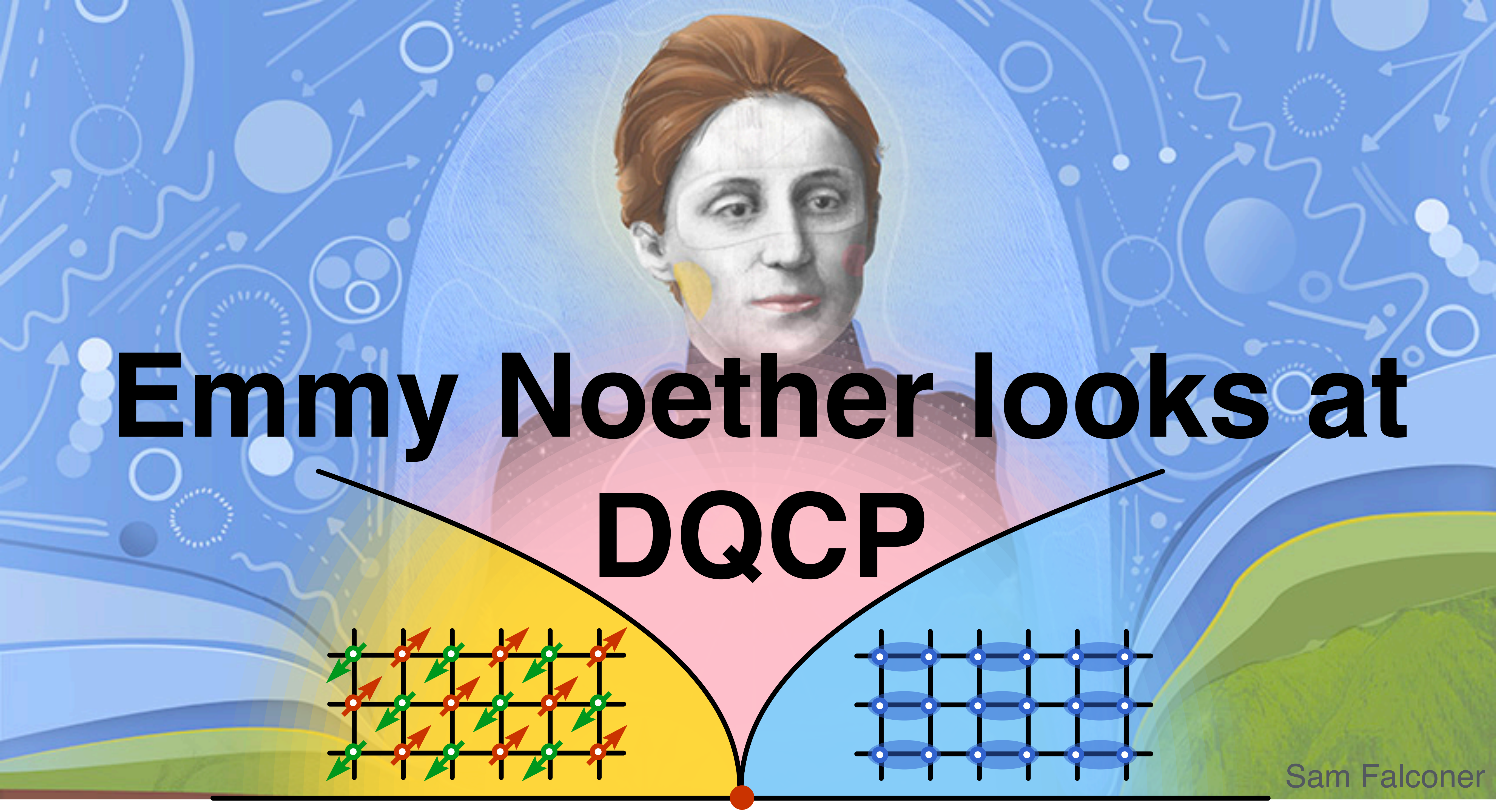
Emmy Noether looks at DQCP



Yi-Zhuang You
(UC San Diego)

Harvard CMSA, April 2019

Symmetry and Conservation Law

- Noether theorem is a profound theorem in physics.
- Continuous Symmetry ⇔ Conservation Law
  - Space/time translation ⇔ momentum/energy
  - Rotation ⇔ angular momentum
  - Internal U(1) ⇔ charge conservation

$$\partial_\mu J^\mu = \partial_t \rho + \nabla \cdot \boldsymbol{J} = 0$$

conserved current

Emmy Noether

- If the internal symmetry ∈ Lie groups, say SO(N),

  O(N) vector: $\boldsymbol{n} = (n_1, n_2, n_3, \cdots)$

  every generator T_{ab} → associated conserved current J^μ_{ab}

$$\begin{bmatrix} n_a \\ n_b \end{bmatrix} \to e^{\mathrm{i}\theta T_{ab}} \begin{bmatrix} n_a \\ n_b \end{bmatrix} \qquad \partial_\mu J^\mu_{ab} = 0$$

Symmetry Breaking and Emergence

- **Spontaneous Symmetry Breaking** (SSB)
  - Hamiltonian: symmetric → ground state: symmetry broken
  - Each broken symmetry generator
    → a Nambu-Goldstone mode (assume Lorentz invariance)
- **Emergent Symmetry**
  - Hamiltonian: no symmetry → low-energy excitations: asymptotically acquire symmetry at long distance

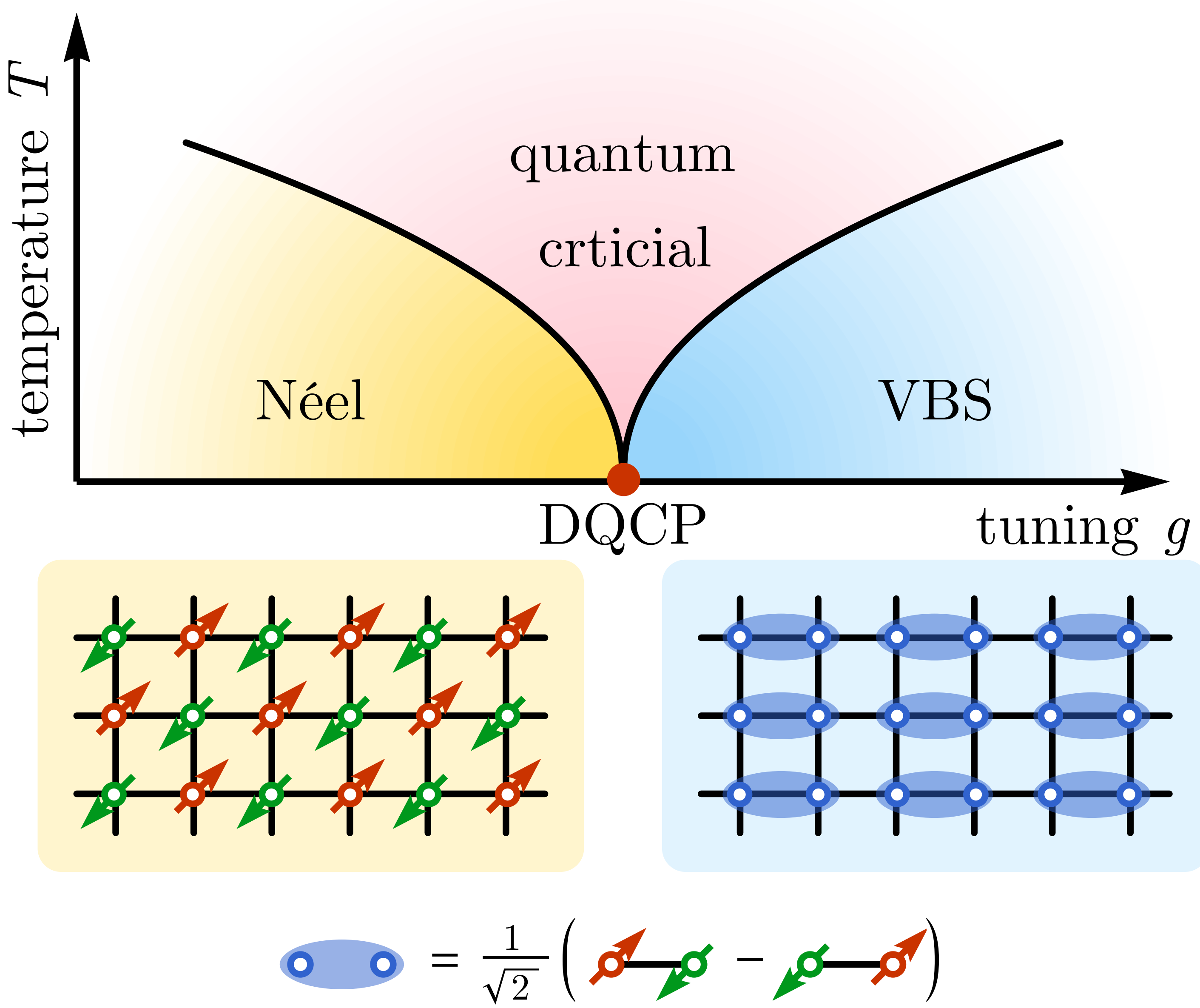
$$\mathcal{L} = \mathcal{L}_{\text{symm.}} + g\mathcal{L}_{\text{anisotropy}}$$

$g \to 0$ under RG

  - Noether theorem: each emergent symmetry generator
    → an emergent conserved current (for internal symmetries)
- Can we observe these consequences?

