

All About Atoms

40-1 PROPERTIES OF ATOMS

Learning Objectives

After reading this module, you should be able to . . .

- 40.01** Discuss the pattern that is seen in a plot of ionization energies versus atomic number Z .
- 40.02** Identify that atoms have angular momentum and magnetism.
- 40.03** Explain the Einstein–de Haas experiment.
- 40.04** Identify the five quantum numbers of an electron in an atom and the allowed values of each.
- 40.05** Determine the number of electron states allowed in a given shell and subshell.
- 40.06** Identify that an electron in an atom has an orbital angular momentum \vec{L} and an orbital magnetic dipole moment $\vec{\mu}_{\text{orb}}$.
- 40.07** Calculate magnitudes for orbital angular momentum \vec{L} and orbital magnetic dipole moment $\vec{\mu}_{\text{orb}}$ in terms of the orbital quantum number ℓ .
- 40.08** Apply the relationship between orbital angular momentum \vec{L} and orbital magnetic dipole moment $\vec{\mu}_{\text{orb}}$.
- 40.09** Identify that \vec{L} and $\vec{\mu}_{\text{orb}}$ cannot be observed (measured) but a component on a measurement axis (usually called the z axis) can.
- 40.10** Calculate the z components L_z of an orbital angular momentum \vec{L} using the orbital magnetic quantum number m_ℓ .
- 40.11** Calculate the z components $\mu_{\text{orb},z}$ of an orbital

magnetic dipole moment $\vec{\mu}_{\text{orb}}$ using the orbital magnetic quantum number m_ℓ and the Bohr magneton μ_B .

- 40.12** For a given orbital state or spin state, calculate the semiclassical angle θ .
- 40.13** Identify that a spin angular momentum \vec{S} (usually simply called spin) and a spin magnetic dipole moment $\vec{\mu}_s$ are intrinsic properties of electrons (and also protons and neutrons).
- 40.14** Calculate magnitudes for spin angular momentum \vec{S} and spin magnetic dipole moment $\vec{\mu}_s$ in terms of the spin quantum number s .
- 40.15** Apply the relationship between the spin angular momentum \vec{S} and the spin magnetic dipole moment $\vec{\mu}_s$.
- 40.16** Identify that \vec{S} and $\vec{\mu}_s$ cannot be observed (measured) but a component on a measurement axis can.
- 40.17** Calculate the z components S_z of the spin angular momentum \vec{S} using the spin magnetic quantum number m_s .
- 40.18** Calculate the z components $\mu_{s,z}$ of the spin magnetic dipole moment $\vec{\mu}_s$ using the spin magnetic quantum number m_s and the Bohr magneton μ_B .
- 40.19** Identify the effective magnetic dipole moment of an atom.

Key Ideas

- Atoms have quantized energies and can make quantum jumps between them. If a jump between a higher energy and a lower energy involves the emission or absorption of a photon, the frequency associated with the light is given by

$$hf = E_{\text{high}} - E_{\text{low}}.$$

- States with the same value of quantum number n form a shell.
- States with the same values of quantum numbers n and ℓ form a subshell.
- The magnitude of the orbital angular momentum of an electron trapped in an atom has quantized values given by

$$L = \sqrt{\ell(\ell + 1)} \hbar, \quad \text{for } \ell = 0, 1, 2, \dots, (n - 1),$$

where \hbar is $h/2\pi$, ℓ is the orbital quantum number, and n is the electron's principal quantum number.

- The component L_z of the orbital angular momentum on a z axis is quantized and given by

$$L_z = m_\ell \hbar, \quad \text{for } m_\ell = 0, \pm 1, \pm 2, \dots, \pm \ell,$$

where m_ℓ is the orbital magnetic quantum number.

- The magnitude μ_{orb} of the orbital magnetic moment of the electron is quantized with the values given by

$$\mu_{\text{orb}} = \frac{e}{2m} \sqrt{\ell(\ell + 1)} \hbar,$$

where m is the electron mass.

- The component $\mu_{\text{orb},z}$ on a z axis is also quantized according to

$$\mu_{\text{orb},z} = -\frac{e}{2m} m_\ell \hbar = -m_\ell \mu_B,$$

where μ_B is the Bohr magneton:

$$\mu_B = \frac{e\hbar}{4\pi m} = \frac{e\hbar}{2m} = 9.274 \times 10^{-24} \text{ J/T.}$$

- Every electron, whether trapped or free, has an intrinsic spin angular momentum \vec{S} with a magnitude that is quantized as

$$S = \sqrt{s(s + 1)} \hbar, \quad \text{for } s = \frac{1}{2},$$

where s is the spin quantum number. An electron is said to be a spin- $\frac{1}{2}$ particle.

- The component S_z on a z axis is also quantized according to

$$S_z = m_s \hbar, \quad \text{for } m_s = \pm s = \pm \frac{1}{2},$$

where m_s is the spin magnetic quantum number.

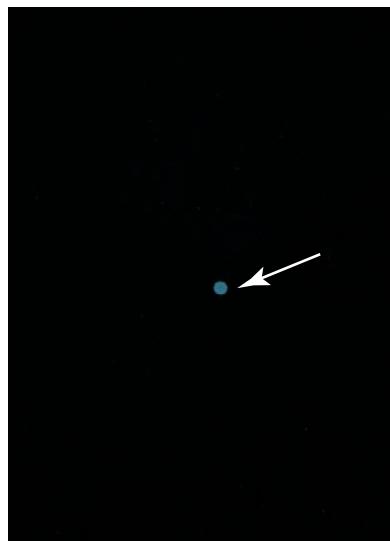
- Every electron, whether trapped or free, has an intrinsic spin magnetic dipole moment $\vec{\mu}_s$ with a magnitude that is

quantized as

$$\mu_s = \frac{e}{m} \sqrt{s(s+1)} \hbar, \quad \text{for } s = \frac{1}{2}.$$

- The component $\mu_{s,z}$ on a z axis is also quantized according to

$$\mu_{s,z} = -2m_s\mu_B, \quad \text{for } m_s = \pm \frac{1}{2}.$$



Courtesy Warren Nagourney

Figure 40-1 The blue dot is a photograph of the light emitted from a single barium ion held for a long time in a trap at the University of Washington. Special techniques caused the ion to emit light over and over again as it underwent transitions between the same pair of energy levels. The dot represents the cumulative emission of many photons.

What Is Physics?

In this chapter we continue with a primary goal of physics—discovering and understanding the properties of atoms. About 100 years ago, researchers struggled to find experiments that would prove the existence of atoms. Now we take their existence for granted and even have photographs (scanning tunneling microscope images) of atoms. We can drag them around on surfaces, such as to make the quantum corral shown in the photograph of Fig. 39-12. We can even hold an individual atom indefinitely in a trap (Fig. 40-1) so as to study its properties when it is completely isolated from other atoms.

Some Properties of Atoms

You may think the details of atomic physics are remote from your daily life. However, consider how the following properties of atoms—so basic that we rarely think about them—affect the way we live in our world.

Atoms are stable. Essentially all the atoms that form our tangible world have existed without change for billions of years. What would the world be like if atoms continually changed into other forms, perhaps every few weeks or every few years?

Atoms combine with each other. They stick together to form stable molecules and stack up to form rigid solids. An atom is mostly empty space, but you can stand on a floor—made up of atoms—without falling through it.

These basic properties of atoms can be explained by quantum physics, as can the three less apparent properties that follow.

Atoms Are Put Together Systematically

Figure 40-2 shows an example of a repetitive property of the elements as a function of their position in the periodic table (Appendix G). The figure is a plot

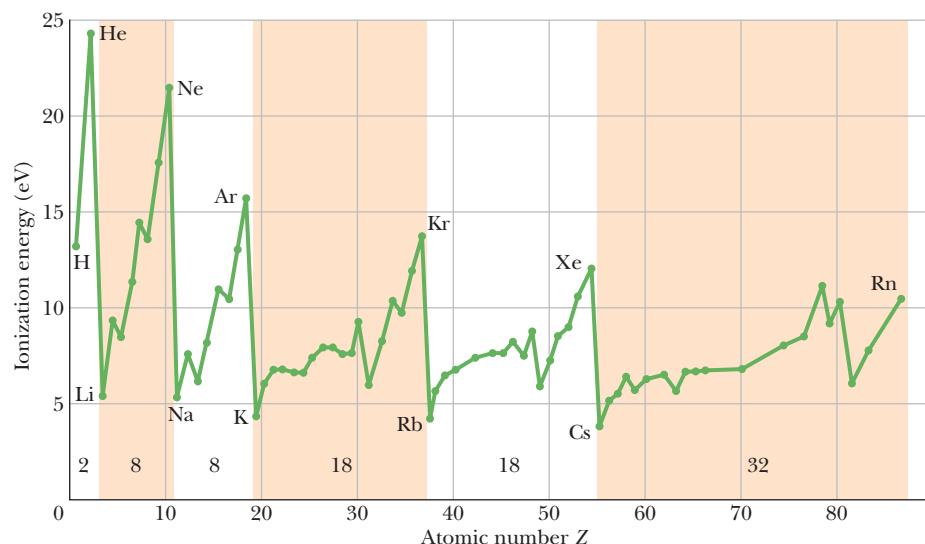


Figure 40-2 A plot of the ionization energies of the elements as a function of atomic number, showing the periodic repetition of properties through the six complete horizontal periods of the periodic table. The number of elements in each of these periods is indicated.

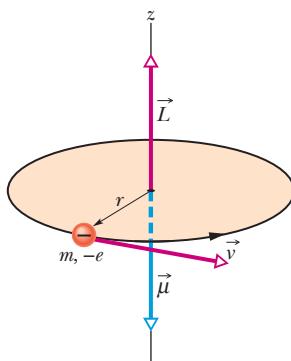


Figure 40-3 A classical model showing a particle of mass m and charge $-e$ moving with speed v in a circle of radius r . The moving particle has an angular momentum \vec{L} given by $\vec{r} \times \vec{p}$, where \vec{p} is its linear momentum $m\vec{v}$. The particle's motion is equivalent to a current loop that has an associated magnetic moment $\vec{\mu}$ that is directed opposite \vec{L} .

of the **ionization energy** of the elements; the energy required to remove the most loosely bound electron from a neutral atom is plotted as a function of the position in the periodic table of the element to which the atom belongs. The remarkable similarities in the chemical and physical properties of the elements in each vertical column of the periodic table are evidence enough that the atoms are constructed according to systematic rules.

The elements are arranged in the periodic table in six complete horizontal **periods** (and a seventh incomplete period): except for the first, each period starts at the left with a highly reactive alkali metal (lithium, sodium, potassium, and so on) and ends at the right with a chemically inert noble gas (neon, argon, krypton, and so on). Quantum physics accounts for the chemical properties of these elements. The numbers of elements in the six periods are

$$2, 8, 8, 18, 18, \text{ and } 32.$$

Quantum physics predicts these numbers.

Atoms Emit and Absorb Light

We have already seen that atoms can exist only in discrete quantum states, each state having a certain energy. An atom can make a transition from one state to another by emitting light (to jump to a lower energy level E_{low}) or by absorbing light (to jump to a higher energy level E_{high}). As we first discussed in Module 39-1, the light is emitted or absorbed as a photon with energy

$$hf = E_{\text{high}} - E_{\text{low}}. \quad (40-1)$$

Thus, the problem of finding the frequencies of light emitted or absorbed by an atom reduces to the problem of finding the energies of the quantum states of that atom. Quantum physics allows us—in principle at least—to calculate these energies.

Atoms Have Angular Momentum and Magnetism

Figure 40-3 shows a negatively charged particle moving in a circular orbit around a fixed center. As we discussed in Module 32-5, the orbiting particle has both an angular momentum \vec{L} and (because its path is equivalent to a tiny current loop) a magnetic dipole moment $\vec{\mu}$. As Fig. 40-3 shows, vectors \vec{L} and $\vec{\mu}$ are both perpendicular to the plane of the orbit but, because the charge is negative, they point in opposite directions.

The model of Fig. 40-3 is strictly classical and does not accurately represent an electron in an atom. In quantum physics, the rigid orbit model has been replaced by the probability density model, best visualized as a dot plot. In quantum physics, however, it is still true that in general, each quantum state of an electron in an atom involves an angular momentum \vec{L} and a magnetic dipole moment $\vec{\mu}$ that have opposite directions (those vector quantities are said to be *coupled*).

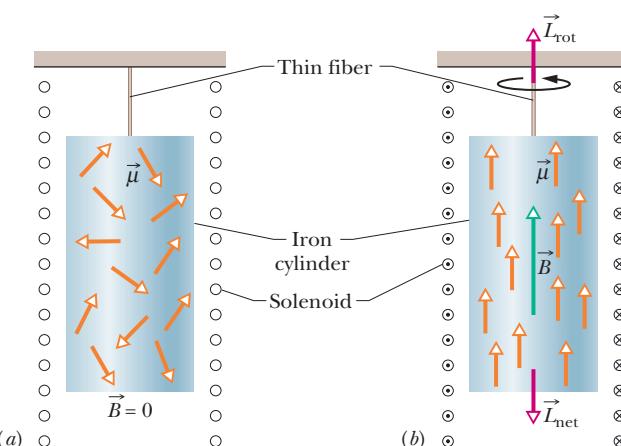


Figure 40-4 The Einstein–de Haas experimental setup. (a) Initially, the magnetic field in the iron cylinder is zero and the magnetic dipole moment vectors $\vec{\mu}$ of its atoms are randomly oriented. (b) When a magnetic field \vec{B} is set up along the cylinder’s axis, the magnetic dipole moment vectors line up parallel to \vec{B} and the cylinder begins to rotate.

Aligning the magnetic moment vectors rotates the cylinder.

The Einstein-de Haas Experiment

In 1915, well before the discovery of quantum physics, Albert Einstein and Dutch physicist W. J. de Haas carried out a clever experiment designed to show that the angular momentum and magnetic moment of individual atoms are coupled.

Einstein and de Haas suspended an iron cylinder from a thin fiber, as shown in Fig. 40-4. A solenoid was placed around the cylinder but not touching it. Initially, the magnetic dipole moments $\vec{\mu}$ of the atoms of the cylinder point in random directions, and so their external magnetic effects cancel (Fig. 40-4a). However, when a current is switched on in the solenoid (Fig. 40-4b) so that a magnetic field \vec{B} is set up parallel to the long axis of the cylinder, the magnetic dipole moments of the atoms of the cylinder reorient themselves, lining up with that field. If the angular momentum \vec{L} of each atom is coupled to its magnetic moment $\vec{\mu}$, then this alignment of the atomic magnetic moments must cause an alignment of the atomic angular momenta opposite the magnetic field.

No external torques initially act on the cylinder; thus, its angular momentum must remain at its initial zero value. However, when \vec{B} is turned on and the atomic angular momenta line up antiparallel to \vec{B} , they tend to give a net angular momentum \vec{L}_{net} to the cylinder as a whole (directed downward in Fig. 40-4b). To maintain zero angular momentum, the cylinder begins to rotate around its central axis to produce an angular momentum \vec{L}_{rot} in the opposite direction (upward in Fig. 40-4b).

The twisting of the fiber quickly produces a torque that momentarily stops the cylinder’s rotation and then rotates the cylinder in the opposite direction as the twisting is undone. Thereafter, the fiber will twist and untwist as the cylinder oscillates about its initial orientation in angular simple harmonic motion.

Observation of the cylinder’s rotation verified that the angular momentum and the magnetic dipole moment of an atom are coupled in opposite directions. Moreover, it dramatically demonstrated that the angular momenta associated with quantum states of atoms can result in *visible* rotation of an object of everyday size.

Angular Momentum, Magnetic Dipole Moments

Every quantum state of an electron in an atom has an associated orbital angular momentum and orbital magnetic dipole moment. Every electron, whether trapped in an atom or free, has a spin angular momentum and a spin magnetic dipole moment that are as intrinsic as its mass and charge. Let’s next discuss these various quantities.

Orbital Angular Momentum

Classically, a moving particle has an angular momentum \vec{L} with respect to any given reference point. In Chapter 11 we wrote this as the cross product

Table 40-1 Electron States for an Atom

Quantum Number	Symbol	Allowed Values	Related to
Principal	n	1, 2, 3, ...	Distance from the nucleus
Orbital	ℓ	0, 1, 2, ..., $(n - 1)$	Orbital angular momentum
Orbital magnetic	m_ℓ	0, $\pm 1, \pm 2, \dots, \pm \ell$	Orbital angular momentum (z component)
Spin	s	$\frac{1}{2}$	Spin angular momentum
Spin magnetic	m_s	$\pm \frac{1}{2}$	Spin angular momentum (z component)

$\vec{L} = \vec{r} \times \vec{p}$, where \vec{r} is a position vector extending to the particle from the reference point and \vec{p} is the particle's linear momentum ($m\vec{v}$). Although an electron in an atom is not a classical moving particle, it too has angular momentum given by $\vec{L} = \vec{r} \times \vec{p}$, with the reference point being the nucleus. However, unlike the classical particle, the electron's *orbital angular momentum* \vec{L} is quantized. For the electron in a hydrogen atom, we can find the quantized (allowed) values by solving Schrödinger's equation. For that situation and any other, we can also find the quantized values by using the appropriate mathematics for a cross product in a quantum situation. (The mathematics is linear algebra, which you may have on your schedule of classes.) Either way we find that the allowed magnitudes of \vec{L} are given by

$$L = \sqrt{\ell(\ell + 1)} \hbar, \quad \text{for } \ell = 0, 1, 2, \dots, (n - 1), \quad (40-2)$$

where \hbar is $h/2\pi$, ℓ is the orbital quantum number (introduced in Table 39-2, which is reproduced in Table 40-1), and n is the electron's principal quantum number.

