Misconceptions of Bell's Theorem And Free Will

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Abstract

In this review, I argue why free will while being an intuitive notion, does not withstand scientific scrutiny. In pursuit of this I give a brief history of the inception of Bell-type inequalities and in order to understand and disentangle the connection between the concept of free will and the assumptions underlying Bell's theorem which are often taken as one and the same. I list notable criticisms of Bell's underlying assumptions and discuss successful experiments and their interpretations. Finally, I examine the implications of the violations of statistical independence in the class of super-deterministic theories and what that would mean for free will in a deterministic setting.

1 Introduction

In he following I will be examining the current state of the issue that is free will. As the literature is rather extensive and can also get very technical at times, this will mostly cover the topics on a surface level. Along the way, I will do my best to illuminate the dubious structures of underlying assumptions and rationales that have led to the concept of free will becoming implicitly assumed, conflated with other terms, and almost swept under the rug. I will start with the layman's definitions and slowly work towards the ontological and quantum-mechanical perspectives. This discussion will be restricted to the intersection of Bell's theorem and free will and will not be covering the measurement problem, contextual models, various interpretations of quantum-mechanics etc.

1.1 Issues With the Laymen Definition

The Cambridge Dictionary defines free will as the following: "The ability to act and make choices independent of any outside influence." A slightly more rigorous definition is given by the philosopher Paul Carus: "Free will is the notional capacity or ability to choose between different possible courses of action unimpeded."
The main statement these and most other definitions are making is that individuals ought to be capable of making free choices. Free may mean without being influenced by fate or destiny which is often used as an argument against determinism[1]. Free may also mean without legislation or other societal factors threatening with consequences. But free can not mean fully conscious and with no outside influence as clearly we humans are the product of our environment not just our genetic code and most brain activity is carried out unconsciously Morsella et al. [2016]. From a scientific standpoint, it would be a lot less problematic to argue we humans have "Will", a higher, conscious form of animalistic impulse and desire rather than making an unproven assumption and using it to dismiss others such as determinism.

It does seem free will is more of a construct that has practical values such as assuming individual accountability and autonomy etc. Unfortunately, many debates around this topic are very reminiscent of those around religion and so it happened that it even found its way into physics. Even though by no means a precise definition we have the ability to choose and decide "freely" will later echo in the misleading scientific work.

2 The Bell Inequality and its Assumptions

2.1 Bells Theorem

Bell's theorem and its inequalities historically originate from a paper by John Bell "on the EPR Paradox" Bell [1964] in which he concerned himself with the work of Einstein, Podolsky, and Rosen titled "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" Einstein et al. [1935]. It was a thought experiment arguing the incompleteness of the quantum mechanical framework. The main issue brought forth was that of information being passed from one particle to another at speeds faster than c. EPR concluded that there have to be inherent hidden parameters that the particles possess which determine the evolution of the system and knowledge thereof would render the system deterministic .
Bell showed given certain assumptions that if such a hidden variable λ indeed existed, the formulus of quantum mechanics would make predictions that do not match the experimental data. His specification of λ (here A) was the following:

> "It is a matter of indifference in the following whether A denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous. However, we write as if A were a single continuous parameter."

While the mathematical conclusion reached from Bell's assumptions is undoubtedly correct, there are some ambivalences regarding the underlying assumptions which will be addressed in the next

[1] In the chapter on super-determinism we will see an argument against this regardless

chapter.
The main insight resting on Bell's theorem and other Bell-like inequalities is the implication that quantum mechanics is fundamentally non-local given certain assumptions. This part seems to be often glanced over or taken for trivial. As a 2016 survey of professional physicists showed that 64% of them believe that Bell's theorem completely ruled out all hidden variable theories and renders local realism impossible. And 34% that believe that it showed merely the impossibility of hidden variables. Neither of which is technically correct Hance et al. [2022].

2.2 Bells Assumptions and Loopholes

Modern Bell tests like many other practical experiments are not free of loopholes. There is quite a variety of them and while some can be successfully prevented by careful experiment design there are others that are hard if not impossible to avoid.
One notorious loophole is the *locality loophole*. Here special relativity teaches us that in principle there could be a causal relationship between a measurement performed by Alice and a signal that travels at speeds $\leq c$ from Bob's measurement or choice and conversely. This is avoided by space-like separation of both the measurement and the setting choice. Note that here lays a clear distinction between the space-like separation of emission events which has been achieved by Scheidl and colleagues and the much more difficult case of separating the past light cones. The latter can almost be achieved by a cosmic Bell test using the light of quasars that is billions of years old and it's wavelength for the detector settings Rauch et al. [2018]. Yet when you turn back time the causal network of space-time can never be disentangled fully, given the fact that it seems to have at one time originate from the same small region.

2.3 Freedom of choice Loophole

The *freedom-of-choice loophole* is another very important one. A fundamental assumption made in Bell's theorem is that λ be not correlated with the measurement setting. This is, so some physicists argue, addressed by not only "free choice" but ultimately the free will of the person conducting the experiment and is nowadays called the freedom of choice assumption Larsson [2014]. A claim that I judge to be incredibly misleading as will be discussed in the following. Though not stated explicitly, the consensus is that the "free choice" assumption was implicit in the formulation of the experiment design. This becomes evident in the way the hidden variable distribution is notated independent from the setting choices as $p(\lambda)$ instead of $p(\lambda|a, b)$.
Then assuming "freedom of choice" equates the two expressions $p(\lambda) = p(\lambda|a, b)$ which is used in the further derivations performed by Bell placing statistical independence as a fundamental assumption Scheidl et al. [2010]. A violation of this sort $p(\lambda) \neq p(\lambda|a, b)$ would therefore be a violation of statistic independence. Statistical independence together with output independence and parameter independence make up what is often referred to as "Bell locality" not be equated with locality in the sense of continuity of action. [2] It follows that a violation of Bell's inequality is a violation of one of these assumptions.

