Measuring and Predicting Running Time

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CSC220 Programming II – Spring 2024

Outline

Running times of different implementations

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- We have two implementations of PhoneDirectory: ArrayBasedPD and SortedPD.

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Running times of different implementations

- We have two implementations of PhoneDirectory: ArrayBasedPD and SortedPD.
- Each has implementations of find, add, and remove.
- Can we compare their speeds?

lookUpEntry

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- ArrayBasedPD.lookupEntry

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  - Look for Vic?

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  - Look for Vic?
  - Calls find.

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  - Have to compare Vic with n entries, where $n =$ size, which is 6.

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- SortedPD.lookupEntry
  - Calls (SortedPD) find.

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  - Who does it compare Vic to?

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  - Better but more helpful when n (size) is large.

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  - Who does it compare Vic to?
  - Better but more helpful when n (size) is large.
  - Requires $\log_2 n$ comparisons

addOrChangeEntry

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  - find uses n comparisons

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  - Let's add Abe.
  - Abe, Ann, Bob, Eve, Ian, Jay, Zoe
  - add uses n array accesses. Actually $n - 1$ reads and n writes, where n is 7. So $2n - 1$.

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  - Let's add Abe.
  - Abe, Ann, Bob, Eve, Ian, Jay, Zoe
  - add uses n array accesses. Actually $n-1$ reads and n writes, where n is 7. So $2n-1$.
  - Total time is $\log_2 n$ comparisons (find) plus $2n-1$ array accesses (add).

removeEntry

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  - Jay, Bob, Zoe, Ian, Ann, Eve
  - Who takes longest to remove? Jay?

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  - Who takes longest to remove? Jay?
  - removeEntry calls find.

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  - removeEntry calls find.
  - find takes 1 comparison to find Jay.

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  - find takes 1 comparison to find Jay.
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  - Jay, Bob, Zoe, Ian, Ann, Eve
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  - Who takes longest to remove? Jay?
  - removeEntry calls find.
  - find takes 1 comparison to find Jay.
  - removeEntry calls remove.
  - Eve, Bob, Zoe, Ian, Ann
  - remove takes 2 array accesses to remove Jay.

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  - remove takes 2 array accesses to remove Jay.
  - Total time for 1 comparison and 2 array accesses.

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  - Total time for 1 comparison and 2 array accesses.
  - What about Eve? (Last entry)

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  - What about Eve? (Last entry)
  - Call to find takes n comparisons.

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  - remove takes 2 array accesses to remove Jay.
  - Total time for 1 comparison and 2 array accesses.
  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
  - add still uses 2 array accesses to “remove” Eve (but it could be smarter).

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  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
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  - So Eve is worst case, requiring time for n comparisons (find) and 2 array accesses (remove).

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  - removeEntry calls find.
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  - Eve, Bob, Zoe, Ian, Ann
  - remove takes 2 array accesses to remove Jay.
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  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
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  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
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  - Eve, Bob, Zoe, Ian, Ann
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  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
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  - Ann, Bob, Eve, Ian, Jay, Zoe
  - Who is the worst to remove?

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  - Jay, Bob, Zoe, Ian, Ann, Eve
  - Who takes longest to remove? Jay?
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  - find takes 1 comparison to find Jay.
  - removeEntry calls remove.
  - Eve, Bob, Zoe, Ian, Ann
  - remove takes 2 array accesses to remove Jay.
  - Total time for 1 comparison and 2 array accesses.
  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
  - add still uses 2 array accesses to "remove" Eve (but it could be smarter).
  - So Eve is worst case, requiring time for n comparisons (find) and 2 array accesses (remove).
- SortedPD.removeEntry
  - Ann, Bob, Eve, Ian, Jay, Zoe
  - Who is the worst to remove?
  - Did you figure out it was Ann?

