① La distancia media entre dos señales periódicas

$x_1(t) \in RC$ y $x_2(t) \in \mathbb{R}\mathbb{C}$; se puede expresar a partir de la potencia media de la diferencia entre ellas

$$d^2(x_1, x_2) = \bar{P}_{x_1 - x_2} = \lim_{T \to \infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 dt$$

Sea $x_1(t)$ dos señales definidas como:

$$x_1(t) = A e^{-jnwot}$$

$$x_2(t) = B e^{jmwot}$$

Con $w_0 = \frac{2\pi}{T}$; $T, A, B \in \mathbb{R}^+$ y $n, m \in \mathbb{Z}$ Determine la distancia entre las dos señales. Compruebe sus resultados con Phthon.

$$P_c(x_2 - x_1) = \underbrace{\frac{1}{T}\int |x_1(t)|^2 dt}_{\bar{P}x_1} - \frac{2}{T}\int |x_1(t)||x_2^*(t)|^2 dt + \underbrace{\frac{1}{T}\int |x_2(t)|^2 dt}_{Px_2}$$

$$Px_1 = \frac{1}{T}\int_0^T |A e^{-jnwot}|^2 dt$$

$$Px_1 = \frac{A^2}{T}\int_0^T |e^{-jnwot}|^2$$

$$Px_1 = \frac{A^2}{T}\int_0^T e^{-jnwot} e^{jnwot} dt$$

$$Px_1 \quad \frac{A^2}{T}\int_0^T e^{-jnwot - jnwot} dt$$

$$Px_1 = \frac{A^2}{T}\int_0^T 1\, dt$$

$$Px_1 = \frac{A^2}{\cancel{T}}(\cancel{T} - 0)$$

$$\boxed{Px_1 = A^2}$$

$$Px_2 = \frac{1}{T}\int_0 |B e^{-jnwot}|^2 dt$$

$$Px_2 = \frac{B^2}{T}\int e^{-jnwot} e^{jnwot} dt$$

$$Px_2 = \frac{B^2}{T}\int_0^T e^{jnwot - jnwot} dt$$

$$Px^2 = \frac{B^2}{T}\left[t \right]_0^T$$

$$Py^2 = \frac{B^2 \cancel{T}}{\cancel{T}} - \frac{B^2(0)}{T}$$

$$\boxed{Px_2 = B^2}$$

$$= -\frac{1}{T}\int_0^T (Ae^{-jnw_0t})\,(B^{*}e^{jmw_0t})^{*}\,dt$$

$$= -\frac{2}{T}\int_0^T Ae^{-jnw_0t}\,Be^{-jmw_0t}\,dt$$

$$= -\frac{2AB}{T}\int_0^T e^{-jnw_0t}\,e^{-jmw_0t}\,dt$$

$$= -\frac{2AB}{T}\int_0^T e^{-jnw_0t+(-jmw_0t)}\,dt$$

$$= -\frac{2AB}{T}\int_0^T e^{-jw_0t(n+m)}\,dt$$

tenemos dos casos $n=-m$ ó $n\neq -m$

$n=-m=0$

$$= -\frac{2AB}{T}\int_0^T e^{0}\,dt \Rightarrow -\frac{2AB}{T}\int_0^T 1\,dt$$

$$= -\frac{2AB}{\cancel{T}}\left[\cancel{T}-0\right] = \boxed{-2AB}$$

$n\neq -m = d$

$$= -\frac{2AB}{T}\int_0^T e^{-jw_0(d)t}\,dt$$

$$= -\frac{2AB}{T}\cdot\frac{1}{-jw_0d}\left[e^{-j\frac{2\pi d\cancel{T}}{\cancel{T}}} - e^{0}\right]$$

$$= \frac{\cancel{-}2AB}{\cancel{T}}\cdot\frac{1}{\cancel{-}j\frac{2\pi}{\cancel{T}}d}\left[e^{-j2\pi d}-1\right]$$

$$= \frac{2BA}{j\pi d}\left[\cos(2\pi d) - j\,\mathrm{sen}(2\pi d) - 1\right]$$

$$= \frac{2AB}{j\pi d}\left[1 - 0 - 1\right]$$

$$= 0$$

$$= d^2 = A^2 + B^2 - 2AB \quad = \boxed{\sqrt{A^2+B^2-2AB}}$$

② Encuentre la señal en tiempo discreto al utilizar un conversor analogico digital con frecuencia de muestreo de 5KHz y 4 bits de capacidad de representación aplicado a la señal continuo:

$$x(t) = 3\cos(1000\pi t) + 5\,\mathrm{sen}(3000\pi t) + 10\cos(11000\pi t).$$

Ralizar la simulación del proceso de discretización (incluyendo al menos tres periodos de $x(t)$, En caso de que la discretización no sea apropiada, diseñe e implemente un conversor adecuado para la señal estudiado

