Sorting

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CSC220 Programming II - Spring 2024







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- There are algorithms like Bucket Sort or Radix Sort that are faster for numbers or strings.
- There are also LOTS of other comparison-based algorithms, like Bubble Sort.
- ▶ But Insertion Sort is the same or faster than Bubble Sort on all inputs.









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Now put the 2 back in:

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We are ready to insert the 6.





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 - What is the worst possible input?







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- We want the first pivot to be 1, the second 2, the third 3, etc.
- Can you figure out the order?







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- ► Swap!





Eventually they pass each other!



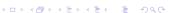
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Now we send k from i to the end. If an element equals the pivot, we swap it with i and increment i.

3 1 2 4 4 9 5 6



Recurse



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Done!







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- Easy to fix $O(n^2)$ case:
 - use a random pivot,
 - but the middle element is usually safe enough.







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- Remember: even though I am writing it like a tree, it is still just an array.

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- ► The 6 has no kids, and neither do 2, 9, nor 5.
- ▶ 1 has 6 as a kid, which is o.k.







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1 5 9 4
```

▶ 1 is o.k. (kids are 1 and 5). 3 is not. Swap with 1:

```
1
3 2
1 5 9 4
```

Still not good, swap with 1 again:

```
1
1 2
3 5 9
```

- Now it is a heap.
- ► That only takes O(n) time!







Now, let's remove the root and put it in the last element.



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- We were going to put the 6 at the root for the removal process anyway
- So swap them:

```
6
1 2
3 5 9 4
```

Now swap down the 6, but ignore the 1 at the bottom. (Decrement size.)

```
1 6 2 3 5 9 4 1 1 3 2 6 5 9 4
```





Swap the 1 and the last element, which is the 4 now, and ignore that 1 thereafter (decrement size):

```
4
3 2
6 5 9 1
1
```



➤ Swap the 1 and the last element, which is the 4 now, and ignore that 1 thereafter (decrement size):

```
4
3 2
6 5 9 1
```

Fix the 4:

```
2
3 4
6 5 9
```

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Can you continue?





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- ► Can you continue?
- ▶ The result is the array sorted in reverse order.





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Fix the 4:

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3 4
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```

- Can you continue?
- The result is the array sorted in reverse order.
- ▶ But if you can do that, you can do it right!







Good:





- ► Good:
 - Guaranteed $O(n \log n)$





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 - Guaranteed O(n log n)Heapifying is O(n), actually.





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 - apparently slower than quick sort in practice







▶ Merge Sort is a little like quick sort but backwards.





- Merge Sort is a little like quick sort but backwards.
- Just split the array in two:

```
3 1 4 1
5 9 2 6
```





- Merge Sort is a little like quick sort but backwards.
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5 9 2 6
```

Sort each recursively:

```
1 1 3 4
2 5 6 9
```





Merging



Merging

Now merge them. You only have to look at the front of each list:

```
1 3 4
2 5 6 9
3 4
2 5 6 9
1 1
3 4
5 6 9
1 1 2
5 6 9
1 1 2 3
```

```
5 6 9
1 1 2 3 4
```



Merging continued



Merging continued

Since the first list is empty, we can just copy the rest of the second list:

1 1 2 3 4 5 6 9





Merge Sort Properties



Merge Sort Properties

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- ► Good:
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- So the running time is the same.







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- So now we have 1134 in one file.





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 - Read in the 5 and 2.



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 - ► The 6 is smaller so write it out.
 - Write out the 9.
 - ▶ We have 2569.





- ▶ 1134
- ▶ 2569





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- Now we need to merge 1134 and 2569 into a single file.





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 - Write out the 5 and read in the 6.
 - Write out the 6 and read in the 9.
 - Write out the 9.
- Result: 11234569.







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- So O(n log n).







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