

AN ELEMENTARY
INTRODUCTION TO THE
**Wolfram
Language**



STEPHEN WOLFRAM

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Table of Contents

Preface	vii	25 Ways to Apply Functions	141
What Is the Wolfram Language?	xi	26 Pure Anonymous Functions	147
Practicalities of Using the Wolfram Language	xiii	27 Applying Functions Repeatedly	153
Other Resources	xv		
1 Starting Out: Elementary Arithmetic	1	28 Tests and Conditionals	163
2 Introducing Functions	3	29 More about Pure Functions	171
3 First Look at Lists	7	30 Rearranging Lists	177
4 Displaying Lists	11	31 Parts of Lists	187
5 Operations on Lists	15	32 Patterns	193
6 Making Tables	19	33 Expressions and Their Structure	197
7 Colors and Styles	25	34 Associations	205
8 Basic Graphics Objects	29	35 Natural Language Understanding	211
9 Interactive Manipulation	33	36 Creating Websites and Apps	217
10 Images	39	37 Layout and Display	227
11 Strings and Text	45	38 Assigning Names to Things	233
12 Sound	53	39 Immediate and Delayed Values	239
13 Arrays, or Lists of Lists	57	40 Defining Your Own Functions	243
14 Coordinates and Graphics	63	41 More about Patterns	249
15 The Scope of the Wolfram Language	73	42 String Patterns and Templates	255
16 Real-World Data	77	43 Storing Things	261
17 Units	87	44 Importing and Exporting	265
18 Geocomputation	93	45 Datasets	271
19 Dates and Times	101	46 Writing Good Code	283
20 Options	107	47 Debugging Your Code	291
21 Graphs and Networks	115	What We Haven't Discussed	295
22 Machine Learning	123	Afterword: Being a Programmer	303
23 More about Numbers	129	Answers to Exercises	305
24 More Forms of Visualization	137	Index	313

Preface

I've been explaining what's now the Wolfram Language to people for more than 25 years, and I finally decided it was time to take what I'd learned and write a minimal introduction that people could read on their own. This book is the result of that effort.

When we first launched Mathematica—the precursor of the Wolfram Language—in 1988, I published a book that provided both a tutorial introduction and reference guide to the system. The book was very popular and I think contributed substantially to the early success of Mathematica. Over the next decade or so, *The Mathematica Book*, as it came to be known, went through five editions, and steadily grew until it was nearly 1500 pages long.

My goal in *The Mathematica Book* was to systematically cover all aspects of the system. But when we released a huge new version in 2007, it became clear that this was simply no longer possible in a single book. Our online documentation had meanwhile steadily grown, and in 2007, with the introduction of a vast number of new examples, it reached the point where a printed version would have been well over 10,000 pages in length.

In 2009 Wolfram | Alpha arrived, with its natural-language interface specifically built for use without explanation or documentation. But then, emerging from Mathematica and Wolfram | Alpha, came the Wolfram Language, and there was once again a need for both explanation and documentation.

I believe that the online documentation for the Wolfram Language—which in printed form would exceed 50,000 pages—does very well at explaining the specifics of how to use all the many capabilities of the system. But particularly for newcomers there's also a need to understand the principles of the language—that I've worked so hard over the years to keep coherent and consistent.

The Wolfram Language is unique among programming languages, and different in many ways. But some time ago, I wrote a Fast Introduction for Programmers (wolfr.am/fifp) that in about 30 pages gives modern programmers at least a basic grounding in the principles of the Wolfram Language.

But what about people who don't already know programming? The Wolfram Language provides a unique opportunity not only to introduce anyone to programming, but to get them quickly to the very frontiers of what can be done with computation today.

That this is possible is a consequence of all the effort we've put into creating the technology of the Wolfram Language over the course of nearly three decades. My goal has been to build a language where anyone can specify as simply as possible what they want to do, then inside, all the details are automatically taken care of to get it done.

For the quick question-answering of Wolfram | Alpha, it's enough just to say in plain English what you want. But if you're going to do more systematic tasks, you need a way to explain them precisely. And that's what the Wolfram Language is for.

So how should people learn the Wolfram Language? One approach is immersion: Be in an environment where the Wolfram Language is used. Explore programs that run, and learn from them as examples. In my observation, this can work very well so long as there is at least occasionally someone around to explain principles and help with issues when they come up.

But what about learning the Wolfram Language entirely on one's own? Here I think what's needed is a systematic introduction that progressively builds from one concept to another, answering every obvious question as it goes. And that's what I'm trying to do in this book.

Learning the Wolfram Language is a bit like learning a human language. There's a mixture of vocabulary and principles, that have to be learned hand in hand. The Wolfram Language is immensely more systematic than human languages—with nothing like irregular verbs to memorize—but still has the same kind of progression towards fluency that comes with more and more practice.

I wondered how to write this book. And eventually I decided to base it loosely on Latin textbooks, of the kind I used when I was a kid. Unlike living languages, Latin cannot be learned by immersion, and so there is no choice but to build step by step, as I do in this book.

In some ways learning programming is a bit like learning mathematics. Both have a certain precision: things are either right or wrong. But with the Wolfram Language, programming is much more concrete: At every step you can see what is happening, and whether what you're doing is right. There are no hidden concepts that have to be explained abstractly from outside and cannot explicitly be seen.

Still, there've been two millennia of development in the teaching of mathematics, that have progressively optimized the sequence of presenting arithmetic, algebra and so on. The problem of teaching the Wolfram Language is something completely new, where everything has to be figured out from scratch. Existing programming education isn't much help, because so much of it is about just the kinds of lower-lower structure that have been automated away in the Wolfram Language.

I view this book as an experiment: an attempt to provide a particular path through learning the Wolfram Language. I am not trying to cover everything in the language, not least because that would take at least 50,000 pages. Instead, I am trying to explain the principles of the language through a limited number of specific examples.

I've chosen the examples to be interesting and useful in practice. But the bigger point is that through the examples, I cover most of the core principles of the language. And knowing these principles, you'll be ready to go to specific documentation to understand any particular aspect of what the language can do.

Needless to say, the Wolfram Language has many sophisticated capabilities. Some of them—like identifying objects in images—are sophisticated on the inside, but easy to explain. But others—like computing Gröbner bases—are also sophisticated to explain, and may require significant outside knowledge of mathematics or computer science.

My goal is to make this book completely self-contained, and to assume nothing beyond everyday common knowledge. I have avoided any explicit use of mathematics beyond basic arithmetic, though those who know advanced mathematics may notice many connections between concepts of mathematics and concepts in the book.

This is certainly not the only elementary introduction to the Wolfram Language that could be written, and I hope there will be many more. It follows a specific—and in many ways arbitrary—path through the vast capabilities of the language, highlighting certain features but not even mentioning many other equally deserving ones.

Still, I hope that the power and beauty of the language that I have nurtured for more than half my life will shine through, and that many students and other people, with many diverse backgrounds, can use this book to get started with the Wolfram Language and get involved with the kind of computational thinking that is quickly becoming a defining feature of our times.

Stephen Wolfram

A handwritten signature in black ink that reads "Stephen Wolfram". The signature is fluid and cursive, with "Stephen" on the top line and "Wolfram" on the bottom line.

What Is the Wolfram Language?

The Wolfram Language is a *computer language*. It gives you a way to communicate with computers, in particular so you can tell them what to do.

There are many computer languages, such as C++, Java, Python and JavaScript. The Wolfram Language is unique in that it's *knowledge based*. That means that it already knows a lot—so you have to tell it much less to get it to do things you want.

In this book, you'll see how to use the Wolfram Language to do a great many things. You'll learn how to think computationally about what you want to do, and how to communicate it to a computer using the Wolfram Language.

Why can't you just say what you want using plain English? That's what you do in Wolfram|Alpha. And it works very well for asking short questions. But if you want to do something more complex, it quickly becomes impractical to describe everything just in plain English. And that's where the Wolfram Language comes in.

It's designed to make it as easy as possible to describe what you want, making use of huge amounts of knowledge that are built into the language. And the crucial thing is that when you use the Wolfram Language to ask for something, the computer immediately knows what you mean, and then can actually do what you want.

I view the Wolfram Language as an optimized tool for turning ideas into reality. You start with an idea of something you want to do. You formulate the idea in computational terms, then you express it in the Wolfram Language. Then it's up to the Wolfram Language to do it as automatically as possible.

You can make things that are visual, textual, interactive or whatever. You can do analyses or figure things out. You can create apps and programs and websites. You can take a very wide variety of ideas and implement them—on your computer, on the web, on a phone, on tiny embedded devices and more.

I started building what's now the Wolfram Language nearly 30 years ago. Along the way, particularly in the form of Mathematica, the Wolfram Language has been extremely widely used in the world's research organizations and universities—and a remarkable range of inventions and discoveries have been made with it.

Today the Wolfram Language has emerged as something else: a new kind of general computer language, which redefines what's practical to do with computers. Among the early users of today's Wolfram Language are many of the world's leading innovators and technology organizations. And there are large and important systems—like Wolfram|Alpha—that are written in the Wolfram Language.

But the very knowledge and automation that makes the Wolfram Language so powerful also makes it accessible to anyone. You don't have to know about the workings of computers, or about technical or mathematical ideas; that's the job of the Wolfram Language. All you need to do is to know the Wolfram Language, so you can tell your computer what you want.

As you work through this book, you'll learn the principles of the Wolfram Language. You'll learn how to use the Wolfram Language to write programs, and you'll see some of the computational thinking it's based on. But most of all, you'll learn a set of powerful skills for turning your ideas into reality. Nobody knows yet all the things that the Wolfram Language will make possible. It's going to be exciting to see—and what you learn in this book will let you become a part of that future.

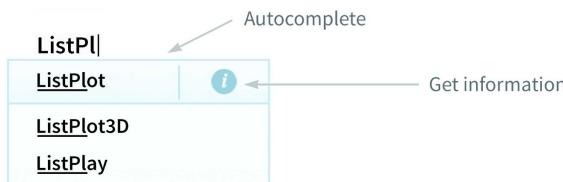
Practicalities of Using the Wolfram Language

The best way to learn the Wolfram Language is to use it. Wolfram Programming Lab is specifically set up for easy access in learning the language, though you can also use other interactive Wolfram Language environments.

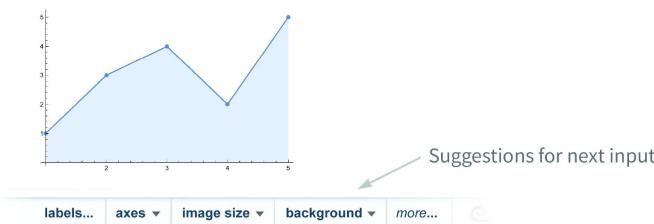
In any of these environments, you enter input in the Wolfram Language, and the system immediately computes output from it. You can do this on desktop, web or mobile. On desktop and web, you typically type **Shift + Enter** to say you've finished your input; on mobile you typically press a button. Your sequence of inputs and outputs—together with any text you may add—all exists in a Wolfram Notebook.

```
In[1]:= 2 + 2 SHIFT + ENTER ← Input
Out[1]= 4 ← Output
```

Within the Wolfram Notebook you'll see a variety of aids to help you enter Wolfram Language input.



plot the sequence 1,3,4,2,5 ← Natural-language input



`Table[Column[Range[n]], {n, 8}]`
`tabla` `columna` `rango` ← Code captions for non-English environments

Wolfram Notebooks, with their interactive sequences of inputs and outputs, are an ideal way to learn, explore and write programs in the Wolfram Language. But the Wolfram Language can also operate without its own interactive interface, in a whole variety of software-engineering configurations. Inside, though, the language is still doing the same computations as in the interactive configuration we discuss in this book.

Q&A**Do I need to know programming to read this book?**

Absolutely not. This book is a self-contained introduction to programming.

What age group is this book suitable for?

Experience suggests that anything above about age 12 is appropriate. I've tried to choose examples that will be relevant and engaging to all age groups, very much including adults.

How much math do I need to know to read this book?

Nothing beyond basic arithmetic. This is a book about programming in the Wolfram Language, not about math.

Do I need to use a computer while reading this book?

You could read it without one, but it will be much better to try things out interactively with a Wolfram Language session, for example in the Wolfram Programming Lab.

Do I have to read this book sequentially?

It will be a lot better that way. I've constructed the book so it progressively builds up a broader and broader base of concepts. If you jump around, you'll probably need to backtrack a lot.

Why are the topics in this book so different from other programming books?

Because the Wolfram Language is a different and higher-level kind of language, that automates away many of the details that programming books about other languages concentrate on.

Is the Wolfram Language an educational language?

It's certainly used for education (and Mathematica is ubiquitous at universities). But it's also very widely used in industry. It's good for education because it's powerful and easy to learn.

Will learning the Wolfram Language help in learning other languages?

Definitely. Knowing the Wolfram Language you'll understand higher-level concepts, which you'll then see played out in lower-level ways in other languages.

The Wolfram Language seems too easy; is it really programming?

Definitely. And because it automates away the drudgery you might associate with programming, you'll be able to go much further, and understand much more.

Can this book be used for a course?

Absolutely! Check out the book webpage (wolfr.am/eiwl) for supplementary material.

Can sections of the book be omitted for a course?

The book is written to provide a sequential presentation of material, so the content may require some patching if sections in the middle are dropped.

How do the exercises relate to the text?

They check and solidify understanding, and show additional implications of the material.

What version of the Wolfram Language does this book assume I will use?

Anything 10.3 and above. Note that even some fairly simple examples (e.g. `Table[x, 5]`) won't work in earlier versions.

Is the code in the book "production grade"?

Usually, yes. Occasionally there is code that is slightly more complicated than it needs to be, because the concepts necessary to make it simpler haven't been introduced by that point in the book.

Other Resources

Wolfram Language Home Page

Broad collection of resources about the Wolfram Language
wolfram.com/language

Wolfram Documentation Center

Documentation on all functions in the Wolfram Language, with extensive examples
reference.wolfram.com/language

Wolfram Programming Lab

Online and desktop access to the Wolfram Language, with educational Explorations
wolfram.com/programming-lab

Fast Introduction for Programmers

Short Wolfram Language tutorial for people with programming experience
wolfram.com/language/fast-introduction-for-programmers

Wolfram Challenges

Dynamic collection of online Wolfram Language programming challenges
challenges.wolfram.com

Wolfram Tweet-a-Program

Many examples of Wolfram Language programs less than 140 characters long
wolfram.com/language/tweet-a-program

Wolfram Demonstrations Project

10,000+ interactive demonstrations written in the Wolfram Language
demonstrations.wolfram.com

Wolfram Community

Online community for learning and discussing Wolfram technology
community.wolfram.com

Wolfram Home Page

The home page of Wolfram Research, the company behind the Wolfram Language
wolfram.com

Stephen Wolfram's Page

The home page for the author of this book
stephenwolfram.com

1 | Starting Out: Elementary Arithmetic

As a first example of how the Wolfram Language operates, let's look at elementary arithmetic.

Add numbers:

In[1]:= **2+2**

Out[1]= **4**

In[2]:= **1234+5678**

Out[2]= **6912**

Multiply numbers:

In[3]:= **1234*5678**

Out[3]= **7006652**

Vocabulary

2 + 2 addition

5 - 2 subtraction

2 * 3 multiplication (2 3 also works)

6 / 2 division

3 ^ 2 raising to a power (e.g. squaring)

Exercises

1.1 Compute $1+2+3$.

1.2 Add the whole numbers from 1 to 5.

1.3 Multiply the whole numbers from 1 to 5.

1.4 Compute 5 squared (i.e. 5×5 or 5 raised to the power 2).

1.5 Compute 3 raised to the fourth power.

1.6 Compute 10 raised to the power 12 (a trillion).

1.7 Compute 2 to the power 2 to the power 2 to the power 2.

1.8 Add parentheses to $4-2*3+4$ to make 14.

1.9 Compute a hundred and twenty thousand multiplied by a hundred and twenty-three.

Q&A

In 2+2 etc. how do I tell the Wolfram Language that I've finished my input?

On a computer, press `Shift + Enter`. On a mobile device, press the  button. See [Practicalities of Using the Wolfram Language](#) for more details.

Why is multiplication indicated by *?

Because the * (“star”, typically typed as `Shift + 8`) looks like a multiplication sign. In the Wolfram Language, you can also just put a space between numbers you want to multiply; the Wolfram Language will then automatically insert a \times multiplication sign.

What does “raised to the power” (^) mean?

6^3 means $6 \times 6 \times 6$ (i.e. 6 multiplied by itself 3 times); 10^5 means $10 \times 10 \times 10 \times 10 \times 10$; etc.

How big can numbers get in the Wolfram Language?

As big as you want—so long as they fit in your computer’s memory.

What is the order of operations in the Wolfram Language?

The same as in ordinary math: powers, multiplication, addition. So $4 * 5^2 + 7$ means $(4 * (5^2)) + 7$. You can use parentheses just like in math. (In math, people sometimes use [...] as well as (...). In the Wolfram Language [...] means something different.)

When I do division, how do I avoid getting fractions in my answer?

If you put in numbers with decimal points, you’ll get out numbers with decimal points. You can also use `N`, as we discuss in Section 23.