The electron can have a definite value of L as given by one of the allowed states in Eq. 40-2, but it cannot have a definite direction for the vector \vec{L} . However, we can measure (detect) definite values of a component L_z along a chosen measurement axis (usually taken to be a z axis) as given by

$$L_z = m_\ell \hbar, \quad \text{for } m_\ell = 0, \pm 1, \pm 2, \dots, \pm \ell, \quad (40-3)$$

where m_ℓ is the orbital magnetic quantum number (Table 40-1). However, if the electron has a definite value of L_z , it does not have definite values for L_x and L_y . We cannot get around this uncertainty by, say, first measuring L_z (getting a definite value) and then measuring L_x (getting a definite value) because the second measurement can change L_z and thus we no longer have a definite value for it. Also, we can never find \vec{L} aligned with an axis because then it would have a definite direction and definite components along the other axes (namely, zero components).

A common way to depict the allowed values for L_z is shown in Fig. 40-5 for the situation in which $\ell = 2$. However, do not take the figure literally because it implies (incorrectly) that \vec{L} has the definite direction of the drawn vector. Still, it allows us to relate the five possible z components to the full vector (which has a magnitude of $\hbar\sqrt{6}$) and to define the *semi-classical angle* θ given by

$$\cos \theta = \frac{L_z}{L}. \quad (40-4)$$

Orbital Magnetic Dipole Moment

Classically, an orbiting charged particle sets up the magnetic field of a magnetic dipole, as we discussed in Module 32-5. From Eq. 32-28, the dipole moment is related to the angular momentum of the classical particle by

$$\vec{\mu}_{\text{orb}} = -\frac{e}{2m} \vec{L}, \quad (40-5)$$

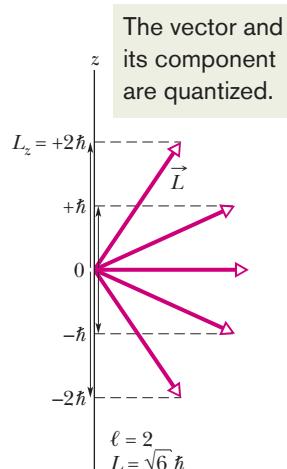


Figure 40-5 The allowed values of L_z for an electron in a quantum state with $\ell = 2$. For every orbital angular momentum vector \vec{L} in the figure, there is a vector pointing in the opposite direction, representing the magnitude and direction of the orbital magnetic dipole moment $\vec{\mu}_{\text{orb}}$.

where m is the mass of the particle, here an electron. The minus sign means that the two vectors in Eq. 40-5 are in opposite directions, which is due to the fact that an electron is negatively charged.

An electron in an atom also has an orbital magnetic dipole moment given by Eq. 40-5, but $\vec{\mu}_{\text{orb}}$ is quantized. We find allowed values of the magnitude by substituting from Eq. 40-2:

$$\mu_{\text{orb}} = \frac{e}{2m} \sqrt{\ell(\ell + 1)} \hbar. \quad (40-6)$$

As with the angular momentum, $\vec{\mu}_{\text{orb}}$ can have a definite magnitude but does not have a definite direction. The best we can do is to measure its component on a z axis, and that component can have a definite value as given by

$$\mu_{\text{orb},z} = -m_\ell \frac{e\hbar}{2m} = -m_\ell \mu_B, \quad (40-7)$$

where μ_B is the *Bohr magneton*:

$$\mu_B = \frac{eh}{4\pi m} = \frac{e\hbar}{2m} = 9.274 \times 10^{-24} \text{ J/T} \quad (\text{Bohr magneton}). \quad (40-8)$$

If the electron has a definite value of $\mu_{\text{orb},z}$, it cannot have definite values of $\mu_{\text{orb},x}$ and $\mu_{\text{orb},y}$.

Spin Angular Momentum

Every electron, whether in an atom or free, has an intrinsic angular momentum that has no classical counterpart (it is *not* of the form $\vec{r} \times \vec{p}$). It is called *spin angular momentum* \vec{S} (or simply *spin*), but the name is misleading because the electron is not spinning. Indeed there is nothing at all rotating in an electron, and yet the electron has angular momentum. The magnitude of \vec{S} is quantized, with values restricted to

$$S = \sqrt{s(s + 1)} \hbar, \quad \text{for } s = \frac{1}{2}, \quad (40-9)$$

where s is the *spin quantum number*. For every electron, $s = \frac{1}{2}$ and the electron is said to be a spin- $\frac{1}{2}$ particle. (Protons and neutrons are also spin- $\frac{1}{2}$ particles.) The language here can be confusing, because both \vec{S} and s are often referred to as spin.

As with the angular momentum associated with motion, this intrinsic angular momentum can have a definite magnitude but does not have a definite direction. The best we can do is to measure its component on a z axis, and that component can have only the definite values given by

$$S_z = m_s \hbar, \quad \text{for } m_s = \pm s = \pm \frac{1}{2}. \quad (40-10)$$

Here m_s is the *spin magnetic quantum number*, which can have only two values: $m_s = +s = +\frac{1}{2}$ (the electron is said to be *spin up*) and $m_s = -s = -\frac{1}{2}$ (the electron is said to be *spin down*). Also, if S_z has a definite value, then S_x and S_y do not. Figure 40-6 is another figure that you should not take literally but it serves to show the possible values of S_z .

The existence of electron spin was postulated on experimental evidence by two Dutch graduate students, George Uhlenbeck and Samuel Goudsmit, from their studies of atomic spectra. The theoretical basis for spin was provided a few years later by British physicist P. A. M. Dirac, who developed a relativistic quantum theory of the electron.

We have now seen the full set of quantum numbers for an electron, as listed in Table 40-1. If an electron is free, it has only its intrinsic quantum numbers s and m_s . If it is trapped in an atom, it has also has the quantum numbers n , ℓ , and m_ℓ .

Spin Magnetic Dipole Moment

As with the orbital angular momentum, a magnetic dipole moment is associated with the spin angular momentum:

$$\vec{\mu}_s = -\frac{e}{m} \vec{S}, \quad (40-11)$$

where the minus sign means that the two vectors are in opposite directions, which is due to the fact that an electron is negatively charged. This $\vec{\mu}_s$ is an intrinsic property of every electron. The vector $\vec{\mu}_s$ does not have a definite direction but it can have a definite magnitude, given by

$$\mu_s = \frac{e}{m} \sqrt{s(s+1)} \hbar. \quad (40-12)$$

The vector can also have a definite component on a z axis, given by

$$\mu_{s,z} = -2m_s \mu_B, \quad (40-13)$$

but that means that it cannot have a definite value of $\mu_{s,x}$ or $\mu_{s,y}$. Figure 40-6 shows the possible values of $\mu_{s,z}$. In the next module we shall discuss the early experimental evidence for the quantized nature in Eq. 40-13.

Shells and Subshells

As we discussed in Module 39-5, all states with the same n form a *shell*, and all states with the same value of n and ℓ form a *subshell*. As displayed in Table 40-1, for a given ℓ , there are $2\ell + 1$ possible values of quantum number m_ℓ and, for each m_ℓ , there are two possible values for the quantum number m_s (spin up and spin down). Thus, there are $2(2\ell + 1)$ states in a subshell. If we count all the states throughout a given shell with quantum number n , we find that the total number in the shell is $2n^2$.

Orbital and Spin Angular Momenta Combined

For an atom containing more than one electron, we define a total angular momentum \vec{J} , which is the vector sum of the angular momenta of the individual electrons—both their orbital and their spin angular momenta. Each element in the periodic table is defined by the number of protons in the nucleus of an atom of the element. This number of protons is defined as being the *atomic number* (or *charge number*) Z of the element. Because an electrically neutral atom contains equal numbers of protons and electrons, Z is also the number of electrons in the neutral atom, and we use this fact to indicate a \vec{J} value for a neutral atom:

$$\vec{J} = (\vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \cdots + \vec{L}_Z) + (\vec{S}_1 + \vec{S}_2 + \vec{S}_3 + \cdots + \vec{S}_Z). \quad (40-14)$$

Similarly, the total magnetic dipole moment of a multielectron atom is the vector sum of the magnetic dipole moments (both orbital and spin) of its individual electrons. However, because of the factor 2 in Eq. 40-13, the resultant magnetic dipole moment for the atom does not have the direction of vector $-\vec{J}$; instead, it makes a certain angle with that vector. The **effective magnetic dipole moment** $\vec{\mu}_{\text{eff}}$ for the atom is the component of the vector sum of the individual magnetic dipole moments in the direction of $-\vec{J}$ (Fig. 40-7). In typical atoms the orbital angular momenta and the spin angular momenta of most of the electrons sum vectorially to zero. Then \vec{J} and $\vec{\mu}_{\text{eff}}$ of those atoms are due to a relatively small number of electrons, often only a single valence electron.

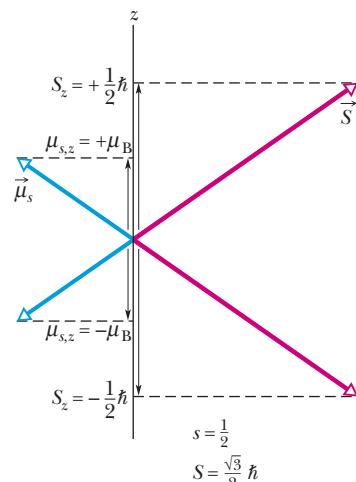


Figure 40-6 The allowed values of S_z and μ_z for an electron.

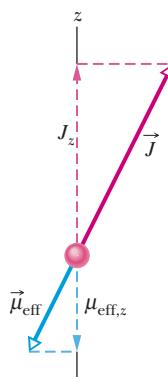


Figure 40-7 A classical model showing the total angular momentum vector \vec{J} and the effective magnetic moment vector $\vec{\mu}_{\text{eff}}$.



Checkpoint 1

An electron is in a quantum state for which the magnitude of the electron's orbital angular momentum \vec{L} is $2\sqrt{3}\hbar$. How many projections of the electron's orbital magnetic dipole moment on a z axis are allowed?

40-2 THE STERN-GERLACH EXPERIMENT

Learning Objectives

After reading this module, you should be able to . . .

40.20 Sketch the Stern–Gerlach experiment and explain the type of atom required, the anticipated result, the actual result, and the importance of the experiment.

Key Ideas

- The Stern–Gerlach experiment demonstrated that the magnetic moment of silver atoms is quantized, experimental proof that magnetic moments at the atomic level are quantized.
- An atom with a magnetic dipole moment experiences a force in a nonuniform magnetic field. If the field changes at

40.21 Apply the relationship between the magnetic field gradient and the force on an atom in a Stern–Gerlach experiment.

the rate of dB/dz along a z axis, then the force is along the z axis and its magnitude is related to the component μ_z of the dipole moment:

$$F_z = \mu_z \frac{dB}{dz}.$$

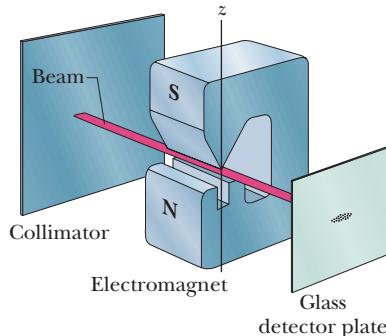


Figure 40-8 Apparatus used by Stern and Gerlach.

The Stern-Gerlach Experiment

In 1922, Otto Stern and Walther Gerlach at the University of Hamburg in Germany showed experimentally that the magnetic moment of silver atoms is quantized. In the Stern–Gerlach experiment, as it is now known, silver is vaporized in an oven, and some of the atoms in that vapor escape through a narrow slit in the oven wall and pass into an evacuated tube. Some of those escaping atoms then pass through a second narrow slit, to form a narrow beam of atoms (Fig. 40-8). (The atoms are said to be *collimated*—made into a beam—and the second slit is called a *collimator*.) The beam passes between the poles of an electromagnet and then lands on a glass detector plate where it forms a silver deposit.

When the electromagnet is off, the silver deposit is a narrow spot. However, when the electromagnet is turned on, the silver deposit should be spread vertically. The reason is that silver atoms are magnetic dipoles, and so vertical magnetic forces act on them as they pass through the vertical magnetic field of the electromagnet; these forces deflect them slightly up or down. Thus, by analyzing the silver deposit on the plate, we can determine what deflections the atoms underwent in the magnetic field. When Stern and Gerlach analyzed the pattern of silver on their detector plate, they found a surprise. However, before we discuss that surprise and its quantum implications, let us discuss the magnetic deflecting force acting on the silver atoms.

The Magnetic Deflecting Force on a Silver Atom

We have not previously discussed the type of magnetic force that deflects the silver atoms in a Stern–Gerlach experiment. It is *not* the magnetic deflecting force that acts on a moving charged particle, as given by Eq. 28-2 ($\vec{F} = q\vec{v} \times \vec{B}$). The reason is simple: A silver atom is electrically neutral (its net charge q is zero), and thus this type of magnetic force is also zero.

The type of magnetic force we seek is due to an interaction between the magnetic field \vec{B} of the electromagnet and the magnetic dipole of the individual silver atom. We can derive an expression for the force in this interaction by starting with the energy U of the dipole in the magnetic field. Equation 28-38 tells us that

$$U = -\vec{\mu} \cdot \vec{B}, \quad (40-15)$$

where $\vec{\mu}$ is the magnetic dipole moment of a silver atom. In Fig. 40-8, the positive direction of the z axis and the direction of \vec{B} are vertically upward. Thus, we can write Eq. 40-15 in terms of the component μ_z of the atom's magnetic dipole

moment along the direction of \vec{B} :

$$U = -\mu_z B. \quad (40-16)$$

Then, using Eq. 8-22 ($F = -dU/dx$) for the z axis shown in Fig. 40-8, we obtain

$$F_z = -\frac{dU}{dz} = \mu_z \frac{dB}{dz}. \quad (40-17)$$

This is what we sought—an equation for the magnetic force that deflects a silver atom as the atom passes through a magnetic field.

The term dB/dz in Eq. 40-17 is the *gradient* of the magnetic field along the z axis. If the magnetic field does not change along the z axis (as in a uniform magnetic field or no magnetic field), then $dB/dz = 0$ and a silver atom is not deflected as it moves between the magnet's poles. In the Stern-Gerlach experiment, the poles are designed to maximize the gradient dB/dz , so as to vertically deflect the silver atoms passing between the poles as much as possible, so that their deflections show up in the deposit on the glass plate.

According to classical physics, the components μ_z of silver atoms passing through the magnetic field in Fig. 40-8 should range in value from $-\mu$ (the dipole moment $\vec{\mu}$ is directed straight down the z axis) to $+\mu$ ($\vec{\mu}$ is directed straight up the z axis). Thus, from Eq. 40-17, there should be a range of forces on the atoms, and therefore a range of deflections of the atoms, from a greatest downward deflection to a greatest upward deflection. This means that we should expect the atoms to land along a vertical line on the glass plate, but they *don't*.

The Experimental Surprise

What Stern and Gerlach found was that the atoms formed two distinct spots on the glass plate, one spot above the point where they would have landed with no deflection and the other spot just as far below that point. The spots were initially too faint to be seen, but they became visible when Stern happened to breathe on the glass plate after smoking a cheap cigar. Sulfur in his breath (from the cigar) combined with the silver to produce a noticeably black silver sulfide.

This two-spot result can be seen in the plots of Fig. 40-9, which shows the outcome of a more recent version of the Stern-Gerlach experiment. In that version, a beam of cesium atoms (magnetic dipoles like the silver atoms in the original Stern-Gerlach experiment) was sent through a magnetic field with a large vertical gradient dB/dz . The field could be turned on and off, and a detector could be moved up and down through the beam.

When the field was turned off, the beam was, of course, undeflected and the detector recorded the central-peak pattern shown in Fig. 40-9. When the field was turned on, the original beam was split vertically by the magnetic field into two smaller beams, one beam higher than the previously undeflected beam and the other beam lower. As the detector moved vertically up through these two smaller beams, it recorded the two-peak pattern shown in Fig. 40-9.

The Meaning of the Results

In the original Stern-Gerlach experiment, two spots of silver were formed on the glass plate, not a vertical line of silver. This means that the component μ_z along \vec{B} (and along z) could not have any value between $-\mu$ and $+\mu$ as classical physics predicts. Instead, μ_z is restricted to only two values, one for each spot on the glass. Thus, the original Stern-Gerlach experiment showed that μ_z is quantized, implying (correctly) that $\vec{\mu}$ is also. Moreover, because the angular momentum \vec{L} of an atom is associated with $\vec{\mu}$, that angular momentum and its component L_z are also quantized.

With modern quantum theory, we can add to the explanation of the two-spot result in the Stern-Gerlach experiment. We now know that a silver atom consists

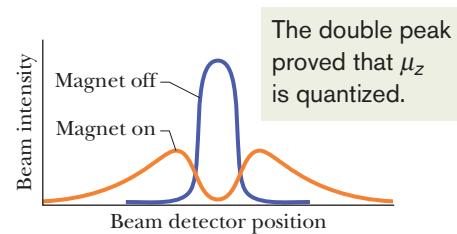


Figure 40-9 Results of a modern repetition of the Stern-Gerlach experiment. With the electromagnet turned off, there is only a single beam; with the electromagnet turned on, the original beam splits into two subbeams. The two subbeams correspond to parallel and antiparallel alignment of the magnetic moments of cesium atoms with the external magnetic field.

of many electrons, each with a spin magnetic moment and an orbital magnetic moment. We also know that all those moments vectorially cancel out *except* for a single electron, and the orbital dipole moment of that electron is zero. Thus, the combined dipole moment $\vec{\mu}$ of a silver atom is the *spin* magnetic dipole moment of that single electron. According to Eq. 40-13, this means that μ_z can have only two components along the z axis in Fig. 40-8. One component is for quantum number $m_s = +\frac{1}{2}$ (the single electron is spin up), and the other component is for quantum number $m_s = -\frac{1}{2}$ (the single electron is spin down). Substituting into Eq. 40-13 gives us

$$\mu_{s,z} = -2(+\frac{1}{2})\mu_B = -\mu_B \quad \text{and} \quad \mu_{s,z} = -2(-\frac{1}{2})\mu_B = +\mu_B. \quad (40-18)$$

Then substituting these expressions for μ_z in Eq. 40-17, we find that the force component F_z deflecting the silver atoms as they pass through the magnetic field can have only the two values

$$F_z = -\mu_B \left(\frac{dB}{dz} \right) \quad \text{and} \quad F_z = +\mu_B \left(\frac{dB}{dz} \right), \quad (40-19)$$

which result in the two spots of silver on the glass. Although no one knew about spin at the time, the Stern–Gerlach results were actually the first experimental evidence of electron spin.