$x(t) \Rightarrow$ 3 periodos

$F_s = 5\,KHz$

$$T_s = \frac{1}{5000}$$

$$x[n] = 3\cos\left(\frac{1000\pi n}{5000}\right) + 5\,\mathrm{sen}\left(\frac{3000\pi n}{5000}\right) + 10\cos\left(\frac{11000\pi n}{5000}\right)$$

$$x[n] = 3\cos\left(\frac{1}{5}\pi n\right) + 5\,\mathrm{sen}\left(\frac{3\pi n}{5}\right) + 10\cos\left(\frac{11}{5}\pi n\right)$$

La señal tiene que estar dentro del rango $(-\pi/2, \pi/2)$

$\frac{11}{5}\pi$ esta por fuera. entonces le restamo 2π.

$$\frac{11\pi}{5} - 2\pi = \frac{10\pi - 11\pi}{5} = \frac{\pi}{5} \Rightarrow \text{Aliasin.}$$

$$x[n] = 3\cos\left(\frac{1}{5}\pi n\right) + 5\,\mathrm{Sen}\left(\frac{3\pi}{5} n\right) + 10\cos\left(\frac{\pi n}{5}\right)$$

$$x[n] = 13\cos\left(\frac{1}{5}\pi n\right) + 5\,\mathrm{sen}\left(\frac{3\pi n}{5}\right)$$

Teorema de Nyquist. Para mirar si la fs es optima.

$F_s \geq 2 f_{max}$ $\qquad f_{max} = \frac{11000\pi}{2\pi} = 5{,}500$

$5000 \geq 2(5500)$

$5000 \geq 11000 \rightarrow$ Esto nos dice que Fs no es adecuado.

3. Sea $x''(t)$ la segunda derivada de la señal $x(t)$ donde $t \in [t_i, t_f]$ Demuestre que los coeficientes de la serie exponencial de fourier se puede calcular segun.

$$C_n = \frac{1}{(t_i - t_f)\, n^2 w_o^2} \int_{t_i}^{t_f} x''(t)\, e^{-Jnw_ot}\, dt \; ; \quad n \in \mathbb{Z}$$

¿Como se puede calcular los coeficientes a_n y b_n desde $x''(t)$ en la serie trigonometrica de fourier.

$$X(t) = \sum_n C_n e^{Jnw_ot}$$

$$\frac{\partial}{\partial t}\left[x(t)\right] = x'(t) = \frac{\partial}{\partial t}\left[\sum_n C_n e^{Jnw_ot}\right]$$

$$x'(t) = \sum_n C_n \frac{\partial}{\partial t}\left[e^{Jnw_ot}\right]$$

$$\frac{\partial}{\partial t}\left[x'(t)\right] = x''(t) = \frac{\partial}{\partial t}\left[\sum_n C_n \frac{\partial}{\partial t}\left[e^{Jnw_ot}\right] e^{Jnw_ot}\right]$$

$$= x''(t) = \sum_n C_n \frac{\partial^2}{\partial t}\left[e^{Jnw_ot}\right]$$

$$* \quad \frac{\partial}{\partial t}\left[e^{Jnw_ot}\right] = Jnw_o\, e^{Jnw_ot}$$

$$* \quad \frac{\partial}{\partial t}\left[Jnw_o\, e^{Jnw_ot}\right] = J^2 n^2 w_o^2\, e^{Jnw_ot}$$