What are the little spaces between digits in results like 7 006 652?

They’re there to help you read the number when it’s displayed; they’re not part of the number.

How do I enter a big number?

Just type the digits, without putting any commas, spaces or other separators (e.g. 1234123511415223).

What happens if I compute 1/0?

Try it! You’ll get a symbolic representation of infinity, on which the Wolfram Language can then do further computations.

More to Explore

[Getting Started in Wolfram Programming Lab \(wolfr.am/eiwl-1-more\)](#)

2 | Introducing Functions

When you type `2 + 2`, the Wolfram Language understands it as `Plus[2, 2]`. `Plus` is a *function*. There are more than 5000 functions built into the Wolfram Language. Arithmetic uses just a very few of these.

Compute $3+4$ using the function `Plus`:

```
In[1]:= Plus[3, 4]
```

```
Out[1]= 7
```

Compute $1+2+3$ using `Plus`:

```
In[2]:= Plus[1, 2, 3]
```

```
Out[2]= 6
```

The function `Times` does multiplication:

```
In[3]:= Times[2, 3]
```

```
Out[3]= 6
```

You can put functions inside other functions:

```
In[4]:= Times[2, Plus[2, 3]]
```

```
Out[4]= 10
```

All functions in the Wolfram Language use square brackets, and have names that start with capital letters.

The function `Max` finds the maximum, or largest, of a collection of numbers.

The maximum of these numbers is 7:

```
In[5]:= Max[2, 7, 3]
```

```
Out[5]= 7
```

The function `RandomInteger` picks a random integer (whole number) between 0 and whatever size you say.

Pick a random whole number between 0 and 100:

```
In[6]:= RandomInteger[100]
```

```
Out[6]= 71
```

Each time you ask, you get another random number:

```
In[7]:= RandomInteger[100]
```

```
Out[7]= 1
```

Vocabulary

<code>Plus[2, 2]</code>	$2+2$	addition
<code>Subtract[5, 2]</code>	$5-2$	subtraction
<code>Times[2, 3]</code>	2×3	multiplication (2 3 also works)
<code>Divide[6, 2]</code>	$6/2$	division
<code>Power[3, 2]</code>	3^2	raising to a power
<code>Max[3, 4]</code>		maximum (largest)
<code>Min[3, 4]</code>		minimum (smallest)
<code>RandomInteger[10]</code>		random whole number

Exercises

- 2.1** Compute $7+6+5$ using the function `Plus`.
- 2.2** Compute $2 \times (3+4)$ using `Times` and `Plus`.
- 2.3** Use `Max` to find the larger of 6×8 and 5×9 .
- 2.4** Find a random number between 0 and 1000.
- 2.5** Use `Max` and `RandomInteger` to generate a number between 10 and 20.

Q&A

Do I have to type the capital letters in `Plus`, `RandomInteger`, etc.?

Yes. In the Wolfram Language, `plus` is not the same as `Plus`. The capital letter in `Plus` signifies that you're talking about the built-in ("official") `plus` function.

Do I have to type square brackets [...] when I use functions?

Yes. Square brackets [...] are for functions; parentheses (...) are for grouping, as in $2*(3+4)$, not for functions.

How does one read `Plus[2, 3]` out loud?

Usually "plus of 2 and 3"; sometimes "plus of 2 comma 3". "[" can be read as "open bracket"; "]" as "close bracket".

Why use `Plus[2, 3]` instead of $2+3$?

For `Plus`, it's not necessary. But for the vast majority of functions—like `Max` or `RandomInteger`—there's no special form like +, so you have to give their names.

Can I mix `Plus[...]` and +?

Yes. Things like `Plus[4+5, 2+3]` or, for that matter, `Plus[4, 5]*5` are just fine.

What does it mean if the Wolfram Language colors some of my input red?

It means you've typed something that the Wolfram Language can't understand. See Section 47 for more information. Start by checking that your open and close brackets are matched.

Tech Notes

- Expressions in the Wolfram Language (see Section 33) consist of nested trees of functions.
- Plus can add any number of numbers, but Subtract only subtracts one number from another (to avoid ambiguities between $(2-3)-4$ and $2-(3-4)$).
- The notion of a function is considerably more general in the Wolfram Language than in either traditional mathematics or computer science. For example, $f[anything]$ is considered a function, whether it evaluates to something definite or remains in symbolic form.

More to Explore

Mathematical Functions in the Wolfram Language (wolfr.am/eiwl-2-more)

3 | First Look at Lists

Lists are a basic way to collect things together in the Wolfram Language. $\{1, 2, 3\}$ is a list of numbers. On their own, lists don't *do* anything; they're just a way to store things. So if you give a list as input, it'll just come back unchanged:

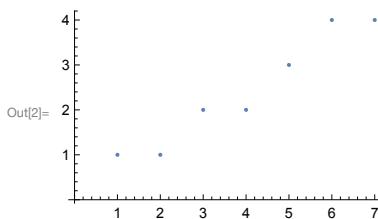
```
In[1]:= {1, 2, 3, 4, a, b, c}
```

```
Out[1]= {1, 2, 3, 4, a, b, c}
```

`ListPlot` is a function that makes a plot of a list of numbers.

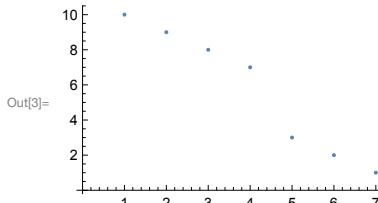
Plot the list of numbers $\{1, 1, 2, 2, 3, 4, 4\}$:

```
In[2]:= ListPlot[{1, 1, 2, 2, 3, 4, 4}]
```



Plot the list of numbers $\{10, 9, 8, 7, 3, 2, 1\}$:

```
In[3]:= ListPlot[{10, 9, 8, 7, 3, 2, 1}]
```



`Range` is a function that makes a list of numbers.

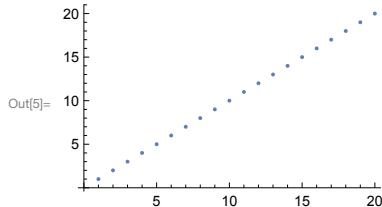
Generate a list of numbers up to 10:

```
In[4]:= Range[10]
```

```
Out[4]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

Generate a list of numbers, then plot it:

```
In[5]:= ListPlot[Range[20]]
```



Reverse reverses the elements in a list.

Reverse the elements in a list:

```
In[6]:= Reverse[{1, 2, 3, 4}]
```

```
Out[6]= {4, 3, 2, 1}
```

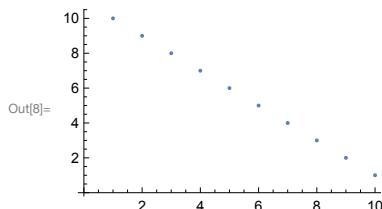
Reverse what Range has generated:

```
In[7]:= Reverse[Range[10]]
```

```
Out[7]= {10, 9, 8, 7, 6, 5, 4, 3, 2, 1}
```

Plot the reversed list:

```
In[8]:= ListPlot[Reverse[Range[10]]]
```



Join joins lists together, making a single list as the result.

Join lists together:

```
In[9]:= Join[{1, 2, 3}, {4, 5}, {6, 7}]
```

```
Out[9]= {1, 2, 3, 4, 5, 6, 7}
```

```
In[10]:= Join[{1, 2, 3}, {1, 2, 3, 4, 5}]
```

```
Out[10]= {1, 2, 3, 1, 2, 3, 4, 5}
```

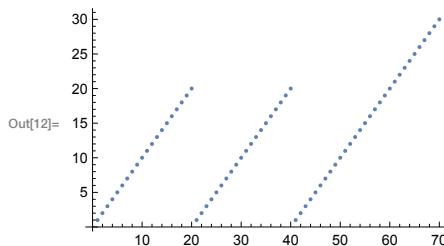
Join two lists made by Range:

```
In[11]:= Join[Range[3], Range[5]]
```

```
Out[11]= {1, 2, 3, 1, 2, 3, 4, 5}
```

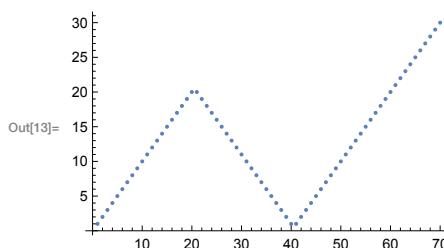
Plot three lists joined together:

```
In[12]:= ListPlot[Join[Range[20], Range[20], Range[30]]]
```



Reverse the list in the middle:

```
In[13]:= ListPlot[Join[Range[20], Reverse[Range[20]], Range[30]]]
```



Vocabulary

<code>{1, 2, 3, 4}</code>	list of elements
---------------------------	------------------

<code>ListPlot[{1, 2, 3, 4}]</code>	plot a list of numbers
-------------------------------------	------------------------

<code>Range[10]</code>	range of numbers
------------------------	------------------

<code>Reverse[{1, 2, 3}]</code>	reverse a list
---------------------------------	----------------

<code>Join[{4, 5, 6}, {2, 3, 2}]</code>	join lists together
---	---------------------

Exercises

- 3.1 Use `Range` to create the list `{1, 2, 3, 4}`.
- 3.2 Make a list of numbers up to 100.
- 3.3 Use `Range` and `Reverse` to create `{4, 3, 2, 1}`.
- 3.4 Make a list of numbers from 1 to 50 in reverse order.
- 3.5 Use `Range`, `Reverse` and `Join` to create `{1, 2, 3, 4, 4, 3, 2, 1}`.
- 3.6 Plot a list that counts up from 1 to 100, then down to 1.
- 3.7 Use `Range` and `RandomInteger` to make a list with a random length up to 10.

- 3.8** Find a simpler form for `Reverse[Reverse[Range[10]]]`.
 - 3.9** Find a simpler form for `Join[{1, 2}, Join[{3, 4}, {5}]]`.
 - 3.10** Find a simpler form for `Join[Range[10], Join[Range[10], Range[5]]]`.
 - 3.11** Find a simpler form for `Reverse[Join[Range[20], Reverse[Range[20]]]]`.
-

Q&A

How does one read `{1, 2, 3}` out loud?

Usually “list 1 2 3”. “{” and “}” are called “braces” or “curly brackets”. “{” is “open brace” and “}” is “close brace”.

Is a list a function?

Yes, `{1, 2, 3}` is `List[1, 2, 3]`. But unlike, say, `Plus`, the function `List` doesn’t actually compute anything; it just comes back unchanged.

What is `ListPlot` plotting?

The values of successive list elements. The *x* value of each point gives the position in the list; the *y* value gives the value of that element.

How long can lists be?

As long as you want, until your computer runs out of memory.

Tech Notes

- `Range[m, n]` generates numbers from *m* to *n*. `Range[m, n, s]` generates numbers from *m* to *n* in steps of *s*.
- Many computer languages have constructs like lists (often called “arrays”). But usually they only allow lists of explicit things, like numbers; you can’t have a list like `{a, b, c}` where you haven’t said what *a*, *b* and *c* are. You can in the Wolfram Language, though, because the Wolfram Language is *symbolic*.
- `{a, b, c}` is a list of elements in a definite order; `{b, c, a}` is a different list.
- Like in math, you can make theorems about Wolfram Language functions. For example, `Reverse[Reverse[x]]` is equal to *x*.

More to Explore

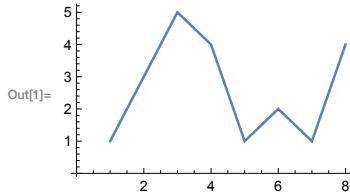
[Guide to Lists in the Wolfram Language](http://wolfr.am/eiwl-3-more) (wolfr.am/eiwl-3-more)

4 | Displaying Lists

`ListPlot` is one way to display, or *visualize*, a list of numbers. There are lots of others. Different ones tend to emphasize different features of a list.

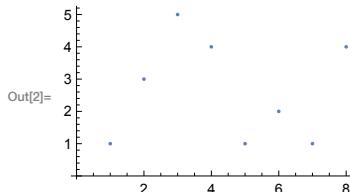
`ListLinePlot` plots a list, joining up values:

```
In[1]:= ListLinePlot[{1, 3, 5, 4, 1, 2, 1, 4}]
```



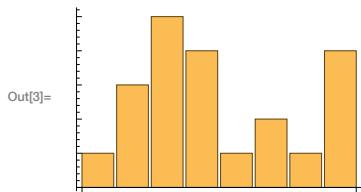
When values jump around, it's usually harder to understand if you don't join them up:

```
In[2]:= ListPlot[{1, 3, 5, 4, 1, 2, 1, 4}]
```



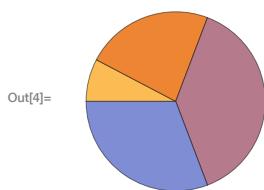
Making a bar chart can be useful too:

```
In[3]:= BarChart[{1, 3, 5, 4, 1, 2, 1, 4}]
```



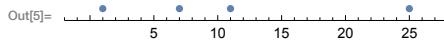
So long as the list isn't too long, a pie chart can be useful:

```
In[4]:= PieChart[{1, 3, 5, 4}]
```



If you just want to know which numbers appear, you can plot them on a number line:

```
In[5]:= NumberLinePlot[{1, 7, 11, 25}]
```



Sometimes you don't want a plot at all; you just want to put the elements of a list in a column:

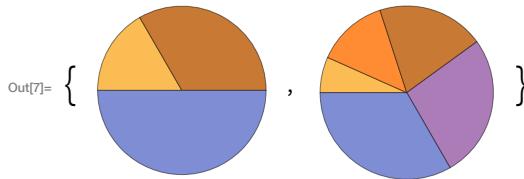
```
In[6]:= Column[{100, 350, 502, 400}]
```

Out[6]=
100
350
502
400

Lists can contain anything, including graphics. So you can combine plots by putting them in lists.

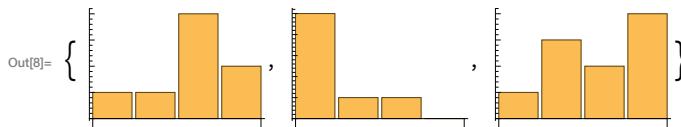
Make a list of two pie charts:

```
In[7]:= {PieChart[Range[3]], PieChart[Range[5]]}
```



Show three bar charts together:

```
In[8]:= {BarChart[{1, 1, 4, 2}], BarChart[{5, 1, 1, 0}], BarChart[{1, 3, 2, 4}]}
```



Vocabulary

ListLinePlot[{1, 2, 5}] values joined by a line

BarChart[{1, 2, 5}] bar chart (values give bar heights)

PieChart[{1, 2, 5}] pie chart (values give wedge sizes)

NumberLinePlot[{1, 2, 5}] numbers arranged on a line

Column[{1, 2, 5}] elements displayed in a column

Exercises

- 4.1** Make a bar chart of $\{1, 1, 2, 3, 5\}$.
- 4.2** Make a pie chart of numbers from 1 to 10.
- 4.3** Make a bar chart of numbers counting down from 20 to 1.
- 4.4** Display numbers from 1 to 5 in a column.
- 4.5** Make a number line plot of the squares $\{1, 4, 9, 16, 25\}$.
- 4.6** Make a list of pie charts with 1, 2 and 3 identical segments.
- 4.7** Make a column of pie charts with 1, 2 and 3 identical segments.

Q&A

How do pie charts work in the Wolfram Language?

As in any pie chart, the wedges have relative sizes determined by the relative sizes of numbers in the list. In the Wolfram Language, the wedge for the first number starts at the 9 o'clock position, and then subsequent wedges read clockwise. The colors of the wedges are chosen in a definite sequence.

How is the vertical scale determined on plots?

It's set up to automatically include all points except distant outliers. Later on (Section 20), we'll talk about the `PlotRange` option, which lets you specify the exact range of the plot.

Tech Note

- Particularly if you're familiar with other computer languages, you may be surprised that a list of plots, for example, can appear as the output of a computation. This is made possible by the crucial fact that the Wolfram Language is *symbolic*. By the way, plots can appear in input as well.

More to Explore

[Data Visualization in the Wolfram Language](#) (wolfr.am/eiwl-4-more)

[Charting and Information Visualization in the Wolfram Language](#) (wolfr.am/eiwl-4-more2)

5 | Operations on Lists

There are thousands of functions in the Wolfram Language that work with lists.

You can do arithmetic with lists:

```
In[1]:= {1, 2, 3} + 10
```

```
Out[1]= {11, 12, 13}
```

```
In[2]:= {1, 1, 2} * {1, 2, 3}
```

```
Out[2]= {1, 2, 6}
```

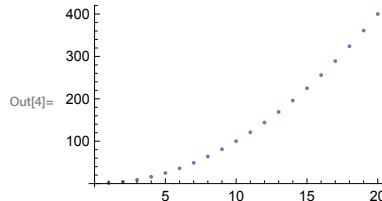
Compute the first 10 squares:

```
In[3]:= Range[10]^2
```

```
Out[3]= {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
```

Plot the first 20 squares:

```
In[4]:= ListPlot[Range[20]^2]
```



`Sort` sorts a list into order:

```
In[5]:= Sort[{4, 2, 1, 3, 6}]
```

```
Out[5]= {1, 2, 3, 4, 6}
```

`Length` finds how long a list is:

```
In[6]:= Length[{5, 3, 4, 5, 3, 4, 5}]
```

```
Out[6]= 7
```

`Total` gives the total from adding up a list:

```
In[7]:= Total[{1, 1, 2, 2}]
```

```
Out[7]= 6
```

Find the total of the numbers from 1 to 10:

```
In[8]:= Total[Range[10]]
```

```
Out[8]= 55
```

`Count` counts the numbers of times something appears in a list.

Count the number of times `a` appears in the list:

```
In[9]:= Count[{a, b, a, a, c, b, a}, a]
```

```
Out[9]= 4
```

It's often useful to be able to get individual elements of a list. `First` gives the first element; `Last` gives the last element. `Part` gives the element at a particular position.

Pick out the first element of a list:

```
In[10]:= First[{7, 6, 5}]
```

```
Out[10]= 7
```

Pick out the last element:

```
In[11]:= Last[{7, 6, 5}]
```

```
Out[11]= 5
```

Pick out element number 2:

```
In[12]:= Part[{7, 6, 5}, 2]
```

```
Out[12]= 6
```

Picking out the first element in a list you've sorted is the same as finding the minimum element:

```
In[13]:= First[Sort[{6, 7, 1, 2, 4, 5}]]
```

```
Out[13]= 1
```

```
In[14]:= Min[{6, 7, 1, 2, 4, 5}]
```

```
Out[14]= 1
```

If you have a number, like 5671, you can make a list of its digits using `IntegerDigits[5671]`.

Break a number into a list of digits:

```
In[15]:= IntegerDigits[1988]
```

```
Out[15]= {1, 9, 8, 8}
```

Find the last digit:

```
In[16]:= Last[IntegerDigits[1988]]
```

```
Out[16]= 8
```

`Take` lets you take a specified number of elements from the beginning of a list.

Take the first 3 elements from a list:

```
In[17]:= Take[{101, 203, 401, 602, 332, 412}, 3]
```

```
Out[17]= {101, 203, 401}
```

Take the first 10 digits of 2 to the power 100:

```
In[18]:= Take[IntegerDigits[2^100], 10]
```

```
Out[18]= {1, 2, 6, 7, 6, 5, 0, 6, 0, 0}
```

`Drop` drops elements from the beginning of a list.

