

Задача 6

4.1.

$A_i \backslash B_j$	B_1	B_2	B_3	B_4	B_5		
A_1	10	8	5	9	16	17	11,0
A_2	4	3	4	11	12	8	4,0
A_3	5	10	9	7	6	10	7,0
A_4	9	2	4	1	3	9	8,0
	6	15	7	8	8		

$$Q(X) = \sum_{i=1}^4 \sum_{j=1}^5 C_{ij} x_{ij} \rightarrow \min(1); x_{ij} - \text{кан-бо из } A_i \text{ к } B_j, \text{ т.е.}$$

$$\begin{aligned} x_{11} &= \min\{6; 17\} \\ x_{12} &= 15; 11 \\ x_{13} &= 4; 8 \\ x_{14} &= 7; 4 \\ x_{15} &= 3; 10 \\ x_{21} &= 8; 7 \\ x_{24} &= \min\{1; 3\} \\ x_{45} &= 8 \end{aligned}$$

$$(2) \begin{cases} x_{11} + x_{12} + x_{13} + x_{14} = 17 \\ x_{21} + x_{22} + x_{23} + x_{24} = 8 \\ x_{31} + x_{32} + x_{33} + x_{34} = 10 \\ x_{41} + x_{42} + x_{43} + x_{44} = 9 \end{cases}$$

$$(3) \begin{cases} x_{11} + x_{21} + x_{31} + x_{41} = 6 \\ x_{12} + x_{22} + x_{32} + x_{42} = 15 \\ x_{13} + x_{23} + x_{33} + x_{43} = 7 \\ x_{14} + x_{24} + x_{34} + x_{44} = 8 \\ x_{15} + x_{25} + x_{35} + x_{45} = 8 \end{cases}$$

$$Q(X) = 10 \cdot 6 + 8 \cdot 11 + 3 \cdot 4 + 4 \cdot 4 + 29 \cdot 3 + 7 \cdot 7 + 1 \cdot 1 + 2 \cdot 8 = 60 + 88 + 12 + 28 + 84 + 7 + 2 + 16 = 332$$

	B ₀	B ₁	B ₂	B ₃	B ₄	B ₅
A ₀			8	5	9	16
A ₁	10		3	4	11	12
A ₂	4	5	10	29	2	6
A ₃	9	2	4	1	3	9
A ₄	6	15	7	8	8	8

$$x_{11} = \min\{8, 9\}$$

$$x_{12} = 15; 1$$

$$x_{22} = 14; 8$$

$$x_{31} = 6; 10$$

$$x_{33} = 7; 17$$

$$x_{35} = 4; 8$$

$$x_{42} = 6; 80$$

$$x_{55} = 4; 4$$

$$f(x_0) = 6 \cdot 8 + 2 \cdot 5 + 4 \cdot 16 + 8 \cdot 3 + 6 \cdot 5 + 4 \cdot 6 + 2 + 8 = 48 + 50 + 64 + 24 + 30 + 24 + 10 = 235$$

$$\begin{aligned} u_1 + v_2 &= 8; & v_2 &= 8; \\ u_2 + v_2 &= 3; & u_2 &= -5; \\ u_4 + v_2 &= 2; & u_4 &= -6; \\ u_4 + v_4 &= 1; & v_4 &= 7; \\ u_1 + v_3 &= 5; & v_3 &= 5; \\ u_1 + v_5 &= 16; & v_5 &= 16; \\ u_3 + v_5 &= 6; & u_3 &= -10; \\ u_3 + v_1 &= 5; & v_1 &= 15; \end{aligned}$$