Sample Problem 40.01 Beam separation in a Stern–Gerlach experiment

In the Stern–Gerlach experiment of Fig. 40-8, a beam of silver atoms passes through a magnetic field gradient dB/dz of magnitude 1.4 T/mm that is set up along the z axis. This region has a length w of 3.5 cm in the direction of the original beam. The speed of the atoms is 750 m/s. By what distance d have the atoms been deflected when they leave the region of the field gradient? The mass M of a silver atom is 1.8×10^{-25} kg.

KEY IDEAS

(1) The deflection of a silver atom in the beam is due to an interaction between the magnetic dipole of the atom and the magnetic field, because of the gradient dB/dz . The deflecting force is directed along the field gradient (along the z axis) and is given by Eqs. 40-19. Let us consider only deflection in the positive direction of z ; thus, we shall use $F_z = \mu_B(dB/dz)$ from Eqs. 40-19.

(2) We assume the field gradient dB/dz has the same value throughout the region through which the silver atoms travel. Thus, force component F_z is constant in that region, and from Newton's second law, the acceleration a_z of an atom along the z axis due to F_z is also constant.

Calculations: Putting these ideas together, we write the acceleration as

$$a_z = \frac{F_z}{M} = \frac{\mu_B(dB/dz)}{M}.$$

Because this acceleration is constant, we can use Eq. 2-15 (from Table 2-1) to write the deflection d parallel to the z axis as

$$d = v_{0z}t + \frac{1}{2}a_z t^2 = 0t + \frac{1}{2} \left(\frac{\mu_B(dB/dz)}{M} \right) t^2. \quad (40-20)$$

Because the deflecting force on the atom acts perpendicular to the atom's original direction of travel, the component v of the atom's velocity along the original direction of travel is not changed by the force. Thus, the atom requires time $t = w/v$ to travel through length w in that direction. Substituting w/v for t into Eq. 40-20, we find

$$\begin{aligned} d &= \frac{1}{2} \left(\frac{\mu_B(dB/dz)}{M} \right) \left(\frac{w}{v} \right)^2 = \frac{\mu_B(dB/dz)w^2}{2Mv^2} \\ &= (9.27 \times 10^{-24} \text{ J/T})(1.4 \times 10^3 \text{ T/m}) \\ &\quad \times \frac{(3.5 \times 10^{-2} \text{ m})^2}{(2)(1.8 \times 10^{-25} \text{ kg})(750 \text{ m/s})^2} \\ &= 7.85 \times 10^{-5} \text{ m} \approx 0.08 \text{ mm}. \end{aligned} \quad (\text{Answer})$$

The separation between the two subbeams is twice this, or 0.16 mm. This separation is not large but is easily measured.



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40-3 MAGNETIC RESONANCE

Learning Objectives

After reading this module, you should be able to ...

- 40.22** For a proton in a magnetic field, sketch the field vector and the proton's magnetic moment vector for the lower energy state and the upper energy state and then include the labels of spin up and spin down.
- 40.23** For a proton in a magnetic field, calculate the energy

difference between the two spin states and find the photon frequency and wavelength required for a transition between the states.

- 40.24** Explain the procedure of producing a nuclear magnetic resonance spectrum.

Key Ideas

- A proton has an intrinsic spin angular momentum \vec{S} and an intrinsic magnetic dipole moment $\vec{\mu}$ that are in the same direction (because the proton is positively charged).
- The magnetic dipole moment $\vec{\mu}$ of a proton in a magnetic field \vec{B} has two quantized components along the field axis: spin up (μ_z is in the direction \vec{B} and spin down μ_z is in the opposite direction).
- Contrary to the situation with an electron, spin up is the lower energy orientation; the difference between the two orientations is $2\mu_z B$.

- The energy required of a photon to spin-flip the proton between the two orientations is

$$hf = 2\mu_z B.$$

- The field is the vector sum of an external field set up by equipment and an internal field set up by the atoms and nuclei surrounding the proton.

- Detection of spin-flips can lead to nuclear magnetic resonance spectra by which specific substances can be identified.

Magnetic Resonance

As we discussed briefly in Module 32-5, a proton has a spin magnetic dipole moment $\vec{\mu}$ that is associated with the proton's intrinsic spin angular momentum \vec{S} . The two vectors are said to be coupled together and, because the proton is positively charged, they are in the same direction. Suppose a proton is located in a magnetic field \vec{B} that is directed along the positive direction of a z axis. Then $\vec{\mu}$ has two possible quantized components along that axis: the component can be $+\mu_z$ if the vector is in the direction of \vec{B} (Fig. 40-10a) or $-\mu_z$ if it is opposite the direction of \vec{B} (Fig. 40-10b).

From Eq. 28-38 ($U(\theta) = -\vec{\mu} \cdot \vec{B}$), recall that an energy is associated with the orientation of any magnetic dipole moment $\vec{\mu}$ located in an external magnetic field \vec{B} . Thus, energy is associated with the two orientations of Figs. 40-10a and b. The orientation in Fig. 40-10a is the lower-energy state ($-\mu_z B$) and is called the *spin-up state* because the proton's spin component S_z (not shown) is also aligned with \vec{B} . The orientation in Fig. 40-10b (the *spin-down state*) is the higher-energy state ($\mu_z B$). Thus, the energy difference between these two states is

$$\Delta E = \mu_z B - (-\mu_z B) = 2\mu_z B. \quad (40-21)$$

If we place a sample of water in a magnetic field \vec{B} , the protons in the hydrogen portions of each water molecule tend to be in the lower-energy state. (We shall not consider the oxygen portions.) Any one of these protons can jump to the higher-energy state by absorbing a photon with an energy hf equal to ΔE . That is, the proton can jump by absorbing a photon of energy

$$hf = 2\mu_z B. \quad (40-22)$$

Such absorption is called **magnetic resonance** or, as originally, **nuclear magnetic resonance** (NMR), and the consequent reversal of S_z is called *spin-flipping*.

In practice, the photons required for magnetic resonance have an associated frequency in the radio-frequency (RF) range and are provided by a small coil wrapped around the sample undergoing resonance. An electromagnetic oscillator called an *RF source* drives a sinusoidal current in the coil at frequency f . The electromagnetic (EM) field set up within the coil and sample also oscillates at

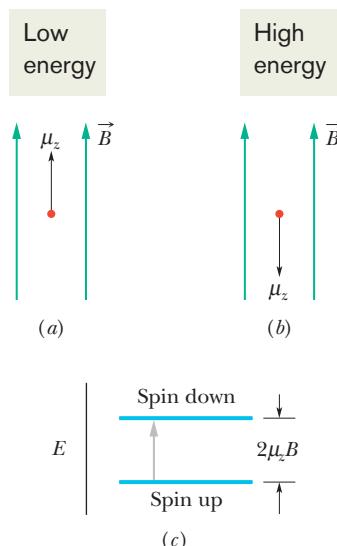


Figure 40-10 The z component of $\vec{\mu}$ for a proton in the (a) lower-energy (spin-up) and (b) higher-energy (spin-down) state. (c) An energy-level diagram for the states, showing the upward quantum jump the proton makes when its spin flips from up to down.

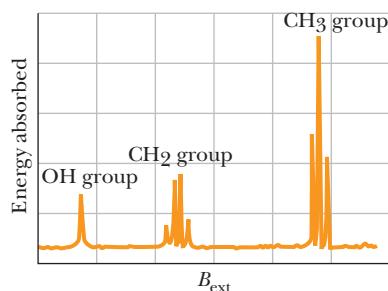


Figure 40-11 A nuclear magnetic resonance spectrum for ethanol, $\text{CH}_3\text{CH}_2\text{OH}$. The spectral lines represent the absorption of energy associated with spin-flips of protons. The three groups of lines correspond, as indicated, to protons in the OH group, the CH_2 group, and the CH_3 group of the ethanol molecule. Note that the two protons in the CH_2 group occupy four different local environments. The entire horizontal axis covers less than 10^{-4} T.

frequency f . If f meets the requirement of Eq. 40-22, the oscillating EM field can transfer a quantum of energy to a proton in the sample via a photon absorption, spin-flipping the proton.

The magnetic field magnitude B that appears in Eq. 40-22 is actually the magnitude of the net magnetic field \vec{B} at the site where a given proton undergoes spin-flipping. That net field is the vector sum of the external field \vec{B}_{ext} set up by the magnetic resonance equipment (primarily a large magnet) and the internal field \vec{B}_{int} set up by the magnetic dipole moments of the atoms and nuclei near the given proton. For practical reasons we do not discuss here, magnetic resonance is usually detected by sweeping the magnitude B_{ext} through a range of values while the frequency f of the RF source is kept at a predetermined value and the energy of the RF source is monitored. A graph of the energy loss of the RF source versus B_{ext} shows a *resonance peak* when B_{ext} sweeps through the value at which spin-flipping occurs. Such a graph is called a *nuclear magnetic resonance spectrum*, or *NMR spectrum*.

Figure 40-11 shows the NMR spectrum of ethanol, which is a molecule consisting of three groups of atoms: CH_3 , CH_2 , and OH. Protons in each group can undergo magnetic resonance, but each group has its own unique magnetic-resonance value of B_{ext} because the groups lie in different internal fields \vec{B}_{int} due to their arrangement within the $\text{CH}_3\text{CH}_2\text{OH}$ molecule. Thus, the resonance peaks in the spectrum of Fig. 40-11 form a unique NMR signature by which ethanol can be identified.

40-4 EXCLUSION PRINCIPLE AND MULTIPLE ELECTRONS IN A TRAP

Learning Objectives

After reading this module, you should be able to . . .

40.25 Identify the Pauli exclusion principle.

40.26 Explain the procedure for placing multiple electrons in traps of one, two, and three dimensions, including the need to obey the exclusion principle and to allow for

degenerate states, and explain the terms empty, partially occupied, and fully occupied.

40.27 For a system of multiple electrons in traps of one, two, and three dimensions, produce energy-level diagrams.

Key Idea

- Electrons in atoms and other traps obey the Pauli exclusion principle, which requires that no two electrons in a trap can have the same set of quantum numbers.

The Pauli Exclusion Principle

In Chapter 39 we considered a variety of electron traps, from fictional one-dimensional traps to the real three-dimensional trap of a hydrogen atom. In all those examples, we trapped only one electron. However, when we discuss traps containing two or more electrons (as we shall below), we must consider a principle that governs any particle whose spin quantum number s is not zero or an integer. This principle applies not only to electrons but also to protons and neutrons, all of which have $s = \frac{1}{2}$. The principle is known as the **Pauli exclusion principle** after Wolfgang Pauli, who formulated it in 1925. For electrons, it states that



No two electrons confined to the same trap can have the same set of values for their quantum numbers.

As we shall discuss in Module 40-5, this principle means that no two electrons in an atom can have the same four values for the quantum numbers n , ℓ , m_ℓ , and m_s . All electrons have the same quantum number $s = \frac{1}{2}$. Thus, any two electrons in an atom must differ in at least one of these other quantum numbers. Were this not true, atoms would collapse, and thus you and the world could not exist.

Multiple Electrons in Rectangular Traps

To prepare for our discussion of multiple electrons in atoms, let us discuss two electrons confined to the rectangular traps of Chapter 39. However, here we shall also include the spin angular momenta. To do this, we assume that the traps are located in a uniform magnetic field. Then according to Eq. 40-10 ($S_z = m_s \hbar$), an electron can be either spin up with $m_s = \frac{1}{2}$ or spin down with $m_s = -\frac{1}{2}$. (We assume that the field is very weak so that the associated energy is negligible.)

As we confine the two electrons to one of the traps, we must keep the Pauli exclusion principle in mind; that is, the electrons cannot have the same set of values for their quantum numbers.

- 1. One-dimensional trap.** In the one-dimensional trap of Fig. 39-2, fitting an electron wave to the trap's width L requires the single quantum number n . Therefore, any electron confined to the trap must have a certain value of n , and its quantum number m_s can be either $+\frac{1}{2}$ or $-\frac{1}{2}$. The two electrons could have different values of n , or they could have the same value of n if one of them is spin up and the other is spin down.
- 2. Rectangular corral.** In the rectangular corral of Fig. 39-13, fitting an electron wave to the corral's widths L_x and L_y requires the two quantum numbers n_x and n_y . Thus, any electron confined to the trap must have certain values for those two quantum numbers, and its quantum number m_s can be either $+\frac{1}{2}$ or $-\frac{1}{2}$; so now there are three quantum numbers. According to the Pauli exclusion principle, two electrons confined to the trap must have different values for at least one of those three quantum numbers.
- 3. Rectangular box.** In the rectangular box of Fig. 39-14, fitting an electron wave to the box's widths L_x , L_y , and L_z requires the three quantum numbers n_x , n_y , and n_z . Thus, any electron confined to the trap must have certain values for these three quantum numbers, and its quantum number m_s can be either $+\frac{1}{2}$ or $-\frac{1}{2}$; so now there are four quantum numbers. According to the Pauli exclusion principle, two electrons confined to the trap must have different values for at least one of those four quantum numbers.

Suppose we add more than two electrons, one by one, to a rectangular trap in the preceding list. The first electrons naturally go into the lowest possible energy level—they are said to *occupy* that level. However, eventually the Pauli exclusion principle disallows any more electrons from occupying that lowest energy level, and the next electron must occupy the next higher level. When an energy level cannot be occupied by more electrons because of the Pauli exclusion principle, we say that level is **full** or **fully occupied**. In contrast, a level that is not occupied by any electrons is **empty** or **unoccupied**. For intermediate situations, the level is **partially occupied**. The *electron configuration* of a system of trapped electrons is a listing or drawing either of the energy levels the electrons occupy or of the set of the quantum numbers of the electrons.

Finding the Total Energy

To find the energy of a system of two or more electrons confined to a trap, we assume that the electrons do not electrically interact with one another; that is, we shall neglect the electric potential energies of pairs of electrons. Then we can calculate the total energy for the system by calculating the energy of each electron (as in Chapter 39) and then summing those energies.

A good way to organize the energy values of a given system of electrons is with an energy-level diagram *for the system*, just as we did for a single electron in the traps of Chapter 39. The lowest level, with energy E_{gr} , corresponds to the ground state of the system. The next higher level, with energy E_{fe} , corresponds to the first excited state of the system. The next level, with energy E_{se} , corresponds to the second excited state of the system, and so on.



Sample Problem 40.02 Energy levels of multiple electrons in a 2D infinite potential well

Seven electrons are confined to a square corral (two-dimensional infinite potential well) with widths $L_x = L_y = L$ (Fig. 39-13). Assume that the electrons do not electrically interact with one another.

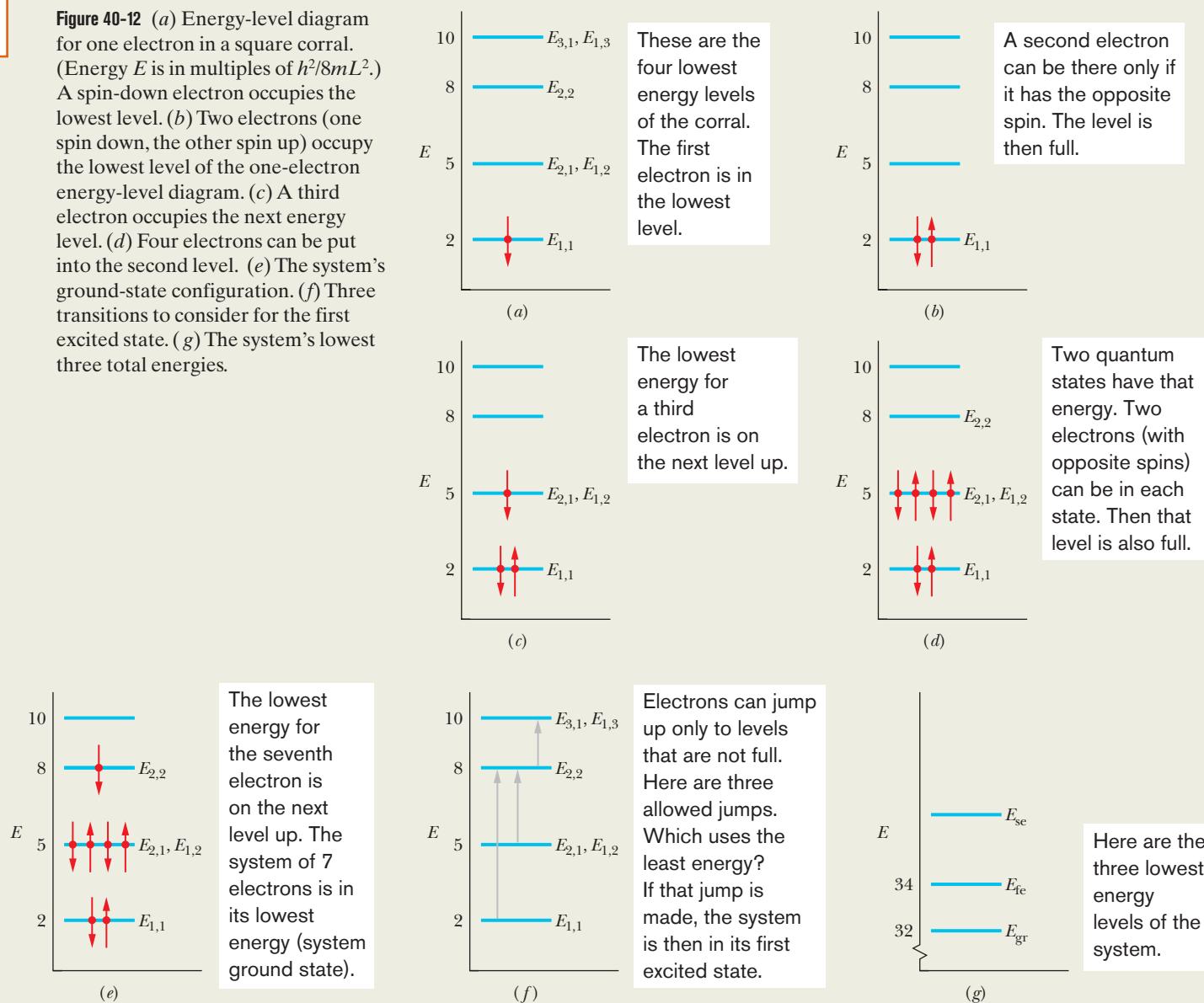
- (a) What is the electron configuration for the ground state of the system of seven electrons?