```
In[19]:= Drop[{101, 203, 401, 602, 332, 412}, 3]
```

```
Out[19]= {602, 332, 412}
```

Vocabulary

<code>{2, 3, 4} + {5, 6, 2}</code>	arithmetic on lists
<code>Sort[{5, 7, 1}]</code>	sort a list into order
<code>Length[{3, 3}]</code>	length of a list (number of elements)
<code>Total[{1, 1, 2}]</code>	total of all elements in a list
<code>Count[{3, 2, 3}, 3]</code>	count occurrences of an element
<code>First[{2, 3}]</code>	first element in a list
<code>Last[{6, 7, 8}]</code>	last element in a list
<code>Part[{3, 1, 4}, 2]</code>	particular part of a list, also written as <code>{3, 1, 4}[[2]]</code>
<code>Take[{6, 4, 3, 1}, 2]</code>	take elements from the beginning of a list
<code>Drop[{6, 4, 3, 1}, 2]</code>	drop elements from the beginning of a list
<code>IntegerDigits[1234]</code>	list of digits in a number

Exercises

- 5.1** Make a list of the first 10 squares, in reverse order.
- 5.2** Find the total of the first 10 squares.
- 5.3** Make a plot of the first 10 squares, starting at 1.
- 5.4** Use `Sort`, `Join` and `Range` to create `{1, 1, 2, 2, 3, 3, 4, 4}`.
- 5.5** Use `Range` and `+` to make a list of numbers from 10 to 20, inclusive.

- 5.6** Make a combined list of the first 5 squares and cubes (numbers raised to the power 3), sorted into order.
- 5.7** Find the number of digits in 2^{128} .
- 5.8** Find the first digit of 2^{32} .
- 5.9** Find the first 10 digits in 2^{100} .
- 5.10** Find the largest digit that appears in 2^{20} .
- 5.11** Find how many zeros appear in the digits of 2^{1000} .
- 5.12** Use `Part`, `Sort` and `IntegerDigits` to find the second-smallest digit in 2^{20} .
- 5.13** Make a line plot of the sequence of digits that appear in 2^{128} .
- 5.14** Use `Take` and `Drop` to get the sequence 11 through 20 from `Range[100]`.

Q&A

Can one add lists of different lengths?

No. `{1, 2} + {1, 2, 3}` won't work. `{1, 2, 0} + {1, 2, 3}` would be fine, if that's what you mean.

Can there be a list with nothing in it?

Yes. `{}` is a list of length 0, with no elements. It's usually called the *null list* or the *empty list*.

Tech Notes

- `IntegerDigits[5671]` gives digits in base 10. `IntegerDigits[5671, 2]` gives digits in base 2. You can use any base you want. `FromDigits[{5, 6, 7, 1}]` reconstructs a number from its list of digits.
- `Rest[list]` gives all the elements of *list* after the first one. `Most[list]` gives all elements other than the last one.

More to Explore

Guide to List Manipulation in the Wolfram Language (wolfr.am/eiwl-5-more)

6 | Making Tables

We've seen a few ways to make lists in the Wolfram Language. You can just type them in. You can use `Range`. And you can use functions like `IntegerDigits`. One of the most common and flexible ways to make lists is with the function `Table`.

In its very simplest form, `Table` makes a list with a single element repeated some specified number of times.

Make a list that consists of 5 repeated 10 times:

```
In[1]:= Table[5, 10]
```

```
Out[1]= {5, 5, 5, 5, 5, 5, 5, 5, 5, 5}
```

This makes a list with `x` repeated 10 times:

```
In[2]:= Table[x, 10]
```

```
Out[2]= {x, x, x, x, x, x, x, x, x, x}
```

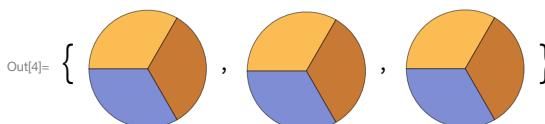
You can repeat lists too:

```
In[3]:= Table[{1, 2}, 10]
```

```
Out[3]= {{1, 2}, {1, 2}, {1, 2}, {1, 2}, {1, 2}, {1, 2}, {1, 2}, {1, 2}, {1, 2}, {1, 2}}
```

Or, actually, anything; here's a list of 3 identical pie charts:

```
In[4]:= Table[PieChart[{1, 1}], 3]
```



But what if we want to make a table where the elements aren't identical? We can do that by introducing a *variable*, and then *iterating* over that variable.

Iterate over `n` to make a list where `n` goes up to 5:

```
In[5]:= Table[a[n], {n, 5}]
```

```
Out[5]= {a[1], a[2], a[3], a[4], a[5]}
```

Here's how this works. To make the first element of the list, `n` is taken to be 1, so `a[n]` is `a[1]`. To make the second element, `n` is taken to be 2, so `a[n]` is `a[2]`, and so on. `n` is called a variable because it's varying as we make the different elements of the list.

Make a table that gives the value of `n + 1` when `n` goes from 1 to 10:

```
In[6]:= Table[n+1, {n, 10}]
```

```
Out[6]= {2, 3, 4, 5, 6, 7, 8, 9, 10, 11}
```

Make a table of the first 10 squares:

```
In[7]:= Table[n^2, {n, 10}]
```

```
Out[7]= {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
```

With `Table`, you can make tables of anything.

Here's a table of successively longer lists produced by `Range`:

```
In[8]:= Table[Range[n], {n, 5}]
```

```
Out[8]= {{1}, {1, 2}, {1, 2, 3}, {1, 2, 3, 4}, {1, 2, 3, 4, 5}}
```

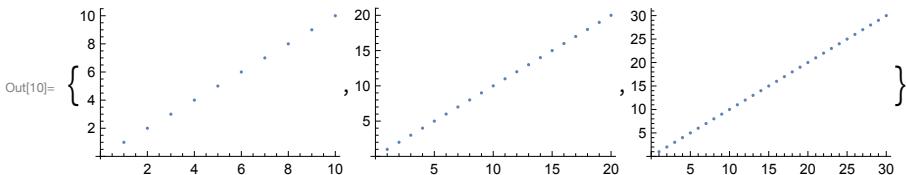
Here each of the lists produced is shown as a column:

```
In[9]:= Table[Column[Range[n]], {n, 8}]
```

```
Out[9]= {{1}, {1, 2}, {1, 2, 3}, {1, 2, 3, 4}, {1, 2, 3, 4, 5}, {1, 2, 3, 4, 5, 6}, {1, 2, 3, 4, 5, 6, 7}, {1, 2, 3, 4, 5, 6, 7, 8}}
```

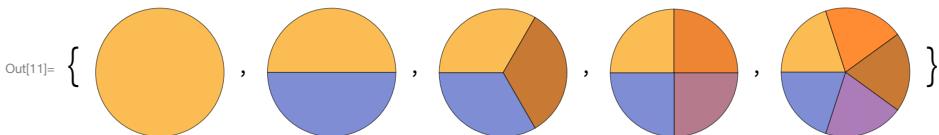
Here's a table of plots of successively longer lists of values:

```
In[10]:= Table[ListPlot[Range[10*n]], {n, 3}]
```



Here are pie charts with successively more segments:

```
In[11]:= Table[PieChart[Table[1, n]], {n, 5}]
```



So far we've always used `n` as our variable. That's a pretty common choice. But we can actually use any (lowercase) letter we want, or any combination of letters.

All that matters is that wherever the variable appears, its name is the same.

`expt` is a perfectly good variable name:

```
In[12]:= Table[2^expt, {expt, 10}]
Out[12]= {2, 4, 8, 16, 32, 64, 128, 256, 512, 1024}
```

Here we're using `x` as the variable name, and it happens to appear several times:

```
In[13]:= Table[{x, x+1, x^2}, {x, 5}]
Out[13]= {{1, 2, 1}, {2, 3, 4}, {3, 4, 9}, {4, 5, 16}, {5, 6, 25}}
```

In `Table[f[n], {n, 5}]`, `n` takes on values 1, 2, 3, 4, 5. `Table[f[n], {n, 3, 5}]` says to start at 3 instead: 3, 4, 5.

This generates a table with `n` going from 1 to 10:

```
In[14]:= Table[f[n], {n, 10}]
Out[14]= {f[1], f[2], f[3], f[4], f[5], f[6], f[7], f[8], f[9], f[10]}
```

This generates a table with `n` going from 4 to 10:

```
In[15]:= Table[f[n], {n, 4, 10}]
Out[15]= {f[4], f[5], f[6], f[7], f[8], f[9], f[10]}
```

This makes `n` go from 4 to 10 in steps of 2:

```
In[16]:= Table[f[n], {n, 4, 10, 2}]
Out[16]= {f[4], f[6], f[8], f[10]}
```

The Wolfram Language emphasizes consistency, so for example `Range` is set up to deal with starting points and steps just like `Table`.

Generate the range of numbers 4 to 10:

```
In[17]:= Range[4, 10]
Out[17]= {4, 5, 6, 7, 8, 9, 10}
```

Generate the range of numbers 4 to 10 going in steps of 2:

```
In[18]:= Range[4, 10, 2]
Out[18]= {4, 6, 8, 10}
```

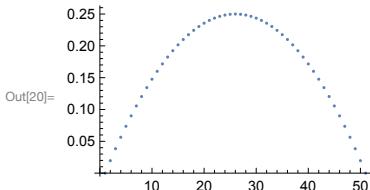
Go from 0 to 1 in steps of 0.1:

```
In[19]:= Range[0, 1, 0.1]
Out[19]= {0., 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.}
```

There are usually many ways to do the same thing in the Wolfram Language. For example, here's how `Table` and `Range` can produce identical plots.

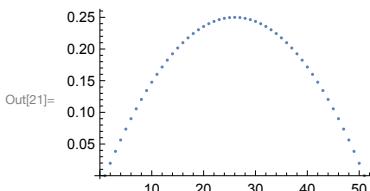
Generate a table and plot it:

```
In[20]:= ListPlot[Table[x-x^2, {x, 0, 1, .02}]]
```



Get the same result by doing arithmetic with the range of values:

```
In[21]:= ListPlot[Range[0, 1, .02] - Range[0, 1, .02]^2]
```



`Table` always separately computes each entry in the list it generates—and you can see this if you use `RandomInteger` in `Table`.

This generates 20 independent random integers with size up to 10:

```
In[22]:= Table[RandomInteger[10], 20]
```

Out[22]= {3, 1, 4, 3, 6, 7, 6, 10, 9, 2, 1, 4, 5, 8, 3, 8, 3, 8, 3, 0}

`RandomInteger` can actually also generate the list directly.

This again generates 20 random integers with size up to 10:

```
In[23]:= RandomInteger[10, 20]
```

Out[23]= {3, 0, 3, 1, 9, 6, 0, 8, 5, 2, 7, 8, 0, 10, 4, 4, 9, 5, 7, 1}

Vocabulary

<code>Table[x, 5]</code>	list of 5 copies of x
<code>Table[f[n], {n, 10}]</code>	list of values of <code>f[n]</code> with n going up to 10
<code>Table[f[n], {n, 2, 10}]</code>	list of values with n going from 2 to 10
<code>Table[f[n], {n, 2, 10, 4}]</code>	list of values with n going from 2 to 10 in steps of 4
<code>Range[5, 10]</code>	list of numbers from 5 to 10
<code>Range[10, 20, 2]</code>	list of numbers from 10 to 20 in steps of 2
<code>RandomInteger[10, 20]</code>	list of 20 random integers up to 10

Exercises

- 6.1** Make a list in which the number 1000 is repeated 5 times.
- 6.2** Make a table of the values of n^3 for n from 10 to 20.
- 6.3** Make a number line plot of the first 20 squares.
- 6.4** Make a list of the even numbers (2, 4, 6, ...) up to 20.
- 6.5** Use `Table` to get the same result as `Range[10]`.
- 6.6** Make a bar chart of the first 10 squares.
- 6.7** Make a table of lists of digits for the first 10 squares.
- 6.8** Make a list line plot of the number of digits in each of the first 100 squares.
- 6.9** Make a table of the first digits of the first 20 squares.
- 6.10** Make a list line plot of the first digits of the first 100 squares.

Q&A

What does the `{...}` (list) in `Table[n^2, {n, 5}]` mean?

A list is always a way of collecting things together. Here what it's collecting is the variable n and its range 5. In the Wolfram Language, this kind of use of a list is called an *iterator specification*.

Why is the `{...}` (list) in `Table[n^2, {n, 5}]` needed?

So one can easily generalize to multidimensional arrays, like `Table[x^2 - y^2, {x, 5}, {y, 5}]`.

What are the constraints on the names of variables?

They can be any sequence of letters or numbers, but they can't start with a number, and—to avoid possible confusion with built-in Wolfram Language functions—they shouldn't start with a capital letter.

Why do you have to name a variable if the name never matters?

Good question! In Section 26, we'll see how to avoid having named variables. It's very elegant, but it's a little more abstract than what we're doing with `Table` here.

Can `Range` deal with negative numbers?

Yes. `Range[-2, 2]` gives $\{-2, -1, 0, 1, 2\}$. `Range[2, -2]` gives $\{\}$, but `Range[2, -2, -1]` gives $\{2, 1, 0, -1, -2\}$.

Tech Notes

- If you specify steps that don't fit evenly in the range you give, `Range` and `Table` just go as far as the steps take them, potentially stopping before the upper limit. (So `Range[1, 6, 2]` gives $\{1, 3, 5\}$, stopping at 5, not 6.)
- Using forms like `Table[x, 20]` requires at least Version 10.2 of the Wolfram Language. In earlier versions, this had to be specified as `Table[x, {20}]`.

More to Explore

The `Table` Function in the Wolfram Language (wolfr.am/eiwl-6-more)

7 | Colors and Styles

The Wolfram Language doesn't just handle things like numbers. It also for example handles things like colors. It lets you refer to common colors by their names.

`Red` represents the color red:

```
In[1]:= Red
```

```
Out[1]= 
```

Make a list of colors:

```
In[2]:= {Red, Green, Blue, Purple, Orange, Black}
```

```
Out[2]= {, , , , , }
```

You can do operations on colors. `ColorNegate` “negates” a color, giving the complementary color. `Blend` blends a list of colors together.

Negating the color yellow gives blue:

```
In[3]:= ColorNegate[Yellow]
```

```
Out[3]= 
```

Here's the result of blending yellow, pink and green:

```
In[4]:= Blend[{Yellow, Pink, Green}]
```

```
Out[4]= 
```

You can specify a color by saying how much red, green and blue it contains. The function `RGBColor` lets you do that, giving the amount of each color, from 0 to 1.

This gives maximum red, with no green or blue:

```
In[5]:= RGBColor[1, 0, 0]
```

```
Out[5]= 
```

Maximum red and green gives yellow:

```
In[6]:= RGBColor[1, 1, 0]
```

```
Out[6]= 
```

This gives a table of colors with maximum red and varying levels of green:

