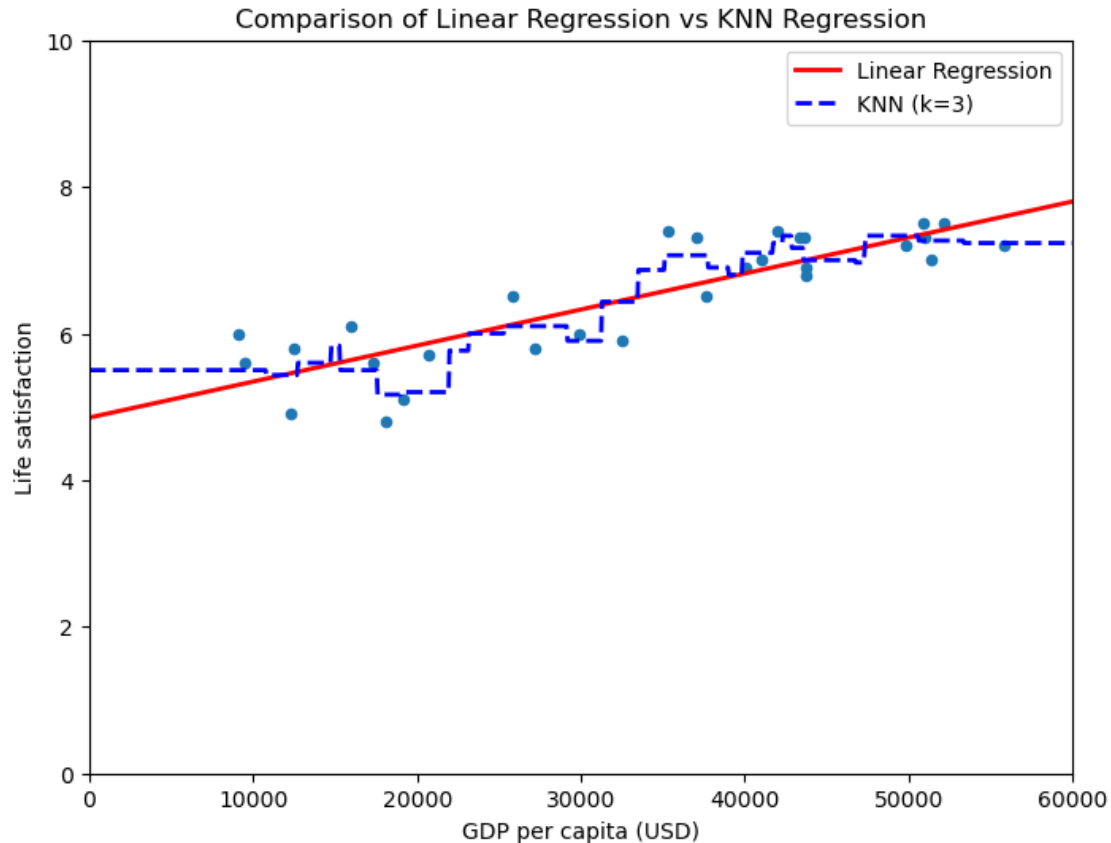
```

```

X.shape= (29, 1)
y.shape= (29, 1)

```



Linear Regression Score (R^2): 0.734

KNN Score (R^2): 0.853

Predictions for Cyprus (GDP = 22587 USD):

Linear Regression: 5.96

KNN (k=3): [5.76666667]

KNN score : 0.8525732853499179

Linear Regression score: 0.7344414355437031

2.3 Qc) Tuning Parameter for k-Nearest Neighbors and A Sanity Check

2.3.1 Answers

K=1 gives score 1 This obviously looks good because of what is discussed earlier about higher R^2 values, but by looking at the graph, it just fits the model 100%, because it just draws a line between each point. This means it has no idea how to predict anything depending on the training data, and it will just predict the training data value closest to whatever value we are trying to predict using the model.

k=5, k=10... The model uses more neighbors to predict new data, where the higher k means it looks at more neighbors to make its prediction. This will make the generalization better, but lower

the training score and the R2 value.

k20... When setting the neighbor count this high, we risk overfitting, where the line just becomes straight and gives bad predictions again.

```
[11]: country_stats.plot(kind='scatter', x="GDP per capita", y='Life satisfaction',
    figsize=(5,3))
plt.axis([0, 60000, 0, 10])

# create an test matrix M, with the same dimensionality as X, and in the range
# [0;60000]
# and a step size of your choice
m=np.linspace(0, 60000, 1000)
M=np.empty([m.shape[0],1])
M[:,0]=m

# from this test M data, predict the y values via the lin.reg. and k-nearest
# models
y_pred_lin = model.predict(M)
y_pred_knn = knn.predict(M) # ASSUMING the variable name 'knn' of your
# KNeighborsRegressor

# use plt.plot to plot x-y into the sample_data plot..
plt.plot(m, y_pred_lin, "r", label="Linear Regression")
plt.plot(m, y_pred_knn, "b", label="KNN (k=3)")
knn_score3 = knn.score(X, y)

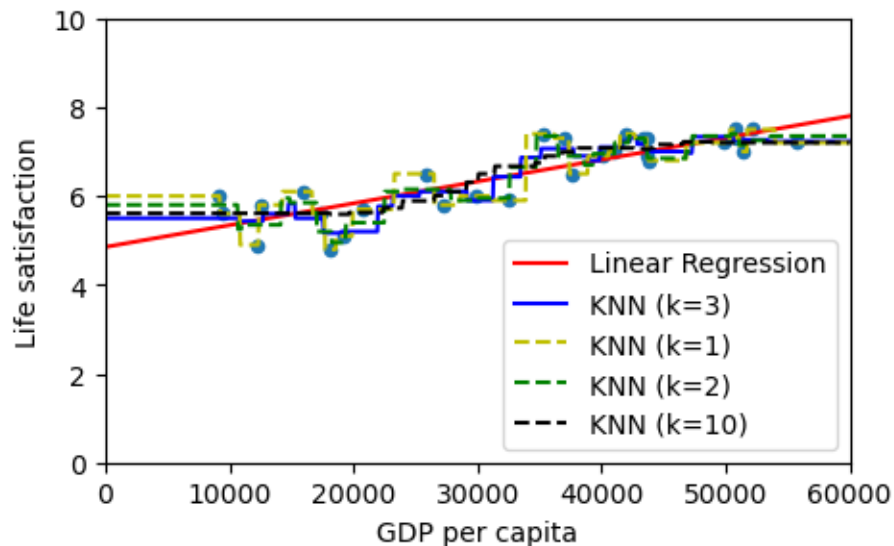
knn = sklearn.neighbors.KNeighborsRegressor(1)
knn.fit(X, y)
knn_score1 = knn.score(X, y)
y_pred_knn1 = knn.predict(M)
plt.plot(m, y_pred_knn1, "y--", label="KNN (k=1)")

knn = sklearn.neighbors.KNeighborsRegressor(2)
knn.fit(X, y)
knn_score2 = knn.score(X, y)
y_pred_knn2 = knn.predict(M)
plt.plot(m, y_pred_knn2, "g--", label="KNN (k=2)")

knn = sklearn.neighbors.KNeighborsRegressor(10)
knn.fit(X, y)
knn_score10 = knn.score(X, y)
y_pred_knn10 = knn.predict(M)
plt.plot(m, y_pred_knn10, "k--", label="KNN (k=10)")

plt.legend()
plt.show()
```

```
#scores
print(f"KNN score (k=1) : {knn_score1}")
print(f"KNN score (k=2) : {knn_score2}")
print(f"KNN score (k=3) : {knn_score3}")
print(f"KNN score (k=10) : {knn_score10}")
print(f"Linear Regression score: {R2_skl}")
```



