Binary Trees, Binary Search Trees, and Heaps

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CSC220 Programming II – Spring 2024

Review: What is a list?

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  - **theElements[**j**]** does not have a predecessor if $j = 0$.

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- Possible list orders:

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- Possible list orders:
  - unsorted,

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- Possible list orders:
  - unsorted,
    - eggs

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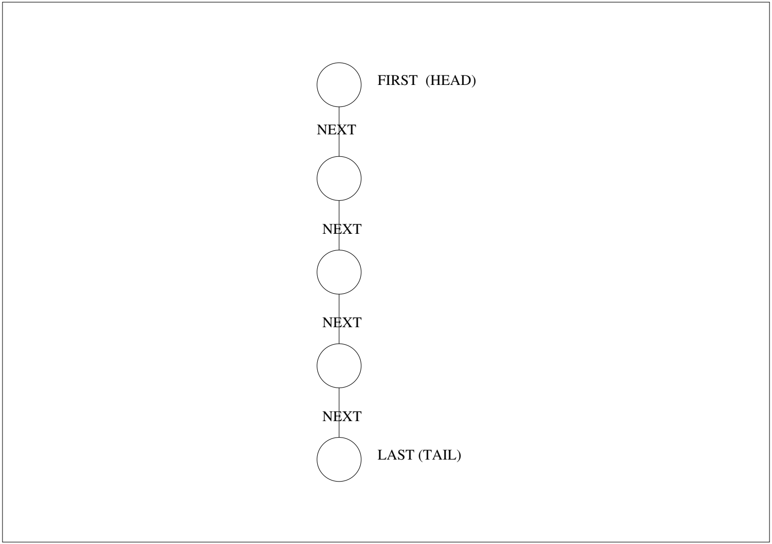
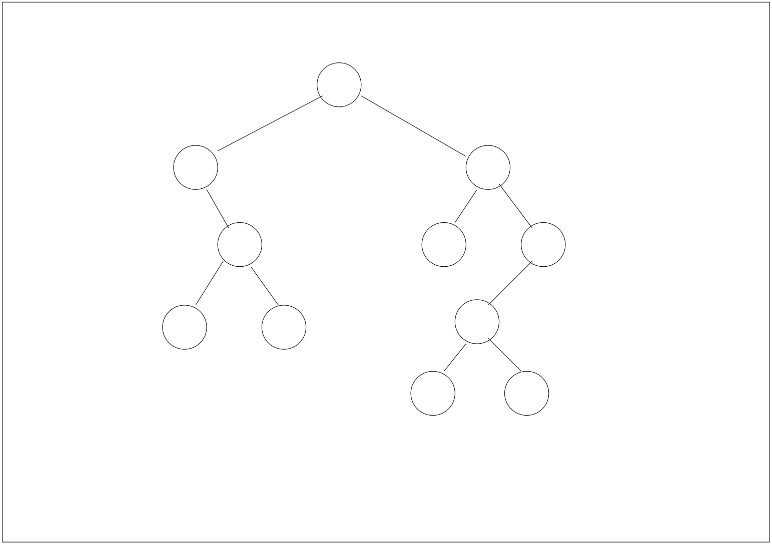
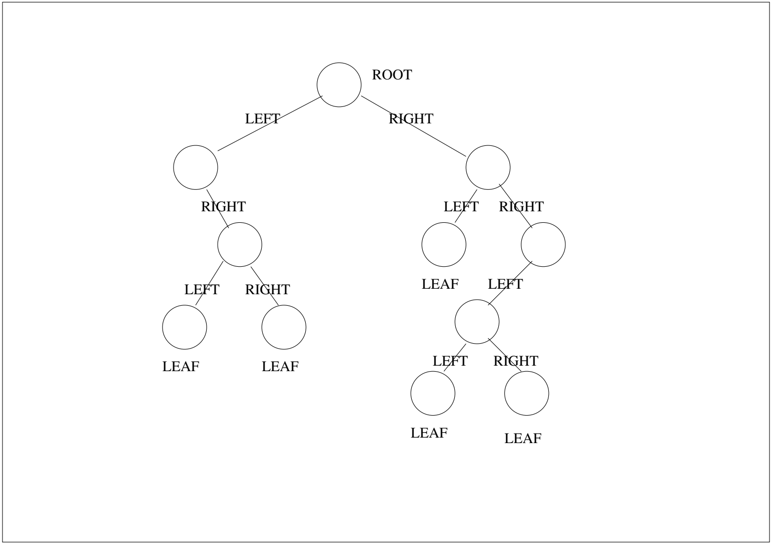
List