```
In[7]:= Table[RGBColor[1, g, 0], {g, 0, 1, 0.05}]
```

```
Out[7]= {, , , , , , , 
```

It's often convenient to specify colors not directly in terms of red, green and blue, but for example instead in terms of *hue*. The function `Hue` lets you do this.

A hue of 0.5 corresponds to cyan:

```
In[8]:= Hue[0.5]
```

```
Out[8]= 
```

Here's a table of colors with hues from 0 to 1:

```
In[9]:= Table[Hue[x], {x, 0, 1, 0.05}]
```

```
Out[9]= {, , , , , , , , , , , }
```

Sometimes you may want to just pick a random color. `RandomColor` lets you do this. When you say `RandomInteger[10]`, you're asking to generate a random integer up to 10. But for a random color you don't have to specify a range, so you can just write `RandomColor[]`—not giving any explicit input to the function.

Generate a random color:

```
In[10]:= RandomColor[]
```

```
Out[10]= 
```

Make a table of 30 random colors:

```
In[11]:= Table[RandomColor[], 30]
```

```
Out[11]= {, , , , , , , , , , , , , , , , , , , , , , , , , , , , , , }
```

Blending together lots of random colors usually gives something muddy:

```
In[12]:= Blend[Table[RandomColor[], 20]]
```

```
Out[12]= 
```

You can use colors in all sorts of places. For example, you can style output with colors.

This gives the number 1000, styled in red:

```
In[13]:= Style[1000, Red]
```

```
Out[13]= 
```

Here are 30 random integers, styled in random colors:

```
In[14]:= Table[Style[RandomInteger[1000], RandomColor[]], 30]
```

```
Out[14]= {, , , , , , , , , , , , , , , , , , , , , , , , , , , , , }
```

Another form of styling is size. You can specify a font size in Style.

Show x styled in 30-point type:

```
In[15]:= Style[x, 30]
```

```
Out[15]= X
```

This styles the number 100 in a sequence of different sizes:

```
In[16]:= Table[Style[100, n], {n, 30}]
```

```
Out[16]= {100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100}
```

You can combine color and size styling; here's x in 25 random colors and sizes:

```
In[17]:= Table[Style[x, RandomColor[], RandomInteger[30]], 25]
```

```
Out[17]= {X, X, X}
```

Vocabulary

Red, Green, Blue, Yellow, Orange, Pink, Purple, ...	colors
RGBColor[0.4, 0.7, 0.3]	red, green, blue color
Hue[0.8]	color specified by hue
RandomColor[]	randomly chosen color
ColorNegate[Red]	negate a color (complement)
Blend[{Red, Blue}]	blend a list of colors
Style[x, Red]	style with a color
Style[x, 20]	style with a size
Style[x, 20, Red]	style with a size and color

Exercises

- 7.1 Make a list of red, yellow and green.
- 7.2 Make a red, yellow, green column (“traffic light”).
- 7.3 Compute the negation of the color orange.
- 7.4 Make a list of colors with hues varying from 0 to 1 in steps of 0.02.
- 7.5 Make a list of colors with maximum red and blue, but with green varying from 0 to 1 in steps of 0.05.
- 7.6 Blend the colors pink and yellow.

- 7.7** Make a list of colors obtained by blending yellow with hues from 0 to 1 in steps of 0.05.
- 7.8** Make a list of numbers from 0 to 1 in steps of 0.1, each with a hue equal to its value.
- 7.9** Make a purple color swatch of size 100.
- 7.10** Make a list of red swatches with sizes from 10 to 100 in steps of 10.
- 7.11** Display the number 999 in red at size 100.
- 7.12** Make a list of the first 10 squares in which each value is styled at its size.
- 7.13** Use `Part` and `RandomInteger` to make a length-100 list in which each element is randomly Red, Yellow or Green.
- 7.14** Use `Part` to make a list of the first 50 digits of 2^{1000} , in which each digit has a size equal to 3 times its value.

Q&A

What named colors does the Wolfram Language have?

Red, Green, Blue, Black, White, Gray, Yellow, Brown, Orange, Pink, Purple, LightRed, etc. In Section 16 we'll see how to use `Ctrl + =` to enter any color name in plain English.

Why can colors be specified by red, green, blue values?

Basically because that's how we humans perceive colors: there are three kinds of cells in our eyes that are roughly sensitive respectively to red, green and blue components of light. (Some other organisms work differently.)

What does color negation do?

It generates *complementary colors*, defined by computing $1 - \text{value}$ (one minus the value) for each RGB component. If you negate the "display (emitted light) primaries" red, green, blue, you get the "print (reflected light) primaries" cyan, magenta, yellow.

What is hue?

It's a way of specifying what are often called *pure colors*, independent of their tint, shade, saturation or brightness. Colors of different hues are often arranged around a *color wheel*. The RGB values for a particular hue are determined by a mathematical formula.

Are there other ways to specify colors than RGB?

Yes. A common one (implemented by `Hue`) is to use the combination of hue, saturation, and brightness. `LABColor` and `XYZColor` are other examples. `GrayLevel` represents shades of gray, with `GrayLevel[0]` being black and `GrayLevel[1]` being white.

Tech Notes

- The little squares of color used to display a color are usually called *swatches*.
- `ChromaticityPlot` and `ChromaticityPlot3D` plot lists of colors in color space.
- You can set lots of other style attributes in the Wolfram Language, like `Bold`, `Italic` and `FontFamily`.

More to Explore

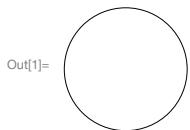
[Color in the Wolfram Language \(wolfr.am/eiwl-7-more\)](http://wolfr.am/eiwl-7-more)

8 | Basic Graphics Objects

In the Wolfram Language, `Circle[]` represents a circle. To display the circle as graphics, use the function `Graphics`. Later, we'll see how to specify the position and size of a circle. But for now, we're just going to deal with a basic circle, which doesn't need any additional input.

Make graphics of a circle:

```
In[1]:= Graphics[Circle[]]
```



Disk represents a filled-in disk:

```
In[2]:= Graphics[Disk[]]
```



`RegularPolygon` gives a regular polygon with however many sides you specify.

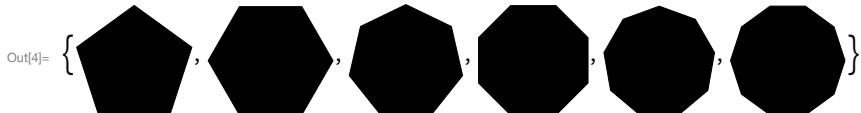
Here's a pentagon (5-sided regular polygon):

```
In[3]:= Graphics[RegularPolygon[5]]
```



Make a table of graphics of regular polygons with between 5 and 10 sides:

```
In[4]:= Table[Graphics[RegularPolygon[n]], {n, 5, 10}]
```



Style works inside `Graphics`, so you can use it to give colors.

Here's an orange pentagon:

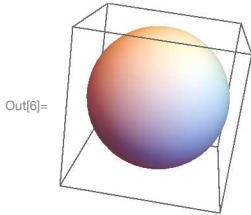
```
In[5]:= Graphics[Style[RegularPolygon[5], Orange]]
```



The Wolfram Language works in 3D as well as 2D, with constructs such as `Sphere`, `Cylinder` and `Cone`. When you have 3D graphics, you can rotate them around interactively to see different angles.

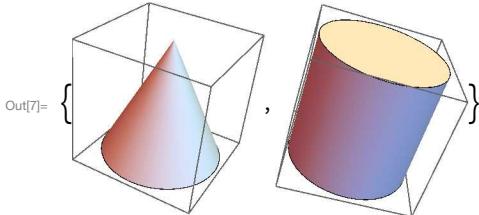
Display a sphere in 3D:

```
In[6]:= Graphics3D[Sphere[]]
```



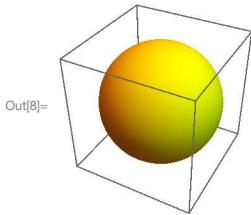
A list of a cone and a cylinder:

```
In[7]:= {Graphics3D[Cone[]], Graphics3D[Cylinder[]]}
```



A yellow sphere:

```
In[8]:= Graphics3D[Style[Sphere[], Yellow]]
```



Vocabulary

<code>Circle[]</code>	specify a circle
<code>Disk[]</code>	specify a filled-in disk
<code>RegularPolygon[n]</code>	specify a regular polygon with n sides
<code>Graphics[object]</code>	display an object as graphics
<code>Sphere[], Cylinder[], Cone[], ...</code>	specify 3D geometric shapes
<code>Graphics3D[object]</code>	display an object as 3D graphics

Exercises

- 8.1** Use `RegularPolygon` to draw a triangle.
- 8.2** Make graphics of a red circle.
- 8.3** Make a red octagon.
- 8.4** Make a list of disks with hue varying from 0 to 1 in steps of 0.1.
- 8.5** Make a column of a red and green triangle.
- 8.6** Make a list of pink regular polygons with between 5 and 10 sides.
- 8.7** Make a graphic of a purple cylinder.
- 8.8** Make a list of randomly colored polygons with 8 down to 3 sides, and apply `Graphics` to the list, to show them all overlaid.

Q&A

How can I make graphics with several objects?

Section 14 will explain. To do so requires understanding *coordinates*.

Why use `Circle[]`, not just `Circle`?

For consistency. As we'll see in Section 14, `Circle[]` is actually short for `Circle[{0, 0}, 1]`, which means a circle of radius 1 and center coordinates {0, 0}.

Why isn't the yellow sphere pure yellow?

Because the Wolfram Language displays it like an actual 3D object, with lighting. If it were pure yellow, you wouldn't see any 3D depth, and it would just look like a 2D disk.

Tech Note

- Another way to specify styles for graphics is to give “directives” in a list, e.g. `{Yellow, Disk[], Black, Circle[]}`.

More to Explore

Guide to Graphics in the Wolfram Language (wolfr.am/eiwl-8-more)

9 | Interactive Manipulation

So far, we've been using the Wolfram Language in a question-answer way: we type input, and the language responds with output. But the language also lets you set up user interfaces where one can continually manipulate a variable. The function `Manipulate` works a lot like `Table`, except that instead of producing a list of results, it gives a slider to interactively choose the value you want.

The slider lets you pick values of n between 1 and 5 (in steps of 1):

```
In[1]:= Manipulate[Table[Orange, n], {n, 1, 5, 1}]
```

Out[1]=

A Manipulate interface with a slider labeled 'n' ranging from 1 to 5. Below the slider is a result area containing a single orange square.

Here's the whole sequence of possible results:

```
In[2]:= Table[Table[Orange, n], {n, 1, 5, 1}]
```

```
Out[2]= {{Orange}, {Orange, Orange}, {Orange, Orange, Orange}, {Orange, Orange, Orange, Orange}, {Orange, Orange, Orange, Orange, Orange}}
```

Here's a similar interface for displaying a column of powers of a number:

```
In[3]:= Manipulate[Column[{n, n^2, n^3}], {n, 1, 10, 1}]
```

Out[3]=

A Manipulate interface with a slider labeled 'n' ranging from 1 to 10. Below the slider is a result area containing the numbers 6, 36, and 216.

And here's the list of possible results in this case:

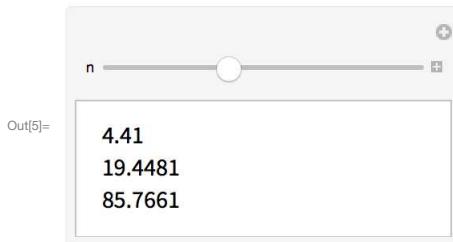
```
In[4]:= Table[Column[{n, n^2, n^3}], {n, 1, 10, 1}]
```

```
Out[4]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
         {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
         {1, 8, 27, 64, 125, 216, 343, 512, 729, 1000}
```

Unlike `Table`, `Manipulate` isn't limited to a fixed set of possible values. If you simply omit the step size in `Manipulate`, it'll assume you want any possible value in the range, not limited to whole numbers.

Without the step size, Manipulate allows any value:

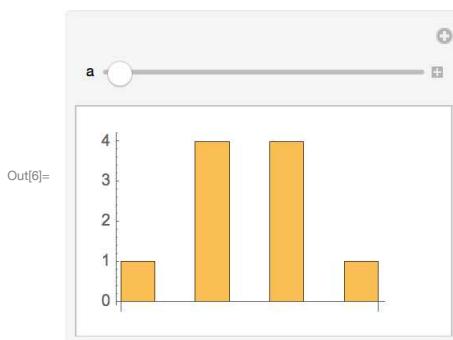
```
In[5]:= Manipulate[Column[{n, n^2, n^3}], {n, 1, 10}]
```



It's very common to have graphics that you want to adjust interactively.

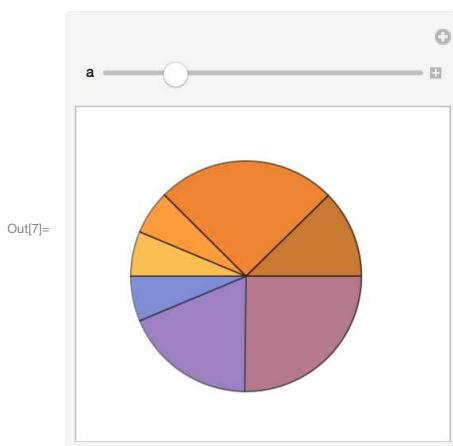
A bar chart that changes as you move the slider:

```
In[6]:= Manipulate[BarChart[{1, a, 4, 2*a, 4, 3*a, 1}], {a, 0, 5}]
```



A pie chart of the same list:

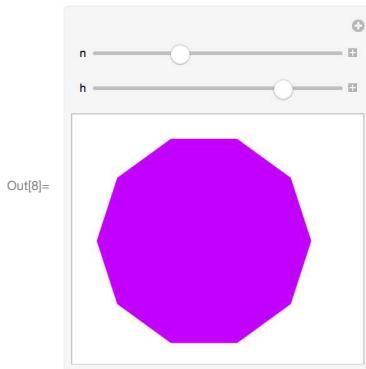
```
In[7]:= Manipulate[PieChart[{1, a, 4, 2*a, 4, 3*a, 1}], {a, 0, 5}]
```



`Manipulate` lets you set up any number of controls. You just give the information for each variable in turn, one after another.

Build an interface to vary the number of sides, and the hue, of a polygon:

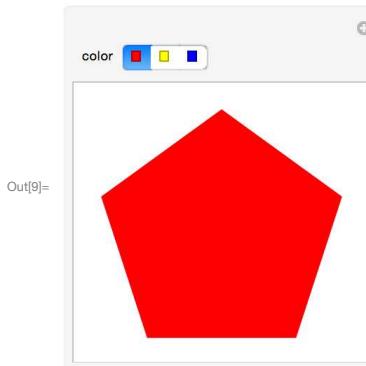
```
In[8]:= Manipulate[Graphics[Style[RegularPolygon[n], Hue[h]]], {n, 5, 20, 1}, {h, 0, 1}]
```



There are many ways to specify controls for `Manipulate`. If you give a list of possibilities, you'll get a *chooser* or *menu*.

Build an interface that lets you choose between three colors:

```
In[9]:= Manipulate[Graphics[Style[RegularPolygon[5], color]], {color, {Red, Yellow, Blue}}]
```



Vocabulary

`Manipulate[anything, {n, 0, 10, 1}]`

manipulate *anything* with *n* varying in steps of 1

`Manipulate[anything, {x, 0, 10}]`

manipulate *anything* with *x* varying continuously

Exercises

- 9.1 Make a Manipulate to show Range[n] with n varying from 0 to 100.
- 9.2 Make a Manipulate to plot the whole numbers up to n, where n can range from 5 to 50.
- 9.3 Make a Manipulate to show a column of between 1 and 10 copies of x.
- 9.4 Make a Manipulate to show a disk with a hue varying from 0 to 1.
- 9.5 Make a Manipulate to show a disk with red, green and blue color controls.
- 9.6 Make a Manipulate to show digit sequences of 4-digit integers (between 1000 and 9999).
- 9.7 Make a Manipulate to create a list of between 5 and 50 equally spaced hues.
- 9.8 Make a Manipulate that shows a list of a variable number of hexagons (between 1 and 10), and with variable hues.
- 9.9 Make a Manipulate that lets you show a regular polygon with between 5 and 20 sides, in red, yellow or blue.
- 9.10 Make a Manipulate that shows a pie chart with a number of equal segments varying from 1 to 10.
- 9.11 Make a Manipulate that gives a bar chart of the 3 digits in integers from 100 to 999.
- 9.12 Make a Manipulate that shows n random colors, where n can range from 1 to 50.
- 9.13 Make a Manipulate to display a column of from 1 to 10 powers of integers from 1 to 25.
- 9.14 Make a Manipulate of a number line of values of x^n for integer x from 1 to 10, with n varying from 0 to 5.
- 9.15 Make a Manipulate to show a sphere that can vary in color from green to red.

Q&A

Does Manipulate work the same on web, mobile and desktop?

Ultimately yes. But it can be significantly slower on the web and on some mobile systems, because every time you move a slider it has to communicate over the internet with a server to find out what to do. On desktop and some mobile systems, everything is happening right there, inside your computer or other device—so it's very fast.

Can I make a standalone app out of a Manipulate?

Yes. To make a web app, for example, you just need to use `CloudDeploy`. We'll talk about this in Section 36.

Can I use random numbers in Manipulate?

Yes, but unless you "seed" them with `SeedRandom`, the random numbers will be different every time you move a slider.

Tech Notes

- Manipulate supports pretty much all standard user interface control types (checkboxes, menus, input fields, color pickers, etc.).
- Sliders in Manipulate often provide a button which opens out to additional controls—including animation, single stepping, and numerical value display.
- Many controls are rendered differently on mouse and touch devices, and some controls only work on one or the other.
- If you’re running natively on your computer, devices like gamepads should immediately work with Manipulate. You can specify which controls should be hooked to what. `ControllerInformation[]` gives information on all your controllers.

More to Explore

The Wolfram Demonstrations Project (wolfr.am/eiwl-9-more): more than 10,000 interactive demonstrations created with Manipulate (wolfr.am/eiwl-9-more2)

10 | Images

Many functions in the Wolfram Language work on *images*. It's easy to get an image into the Wolfram Language, for example by copying or dragging it from the web or from a photo collection. You can also just capture an image directly from your camera using the function `CurrentImage`.

Get the current image from your computer's camera (here, me working on this book):

```
In[1]:= CurrentImage[]
```



You can apply functions to images just like you apply functions to numbers or lists or anything else. The function `ColorNegate` that we saw in connection with colors also works on images, giving a “negative image”.

Negate the colors in the image (making me look pretty weird):

```
In[2]:= ColorNegate[]
```



Blur the image:

```
In[3]:= Blur[]
```



The number says how much to blur the image:

```
In[4]:= Blur[, 10]
```

```
Out[4]= 
```

You can make a table of the results of different amounts of blurring:

```
In[5]:= Table[Blur[, n], {n, 0, 15, 5}]
```

```
Out[5]= 
```

`ImageCollage` puts images together:

```
In[6]:= ImageCollage[Table[Blur[, n], {n, 0, 15, 5}]]
```

```
Out[6]= 
```

There's lots of analysis one can do on images. For example, `DominantColors` finds a list of the most important colors in an image.