removeEntry

- ArrayBasedPD.removeEntry
  - Jay, Bob, Zoe, Ian, Ann, Eve
  - Who takes longest to remove? Jay?
  - removeEntry calls find.
  - find takes 1 comparison to find Jay.
  - removeEntry calls remove.
  - Eve, Bob, Zoe, Ian, Ann
  - remove takes 2 array accesses to remove Jay.
  - Total time for 1 comparison and 2 array accesses.
  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
  - add still uses 2 array accesses to “remove” Eve (but it could be smarter).
  - So Eve is worst case, requiring time for n comparisons (find) and 2 array accesses (remove).
- SortedPD.removeEntry
  - Ann, Bob, Eve, Ian, Jay, Zoe
  - Who is the worst to remove?
  - Did you figure out it was Ann?
  - find takes $\log_2 n$ comparisons to locate Ann.

removeEntry

- ArrayBasedPD.removeEntry
  - Jay, Bob, Zoe, Ian, Ann, Eve
  - Who takes longest to remove? Jay?
  - removeEntry calls find.
  - find takes 1 comparison to find Jay.
  - removeEntry calls remove.
  - Eve, Bob, Zoe, Ian, Ann
  - remove takes 2 array accesses to remove Jay.
  - Total time for 1 comparison and 2 array accesses.
  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
  - add still uses 2 array accesses to "remove" Eve (but it could be smarter).
  - So Eve is worst case, requiring time for n comparisons (find) and 2 array accesses (remove).
- SortedPD.removeEntry
  - Ann, Bob, Eve, Ian, Jay, Zoe
  - Who is the worst to remove?
  - Did you figure out it was Ann?
  - find takes $\log_2 n$ comparisons to locate Ann.
  - add takes n array reads and writes to move everyone else back.

removeEntry

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  - Jay, Bob, Zoe, Ian, Ann, Eve
  - Who takes longest to remove? Jay?
  - removeEntry calls find.
  - find takes 1 comparison to find Jay.
  - removeEntry calls remove.
  - Eve, Bob, Zoe, Ian, Ann
  - remove takes 2 array accesses to remove Jay.
  - Total time for 1 comparison and 2 array accesses.
  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
  - add still uses 2 array accesses to “remove” Eve (but it could be smarter).
  - So Eve is worst case, requiring time for n comparisons (find) and 2 array accesses (remove).
- SortedPD.removeEntry
  - Ann, Bob, Eve, Ian, Jay, Zoe
  - Who is the worst to remove?
  - Did you figure out it was Ann?
  - find takes $\log_2 n$ comparisons to locate Ann.
  - add takes n array reads and writes to move everyone else back.
  - Bob, Eve, Ian, Jay, Zoe

removeEntry

- ArrayBasedPD.removeEntry
  - Jay, Bob, Zoe, Ian, Ann, Eve
  - Who takes longest to remove? Jay?
  - removeEntry calls find.
  - find takes 1 comparison to find Jay.
  - removeEntry calls remove.
  - Eve, Bob, Zoe, Ian, Ann
  - remove takes 2 array accesses to remove Jay.
  - Total time for 1 comparison and 2 array accesses.
  - What about Eve? (Last entry)
  - Call to find takes n comparisons.
  - add still uses 2 array accesses to “remove” Eve (but it could be smarter).
  - So Eve is worst case, requiring time for n comparisons (find) and 2 array accesses (remove).
- SortedPD.removeEntry
  - Ann, Bob, Eve, Ian, Jay, Zoe
  - Who is the worst to remove?
  - Did you figure out it was Ann?
  - find takes $\log_2 n$ comparisons to locate Ann.
  - add takes n array reads and writes to move everyone else back.
  - Bob, Eve, Ian, Jay, Zoe
  - Total is $\log_2 n$ comparisons (find) and $2n$ array accesses (remove).

