Linked Lists

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Outline

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- A linked list is just as slow as an array (actually slower).
- But a skip list is MUCH faster.

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    - No hope of a fast addOrChange method for large n.

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  - References to the next and previous entries in the list.

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- The slides show how to use this structure to implement a phone directory.

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  - Two possibilities.

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  - Similar to entries in array with index bigger than size.

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- Draw the diagram of what should happen.
- Write the line that makes that change happen.

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- add is now $\mathrm{O}(1)$

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- One step forward, two steps back!

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    - with only two variables.
    - Keep each step simple!

This week’s application

This week's application

- We need a nice application for our Map.

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  - Yet another game!

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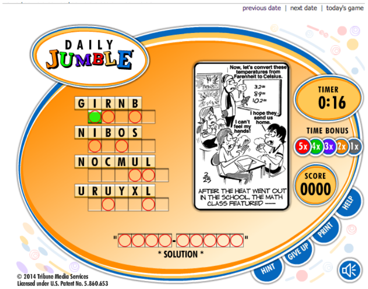

previous date | next date | today's game
DAILY JUMBLE
GIRNB
NIBOS
NOCMUL
URUYXL
Now, let's convert these temperatures from Farenheit to Celsius.
32= 89= 102=
I hope they send us home.
I can't feel my hands!
AFTER THE HEAT WENT OUT IN THE SCHOOL, THE MATH CLASS FEATURED ——
TIMER 0:16
TIME BONUS 5x 4x 3x 2x 1x
SCORE 0000
HELP
PRINT
GIVE UP
HINT
* SOLUTION *
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  - Daily Jumble

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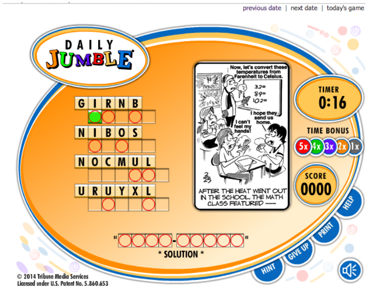
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    - Daily Jumble
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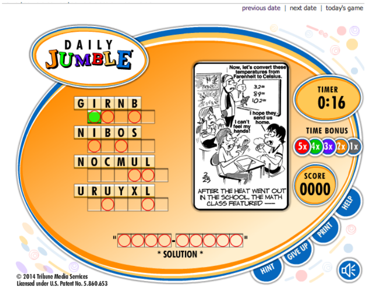
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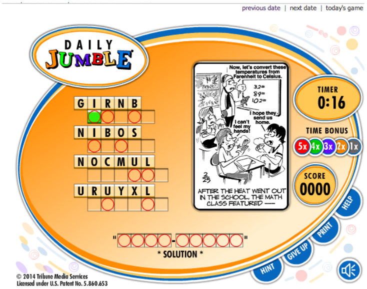
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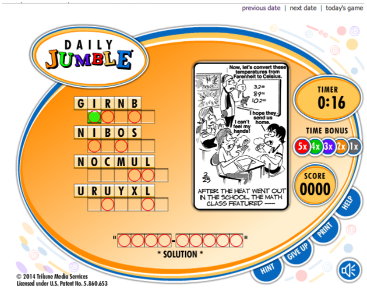
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  - Puzzle has “rtpocmue”?
  - Unscrambled is “computer”.
  - How can a Map help us to do that?

Slow Way

Slow Way

- We have a dictionary file.

Slow Way

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Slow Way

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  - But the number of orderings is 8! = 40,320, bad!.

Using a Map

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  - How about the letters in alphabetical order?

Using a Map

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  - will all be stored under the key "ader".
  - So the value will be "read" because it is last.
  - Solution is to use **List<String>** as the value type.
  - But we won't do that this time.

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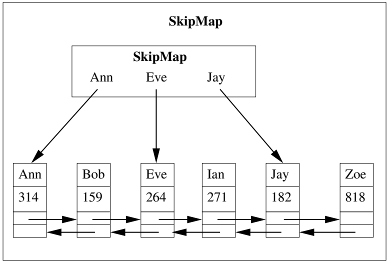
- A computer that can do a billion operations in a second
- will take 233 seconds times the number of operations per find.