```
KNN score (k=1) : 1.0
KNN score (k=2) : 0.9091881835016248
KNN score (k=3) : 0.8525732853499179
KNN score (k=10) : 0.7833080605150065
Linear Regression score: 0.7344414355437031
```

2.4 Qd) Trying out a Neural Network

```
[12]: from sklearn.neural_network import MLPRegressor

# Setup MLPRegressor
mlp = MLPRegressor( hidden_layer_sizes=(10,), solver='adam', activation='relu',
    tol=1E-5, max_iter=100000, verbose=True)
mlp.fit(X, y.ravel())

# lets make a MLP regressor prediction and redo the plots
y_pred_mlp = mlp.predict(M)

mlp_score = mlp.score(X, y)
```

```

plt.plot(m, y_pred_lin, "r", label="Linear Regression")
plt.plot(m, y_pred_knn, "b", label="KNN (k=3)")
plt.plot(m, y_pred_mlp, "k", label="MLP (h=10)")

plt.legend()
plt.show()

y_nn_pred = mlp.predict(X_cyprus)
print(f"Predictions for Cyprus (GDP = 22587 USD): {y_nn_pred[0]}")

# Scores
print(f"KNN score (k=3)      : {knn_score3}")
print(f"Linear Regression score: {R2_skl}")
print(f"MLP score              : {mlp_score}")

```

