

# Hash Tables

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- ▶ 2-3 Tree (BTree): add, find, and remove in  $O(\log n)$  worst case time.



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- ▶ When you look up a name, it is nice to be able to go forward or back a few names in case you misspelled it.
- ▶ If you want Milenkovic in a hash table, you better not look for Milenkovich because it will be far far away.



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public int hashCode()
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Returns a hash code for this string. The hash code for a String object is computed as

$$s[0]*31^{(n-1)} + s[1]*31^{(n-2)} + \dots + s[n-1]$$

using int arithmetic, where  $s[i]$  is the  $i$ th character of the string,  $n$  is the length of the string, and  $^$  indicates exponentiation. (The hash value of the empty string is zero.)



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An int can only hold integers in the range from -2147483648 to 2147483647.





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int hashIndex (String name, int m) {  
    int code = name.hashCode();  
    int index = code % m;  
    if (index < 0)  
        index += m;  
    return index;  
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- ▶ Entries in the same list in the first table will be in different lists in the second table.



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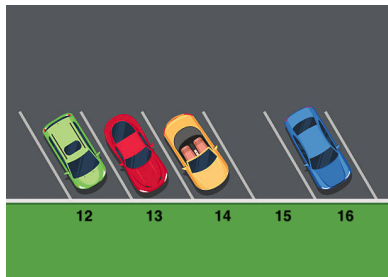
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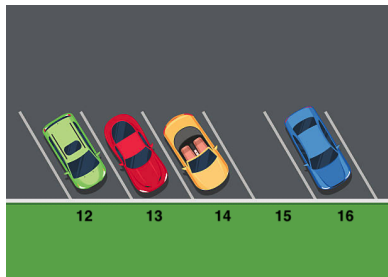




# Add

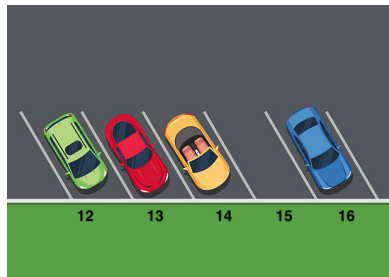


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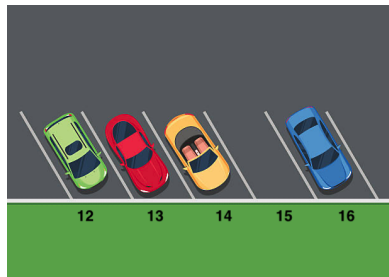
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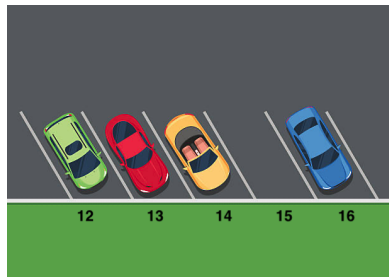
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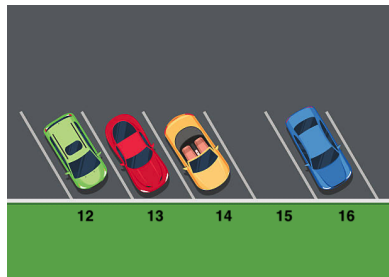
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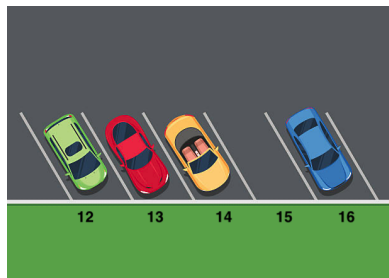
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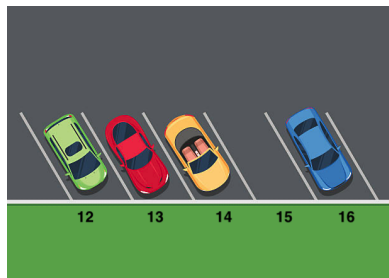
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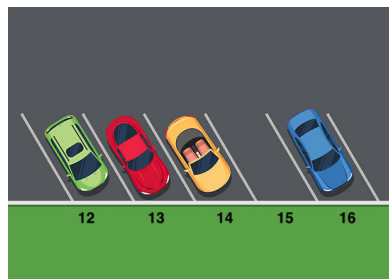
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Here is how to find Victor.