SkipMap


SkipMap
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Ann
Eve
Jay
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314
Bob
159
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264
Ian
271
Jay
182
Zoe
818

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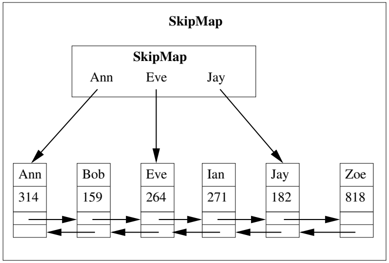

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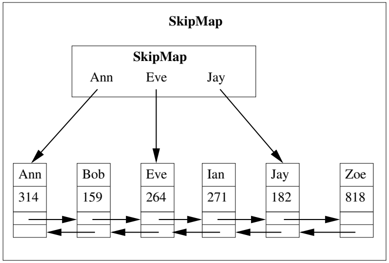

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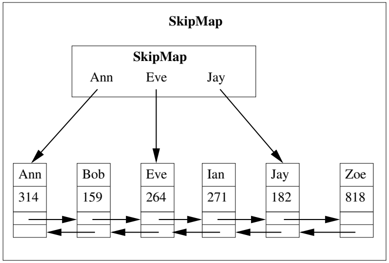

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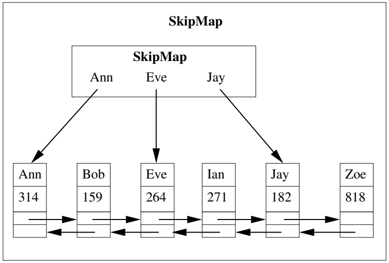

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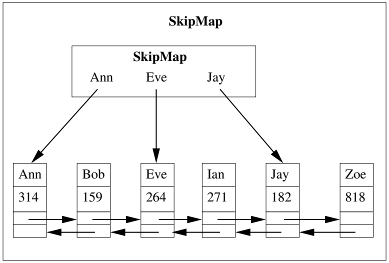

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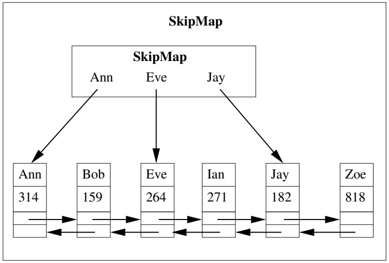

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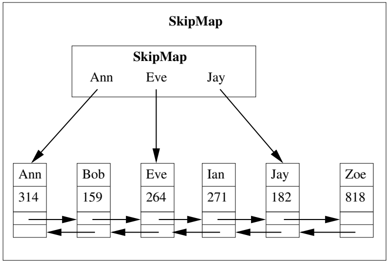
- The previous Entry, Eve, in the SkipMap is less than Ian.

SkipMap



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- Jay’s value in the smaller SkipMap is the Jay entry in the linked list.
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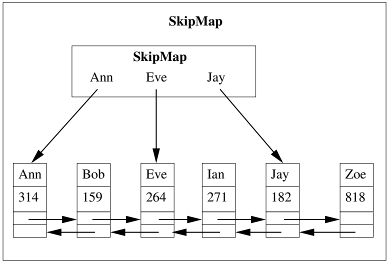
- The previous Entry, Eve, in the SkipMap is less than Ian.
- How many people do you expect between Eve and Jay?

SkipMap



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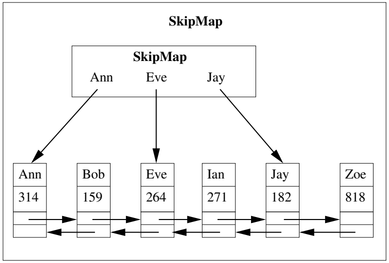

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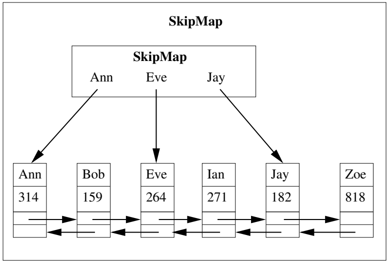

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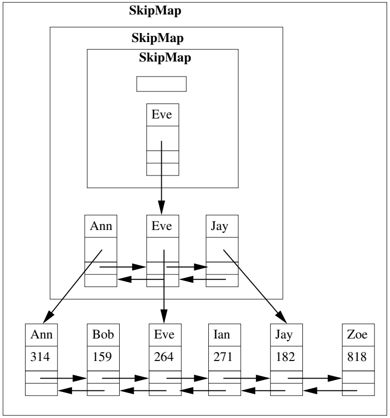
- So lookup in a SkipMap requires lookup in a SkipMap half the size plus one extra step.
- Gold coin idea! Find and get are $O(\log n)$.