One-electron diagram: We can determine the electron configuration of the system by placing the seven electrons in the corral one by one, to build up the system. Because we assume the electrons do not electrically interact with one another, we can use the energy-level diagram for a single trapped electron

in order to keep track of how we place the seven electrons in the corral. That *one-electron energy-level diagram* is given in Fig. 39-15 and partially reproduced here as Fig. 40-12a. Recall that the levels are labeled as $E_{nx,ny}$ for their associated energy. For example, the lowest level is for energy $E_{1,1}$, where quantum number n_x is 1 and quantum number n_y is 1.

Pauli principle: The trapped electrons must obey the Pauli exclusion principle; that is, no two electrons can have the same set of values for their quantum numbers n_x , n_y , and m_s . The first electron goes into energy level $E_{1,1}$ and can have $m_s = \frac{1}{2}$ or $m_s = -\frac{1}{2}$. We arbitrarily choose the latter

Figure 40-12 (a) Energy-level diagram for one electron in a square corral. (Energy E is in multiples of $\hbar^2/8mL^2$.) A spin-down electron occupies the lowest level. (b) Two electrons (one spin down, the other spin up) occupy the lowest level of the one-electron energy-level diagram. (c) A third electron occupies the next energy level. (d) Four electrons can be put into the second level. (e) The system's ground-state configuration. (f) Three transitions to consider for the first excited state. (g) The system's lowest three total energies.



and draw a down arrow (to represent spin down) on the $E_{1,1}$ level in Fig. 40-12a. The second electron also goes into the $E_{1,1}$ level but must have $m_s = +\frac{1}{2}$ so that one of its quantum numbers differs from those of the first electron. We represent this second electron with an up arrow (for spin up) on the $E_{1,1}$ level in Fig. 40-12b.

Electrons, one by one: The level for energy $E_{1,1}$ is fully occupied, and thus the third electron cannot have that energy. Therefore, the third electron goes into the next higher level, which is for the equal energies $E_{2,1}$ and $E_{1,2}$ (the level is degenerate). This third electron can have quantum numbers n_x and n_y of either 1 and 2 or 2 and 1, respectively. It can also have a quantum number m_s of either $+\frac{1}{2}$ or $-\frac{1}{2}$. Let us arbitrarily assign it the quantum numbers $n_x = 2$, $n_y = 1$, and $m_s = -\frac{1}{2}$. We then represent it with a down arrow on the level for $E_{1,2}$ and $E_{2,1}$ in Fig. 40-12c.

You can show that the next three electrons can also go into the level for energies $E_{2,1}$ and $E_{1,2}$, provided that no set of three quantum numbers is completely duplicated. That level then contains four electrons (Fig. 40-12d), with quantum numbers (n_x, n_y, m_s) of

$$(2, 1, -\frac{1}{2}), (2, 1, +\frac{1}{2}), (1, 2, -\frac{1}{2}), (1, 2, +\frac{1}{2}),$$

and the level is fully occupied. Thus, the seventh electron goes into the next higher level, which is the $E_{2,2}$ level. Let us assume this electron is spin down, with $m_s = -\frac{1}{2}$.

Figure 40-12e shows all seven electrons on a one-electron energy-level diagram. We now have seven electrons in the corral, and they are in the configuration with the lowest energy that satisfies the Pauli exclusion principle. Thus, the ground-state configuration of the system is that shown in Fig. 40-12e and listed in Table 40-2.

(b) What is the total energy of the seven-electron system in its ground state, as a multiple of $h^2/8mL^2$?

KEY IDEA

The total energy E_{gr} is the sum of the energies of the individual electrons in the system's ground-state configuration.

Ground-state energy: The energy of each electron can be read from Table 39-1, which is partially reproduced in Table 40-2, or from Fig. 40-12e. Because there are two electrons in the first (lowest) level, four in the second level, and one in the third level, we have

$$\begin{aligned} E_{\text{gr}} &= 2\left(2 \frac{h^2}{8mL^2}\right) + 4\left(5 \frac{h^2}{8mL^2}\right) + 1\left(8 \frac{h^2}{8mL^2}\right) \\ &= 32 \frac{h^2}{8mL^2}. \end{aligned} \quad (\text{Answer})$$

(c) How much energy must be transferred to the system for it to jump to its first excited state, and what is the energy of that state?

KEY IDEAS

- If the system is to be excited, one of the seven electrons must make a quantum jump up the one-electron energy-level diagram of Fig. 40-12e.
- If that jump is to occur, the energy change ΔE of the electron (and thus of the system) must be $\Delta E = E_{\text{high}} - E_{\text{low}}$ (Eq. 39-5), where E_{low} is the energy of the level where the jump begins and E_{high} is the energy of the level where the jump ends.
- The Pauli exclusion principle must still apply; an electron *cannot* jump to a level that is fully occupied.

First-excited-state energy: Let us consider the three jumps shown in Fig. 40-12f; all are allowed by the Pauli exclusion principle because they are jumps to either empty or partially occupied states. In one of those possible jumps, an electron jumps from the $E_{1,1}$ level to the partially occupied $E_{2,2}$ level. The change in the energy is

$$\Delta E = E_{2,2} - E_{1,1} = 8 \frac{h^2}{8mL^2} - 2 \frac{h^2}{8mL^2} = 6 \frac{h^2}{8mL^2}.$$

(We shall assume that the spin orientation of the electron making the jump can change as needed.)

In another of the possible jumps in Fig. 40-12f, an electron jumps from the degenerate level of $E_{2,1}$ and $E_{1,2}$ to the partially occupied $E_{2,2}$ level. The change in the energy is

$$\Delta E = E_{2,2} - E_{2,1} = 8 \frac{h^2}{8mL^2} - 5 \frac{h^2}{8mL^2} = 3 \frac{h^2}{8mL^2}.$$

In the third possible jump in Fig. 40-12f, the electron in the $E_{2,2}$ level jumps to the unoccupied, degenerate level of $E_{1,3}$ and $E_{3,1}$. The change in energy is

$$\Delta E = E_{1,3} - E_{2,2} = 10 \frac{h^2}{8mL^2} - 8 \frac{h^2}{8mL^2} = 2 \frac{h^2}{8mL^2}.$$

Table 40-2 Ground-State Configuration and Energies

n_x	n_y	m_s	Energy ^a
2	2	$-\frac{1}{2}$	8
2	1	$+\frac{1}{2}$	5
2	1	$-\frac{1}{2}$	5
1	2	$+\frac{1}{2}$	5
1	2	$-\frac{1}{2}$	5
1	1	$+\frac{1}{2}$	2
1	1	$-\frac{1}{2}$	$\frac{2}{2}$
			Total 32

^aIn multiples of $h^2/8mL^2$.

Of these three possible jumps, the one requiring the least energy change ΔE is the last one. We could consider even more possible jumps, but none would require less energy. Thus, for the system to jump from its ground state to its first excited state, the electron in the $E_{2,2}$ level must jump to the unoccupied, degenerate level of $E_{1,3}$ and $E_{3,1}$, and the required energy is

$$\Delta E = 2 \frac{h^2}{8mL^2}. \quad (\text{Answer})$$



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The energy E_{fe} of the first excited state of the system is then

$$E_{\text{fe}} = E_{\text{gr}} + \Delta E \\ = 32 \frac{h^2}{8mL^2} + 2 \frac{h^2}{8mL^2} = 34 \frac{h^2}{8mL^2}. \quad (\text{Answer})$$

We can represent this energy and the energy E_{gr} for the ground state of the system on an energy-level diagram *for the system*, as shown in Fig. 40-12g.

40-5 BUILDING THE PERIODIC TABLE

Learning Objectives

After reading this module, you should be able to . . .

- 40.28** Identify that all states in a subshell have the same energy that is determined primarily by quantum number n but to a lesser extent by quantum number ℓ .
- 40.29** Identify the labeling system for the orbital angular momentum quantum number.
- 40.30** Identify the procedure for filling up the shells and subshells in building up the periodic table for as long as the electron-electron interaction can be neglected.

Key Ideas

- In the periodic table, the elements are listed in order of increasing atomic number Z , where Z is the number of protons in the nucleus. For a neutral atom, Z is also the number of electrons.
- States with the same value of quantum number n form a shell.

40.31 Distinguish the noble gases from the other elements in terms of chemical interactions, net angular momentum, and ionization energy.

40.32 For a transition between two given atomic energy levels, for either emission or absorption of light, apply the relationship between the energy difference and the frequency and wavelength of the light.

- States with the same values of quantum numbers n and ℓ form a subshell.
- A closed shell and a closed subshell contain the maximum number of electrons as allowed by the Pauli exclusion principle. The net angular momentum and net magnetic moment of such closed structures are zero.

Building the Periodic Table

The four quantum numbers n , ℓ , m_ℓ , and m_s identify the quantum states of individual electrons in a multielectron atom. The wave functions for these states, however, are not the same as the wave functions for the corresponding states of the hydrogen atom because, in multielectron atoms, the potential energy associated with a given electron is determined not only by the charge and position of the atom's nucleus but also by the charges and positions of all the other electrons in the atom. Solutions of Schrödinger's equation for multielectron atoms can be carried out numerically—in principle at least—using a computer.

Shells and Subshells

As we discussed in Module 40-1, all states with the same n form a *shell*, and all states with the same value of n and ℓ form a *subshell*. For a given ℓ , there are $2\ell + 1$ possible values of quantum number m_ℓ and, for each m_ℓ , there are two possible values for the quantum number m_s (spin up and spin down). Thus, there are $2(2\ell + 1)$

states in a subshell. If we count all the states throughout a given shell with quantum number n , we find that the total number in the shell is $2n^2$. All states in a given subshell have about the same energy, which depends primarily on the value of n , but it also depends somewhat on the value of ℓ .

For the purpose of labeling subshells, the values of ℓ are represented by letters:

$$\begin{array}{ccccccc} \ell = 0 & 1 & 2 & 3 & 4 & 5 & \dots \\ s & p & d & f & g & h & \dots \end{array}$$

For example, the $n = 3, \ell = 2$ subshell would be labeled the $3d$ subshell.

When we assign electrons to states in a multielectron atom, we must be guided by the Pauli exclusion principle of Module 40-4; that is, no two electrons in an atom can have the same set of the quantum numbers n, ℓ, m_ℓ , and m_s . If this important principle did not hold, *all* the electrons in any atom could jump to the atom's lowest energy level, which would eliminate the chemistry of atoms and molecules, and thus also eliminate biochemistry and us. Let us examine the atoms of a few elements to see how the Pauli exclusion principle operates in the building up of the periodic table.

Neon

The neon atom has 10 electrons. Only two of them fit into the lowest-energy subshell, the $1s$ subshell. These two electrons both have $n = 1, \ell = 0$, and $m_\ell = 0$, but one has $m_s = +\frac{1}{2}$ and the other has $m_s = -\frac{1}{2}$. The $1s$ subshell contains $2[2(0) + 1] = 2$ states. Because this subshell then contains all the electrons permitted by the Pauli principle, it is said to be **closed**.

Two of the remaining eight electrons fill the next lowest energy subshell, the $2s$ subshell. The last six electrons just fill the $2p$ subshell, which, with $\ell = 1$, holds $2[2(1) + 1] = 6$ states.

In a closed subshell, all allowed z projections of the orbital angular momentum vector \vec{L} are present and, as you can verify from Fig. 40-5, these projections cancel for the subshell as a whole; for every positive projection there is a corresponding negative projection of the same magnitude. Similarly, the z projections of the spin angular momenta also cancel. Thus, a closed subshell has no angular momentum and no magnetic moment of any kind. Furthermore, its probability density is spherically symmetric. Then neon with its three closed subshells ($1s$, $2s$, and $2p$) has no “loosely dangling electrons” to encourage chemical interaction with other atoms. Neon, like the other **noble gases** that form the right-hand column of the periodic table, is almost chemically inert.

Sodium

Next after neon in the periodic table comes sodium, with 11 electrons. Ten of them form a closed neon-like core, which, as we have seen, has zero angular momentum. The remaining electron is largely outside this inert core, in the $3s$ subshell—the next lowest energy subshell. Because this **valence electron** of sodium is in a state with $\ell = 0$ (that is, an s state using the lettering system above), the sodium atom's angular momentum and magnetic dipole moment must be due entirely to the spin of this single electron.

Sodium readily combines with other atoms that have a “vacancy” into which sodium's loosely bound valence electron can fit. Sodium, like the other **alkali metals** that form the left-hand column of the periodic table, is chemically active.

Chlorine

The chlorine atom, which has 17 electrons, has a closed 10-electron, neon-like core, with 7 electrons left over. Two of them fill the $3s$ subshell, leaving five to be assigned to the $3p$ subshell, which is the subshell next lowest in energy. This subshell, which has $\ell = 1$, can hold $2[2(1) + 1] = 6$ electrons, and so there is a vacancy, or a “hole,” in this subshell.

Chlorine is receptive to interacting with other atoms that have a valence electron that might fill this hole. Sodium chloride (NaCl), for example, is a very stable compound. Chlorine, like the other **halogens** that form column VIIA of the periodic table, is chemically active.

Iron

The arrangement of the 26 electrons of the iron atom can be represented as follows:

$$\begin{array}{cccccc} 1s^2 & 2s^2 & 2p^6 & 3s^2 & \underbrace{3p^6 & 3d^6} & 4s^2 \end{array}$$

The subshells are listed in numerical order and, following convention, a superscript gives the number of electrons in each subshell. From Table 40-1 we can see that an *s* subshell ($\ell = 0$) can hold 2 electrons, a *p* subshell ($\ell = 1$) can hold 6, and a *d* subshell ($\ell = 2$) can hold 10. Thus, iron's first 18 electrons form the five filled subshells that are marked off by the bracket, leaving 8 electrons to be accounted for. Six of the eight go into the *3d* subshell, and the remaining two go into the *4s* subshell.

The reason the last two electrons do not also go into the *3d* subshell (which can hold 10 electrons) is that the $3d^6\ 4s^2$ configuration results in a lower-energy state for the atom as a whole than would the $3d^8$ configuration. An iron atom with 8 electrons (rather than 6) in the *3d* subshell would quickly make a transition to the $3d^6\ 4s^2$ configuration, emitting electromagnetic radiation in the process. The lesson here is that except for the simplest elements, the states may not be filled in what we might think of as their “logical” sequence.

40-6 X RAYS AND THE ORDERING OF THE ELEMENTS

Learning Objectives

After reading this module, you should be able to . . .

- 40.33** Identify where x rays are located in the electromagnetic spectrum.
- 40.34** Explain how x rays are produced in a laboratory or medical setting.
- 40.35** Distinguish between a continuous x-ray spectrum and a characteristic x-ray spectrum.
- 40.36** In a continuous x-ray spectrum, identify the cause of the cutoff wavelength λ_{\min} .
- 40.37** Identify that in an electron–atom collision, energy and momentum are conserved.
- 40.38** Apply the relationship between a cutoff wavelength λ_{\min} and the kinetic energy K_0 of the incident electrons.

Key Ideas

- When a beam of high-energy electrons impact a target, the electrons can lose their energy by scattering from atoms and emitting a continuous spectrum of x rays.
- The shortest wavelength in the spectrum is the cutoff wavelength λ_{\min} , which is emitted when an incident electron loses its full kinetic energy K_0 in a single collision:

$$\lambda_{\min} = \frac{hc}{K_0}$$

- 40.39** Draw an energy-level diagram for holes and identify (with labels) the transitions that produce x rays.
- 40.40** For a given hole transition, calculate the wavelength of the emitted x ray.
- 40.41** Explain the importance of Moseley's work with regard to the periodic table.
- 40.42** Sketch a Moseley plot.
- 40.43** Describe the screening effect in a multielectron atom.
- 40.44** Apply the relationship between the frequency of the emitted K-alpha x rays and the atomic number Z of the atoms.

- The characteristic x-ray spectrum is produced when incident electrons eject low-lying electrons in the target atoms and electrons from upper levels jump down to the resulting holes, emitting light.
- A Moseley plot is a graph of the square root of the characteristic-emission frequencies \sqrt{f} versus atomic number Z of the target atoms. The straight-line plot reveals that the position of an element in the periodic table is set by Z and not the atomic weight.

X Rays and the Ordering of the Elements

When a solid target, such as solid copper or tungsten, is bombarded with electrons whose kinetic energies are in the kiloelectron-volt range, electromagnetic radiation called **x rays** is emitted. Our concern here is what these rays can teach us about the atoms that absorb or emit them. Figure 40-13 shows the wavelength spectrum of the x rays produced when a beam of 35 keV electrons falls on a molybdenum target. We see a broad, continuous spectrum of radiation on which are superimposed two peaks of sharply defined wavelengths. The continuous spectrum and the peaks arise in different ways, which we next discuss separately.