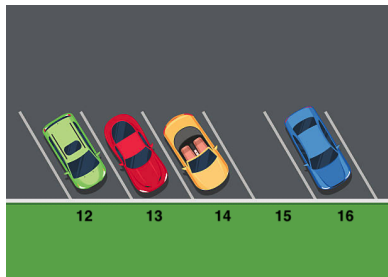
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Here is how to find Victor.

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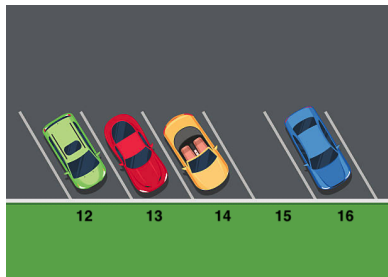
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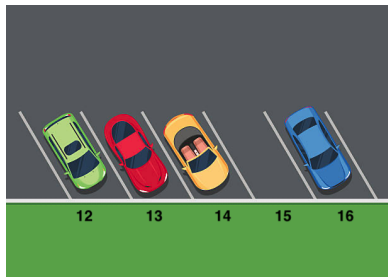
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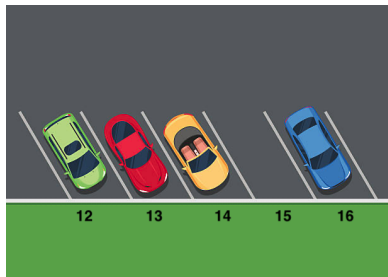
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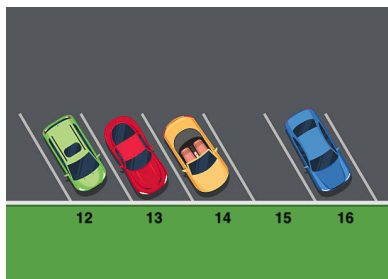
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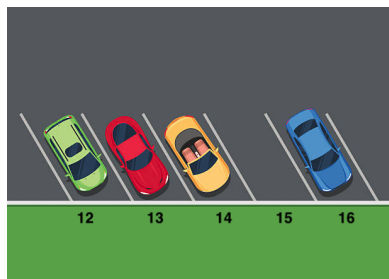
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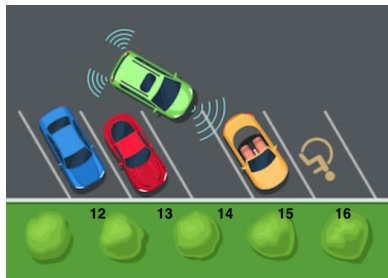


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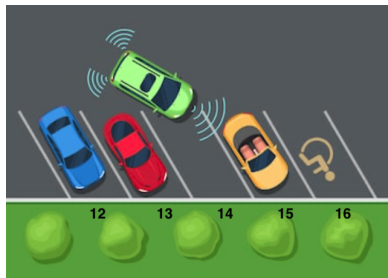


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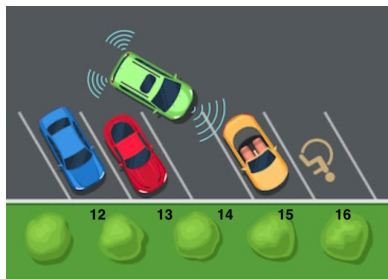