Deconfined Quantum Critical Point (DQCP)

- Exotic quantum critical point between two SSB phases: Néel and valence bond solid (VBS) in (2+1)D



- Beyond Landau-Ginzburg-Wilson (LGW) paradigm.
- Deconfinement: fractionalized spinons and emergent gauge fluctuations
- Emergent continuous symmetry: SO(5), O(4) …

Senthil, Vishwanath, Balents, Sachdev, Fisher (2004)

Easy-Plane J-Q (EPJQ) Model

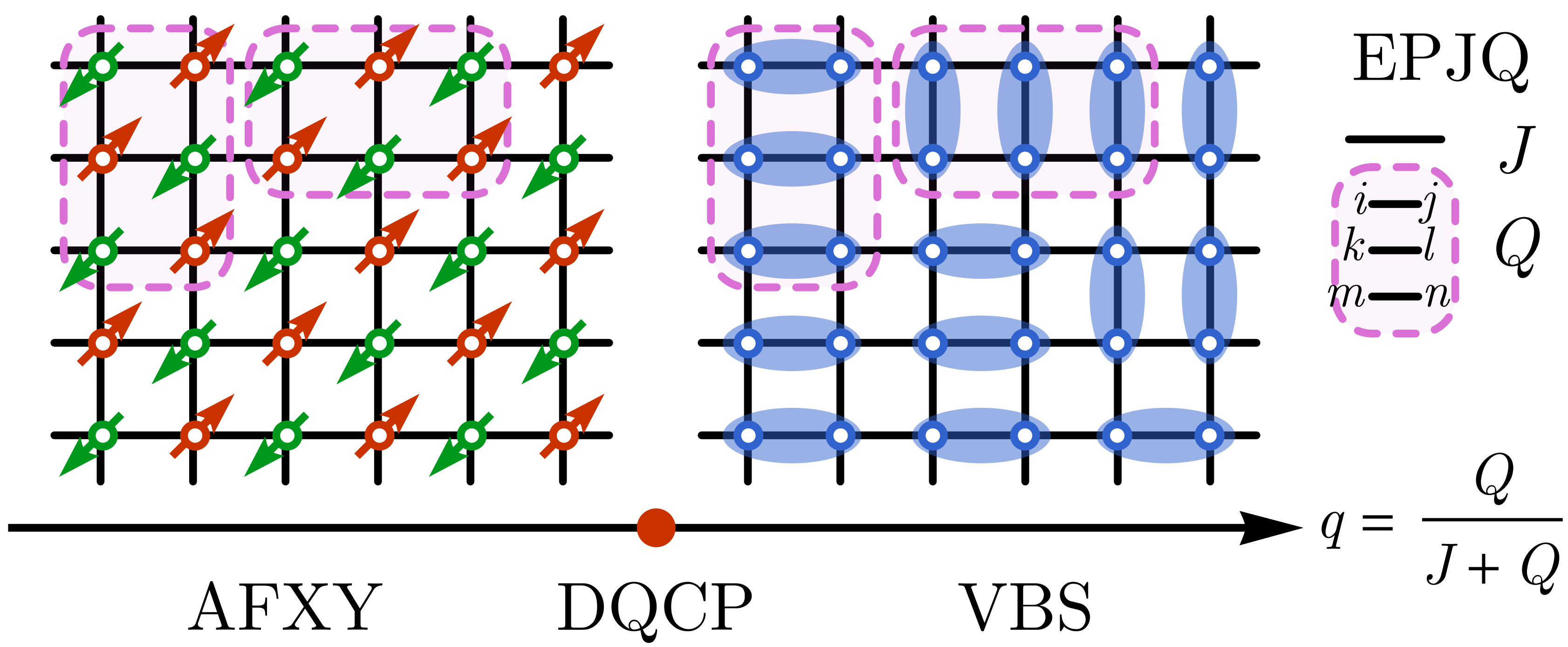
- Easy-Plane J-Q (EPJQ) Model

$$H_{\mathrm{EPJQ}} = -J\sum_{\langle ij\rangle}(P_{ij} + \Delta S_i^z S_j^z) - Q \sum_{\begin{bmatrix} i & j \\ k & l \\ m & n \end{bmatrix}} P_{ij}P_{kl}P_{mn}$$

$$\downarrow$$

$$J(S_i^x S_j^x + S_i^y S_j^y + (1-\Delta) S_i^z S_j^z)$$

  - Square lattice, spin-1/2 $\boldsymbol{S}_i$ per site, anisotropy $\Delta = 0.5$
  - Singlet projection operator on bond $P_{ij} = \frac{1}{4} - \boldsymbol{S}_i \cdot \boldsymbol{S}_j$

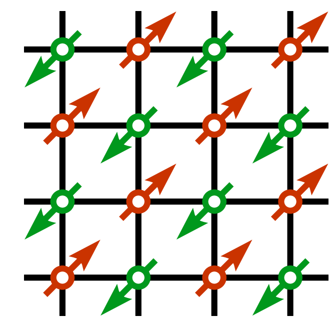


Qin, He, You, Lu, Sen, Sandvik, Xu, Meng (2017)

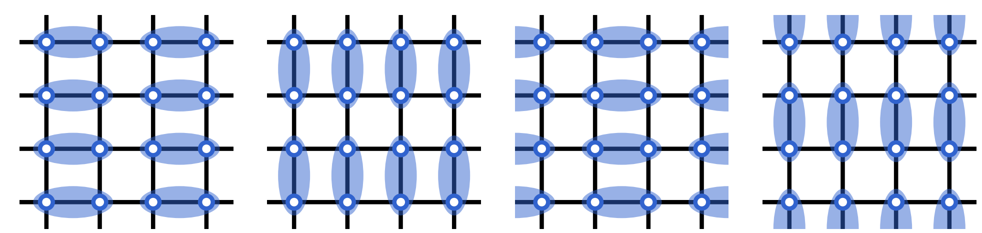
O(4) Non-Linear σ-Model

- Order parameters $\boldsymbol{n} = (n_1, n_2, n_3, n_4)$

$$\left.\begin{aligned} n_1 &\sim (-)^{x_i+y_i} S_i^x \\ n_2 &\sim (-)^{x_i+y_i} S_i^y \end{aligned}\right\} \text{AFXY} \longleftarrow \text{spin U(1)}$$



$$\left.\begin{aligned} n_3 &\sim (-)^{x_i}\left(\tfrac{1}{4} - \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+\hat{x}}\right) \\ n_4 &\sim (-)^{y_i}\left(\tfrac{1}{4} - \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+\hat{y}}\right) \end{aligned}\right\} \text{VBS} \longleftarrow \text{lattice } \mathbb{Z}_4$$



- O(4) NLSM at Θ = π

$$\mathcal{L}[\boldsymbol{n}] = \frac{1}{g}(\partial_\mu \boldsymbol{n})^2 + \frac{\mathrm{i}\Theta}{2\pi^2}\epsilon^{abcd} n_a \partial_\tau n_b \partial_x n_c \partial_y n_d$$

- Θ term: vortex of (n_1, n_2) carries $\mathbb{Z}_4$ rep. (n_3, n_4)
  vortex of (n_3, n_4) carries U(1) rep. (n_1, n_2)
- Vortex condensation: destroy one order, establish another

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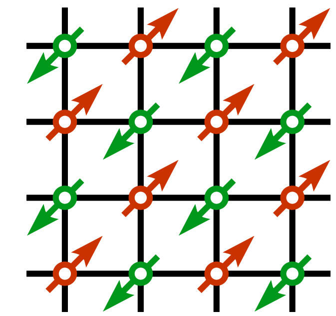

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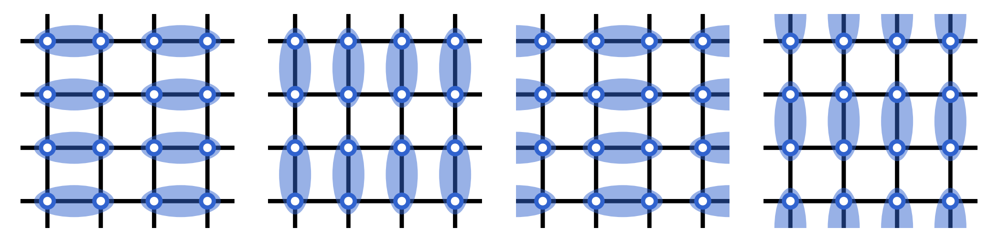

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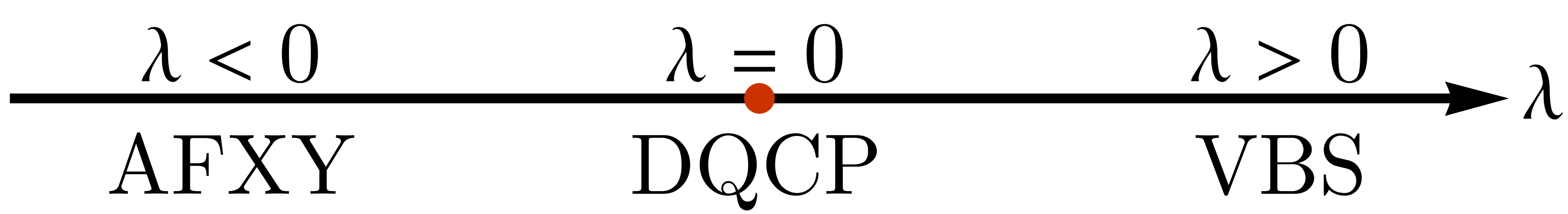
$$+ \lambda(n_1^2 + n_2^2 - n_3^2 - n_4^2)$$



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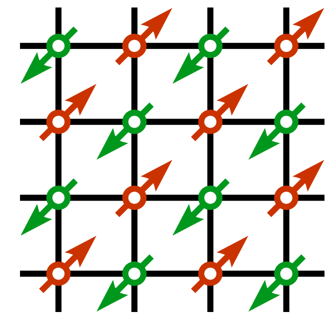

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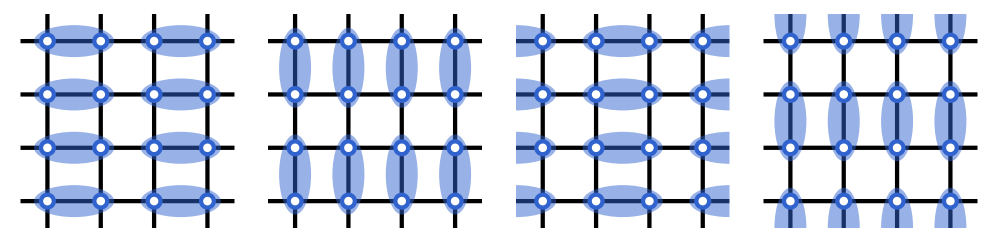