$u_i \backslash v_i$	15	8	5	7	16
0	10	8	5	9	16
-5	4	3	4	11	12
-10	5	10	29	7	6
-6	9	2	4	1	3

$$\begin{aligned}
 \Delta_{12} &= 10 - (15 + 0) = -5 < 0 \\
 \Delta_{14} &= 9 - (7 + 0) = 2 > 0 \\
 \Delta_{21} &= 4 - (-5 + 15) = -6 < 0 \\
 \Delta_{23} &= 4 - (5 - 5) = 4 > 0 \\
 \Delta_{24} &= 12 - (7 - 5) = 10 > 0 \\
 \Delta_{25} &= 12 - (16 - 5) = 1 > 0 \\
 \Delta_{31} &= 5 - (5 + 0) = 0 \\
 \Delta_{32} &= 19 - (8 + 10) = 1 > 0 \\
 \Delta_{34} &= 7 - (7 + 10) = -10 < 0 \\
 \Delta_{35} &= 9 - (8 + 5) = -4 < 0 \\
 \Delta_{41} &= 4 - (5 + 6) = -7 < 0 \\
 \Delta_{45} &= 3 - (16 - 0) = -13 < 0
 \end{aligned}$$

34. $(0, 5) - \text{source}$

Usser $(4, 5) \rightarrow (4, 2) \rightarrow (1, 2) \rightarrow (1, 5)$

$$\min(4, 2) = 2$$

	B_0	B_1	B_2	B_3	B_4	B_5	
A_0		6	15	7	8	8	
A_1		10	2	5	9	16	u_1
A_2		4	3	4	11	12	u_2
A_3		5	10	23	7	6	u_3
A_4		9	2	4	1	3	u_4
A_5		15	8	5	14	18	
	v_1	v_2	v_3	v_4	v_5		

$$\Delta_{11} = 10 - 15 = -5 < 0$$

$$\Delta_{14} = 9 - 4 = 5 > 0$$

$$\Delta_{22} = 2 - (7 - 3 + 0) = -2 < 0$$

$$\Delta_{21} = -6 < 0$$

$$\min\{(2, 2), (3, 1), (1, 4)\} = 1$$

$$U_{\text{user}} (2, 1) \rightarrow (2, 2) \rightarrow (1, 2) \rightarrow (1, 5) \rightarrow (3, 5) \rightarrow (3, 1)$$

	B_1	B_2	B_3	B_4	B_5	
A_1	6	15	7	8	8	u_1
A_2	10	8	5	9	16	u_2
A_3	4	10	7			0
A_4	3	5	4	11	12	u_4
A_5	8	3	5	10	29	7
A_6	10	3	9	2	4	1
A_7	9	8	5	8	10	

$\text{cykl}(2,1) = 4$

$$\varphi = 8 \cdot 10 + 5 \cdot 7 + 4 \cdot 3 + 3 \cdot 5 + 5 \cdot 3 + 6 \cdot 7 + 8 + 3 = 210$$

	B_1	B_2	B_3	B_4	B_5	
A_1	4	5	10	11	1	8
A_2	3	5	3	10	11	9
A_3	12	14	15	10	11	7
A_4	2	5	6	7	10	16
A_5	14	6	10	2	8	

$$\varphi(X) = \sum_{i=1}^4 \sum_{j=1}^5 C_{ij} X_{ij} \rightarrow \max;$$

$$\begin{cases} X_{11} + X_{12} + X_{13} + X_{14} + X_{15} \geq 8; \\ X_{21} + X_{22} + X_{23} + X_{24} + X_{25} \geq 9; \\ X_{31} + X_{32} + X_{33} + X_{34} + X_{35} \geq 16; \\ X_{41} + X_{42} + X_{43} + X_{44} + X_{45} \geq 7; \end{cases}$$

$$X_{ij} \geq 0;$$

$$\begin{cases} X_{11} + X_{21} + X_{31} + X_{41} \leq 14; \\ X_{12} + X_{22} + X_{32} + X_{42} \leq 6; \\ X_{13} + X_{23} + X_{33} + X_{43} \leq 12; \\ X_{14} + X_{24} + X_{34} + X_{44} \leq 2; \\ X_{15} + X_{25} + X_{35} + X_{45} \leq 8; \end{cases}$$