```

Iteration 1, loss = 241269671.70241207
Iteration 2, loss = 238627172.97791645
Iteration 3, loss = 236002859.05747169
Iteration 4, loss = 233396981.59040323
Iteration 5, loss = 230809782.89627913
Iteration 6, loss = 228241495.31556553
Iteration 7, loss = 225692340.60716501
Iteration 8, loss = 223162529.39726055
Iteration 9, loss = 220652260.68325421
Iteration 10, loss = 218161721.39591217
Iteration 11, loss = 215691086.02211234
Iteration 12, loss = 213240516.28986719
Iteration 13, loss = 210810160.91656119
Iteration 14, loss = 208400155.42062542
Iteration 15, loss = 206010621.99617901
Iteration 16, loss = 203641669.44951037
Iteration 17, loss = 201293393.19566348
Iteration 18, loss = 198965875.31284085
Iteration 19, loss = 196659184.65184656
Iteration 20, loss = 194373376.99736831
Iteration 21, loss = 192108495.27754754
Iteration 22, loss = 189864569.81800640
Iteration 23, loss = 187641618.63629106
Iteration 24, loss = 185439647.77255344
Iteration 25, loss = 183258651.65221578
Iteration 26, loss = 181098613.47635674
Iteration 27, loss = 178959505.63559490
Iteration 28, loss = 176841290.14334562
Iteration 29, loss = 174743919.08446059
Iteration 30, loss = 172667335.07543904
Iteration 31, loss = 170611471.73260182
Iteration 32, loss = 168576254.14485231
Iteration 33, loss = 166561599.34789708

```

Iteration 34, loss = 164567416.79705611
Iteration 35, loss = 162593608.83606538
Iteration 36, loss = 160640071.15954152
Iteration 37, loss = 158706693.26704603
Iteration 38, loss = 156793358.90695116
Iteration 39, loss = 154899946.50856197
Iteration 40, loss = 153026329.60119081
Iteration 41, loss = 151172377.21911317
Iteration 42, loss = 149337954.29154515
Iteration 43, loss = 147522922.01698315
Iteration 44, loss = 145727138.22143203
Iteration 45, loss = 143950457.70020947
Iteration 46, loss = 142192732.54316923
Iteration 47, loss = 140453812.44331515
Iteration 48, loss = 138733544.98889995
Iteration 49, loss = 137031775.93920210
Iteration 50, loss = 135348349.48426476
Iteration 51, loss = 133683108.48895615
Iteration 52, loss = 132035894.72177133
Iteration 53, loss = 130406549.06884965
Iteration 54, loss = 128794911.73372027
Iteration 55, loss = 127200822.42332166
Iteration 56, loss = 125624120.52086209
Iteration 57, loss = 124064645.24610542
Iteration 58, loss = 122522235.80367298
Iteration 59, loss = 120996731.51995675
Iteration 60, loss = 119487971.96923667
Iteration 61, loss = 117995797.08958735
Iteration 62, loss = 116520047.28915091
Iteration 63, loss = 115060563.54333788
Iteration 64, loss = 113617187.48350392
Iteration 65, loss = 112189761.47763158
Iteration 66, loss = 110778128.70352770
Iteration 67, loss = 109382133.21502563
Iteration 68, loss = 108001620.00166319
Iteration 69, loss = 106636435.04228225
Iteration 70, loss = 105286425.35297588
Iteration 71, loss = 103951439.02978800
Iteration 72, loss = 102631325.28654654
Iteration 73, loss = 101325934.48819205
Iteration 74, loss = 100035118.17994155
Iteration 75, loss = 98758729.11260703
Iteration 76, loss = 97496621.26436983
Iteration 77, loss = 96248649.85929106
Iteration 78, loss = 95014671.38282225
Iteration 79, loss = 93794543.59456091
Iteration 80, loss = 92588125.53848058
Iteration 81, loss = 91395277.55084935

Iteration 82, loss = 90215861.26603399
Iteration 83, loss = 89049739.62037455
Iteration 84, loss = 87896776.85430108
Iteration 85, loss = 86756838.51284938
Iteration 86, loss = 85629791.44472325
Iteration 87, loss = 84515503.80003873
Iteration 88, loss = 83413845.02687503
Iteration 89, loss = 82324685.86674777
Iteration 90, loss = 81247898.34911059
Iteration 91, loss = 80183355.78498366
Iteration 92, loss = 79130932.75979878
Iteration 93, loss = 78090505.12554350
Iteration 94, loss = 77061949.99228118
Iteration 95, loss = 76045145.71911576
Iteration 96, loss = 75039971.90466595
Iteration 97, loss = 74046309.37710661
Iteration 98, loss = 73064040.18383175
Iteration 99, loss = 72093047.58078739
Iteration 100, loss = 71133216.02151915
Iteration 101, loss = 70184431.14597580
Iteration 102, loss = 69246579.76910537
Iteration 103, loss = 68319549.86927830
Iteration 104, loss = 67403230.57656796
Iteration 105, loss = 66497512.16091702
Iteration 106, loss = 65602286.02021486
Iteration 107, loss = 64717444.66830928
Iteration 108, loss = 63842881.72297354
Iteration 109, loss = 62978491.89384738
Iteration 110, loss = 62124170.97036944
Iteration 111, loss = 61279815.80971681
Iteration 112, loss = 60445324.32476501
Iteration 113, loss = 59620595.47208157
Iteration 114, loss = 58805529.23996442
Iteration 115, loss = 58000026.63653474
Iteration 116, loss = 57203989.67789454
Iteration 117, loss = 56417321.37635545
Iteration 118, loss = 55639925.72874756
Iteration 119, loss = 54871707.70481382
Iteration 120, loss = 54112573.23569584
Iteration 121, loss = 53362429.20251665
Iteration 122, loss = 52621183.42506416
Iteration 123, loss = 51888744.65057988
Iteration 124, loss = 51165022.54265586
Iteration 125, loss = 50449927.67024303
Iteration 126, loss = 49743371.49677356
Iteration 127, loss = 49045266.36939914
Iteration 128, loss = 48355525.50834722
Iteration 129, loss = 47674062.99639702

Iteration 130, loss = 47000793.76847580
Iteration 131, loss = 46335633.60137734
Iteration 132, loss = 45678499.10360270
Iteration 133, loss = 45029307.70532422
Iteration 134, loss = 44387977.64847327
Iteration 135, loss = 43754427.97695119
Iteration 136, loss = 43128578.52696498
Iteration 137, loss = 42510349.91748600
Iteration 138, loss = 41899663.54083306
Iteration 139, loss = 41296441.55337832
Iteration 140, loss = 40700606.86637669
Iteration 141, loss = 40112083.13691761
Iteration 142, loss = 39530794.75899878
Iteration 143, loss = 38956666.85472170
Iteration 144, loss = 38389625.26560767
Iteration 145, loss = 37829596.54403398
Iteration 146, loss = 37276507.94478952
Iteration 147, loss = 36730287.41674873
Iteration 148, loss = 36190863.59466326
Iteration 149, loss = 35658165.79107032
Iteration 150, loss = 35132123.98831711
Iteration 151, loss = 34612668.83069985
Iteration 152, loss = 34099731.61671689
Iteration 153, loss = 33593244.29143488
Iteration 154, loss = 33093139.43896661
Iteration 155, loss = 32599350.27506009
Iteration 156, loss = 32111810.63979723
Iteration 157, loss = 31630454.99040159
Iteration 158, loss = 31155218.39415381
Iteration 159, loss = 30686036.52141373
Iteration 160, loss = 30222845.63874827
Iteration 161, loss = 29765582.60216374
Iteration 162, loss = 29314184.85044175
Iteration 163, loss = 28868590.39857747
Iteration 164, loss = 28428737.83131916
Iteration 165, loss = 27994566.29680800
Iteration 166, loss = 27566015.50031687
Iteration 167, loss = 27143025.69808733
Iteration 168, loss = 26725537.69126338
Iteration 169, loss = 26313492.81992099
Iteration 170, loss = 25906832.95719263
Iteration 171, loss = 25505500.50348486
Iteration 172, loss = 25109438.38078905
Iteration 173, loss = 24718590.02708301
Iteration 174, loss = 24332899.39082321
Iteration 175, loss = 23952310.92552606
Iteration 176, loss = 23576769.58443736
Iteration 177, loss = 23206220.81528873

Iteration 178, loss = 22840610.55513996
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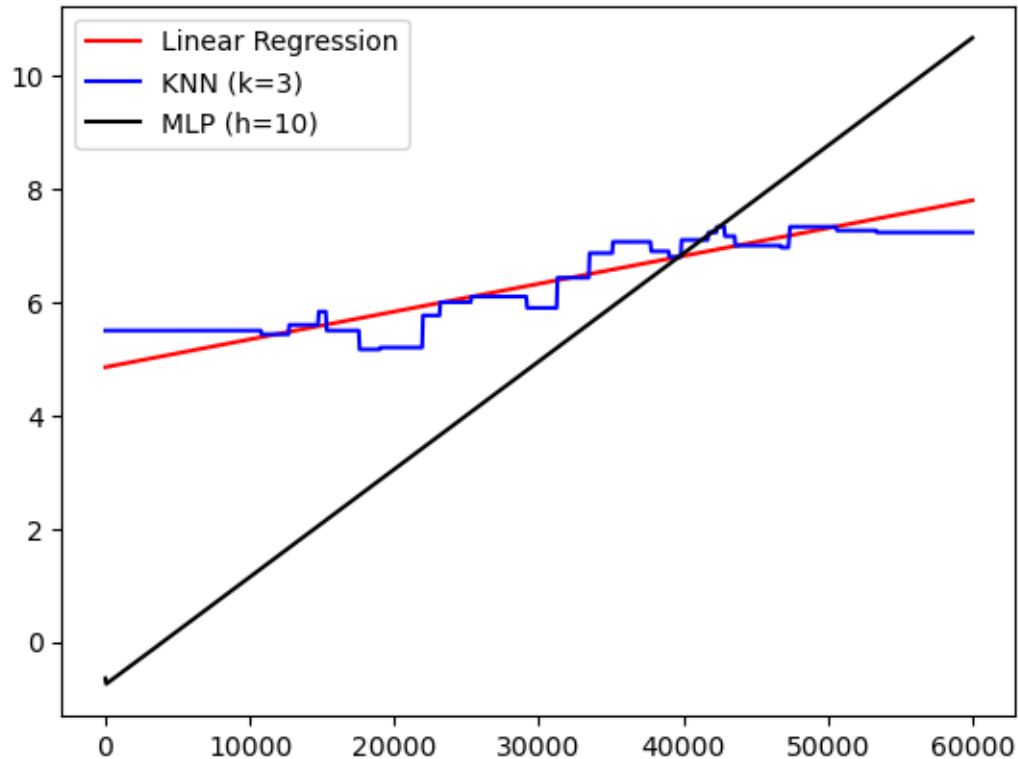
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Iteration 724, loss = 4.59476828
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Iteration 726, loss = 4.42133244
Iteration 727, loss = 4.34007482
Iteration 728, loss = 4.26225908
Iteration 729, loss = 4.18774384
Iteration 730, loss = 4.11639335
Iteration 731, loss = 4.04807724
Iteration 732, loss = 3.98267036
Iteration 733, loss = 3.92005253
Iteration 734, loss = 3.86010840
Iteration 735, loss = 3.80272721
Iteration 736, loss = 3.74780266
Iteration 737, loss = 3.69523270
Iteration 738, loss = 3.64491941
Iteration 739, loss = 3.59676877
Iteration 740, loss = 3.55069060
Iteration 741, loss = 3.50659832
Iteration 742, loss = 3.46440888
Iteration 743, loss = 3.42404257
Iteration 744, loss = 3.38542293
Iteration 745, loss = 3.34847660
Iteration 746, loss = 3.31313320
Iteration 747, loss = 3.27932523
Iteration 748, loss = 3.24698792
Iteration 749, loss = 3.21605916
Iteration 750, loss = 3.18647939
Iteration 751, loss = 3.15819148
Iteration 752, loss = 3.13114064
Iteration 753, loss = 3.10527433

Iteration 754, loss = 3.08054219
Iteration 755, loss = 3.05689591
Iteration 756, loss = 3.03428920
Iteration 757, loss = 3.01267766
Iteration 758, loss = 2.99201873
Iteration 759, loss = 2.97227164
Iteration 760, loss = 2.95339728
Iteration 761, loss = 2.93535818
Iteration 762, loss = 2.91811843
Iteration 763, loss = 2.90164361
Iteration 764, loss = 2.88590075
Iteration 765, loss = 2.87085825
Iteration 766, loss = 2.85648582
Iteration 767, loss = 2.84275446
Iteration 768, loss = 2.82963639
Iteration 769, loss = 2.81710497
Iteration 770, loss = 2.80513471
Iteration 771, loss = 2.79370119
Iteration 772, loss = 2.78278100
Iteration 773, loss = 2.77235175
Iteration 774, loss = 2.76239198
Iteration 775, loss = 2.75288113
Iteration 776, loss = 2.74379955
Iteration 777, loss = 2.73512838
Iteration 778, loss = 2.72684959
Iteration 779, loss = 2.71894593
Iteration 780, loss = 2.71140085
Iteration 781, loss = 2.70419854
Iteration 782, loss = 2.69732385
Iteration 783, loss = 2.69076229
Iteration 784, loss = 2.68449998
Iteration 785, loss = 2.67852366
Iteration 786, loss = 2.67282061
Iteration 787, loss = 2.66737867
Iteration 788, loss = 2.66218623
Iteration 789, loss = 2.65723215
Iteration 790, loss = 2.65250578
Iteration 791, loss = 2.64799694
Iteration 792, loss = 2.64369589
Iteration 793, loss = 2.63959330
Iteration 794, loss = 2.63568028
Iteration 795, loss = 2.63194829
Iteration 796, loss = 2.62838918
Iteration 797, loss = 2.62499516
Iteration 798, loss = 2.62175877
Iteration 799, loss = 2.61867289
Iteration 800, loss = 2.61573071
Iteration 801, loss = 2.61292570

Iteration 802, loss = 2.61025165
Iteration 803, loss = 2.60770260
Iteration 804, loss = 2.60527286
Iteration 805, loss = 2.60295700
Iteration 806, loss = 2.60074981
Iteration 807, loss = 2.59864634
Iteration 808, loss = 2.59664183
Iteration 809, loss = 2.59473175
Iteration 810, loss = 2.59291178
Iteration 811, loss = 2.59117776
Iteration 812, loss = 2.58952574
Iteration 813, loss = 2.58795195
Iteration 814, loss = 2.58645278
Iteration 815, loss = 2.58502479
Iteration 816, loss = 2.58366467
Iteration 817, loss = 2.58236929
Iteration 818, loss = 2.58113564
Iteration 819, loss = 2.57996086
Iteration 820, loss = 2.57884221
Iteration 821, loss = 2.57777708
Iteration 822, loss = 2.57676297
Iteration 823, loss = 2.57579749
Iteration 824, loss = 2.57487839
Iteration 825, loss = 2.57400348
Iteration 826, loss = 2.57317069
Iteration 827, loss = 2.57237805
Iteration 828, loss = 2.57162367
Iteration 829, loss = 2.57090575
Iteration 830, loss = 2.57022257
Iteration 831, loss = 2.56957249
Iteration 832, loss = 2.56895395
Iteration 833, loss = 2.56836545
Iteration 834, loss = 2.56780558
Iteration 835, loss = 2.56727297
Iteration 836, loss = 2.56676634
Iteration 837, loss = 2.56628443
Iteration 838, loss = 2.56582608
Iteration 839, loss = 2.56539017
Iteration 840, loss = 2.56497562
Iteration 841, loss = 2.56458140
Iteration 842, loss = 2.56420656
Iteration 843, loss = 2.56385015
Iteration 844, loss = 2.56351129
Iteration 845, loss = 2.56318914
Iteration 846, loss = 2.56288290
Iteration 847, loss = 2.56259180
Iteration 848, loss = 2.56231510
Iteration 849, loss = 2.56205211

Iteration 850, loss = 2.56180217
Iteration 851, loss = 2.56156465
Iteration 852, loss = 2.56133894
Iteration 853, loss = 2.56112447
Iteration 854, loss = 2.56092069
Iteration 855, loss = 2.56072708
Iteration 856, loss = 2.56054314
Iteration 857, loss = 2.56036841
Iteration 858, loss = 2.56020244
Iteration 859, loss = 2.56004478
Iteration 860, loss = 2.55989504
Iteration 861, loss = 2.55975283
Iteration 862, loss = 2.55961778
Iteration 863, loss = 2.55948953
Iteration 864, loss = 2.55936775
Iteration 865, loss = 2.55925212
Iteration 866, loss = 2.55914234
Iteration 867, loss = 2.55903811
Iteration 868, loss = 2.55893916
Iteration 869, loss = 2.55884524
Iteration 870, loss = 2.55875608
Iteration 871, loss = 2.55867145
Iteration 872, loss = 2.55859114
Iteration 873, loss = 2.55851491
Iteration 874, loss = 2.55844257
Iteration 875, loss = 2.55837392
Iteration 876, loss = 2.55830879
Iteration 877, loss = 2.55824698
Iteration 878, loss = 2.55818834
Iteration 879, loss = 2.55813270
Iteration 880, loss = 2.55807992
Iteration 881, loss = 2.55802985
Iteration 882, loss = 2.55798235
Iteration 883, loss = 2.55793730
Iteration 884, loss = 2.55789456
Iteration 885, loss = 2.55785403
Iteration 886, loss = 2.55781559
Iteration 887, loss = 2.55777913
Iteration 888, loss = 2.55774456
Iteration 889, loss = 2.55771177
Iteration 890, loss = 2.55768068
Iteration 891, loss = 2.55765120
Iteration 892, loss = 2.55762324
Iteration 893, loss = 2.55759674
Iteration 894, loss = 2.55757161
Iteration 895, loss = 2.55754778
Iteration 896, loss = 2.55752519
Iteration 897, loss = 2.55750377

Iteration 898, loss = 2.55748346
Iteration 899, loss = 2.55746422
Iteration 900, loss = 2.55744597
Iteration 901, loss = 2.55742867
Iteration 902, loss = 2.55741227
Iteration 903, loss = 2.55739672
Iteration 904, loss = 2.55738198
Iteration 905, loss = 2.55736801
Iteration 906, loss = 2.55735476
Iteration 907, loss = 2.55734221
Iteration 908, loss = 2.55733030
Iteration 909, loss = 2.55731902
Iteration 910, loss = 2.55730832
Iteration 911, loss = 2.55729817
Iteration 912, loss = 2.55728855
Iteration 913, loss = 2.55727943
Iteration 914, loss = 2.55727079
Iteration 915, loss = 2.55726259
Iteration 916, loss = 2.55725481
Iteration 917, loss = 2.55724743
Iteration 918, loss = 2.55724044
Iteration 919, loss = 2.55723380
Iteration 920, loss = 2.55722751
Iteration 921, loss = 2.55722154
Iteration 922, loss = 2.55721587
Training loss did not improve more than tol=0.000010 for 10 consecutive epochs.
Stopping.