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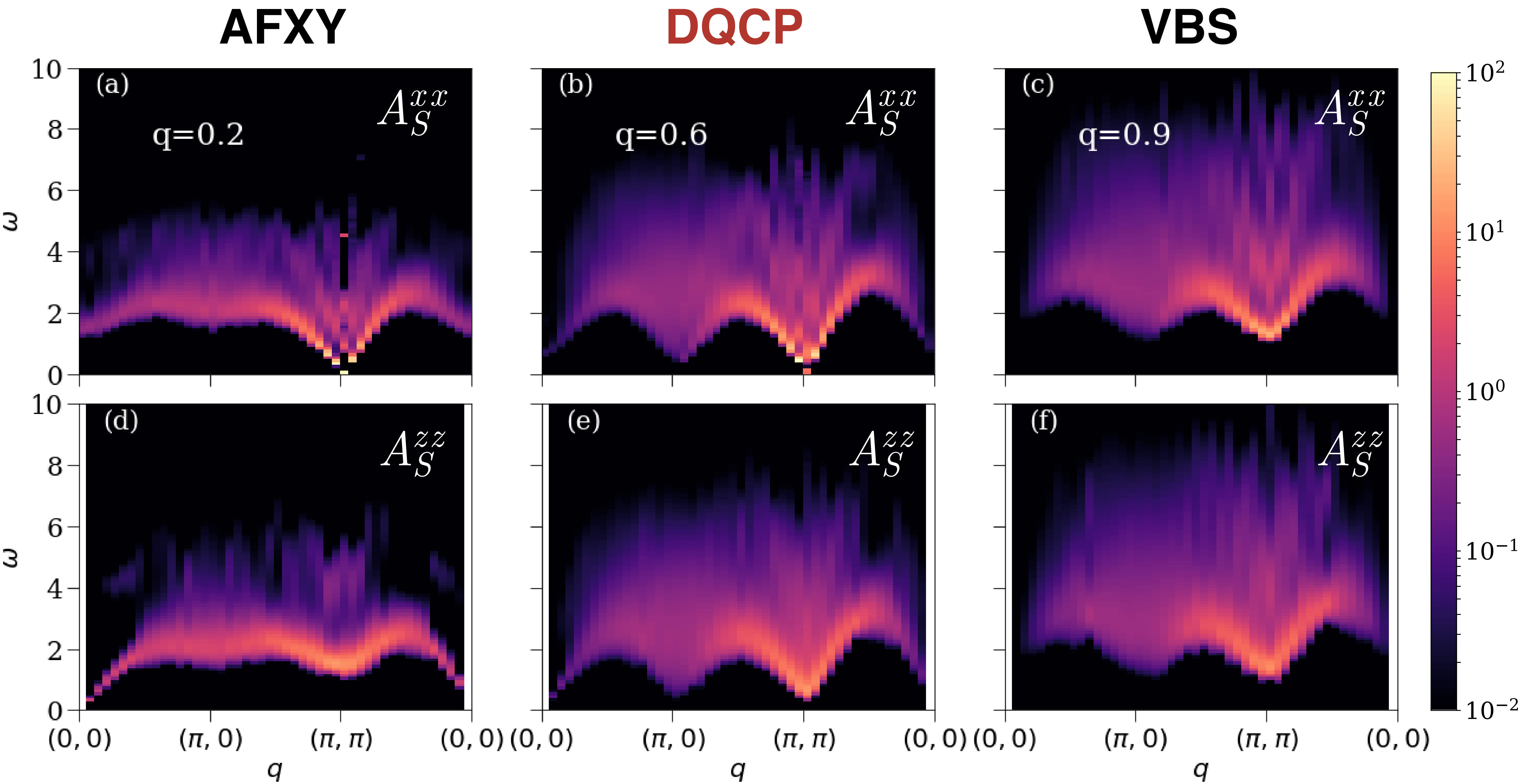
- At DQCP (λ = 0), other anisotropies are expected to be irrelevant, e.g $(n_1^2 + n_2^2)^2, n_3 n_4 (n_3^2 - n_4^2) \cdots$
- Emergent O(4) symmetry ⇒ O(4) conserved currents …

Spin Excitation Spectrum

- Spin excitation spectrum $A_S^{ab}(\boldsymbol{q},\omega) \sim 2\,\mathrm{Im}\langle S^a S^b\rangle(\boldsymbol{q},\omega+\mathrm{i}0_+)$



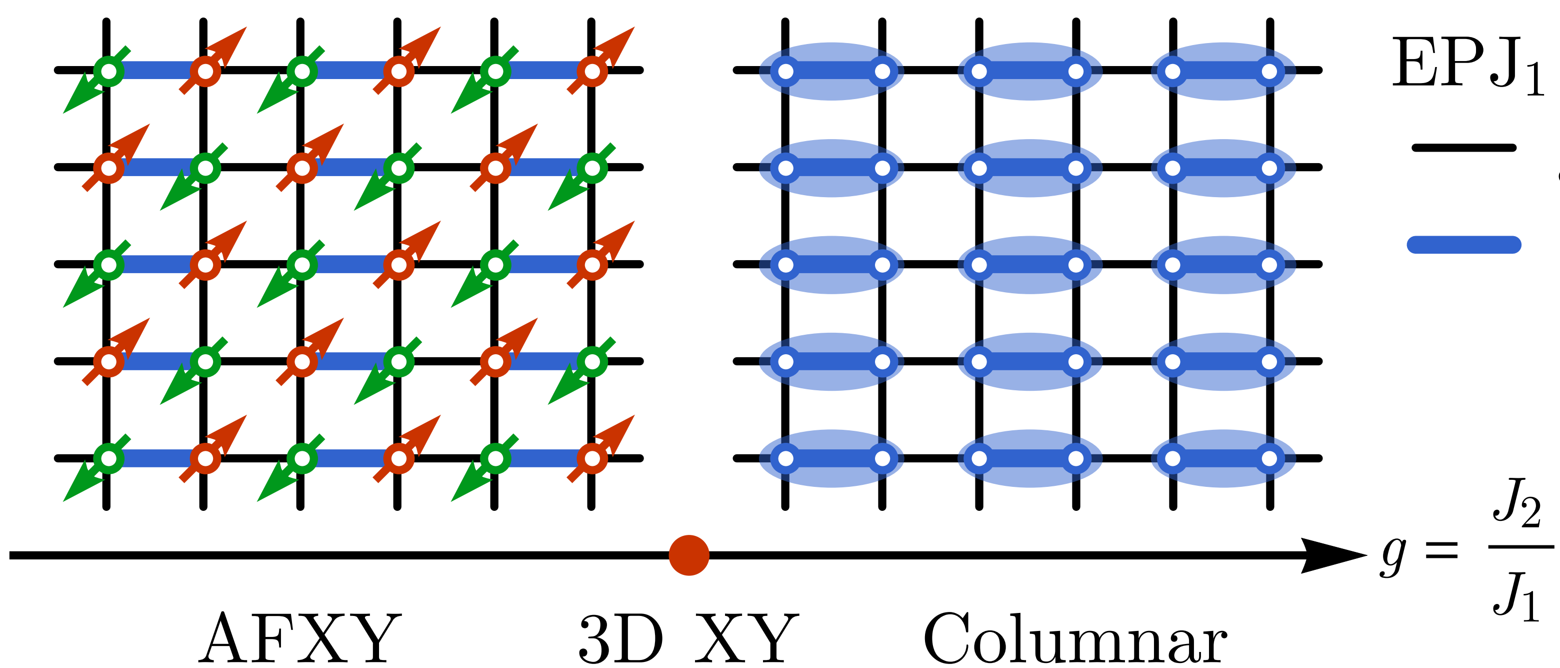
Ma, Sun, You, Xu, Vishwanath, Sandvik, Meng, arXiv:1803.01180

Easy-plane J_1-J_2 (EPJ_1J_2) Model

- To identify the unique spectral feature of DQCP, we introduce a "control group" model → Easy-plane J_1-J_2 (EPJ_1J_2) Model

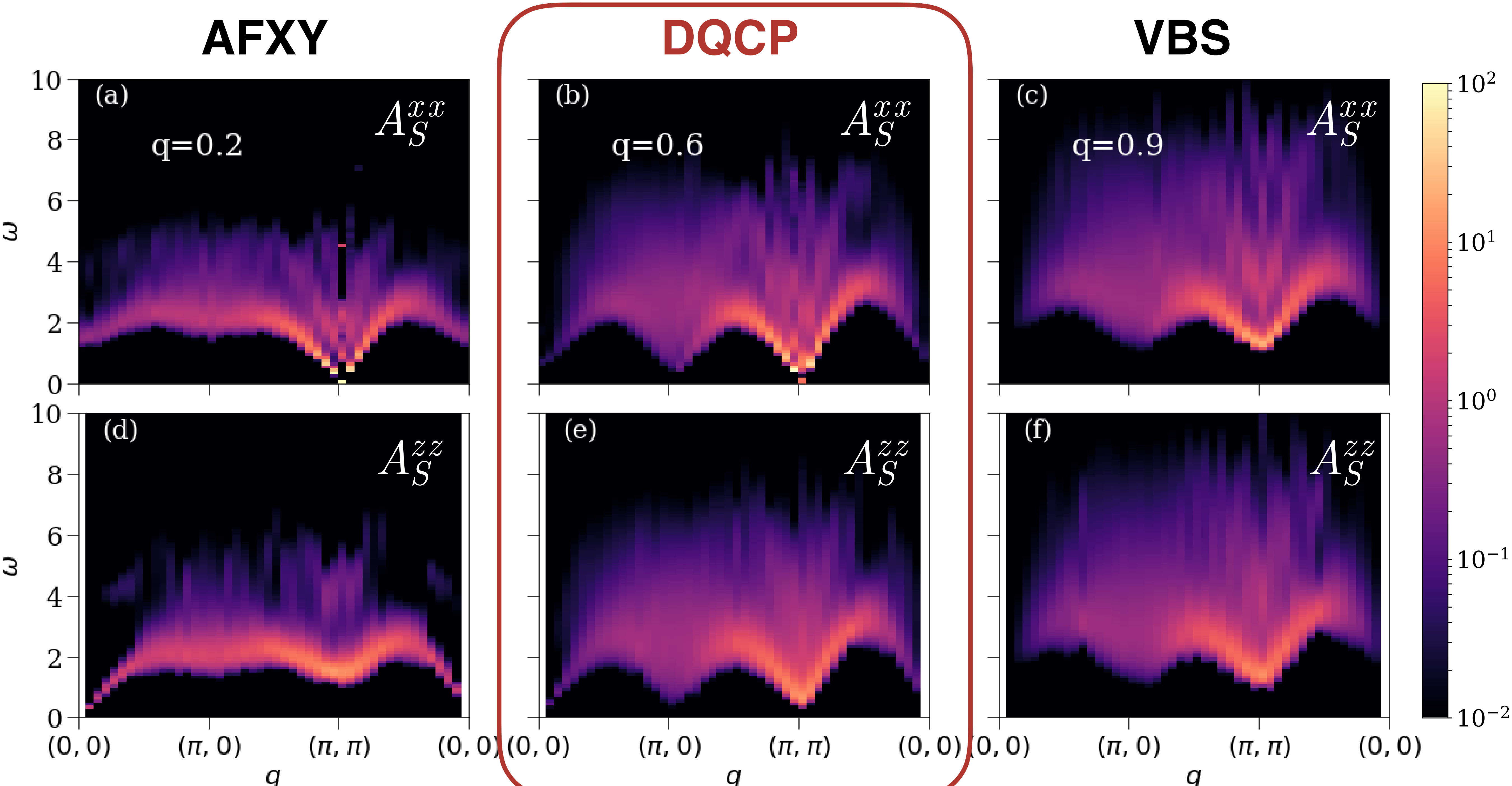
$$H_{\mathrm{EPJ_1J_2}} = J_1 \sum_{\langle ij \rangle} \mathrm{XXZ}_{ij} + J_2 \sum_{\langle ij \rangle'} \mathrm{XXZ}_{ij}$$

- Square lattice, spin-1/2 $\boldsymbol{S}_i$ per site + easy-plane anisotropy
- XXZ coupling on bond $\mathrm{XXZ}_{ij} = S_i^x S_j^x + S_i^y S_j^y + \frac{1}{2} S_i^z S_j^z$



