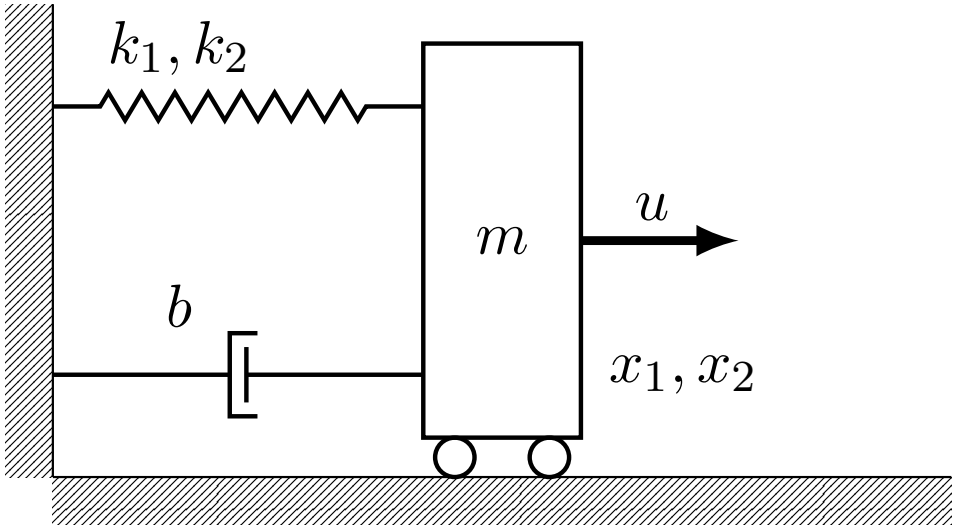
Example: Mass-Spring-Damper System

Consider the following mass-spring-damper system *. A mass m moving with position x_1 and velocity x_2, is pulled by an external force u. The mass is connected to a nonlinear spring giving the force

$$F_s = k_1 x_1 + k_2 x_1^3$$

and to a linear damper giving the force

$$F_d = b x_2.$$



* Example 5.5 in *Control of Nonlinear Systems* by S. T. Glad, 2009. `https://www.control.isy.liu.se/student/graduate/nonlin/book.pdf`.

Example: Mass-Spring-Damper System

Then the equations of motion are given by

$$\dot{x}_1 = x_2,$$
$$m\dot{x}_2 = u - k_1 x_1 - k_2 x_1^3 - b x_2.$$

Let $x = (x_1, x_2) \in \mathbb{R}^2$. The system model is in the form $\dot{x} = f(x) + g(x)u$ with

$$f(x) = \begin{bmatrix} x_2 \\ \frac{1}{m}(-k_1 x_1 - k_2 x_1^3 - b x_2) \end{bmatrix}, \quad g(x) = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}.$$

Let the cost to be minimized be

$$\int_0^\infty (q(x) + ru^2)\, \mathrm{d}t.$$

Choose

$$k_1 = 3, \quad k_2 = 2, \quad b = 2, \quad m = 5,$$

and

$$q(x) = 5x_1^2 + 3x_2^2, \quad r = 2.$$