SkipMap – Gold Coin?



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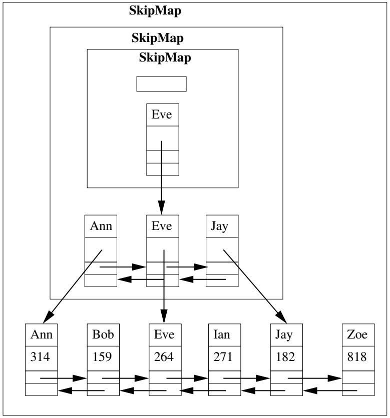

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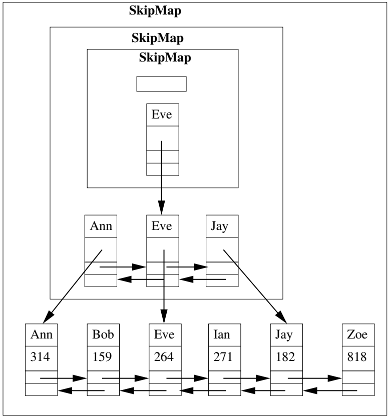

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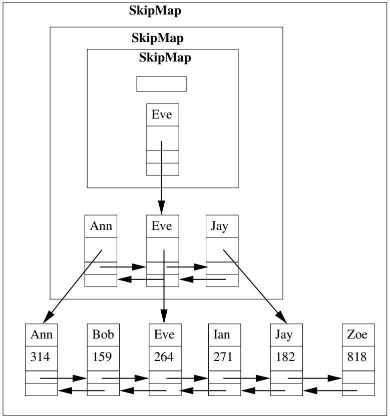

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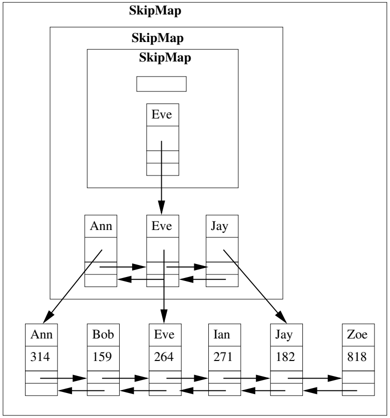

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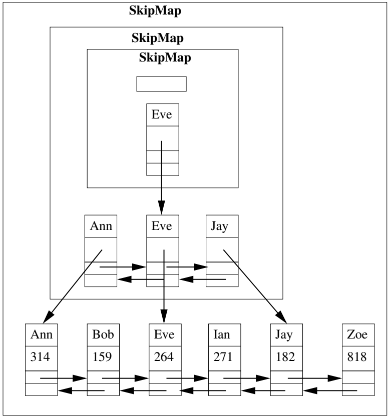

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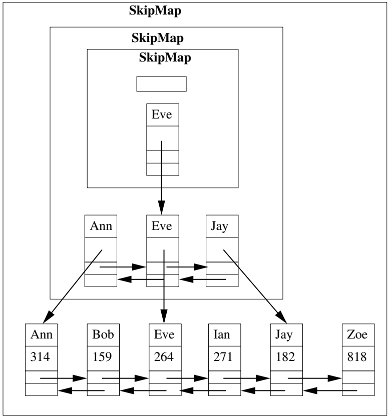

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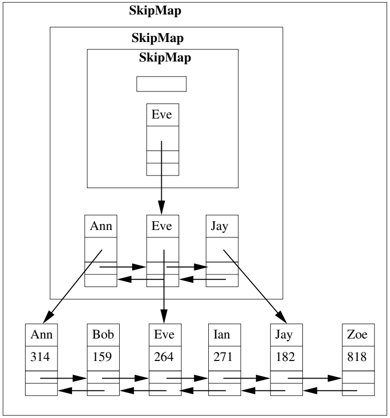

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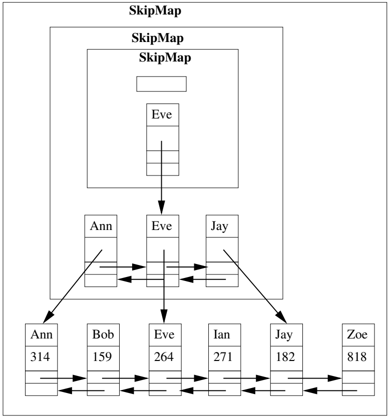
SkipMap – Gold Coin?