Spin Excitation Spectrum

- Spin excitation spectrum for EPJQ model

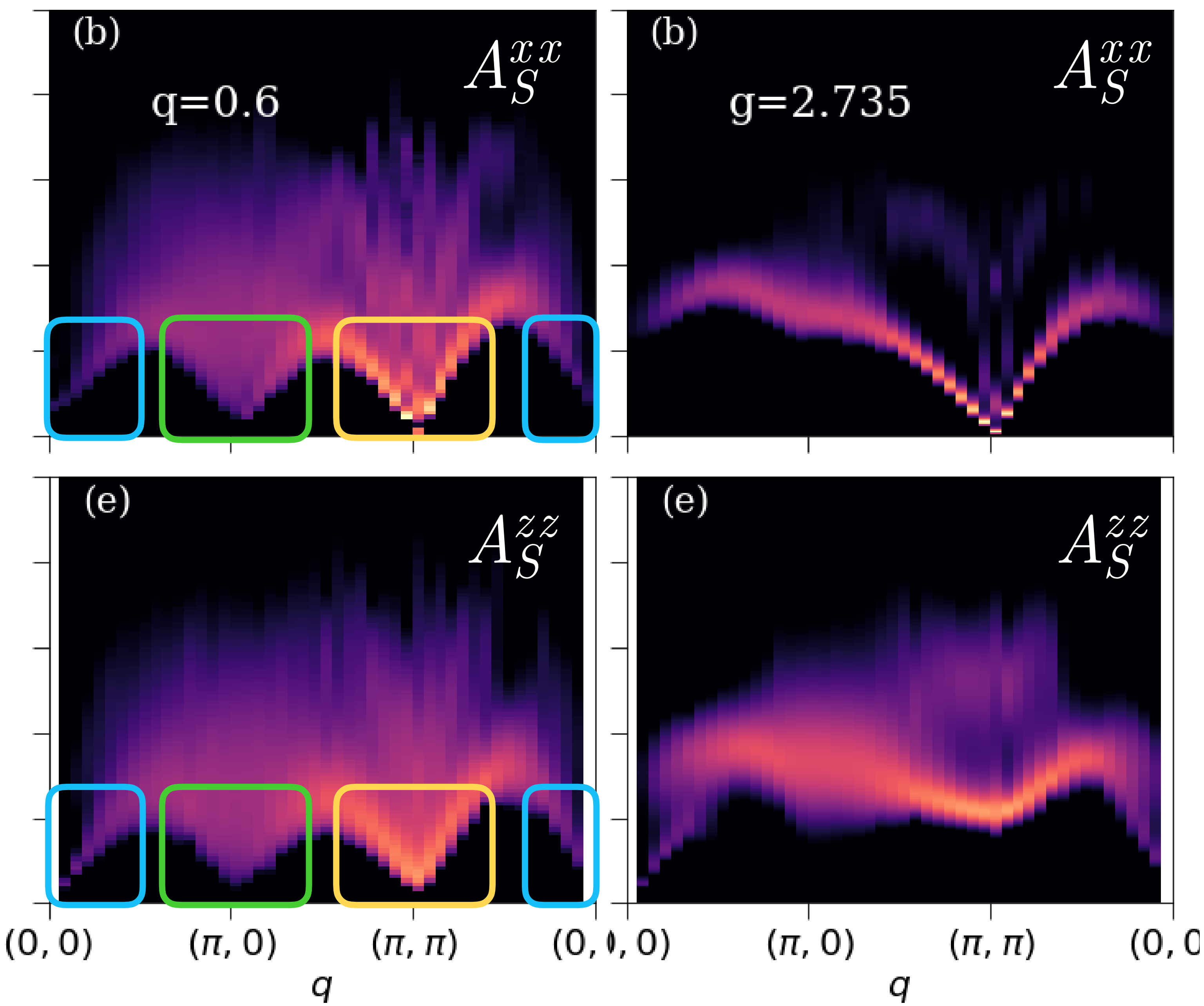


Ma, Sun, You, Xu, Vishwanath, Sandvik, Meng, arXiv:1803.01180

Deconfined Quantum Critical Point (DQCP)

- Spectrum: DQCP v.s. 3D XY

EPJQ (**DQCP**) EPJ$_1$J$_2$ (3DXY)



$$\overbrace{N_x, N_y, N_z}^{\text{Néel}}, \overbrace{D_x, D_y}^{\text{VBS}}$$

- **Spinon continuum** at momentum (π,π)

$$\boldsymbol{N} \sim (-)^i \boldsymbol{S}_i \sim z^\dagger \boldsymbol{\sigma} z$$

- **Spin density** at momentum $(0,0)$

$$\boldsymbol{N} \times \partial_t \boldsymbol{N}$$

- **XY-VBS current** at momenta $(\pi,0), (0,\pi)$

$$\boldsymbol{N}(\nabla \cdot \boldsymbol{D}) - (\boldsymbol{D} \cdot \nabla)\boldsymbol{N}$$

Identify Noether Currents

- Emergent O(4) symmetry rotates the O(4) vector

$$\boldsymbol{n} = (n_1, n_2, n_3, n_4) = (N_x, N_y, D_x, D_y)$$

$$\mathcal{L}[\boldsymbol{n}] = \frac{1}{g}(\partial_\mu \boldsymbol{n})^2 + \frac{\mathrm{i}\Theta}{2\pi^2}\epsilon^{abcd} n_a \partial_\tau n_b \partial_x n_c \partial_y n_d$$

- Noether theorem
  - Each generator T_{ab}: $\boldsymbol{n} \to e^{\mathrm{i}\theta T_{ab}}\boldsymbol{n} \quad (a, b = 1, 2, 3, 4)$
  - Associate with a conserved current J^μ_{ab} with $\partial_\mu J^\mu_{ab} = 0$

$$J^\mu_{ab} = \frac{\delta \mathcal{L}}{\delta(\partial_\mu \boldsymbol{n})} \cdot (\mathrm{i}T_{ab}\boldsymbol{n}) = n_a \partial_\mu n_b - n_b \partial_\mu n_a$$

  - SO(4) has 6 generators, each current has (2+1) space-time components → altogether 18 components
  - Not all of them appear in the spin excitation spectrum …

Conserved Currents as Spin Excitations

- In the spin excitation (S = 1) spectrum, 5 components of the Noether current can be observed

T_{ab}	J^{μ}_{ab}	expression	spin	$\boldsymbol{Q}$
T_{14}	J^2_{14}	$N_x \partial_y D_y - D_y \partial_y N_x$	S^x	$(\pi, 0)$
T_{13}	J^1_{13}	$N_x \partial_x D_x - D_x \partial_x N_x$	S^x	$(0, \pi)$
T_{24}	J^2_{24}	$N_y \partial_y D_y - D_y \partial_y N_y$	S^y	$(\pi, 0)$
T_{23}	J^1_{23}	$N_y \partial_x D_x - D_x \partial_y N_y$	S^y	$(0, \pi)$
T_{12}	J^0_{12}	$N_x \partial_t N_y - N_y \partial_t N_x$	S^z	$(0, 0)$

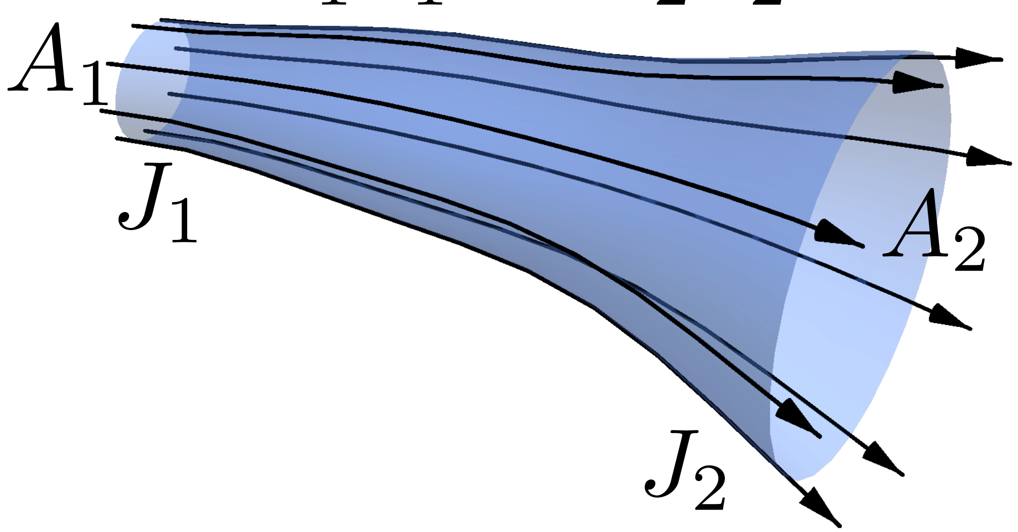
To verify the emergent O(4) symmetry, we need to test if these currents are indeed conserved

		spin	$\boldsymbol{Q}$
n_1	N_x	S^x	(π, π)
n_2	N_y	S^y	(π, π)
n_3	D_x	$-$	$(\pi, 0)$
n_4	D_y	$-$	$(0, \pi)$

Conservation Law from Scaling Dimension

- In (2+1)D space-time, conserved current must scale as

$$A_1 J_1 = A_2 J_2 \quad \Rightarrow \quad J \sim 1/A \sim 1/r^2$$



Current-current correlation must decay with exact power

$$\langle J^\mu_{ab}(r) J^\mu_{ab}(0)\rangle \sim \frac{1}{r^4}$$

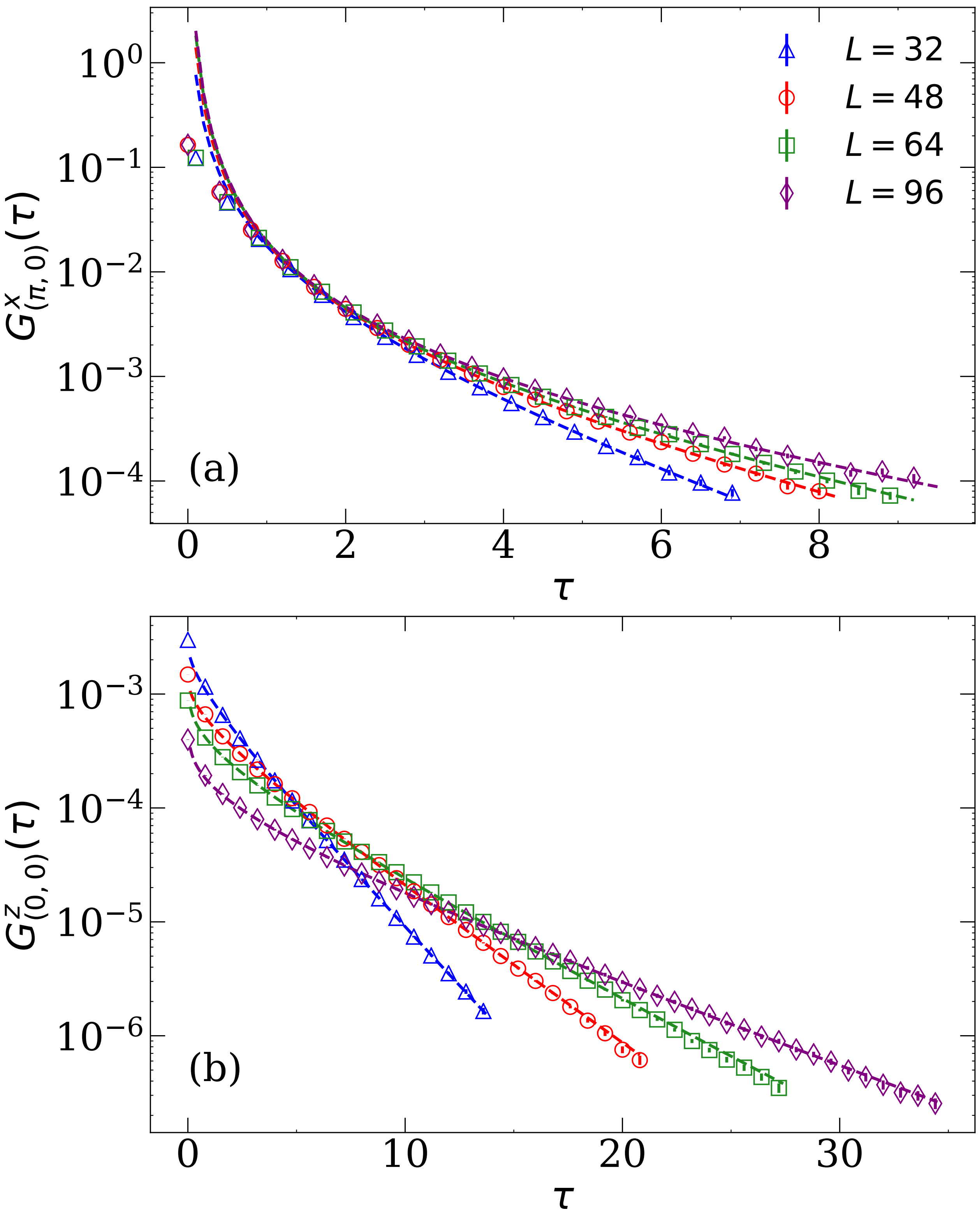
- Non-conserved current will not follow the precise scaling