$$\begin{aligned} \Delta_{11} &= 10 - (3) = 7; \\ \Delta_{12} &= 9 - 8 = 1; \\ \Delta_{13} &= 16 - 10 = 6; \\ \Delta_{14} &= 4 - (5 + 5) = 4; \\ \Delta_{15} &= 11 - (5 + 10) = 8; \\ \Delta_{21} &= 12 - (5 + 10) = 7; \\ \Delta_{22} &= 10 - (4 + 6) = 6; \\ \Delta_{23} &= 29 - (4 + 3) = 28; \\ \Delta_{24} &= 7 - (4 + 8) = 3; \\ \Delta_{25} &= 9 - (3 + 3) = 3; \\ \Delta_{31} &= 2 - (7 + 6) = 1; \\ \Delta_{32} &= 4 - (7 + 5) = 6; \end{aligned}$$

$\Delta_{ij} \neq 0; \Delta_{25} > 0 \Rightarrow \text{J. op. n. l.}$

$$\begin{aligned} x_{10} &= \min(14, 8) \\ x_{11} &= \min(6, 9) \\ x_{12} &= \min(6, 3) \\ x_{13} &= \min(3, 7) \\ x_{33} &= \min(10, 4) \\ x_{43} &= \min(6, 16) \\ x_{44} &= \min(2, 10) \\ x_{45} &= 8 \end{aligned}$$

$$\begin{cases} x_{11} + x_{12} + x_{13} + x_{14} = x_{15} \geq 0 \\ x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 14 \\ x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 6 \\ x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 3 \end{cases}$$

$$\varphi(x) = 4 \cdot 8 + 3 \cdot 6 + 5 \cdot 3 + 14 \cdot 3 + 15 \cdot 4 + 6 \cdot 6 + 7 \cdot 2 + 10 \cdot 8 = 280$$

	B ₁	B ₂	B ₃	B ₄	B ₅	
A ₁						8
A ₂	4	5	10	11	1	30
A ₃	3	3	3	10	11	9
A ₄	12	14	15	10	11	7
A ₅	2	5	6	7	10	16
A ₆	10	6				10
	14	6	10	2	9	

$$\begin{aligned} x_{33} &= \min(10, 2) \\ x_{35} &= 8, 9 \\ x_{42} &= 6, 16 \\ x_{12} &= 11, 1 \\ x_{14} &= 2, 8 \\ x_{13} &= 3, 6 \\ x_{11} &= 14, 3 \\ x_{12} &= 10 \end{aligned}$$

$$\begin{aligned} \Delta_{12} &= 5 - 7 = -2 \leq 0; \\ \Delta_{15} &= 1 - 12 = -11 \leq 0; \\ \Delta_{21} &= 5 - (-1 + 7) = -1 \leq 0; \\ \Delta_{23} &= 3 - (-1 + 10) = -6 \leq 0; \\ \Delta_{24} &= 10 - (-1 + 11) = 0 \leq 0; \\ \Delta_{31} &= 12 - (5 + 4) = 3 > 0; \\ \Delta_{32} &= 14 - (5 + 7) = 2 > 0; \\ \Delta_{34} &= 10 - (5 + 11) = -6 \leq 0; \\ \Delta_{35} &= 11 - (5 + 12) = -6 \leq 0; \\ \Delta_{43} &= 6 - (-2 + 10) = -2 \leq 0; \\ \Delta_{44} &= 7 - (-2 + 11) = -2 \leq 0; \\ \Delta_{45} &= 10 - (-2 + 12) = 0 \leq 0 \end{aligned}$$

$$\begin{aligned} \max(2, 3) &= 3; \\ \min(3, 1) &= 1 \end{aligned}$$

$$\varphi(x) = 4 \cdot 3 + 10 \cdot 3 + 11 \cdot 2 + 3 \cdot 1 + 11 \cdot 8 + 15 \cdot 7 + 2 \cdot 10 + 5 \cdot 6 = 340$$