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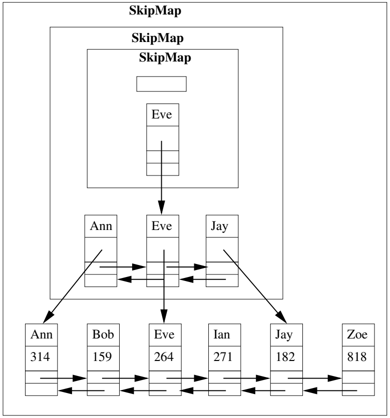
- Finding the gold coin in 1000 required
- 1 weighing

SkipMap – Gold Coin?



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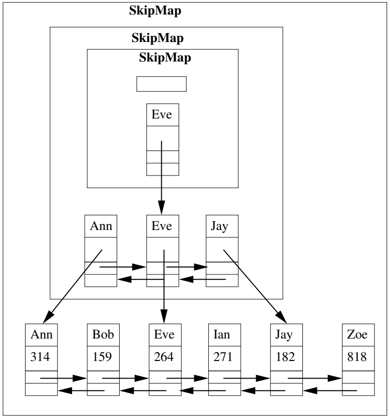
SkipMap – Gold Coin?



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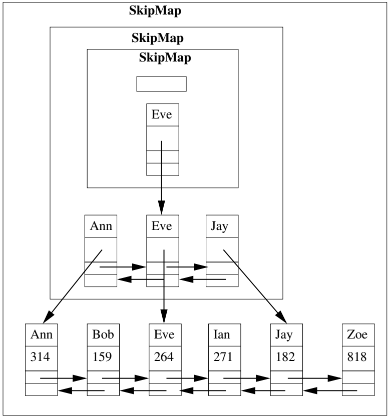

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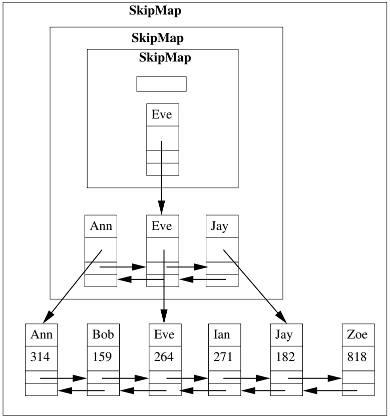

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- Finding the gold coin in 1000 required
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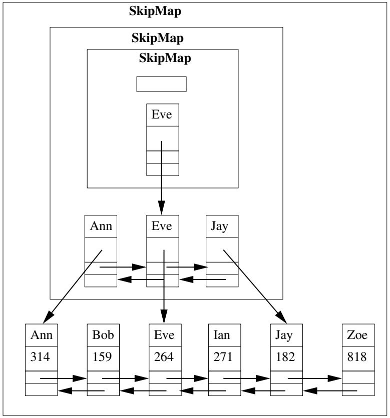
- Would the time be different if it were
- find in 500 plus 1 weighing (step forward)?

SkipMap add and remove



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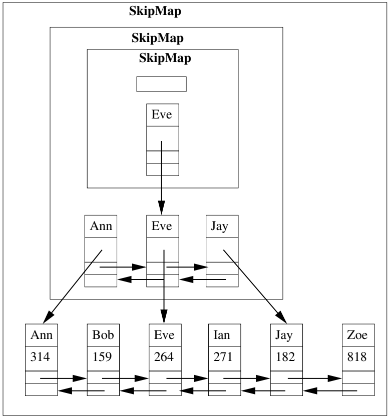

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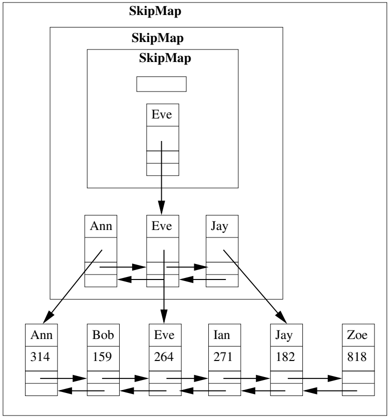
SkipMap add and remove