$$\langle J^\mu_{ab}(r) J^\mu_{ab}(0)\rangle \sim \frac{1}{r^{4+\eta}}$$

"anomalous" dimension

  - A vanishing η indicates the current conservation
  - This exponent can be measured from spin-spin correlation, given their correspondence to the O(4) current

Measurement of Anomalous Dimension



Ma, You, Meng, arXiv:1811.08823

- General form of current-current correlation

$$\langle J^{\mu}_{ab} J^{\mu}_{ab} \rangle \sim |q|^{1+\eta} \Big(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{|q|^2} \Big)$$

- Fourier transform to imaginary time domain

$$G^{xx}_{(\pi,0)}(\tau, \boldsymbol{q}) \propto q_2^2 F_{\frac{\eta}{2}}(\tau, \boldsymbol{q}) + \frac{\eta+1}{2} F_{\frac{\eta}{2}+1}(\tau, \boldsymbol{q}))$$

$$G^{zz}_{(0,0)}(\tau, \boldsymbol{q}) \propto \boldsymbol{q}^2 F_{\frac{\eta}{2}}(\tau, \boldsymbol{q})$$

$$F_\alpha(\tau, \boldsymbol{q}) = |\frac{2\boldsymbol{q}}{\tau}|^\alpha K_\alpha(|\boldsymbol{q}\tau|)$$

- Determine η by fitting
- Finite-size scaling

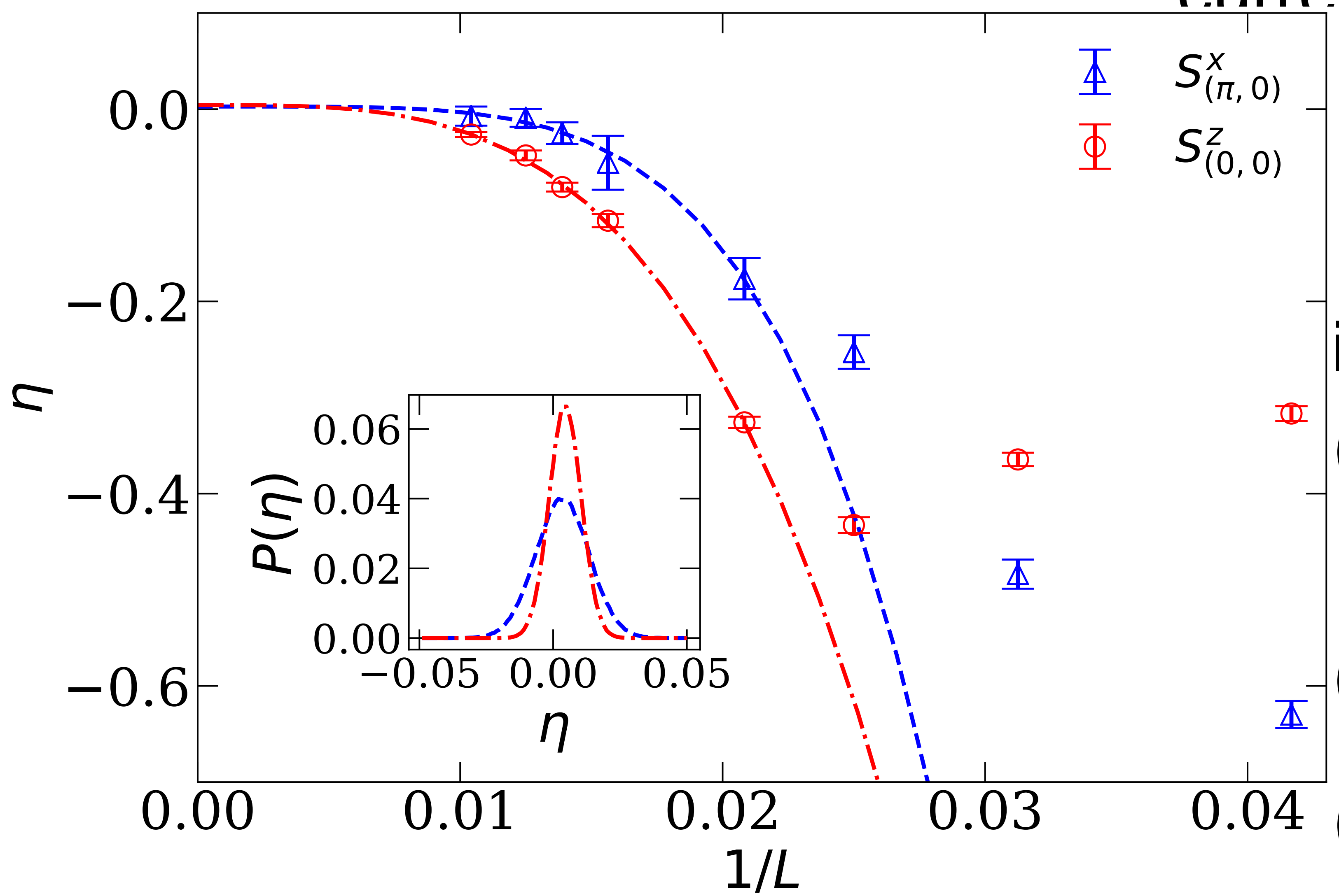
Measurement of Anomalous Dimension

- General form of current-current correlation

$$\sim |q|^{1+\eta}\Big(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{|q|^2}\Big)$$

er transform to inary time domain

$$(\tau, \boldsymbol{q}) \propto q_2^2 F_{\frac{\eta}{2}}(\tau, \boldsymbol{q})$$
$$+ \frac{\eta+1}{2} F_{\frac{\eta}{2}+1}(\tau, \boldsymbol{q}))$$
$$(\tau, \boldsymbol{q}) \propto \boldsymbol{q}^2 F_{\frac{\eta}{2}}(\tau, \boldsymbol{q})$$
$$(\tau, \boldsymbol{q}) = |\frac{2\boldsymbol{q}}{\tau}|^\alpha K_\alpha(|\boldsymbol{q}\tau|)$$



Ma, You, Meng, arXiv:1811.08823

- Determine η by fitting
- Finite-size scaling

Conserved Currents as Spin Excitations

- The channels we have measured are: $G^{xx}_{(\pi,0)}, G^{zz}_{(0,0)}$

	T_{ab}	J^{μ}_{ab}	expression	spin	$\boldsymbol{Q}$
✔	T_{14}	J^2_{14}	$N_x\partial_y D_y - D_y\partial_y N_x$	S^x	$(\pi, 0)$
✔	T_{13}	J^1_{13}	$N_x\partial_x D_x - D_x\partial_x N_x$	S^x	$(0, \pi)$
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✔	T_{23}	J^1_{23}	$N_y\partial_x D_x - D_x\partial_y N_y$	S^y	$(0, \pi)$
✔	T_{12}	J^0_{12}	$N_x\partial_t N_y - N_y\partial_t N_x$	S^z	$(0, 0)$

- The missing generator is

✔ $T_{34} \rightarrow (D_x, D_y)$ rotation (S = 0), invisible in spin channel

$$\frac{1}{2\mathrm{i}}[T_{13}, T_{14}] = \frac{1}{2\mathrm{i}}[T_{23}, T_{24}] = T_{34}$$

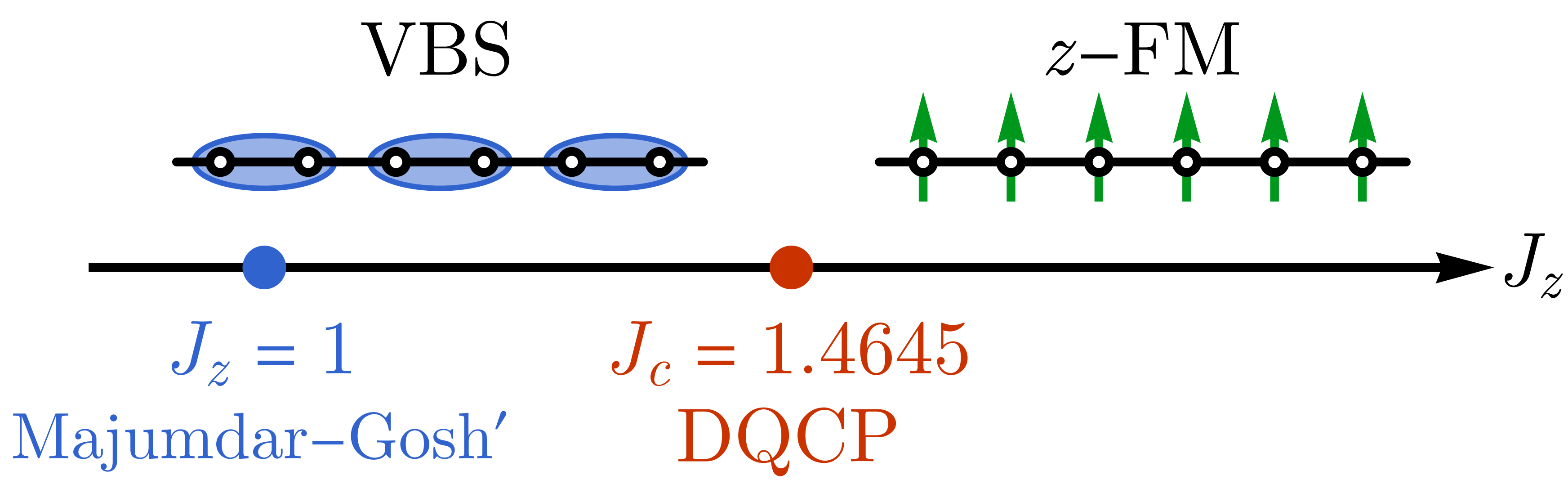
- All generators are checked → SO(4) symmetry confirmed
- Improper Z_2 symmetry is microscopic → Emergent O(4)

(1+1)D Analog of DQCP

- A recent proposal: 1D DQCP (Ising-DQCP) Jiang, Motrunich (2019)

$$H = \sum_i (-J_x S_i^x S_{i+1}^x - J_z S_i^z S_{i+1}^z + K_x S_i^x S_{i+2}^x + K_z S_i^z S_{i+2}^z)$$

  - 1D spin chain, on-site symmetry $\mathbb{Z}_2^x \times \mathbb{Z}_2^z$
  - Fix $K_x = K_z = 1/2$, $J_x = 1$, tune J_z



  - Direct continuous transition between two Ising ordered phases → similar to DQCP in 2D.