The Continuous X-Ray Spectrum

Here we examine the continuous x-ray spectrum of Fig. 40-13, ignoring for the time being the two prominent peaks that rise from it. Consider an electron of initial kinetic energy K_0 that collides (interacts) with one of the target atoms, as in Fig. 40-14. The electron may lose an amount of energy ΔK , which will appear as the energy of an x-ray photon that is radiated away from the site of the collision. (Very little energy is transferred to the recoiling atom because of the relatively large mass of the atom; here we neglect that transfer.)

The scattered electron in Fig. 40-14, whose energy is now less than K_0 , may have a second collision with a target atom, generating a second photon, with a different photon energy. This electron-scattering process can continue until the electron is approximately stationary. All the photons generated by these collisions form part of the continuous x-ray spectrum.

A prominent feature of that spectrum in Fig. 40-13 is the sharply defined **cutoff wavelength** λ_{\min} , below which the continuous spectrum does not exist. This minimum wavelength corresponds to a collision in which an incident electron loses *all* its initial kinetic energy K_0 in a single head-on collision with a target atom. Essentially all this energy appears as the energy of a single photon, whose associated wavelength—the minimum possible x-ray wavelength—is found from

$$K_0 = hf = \frac{hc}{\lambda_{\min}},$$

or

$$\lambda_{\min} = \frac{hc}{K_0} \quad (\text{cutoff wavelength}). \quad (40-23)$$

The cutoff wavelength is totally independent of the target material. If we were to switch from a molybdenum target to a copper target, for example, all features of the x-ray spectrum of Fig. 40-13 would change *except* the cutoff wavelength.



Checkpoint 2

Does the cutoff wavelength λ_{\min} of the continuous x-ray spectrum increase, decrease, or remain the same if you (a) increase the kinetic energy of the electrons that strike the x-ray target, (b) allow the electrons to strike a thin foil rather than a thick block of the target material, (c) change the target to an element of higher atomic number?

The Characteristic X-Ray Spectrum

We now turn our attention to the two peaks of Fig. 40-13, labeled K_α and K_β . These (and other peaks that appear at wavelengths beyond the range displayed in Fig. 40-13) form the **characteristic x-ray spectrum** of the target material.

The peaks arise in a two-part process. (1) An energetic electron strikes an atom in the target and, while it is being scattered, the incident electron knocks out one of the atom's deep-lying (low n value) electrons. If the deep-lying elec-

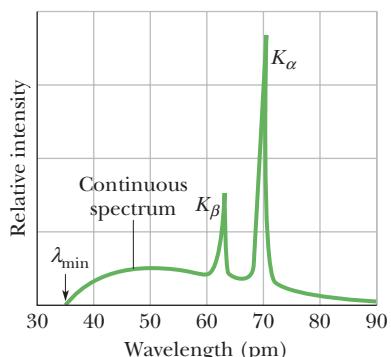


Figure 40-13 The distribution by wavelength of the x rays produced when 35 keV electrons strike a molybdenum target. The sharp peaks and the continuous spectrum from which they rise are produced by different mechanisms.

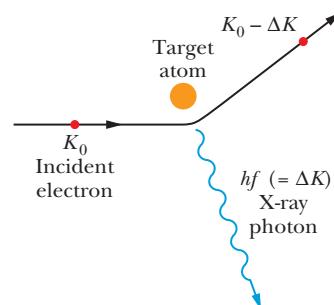


Figure 40-14 An electron of kinetic energy K_0 passing near an atom in the target may generate an x-ray photon, the electron losing part of its energy in the process. The continuous x-ray spectrum arises in this way.

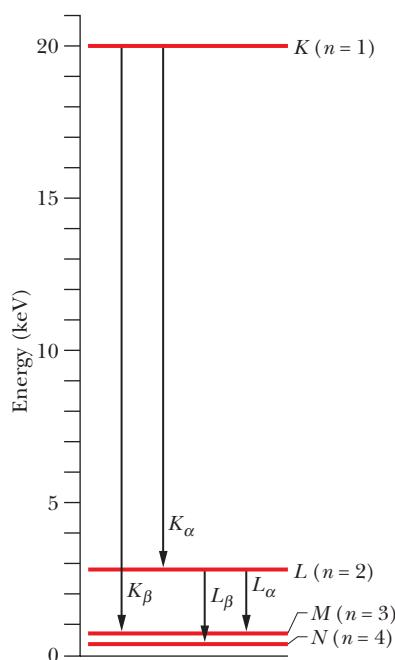


Figure 40-15 A simplified energy-level diagram for a molybdenum atom, showing the transitions (of holes rather than electrons) that give rise to some of the characteristic x rays of that element. Each horizontal line represents the energy of the atom with a hole (a missing electron) in the shell indicated.

tron is in the shell defined by $n = 1$ (called, for historical reasons, the K shell), there remains a vacancy, or *hole*, in this shell. (2) An electron in one of the shells with a higher energy jumps to the K shell, filling the hole in this shell. During this jump, the atom emits a characteristic x-ray photon. If the electron that fills the K -shell vacancy jumps from the shell with $n = 2$ (called the L shell), the emitted radiation is the K_{α} line of Fig. 40-13; if it jumps from the shell with $n = 3$ (called the M shell), it produces the K_{β} line, and so on. The hole left in either the L or M shell will be filled by an electron from still farther out in the atom.

In studying x rays, it is more convenient to keep track of where a hole is created deep in the atom's "electron cloud" than to record the changes in the quantum state of the electrons that jump to fill that hole. Figure 40-15 does exactly that; it is an energy-level diagram for molybdenum, the element to which Fig. 40-13 refers. The baseline ($E = 0$) represents the neutral atom in its ground state. The level marked K (at $E = 20$ keV) represents the energy of the molybdenum atom with a hole in its K shell, the level marked L (at $E = 2.7$ keV) represents the atom with a hole in its L shell, and so on.

The transitions marked K_{α} and K_{β} in Fig. 40-15 are the ones that produce the two x-ray peaks in Fig. 40-13. The K_{α} spectral line, for example, originates when an electron from the L shell fills a hole in the K shell. To state this transition in terms of what the arrows in Fig. 40-15 show, a hole originally in the K shell moves to the L shell.

Ordering the Elements

In 1913, British physicist H. G. J. Moseley generated characteristic x rays for as many elements as he could find—he found 38—by using them as targets for electron bombardment in an evacuated tube of his own design. By means of a trolley manipulated by strings, Moseley was able to move the individual targets into the path of an electron beam. He measured the wavelengths of the emitted x rays by the crystal diffraction method described in Module 36-7.

Moseley then sought (and found) regularities in these spectra as he moved from element to element in the periodic table. In particular, he noted that if, for a given spectral line such as K_{α} , he plotted for each element the square root of the frequency f against the position of the element in the periodic table, a straight line resulted. Figure 40-16 shows a portion of his extensive data. Moseley's conclusion was this:

We have here a proof that there is in the atom a fundamental quantity, which increases by regular steps as we pass from one element to the next. This quantity can only be the charge on the central nucleus.

As a result of Moseley's work, the characteristic x-ray spectrum became the universally accepted signature of an element, permitting the solution of a number of

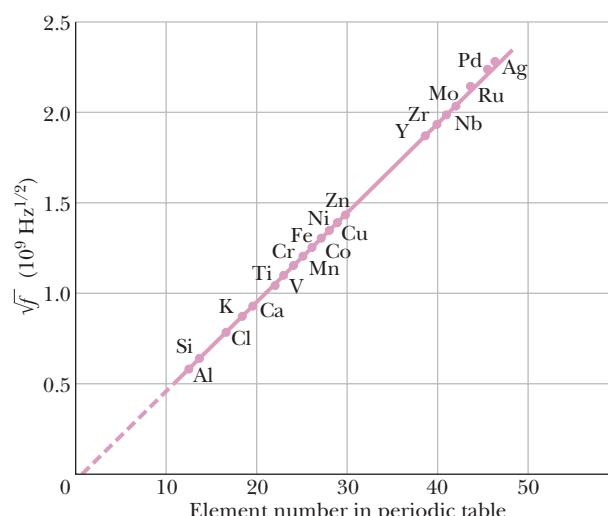


Figure 40-16 A Moseley plot of the K_{α} line of the characteristic x-ray spectra of 21 elements. The frequency is calculated from the measured wavelength.

periodic table puzzles. Prior to that time (1913), the positions of elements in the table were assigned in order of atomic *mass*, although it was necessary to invert this order for several pairs of elements because of compelling chemical evidence; Moseley showed that it is the nuclear charge (that is, atomic number Z) that is the real basis for ordering the elements.

In 1913 the periodic table had several empty squares, and a surprising number of claims for new elements had been advanced. The x-ray spectrum provided a conclusive test of such claims. The lanthanide elements, often called the rare earth elements, had been sorted out only imperfectly because their similar chemical properties made sorting difficult. Once Moseley's work was reported, these elements were properly organized.

It is not hard to see why the characteristic x-ray spectrum shows such impressive regularities from element to element whereas the optical spectrum in the visible and near-visible region does not: The key to the identity of an element is the charge on its nucleus. Gold, for example, is what it is because its atoms have a nuclear charge of $+79e$ (that is, $Z = 79$). An atom with one more elementary charge on its nucleus is mercury; with one fewer, it is platinum. The K electrons, which play such a large role in the production of the x-ray spectrum, lie very close to the nucleus and are thus sensitive probes of its charge. The optical spectrum, on the other hand, involves transitions of the outermost electrons, which are heavily screened from the nucleus by the remaining electrons of the atom and thus are *not* sensitive probes of nuclear charge.

Accounting for the Moseley Plot

Moseley's experimental data, of which the Moseley plot of Fig. 40-16 is but a part, can be used directly to assign the elements to their proper places in the periodic table. This can be done even if no theoretical basis for Moseley's results can be established. However, there is such a basis.

According to Eq. 39-33, the energy of the hydrogen atom is

$$E_n = -\frac{me^4}{8\epsilon_0^2 h^2} \frac{1}{n^2} = -\frac{13.60 \text{ eV}}{n^2}, \quad \text{for } n = 1, 2, 3, \dots \quad (40-24)$$

Consider now one of the two innermost electrons in the K shell of a multi-electron atom. Because of the presence of the other K -shell electron, our electron "sees" an effective nuclear charge of approximately $(Z - 1)e$, where e is the elementary charge and Z is the atomic number of the element. The factor e^4 in Eq. 40-24 is the product of e^2 —the square of hydrogen's nuclear charge—and $(-e)^2$ —the square of an electron's charge. For a multielectron atom, we can approximate the effective energy of the atom by replacing the factor e^4 in Eq. 40-24 with $(Z - 1)^2 e^2 \times (-e)^2$, or $e^4(Z - 1)^2$. That gives us

$$E_n = -\frac{(13.60 \text{ eV})(Z - 1)^2}{n^2}. \quad (40-25)$$

We saw that the K_α x-ray photon (of energy hf) arises when an electron makes a transition from the L shell (with $n = 2$ and energy E_2) to the K shell (with $n = 1$ and energy E_1). Thus, using Eq. 40-25, we may write the energy change as

$$\begin{aligned} \Delta E &= E_2 - E_1 \\ &= \frac{-(13.60 \text{ eV})(Z - 1)^2}{2^2} - \frac{-(13.60 \text{ eV})(Z - 1)^2}{1^2} \\ &= (10.2 \text{ eV})(Z - 1)^2. \end{aligned}$$

Then the frequency f of the K_α line is

$$\begin{aligned} f &= \frac{\Delta E}{h} = \frac{(10.2 \text{ eV})(Z - 1)^2}{(4.14 \times 10^{-15} \text{ eV} \cdot \text{s})} \\ &= (2.46 \times 10^{15} \text{ Hz})(Z - 1)^2. \end{aligned} \quad (40-26)$$

Taking the square root of both sides yields

$$\sqrt{f} = CZ - C, \quad (40-27)$$

in which C is a constant ($= 4.96 \times 10^7 \text{ Hz}^{1/2}$). Equation 40-27 is the equation of a straight line. It shows that if we plot the square root of the frequency of the K_α x-ray spectral line against the atomic number Z , we should obtain a straight line. As Fig. 40-16 shows, that is exactly what Moseley found.



Sample Problem 40.03 Characteristic spectrum in x-ray production

A cobalt target is bombarded with electrons, and the wavelengths of its characteristic x-ray spectrum are measured. There is also a second, fainter characteristic spectrum, which is due to an impurity in the cobalt. The wavelengths of the K_α lines are 178.9 pm (cobalt) and 143.5 pm (impurity), and the proton number for cobalt is $Z_{\text{Co}} = 27$. Determine the impurity using only these data.

KEY IDEA

The wavelengths of the K_α lines for both the cobalt (Co) and the impurity (X) fall on a K_α Moseley plot, and Eq. 40-27 is the equation for that plot.

Calculations: Substituting c/λ for f in Eq. 40-27, we obtain

$$\sqrt{\frac{c}{\lambda_{\text{Co}}}} = CZ_{\text{Co}} - C \quad \text{and} \quad \sqrt{\frac{c}{\lambda_X}} = CZ_X - C.$$

Dividing the second equation by the first neatly eliminates C , yielding

$$\sqrt{\frac{\lambda_{\text{Co}}}{\lambda_X}} = \frac{Z_X - 1}{Z_{\text{Co}} - 1}.$$

Substituting the given data yields

$$\sqrt{\frac{178.9 \text{ pm}}{143.5 \text{ pm}}} = \frac{Z_X - 1}{27 - 1}.$$

Solving for the unknown, we find that

$$Z_X = 30.0. \quad (\text{Answer})$$

Thus, the number of protons in the impurity nucleus is 30, and a glance at the periodic table identifies the impurity as zinc. Note that with a larger value of Z than cobalt, zinc has a smaller value of the K_α line. This means that the energy associated with that jump must be greater in zinc than cobalt.



Additional examples, video, and practice available at WileyPLUS

40-7 LASERS

Learning Objectives

After reading this module, you should be able to . . .

- 40.45 Distinguish the light of a laser from the light of a common lightbulb.
- 40.46 Sketch energy-level diagrams for the three basic ways that light can interact with matter (atoms) and identify which is the basis of lasing.
- 40.47 Identify metastable states.
- 40.48 For two energy states, apply the relationship between the relative number of atoms in the higher state due to thermal agitation, the energy difference, and the temperature.

Key Ideas

- In stimulated emission, an atom in an excited state can be induced to de-excite to a lower energy state by emitting a photon if an identical photon passes the atom.
- The light emitted in stimulated emission is in phase with and travels in the direction of the light causing the emission.

- 40.49 Identify population inversion, explain why it is required in a laser, and relate it to the lifetimes of the states.
- 40.50 Discuss how a helium–neon laser works, pointing out which gas lasers and explaining why the other gas is required.
- 40.51 For stimulated emission, apply the relationships between energy change, frequency, and wavelength.
- 40.52 For stimulated emission, apply the relationships between energy, power, time, intensity, area, photon energy, and rate of photon emission.

- A laser can emit light via stimulated emission provided that its atoms are in a population inversion. That is, for the pair of levels involved in the stimulated emission, more atoms must be in the upper level than the lower level so that there is more stimulated emission than just absorption.

Lasers and Laser Light

In the early 1960s, quantum physics made one of its many contributions to technology: the **laser**. Laser light, like the light from an ordinary lightbulb, is emitted when atoms make a transition from one quantum state to a lower one. However, in a lightbulb the emissions are random, both in time and direction, and in a laser they are coordinated so that the emissions are at the same time and in the same direction. As a result, laser light has the following characteristics:

- 1. Laser light is highly monochromatic.** Light from an ordinary incandescent lightbulb is spread over a continuous range of wavelengths and is certainly not monochromatic. The radiation from a fluorescent neon sign is monochromatic, true, to about 1 part in 10^6 , but the sharpness of definition of laser light can be many times greater, as much as 1 part in 10^{15} .
- 2. Laser light is highly coherent.** Individual long waves (*wave trains*) for laser light can be several hundred kilometers long. When two separated beams that have traveled such distances over separate paths are recombined, they “remember” their common origin and are able to form a pattern of interference fringes. The corresponding *coherence length* for wave trains emitted by a lightbulb is typically less than a meter.
- 3. Laser light is highly directional.** A laser beam spreads very little; it departs from strict parallelism only because of diffraction at the exit aperture of the laser. For example, a laser pulse used to measure the distance to the Moon generates a spot on the Moon’s surface with a diameter of only a few kilometers. Light from an ordinary bulb can be made into an approximately parallel beam by a lens, but the beam divergence is much greater than for laser light. Each point on a lightbulb’s filament forms its own separate beam, and the angular divergence of the overall composite beam is set by the size of the filament.
- 4. Laser light can be sharply focused.** If two light beams transport the same amount of energy, the beam that can be focused to the smaller spot will have the greater intensity (power per unit area) at that spot. For laser light, the focused spot can be so small that an intensity of 10^{17} W/cm^2 is readily obtained. An oxyacetylene flame, by contrast, has an intensity of only about 10^3 W/cm^2 .

Lasers Have Many Uses

The smallest lasers, used for voice and data transmission over optical fibers, have as their active medium a semiconducting crystal about the size of a pinhead. Small as they are, such lasers can generate about 200 mW of power. The largest lasers, used for nuclear fusion research and for astronomical and military applications, fill a large building. The largest such laser can generate brief pulses of laser light with a power level, during the pulse, of about 10^{14} W . This is a few hundred times greater than the total electrical power generating capacity of the United States. To avoid a brief national power blackout during a pulse, the energy required for each pulse is stored up at a steady rate during the relatively long interpulse interval.

Among the many uses of lasers are reading bar codes, manufacturing and reading compact discs and DVDs, performing surgery of many kinds (both as a surgical aid as in Fig. 40-17 and as a cutting and cauterizing tool), surveying, cutting cloth in the garment industry (several hundred layers at a time), welding auto bodies, and generating holograms.