```
Predictions for Cyprus (GDP = 22587 USD): 3.544095102346475
KNN score (k=3) : 0.8525732853499179
Linear Regression score: 0.7344414355437031
MLP score : -6.514105598162307
```

2.4.1 Answers

Can the score for MLP be compared with LinReg and KNN?

Yes, in the docs for the MLP's score function, it is calculated the same way as they are in both other fits, so they can be compared. For the MLP, we get a pretty bad fit, as we can see in both the negative R2 score, and by looking at the plot.

https://scikit-learn.org/stable/modules/generated/sklearn.neural_network.MLPRegressor.html#sklearn.neural_network.MLPRegressor

3 Cost Function

3.1 Qa Given the following $x^{(i)}$'s, construct and print the X matrix in python.

Definitions of $x^{(i)}$'s is stripped from this pdf - can be found in original exercise notebook.

```
[13]: import numpy as np

y_true = np.array([1,2,3,4]) # NOTE: you'll need this later
```

```
X = np.array([[1,2,3],[4,2,1],[3,8,5],[-9,-1,0]])

print("X_true = \n", X)
```

```
X_true =
[[ 1  2  3]
 [ 4  2  1]
 [ 3  8  5]
 [-9 -1  0]]
```

3.2 Qb Implement the L1 and L2 norms for vectors in python.

Defined without using any methods from libraries, og python primitives.

