Problem Set

Section I (Oct. 4)

1. Consider two kets $|\alpha\rangle$ and $|\beta\rangle$. Let $|i\rangle$ be a complete set of basis ket states. Suppose $\langle i \mid \alpha\rangle$ and $\langle i \mid \beta\rangle$ are known for all basis states $|i\rangle$. Find the matrix representation of the operator $|\alpha\rangle\langle\beta|$ in that basis.

2. We now consider a qubit system and let $|\alpha\rangle = |\sigma^z = +1\rangle$ and $|\beta\rangle = |\sigma^x = +1\rangle$. Write down the explicit square matrix that corresponds to $|\alpha\rangle\langle\beta|$ in the σ^z basis.

3. Construct the state $|\boldsymbol{n}\cdot\boldsymbol{\sigma} = +1\rangle$ such that

$$\boldsymbol{n}\cdot\boldsymbol{\sigma}\,|\boldsymbol{n}\cdot\boldsymbol{\sigma} = +1\rangle = (+1)\,|\boldsymbol{n}\cdot\boldsymbol{\sigma} = +1\rangle, \tag{1}$$

where $\boldsymbol{n} = (n_x, n_y, n_z)$ is a unit vector.

- $\boldsymbol{n}\cdot\boldsymbol{\sigma}$ is an *operator*

$$\boldsymbol{n}\cdot\boldsymbol{\sigma} = n_x\,\sigma^x + n_y\,\sigma^y + n_z\,\sigma^z. \tag{2}$$

- $\boldsymbol{\sigma} = (\sigma^x, \sigma^y, \sigma^z)$ is a *vector* of *operators*, i.e. each component of the vector $\boldsymbol{\sigma}$ is an operator.

If we treat the qubit as a spin, the spin operators are related by

$$\boldsymbol{S} = \frac{\hbar}{2}\,\boldsymbol{\sigma}. \tag{3}$$

4. A beam of electrons goes through a series of Stern-Gerlach measurements as follows: (a) the first measurement accepts $s_z = \hbar/2$ atoms and rejects $s_z = -\hbar/2$ atoms; (b) the second measurement accepts $s_{\boldsymbol{n}} = \hbar/2$ atoms and rejects $s_{\boldsymbol{n}} = -\hbar/2$ atoms, where $s_{\boldsymbol{n}}$ is the eigenvalue of the operator $\boldsymbol{n}\cdot\boldsymbol{S}$; (c) the third measurement accepts $s_z = -\hbar/2$ atoms and rejects $s_z = \hbar/2$ atoms. What is the intensity of the final $s_z = -\hbar/2$ beam when the $s_z = \hbar/2$ beam surviving the first measurement is normalized to unity? How must we orient the second measuring apparatus if we are to maximizing the intensity of the final $s_z = -\hbar/2$ beam?