Figure 40-17 A patient’s head is scanned and mapped by (red) laser light in preparation for brain surgery. During the surgery, the laser-derived image of the head will be superimposed on the model of the brain shown on the monitor, to guide the surgical team into the region shown in green (lower right) on the model displayed on the screen.



Sam Ogden/Photo Researchers, Inc.

How Lasers Work

Because the word “laser” is an acronym for “light amplification by the stimulated emission of radiation,” you should not be surprised that stimulated emission is the key to laser operation. Einstein introduced this concept in 1917 in the paper where he explained the Planck formula for an ideal blackbody radiator (Eq. 38-14). Although the world had to wait until 1960 to see an operating laser, the groundwork for its development was put in place decades earlier.

Consider an isolated atom that can exist either in its state of lowest energy (its ground state), whose energy is E_0 , or in a state of higher energy (an excited state), whose energy is E_x . Here are three processes by which the atom can move from one of these states to the other:

- 1. Absorption.** Figure 40-18a shows the atom initially in its ground state. If the atom is placed in an electromagnetic field that is alternating at frequency f , the atom can absorb an amount of energy hf from that field and move to the higher-energy state. From the principle of conservation of energy we have

$$hf = E_x - E_0. \quad (40-28)$$

We call this process **absorption**.

- 2. Spontaneous emission.** In Fig. 40-18b the atom is in its excited state and no external radiation is present. After a time, the atom will de-excite to its ground state, emitting a photon of energy hf in the process. We call this process **spontaneous emission**—*spontaneous* because the event is random and set by chance. The light from the filament of an ordinary lightbulb or any other common light source is generated in this way.

Normally, the mean life of excited atoms before spontaneous emission occurs is about 10^{-8} s. However, for some excited states, this mean life is perhaps as much as 10^5 times longer. We call such long-lived states **metastable**; they play an important role in laser operation.

- 3. Stimulated emission.** In Fig. 40-18c the atom is again in its excited state, but this time radiation with a frequency given by Eq. 40-28 is present. A photon of energy hf can stimulate the atom to move to its ground state, during which process the atom emits an additional photon, whose energy is also hf . We call this process **stimulated emission**—*stimulated* because the event is triggered by the external photon. The emitted photon is in every way identical to the stimulating photon. Thus, the waves associated with the photons have the same energy, phase, polarization, and direction of travel.

Figure 40-18 The interaction of radiation and matter in the processes of (a) absorption, (b) spontaneous emission, and (c) stimulated emission. An atom (matter) is represented by the red dot; the atom is in either a lower quantum state with energy E_0 or a higher quantum state with energy E_x . In (a) the atom absorbs a photon of energy hf from a passing light wave. In (b) it emits a light wave by emitting a photon of energy hf . In (c) a passing light wave with photon energy hf causes the atom to emit a photon of the same energy, increasing the energy of the light wave.

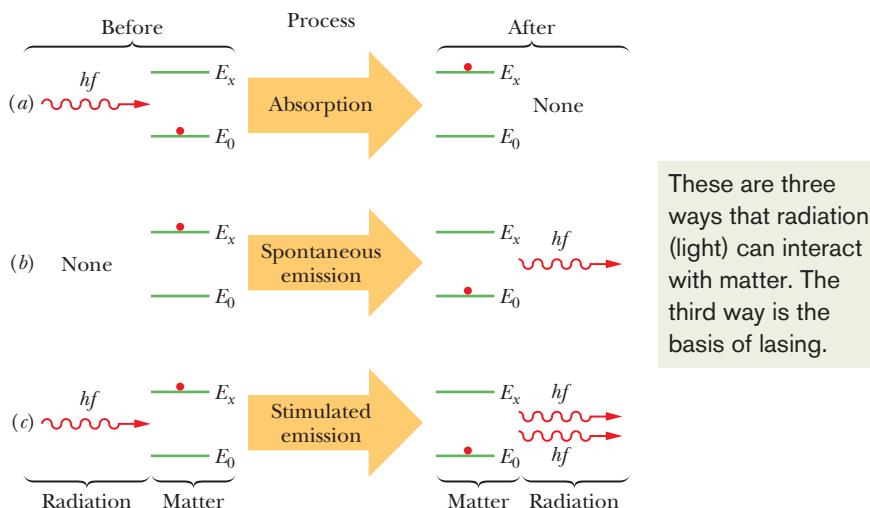


Figure 40-18c describes stimulated emission for a single atom. Suppose now that a sample contains a large number of atoms in thermal equilibrium at temperature T . Before any radiation is directed at the sample, a number N_0 of these atoms are in their ground state with energy E_0 and a number N_x are in a state of higher energy E_x . Ludwig Boltzmann showed that N_x is given in terms of N_0 by

$$N_x = N_0 e^{-(E_x - E_0)/kT}, \quad (40-29)$$

in which k is Boltzmann's constant. This equation seems reasonable. The quantity kT is the mean kinetic energy of an atom at temperature T . The higher the temperature, the more atoms—on average—will have been “bumped up” by thermal agitation (that is, by atom–atom collisions) to the higher energy state E_x . Also, because $E_x > E_0$, Eq. 40-29 requires that $N_x < N_0$; that is, there will always be fewer atoms in the excited state than in the ground state. This is what we expect if the level populations N_0 and N_x are determined only by the action of thermal agitation. Figure 40-19a illustrates this situation.

If we now flood the atoms of Fig. 40-19a with photons of energy $E_x - E_0$, photons will disappear via absorption by ground-state atoms and photons will be generated largely via stimulated emission of excited-state atoms. Einstein showed that the probabilities per atom for these two processes are identical. Thus, because there are more atoms in the ground state, the *net* effect will be the absorption of photons.

To produce laser light, we must have more photons emitted than absorbed; that is, we must have a situation in which stimulated emission dominates. Thus, we need more atoms in the excited state than in the ground state, as in Fig. 40-19b. However, because such a **population inversion** is not consistent with thermal equilibrium, we must think up clever ways to set up and maintain one.

The Helium-Neon Gas Laser

Figure 40-20 shows a common type of laser developed in 1961 by Ali Javan and his coworkers. The glass discharge tube is filled with a 20 : 80 mixture of helium and neon gases, neon being the medium in which laser action occurs.

Figure 40-21 shows simplified energy-level diagrams for the two types of atoms. An electric current passed through the helium–neon gas mixture serves—through collisions between helium atoms and electrons of the current—to raise many helium

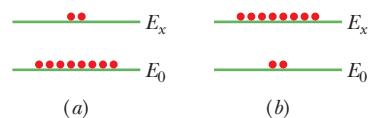


Figure 40-19 (a) The equilibrium distribution of atoms between the ground state E_0 and excited state E_x accounted for by thermal agitation. (b) An inverted population, obtained by special methods. Such a population inversion is essential for laser action.

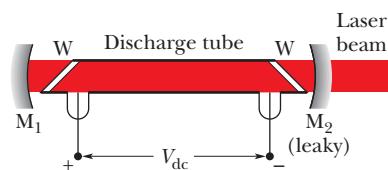


Figure 40-20 The elements of a helium–neon gas laser. An applied potential V_{dc} sends electrons through a discharge tube containing a mixture of helium gas and neon gas. Electrons collide with helium atoms, which then collide with neon atoms, which emit light along the length of the tube. The light passes through transparent windows W and reflects back and forth through the tube from mirrors M_1 and M_2 to cause more neon atom emissions. Some of the light leaks through mirror M_2 to form the laser beam.

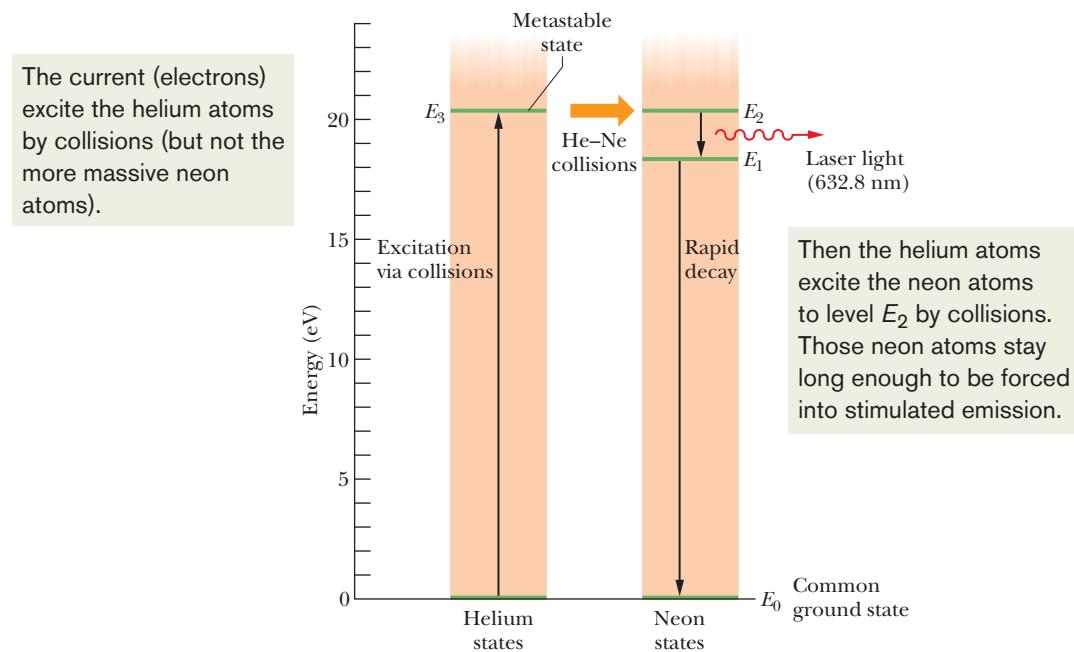


Figure 40-21 Five essential energy levels for helium and neon atoms in a helium–neon gas laser. Laser action occurs between levels E_2 and E_1 of neon when more atoms are at the E_2 level than at the E_1 level.

atoms to state E_3 , which is metastable with a mean life of at least $1\ \mu\text{s}$. (The neon atoms are too massive to be excited by collisions with the (low-mass) electrons.)

The energy of helium state E_3 (20.61 eV) is very close to the energy of neon state E_2 (20.66 eV). Thus, when a metastable (E_3) helium atom and a ground-state (E_0) neon atom collide, the excitation energy of the helium atom is often transferred to the neon atom, which then moves to state E_2 . In this manner, neon level E_2 (with a mean life of 170 ns) can become more heavily populated than neon level E_1 (which, with a mean life of only 10 ns, is almost empty).

This population inversion is relatively easy to set up because (1) initially there are essentially no neon atoms in state E_1 , (2) the long mean life of helium level E_3 means that there is always a good chance that collisions will excite neon atoms to their E_2 level, and (3) once those neon atoms undergo stimulated emission and fall to their E_1 level, they almost immediately fall down to their ground state (via intermediate levels not shown) and are then ready to be re-excited by collisions.

Suppose now that a single photon is spontaneously emitted as a neon atom transfers from state E_2 to state E_1 . Such a photon can trigger a stimulated emission event, which, in turn, can trigger other stimulated emission events. Through such a chain reaction, a coherent beam of laser light, moving parallel to the tube axis, can build up rapidly. This light, of wavelength 632.8 nm (red), moves through the discharge tube many times by successive reflections from mirrors M_1 and M_2 shown in Fig. 40-20, accumulating additional stimulated emission photons with each passage. M_1 is totally reflecting, but M_2 is slightly “leaky” so that a small fraction of the laser light escapes to form a useful external beam.



Checkpoint 3

The wavelength of light from laser A (a helium–neon gas laser) is 632.8 nm; that from laser B (a carbon dioxide gas laser) is $10.6\ \mu\text{m}$; that from laser C (a gallium arsenide semiconductor laser) is 840 nm. Rank these lasers according to the energy interval between the two quantum states responsible for laser action, greatest first.



Sample Problem 40.04 Population inversion in a laser

In the helium–neon laser of Fig. 40-20, laser action occurs between two excited states of the neon atom. However, in many lasers, laser action (*lasing*) occurs between the ground state and an excited state, as suggested in Fig. 40-19b.

- (a) Consider such a laser that emits at wavelength $\lambda = 550\ \text{nm}$. If a population inversion is not generated, what is the ratio of the population of atoms in state E_x to the population in the ground state E_0 , with the atoms at room temperature?

KEY IDEAS

- (1) The naturally occurring population ratio N_x/N_0 of the two states is due to thermal agitation of the gas atoms (Eq. 40-29):

$$N_x/N_0 = e^{-(E_x - E_0)/kT}. \quad (40-30)$$

To find N_x/N_0 with Eq. 40-30, we need to find the energy separation $E_x - E_0$ between the two states. (2) We can obtain $E_x - E_0$ from the given wavelength of 550 nm for the lasing between those two states.

Calculation: The lasing wavelength gives us

$$\begin{aligned} E_x - E_0 &= hf = \frac{hc}{\lambda} \\ &= \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})(3.00 \times 10^8\ \text{m/s})}{(550 \times 10^{-9}\ \text{m})(1.60 \times 10^{-19}\ \text{J/eV})} \\ &= 2.26\ \text{eV}. \end{aligned}$$

To solve Eq. 40-30, we also need the mean energy of thermal agitation kT for an atom at room temperature (assumed to be 300 K), which is

$$kT = (8.62 \times 10^{-5}\ \text{eV/K})(300\ \text{K}) = 0.0259\ \text{eV},$$

in which k is Boltzmann’s constant.

Substituting the last two results into Eq. 40-30 gives us the population ratio at room temperature:

$$\begin{aligned} N_x/N_0 &= e^{-(2.26\ \text{eV})/(0.0259\ \text{eV})} \\ &\approx 1.3 \times 10^{-38}. \quad (\text{Answer}) \end{aligned}$$

This is an extremely small number. It is not unreasonable, however. Atoms with a mean thermal agitation energy of

only 0.0259 eV will not often impart an energy of 2.26 eV to another atom in a collision.

(b) For the conditions of (a), at what temperature would the ratio N_x/N_0 be 1/2?

Calculation: Now we want the temperature T such that thermal agitation has bumped enough neon atoms up to the higher-energy state to give $N_x/N_0 = 1/2$. Substituting that ratio into Eq. 40-30, taking the natural logarithm of both sides, and solving for T yield



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$$T = \frac{E_x - E_0}{k(\ln 2)} = \frac{2.26 \text{ eV}}{(8.62 \times 10^{-5} \text{ eV/K})(\ln 2)} = 38000 \text{ K.}$$

(Answer)

This is much hotter than the surface of the Sun. Thus, it is clear that if we are to invert the populations of these two levels, some specific mechanism for bringing this about is needed—that is, we must “pump” the atoms. No temperature, however high, will naturally generate a population inversion by thermal agitation.

Review & Summary

Some Properties of Atoms Atoms have quantized energies and can make quantum jumps between them. If a jump between a higher energy and a lower energy involves the emission or absorption of a photon, the frequency associated with the light is given by

$$hf = E_{\text{high}} - E_{\text{low}}. \quad (40-1)$$

States with the same value of quantum number n form a shell. States with the same values of quantum numbers n and ℓ form a subshell.

Orbital Angular Momentum and Magnetic Dipole Moments The magnitude of the orbital angular momentum of an electron trapped in an atom has quantized values given by

$$L = \sqrt{\ell(\ell + 1)} \hbar, \quad \text{for } \ell = 0, 1, 2, \dots, (n - 1), \quad (40-2)$$

where \hbar is $h/2\pi$, ℓ is the orbital magnetic quantum number, and n is the electron's principal quantum number. The component L_z of the orbital angular momentum on a z axis is quantized and given by

$$L_z = m_\ell \hbar, \quad \text{for } m_\ell = 0, \pm 1, \pm 2, \dots, \pm \ell, \quad (40-3)$$

where m_ℓ is the orbital magnetic quantum number. The magnitude μ_{orb} of the orbital magnetic moment of the electron is quantized with the values given by

$$\mu_{\text{orb}} = \frac{e}{2m} \sqrt{\ell(\ell + 1)} \hbar, \quad (40-6)$$

where m is the electron mass. The component $\mu_{\text{orb},z}$ on a z axis is also quantized according to

$$\mu_{\text{orb},z} = -\frac{e}{2m} m_\ell \hbar = -m_\ell \mu_B, \quad (40-7)$$

where μ_B is the Bohr magneton:

$$\mu_B = \frac{eh}{4\pi m} = \frac{e\hbar}{2m} = 9.274 \times 10^{-24} \text{ J/T}. \quad (40-8)$$

Spin Angular Momentum and Magnetic Dipole Moment Every electron, whether trapped or free, has an intrinsic spin angular momentum \vec{S} with a magnitude that is quantized as

$$S = \sqrt{s(s + 1)} \hbar, \quad \text{for } s = \frac{1}{2}, \quad (40-9)$$

where s is the spin quantum number. An electron is said to be a

spin- $\frac{1}{2}$ particle. The component S_z on a z axis is also quantized according to

$$S_z = m_s \hbar, \quad \text{for } m_s = \pm s = \pm \frac{1}{2}, \quad (40-10)$$

where m_s is the spin magnetic quantum number. Every electron, whether trapped or free, has an intrinsic spin magnetic dipole moment $\vec{\mu}_s$ with a magnitude that is quantized as

$$\mu_s = \frac{e}{m} \sqrt{s(s + 1)} \hbar, \quad \text{for } s = \frac{1}{2}. \quad (40-12)$$

The component $\mu_{s,z}$ on a z axis is also quantized according to

$$\mu_{s,z} = -2m_s \mu_B, \quad \text{for } m_s = \pm \frac{1}{2}. \quad (40-13)$$

Stern-Gerlach Experiment The Stern-Gerlach experiment demonstrated that the magnetic moment of silver atoms is quantized, experimental proof that magnetic moments at the atomic level are quantized. An atom with magnetic dipole moment experiences a force in a nonuniform magnetic field. If the field changes at the rate of dB/dz along a z axis, then the force is along the z axis and is related to the component μ_z of the dipole moment:

$$F_z = \mu_z \frac{dB}{dz}. \quad (40-17)$$

A proton has an intrinsic spin angular momentum \vec{S} and an intrinsic magnetic dipole moment $\vec{\mu}$ that are in the same direction.