```
[14]: import math

def L1(x):
    if x.ndim != 1:
        raise ValueError("expected x to be of ndim=1, got ndim=",X.ndim)
    sum = 0
    for i in x:
        if i > 0:
            sum += i
        else:
            sum += -i
    return sum

def L2(x):
    if x.ndim != 1:
        raise ValueError("expected x to be of ndim=1, got ndim=",X.ndim)
    sum = 0
    for i in x:
        sum += i**2
    sum = sum**0.5
    return sum

def L2Dot(x):
    assert x.ndim == 1 and isinstance(x, np.ndarray)
    return x.dot(x)**0.5

# TEST vectors: here I test your implementation...calling your L1() and L2()
↳ functions
tx=np.array([1, 2, 3, -1])
ty=np.array([3,-1, 4, 1])

expected_d1=8.0
expected_d2=4.242640687119285
```

```

d1=L1(tx-ty)
d2=L2(tx-ty)

print(f"tx-ty={tx-ty}, d1-expected_d1={d1-expected_d1},  

    ↪d2-expected_d2={d2-expected_d2}")

eps=1E-9
assert math.fabs(d1-expected_d1)<eps, "L1 dist seems to be wrong"
assert math.fabs(d2-expected_d2)<eps, "L2 dist seems to be wrong"

print("OK(part-1)")

d2dot=L2Dot(tx-ty)
print("d2dot-expected_d2=",d2dot-expected_d2)
assert math.fabs(d2dot-expected_d2)<eps, "L2Ddot dist seem to be wrong"
print("OK(part-2)")

```

```

tx-ty=[-2  3 -1 -2], d1-expected_d1=0.0, d2-expected_d2=0.0
OK(part-1)
d2dot-expected_d2= 0.0
OK(part-2)

```

3.3 Qc Construct the Root Mean Square Error (RMSE) function (Equation 2-1 [HOML]).

```

[15]: def RMSE(y_pred, y_true):
    assert len(y_pred) == len(y_true) and y_pred.ndim == 1 and y_true.ndim == 1
    err_vec = y_pred - y_true
    l2 = L2(err_vec)
    return l2 / len(err_vec)**0.5

# Dummy h function:
def h(X):
    if X.ndim!=2:
        raise ValueError("excpeted X to be of ndim=2, got ndim=",X.ndim)
    if X.shape[0]==0 or X.shape[1]==0:
        raise ValueError("X got zero data along the 0/1 axis, cannot continue")
    return X[:,0]

# Calls your RMSE() function:
r=RMSE(h(X), y_true)

eps=1E-9
expected=6.57647321898295
print(f"RMSE={r}, diff={r-expected}")
assert math.fabs(r-expected)<eps, "your RMSE dist seems to be wrong"

```

```
print("OK")
```

RMSE=6.576473218982953, diff=2.6645352591003757e-15

OK

3.4 Qd Similar construct the Mean Absolute Error (MAE) function (Equation 2-2 [HOML]) and evaluate it.

```
[16]: def MAE(y_pred, y_true):
    assert len(y_pred) == len(y_true) and y_pred.ndim == 1 and y_true.ndim == 1
    err_vec = y_pred - y_true
    return L1(err_vec) / len(err_vec)

# Calls your MAE function:
r=MAE(h(X), y_true)

# TEST vector:
expected=3.75
print(f"MAE={r}, diff={r-expected}")
assert math.fabs(r-expected)<eps, "MAE dist seems to be wrong"

print("OK")
```

MAE=3.75, diff=0.0

OK

3.5 Qe Robust Code

The functions above in this journal section are already made robust with asserts.

3.6 Qf Conclusion

3.6.1 Answer

Cost functions are at the basis of what the algorithms view as a success or not. A cost function is essentially, what the measure the machine should look for when training, or try to minimize. What lies behind the cost function is therefore essential. The L1 and L2 calculations are some of the key ingredients in the basic cost functions, and understanding how they work, and what they represent is key.

When L1 is used, the absolute errors are simply summed up. But the L2 squares all of the errors, which means it is more sensitive to outlier data, and is more punishing at larger errors.

4 Dummy Classifier

4.1 Qa Load and display the MNIST data

We create the `MNIST_GetDataSet()` and `MNIST_PlotDigit()` functions, so they can be reused later.

```
[17]: import matplotlib.pyplot as plt
import matplotlib
from sklearn.datasets import fetch_openml

def MNIST_GetDataSet():
    X, y = fetch_openml('mnist_784', return_X_y=True, cache=True,
↳ as_frame=False)
    return X, y

X, y = MNIST_GetDataSet()
print(f"Shape of X: {X.shape}")
```

Shape of X: (70000, 784)

```
[18]: def MNIST_PlotDigit(data):
    image = data.reshape(28, 28)
    plt.imshow(image, cmap = matplotlib.cm.binary)
    plt.axis("off")

MNIST_PlotDigit(X[0])
plt.show()
```



4.2 Qb Add a Stochastic Gradient Decent [SGD] Classifier

Below the data is split into train and test sets, and the SGD classifier is trained.

```
[19]: from sklearn.linear_model import SGDClassifier

X_train, X_test, y_train, y_test = X[:50000], X[50000:], y[:50000], y[50000:]

y_train_5 = (y_train == '5')
y_test_5 = (y_test == '5')

sgd_clf = SGDClassifier(random_state=42)
sgd_clf.fit(X_train, y_train_5)

sgd_predict_5 = sgd_clf.predict(X_test)
```

In order to print some correctly classified and incorrectly classified digits, we first get some of the indices of these digits:

```
[20]: true_positives = []
false_positives = []
false_negatives = []
for i in range(len(y_test_5)):
    if y_test_5[i] == True and sgd_predict_5[i] == True:
```

```

        true_positives.append(i)
    elif y_test_5[i] == False and sgd_predict_5[i] == True:
        false_positives.append(i)
    elif y_test_5[i] == True and sgd_predict_5[i] == False:
        false_negatives.append(i)
print(f"Number of true positives : {len(true_positives)}")
print(f"Number of false positives: {len(false_positives)}")
print(f"Number of false negatives: {len(false_negatives)}")

```

Number of true positives : 1523

Number of false positives: 615

Number of false negatives: 284

Now we plot some digits:

```

[21]: # Plotting some true positives:
def PlotMultiple(amount, indicies, X):
    fig, axes = plt.subplots(1, amount, figsize=(12, 4))
    for i in range(amount):
        plt.subplot(1, amount, i + 1)
        MNIST_PlotDigit(X[indicies[i]])

    plt.tight_layout()
    plt.show()

PlotMultiple(10, true_positives, X_test)

```



```

[22]: # Plotting some false positives
PlotMultiple(10, false_positives, X_test)

```



```

[23]: # Plotting some false negatives
PlotMultiple(10, false_negatives, X_test)

```



4.3 Qc Implement a dummy binary classifier

Below is a (very stupid) DummyClassifier, that simply takes in a 'strategy' and classifies everything as that strategy:

```
[24]: from sklearn.metrics import accuracy_score
      from sklearn.base import BaseEstimator, ClassifierMixin
      import numpy as np

      class DummyClassifier(BaseEstimator, ClassifierMixin):
          def __init__(self, strategy):
              self.strategy = strategy

          def fit(self, X, y=None):
              # Actually do nothing
              return self

          def predict(self, X):
              n_samples = X.shape[0]
              return np.full(n_samples, self.strategy)

          def score(self, X, y):
              y_pred = self.predict(X)
              return accuracy_score(y, y_pred)
```

Testing the classifier and printing the score:

```
[25]: dummy = DummyClassifier(False)
      dummy.fit(X_train, y_train_5)

      dummy_pred = dummy.predict(X_test)
      dummy_score = dummy.score(X_test, y_test_5)
      print(f"dummy_score: {dummy_score}")
```

dummy_score: 0.90965

4.3.1 Comparison with book result

Our score is 0.909, which fits perfectly with the books result of 0.909 as well. They are of course compatible as both classifiers are just guessing false on all images. This should theoretically give us an accuracy of 90%, which fits

(10 numbers, 1 of those is 5..., 90%)

4.4 Qd Conclusion

4.4.1 Answer

In general, this exercise was mostly about getting the basic machine learning and sklearn techniques into our fingers. Splitting the MNIST test set for common ML best practices, and creating a very basic binary classifier adds greatly to the understanding of how ML works - and also that it is not magic. Even a fairly simple human categorization of “5 or not 5” turns out to be not so easy, as can be seen on the plots of the false positives and false negatives. This was quite revealing, as some of the 5’s looks like they should be easily recognizable.

Also, this exercise gave some insight into how python classes work together with the sklearn library, when creating our own classifier. Something that might be useful when a very specific type of classification is needed, or later in the course.

Lastly, we can also conclude that it is important to think about what kind of data and classification that you are working with. Just because you have an accuracy of 90%, does not necessarily make the model good, as we see when using the dummy classifier.

5 Performance Metrics

5.1 Qa Implement the Accuracy function and test it on the MNIST data.

We added the assert at the top of UnpackPerfMetrics(), to make sure the denom is above 0.