$$x''(t) = \sum_n \underbrace{C_n(-n^2 w_o^2}_{\tilde{C}_n} e^{Jnw_ot})$$

$$x''(t) = \sum_n \tilde{C}_n e^{Jnw_ot}.$$

Sabemos que para calcular C_n y aplicamos con $x''(t)$

$$C_n = \frac{1}{T}\int_{t_i}^{t_f} x(t)\, e^{Jnw_ot}\, dt$$

$$\tilde{C}_n = \frac{1}{T}\int_{t_i}^{t_f} x''(t)\, e^{Jnw_ot}\, dt.$$

Remplazamos $\widetilde{C_n}$

$$-C_n n^2 w_o^2 = \frac{1}{T}\int_{T_i}^{t_f} x''(t)\, e^{Jnw_ot}\, dt.$$

$$C_n = -\frac{1}{T n^2 w_o^2}\int_{t_i}^{t_f} x''(t)\, e^{Jnw_ot}\, dt$$

$$C_n = -\frac{1}{(t_i - t_f) n^2 w_o^2}\int_{t_i}^{t_f} x''(t)\, e^{Jnw_ot}\, dt.$$

Calculamos los coeficientes a_n y b_n.

$$x(t) = \sum_{n=0}^{N} a_n \cos(nw_ot) + b_n \operatorname{sen}(nw_ot)$$

$$x'(t) = \sum_{n=0}^{N} -a_n(nw_o)\operatorname{sen}(nw_ot) + b_n(nw_o)\cos(nw_ot).$$

$$x''(t) = \sum_{n=0}^{N} \underbrace{-a_n(n^2w_o^2)}\cos(nw_ot) - \underbrace{b_n(n^2w_o^2)}\operatorname{sen}(nw_ot).$$

Calculamos a_n y b_n.

$$-a_n n^2 w_o^2 = \frac{2}{T}\int x''(t)\cos(nw_ot)\, dt$$

$$a_n = \frac{-2}{T n^2 w_o^2}\int x''(t)\cos(nw_ot)\, dt.$$

$$-b_n n^2 w_o^2 = \frac{2}{T}\int x''(t)\operatorname{sen}(nw_ot)\, dt.$$

$$b_n = \frac{-2}{T n^2 w_o^2}\int x''(t)\operatorname{sen}(nw_ot)\, dt.$$

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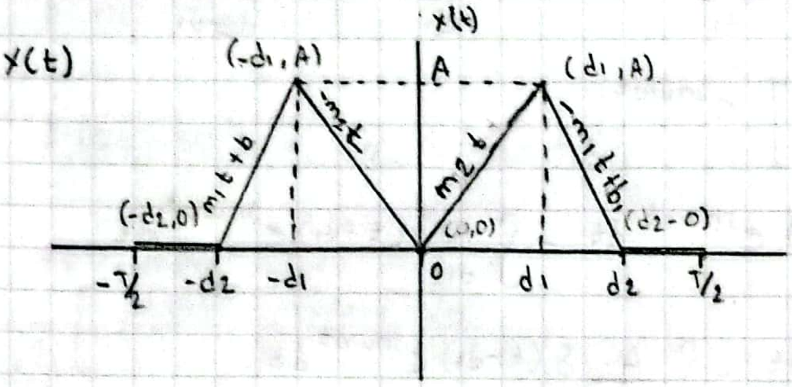
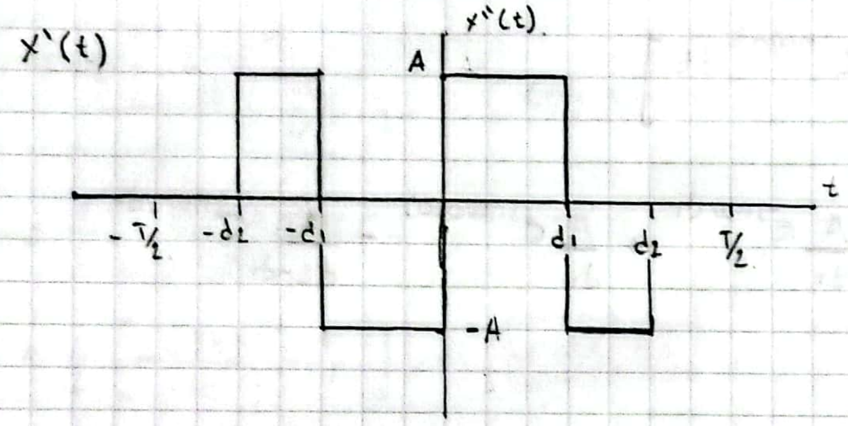
Encuentre el espectro de Fourier, su parte real, imaginaria, magnitud, fase y el error relativo para $n \in \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$ a partir de $x''(t)$ para la señal $x(t)$ en la figura 1