O(4) Non-Linear σ-Model

- Order parameters

$$x\text{-FM: } n_1 \sim S_i^x$$

$$y\text{-AFM: } n_2 \sim (-)^i S_i^y$$

$$z\text{-FM: } n_3 \sim S_i^z$$

$$\text{VBS: } n_4 \sim (-)^i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

for AFM Heisenberg chain:

$$x\text{-AFM: } n_1 \sim (-)^i S_i^x$$

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Affleck, Haldane (1987)

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- Field theory: O(4) NLSM with k=1 WZW term + anisotropies

$$\mathcal{L}[\boldsymbol{n}] = \frac{1}{2\kappa}(\partial_\mu \boldsymbol{n})^2 + \frac{\mathrm{i}k}{\pi}\epsilon^{abcd} n_a \partial_\tau n_b \partial_x n_c \partial_u n_d$$
$$+ \lambda_1 n_1^2 + \lambda_2 n_2^2 + \lambda_3 n_3^2 + \lambda_4 n_4^2$$

- Translation, reflection, spin $\mathbb{Z}_2^x \times \mathbb{Z}_2^z$ provide 4 independent $\mathbb{Z}_2$ symmetries, effectively flipping each component of $\boldsymbol{n}$

O(4) Non-Linear σ-Model

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$$
\begin{aligned}
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\end{aligned}
$$

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$$
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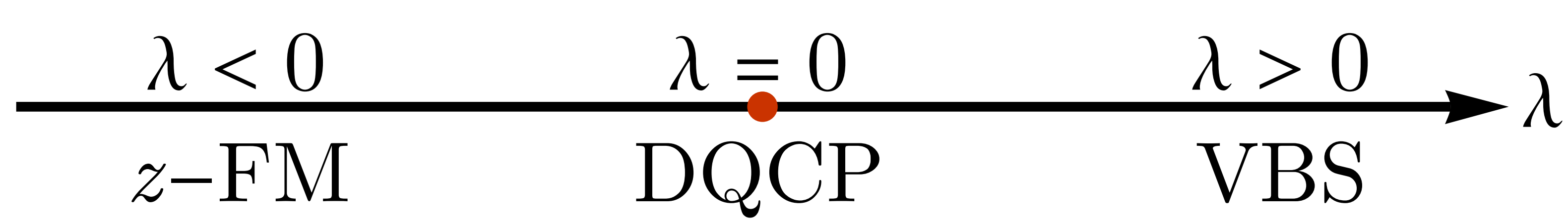
&+ \lambda(n_3^2 - n_4^2) + \lambda'(n_1^2 - n_2^2) \\
&+ \mu(n_1^2 + n_2^2 - n_3^2 - n_4^2) + \cdots \quad (\mu > 0)
\end{aligned}
$$



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- At the DQCP, emergent O(2)xO(2) symmetry (from $\mathbb{Z}_2^4$)

Identify Noether Currents

- Proposed emergent symmetry: $\mathrm{O}(2)_\phi \times \mathrm{O}(2)_\theta$

$$n_1 + \mathrm{i}n_2 \sim e^{\mathrm{i}\phi} : J^\mu_\phi = \mathrm{i}\partial_\mu \phi = n_1 \partial_\mu n_2 - n_2 \partial_\mu n_1$$

$$n_3 + \mathrm{i}n_4 \sim e^{\mathrm{i}\theta} : J^\mu_\theta = \mathrm{i}\partial_\mu \theta = n_3 \partial_\mu n_4 - n_4 \partial_\mu n_3$$

	T_x	g_x	g_z	$\mathcal{T}$	$\mathcal{P}$
n_1	n_1	n_1	$-n_1$	$-n_1$	n_1
n_2	$-n_2$	$-n_2$	$-n_2$	$-n_2$	n_2
n_3	n_3	$-n_3$	n_3	$-n_3$	n_3
n_4	$-n_4$	n_4	n_4	n_4	$-n_4$
J^0_ϕ	$-J^0_\phi$	$-J^0_\phi$	J^0_ϕ	$-J^0_\phi$	J^0_ϕ
J^1_ϕ	$-J^1_\phi$	$-J^1_\phi$	J^1_ϕ	J^1_ϕ	$-J^1_\phi$
J^0_θ	$-J^0_\theta$	$-J^0_\theta$	J^0_θ	J^0_θ	$-J^0_\theta$
J^1_θ	$-J^1_\theta$	$-J^1_\theta$	J^1_θ	$-J^1_\theta$	J^1_θ

Identify Noether Currents

- On the lattice model level, we check the symmetry properties of spin and dimmer operators

z-AFM

$$S^z_\pi \sim (-)^i S^z_i$$

xy-VBS

$$\Gamma_\pi \sim (-)^i (S^x_i S^y_{i+1} + \text{h.c.})$$

	T_x	g_x	g_z	$\mathcal{T}$	$\mathcal{P}$
S^x_i	S^x_{i+1}	S^x_i	$-S^x_i$	$-S^x_i$	S^x_{-i}
S^y_i	S^y_{i+1}	$-S^y_i$	$-S^y_i$	$-S^y_i$	S^y_{-i}
S^z_i	S^z_{i+1}	$-S^z_i$	S^z_i	$-S^z_i$	S^z_{-i}
S^z_π	$-S^z_\pi$	$-S^z_\pi$	S^z_π	$-S^z_\pi$	S^z_π
Γ_π	$-\Gamma_\pi$	$-\Gamma_\pi$	Γ_π	Γ_π	$-\Gamma_\pi$

Identify Noether Currents

- By comparing symmetry representations,

	T_x	g_x	g_z	$\mathcal{T}$	$\mathcal{P}$
J_ϕ^0	$-J_\phi^0$	$-J_\phi^0$	J_ϕ^0	$-J_\phi^0$	J_ϕ^0
J_ϕ^1	$-J_\phi^1$	$-J_\phi^1$	J_ϕ^1	J_ϕ^1	$-J_\phi^1$
J_θ^0	$-J_\theta^0$	$-J_\theta^0$	J_θ^0	J_θ^0	$-J_\theta^0$
J_θ^1	$-J_\theta^1$	$-J_\theta^1$	J_θ^1	$-J_\theta^1$	J_θ^1
S_π^z	$-S_\pi^z$	$-S_\pi^z$	S_π^z	$-S_\pi^z$	S_π^z
Γ_π	$-\Gamma_\pi$	$-\Gamma_\pi$	Γ_π	Γ_π	$-\Gamma_\pi$

we can make the identification:

$$J_\phi^0 \sim J_\theta^1 \sim S_\pi^z \sim (-)^i S_i^z$$

$$J_\phi^1 \sim J_\theta^0 \sim \Gamma_\pi \sim (-)^i (S_i^x S_{i+1}^y + \mathrm{h.c.})$$