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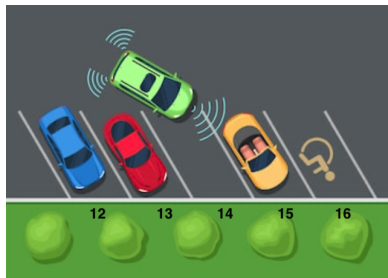
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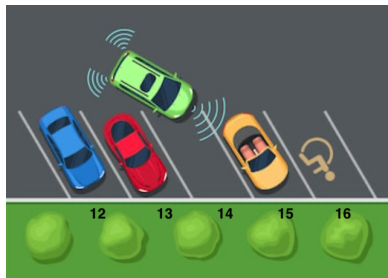
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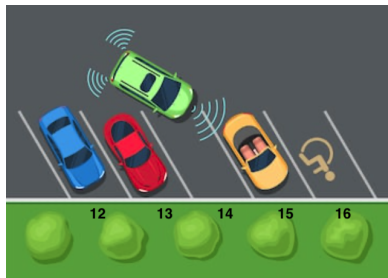
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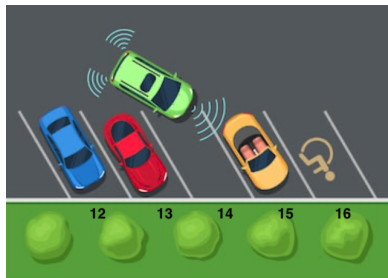
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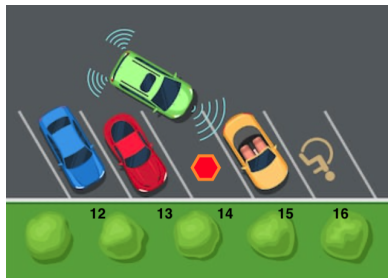
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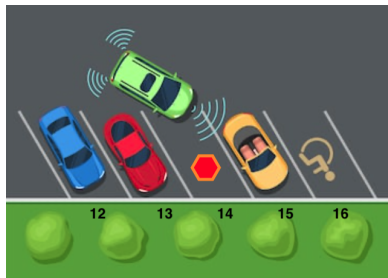
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- ▶ How do we fix this?

# Marking DELETED spaces



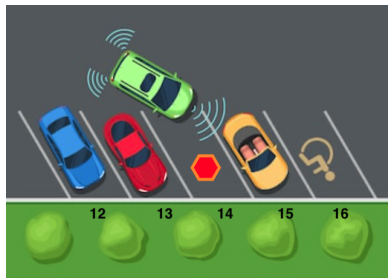
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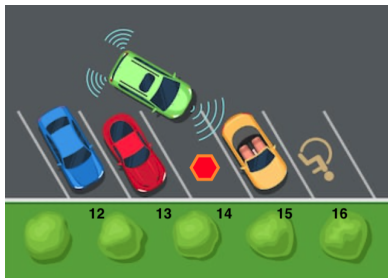
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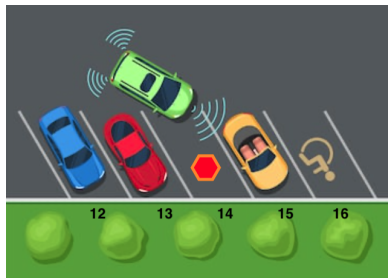
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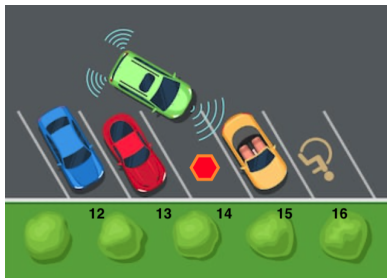
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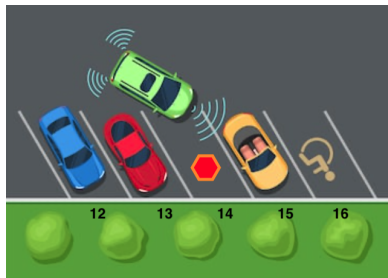
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- ▶  $O(m)$  right? So this is not like separate chaining.
- ▶ You have to keep  $n \leq m/2$ .



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- ▶ Needs to rehash and possibly reallocate when less than  $m/2$  spaces are empty.



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