Magnetic Resonance The magnetic dipole moment of a proton in a magnetic field \vec{B} along a z axis has two quantized components on that axis: spin up (μ_z is in the direction \vec{B}) and spin down (μ_z is in the opposite direction). Contrary to the situation with an electron, spin up is the lower energy orientation; the difference between the two orientations is $2\mu_z B$. The energy required of a photon to spin-flip the proton between the two orientations is

$$hf = 2\mu_z B. \quad (40-22)$$

The field is the vector sum of an external field set up by equipment and an internal field set up by the atoms and nuclei surrounding the proton. Detection of spin-flips can lead to nuclear magnetic resonance spectra by which specific substances can be identified.

Pauli Exclusion Principle Electrons in atoms and other traps obey the Pauli exclusion principle, which requires that no two electrons in a trap can have the same set of quantum numbers.

Building the Periodic Table In the periodic table, the elements are listed in order of increasing atomic number Z , where Z is the number of protons in the nucleus. For a neutral atom, Z is also the number of electrons. States with the same value of quantum number n form a shell. States with the same values of quantum numbers n and ℓ form a subshell. A closed shell and a closed subshell contain the maximum number of electrons as allowed by the Pauli exclusion principle. The net angular momentum and net magnetic moment of such closed structures is zero.

X Rays and the Numbering of the Elements When a beam of high-energy electrons impacts a target, the electrons can lose their energy by emitting x rays when they scatter from atoms in the target. The emission is over a range of wavelengths, said to be a continuous spectrum. The shortest wavelength in the spectrum is the cutoff wavelength λ_{\min} , which is emitted when an incident electron loses its full kinetic energy K_0 in a single scattering event, with a single x-ray emission:

$$\lambda_{\min} = \frac{hc}{K_0}.$$

The characteristic x-ray spectrum is produced when incident electrons eject low-lying electrons in the target atoms and electrons from upper levels jump down to the resulting holes, emitting light. A Moseley plot is a graph of the square root of the characteristic-emission frequencies \sqrt{f} versus atomic number Z of the target atoms. The straight-line plot reveals that the position of an element in the periodic table is set by Z and not by the atomic weight.

Lasers In stimulated emission, an atom in an excited state can be induced to de-excite to a lower energy state by emitting a photon if an identical photon passes the atom. The light emitted in stimulated emission is in phase with and travels in the direction of the light causing the emission.

A laser can emit light via stimulated emission provided that its atoms are in population inversion. That is, for the pair of levels involved in the stimulated emission, more atoms must be in the upper level than the lower level so that there is more stimulated emission than just absorption.

Questions

1 How many (a) subshells and (b) electron states are in the $n = 2$ shell? How many (c) subshells and (d) electron states are in the $n = 5$ shell?

2 An electron in an atom of gold is in a state with $n = 4$. Which of these values of ℓ are possible for it: $-3, 0, 2, 3, 4, 5$?

3 Label these statements as true or false: (a) One (and only one) of these subshells cannot exist: $2p, 4f, 3d, 1p$. (b) The number of values of m_ℓ that are allowed depends only on ℓ and not on n . (c) There are four subshells with $n = 4$. (d) The smallest value of n for a given value of ℓ is $\ell + 1$. (e) All states with $\ell = 0$ also have $m_\ell = 0$. (f) There are n subshells for each value of n .

4 An atom of uranium has closed $6p$ and $7s$ subshells. Which subshell has the greater number of electrons?

5 An atom of silver has closed $3d$ and $4d$ subshells. Which subshell has the greater number of electrons, or do they have the same number?

6 From which atom of each of the following pairs is it easier to remove an electron: (a) krypton or bromine, (b) rubidium or cerium, (c) helium or hydrogen?

7 An electron in a mercury atom is in the $3d$ subshell. Which of the following m_ℓ values are possible for it: $-3, -1, 0, 1, 2$?

8 Figure 40-22 shows three points at which a spin-up electron can be placed in a nonuniform magnetic field (there is a gradient along the z axis). (a) Rank the three points according to the energy U of the electron's intrinsic magnetic dipole moment $\vec{\mu}_s$, most positive first. (b) What is the direction of the force on the electron due to the magnetic field if the spin-up electron is at point 2?

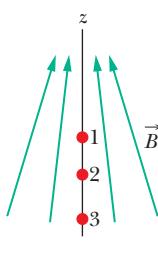


Figure 40-22
Question 8.

9 The K_α x-ray line for any element arises because of a transition between the K shell ($n = 1$) and the L shell ($n = 2$). Figure 40-13 shows this line (for a molybdenum target) occurring at a single wavelength. With higher resolution, however, the line splits into several wavelength components because the L shell does not have a unique energy. (a) How many components does the K_α line have? (b) Similarly, how many components does the K_β line have?

10 Consider the elements krypton and rubidium. (a) Which is more suitable for use in a Stern–Gerlach experiment of the kind described in connection with Fig. 40-8? (b) Which, if either, would not work at all?

11 On which quantum numbers does the energy of an electron depend in (a) a hydrogen atom and (b) a vanadium atom?

12 Which (if any) of the following are essential for laser action to occur between two energy levels of an atom? (a) There are more atoms in the upper level than in the lower. (b) The upper level is metastable. (c) The lower level is metastable. (d) The lower level is the ground state of the atom. (e) The lasing medium is a gas.

13 Figure 40-21 shows partial energy-level diagrams for the helium and neon atoms that are involved in the operation of a helium–neon laser. It is said that a helium atom in state E_3 can collide with a neon atom in its ground state and raise the neon atom to state E_2 . The energy of helium state E_3 (20.61 eV) is close to, but not exactly equal to, the energy of neon state E_2 (20.66 eV). How can the energy transfer take place if these energies are not exactly equal?

14 The x-ray spectrum of Fig. 40-13 is for 35.0 keV electrons striking a molybdenum ($Z = 42$) target. If you substitute a silver ($Z = 47$) target for the molybdenum target, will (a) λ_{\min} , (b) the wavelength for the K_α line, and (c) the wavelength for the K_β line increase, decrease, or remain unchanged?



Problems



Tutoring problem available (at instructor's discretion) in WileyPLUS and WebAssign



Worked-out solution available in Student Solutions Manual



Number of dots indicates level of problem difficulty



Additional information available in *The Flying Circus of Physics* and at flyingcircusofphysics.com



Worked-out solution is at



Interactive solution is at

<http://www.wiley.com/college/halliday>

Module 40-1 Properties of Atoms

•1 An electron in a hydrogen atom is in a state with $\ell = 5$. What is the minimum possible value of the semiclassical angle between \vec{L} and L_z ?

•2 How many electron states are there in a shell defined by the quantum number $n = 5$?

•3 (a) What is the magnitude of the orbital angular momentum in a state with $\ell = 3$? (b) What is the magnitude of its largest projection on an imposed z axis?

•4 How many electron states are there in the following shells: (a) $n = 4$, (b) $n = 1$, (c) $n = 3$, (d) $n = 2$?

•5 (a) How many ℓ values are associated with $n = 3$? (b) How many m_ℓ values are associated with $\ell = 1$?

•6 How many electron states are in these subshells: (a) $n = 4$, $\ell = 3$; (b) $n = 3$, $\ell = 1$; (c) $n = 4$, $\ell = 1$; (d) $n = 2$, $\ell = 0$?

•7 An electron in a multielectron atom has $m_\ell = +4$. For this electron, what are (a) the value of ℓ , (b) the smallest possible value of n , and (c) the number of possible values of m_s ?

•8 In the subshell $\ell = 3$, (a) what is the greatest (most positive) m_ℓ value, (b) how many states are available with the greatest m_ℓ value, and (c) what is the total number of states available in the subshell?

•9 **SSM** **WWW** An electron is in a state with $\ell = 3$. (a) What multiple of \hbar gives the magnitude of \vec{L} ? (b) What multiple of μ_B gives the magnitude of $\vec{\mu}$? (c) What is the largest possible value of m_ℓ , (d) what multiple of \hbar gives the corresponding value of L_z , and (e) what multiple of μ_B gives the corresponding value of $\mu_{\text{orb},z}$? (f) What is the value of the semiclassical angle θ between the directions of L_z and \vec{L} ? What is the value of angle θ for (g) the second largest possible value of m_ℓ and (h) the smallest (that is, most negative) possible value of m_ℓ ?

•10 An electron is in a state with $n = 3$. What are (a) the number of possible values of ℓ , (b) the number of possible values of m_ℓ , (c) the number of possible values of m_s , (d) the number of states in the $n = 3$ shell, and (e) the number of subshells in the $n = 3$ shell?

•11 **SSM** If orbital angular momentum \vec{L} is measured along, say, a z axis to obtain a value for L_z , show that

$$(L_x^2 + L_y^2)^{1/2} = [\ell(\ell + 1) - m_z^2]^{1/2}\hbar$$

is the most that can be said about the other two components of the orbital angular momentum.

•12 **GO** A magnetic field is applied to a freely floating uniform iron sphere with radius $R = 2.00$ mm. The sphere initially had no net magnetic moment, but the field aligns 12% of the magnetic moments of the atoms (that is, 12% of the magnetic moments of the loosely bound electrons in the sphere, with one such electron per atom). The magnetic moment of those aligned electrons is the sphere's intrinsic magnetic moment $\vec{\mu}_s$. What is the sphere's resulting angular speed ω ?

Module 40-2 The Stern–Gerlach Experiment

•13 **SSM** What is the acceleration of a silver atom as it passes through the deflecting magnet in the Stern–Gerlach experiment of Fig. 40-8 if the magnetic field gradient is 1.4 T/mm ?

•14 Suppose that a hydrogen atom in its ground state moves 80 cm through and perpendicular to a vertical magnetic field that has a magnetic field gradient $dB/dz = 1.6 \times 10^2 \text{ T/m}$. (a) What is the magnitude of force exerted by the field gradient on the atom due to the magnetic moment of the atom's electron, which we take to be 1 Bohr magneton? (b) What is the vertical displacement of the atom in the 80 cm of travel if its speed is $1.2 \times 10^5 \text{ m/s}$?

•15 Calculate the (a) smaller and (b) larger value of the semiclassical angle between the electron spin angular momentum vector and the magnetic field in a Stern–Gerlach experiment. Bear in mind that the orbital angular momentum of the valence electron in the silver atom is zero.

•16 Assume that in the Stern–Gerlach experiment as described for neutral silver atoms, the magnetic field \vec{B} has a magnitude of 0.50 T. (a) What is the energy difference between the magnetic moment orientations of the silver atoms in the two subbeams? (b) What is the frequency of the radiation that would induce a transition between these two states? (c) What is the wavelength of this radiation, and (d) to what part of the electromagnetic spectrum does it belong?

Module 40-3 Magnetic Resonance

•17 In an NMR experiment, the RF source oscillates at 34 MHz and magnetic resonance of the hydrogen atoms in the sample being investigated occurs when the external field \vec{B}_{ext} has magnitude 0.78 T. Assume that \vec{B}_{int} and \vec{B}_{ext} are in the same direction and take the proton magnetic moment component μ_z to be $1.41 \times 10^{-26} \text{ J/T}$. What is the magnitude of \vec{B}_{int} ?

•18 A hydrogen atom in its ground state actually has two possible, closely spaced energy levels because the electron is in the magnetic field \vec{B} of the proton (the nucleus). Accordingly, an energy is associated with the orientation of the electron's magnetic moment $\vec{\mu}$ relative to \vec{B} , and the electron is said to be either spin up (higher energy) or spin down (lower energy) in that field. If the electron is excited to the higher-energy level, it can de-excite by spin-flipping and emitting a photon. The wavelength associated with that photon is 21 cm. (Such a process occurs extensively in the Milky Way galaxy, and reception of the 21 cm radiation by radio telescopes reveals where hydrogen gas lies between stars.) What is the effective magnitude of \vec{B} as experienced by the electron in the ground-state hydrogen atom?

•19 What is the wavelength associated with a photon that will induce a transition of an electron spin from parallel to antiparallel orientation in a magnetic field of magnitude 0.200 T? Assume that $\ell = 0$.

Module 40-4 Exclusion Principle and Multiple Electrons in a Trap

•20 A rectangular corral of widths $L_x = L$ and $L_y = 2L$ contains seven electrons. What multiple of $h^2/8mL^2$ gives the energy of the

ground state of this system? Assume that the electrons do not interact with one another, and do not neglect spin.

- 21 Seven electrons are trapped in a one-dimensional infinite potential well of width L . What multiple of $\hbar^2/8mL^2$ gives the energy of the ground state of this system? Assume that the electrons do not interact with one another, and do not neglect spin.

•22 Figure 40-23 is an energy-level diagram for a fictitious infinite potential well that contains one electron. The number of degenerate states of the levels are indicated: “non” means nondegenerate (which includes the ground state of the electron), “double” means 2 states, and “triple” means 3 states. We put a total of 11 electrons in the well. If the electrostatic forces between the electrons can be neglected, what multiple of $\hbar^2/8mL^2$ gives the energy of the first excited state of the 11-electron system?

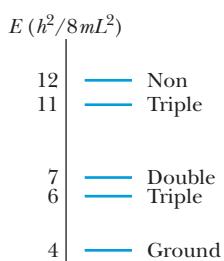


Figure 40-23
Problem 22.

- 23 **SSM** A cubical box of widths $L_x = L_y = L_z = L$ contains eight electrons. What multiple of $\hbar^2/8mL^2$ gives the energy of the ground state of this system? Assume that the electrons do not interact with one another, and do not neglect spin.

- 24 For Problem 20, what multiple of $\hbar^2/8mL^2$ gives the energy of (a) the first excited state, (b) the second excited state, and (c) the third excited state of the system of seven electrons? (d) Construct an energy-level diagram for the lowest four energy levels.

- 25 For the situation of Problem 21, what multiple of $\hbar^2/8mL^2$ gives the energy of (a) the first excited state, (b) the second excited state, and (c) the third excited state of the system of seven electrons? (d) Construct an energy-level diagram for the lowest four energy levels of the system.

- 26 For the situation of Problem 23, what multiple of $\hbar^2/8mL^2$ gives the energy of (a) the first excited state, (b) the second excited state, and (c) the third excited state of the system of eight electrons? (d) Construct an energy-level diagram for the lowest four energy levels of the system.

Module 40-5 Building the Periodic Table

- 27 **SSM** **WWW** Two of the three electrons in a lithium atom have quantum numbers (n, ℓ, m_ℓ, m_s) of $(1, 0, 0, +\frac{1}{2})$ and $(1, 0, 0, -\frac{1}{2})$. What quantum numbers are possible for the third electron if the atom is (a) in the ground state and (b) in the first excited state?

- 28 Show that the number of states with the same quantum number n is $2n^2$.

- 29 A recently named element is darmstadtium (Ds), which has 110 electrons. Assume that you can put the 110 electrons into the atomic shells one by one and can neglect any electron-electron interaction. With the atom in ground state, what is the spectroscopic notation for the quantum number ℓ for the last electron?

- 30 For a helium atom in its ground state, what are quantum numbers $(n, \ell, m_\ell, \text{ and } m_s)$ for the (a) spin-up electron and (b) spin-down electron?

- 31 Consider the elements selenium ($Z = 34$), bromine ($Z = 35$), and krypton ($Z = 36$). In their part of the periodic table, the sub-

shells of the electronic states are filled in the sequence

$$1s\ 2s\ 2p\ 3s\ 3p\ 3d\ 4s\ 4p \dots$$

What are (a) the highest occupied subshell for selenium and (b) the number of electrons in it, (c) the highest occupied subshell for bromine and (d) the number of electrons in it, and (e) the highest occupied subshell for krypton and (f) the number of electrons in it?

- 32 Suppose two electrons in an atom have quantum numbers $n = 2$ and $\ell = 1$. (a) How many states are possible for those two electrons? (Keep in mind that the electrons are indistinguishable.) (b) If the Pauli exclusion principle did not apply to the electrons, how many states would be possible?

Module 40-6 X Rays and the Ordering of the Elements

- 33 Through what minimum potential difference must an electron in an x-ray tube be accelerated so that it can produce x rays with a wavelength of 0.100 nm?

- 34 The wavelength of the K_α line from iron is 193 pm. What is the energy difference between the two states of the iron atom that give rise to this transition?

- 35 **SSM** **WWW** In Fig. 40-13, the x rays shown are produced when 35.0 keV electrons strike a molybdenum ($Z = 42$) target. If the accelerating potential is maintained at this value but a silver ($Z = 47$) target is used instead, what values of (a) λ_{\min} , (b) the wavelength of the K_α line, and (c) the wavelength of the K_β line result? The K , L , and M atomic x-ray levels for silver (compare Fig. 40-15) are 25.51, 3.56, and 0.53 keV.

- 36 When electrons bombard a molybdenum target, they produce both continuous and characteristic x rays as shown in Fig. 40-13. In that figure the kinetic energy of the incident electrons is 35.0 keV. If the accelerating potential is increased to 50.0 keV, (a) what is the value of λ_{\min} , and (b) do the wavelengths of the K_α and K_β lines increase, decrease, or remain the same?

- 37 Show that a moving electron cannot spontaneously change into an x-ray photon in free space. A third body (atom or nucleus) must be present. Why is it needed? (Hint: Examine the conservation of energy and momentum.)

- 38 Here are the K_α wavelengths of a few elements:

Element	λ (pm)	Element	λ (pm)
Ti	275	Co	179
V	250	Ni	166
Cr	229	Cu	154
Mn	210	Zn	143
Fe	193	Ga	134

Make a Moseley plot (like that in Fig. 40-16) from these data and verify that its slope agrees with the value given for C in Module 40-6.