Numerical Result

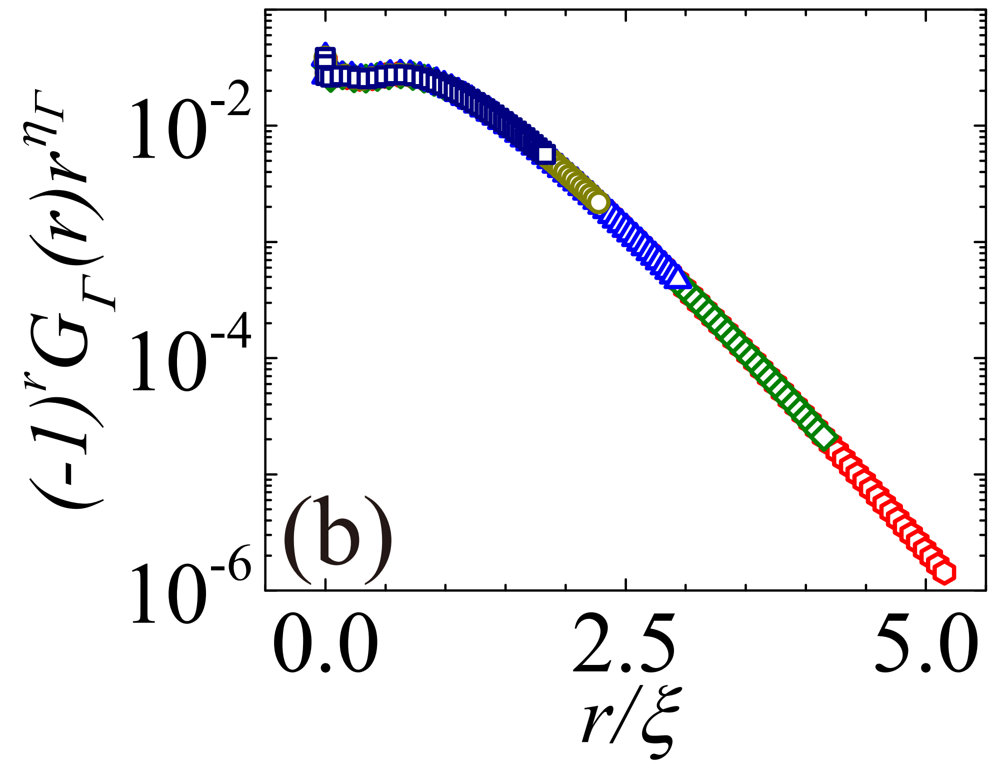
- By data collapse and fitting

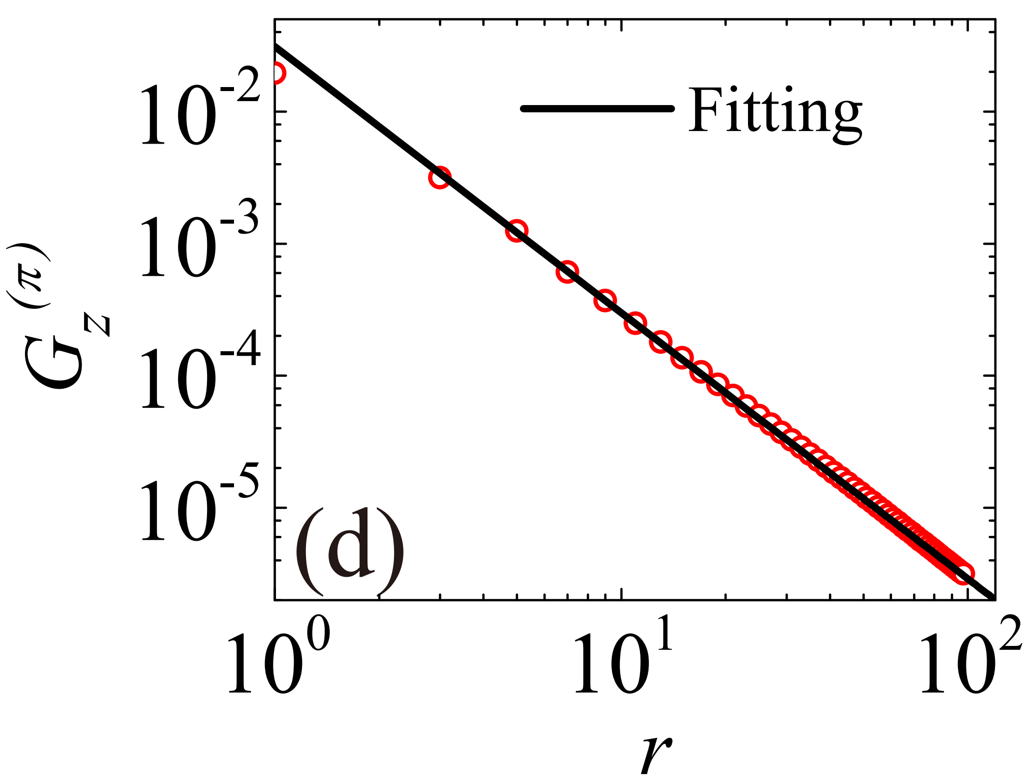
$$\langle S_i^z S_{i+r}^z \rangle \sim \frac{1}{r^{0.68(3)}} + \frac{(-1)^r}{r^{2.02(6)}}$$

$$\langle \Gamma_i \Gamma_{i+r} \rangle \sim \frac{(-1)^r}{r^{2.00(5)}}$$

$\uparrow$

$\Gamma_i = S_i^x S_{i+1}^y$ (xy-dimer)





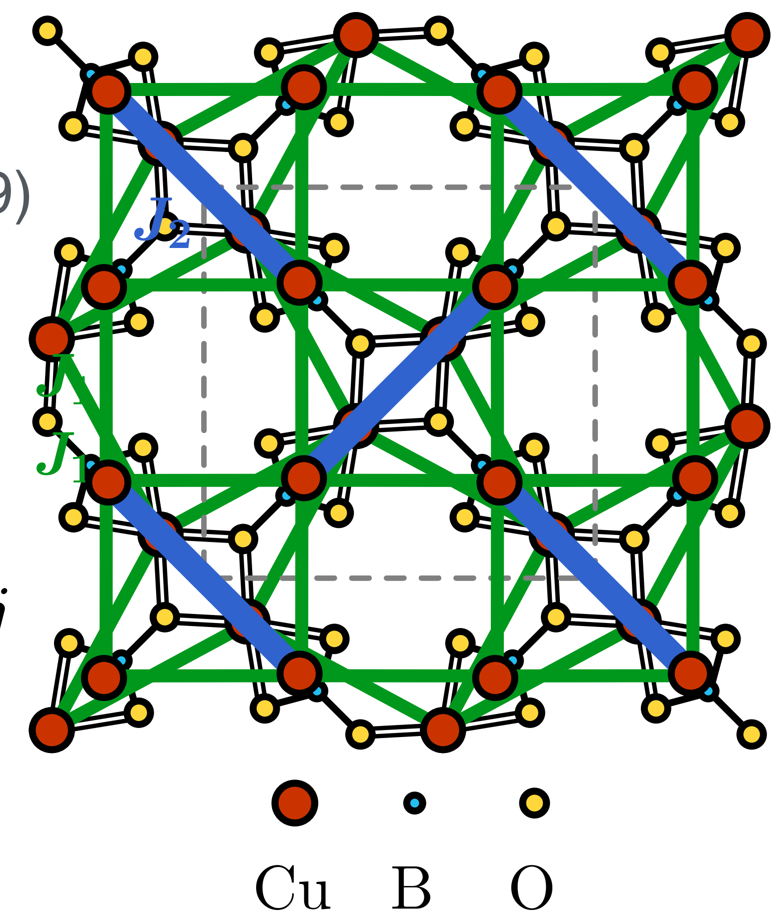
- Exponents are consistent with 2 (within error range)
  → conservation of Noether currents $\partial_\mu J^\mu_\phi = \partial_\mu J^\mu_\theta = 0$
- Improper Z_2 subgroups are microscopic symmetries
  → emergent O(2)xO(2) symmetry

Huang, Lu, You, Meng, Xiang; Roberts, Jiang, Motrunich (arXiv today)

Potential Realization of DQCP in $SrCu_2(BO_3)_2$

- Layered quantum magnet
- Cu site carries spin-1/2
- Heisenberg by superexchange
  → Shastry-Sutherland model

Kageyama et.al.
PRL 82, 3168 (1999)



$$H = J_1 \sum_{ij \in \mathrm{n.n.}} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{ij \in \mathrm{dimer}} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

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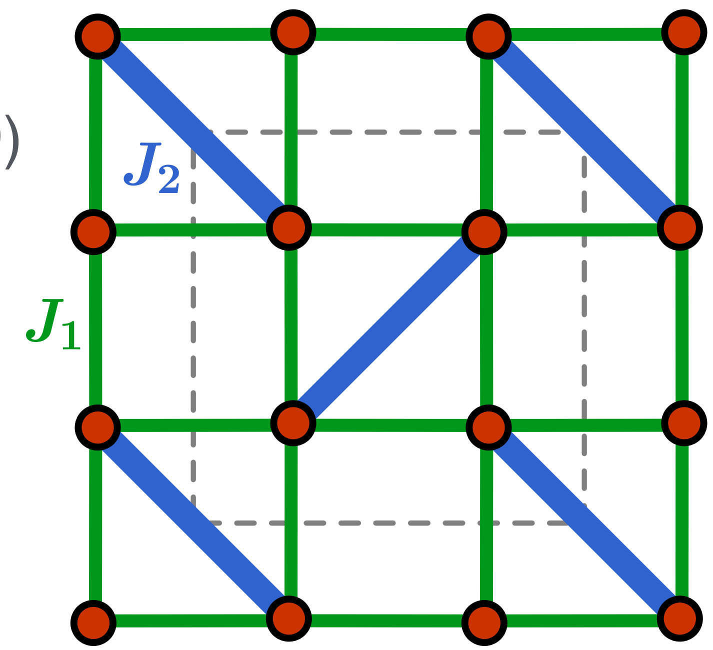
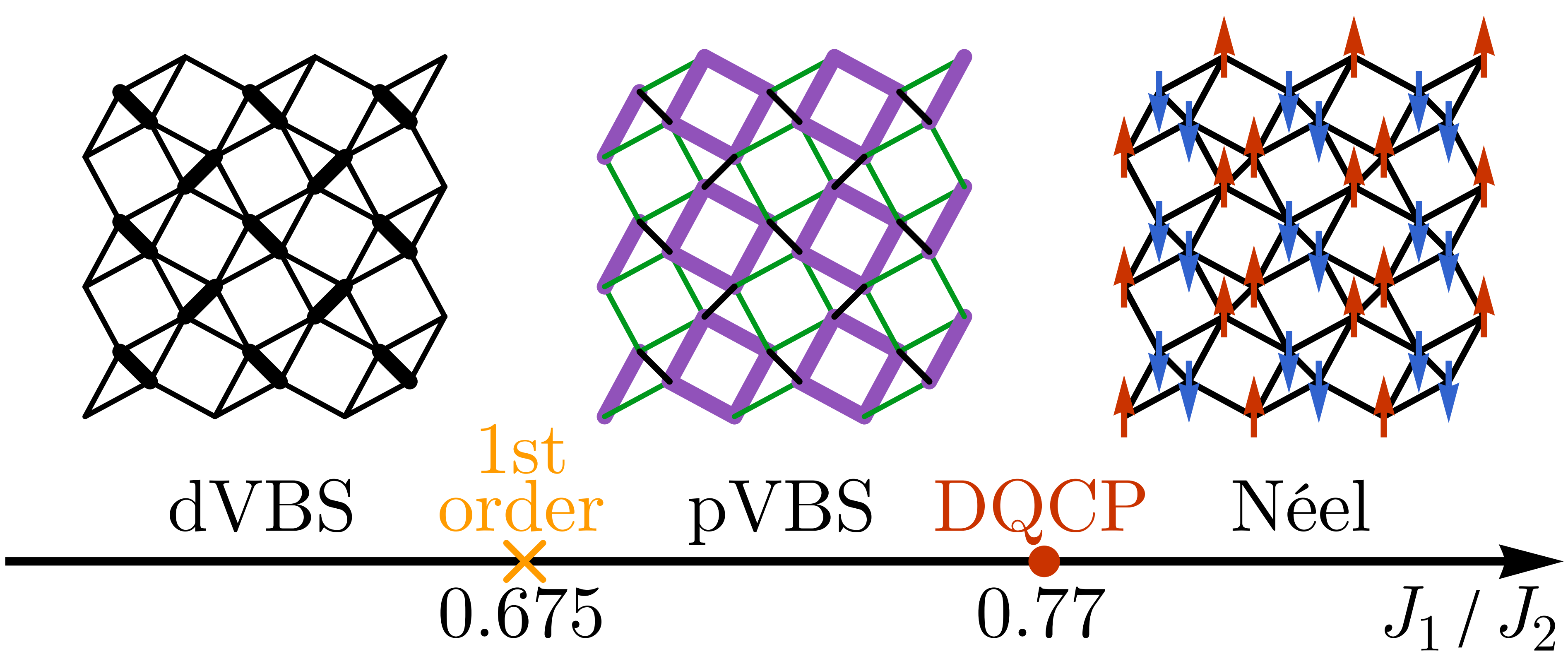