Now recall the earlier definitions of free choice in 1.1 and notice that the requirement of not being causally connected is the same as the one needed to close the freedom of choice loophole which is that the measurement settings and choice not correlate.
This similarity and heavy association Feldman et al. [2014] may have been the cause of the conflation of the concept of free will, which people have strong feelings for, with "free choice" under the guise of statistical independence and has done science a disservice as it created a confusing and heavy bias towards "free choice".
In an attempt to close this loophole it remains a topic of discussion whether Bell tests like the cosmic Bell test or the BIG Bell test big [2018] can do so. Hossenfelder and Palmer scrutinize the first by arguing that the cosmic Bell test merely rules out that the correlations were locally caused by past events. Furthermore, it is maintained that a small change in initial conditions in this case somewhere in the entanglement of the hyper-surface within the past light cones billions of years ago, can lead to a large change in the observed wavelength of light. Pointing

[2] A term often used in this context is local realism and refers to Bell's locality

to our lack of intuition that the metric of a state space may not have to be Euclidean. While the latter "proves freedom of choice by assuming freedom of choice" Hossenfelder and Palmer [2020].

Our modern rigorous scientific formulation of free will also rests on this freedom of choice. For example, the free will theorem is an elaborate formulation that among other things states that if experimenters are to make a "free choice" about what to measure, the results can not be determined by the past information. Conway and Kochen [2006]
In the end "free choice" and the concept of free will really should not be used interchangeably within the scientific community.

3 Determinism and Superdeterminism

3.1 Superdeterminism

We have now disentangled the existence of free will from "free choice" which is the necessary condition for statistical independence. This is also why fundamentally super-determinism and determinism theories are often dismissed in the light of Bell's theorem with the claim that they provide an unclosable loophole in the assumptions by violating statistical independence
't Hooft [2009].
While a super-deterministic theory as introduced here is deterministic and therefore reversible, it is also Psi-epistemic, considering the wave function to be an emergent phenomenon and not fundamental. If one looks at the probabilistic mathematical foundation of quantum mechanics it is not hard to believe that it might be an average representation of something we have not quite figured out.
Superdeterminism also falls under the category of hidden variable theory. Hossenfelder and Palmer argue that even though it may be deterministic in nature, our lack of knowledge of all states renders it for all intents and purposes nondeterministic. As the state information could be encoded in a widely distributed manner.
It does seem like super-deterministic theories might not be such a bad deal afterall.

Another common critique is the conspiracy argument which states that upon the execution of any measurement because super-deterministic theories are reversible, we can reverse the time evolution back to the initial state responsible for the current outcome. Then we would have explained away said observation. This hardly follows the spirit of the scientific methodology of finding laws that apply to larger domains. Regardless the postulation is that the universe conspired against the observer by fine-tuning of the initial conditions. While Bell himself questioned if these ab-initio types of "conspiracies" or "fine-tuning" are not themselves more digestible than non-localities. (cite) Other scientists argue that this issue is foundation enough to outright dismiss any super-deterministic and thus statistically independent setting.

3.2 "Free Will" in Deterministic Theories

Now assuming we live in a super-deterministic or deterministic setting what does this mean for our understanding of free will, does society crumble because our fate is inevitable? Yes, but no. It turns out we can still reconstruct it without using "free choice". The following argument can be made in many ways one of them being with the concept of computational irreducibility first introduced in Wolfram's a new kind of science Wolfram [2002]. This concept makes the following empiric meta-statement about deterministic processes: Sufficiently complex dynamic systems can exhibit unpredictable behaviour that can only be simulated by running the computation itself. This is very much in the same spirit as the earlier statement of how just because something is deterministic it does not mean we can predict the future. This is one of the many ways compatibilists can restore free will. But this is also the point where this discussion would drift off into the philosophically. The question of whether we should act like we have free will because otherwise society would not function and other related speculations can not be covered in this report. The intention is to show that nothing of value is lost in super-deterministic settings.
Both t'Hooft and Wolfram have proposed discrete computational models for understanding physics which are super-deterministic. With the latter gaining traction in recent years and having surprising success at deriving modern-day physics Gorard [2020].

It may also be of note that Gorard has claimed that these deterministic models can also reproduce violations of Bell's theorem which points.

4 Conclusion

While this topic is by no means completely covered I hope it at least gets the point across that it is an open issue. Since as we have seen the question of free will has nothing to do with Bell's theorem and is a conflation of one of Bell's assumptions called statistical independence where the experimenter is capable of making uncorrelated choices with the idea that free will means that we can make free choices. The concept of computational irreducibly itself could already make free will as we think we know it emerges in humans in a completely deterministic setting. Furthermore, we have cleared up a few common misunderstandings about what Bell's theorem really states. Namely that given statistical independence, output independence, and parameter independence we can observe violations that rule out local hidden variable theories as a whole since there is no telling which assumption was violated. From this, we have learned to always be careful on which axioms our frameworks are operating, and that we might benefit from examining super-deterministic theories with an open mind.

5 Appendix

I hereby guarantee that this entire text is 100% human generated.

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