- 39 **SSM** Calculate the ratio of the wavelength of the K_α line for niobium (Nb) to that for gallium (Ga). Take needed data from the periodic table of Appendix G.

- 40 (a) From Eq. 40-26, what is the ratio of the photon energies due to K_α transitions in two atoms whose atomic numbers are Z and Z' ? (b) What is this ratio for uranium and aluminum? (c) For uranium and lithium?

••41 The binding energies of K -shell and L -shell electrons in copper are 8.979 and 0.951 keV, respectively. If a K_α x ray from copper is incident on a sodium chloride crystal and gives a first-order Bragg reflection at an angle of 74.1° measured relative to parallel planes of sodium atoms, what is the spacing between these parallel planes?

••42 From Fig. 40-13, calculate approximately the energy difference $E_L - E_M$ for molybdenum. Compare it with the value that may be obtained from Fig. 40-15.

••43 A tungsten ($Z = 74$) target is bombarded by electrons in an x-ray tube. The K , L , and M energy levels for tungsten (compare Fig. 40-15) have the energies 69.5, 11.3, and 2.30 keV, respectively. (a) What is the minimum value of the accelerating potential that will permit the production of the characteristic K_α and K_β lines of tungsten? (b) For this same accelerating potential, what is λ_{\min} ? What are the (c) K_α and (d) K_β wavelengths?

••44 A 20 keV electron is brought to rest by colliding twice with target nuclei as in Fig. 40-14. (Assume the nuclei remain stationary.) The wavelength associated with the photon emitted in the second collision is 130 pm greater than that associated with the photon emitted in the first collision. (a) What is the kinetic energy of the electron after the first collision? What are (b) the wavelength λ_1 and (c) the energy E_1 associated with the first photon? What are (d) λ_2 and (e) E_2 associated with the second photon?

••45 X rays are produced in an x-ray tube by electrons accelerated through an electric potential difference of 50.0 kV. Let K_0 be the kinetic energy of an electron at the end of the acceleration. The electron collides with a target nucleus (assume the nucleus remains stationary) and then has kinetic energy $K_1 = 0.500K_0$. (a) What wavelength is associated with the photon that is emitted? The electron collides with another target nucleus (assume it, too, remains stationary) and then has kinetic energy $K_2 = 0.500K_1$. (b) What wavelength is associated with the photon that is emitted?

••46 Determine the constant C in Eq. 40-27 to five significant figures by finding C in terms of the fundamental constants in Eq. 40-24 and then using data from Appendix B to evaluate those constants. Using this value of C in Eq. 40-27, determine the theoretical energy E_{theory} of the K_α photon for the low-mass elements listed in the following table. The table includes the value (eV) of the measured energy E_{exp} of the K_α photon for each listed element. The percentage deviation between E_{theory} and E_{exp} can be calculated as

$$\text{percentage deviation} = \frac{E_{\text{theory}} - E_{\text{exp}}}{E_{\text{exp}}} \cdot 100.$$

What is the percentage deviation for (a) Li, (b) Be, (c) B, (d) C, (e) N, (f) O, (g) F, (h) Ne, (i) Na, and (j) Mg?

Li	54.3	O	524.9
Be	108.5	F	676.8
B	183.3	Ne	848.6
C	277	Na	1041
N	392.4	Mg	1254

(There is actually more than one K_α ray because of the splitting of the L energy level, but that effect is negligible for the elements listed here.)

Module 40-7 Lasers

•47 The active volume of a laser constructed of the semiconductor GaAlAs is only $200 \mu\text{m}^3$ (smaller than a grain of sand), and yet the laser can continuously deliver 5.0 mW of power at a wavelength of $0.80 \mu\text{m}$. At what rate does it generate photons?

•48 A high-powered laser beam ($\lambda = 600 \text{ nm}$) with a beam diameter of 12 cm is aimed at the Moon, $3.8 \times 10^5 \text{ km}$ distant. The beam spreads only because of diffraction. The angular location of the edge of the central diffraction disk (see Eq. 36-12) is given by

$$\sin \theta = \frac{1.22\lambda}{d},$$

where d is the diameter of the beam aperture. What is the diameter of the central diffraction disk on the Moon's surface?

•49 Assume that lasers are available whose wavelengths can be precisely “tuned” to anywhere in the visible range—that is, in the range $450 \text{ nm} < \lambda < 650 \text{ nm}$. If every television channel occupies a bandwidth of 10 MHz, how many channels can be accommodated within this wavelength range?

•50 A hypothetical atom has only two atomic energy levels, separated by 3.2 eV. Suppose that at a certain altitude in the atmosphere of a star there are $6.1 \times 10^{13}/\text{cm}^3$ of these atoms in the higher-energy state and $2.5 \times 10^{15}/\text{cm}^3$ in the lower-energy state. What is the temperature of the star's atmosphere at that altitude?

•51 SSM A hypothetical atom has energy levels uniformly separated by 1.2 eV. At a temperature of 2000 K, what is the ratio of the number of atoms in the 13th excited state to the number in the 11th excited state?

•52 GO A laser emits at 424 nm in a single pulse that lasts 0.500 μs . The power of the pulse is 2.80 MW. If we assume that the atoms contributing to the pulse underwent stimulated emission only once during the 0.500 μs , how many atoms contributed?

•53 A helium-neon laser emits laser light at a wavelength of 632.8 nm and a power of 2.3 mW. At what rate are photons emitted by this device?

•54 A certain gas laser can emit light at wavelength 550 nm, which involves population inversion between ground state and an excited state. At room temperature, how many moles of neon are needed to put 10 atoms in that excited state by thermal agitation?

•55 A pulsed laser emits light at a wavelength of 694.4 nm. The pulse duration is 12 ps, and the energy per pulse is 0.150 J. (a) What is the length of the pulse? (b) How many photons are emitted in each pulse?

•56 A population inversion for two energy levels is often described by assigning a negative Kelvin temperature to the system. What negative temperature would describe a system in which the population of the upper energy level exceeds that of the lower level by 10% and the energy difference between the two levels is 2.26 eV?

•57 A hypothetical atom has two energy levels, with a transition wavelength between them of 580 nm. In a particular sample at 300 K, 4.0×10^{20} such atoms are in the state of lower energy. (a) How many atoms are in the upper state, assuming conditions of thermal equilibrium? (b) Suppose, instead, that 3.0×10^{20} of these atoms are “pumped” into the upper state by an external process, with 1.0×10^{20} atoms remaining in the lower state. What is the maxi-

mum energy that could be released by the atoms in a single laser pulse if each atom jumps once between those two states (either via absorption or via stimulated emission)?

••58 The mirrors in the laser of Fig. 40-20, which are separated by 8.0 cm, form an optical cavity in which standing waves of laser light can be set up. Each standing wave has an integral number n of half wavelengths in the 8.0 cm length, where n is large and the waves differ slightly in wavelength. Near $\lambda = 533$ nm, how far apart in wavelength are the standing waves?

••59 Figure 40-24 shows the energy levels of two types of atoms. Atoms A are in one tube, and atoms B are in another tube. The energies (relative to a ground-state energy of zero) are indicated; the average lifetime of atoms in each level is also indicated. All the atoms are initially pumped to levels higher than the levels shown in the figure. The atoms then drop down through the levels, and many become “stuck” on certain levels, leading to population inversion and lasing. The light emitted by A illuminates B and can cause stimulated emission of B. What is the energy per photon of that stimulated emission of B?

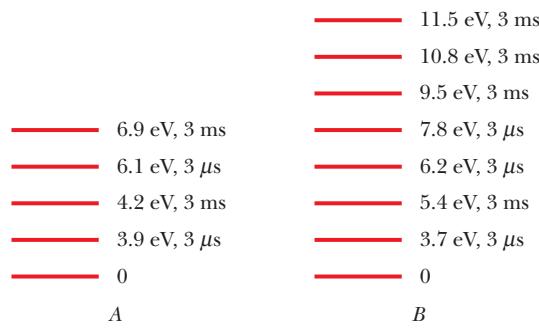


Figure 40-24 Problem 59.

••60 The beam from an argon laser (of wavelength 515 nm) has a diameter d of 3.00 mm and a continuous energy output rate of 5.00 W. The beam is focused onto a diffuse surface by a lens whose focal length f is 3.50 cm. A diffraction pattern such as that of Fig. 36-10 is formed, the radius of the central disk being given by

$$R = \frac{1.22 f \lambda}{d}$$

(see Eq. 36-12 and Fig. 36-14). The central disk can be shown to contain 84% of the incident power. (a) What is the radius of the central disk? (b) What is the average intensity (power per unit area) in the incident beam? (c) What is the average intensity in the central disk?

••61 The active medium in a particular laser that generates laser light at a wavelength of 694 nm is 6.00 cm long and 1.00 cm in diameter. (a) Treat the medium as an optical resonance cavity analogous to a closed organ pipe. How many standing-wave nodes are there along the laser axis? (b) By what amount Δf would the beam frequency have to shift to increase this number by one? (c) Show that Δf is just the inverse of the travel time of laser light for one round trip back and forth along the laser axis. (d) What is the corresponding fractional frequency shift $\Delta f/f$? The appropriate index of refraction of the lasing medium (a ruby crystal) is 1.75.

••62 Ruby lasers at a wavelength of 694 nm. A certain ruby crystal has 4.00×10^{19} Cr ions (which are the atoms that lase). The lasing transition is between the first excited state and the ground state, and the output is a light pulse lasting 2.00 μ s. As the pulse begins, 60.0% of the Cr ions are in the first excited state and the rest

are in the ground state. What is the average power emitted during the pulse? (Hint: Don't just ignore the ground-state ions.)

Additional Problems

63 Figure 40-25 is an energy-level diagram for a fictitious three-dimensional infinite potential well that contains one electron. The number of degenerate states of the levels are indicated: “non” means nondegenerate (which includes the ground state) and “triple” means 3 states. If we put a total of 22 electrons in the well, what multiple of $\hbar^2/8mL^2$ gives the energy of the ground state of the 22-electron system? Assume that the electrostatic forces between the electrons are negligible.

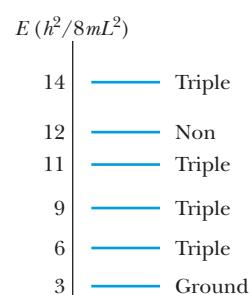


Figure 40-25
Problem 63.

64 *Martian CO₂ laser.* Where sunlight shines on the atmosphere of Mars, carbon dioxide molecules at an altitude of about 75 km undergo natural laser action. The energy levels involved in the action are shown in Fig. 40-26; population inversion occurs between energy levels E_2 and E_1 . (a) What wavelength of sunlight excites the molecules in the lasing action? (b) At what wavelength does lasing occur? (c) In what region of the electromagnetic spectrum do the excitation and lasing wavelengths lie?

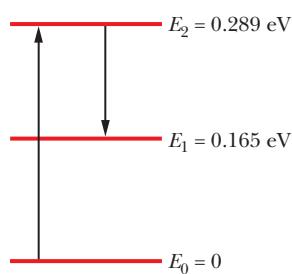


Figure 40-26 Problem 64.

65 Excited sodium atoms emit two closely spaced spectrum lines called the *sodium doublet* (Fig. 40-27) with wavelengths 588.995 nm and 589.592 nm. (a) What is the difference in energy between the two upper energy levels ($n = 3, \ell = 1$)? (b) This energy difference occurs because the electron's spin magnetic moment can be oriented either parallel or antiparallel to the internal magnetic field associated with the electron's orbital motion. Use your result in (a) to find the magnitude of this internal magnetic field.

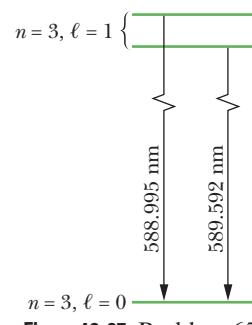


Figure 40-27 Problem 65.

66 *Comet stimulated emission.* When a comet approaches the Sun, the increased warmth evaporates water from the ice on the surface of the comet nucleus, producing a thin atmosphere of water vapor around the nucleus. Sunlight can then dissociate H₂O molecules in the vapor to H atoms and OH molecules. The sunlight can also excite the OH molecules to higher energy levels.

When the comet is still relatively far from the Sun, the sunlight causes equal excitation to the E_2 and E_1 levels (Fig. 40-28a). Hence, there is no population inversion between the two levels. However, as the comet approaches the Sun, the excitation to the E_1 level decreases and population inversion occurs. The reason has to do with one of the many wavelengths—said to be *Fraunhofer lines*—that are missing in sunlight because, as the light travels outward through the Sun's atmosphere, those particular wavelengths are absorbed by the atmosphere.

As a comet approaches the Sun, the Doppler effect due to the comet's speed relative to the Sun shifts the Fraunhofer lines in

wavelength, apparently overlapping one of them with the wavelength required for excitation to the E_1 level in OH molecules. Population inversion then occurs in those molecules, and they radiate stimulated emission (Fig. 40-28b). For example, as comet Kouhoutek approached the Sun in December 1973 and January 1974, it radiated stimulated emission at about 1666 MHz during mid-January. (a) What was the energy difference $E_2 - E_1$ for that emission? (b) In what region of the electromagnetic spectrum was the emission?

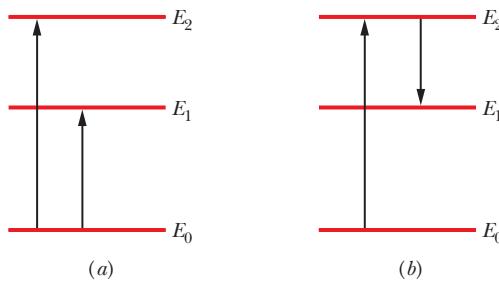


Figure 40-28 Problem 66.

67 Show that the cutoff wavelength (in picometers) in the continuous x-ray spectrum from any target is given by $\lambda_{\min} = 1240/V$, where V is the potential difference (in kilovolts) through which the electrons are accelerated before they strike the target.

68 By measuring the go-and-return time for a laser pulse to travel from an Earth-bound observatory to a reflector on the Moon, it is possible to measure the separation between these bodies. (a) What is the predicted value of this time? (b) The separation can be measured to a precision of about 15 cm. To what uncertainty in travel time does this correspond? (c) If the laser beam forms a spot on the Moon 3 km in diameter, what is the angular divergence of the beam?

69 SSM Can an incoming intercontinental ballistic missile be destroyed by an intense laser beam? A beam of intensity 10^8 W/m^2 would probably burn into and destroy a nonspinning missile in 1 s. (a) If the laser had 5.0 MW power, 3.0 μm wavelength, and a 4.0 m beam diameter (a very powerful laser indeed), would it destroy a missile at a distance of 3000 km? (b) If the wavelength could be changed, what maximum value would work? Use the equation for the central diffraction maximum as given by Eq. 36-12 ($\sin \theta = 1.22\lambda/d$).

70 A molybdenum ($Z = 42$) target is bombarded with 35.0 keV electrons and the x-ray spectrum of Fig. 40-13 results. The K_β and K_α wavelengths are 63.0 and 71.0 pm, respectively. What photon energy corresponds to the (a) K_β and (b) K_α radiation? The two radiations are to be filtered through one of the substances in the following table such that the substance absorbs the K_β line more strongly than the K_α line. A substance will absorb radiation x_1 more strongly than it absorbs radiation x_2 if a photon of x_1 has enough en-

ergy to eject a K electron from an atom of the substance but a photon of x_2 does not. The table gives the ionization energy of the K electron in molybdenum and four other substances. Which substance in the table will serve (c) best and (d) second best as the filter?

	Zr	Nb	Mo	Tc	Ru
Z	40	40	42	43	44
$E_K(\text{keV})$	18.00	18.99	20.00	21.04	22.12

71 An electron in a multielectron atom is known to have the quantum number $\ell = 3$. What are its possible n, m_ℓ , and m_s quantum numbers?

72 Show that if the 63 electrons in an atom of europium were assigned to shells according to the “logical” sequence of quantum numbers, this element would be chemically similar to sodium.

73 SSM Lasers can be used to generate pulses of light whose durations are as short as 10 fs. (a) How many wavelengths of light ($\lambda = 500 \text{ nm}$) are contained in such a pulse? (b) In

$$\frac{10 \text{ fs}}{1 \text{ s}} = \frac{1 \text{ s}}{X},$$

what is the missing quantity X (in years)?

74 Show that $\hbar = 1.06 \times 10^{-34} \text{ J}\cdot\text{s} = 6.59 \times 10^{-16} \text{ eV}\cdot\text{s}$.

75 Suppose that the electron had no spin and that the Pauli exclusion principle still held. Which, if any, of the present noble gases would remain in that category?

76 (A correspondence principle problem.) Estimate (a) the quantum number ℓ for the orbital motion of Earth around the Sun and (b) the number of allowed orientations of the plane of Earth’s orbit. (c) Find θ_{\min} , the half-angle of the smallest cone that can be swept out by a perpendicular to Earth’s orbit as Earth revolves around the Sun.

77 Knowing that the minimum x-ray wavelength produced by 40.0 keV electrons striking a target is 31.1 pm, determine the Planck constant \hbar .

78 Consider an atom with two closely spaced excited states A and B . If the atom jumps to ground state from A or from B , it emits a wavelength of 500 nm or 510 nm, respectively. What is the energy difference between states A and B ?

79 In 1911, Ernest Rutherford modeled an atom as being a point of positive charge Ze surrounded by a negative charge $-Ze$ uniformly distributed in a sphere of radius R centered at the point. At distance r within the sphere, the electric potential is

$$V = \frac{Ze}{4\pi\epsilon_0} \left(\frac{1}{r} - \frac{3}{2R} + \frac{r^2}{2R^3} \right).$$

(a) From this formula, determine the magnitude of electric field for $0 \leq r \leq R$. What are the (b) electric field and (c) potential for $r \geq R$?