```
[26]: import math

def UnpackPerfMetrics(y_true, y_pred):
    assert y_true.shape == y_pred.shape and y_true.shape[0] > 0
    TP, TN, FP, FN = 0, 0, 0, 0
    for i, _ in enumerate(y_pred):
        if y_true[i] == True and y_pred[i] == True:
            TP += 1
        elif y_true[i] == False and y_pred[i] == False:
            TN += 1
        elif y_true[i] == True and y_pred[i] == False:
            FN += 1
        else:
            FP += 1
    return TP, TN, FP, FN

def MyAccuracy(y_true, y_pred):
    TP, TN, FP, FN = UnpackPerfMetrics(y_true, y_pred)
    accuracy = (TP + TN) / y_true.shape[0]
    return accuracy

from sklearn.metrics import accuracy_score

def TestAccuracy(y_true, y_pred):
```

```

a0=MyAccuracy(y_true, y_pred)
a1=accuracy_score(y_true, y_pred)

print(f"MyAccuracy      = {a0}")
print(f"accuracy_score = {a1}")

eps = 1E-9
if math.fabs(a0 - a1) > eps:
    raise ValueError("Difference in MyAccuracy and accuracy_score too big!")

TestAccuracy(y_test_5, sgd_predict_5)
TestAccuracy(y_test_5, dummy_pred)

```

```

MyAccuracy      = 0.95505
accuracy_score = 0.95505
MyAccuracy      = 0.90965
accuracy_score = 0.90965

```

5.2 Qb Implement Precision, Recall and F_1 -score and test it on the MNIST data for both the SGD and Dummy classifier models

Check for denom = 0 is in UnpackPerfMetrics(), as shown in Qa.

```

[27]: from sklearn.metrics import precision_score, recall_score, f1_score
def MyPrecision(y_true, y_pred):
    TP, TN, FP, FN = UnpackPerfMetrics(y_true, y_pred)
    if TP + FP == 0: return 0.0
    return TP / (TP + FP)

def MyRecall(y_true, y_pred):
    TP, TN, FP, FN = UnpackPerfMetrics(y_true, y_pred)
    if TP + FN == 0: return 0.0
    return TP / (TP + FN)

def MyF1Score(y_true, y_pred):
    precision = MyPrecision(y_true, y_pred)
    recall = MyRecall(y_true, y_pred)
    if precision == 0 or recall == 0: return 0.0
    return 2 / (1/precision + 1/recall)

def TestMetrics(y_true, y_pred):
    p0 = MyPrecision(y_true, y_pred)
    p1 = precision_score(y_true, y_pred)

    r0 = MyRecall(y_true, y_pred)
    r1 = recall_score(y_true, y_pred)

    f1_0 = MyF1Score(y_true, y_pred)

```

```

f1_1 = f1_score(y_true, y_pred)

eps = 1E-9

print(f"MyPrecision      = {p0}")
print(f"precision_score = {p1}")
if math.fabs(p0 - p1) > eps:
    raise ValueError("Difference in MyPrecision and precision_score too big!
↪")

print(f"MyRecall        = {r0}")
print(f"recall_score    = {r1}")
if math.fabs(r0 - r1) > eps:
    raise ValueError("Difference in MyRecall and recall_score too big!")

print(f"MyF1Score       = {f1_0}")
print(f"f1_score         = {f1_1}")
if math.fabs(f1_0 - f1_1) > eps:
    raise ValueError("Difference in MyF1Score and f1_score too big!")

print("SGD Performance Metrics")
TestMetrics(y_test_5, sgd_predict_5)
print("=====")
print("Dummy Performance Metrics")
TestMetrics(y_test_5, dummy_pred)

```

SGD Performance Metrics

```

MyPrecision      = 0.7123479887745556
precision_score  = 0.7123479887745556
MyRecall         = 0.8428334255672385
recall_score     = 0.8428334255672385
MyF1Score        = 0.7721166032953104
f1_score         = 0.7721166032953105
=====

```

Dummy Performance Metrics

```

/home/lassebp7/anaconda3/lib/python3.13/site-
packages/sklearn/metrics/_classification.py:1565: UndefinedMetricWarning:
Precision is ill-defined and being set to 0.0 due to no predicted samples. Use
`zero_division` parameter to control this behavior.
  _warn_prf(average, modifier, f"{metric.capitalize()} is", len(result))

```

```

MyPrecision      = 0.0
precision_score  = 0.0
MyRecall         = 0.0
recall_score     = 0.0
MyF1Score        = 0.0
f1_score         = 0.0

```

5.3 Qc The Confusion Matrix

```
[28]: from sklearn.metrics import confusion_matrix

M_dummy = confusion_matrix(y_test_5, dummy_pred)
M_SGD = confusion_matrix(y_test_5, sgd_predict_5)

print("M_dummy:\n", M_dummy)
print("M_SGD:\n", M_SGD)

M_dummy_swapped = confusion_matrix(dummy_pred, y_test_5)
M_SGD_swapped = confusion_matrix(sgd_predict_5, y_test_5)

print("M_dummy_swapped:\n", M_dummy_swapped)
print("M_SGD_swapped:\n", M_SGD_swapped)
```