Summary

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- ArrayBasedPD

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- ArrayBasedPD
  - find: n comparisons

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- ArrayBasedPD
  - find: n comparisons
  - add: 1 array access (usually)

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  - find: *n* comparisons
  - add: 1 array access (usually)
  - remove: 2 array accesses

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  - find: n comparisons
  - add: 1 array access (usually)
  - remove: 2 array accesses
  - lookupEntry: n comparisons

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  - find: *n* comparisons
  - add: 1 array access (usually)
  - remove: 2 array accesses
  - lookupEntry: *n* comparisons
  - addOrChangeEntry: *n* comparisons plus 1 array access (usually)

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  - find: n comparisons
  - add: 1 array access (usually)
  - remove: 2 array accesses
  - lookupEntry: n comparisons
  - addOrChangeEntry: n comparisons plus 1 array access (usually)
  - removeEntry: n comparisons plus 2 array accesses

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- ArrayBasedPD
  - find: n comparisons
  - add: 1 array access (usually)
  - remove: 2 array accesses
  - lookupEntry: n comparisons
  - addOrChangeEntry: n comparisons plus 1 array access (usually)
  - removeEntry: n comparisons plus 2 array accesses
- SortedPD

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- By the time you have opened the garage door, got in, started it up, etc., you will spend more time than just walking there.

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- So 75 microseconds.

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- So 75 microseconds.
- Notice that I used the same log base 10. You can't switch log bases in the middle, or you will get a different (and wrong) answer.

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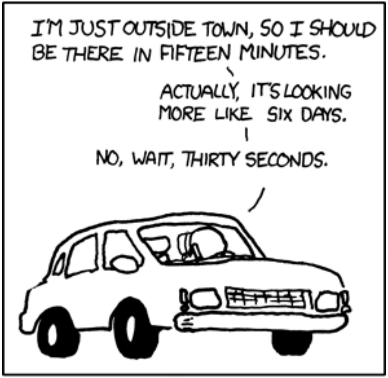
- Different log. Same answer!



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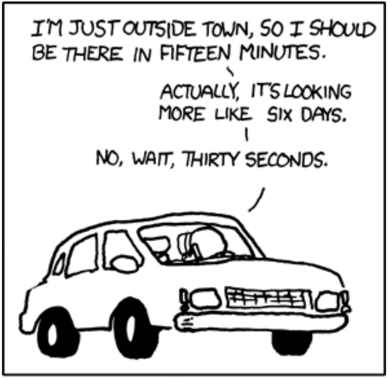

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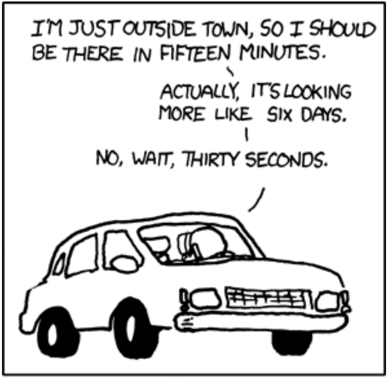

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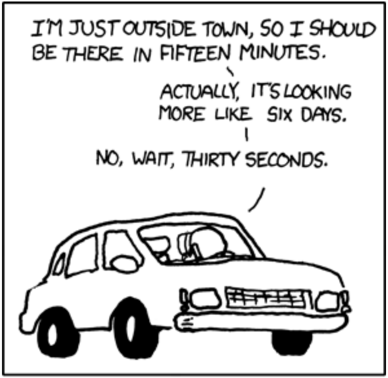

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- Answer: repeat the experiment many times and take the average.

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- Let's say it takes 1,010,203 microseconds seconds to run it 20,000 times.
- The average time 1010203 / 20000 = 50.51015 microseconds.
- Much more accurate. We can trust 5 digits (maybe).

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- Accurate predictions can make or break a business and save millions of dollars.
- To improve the accuracy of a measurement, repeat it many times and take an average.
- For example, run it once to get an approximate time. Figure out how many times you can run it in one second. Run it that many times and take the average running time.