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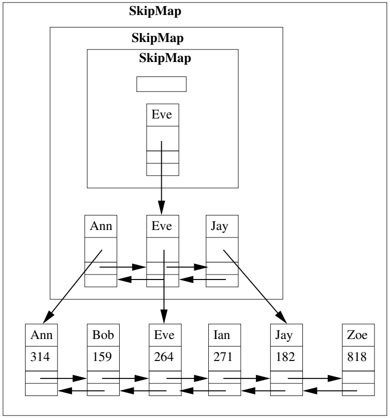

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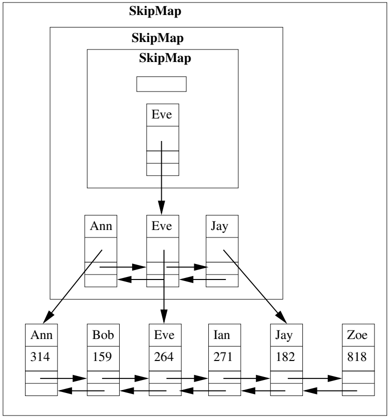

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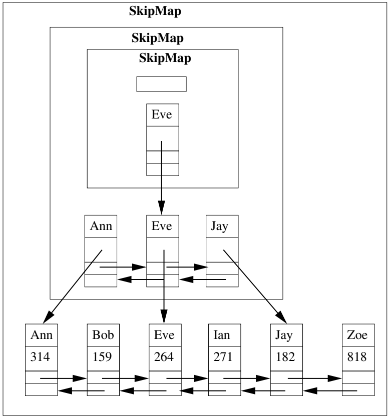

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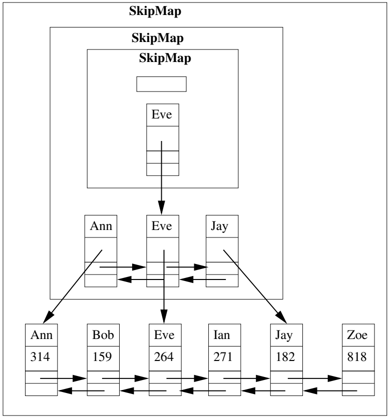

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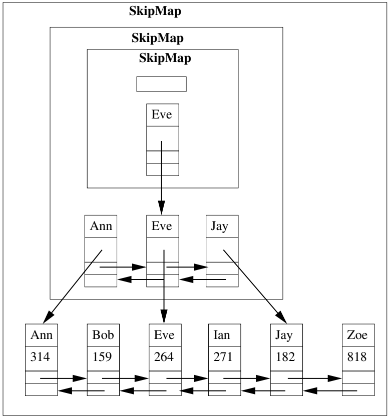

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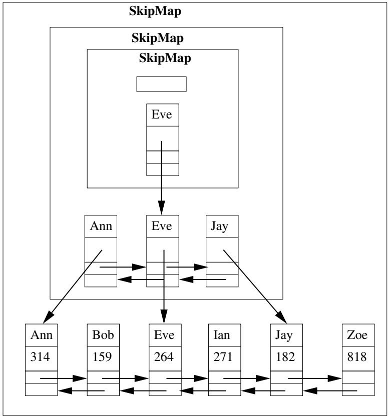

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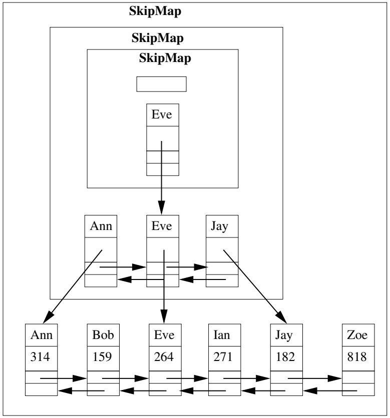

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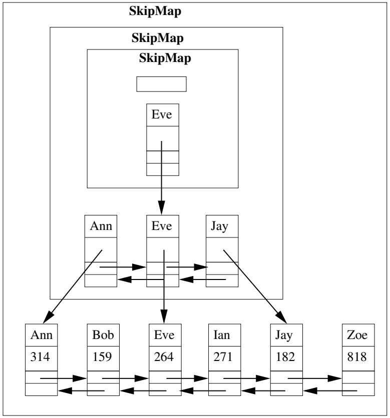

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- This also means a SkipMap only requires twice as much space as a LinkedMap.