Compruebe el espectro obtenido con la estimación a partir de $x(t)$. presente las simulaciones de python respectivas



$$x(t) = \begin{cases} 0 & -T/2 \le t < d_2 \\ m_1 t + b_1 & -d_2 \le t < d_1 \\ -m_2 t + b_2 & -d_1 \le t \le 0 \\ m_2 t + b_2 & 0 \le t < d_1 \\ -m_1 t + b_1 & d_1 \le t \le d_2 \\ 0 & d_2 \le t \le T/2 \end{cases}$$



$m_1 = \dfrac{A}{d_2 - d_1}$ $\quad m_2 = \dfrac{A}{d_1}$

$$b_1 = x(t) - m_1 t$$

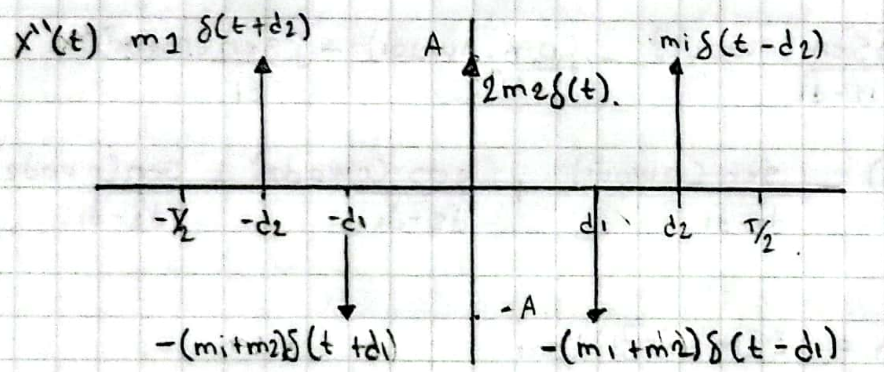
$$= A - \frac{A}{d_2 - d_1}(-d_1)$$

$$b_1 = A\left(1 + \frac{d_1}{(d_2 - d_1)}\right) =$$

$$= \frac{d_2 - \cancel{d_1} + \cancel{d_1}}{d_2 - d_1} = \frac{A d_2}{d_2 - d_1}$$

$$b_1 = m_1 d_2$$

$b_2 = 0$. Pasa por el origen.



$$x''(t) = m_1 \delta(t + d_2) - (m_1 + m_2)\delta(t + d_1) + 2 m_2 \delta(t) - (m_1 + m_2)\delta(t - d_1)$$
$$+ m_1 \delta(t - d_2)$$

$$x''(t) = m_1(\delta(t + d_2) + \delta(t - d_2)) - (m_1 + m_2)(\delta(t + d_1) + \delta(t - d_1))$$
$$+ 2 m_2 \delta(t)$$

evaluamos la $x''(t)$.

$$C_n = \frac{1}{Tn^2 w_o^2}\left[x''(t)\, e^{-jnw_ot}\, dt \right]$$

$$C_n = \frac{1}{Tn^2 w_o^2}\left[\int_T \Big(m_1[(\delta(t+d_2) + \delta(t-d_2)] - [(m_1+m_2)(\delta(t+d_1) + \delta(t-d_1)] \right.$$

$$\left. + 2m_2\delta(t)\Big) e^{-jnw_ot} \right.$$

$$C_n = \frac{1}{Tn^2 w_o^2}\left[\int_T m_1[\delta(t+d_2) + \delta(t-d_2)\, e^{-jnw_ot}\, dt \right.$$

$$- \int_T (m_1+m_2)(\delta(t+d_1) + \delta(t-d_1)\, e^{-jnw_ot}\, dt$$

$$+ \int_T 2m^2\delta(t)\, e^{-jnw_ot}$$

$$\boxed{\int_T F(t)\,\delta(t \pm t_o)\, dt = F(\pm t_o)}$$

$$C_n = \frac{1}{Tn^2 w_o^2}\left[m_1\left(\int_T \delta(t+d_2)\, e^{-jnw_ot}\, dt + \int_T \delta(t-d_2)\, e^{-jnw_ot}\, dt\right) \right.$$