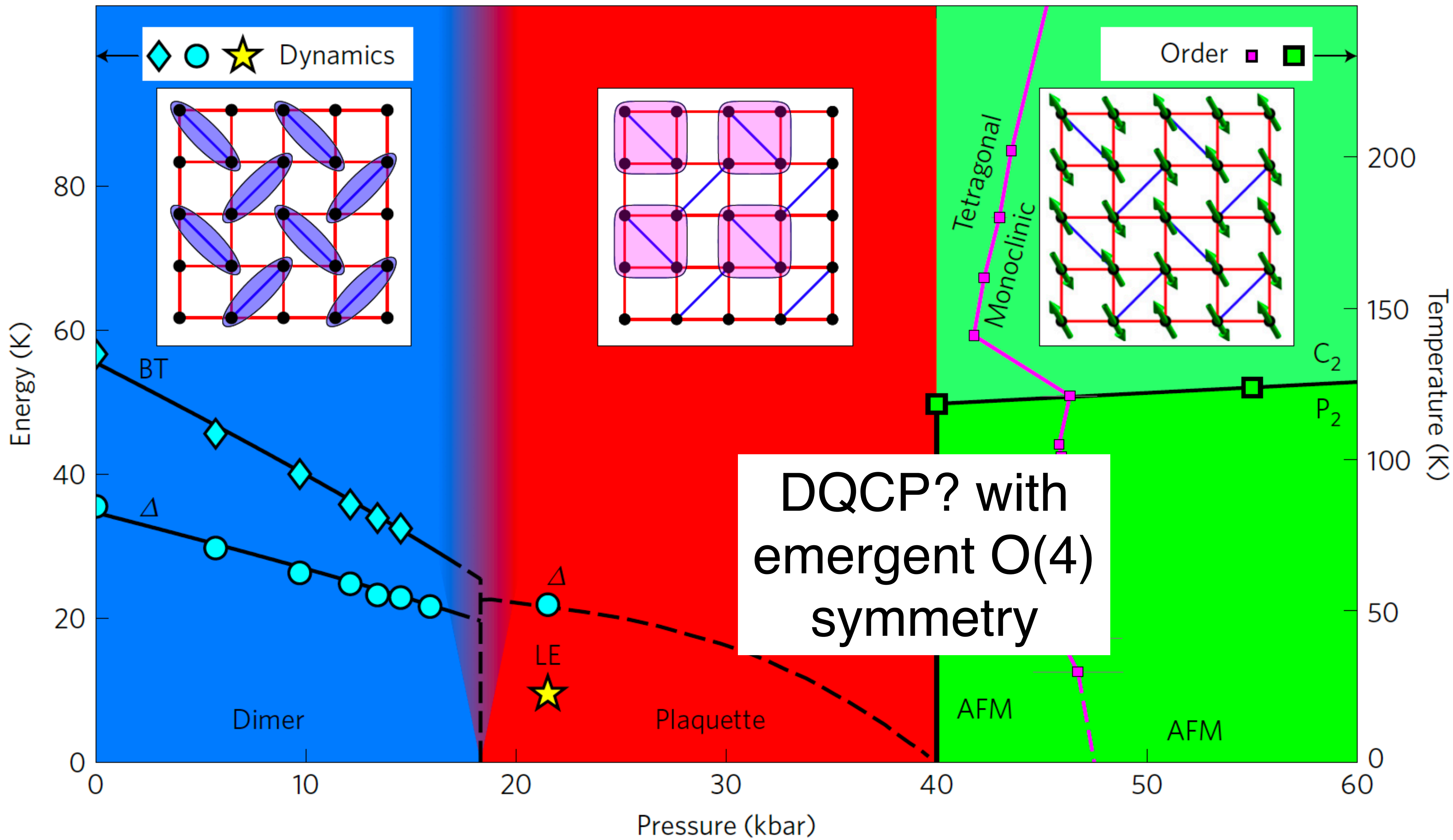
$$H = J_1 \sum_{ij \in \mathrm{n.n.}} \boldsymbol{S}_i \cdot \boldsymbol{S}_j + J_2 \sum_{ij \in \mathrm{dimer}} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$



Lee, You, Vishwanath (to appear)

Potential Realization of DQCP in $SrCu_2(BO_3)_2$

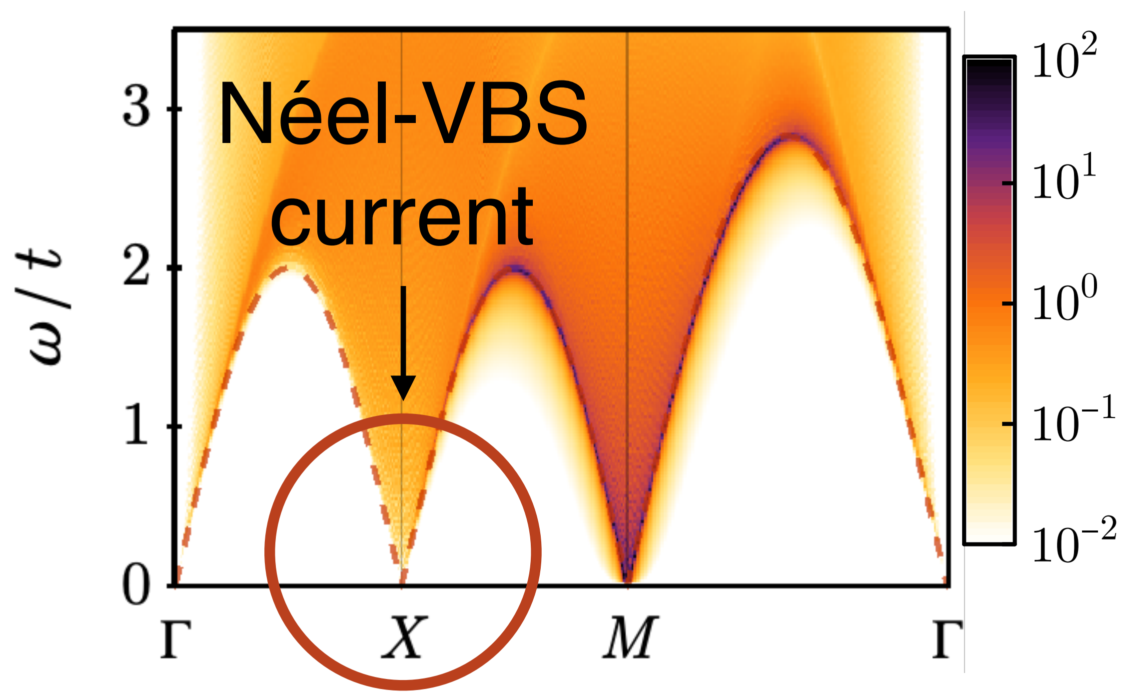
- Apply uniaxial strain effectively tunes J_1/J_2 ratio

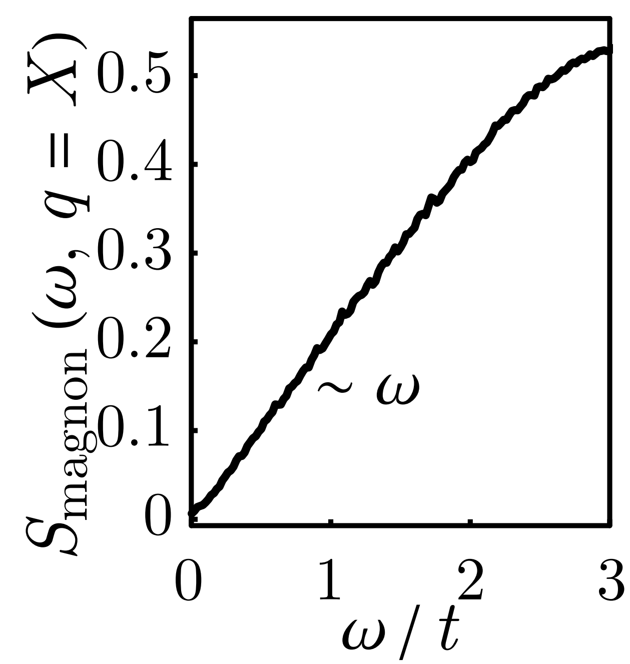


Zayed et.al. Nature Physics 13, 962-966 (2017)

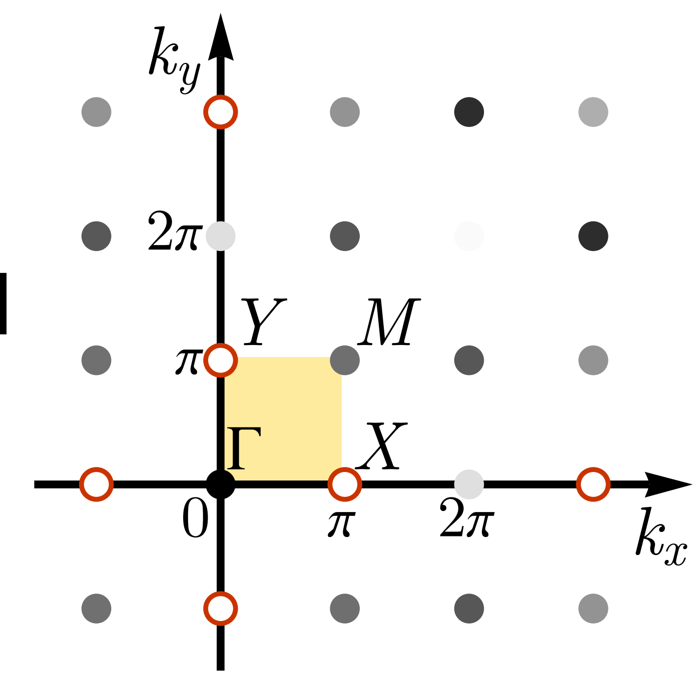
Spectral Signature of O(4) Currents

- Emergent O(4) currents should appear in the spin excitation spectrum with spectral weight ~ ω





- The momentum point X is an extinction point of diffraction from Cu sites (protected by the glide reflection symmetry)
  → No elastic scattering at low-energy



Lee, You, Vishwanath (to appear)

Summary

- Noether theorem: emergent continuous symmetry → emergent conserved currents, which can be observed in the low-energy excitation spectrum
- Apart from spin/boson systems, fermion systems can also have “DQCP” (e.g. symmetric mass generation) with emergent symmetry. Noether current provides a universal probe for these exotic quantum phase transitions.
- QMC + SAC allows us to explore spectral features of DQCP in both bosonic and (sign-problem-free) fermionic systems.
- The spectral features are relatively easy to probe by INS, RIXS or NMR, and are robust in a range of temperature, which may guide the search for DQCP in real materials.

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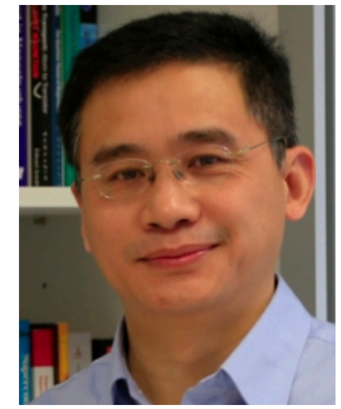

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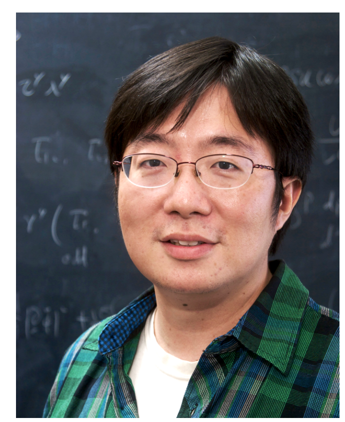

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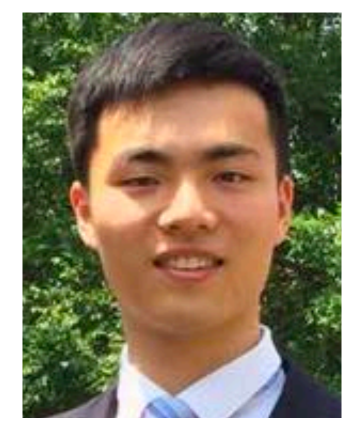

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