```
M_dummy:
[[18193    0]
 [ 1807    0]]
M_SGD:
[[17578   615]
 [  284 1523]]
M_dummy_swapped:
[[18193  1807]
 [    0    0]]
M_SGD_swapped:
[[17578   284]
 [  615 1523]]
```

5.3.1 Answer

From the result of running the code, the matrix returned from confusion matrix must be

[[TN, FP]

[FN, TP]]

As the dummy model never predicts true, no positives are to be found, and the second column will be all 0's

In the SDG we see that it predicts some positives, so the second column has numbers. It is mostly correct, but there are still some false positives and negatives

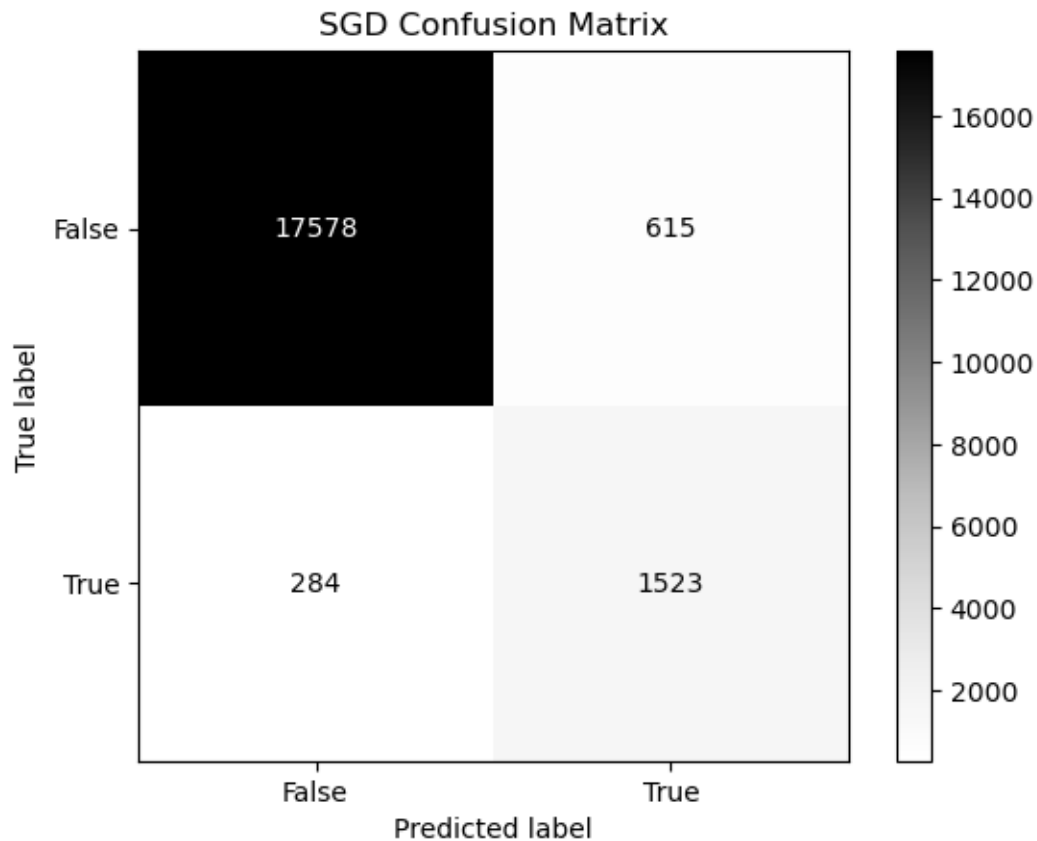
Swapped variable order Swapping the variable order swaps the places of FP and FN, and can therefore lead to some quite different conclusions.

5.4 Qd A Confusion Matrix Heat-map

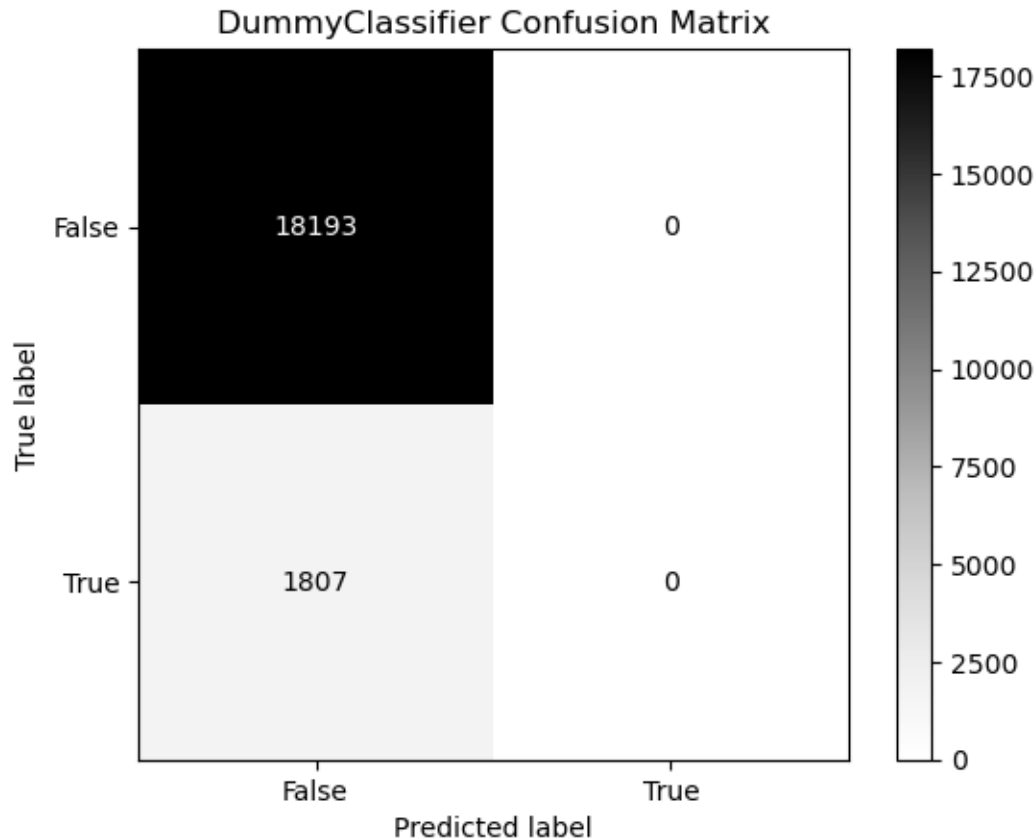
First we created heat maps for the 2x2 confusion matrixes, seen below

```
[29]: from sklearn.metrics import ConfusionMatrixDisplay
import matplotlib.pyplot as plt

ConfusionMatrixDisplay.from_predictions(y_test_5, sgd_predict_5, cmap="Grays")
plt.title("SGD Confusion Matrix")
plt.figure()
ConfusionMatrixDisplay.from_predictions(y_test_5, dummy_pred, cmap="Grays")
plt.title("DummyClassifier Confusion Matrix")
plt.show()
```



<Figure size 640x480 with 0 Axes>



But as they don't tell us much about where the errors occur, we also created them on the 10x10 data

```
[30]: from sklearn.metrics import ConfusionMatrixDisplay
from sklearn.model_selection import cross_val_predict
from sklearn.preprocessing import StandardScaler

# had to reduce size due to taking too long to create plots

scaler = StandardScaler()

X_train_small = X_train[:10000]
y_train_small = y_train[:10000]

X_train_scaled_small = scaler.fit_transform(X_train_small.astype(np.float64))