$B_i \backslash B_j$	B_1	B_2	B_3	B_4	B_5
A_1	14	6	10	2	8
A_1	4	5	10	11	1
A_2	8	3	5	13	10
A_2	9	1	12	14	15
A_3	7	2	5	6	7
A_4	16	10	6	2	10
	7	10	10	11	15
	v_1	v_2	v_3	v_4	v_5

$$\begin{aligned}
 a_{11} &= -3 \\
 a_{12} &= -5 \\
 a_{13} &= -14 \\
 a_{22} &= -1 \\
 a_{23} &= -3 \\
 a_{24} &= -11 > 0 \\
 a_{32} &= -1 \\
 a_{34} &= -6 \\
 a_{35} &= -9 \\
 a_{43} &= 1 > 0 \\
 a_{44} &= 1 > 0 \\
 a_{45} &= 0
 \end{aligned}$$

$$(2, 4) \rightarrow (2, 1) \rightarrow (3, 1) \rightarrow (3, 3) \rightarrow (1, 3) \rightarrow (1, 4)$$

$B_i \backslash B_j$	B_1	B_2	B_3	B_4	B_5
A_1	14	6	6	2	8
A_1	4	5	10	11	1
A_2	8	3	5	3	10
A_2	9	1	12	14	15
A_3	7	2	5	6	7
A_4	16	10	6	2	10
	7	10	10	11	12
	v_1	v_2	v_3	v_4	v_5

$$\begin{aligned}
 a_{11} &= 3 - (-1 + 7) = -3 \\
 a_{22} &= -4 \\
 a_{23} &= -6 \\
 a_{35} &= -11 \\
 a_{35} &= -6 \\
 a_{45} &= 3 > 0 \\
 a_{43} &= 1 \\
 a_{44} &= 1 \\
 \text{max} &
 \end{aligned}$$

$$(4, 5) \rightarrow (4, 1) \rightarrow (3, 1) \rightarrow (3, 3) \rightarrow (1, 3) \rightarrow (1, 4) \rightarrow (2, 4) \rightarrow (2, 5)$$

$A_i \backslash B_j$	B_1	B_2	B_3	B_4	B_5	
A_1	4	5	10	11	1	u_1
A_2	3	5	3	10	11	u_2
A_3	12	14	15	10	11	u_3
A_4	2	5	6	7	10	u_4
	7	10	10	14	15	
	v_1	v_2	v_3	v_4	v_5	

$$\begin{aligned} \Delta_{11} &= 3 - ((-4) + 10) = -3, \\ \Delta_{12} &= -1, \\ \Delta_{13} &= -3, \\ \Delta_{14} &= -3, \\ \Delta_{15} &= -14, \\ \Delta_{21} &= -9, \\ \Delta_{22} &= -9, \\ \Delta_{23} &= -2, \\ \Delta_{24} &= 1 + 0 = -5, \text{ - best } \end{aligned}$$

$$(4,3) \rightarrow (4,1) \rightarrow (3,1) \rightarrow (3,3)$$

$A_i \backslash B_j$	B_1	B_2	B_3	B_4	B_5	
A_1	4	5	10	11	1	u_1
A_2	3	5	3	10	11	u_2
A_3	12	14	15	10	11	u_3
A_4	2	5	6	7	10	u_4
	6	9	10	13	14	
	v_1	v_2	v_3	v_4	v_5	

$$\begin{aligned} \Delta_{11} &= -2, \\ \Delta_{12} &= -4, \\ \Delta_{14} &= -2, \\ \Delta_{13} &= -12, \\ \Delta_{21} &= 0, \Rightarrow \text{minima ke eq.} \\ \Delta_{22} &= 1, \\ \Delta_{23} &= -10, \\ \Delta_{32} &= -1, \\ \Delta_{33} &= -1, \\ \Delta_{34} &= -9, \\ \Delta_{35} &= -9, \\ \Delta_{44} &= -2, \end{aligned}$$

$$\varphi = 0.8 + 10.2 + 11.7 + 12.7 + 2.7 + 5.6 + 6.2 + 10 = 322$$