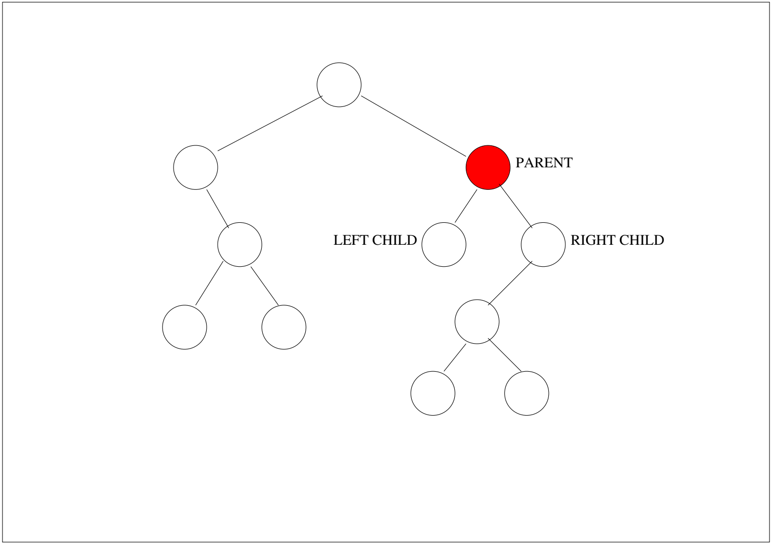
Binary Tree



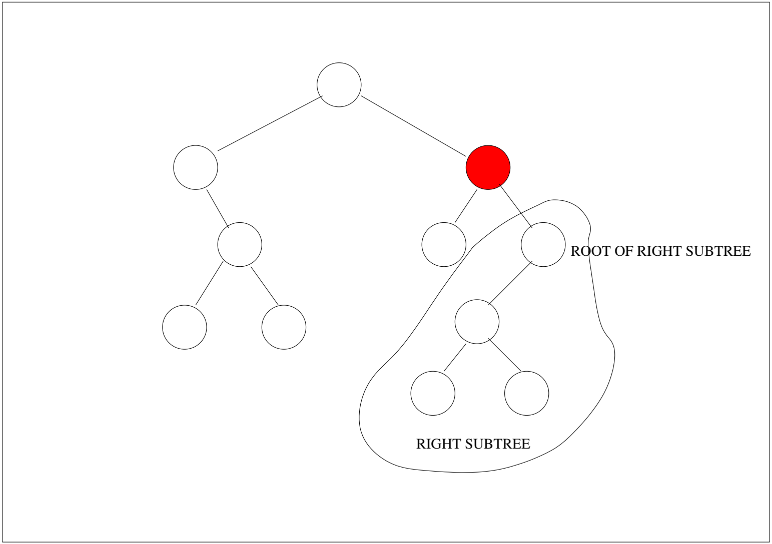
Roots and Leaves



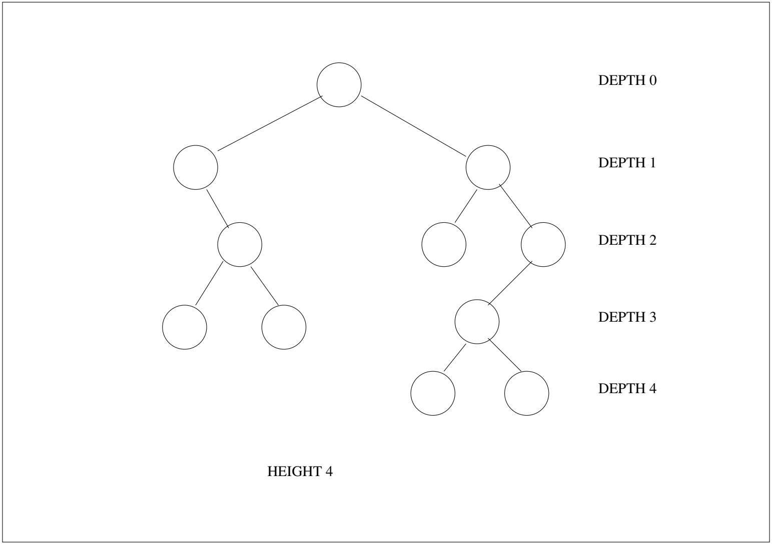
Parent, Left Child, Right Child



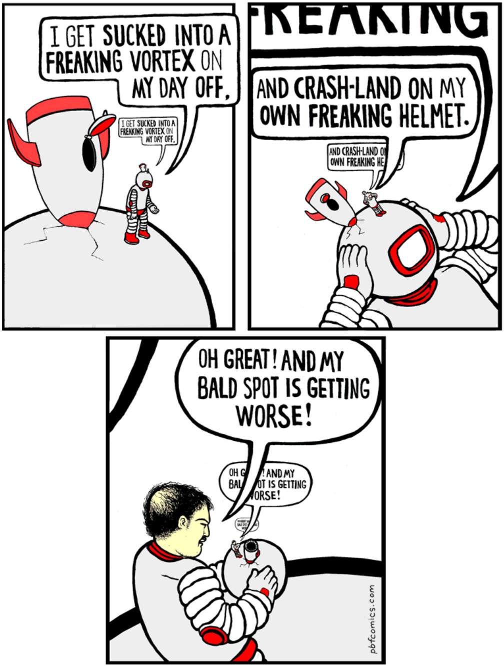
Subtree



Depth of Element and Height of Tree

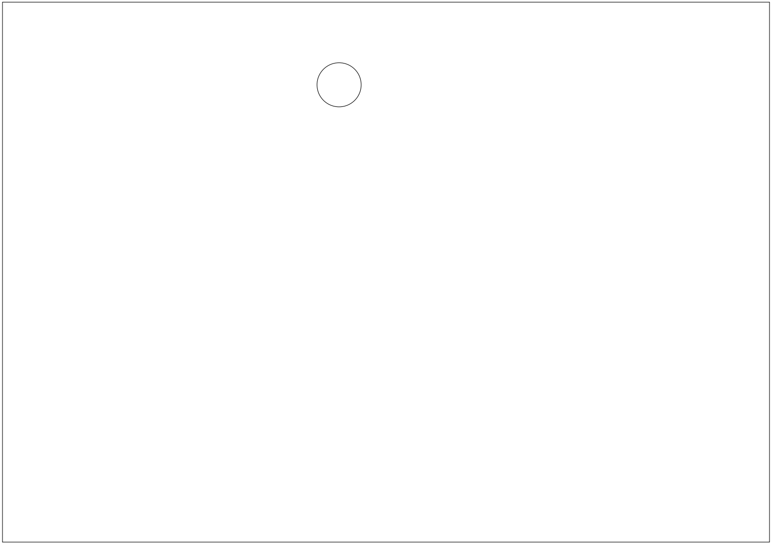


Trees can be defined RECURSIVELY

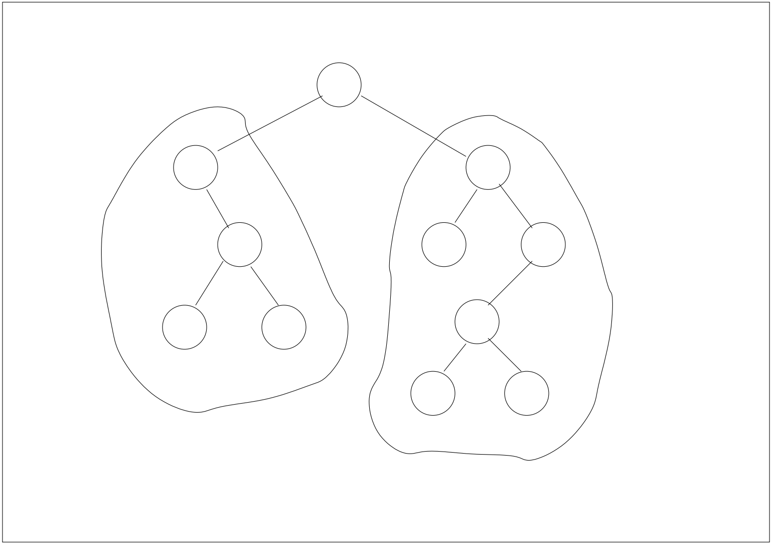


Recursive Definition: Empty

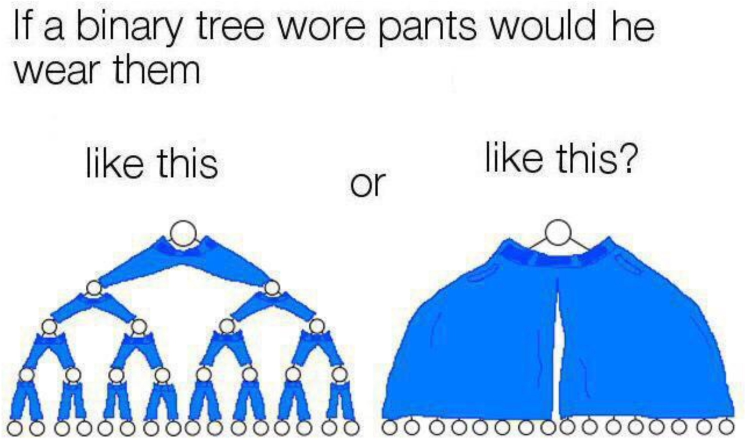
or a single element



with a left and right (sub)tree



Question


If a binary tree wore pants would he wear them
like this
or
like this?

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  - **root** variable points to root entry.

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  - Left child of **theElements[i]** is at **theElements[$2i+1$]**, but only if $2i+1 <$size. Otherwise, there is no left child.

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  - For example, at a hospital emergency room serve in order of minutes until death.

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