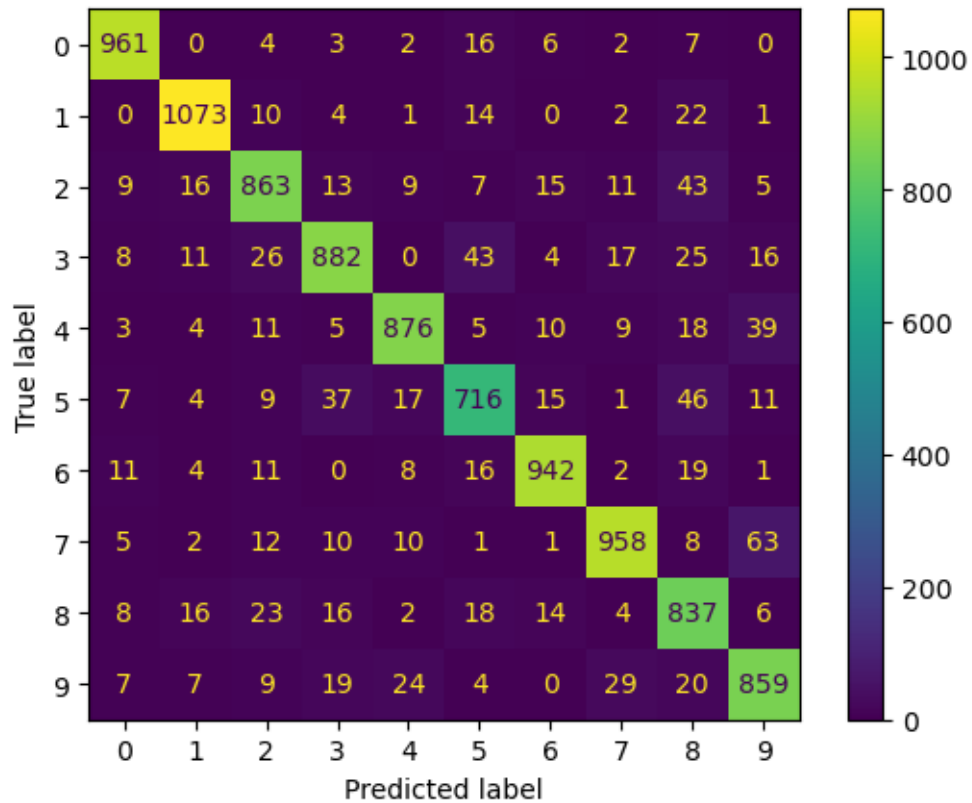
y_train_pred = cross_val_predict(sgd_clf, X_train_scaled_small, y_train_small,
                                cv=3, n_jobs=4)
ConfusionMatrixDisplay.from_predictions(y_train_small, y_train_pred)
plt.show()
```

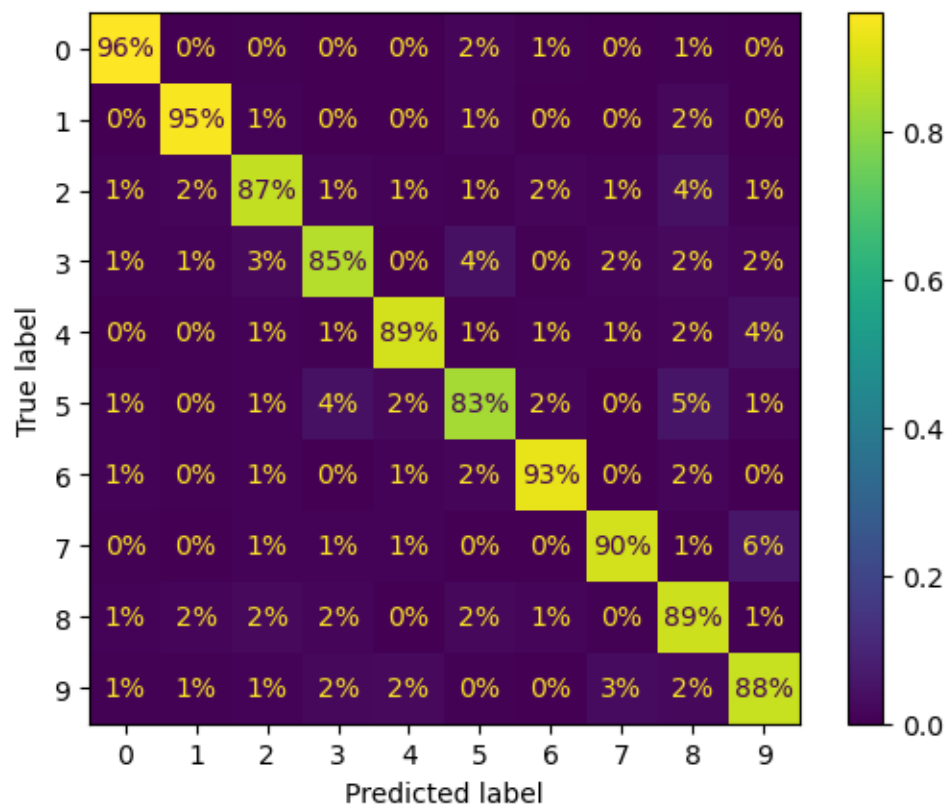
```

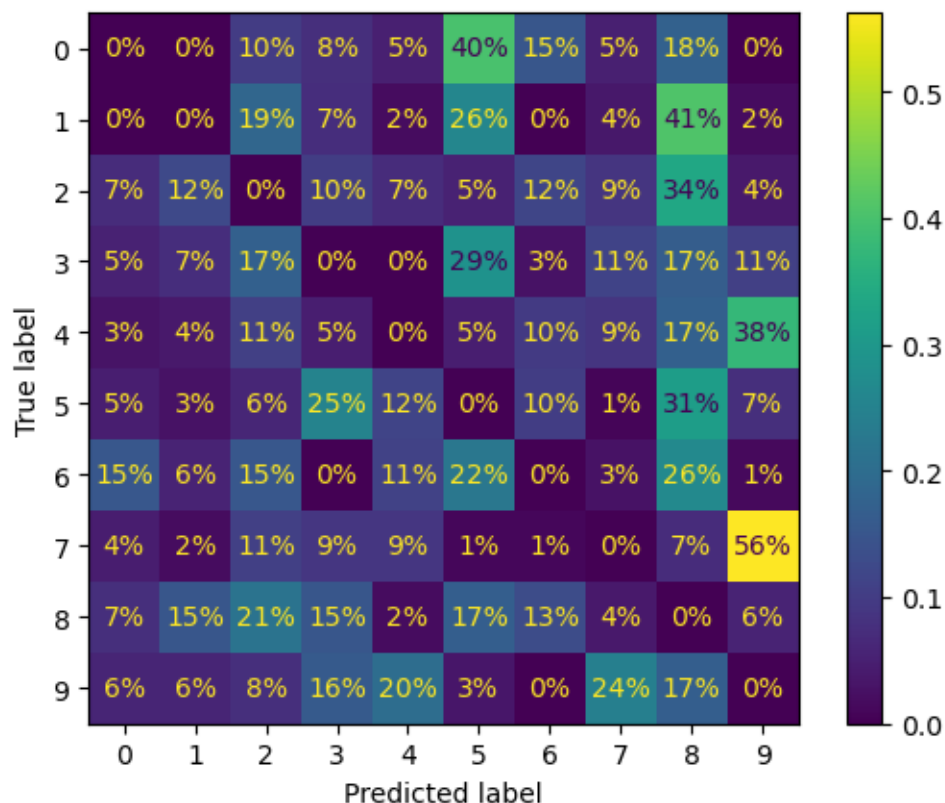
ConfusionMatrixDisplay.from_predictions(y_train_small, y_train_pred,
normalize="true", values_format=".0%")
plt.show()

sample_weight = (y_train_pred != y_train_small)
ConfusionMatrixDisplay.from_predictions(y_train_small, y_train_pred,
↪sample_weight=sample_weight, normalize="true", values_format=".0%")
plt.show()

```







In the first heatmap, we see each row being the true digit label, and the column being the predicted label.

The diagonal are the true positives, and the other numbers in each row are the falsely classified numbers, and what else they were identified as.

This is then elaborated on in the other heatmaps, in total percentages, and percentages per row. This helps us see where our model typically classifies wrong, thereby where it should be improved.

5.5 Qe Conclusion

In this exercise notebook we have looked at different types of metrics, that we can use to evaluate our models. We went through some of the functions that are used for this, such as recall, F1 and precision, and implemented them by hand, to better our understanding of how they work behind the scenes.

After that we looked at confusion matrices, that we can use to look at the numbers directly, to see how many TP, FP, TN and TP we are getting from our dataset through our model. We also compared our SGD model to the dummy model, to directly see how the dummy model works. We could see from the numbers produced here that due to how our dataset is structured and how our classification is made, why the dummy model achieved a high accuracy in the previous exercise.

Lastly, we looked at ways to visualize these heatmaps in different versions, to analyze where our model makes the most mistakes. This information can then be used to see where it needs to be

improved to achieve better results.