$$- (m_1+m_2)\left(\int_T \delta(t+d_1)\, e^{-jnw_ot}\, dt + \int_T \delta(t-d_1)\, e^{-jnw_ot}\, dt\right)$$

$$\left. + 2m_2 \int_T \delta(t)\, e^{-jnw_ot}\, dt \right]$$

$$C_n = \frac{1}{Tn^2 w_o^2}\left[m_1\left(e^{-jnw_od_2} + e^{jnw_od_2}\right) - (m_1+m_2)\left(e^{-jnw_od_1} + e^{jnw_od_1}\right) \right.$$

$$\left. + 2m_2 e^0 \right]$$

$$C_n = \frac{1}{Tn^2 w_o^2}\left[m_1\left(\cos(nw_od_2) \cancel{-j\,sen(nw_od_2)} + \cos(nw_o d_2) \cancel{+j\,sen(nw_od_2)}\right) \right.$$

$$- (m_1+m_2)\left(\cos(nw_od_1 \cancel{-j\,sen(nw_od_1} + \cos(nw_od_1 \cancel{+j\,sen(nw_od_1)}\right)$$

$$\left. + 2m_2 \right]$$

$$C_n = \frac{-1}{Tn^2 w_0^2} \cdot \left[2m_1 \cos(n w_0 d_2) - (m_1 + m_2)\cos(n w_0 d_1) + 2m_2 \right]$$

C_n no tiene números complejos por lo tanto su parte imaginaria es 0.

Para el angulo de $C_n = \tan^{-1}\left\{\frac{Im\{cn\}}{Re\{cn\}}\right] = \frac{0}{Re\{cn\}} = 0.$

$\measuredangle C_n = 0$

$$|C_n| = \sqrt{Re^2\{cn\} + \underbrace{Im^2\{cn\}}_{0}}$$

$$|C_n| = \sqrt{Re^2\{cn\}} = |Re\{cn\}$$

$$|C_n| = \left| \frac{-1}{Tn^2 w_0^2} \left[2m_1 \cos(n w_0 d_2) - (m_1 + m_2)\cos(n w_0 d_1) + 2m_2 \right] \right|$$

Calculamos el error.

$$e_r = [\%] = \left(1 - \frac{\sum_{n=-N}^{N} |C_n|^2 P_n}{\overline{P_x}} \right) \cdot 100$$

$$\overline{P_x} = \frac{1}{T} \int_T |x(t)|^2 \, dt$$

Como es simetrica es $2\overline{P_x}$

$$\overline{P_x} = \frac{2}{T} \int_0^{T/2} (x(t))^2 dt = \frac{2}{T}\left[\int_0^{d_1} (m_2 t)^2 dt + \int_{d_1}^{d_2} (-m_1 t + b_1)^2 dt \right]$$

$$\overline{P_x} = \frac{2}{T}\left[m_2^2 \cdot \frac{t^3}{3} \Big|_0^{d_1} + \int (-m_1 t^2 + 2m_1 b_1 t + b_1^2) \, dt \right]$$

$$\overline{P}x = \frac{2}{T}\left[\frac{m_2^2 d_1^2}{3} + b_1^2 t \Big|_{d_1}^{d_2} - 2b_1 m_1 \cdot \frac{t^2}{2}\Big|_{d_1}^{d_2} + m_1^2 \cdot \frac{t^3}{3}\Big|_{d_1}^{d_2}\right]$$

$$\overline{P}x = \frac{2}{T}\left[\frac{m_2^2 d_1^2}{3} + b_1^2(d_2 - d_1) - 2b_1 m_1\left(\frac{d_2^2}{2} - \frac{d_1^2}{2}\right) + m_1^2\left(\frac{d_2^3}{3} - \frac{d_1^3}{3}\right)\right)$$

$$\overline{P}x = \frac{2}{T}\left[\frac{A^2 d_1}{3} + \frac{A^2(d_2 - d_1)}{3}\right] = \frac{2}{T}\left[\frac{A^2 d_2}{3}\right]$$

$$e_r[\%] = \left(1 - \frac{\sum_{n=N}^{N} |C_n|^2 \cdot 1}{\frac{2A^2 d_2}{3T}}\right) \cdot 100$$