```
In[7]:= DominantColors[]
```

```
Out[7]= {■, ■, ■, ■}
```

`Binarize` makes an image black and white:

```
In[8]:= Binarize[]
```

```
Out[8]= 
```

Not surprisingly, the dominant colors in the result are black and white:

```
In[9]:= DominantColors[Binarize[]]
```

```
Out[9]= {■, ■}
```

Another type of analysis is *edge detection*: finding where in the image there are sharp changes in color. The result looks a bit like a sketch derived from the original image.

Pick out edges in the original image:

```
In[10]:= EdgeDetect[]
```

```
Out[10]= 
```

“Add” the original image to the result of the edge detection:

```
In[11]:= ImageAdd[, 

Out[11]= 


```

It’s often convenient to do image processing interactively, creating interfaces using `Manipulate`. For example, `Binarize` lets you specify a threshold for what will be turned black as opposed to white. And often the best way to find the right threshold is just to interactively experiment with it.

Make an interface to adjust the threshold for binarizing an image:

```
In[12]:= Manipulate[Binarize[, t], {t, 0, 1}]
```

Out[12]= 

Vocabulary

<code>CurrentImage[]</code>	capture the current image from your computer, etc.
<code>ColorNegate[]</code>	negate the colors in an image
<code>Binarize[]</code>	convert an image to black and white
<code>Blur[, 5]</code>	blur an image
<code>EdgeDetect[]</code>	detect the edges in an image
<code>DominantColors[]</code>	get a list of dominant colors in an image
<code>ImageCollage[{, , }]</code>	put together images in a collage
<code>ImageAdd[, ]</code>	add color values of two images

Exercises

- 10.1** Color negate the result of edge detecting an image.
- 10.2** Use `Manipulate` to make an interface for blurring an image from 0 to 20.
- 10.3** Make a table of the results from edge detecting an image with blurring from 1 to 10.
- 10.4** Make an image collage of an image together with the results of blurring, edge detecting and binarizing it.
- 10.5** Add an image to a binarized version of it.
- 10.6** Create a `Manipulate` to display edges of an image as it gets blurred from 0 to 20.
- 10.7** Image operations work on `Graphics` and `Graphics3D`. Edge detect a picture of a sphere.
- 10.8** Make a `Manipulate` to make an interface for blurring a purple pentagon from 0 to 20.
- 10.9** Create a collage of 9 images of disks, each with a random color.
- 10.10** Use `ImageCollage` to make a combined image of spheres with hues from 0 to 1 in steps of 0.2.
- 10.11** Make a table of blurring a disk by an amount from 0 to 30 in steps of 5.
- 10.12** Use `ImageAdd` to add an image to an image of a disk.
- 10.13** Use `ImageAdd` to add an image to an image of a red octagon.
- 10.14** Add an image to the color negated version of the edge detected image.

Q&A**CurrentImage gives me a dark image. What should I do?**

Just run it again. When a computer camera is first switched on, it may not have adjusted itself for light levels.

What does the number in Blur mean?

It's the range of pixels that get blurred together.

How does Binarize decide what's black and what's white?

If you don't tell it a threshold, it will pick one based on analyzing the distribution of colors in the image.

Tech Notes

- The very powerful fact that images can appear directly in Wolfram Language code is another consequence of the fact that the Wolfram Language is symbolic.
- A convenient way to get collections of images, say related to crocodiles, is to use `WikipediaData["crocodiles", "ImageList"]`.

More to Explore

Guide to Image Computation in the Wolfram Language (wolfr.am/eiwl-10-more)

11 | Strings and Text

Another thing the Wolfram Language lets you compute with is text. You enter text as a *string*, indicated by quotes ("").

Enter a string:

```
In[1]:= "This is a string."
```

```
Out[1]= This is a string.
```

Just like when you enter a number, a string on its own comes back unchanged—except that the quotes aren't visible when the string is displayed. There are many functions that work on strings. Like `StringLength`, which gives the length of a string.

`StringLength` counts the number of characters in a string:

```
In[2]:= StringLength["hello"]
```

```
Out[2]= 5
```

`StringReverse` reverses the characters in a string:

```
In[3]:= StringReverse["hello"]
```

```
Out[3]= olleh
```

`ToUpperCase` makes all the characters in a string upper case (capital letters):

```
In[4]:= ToUpperCase["I'm coding in the Wolfram Language!"]
```

```
Out[4]= I'M CODING IN THE WOLFRAM LANGUAGE!
```

`StringTake` takes a certain number of characters from the beginning of a string:

```
In[5]:= StringTake["this is about strings", 10]
```

```
Out[5]= this is ab
```

If you take 10 characters, you get a string of length 10:

```
In[6]:= StringLength[StringTake["this is about strings", 10]]
```

```
Out[6]= 10
```

`StringJoin` joins strings (don't forget spaces if you want to separate words):

```
In[7]:= StringJoin["Hello", " ", "there!", " How are you?"]
```

```
Out[7]= Hello there! How are you?
```

You can make lists of strings, then apply functions to them.

A list of strings:

```
In[8]:= {"apple", "banana", "strawberry"}
```

```
Out[8]= {apple, banana, strawberry}
```

Get the first two characters from each string:

```
In[9]:= StringTake[{"apple", "banana", "strawberry"}, 2]
Out[9]= {ap, ba, st}
```

`StringJoin` joins the strings in a list:

```
In[10]:= StringJoin[{"apple", "banana", "strawberry"}]
Out[10]= applebananastrawberry
```

Sometimes it's useful to turn strings into lists of their constituent characters. Each character is actually a string itself, of length 1.

`Characters` breaks a string into a list of its characters:

```
In[11]:= Characters["a string is made of characters"]
Out[11]= {a, , s, t, r, i, n, g, , i, s, , m, a, d, e, , o, f, , c, h, a, r, a, c, t, e, r, s}
```

Once you've broken a string into a list of characters, you can use all the usual list functions on it.

`Sort` the characters in a string:

```
In[12]:= Sort[Characters["a string of characters"]]
Out[12]= {, , , a, a, a, c, c, e, f, g, h, i, n, o, r, r, r, s, s, t, t}
```

The invisible elements at the beginning of the list are space characters. If you want to see strings in the form you'd input them, complete with "...", use `InputForm`.

`InputForm` shows strings as you would input them, including quotes:

```
In[13]:= InputForm[Sort[Characters["a string of characters"]]]
Out[13]= {" ", " ", " ", "a", "a", "a", "c", "c", "e", "f", "g", "h", "i", "n", "o", "r",
"r", "r", "s", "s", "t", "t"}
```

Functions like `StringJoin` and `Characters` work on strings of any kind; it doesn't matter if they're meaningful text or not. There are other functions, like `TextWords`, that specifically work on meaningful text, written, say, in English.

`TextWords` gives a list of the words in a string of text:

```
In[14]:= TextWords["This is a sentence. Sentences are made of words."]
Out[14]= {This, is, a, sentence, Sentences, are, made, of, words}
```

This gives the length of each word:

```
In[15]:= StringLength[TextWords["This is a sentence. Sentences are made of words."]]
Out[15]= {4, 2, 1, 8, 9, 3, 4, 2, 5}
```

`TextSentences` breaks a text string into a list of sentences:

```
In[16]:= TextSentences["This is a sentence. Sentences are made of words."]
Out[16]= {This is a sentence., Sentences are made of words.}
```

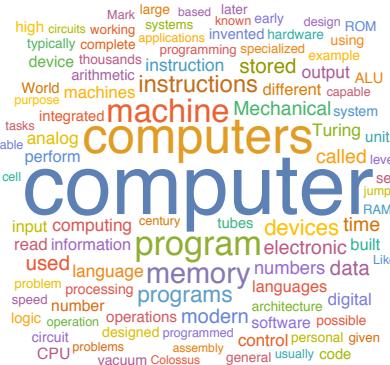
There are lots of ways to get text into the Wolfram Language. One example is the `WikipediaData` function, which gets the current text of Wikipedia articles.

Get the first 100 characters of the Wikipedia article about “computers”:

```
In[17]:= StringTake[WikipediaData["computers"], 100]
Out[17]= A computer is a general-purpose device
          that can be programmed to carry out a set of arithmetic or lo
```

A convenient way to get a sense of what’s in a piece of text is to create a word cloud. The function `WordCloud` does this. To avoid seeing “uninteresting” words like “the” and “and” (often called *stopwords*), we use the function `DeleteStopwords`.

Create a word cloud for the Wikipedia article on “computers”:

```
In[18]:= WordCloud[DeleteStopwords[WikipediaData["computers"]]]
Out[18]= 
```

Not surprisingly, “computer” and “computers” are the most common (non-stopword) words in the article.

The Wolfram Language has lots of built-in knowledge about words that appear in English and other languages. `WordList` gives lists of words.

Get the first 20 words from a list of common English words:

```
In[19]:= Take[WordList[], 20]
Out[19]= {a, aah, aardvark, aback, abacus, abaft, abalone,
          abandon, abandoned, abandonment, abase, abasement, abash,
          abashed, abashment, abate, abatement, abattoir, abbe, abbess}
```

Make a word cloud from the first letters of all the words:

```
In[20]:= WordCloud[StringTake[WordList[], 1]]
```



Strings don't have to contain text. In a juxtaposition of ancient and modern, we can for example generate Roman numerals as strings.

Generate the Roman numeral string for 1988:

```
In[21]:= RomanNumeral[1988]
```

```
Out[21]= MCMLXXXVIII
```

Make a table of the Roman numerals for numbers up to 20:

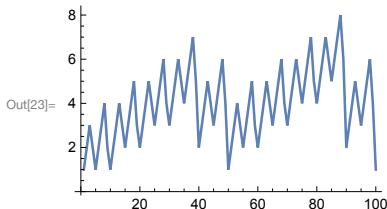
```
In[22]:= Table[RomanNumeral[n], {n, 20}]
```

```
Out[22]= {I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII, XIII, XIV, XV, XVI, XVII, XVIII, XIX, XX}
```

As with everything, we can do computations on these strings. For example, we can plot the lengths of successive Roman numerals.

Plot the lengths of the Roman numerals for numbers up to 100:

```
In[23]:= ListLinePlot[Table[StringLength[RomanNumeral[n]], {n, 100}]]
```



IntegerName gives the English name of an integer.

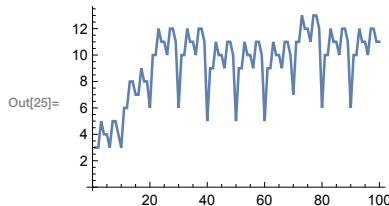
Generate a string giving the name of the integer 56:

```
In[24]:= IntegerName[56]
```

```
Out[24]= fifty-six
```

Here's a plot of the lengths of integer names in English:

```
In[25]:= ListLinePlot[Table[StringLength[IntegerName[n]], {n, 100}]]
```



There are various ways to turn letters into numbers (and vice versa).

`Alphabet` gives the alphabet:

```
In[26]:= Alphabet[]
```

```
Out[26]= {a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}
```

`LetterNumber` tells you where in the alphabet a letter appears:

```
In[27]:= LetterNumber[{"a", "b", "x", "y", "z"}]
```

```
Out[27]= {1, 2, 24, 25, 26}
```

`FromLetterNumber` does the opposite:

```
In[28]:= FromLetterNumber[{10, 11, 12, 13, 14, 15}]
```

```
Out[28]= {j, k, l, m, n, o}
```

`Alphabet` knows about non-English alphabets too:

```
In[29]:= Alphabet["Russian"]
```

```
Out[29]= {а, б, в, г, д, е, ё, ж, з, и, й, к, л, м, н, о, п, р, с, т, у, ф, х, ц, ч, щ, ў, Ѣ, Ѥ, ѧ, ѩ, Ѫ, Ѭ, Ѱ, Ѵ}
```

If you want to, you can also turn text into images, which you can then manipulate using image processing. The function `Rasterize` makes a *raster*, or *bitmap*, of something.

Generate an image of a piece of text:

```
In[30]:= Rasterize[Style["ABC", 100]]
```

```
Out[30]= ABC
```

Do image processing on it:

```
In[31]:= EdgeDetect[Rasterize[Style["ABC", 100]]]
```

```
Out[31]= ABC
```

Vocabulary

<code>"string"</code>	a string
<code>StringLength["string"]</code>	length of a string
<code>StringReverse["string"]</code>	reverse a string
<code>StringTake["string", 4]</code>	take characters at the beginning of a string
<code>StringJoin["string", "string"]</code>	join strings together
<code>StringJoin[{"string", "string"}]</code>	join a list of strings
<code>ToUpperCase["string"]</code>	convert characters to upper case
<code>Characters["string"]</code>	convert a string to a list of characters
<code>TextWords["string"]</code>	list of words from a string
<code>TextSentences["string"]</code>	list of sentences
<code>WikipediaData["topic"]</code>	Wikipedia article about a topic
<code>WordCloud["text"]</code>	word cloud based on word frequencies
<code>DeleteStopwords["string"]</code>	delete uninteresting words from a string (a, and, ...)
<code>WordList[]</code>	list of common words in English
<code>Alphabet[]</code>	list of letters of the alphabet
<code>LetterNumber["c"]</code>	where a letter appears in the alphabet
<code>FromLetterNumber[n]</code>	letter appearing at a position in the alphabet
<code>RomanNumeral[n]</code>	convert a number to its Roman numeral string
<code>IntegerName[n]</code>	convert a number to its English name string
<code>InputForm["string"]</code>	show a string with quotes
<code>Rasterize["string"]</code>	make a bitmap image

Exercises

- 11.1 Join two copies of the string "Hello".
- 11.2 Make a single string of the whole alphabet, in upper case.
- 11.3 Generate a string of the alphabet in reverse order.
- 11.4 Join 100 copies of the string "AGCT".
- 11.5 Use `StringTake`, `StringJoin` and `Alphabet` to get "abcdef".
- 11.6 Create a column with increasing numbers of letters from the string "this is about strings".
- 11.7 Make a bar chart of the lengths of the words in "A long time ago, in a galaxy far, far away".
- 11.8 Find the string length of the Wikipedia article for "computer".
- 11.9 Find how many words are in the Wikipedia article for "computer".
- 11.10 Find the first sentence in the Wikipedia article about "strings".
- 11.11 Make a string from the first letters of all sentences in the Wikipedia article about computers.
- 11.12 Find the maximum word length among English words from `WordList[]`.
- 11.13 Count the number of words in `WordList[]` that start with "q".
- 11.14 Make a line plot of the lengths of the first 1000 words from `WordList[]`.
- 11.15 Use `StringJoin` and `Characters` to make a word cloud of all letters in the words from `WordList[]`.
- 11.16 Use `StringReverse` to make a word cloud of the last letters in the words from `WordList[]`.
- 11.17 Find the Roman numerals for the year 1959.
- 11.18 Find the maximum string length of any Roman-numeral year from 1 to 2020.
- 11.19 Make a word cloud from the first characters of the Roman numerals up to 100.
- 11.20 Use `Length` to find the length of the Russian alphabet.
- 11.21 Generate the uppercase Greek alphabet.
- 11.22 Make a bar chart of the letter numbers in "wolfram".
- 11.23 Use `FromLetterNumber` to make a string of 1000 random letters.
- 11.24 Make a list of 100 random 5-letter strings.
- 11.25 Make a white-on-black size-200 letter "A".
- 11.26 Use `Manipulate` to make an interactive selector of size-100 characters from the alphabet, controlled by a slider.
- 11.27 Use `Manipulate` to make an interactive selector of black-on-white outlines of size-100 characters from the alphabet.
- 11.28 Use `Manipulate` to create a "vision simulator" that blurs a size-200 letter "A" by an amount from 0 to 50.

Q&A

What is the difference between "x" and x?

"x" is a string; x is a Wolfram Language symbol, just like Plus or Max, that can be defined to actually do computations. We'll talk much more about symbols later.

How do I enter characters that aren't on my keyboard?

You can use whatever methods your computer provides, or you can do it directly with the Wolfram Language using constructs such as \[Alpha].

How do I put quotes ("") inside a string?

Use \" (and if you want to put \" literally in the string, use \\\"). (You'll use a lot of backslashes if you want to put \\\" in: \\\"\\\"\\\".)

What about letters that aren't English? How are they numbered?

LetterNumber["a", "Greek"] gives numbering in the Greek alphabet. All characters are assigned a *character code*. You can find it using ToCharacterCode.

How are the colors of elements in word clouds determined?

By default it's random within a certain color palette. You can specify it if you want to.

How come the word cloud shows "s" as the most common letter?

Because it is the most common *first* letter for common words in English. If you look at all letters, the most common is "e".

Tech Notes

- StringTake["string", -2] takes 2 characters from the end of the string.
- Every character, whether "a", "α" or "狼" is represented by a Unicode character code, found with ToCharacterCode. You can explore "Unicode space" with FromCharacterCode.
- If you get a different result from WikipediaData, that's because Wikipedia has been changed.

More to Explore

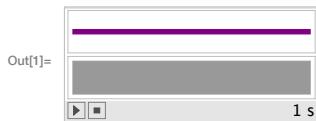
Guide to String Manipulation in the Wolfram Language (wolfr.am/eiwl-11-more)

12 | Sound

In the Wolfram Language, sound works a lot like graphics, except that instead of having things like circles, one has sound notes. Press the play ▶ button to actually play sounds. If you don't say otherwise, the Wolfram Language will make the notes sound as if they were played on a piano.

Generate a middle C note:

```
In[1]:= Sound[SoundNote["C"]]
```



You can specify a sequence of notes by giving them in a list.

Play three notes in sequence:

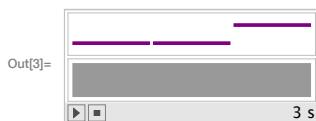
```
In[2]:= Sound[{SoundNote["C"], SoundNote["C"], SoundNote["G"]} ]
```



Instead of giving names of notes, you can give a number to specify their pitch. Middle C is 0. Each semitone above middle C goes up by 1. Middle G is 7 semitones above middle C, so it's specified by the number 7. (An octave is 12 semitones.)

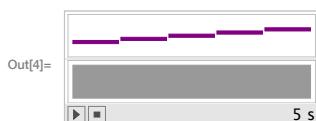
Specify the notes by numbers:

```
In[3]:= Sound[{SoundNote[0], SoundNote[0], SoundNote[7]}]
```



Use `Table` to generate a sequence of 5 notes:

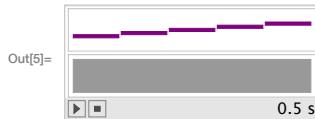
```
In[4]:= Sound[Table[SoundNote[n], {n, 5}]]
```



If you don't say otherwise, each note lasts 1 second. Use `SoundNote[pitch, length]` to get a different length.

Play each note for 0.1 seconds:

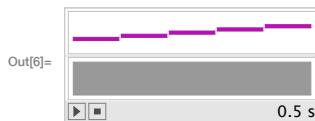
```
In[5]:= Sound[Table[SoundNote[n, 0.1], {n, 5}]]
```



In addition to piano, `SoundNote` can handle a long list of possible instruments. The name of each instrument is a string.

Play notes on a simulated violin:

```
In[6]:= Sound[Table[SoundNote[n, 0.1, "Violin"], {n, 5}]]
```



It's easy to make "random music"—different every time you generate it.

Play a sequence of 20 notes with random pitches:

```
In[7]:= Sound[Table[SoundNote[RandomInteger[12], 0.1, "Violin"], 20]]
```



Vocabulary

Sound[...] create a sound from notes

SoundNote["C"] a named note

SoundNote[5] a note with a numbered pitch

SoundNote[5, 0.1] a note played for a specified time

SoundNote[5, 0.1, "Guitar"] a note played on a certain instrument

Exercises

- 12.1** Generate the sequence of notes with pitches 0, 4, and 7.
- 12.2** Generate 2 seconds of playing middle A on a cello.
- 12.3** Generate a “piano riff” of notes from pitch 0 to pitch 48 in steps of 1, with each note lasting 0.05 seconds.
- 12.4** Generate a sequence of notes going from pitch 12 down to 0 in steps of 1.
- 12.5** Generate a sequence of 5 notes starting with middle C, then successively going up by an octave at a time.
- 12.6** Generate a sequence of 10 notes on a trumpet with random pitches from 0 to 12 and duration 0.2 seconds.
- 12.7** Generate a sequence of 10 notes with random pitches up to 12 and random durations up to 10 tenths of a second.
- 12.8** Generate 0.1-second notes with pitches given by the digits of 2^{31} .
- 12.9** Create a sound from the letters in CABBAGE, each playing for 0.3 seconds sounding like a guitar.
- 12.10** Generate 0.1-second notes with pitches given by the letter numbers of the characters in “wolfram”.

Q&A

How do I know which instruments are available?

Look at the list under “Details and Options” in the SoundNote reference page, or just start typing and see the completions you’re offered. You can also use instrument numbers, from 1 to 128. All the standard MIDI instruments are there, including percussion.

How do I play notes below middle C?

Just use negative numbers, like SoundNote[-10].

What are sharp and flat notes called?

E \sharp (E sharp), Ab (A flat), etc. They also have numbers (e.g. E \sharp is 5).

How do I make a chord?

Put note names in a list, as in SoundNote[{"C", "G"}].

How do I make a rest?

For a 0.2-second rest, use SoundNote[None, 0.2].

How do I get a sound to play immediately, without having to press the play button?

Use EmitSound, as in EmitSound[Sound[SoundNote["C"]]], etc.

Why do I need quotes in the name of a note like “C”?

Because the name is a Wolfram Language string. If you typed just C, it would be interpreted as a function named C, which isn’t what you want.

Tech Notes

- **SoundNote** corresponds to MIDI sound. The Wolfram Language also supports “sampled sound”, for example using functions like **ListPlay**.
- To get spoken output, use **Speak**. To make a beep, use **Beep**.

More to Explore

Guide to Sound Generation in the Wolfram Language (wolfr.am/eiwl-12-more)

13 | Arrays, or Lists of Lists

We've seen how `Table` can be used to make lists. Now let's see how `Table` can be used to create higher-dimensional arrays of values.

Make a list of 4 copies of `x`:

```
In[1]:= Table[x, 4]
```

```
Out[1]= {x, x, x, x}
```

Make a list of 4 copies of a list that contains 5 copies of `x`:

```
In[2]:= Table[x, 4, 5]
```

```
Out[2]= {{x, x, x, x, x}, {x, x, x, x, x}, {x, x, x, x, x}, {x, x, x, x, x}}
```

Use `Grid` to display the result in a grid:

```
In[3]:= Grid[Table[x, 4, 5]]
```

```
X X X X X  
X X X X X  
Out[3]= X X X X X  
X X X X X
```

You can use `Table` with two variables to make a 2D array. The first variable corresponds to the row; the second to the column.

Make an array of colors: red going down, blue going across:

```
In[4]:= Grid[Table[RGBColor[r, 0, b], {r, 0, 1, .2}, {b, 0, 1, .2}]]
```

```
Out[4]=
```

Show every array element as its row number:

```
In[5]:= Grid[Table[i, {i, 4}, {j, 5}]]
```

```
1 1 1 1 1  
2 2 2 2 2  
Out[5]= 3 3 3 3 3  
4 4 4 4 4
```

Show every array element as its column number:

```
In[6]:= Grid[Table[j, {i, 4}, {j, 5}]]
```

```
1 2 3 4 5
1 2 3 4 5
Out[6]= 1 2 3 4 5
1 2 3 4 5
```

Generate an array in which each element is the sum of its row and column number:

```
In[7]:= Grid[Table[i+j, {i, 5}, {j, 5}]]
```

```
2 3 4 5 6
3 4 5 6 7
Out[7]= 4 5 6 7 8
5 6 7 8 9
6 7 8 9 10
```

Generate a multiplication table:

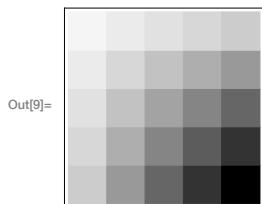
```
In[8]:= Grid[Table[i*j, {i, 5}, {j, 5}]]
```

```
1 2 3 4 5
2 4 6 8 10
Out[8]= 3 6 9 12 15
4 8 12 16 20
5 10 15 20 25
```

`ArrayPlot` lets you visualize values in an array. Larger values are shown darker.

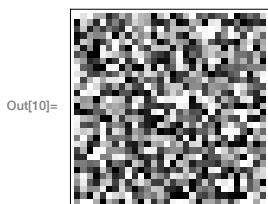
Visualize a multiplication table:

```
In[9]:= ArrayPlot[Table[i*j, {i, 5}, {j, 5}]]
```



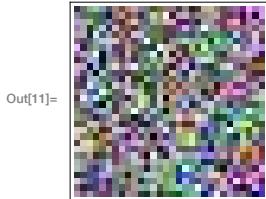
Generate and plot an array of random values:

```
In[10]:= ArrayPlot[Table[RandomInteger[10], 30, 30]]
```



`ArrayPlot` also lets you put colors as values:

```
In[11]:= ArrayPlot[Table[RandomColor[], 30, 30]]
```



Images are ultimately arrays of pixels. Color images make each pixel have red, green and blue values. Black-and-white images have pixels with values 0 (black) or 1 (white). You can get the actual pixel values using `ImageData`.

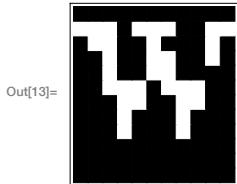
Find the value of pixels in an image of a “W”:

```
In[12]:= ImageData[Binarize[Rasterize["W"]]]
```

```
Out[12]= {{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}, {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1},  
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}, {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1}, {1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1},  
{0, 0, 1, 1, 1, 0, 0, 1, 1, 1, 0, 0}, {1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1}, {1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 1},  
{1, 0, 1, 1, 0, 0, 0, 1, 1, 0, 1}, {1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 1}, {1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 1},  
{1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1}, {1, 1, 0, 0, 1, 1, 0, 0, 1, 1}, {1, 1, 0, 0, 1, 1, 1, 0, 0, 1, 1},  
{1, 1, 0, 0, 1, 1, 1, 0, 0, 1}, {1, 1, 1, 1, 1, 1, 1, 1, 1, 1}, {1, 1, 1, 1, 1, 1, 1, 1, 1, 1},  
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1}}
```

Use `ArrayPlot` to visualize the array of values:

```
In[13]:= ArrayPlot[ImageData[Binarize[Rasterize["W"]]]]
```



The image is of very low resolution, because that’s how `Rasterize` made it in this case. It’s also white-on-black instead of black-on-white. That’s because in an image 0 is black and 1 is white (like in `RGBColor`), while `ArrayPlot`’s default is to make larger values darker.

You can do arithmetic with arrays, just like lists. That means it's easy to swap 0 and 1 in this array: Just subtract everything from 1, so every 0 becomes $1 - 0 = 1$, and every 1 becomes $1 - 1 = 0$.

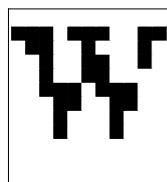
Find pixel values, then do arithmetic to swap 0 and 1 in the array:

```
In[14]:= 1 -ImageData[Binarize[Rasterize["W"]]]
```

```
Out[14]= {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {1, 1, 1, 0, 1, 1, 1, 0, 0, 1}, {0, 1, 1, 0, 0, 1, 0, 0, 0, 1}, {0, 0, 1, 0, 0, 1, 1, 0, 0, 0}, {0, 0, 1, 0, 0, 1, 1, 0, 0, 0}, {0, 0, 1, 1, 1, 0, 1, 1, 0, 0}, {0, 0, 0, 1, 1, 0, 0, 1, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 1, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}}
```

The result is black-on-white:

```
In[15]:= ArrayPlot[1 -ImageData[Binarize[Rasterize["W"]]]]
```



Vocabulary

Table [<i>x</i> , 4, 5]	make a 2D array of values
---------------------------------	---------------------------

Grid [<i>array</i>]	lay out values from an array in a grid
------------------------------	--

ArrayPlot [<i>array</i>]	visualize the values in an array
-----------------------------------	----------------------------------

ImageData [<i>image</i>]	get the array of pixel values from an image
-----------------------------------	---

Exercises

13.1 Make a 12×12 multiplication table.

13.2 Make a 5×5 multiplication table for Roman numerals.

13.3 Make a 10×10 grid of random colors.

13.4 Make a 10×10 grid of randomly colored random integers between 0 and 10.

13.5 Make a grid of all possible strings consisting of pairs of letters of the alphabet ("aa", "ab", etc.).

13.6 Visualize $\{1, 4, 3, 5, 2\}$ with a pie chart, number line, line plot, and a bar chart. Place these in a 2×2 grid.

13.7 Make an array plot of hue values $x * y$, where x and y each run from 0 to 1 in steps of 0.05.

13.8 Make an array plot of hue values x/y , where x and y each run from 1 to 50 in steps of 1.

13.9 Make an array plot of the lengths of Roman numeral strings in a multiplication table up to 100×100 .

Q&A**Can the limits of one variable in a table depend on another?**

Yes, later ones can depend on earlier ones. `Table[x, {i, 4}, {j, i}]` makes a “ragged” triangular array.

Can I make tables that are lists of lists of lists?

Yes, you can make tables of any dimension. `Image3D` gives a way to visualize 3D arrays.

Why does 0 correspond to black, and 1 to white, in images?

0 means zero intensity of light, i.e. black. 1 means maximum intensity, i.e. white.

How do I get the original image back from the output of `ImageData`?

Just apply the function `Image` to it.

Tech Notes

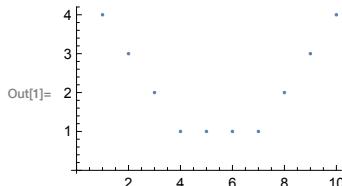
- Arrays in the Wolfram Language are just lists in which each element is itself a list. The Wolfram Language also allows much more general structures, that mix lists and other things.
- Lists in the Wolfram Language correspond to mathematical *vectors*; lists of equal-length lists correspond to *matrices*.
- If most of the entries in an array are 0 (or some other fixed value), you can use `SparseArray` to construct an array just by giving the positions and values of nonzero elements.

14 | Coordinates and Graphics

We've used `ListPlot` and `ListLinePlot` to plot lists of values, where each value appears after the one before. But by giving lists containing *pairs of coordinates* instead of single values, we can use these functions to plot points at arbitrary positions.

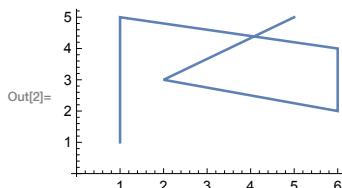
Plot a list of values where each value appears after the one before:

```
In[1]:= ListPlot[{4, 3, 2, 1, 1, 1, 2, 3, 4}]
```



Plot a sequence of arbitrary points specified by $\{x, y\}$ coordinates:

```
In[2]:= ListLinePlot[{{1, 1}, {1, 5}, {6, 4}, {6, 2}, {2, 3}, {5, 5}}]
```



The position of each point here is specified by $\{x, y\}$ coordinates. Following the standard convention in mathematics, the x value says how far across horizontally the point should be; the y value says how far up vertically it should be.

Generate a sequence of random $\{x, y\}$ coordinates:

```
In[3]:= Table[RandomInteger[20], 10, 2]
```

```
Out[3]= {{19, 8}, {11, 20}, {14, 15}, {5, 8}, {6, 4}, {16, 14}, {1, 17}, {10, 7}, {5, 6}, {17, 2}}
```

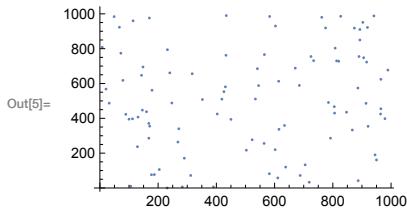
Another way to get random coordinates:

```
In[4]:= RandomInteger[20, {10, 2}]
```

```
Out[4]= {{2, 2}, {20, 18}, {16, 2}, {13, 13}, {6, 15}, {11, 18}, {10, 20}, {17, 20}, {8, 14}, {2, 10}}
```

Plot 100 points at random coordinates:

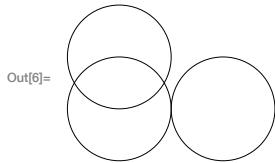
```
In[5]:= ListPlot[Table[RandomInteger[1000], 100, 2]]
```



We can use coordinates to construct graphics. Earlier we saw how to make graphics of one circle. To make graphics of more than one circle we have to say where each circle is, which we can do by giving the coordinates of their centers.

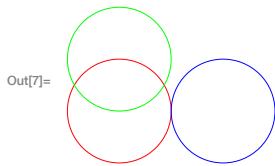
Place circles by giving the coordinates of their centers:

```
In[6]:= Graphics[{Circle[{1, 1}], Circle[{1, 2}], Circle[{3, 1}]}]
```



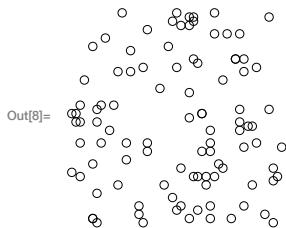
If we apply color styles it's easier to see which circle is which:

```
In[7]:= Graphics[{Style[Circle[{1, 1}], Red], Style[Circle[{1, 2}], Green], Style[Circle[{3, 1}], Blue]}]
```



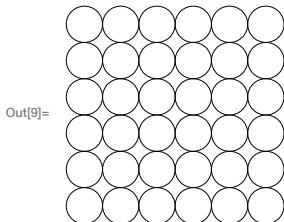
Make a graphic with 100 randomly placed circles, each with center coordinates up to 50:

```
In[8]:= Graphics[Table[Circle[RandomInteger[50, 2]], 100]]
```



A 2D array of circles, arranged so that they just touch:

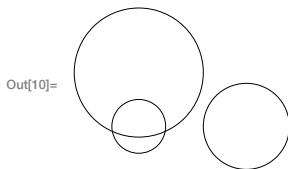
```
In[9]:= Graphics[Table[Circle[{x, y}], {x, 0, 10, 2}, {y, 0, 10, 2}]]
```



`Circle[{x, y}]` means a circle centered at position $\{x, y\}$. If you don't say otherwise, the circle is given radius 1. You can make a circle of any radius using `Circle[{x, y}, r]`.

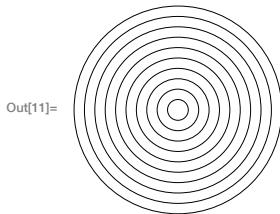
Use different radii for different circles:

```
In[10]:= Graphics[{Circle[{1, 1}, 0.5], Circle[{1, 2}, 1.2], Circle[{3, 1}, 0.8]}]
```



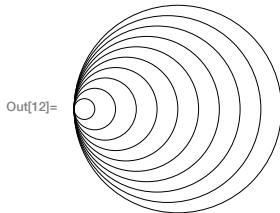
Make 10 concentric circles:

```
In[11]:= Graphics[Table[Circle[{0, 0}, r], {r, 10}]]
```



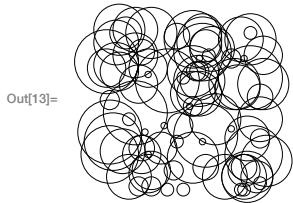
Draw larger circles with centers that shift progressively to the right:

```
In[12]:= Graphics[Table[Circle[{x, 0}, x], {x, 10}]]
```



Pick both positions and radii at random:

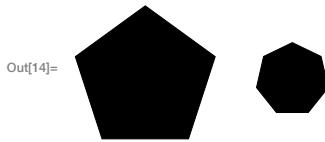
```
In[13]:= Graphics[Table[Circle[RandomInteger[50, 2], RandomInteger[10]], 100]]
```



`RegularPolygon` works much the same as `Circle` and `Disk`, except that in addition to giving the position of the center and the size, you also have to specify how many sides the polygon should have.

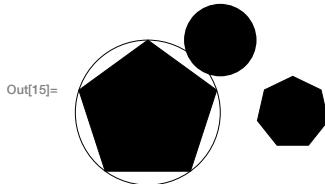
Make graphics of a size-1 regular pentagon and a size-0.5 regular heptagon:

```
In[14]:= Graphics[{RegularPolygon[{1, 1}, 1, 5], RegularPolygon[{3, 1}, 0.5, 7]}]
```



You can mix different kinds of graphics objects:

```
In[15]:= Graphics[{RegularPolygon[{1, 1}, 1, 5],
  Circle[{1, 1}, 1], RegularPolygon[{3, 1}, .5, 7], Disk[{2, 2}, .5]}]
```



To make arbitrary graphics, you need the basic *graphics primitives* `Point`, `Line` and `Polygon`. `Point[{x, y}]` represents a point at coordinate position $\{x, y\}$. To get multiple points, you can either give a list of `Point[{x, y}]`s, or you can give a list of coordinate positions inside a single `Point`.

Graphics of three points at specified positions:

```
In[16]:= Graphics[{Point[{0, 0}], Point[{2, 0}], Point[{1, 1.5}]}]
```

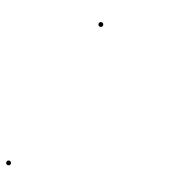
Out[16]=



An alternative form, where all the coordinate positions are collected in a single list:

```
In[17]:= Graphics[Point[{{0, 0}, {2, 0}, {1, 1.5}}]]
```

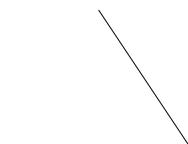
Out[17]=



Make a line joining the positions:

```
In[18]:= Graphics[Line[{{0, 0}, {2, 0}, {1, 1.5}}]]
```

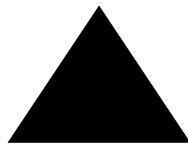
Out[18]=



Make a polygon with corners at the positions you give:

```
In[19]:= Graphics[Polygon[{{0, 0}, {2, 0}, {1, 1.5}}]]
```

Out[19]=

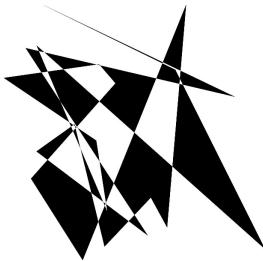


`RegularPolygon` makes a regular polygon in which all sides and angles are the same. `Polygon` lets you make any polygon, even strange ones that fold over themselves.

A polygon with 20 corners at random coordinates below 100; the polygon folds over itself:

```
In[20]:= Graphics[Polygon[Table[RandomInteger[100], 20, 2]]]
```

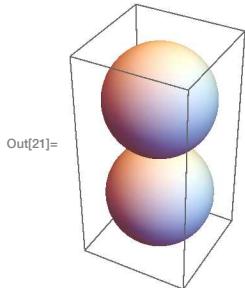
Out[20]=



The things we've done so far here immediately generalize to 3D. Instead of having two coordinates $\{x, y\}$ we have three: $\{x, y, z\}$. In the Wolfram Language, x goes across the screen, y goes "into" the screen, and z goes up the screen.

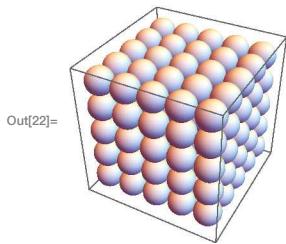
Two spheres stacked on top of each other:

```
In[21]:= Graphics3D[{Sphere[{0, 0, 0}], Sphere[{0, 0, 2}]}]
```



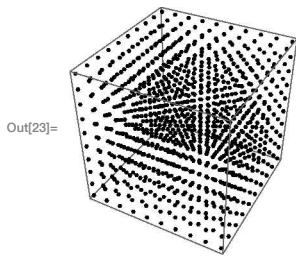
A 3D array of spheres (radius 1/2 makes them just touch):

```
In[22]:= Graphics3D[Table[Sphere[{x, y, z}, 1/2], {x, 5}, {y, 5}, {z, 5}]]
```



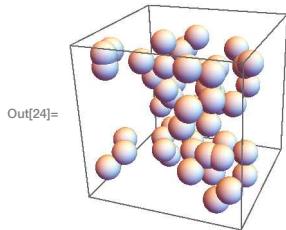
A 3D array of points:

```
In[23]:= Graphics3D[Table[Point[{x, y, z}], {x, 10}, {y, 10}, {z, 10}]]
```



50 spheres at random 3D positions, with each coordinate running up to 10:

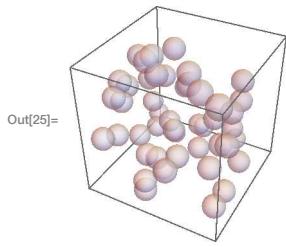
```
In[24]:= Graphics3D[Table[Sphere[RandomInteger[10, 3]], 50]]
```



If you don't say otherwise, 3D objects like spheres are drawn solid, so you can't see through them. But just like you can specify what color something is, you can also specify its *opacity*. Opacity 1 means completely opaque, so you can't see through it at all; opacity 0 means completely transparent.

Specify opacity 0.5 for all spheres:

```
In[25]:= Graphics3D[Table[Style[Sphere[RandomInteger[10, 3]], Opacity[0.5]], 50]]
```



You can use Manipulate to make graphics—in 2D or 3D—that can be manipulated.

Manipulate the position and opacity of the second sphere:

```
In[26]:= Manipulate[
 Graphics3D[{Sphere[{0, 0, 0}], Style[Sphere[{x, 0, 0}], Opacity[o]]}], {x, 1, 3}, {o, 0.5, 1}]
```

Out[26]=

Vocabulary

Point [$\{x, y\}$]	a point at coordinates $\{x, y\}$
Line [$\{\{1, 1\}, \{2, 4\}, \{1, 2\}\}$]	a line connecting specified coordinates
Circle [$\{x, y\}$]	a circle with center at $\{x, y\}$
Circle [$\{x, y\}, r$]	a circle with center at $\{x, y\}$ and radius r
RegularPolygon [$\{x, y\}, s, n$]	a regular polygon with center $\{x, y\}$ and n sides each s long
Polygon [$\{\{1, 1\}, \{2, 4\}, \{1, 2\}\}$]	a polygon with the specified corners
Sphere [$\{x, y, z\}$]	a sphere with center at $\{x, y, z\}$
Sphere [$\{x, y, z\}, r$]	a sphere with center at $\{x, y, z\}$ and radius r
Opacity [<i>level</i>]	specify an opacity level (0: transparent; 1: solid)

Exercises

- 14.1** Make graphics of 5 concentric circles centered at $\{0, 0\}$ with radii $1, 2, \dots, 5$.
- 14.2** Make 10 concentric circles with random colors.
- 14.3** Make graphics of a 10×10 grid of circles with radius 1 centered at integer points $\{x, y\}$.
- 14.4** Make a 10×10 grid of points with coordinates at integer positions up to 10.
- 14.5** Make a Manipulate with between 1 and 20 concentric circles.
- 14.6** Place 50 spheres with random colors at random coordinates up to 10.
- 14.7** Make a $10 \times 10 \times 10$ array of spheres with RGB components ranging from 0 to 1.
- 14.8** Make a Manipulate with a varying between -2 and +2 that contains circles centered at $\{a \cdot x, 0\}$ with x going from 1 to 10.
- 14.9** Make a 5×5 array of regular hexagons with size $1/2$, centered at integer points.
- 14.10** Make a line in 3D that goes through 50 random points with coordinates randomly chosen up to 50.

Q&A**What determines the range of coordinates shown?**

By default it's picked automatically, but you can set it explicitly using the `PlotRange` option, as discussed in Section 20.

How can I put axes on graphics?

Use the option (see Section 20) `Axes → True`.

How do I change the appearance of the edges of a polygon or disk?

Use `EdgeForm` inside `Style`.

What other graphics constructs are there?

Quite a few. Examples include `Text` (for placing text inside graphics), `Arrow` (for putting arrowheads on lines etc.), `Inset` (for putting graphics inside graphics) and `FilledCurve`.

How do I get rid of the box around 3D graphics?

Use the option (see Section 20) `Boxed → False`.

Tech Notes

- The random circles here are drawn at integer coordinates. You can use `RandomReal` to place them at arbitrary coordinates.
- Instead of using `Style`, you can give directives for graphics in a list, like `{Red, Disk[]}`. A particular directive will affect every graphics object that appears after it in the list.
- In 2D graphics, objects are drawn in whatever order you give them, so later ones can cover up earlier ones.
- You can apply geometric transformations to graphics objects using functions like `Translate`, `Scale` and `Rotate`.
- Polygons that fold over themselves (say, to make a bowtie shape) are displayed using an even-odd rule.
- 3D graphics can include `Cuboid`, `Tetrahedron` and polyhedra specified by `PolyhedronData`, as well as arbitrary shapes defined by meshes.

More to Explore

[Guide to Graphics in the Wolfram Language \(wolfr.am/eiwl-14-more\)](http://wolfr.am/eiwl-14-more)

15 | The Scope of the Wolfram Language

In the past 14 sections, we've seen many things the Wolfram Language can do. But what we've seen is only the very beginning. We've discussed around 85 built-in functions—but altogether the language has more than 5000.

You can go to the Documentation Center to start exploring all these functions.

The front page of the Wolfram Language Documentation Center:

Wolfram Language & System Documentation Center

search documentation

Core Language & Structure	$f[x]$	Data Manipulation & Analysis	x^2+y	Visualization & Graphics
Symbolic & Numeric Computation		Strings & Text	a to our and by large <small>they should on the</small>	Graphs & Networks
Images		Geometry		Sound
Time-Related Computation		Geographic Data & Computation		Scientific and Medical Data & Computation
Engineering Data & Computation		Financial Data & Computation		Social, Cultural & Linguistic Data
Higher Mathematical Computation	$\sum_{k=0}^{\infty} (a_k)_i$	Documents & Presentation		User Interface Construction
System Operation & Setup		External Interfaces & Connections		Cloud & Deployment

Fast Introduction for Programmers » Wolfram|Alpha Knowledgebase Examples » Legacy Documentation »

Let's pick Geometry as an example.

Open the Geometry tile:



The Documentation Center has *guide pages* with overviews of functions related to particular topics.

The guide page for Plane Geometry:

WOLFRAM LANGUAGE GUIDE Functions ▾ Related Guides ▾

Plane Geometry

The Wolfram Language provides fully integrated support for plane geometry, including basic regions such as points, lines, triangles, and disks; functions for computing basic properties such as arc length and area; and nearest points to solvers to find the intersection of regions or integrals over regions.

▼ Reference

Geometrical Objects »

SSSTriangle — a triangle specified by the length of its sides
Point = **Line** = **HalfLine** = **InfiniteLine** = **Circle**
SASTriangle = **ASATriangle** = **AASTriangle** = **Triangle** = **Rectangle** = **Parallelogram** =
Polygon = **HalfPlane** = **InfinitePlane** = **Disk**

Visualization
Graphics — visualize regions with different styles

Measures & Tests »

ArcLength — length of a curve
Area — area of a region
RegionMember — test whether a point is in a region
RegionNearest — nearest point in a region to a given point
RegionQ = **RegionDimension** = **RegionDistance** = ...

Solving with Regions »

FindInstance — find examples of points in a region
Solve — find curve crossings etc.
NSolve = **Reduce** = **Minimize** = **NMinimize** = ...

Now you can visit the *function page* for a particular function, say **Parallelogram**.

The function page for **Parallelogram**:

The screenshot shows the 'Parallelogram' function page. At the top, it says 'BUILT-IN WOLFRAM LANGUAGE SYMBOL'. Below that is the title 'Parallelogram'. Under the title is a yellow box containing the function signature `Parallelogram[p, {v1, v2}]` and a description: 'represents a parallelogram with origin p and directions v_1 and v_2 '. There are sections for 'Details' and 'Examples'. The 'Examples' section is expanded, showing a 'Basic Examples' section with three items. The first item shows code: `In[1]:= Graphics[Parallelogram[]]` and output: `Out[1]=`, followed by a black parallelogram graphic.

There's a summary at the top, then lots of examples of how the **Parallelogram** function can be used, as well as a section to open to get more details.

Run the first example from the **Parallelogram** function page:

```
In[1]:= Graphics[Parallelogram[]]
```

Out[1]=



Whenever you start typing the name of a function, you'll see an autocomplete menu.

Click the to see the function page for a particular function.

Select from the autocomplete menu to pick a function:

The screenshot shows an autocomplete menu for the prefix 'Parall'. The first item, 'Parallelogram', is highlighted and has a blue border. To its right is an info icon (). Below 'Parallelogram' are other suggestions: 'Parallelize', 'Parallelepiped', 'ParallelMap', and 'ParallelTable'. At the bottom of the menu is a double arrow icon.

When the name is complete you'll see:

Parallelogram



Press the to see:

Parallelogram



`Parallelogram[p, {v1, v2}]`

represents a parallelogram with origin p and directions v_1 and v_2 .

All the functions in the Wolfram Language follow the same principles—so for example the `Parallelogram` function works very much like the `RegularPolygon` function that we already discussed.

Q&A

How do I get to the Documentation Center?

It depends where you're running the Wolfram Language. If you're using the web or mobile, click the icon. On the desktop, go to the Help menu.

How do I try examples in the documentation?

Often you can just run them right inside the documentation. Alternatively, copy them to your working notebook and run them there.

How long does it take to learn the whole Wolfram Language?

As in learning a human language, it takes some time to become fluent, and to know the principles of the language. One can go on learning specific vocabulary almost indefinitely.

How much of the Wolfram Language does one need to know to use it in practice?

One needs to know its principles, as they are covered, for example, in this book. As with a human language, one can have a comparatively small working vocabulary, not least because there are usually many ways to express a given thing. It's easy to expand vocabulary in any particular case just by looking at documentation.

How can I read Wolfram Language code if I'm not fluent in English?

It'll help if you switch on *code captions*—which give short descriptions for every function name that appears, and are available for many languages.

Tech Note

- There's computable data about the structure of the Wolfram Language right in the Wolfram Language, accessible from `WolframLanguageData`.

More to Explore

The Wolfram Language Home Page (wolfr.am/eiwl-15-more)

Wolfram Language Documentation Center (wolfr.am/eiwl-15-more2)

16 | Real-World Data

A central feature of the Wolfram Language is that it's got immense amounts of real-world data built in. It's got data on countries and animals and movies, and lots more. It gets all this from the Wolfram Knowledgebase, which is being updated all the time—and is what powers Wolfram|Alpha and services based on it.

But how can you talk about a country in the Wolfram Language? The easiest way is just to use plain English. You can tell the Wolfram Language you're going to be giving it plain English by pressing **Ctrl**+**=** (hold down the Control key and press the **=** key), or on a touch device, by pressing the  button.



Enter the plain English “united states”:

 **united states**

As soon as you press **Return** (or click away), the Wolfram Language will try to interpret what you typed. Assuming it succeeds, it'll display a little yellow box that represents a *Wolfram Language entity*. In this case, it's the entity corresponding to the United States.

 **United States (country)** 

Press the check mark to confirm that's what you want:

 **United States (country)**

Now you can ask for lots of *properties* of this entity. Like you could ask for the US flag.

Ask for the flag property of the United States:

In[1]:=  **United States (country)** **["Flag"]**



The result you get is something you can go on doing computation with—like in this case image processing.

Color-negate the US flag:

In[2]:= **ColorNegate**[ **United States (country)** **["Flag"]**]



If all you want to do is to get the US flag, you can just ask for it in English.

In[3]:= `US flag`



`EntityValue` is a more flexible way to ask for the values of properties.

Use `EntityValue` to get the US flag:

In[4]:= `EntityValue[US flag, "Flag"]`



`EntityValue` also works with lists of entities.

Get flags for a list of countries:

In[5]:= `EntityValue[{US, brazil, china}, "Flag"]`



The Wolfram Language has deep knowledge about countries, as about many other things.

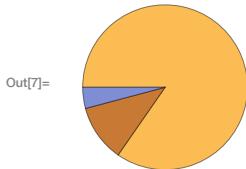
Find out how many radio stations there are in the list of countries:

In[6]:= `EntityValue[{US, brazil, china}, "RadioStations"]`

Out[6]= {13 769, 1822, 673}

Make a pie chart of the results:

In[7]:= `PieChart[EntityValue[{US, brazil, china}, "RadioStations"]]`



Find countries that border Switzerland:

```
In[8]:=  switzerland ["BorderingCountries"]
```

```
Out[8]= { Austria,  France,  Germany,  Italy,  Liechtenstein}
```

Find their flags:

```
In[9]:= EntityValue[ switzerland ["BorderingCountries"], "Flag"]
```

```
Out[9]= { ,  ,  ,  ,  }
```

Sometimes you'll want to talk about a class of entities—like, say, planets.

Ask for planets, and get the class of entities corresponding to planets:

```
In[10]:=  planets
```

```
Out[10]= 
```

Classes of entities are indicated by . You can get a list of all entities in a class using `EntityList`.

Get the list of planets:

```
In[11]:= EntityList[ planets]
```

```
Out[11]= { Mercury,  Venus,  Earth,  Mars,  Jupiter,  Saturn,  Uranus,  Neptune}
```

Get images of all of the planets:

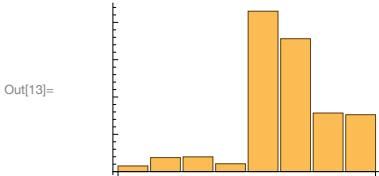
```
In[12]:= EntityValue[ planets, "Image"]
```

```
Out[12]= {, , , , , , , }
```

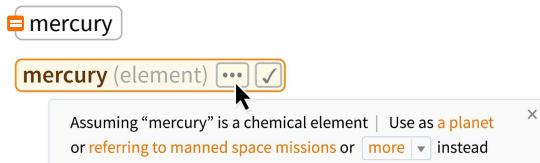
`EntityValue` can actually handle entity classes directly, so you don't need to use `EntityList` with it.

Get the radius of each of the planets, and make a bar chart of them:

```
In[13]:= BarChart[EntityValue[planets, "Radius"]]
```



It's very convenient to use plain English to describe things. But a downside is that it can be ambiguous. If you say “mercury”, do you mean the planet mercury or the chemical element mercury or something else called “mercury”? When you use `Ctrl + =`, it'll always make an initial choice. But if you press the `...` you can change to another choice. Press the check mark to accept a choice.



To see how the Wolfram Language internally represents entities you can use `InputForm`.

Show the internal form of the entity that represents the United States:

```
In[14]:= InputForm[USA]
```

```
Out[14]= Entity["Country", "UnitedStates"]
```

Show the internal form for New York City:

```
In[15]:= InputForm[nyc]
```

```
Out[15]= Entity["City", {"NewYork", "NewYork", "UnitedStates"}]
```

There are millions of entities in the Wolfram Language, each with a definite internal form. In principle you could enter any entity using its internal form. But unless you're using the same entity over and over again, it's much more practical just to use `Ctrl + =` and enter a name for the entity in plain English.

There are thousands of different types of entities in the Wolfram Language, covering all sorts of areas of knowledge. To find out about them, check out the Wolfram Language documentation, or the Wolfram | Alpha examples pages. Each type of entity then has a list of properties—often hundreds of them. One way to find this list is to use `EntityProperties`.

Possible properties for amusement parks:

In[16]:= `EntityProperties["AmusementPark"]`

Out[16]= `{ "administrative division", "type", "area", "city", "closing date",
"country", "image", "latitude", "longitude", "name", "number of rides",
"opening date", "owner", "coordinates", "slogan", "rides", "status" }`

In practice, though, a good approach is to ask in plain English for a property of some entity, then to look at the interpretation that's found, and re-use the property from it.

Ask for the height of the Eiffel Tower:

`entity[eiffel tower, "Height"]`

In[17]:= `eiffel tower ["Height"]`

Out[17]= `1062.99 ft`

Re-use the "Height" property, applied to the Great Pyramid:

In[18]:= `entity[pyramid of giza, "Height"]`

Out[18]= `456.037 ft`

Different types of entities have different properties. One common property for many types of entities is "Image".

Get images of various entities:

In[19]:= `entity[koala, "Image"]`

Out[19]=



In[20]:= `entity[eiffel tower, "Image"]`

Out[20]=



In[21]:=  starry night ["Image"]



Out[21]=

In[22]:=  stephen wolfram ["Image"]

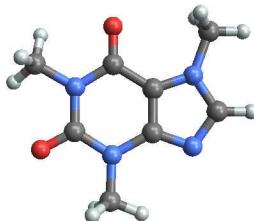


Out[22]=

Other types of objects have other properties.

A plot of a caffeine molecule:

In[23]:=  caffeine ["MoleculePlot"]



Out[23]=

Rotatable 3D graphics of a skull:

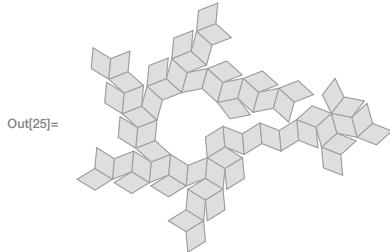
In[24]:=  skull ["Graphics3D"]



Out[24]=

A net that folds up into our 3D company logo:

In[25]:=  rhombic hexecontahedron ["NetImage"]



Vocabulary

<code>Ctrl + =</code>	plain English input
<code>EntityList[class]</code>	entities in a class
<code>EntityValue[entities, property]</code>	value of a property of an entity
<code>EntityProperties[type]</code>	list of properties for an entity type
<code>InputForm[entity]</code>	internal Wolfram Language representation of an entity

Exercises

- 16.1 Find the flag of Switzerland.
- 16.2 Get an image of an elephant.
- 16.3 Use the "Mass" property to generate a list of the masses of the planets.
- 16.4 Make a bar chart of the masses of the planets.
- 16.5 Make an image collage of images of the planets.
- 16.6 Edge detect the flag of China.
- 16.7 Find the height of the Empire State Building.
- 16.8 Compute the height of the Empire State Building divided by the height of the Great Pyramid.
- 16.9 Compute the elevation of Mount Everest divided by the height of the Empire State Building.
- 16.10 Find the dominant colors in the painting *The Starry Night*.
- 16.11 Find the dominant colors in an image collage of the flag images of all countries in Europe.
- 16.12 Make a pie chart of the GDP of countries in Europe.
- 16.13 Add an image of a koala to an image of the Australian flag.

Q&A**Where does the Wolfram Language get its real-world data?**

It's all from the central Wolfram Knowledgebase. We've been building this knowledgebase for many years, carefully curating data from thousands of primary sources.

Is the data in the Wolfram Language regularly updated?

Yes. We put a lot of effort into keeping it all up to date. And in fact there's new data flowing in every second—about market prices, weather, earthquakes, aircraft positions and lots more.

How accurate is the data in the Wolfram Language?

We go to a lot of trouble to make it as accurate as possible, and we check it extensively. But ultimately we often have to rely on what governments and other outside organizations report.

What is the relation to Wolfram|Alpha?

Wolfram|Alpha uses the same knowledgebase as the Wolfram Language.

How should I refer to a particular entity?

However you want to. The Wolfram Language is set up to understand all common ways to refer to entities. ("New York City", "NYC", "the big apple", etc., all work.)

How can I find all properties and values for a given entity?

Use `entity["PropertyAssociation"]`.

What does it mean if EntityValue gives Missing[...]?

It means the value you've asked for isn't known, or at least isn't in the Wolfram Knowledgebase. Use `DeleteMissing` to delete `Missing[...]` elements in a list.

Tech Notes

- The Wolfram Knowledgebase is stored in the cloud, so even if you're using a desktop version of the Wolfram Language, you'll need to connect to the network to start getting real-world data.
- The Wolfram Knowledgebase contains many trillions of specific facts and values, stored in a Wolfram Language symbolic framework, with a variety of underlying database technologies.
- The Wolfram Knowledgebase has been systematically built and curated from large numbers of primary sources of data. It doesn't come from web searching.
- Real-world data often involves *units*, which we'll discuss in the next section.
- Instead of using natural language, you can access the Wolfram Knowledgebase through specific functions like `CountryData` and `MovieData`. Sometimes this may be faster.
- If you want to find the original source of a particular piece of data, you can look at documentation (e.g. for `CountryData`, etc.), or you can ask for the data in Wolfram|Alpha and follow source links.
- Sometimes you want to talk about a special instance of an entity, like a country in a particular year, or a certain amount of a substance. You can do this using `EntityInstance`.
- `RandomEntity` finds random entities of a given type.

More to Explore

Major Areas Covered by the Wolfram Knowledgebase (wolfr.am/eiwl-16-more)

Geographic Data and Computation (wolfr.am/eiwl-16-more2)

Scientific and Medical Data and Computation (wolfr.am/eiwl-16-more3)

Engineering Data and Computation (wolfr.am/eiwl-16-more4)

Social, Cultural and Linguistic Data (wolfr.am/eiwl-16-more5)

17 | Units

As soon as you're dealing with real-world quantities, it's inevitable that you'll run into *units*. In the Wolfram Language, you can enter quantities with units using **Ctrl + =**.

Enter a quantity of time in units of hours:

 2.6 hours

 2.6 h

Press the check mark to accept the interpretation:

In[1]:= 2.6 h

You can use **InputForm** to see how the Wolfram Language internally represents this.

Show the internal form of a quantity:

In[2]:= **InputForm**[ 2.6 hours]

Out[2]= Quantity[2.6, "Hours"]

You can always enter quantities directly like this, or you can use **Ctrl + =**, either for the whole thing, or just for the units.

The Wolfram Language knows about all the 10,000 or so common kinds of units. **UnitConvert** converts between them.

Convert from hours to minutes:

In[3]:= **UnitConvert**[2.6 h , "Minutes"]

Out[3]= 156. min

You can do arithmetic with quantities even if they have different units.

Add a length in feet to one in centimeters:

In[4]:=  +  7.5 ft + 14 cm

Out[4]= 242.6 cm

Divide one length by another:

In[5]:=  /  7.5 ft / 14 cm

Out[5]= 16.3286

You can compute with money too.

Use dollars in a computation:

In[6]:= $7.5 * \$3 + 2.51 * \8

Out[6]= \$42.58

Multiply a price per pound by a weight in kilograms:

In[7]:= $\$15/\text{lb} * 5.6 \text{ kg}$

Out[7]= \$185.19

You can convert between currencies. The Wolfram Language always knows the latest conversion rate.

In[8]:= **CurrencyConvert**[100 euros, US dollars]

Out[8]= \$112.10

There are many places where units show up. Another example is in angles—and this is such a common case that the Wolfram Language lets you handle it in a special way. You can enter an angle like 30 degrees either as 30 Degree, or as 30° , where the $^\circ$ can be typed as Esc deg Esc.

Display a string rotated by 30 degrees:

In[9]:= **Rotate**["hello", 30 °]

Out[9]= hello

If you leave off the Degree or $^\circ$, the Wolfram Language will assume you're talking about radians, which go from 0 to 2π (about 6.28) around a circle, rather than degrees, which go from 0 to 360.

$\pi/2$ radians is equivalent to 90° :

In[10]:= **Rotate**["hello", Pi/2]

Out[10]= hello

Make a list of rotations in degrees from 0 to 360:

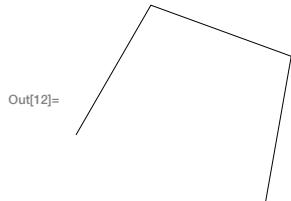
In[11]:= **Table**[**Rotate**[n, n Degree], {n, 0, 360, 30}]

Out[11]= {0, 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, 360}

There's lots to do with angles. For example, `AnglePath` gives the path you'd follow if you successively turned by a sequence of angles.

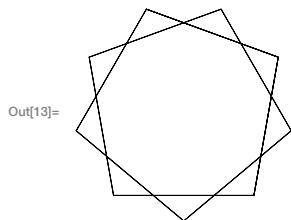
Start off horizontal, then turn three times by 80° :

```
In[12]:= Graphics[Line[AnglePath[{0 °, 80 °, 80 °, 80 °}]]]
```



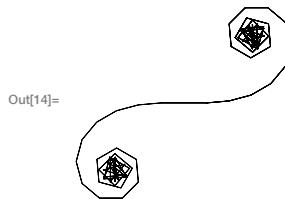
Keep turning by 80° and you'll eventually get back to where you started:

```
In[13]:= Graphics[Line[AnglePath[Table[80 °, 20]]]]
```



If you keep increasing the angle, you get an interesting pattern:

```
In[14]:= Graphics[Line[AnglePath[Table[n*5 °, {n, 200}]]]]
```



Vocabulary

<code>UnitConvert[quantity, unit]</code>	convert between units
<code>CurrencyConvert[amount, unit]</code>	convert between currencies
<code>30 Degree</code>	angle in degrees
<code>30 °</code>	angle in degrees entered with <code>Esc deg Esc</code>
<code>Rotate[expr, angle]</code>	rotate on the screen
<code>AnglePath[{angle1, angle2, ...}]</code>	path derived from a sequence of turns

Exercises

- 17.1** Convert 4.5 lbs (pounds) to kilograms.
- 17.2** Convert 60.25 mph to kilometers per hour.
- 17.3** Find the height of the Eiffel Tower in miles.
- 17.4** Find the height of Mount Everest divided by the height of the Eiffel Tower.
- 17.5** Find the mass of the Earth divided by the mass of the Moon.
- 17.6** Convert 2500 Japanese yen to US dollars.
- 17.7** Find the total of 35 ounces, 1/4 ton, 45 lbs and 9 stone in kilograms.
- 17.8** Get a list of the distance to each planet using the "DistanceFromEarth" property, and convert all the results to light minutes.
- 17.9** Rotate the string "hello" by 180°.
- 17.10** Make a table of a size-100 "A" rotated by 0° through 360° in steps of 30°.
- 17.11** Make a Manipulate to rotate an image of a cat between 0° and 180°.
- 17.12** Generate graphics for a path obtained by turning 0°, 1°, 2°, ..., 180°.
- 17.13** Make graphics of the path obtained by turning a constant angle 100 times, controlling the angle from 0° to 360° with a Manipulate.
- 17.14** Make graphics of the path obtained by successively turning by the digits of 2^{10000} multiplied by 30°.

Q&A

What unit abbreviations does the Wolfram Language understand?

Pretty much any common abbreviation, whether it's miles/hr or mph or mi/h, etc. (If you're wondering if an abbreviation will work, just try it.)

Does the Wolfram Language pick units based on what country I'm in?

Yes. For example, it'll tend to use inches if you're in the US, and centimeters if you're in continental Europe.

Do I have to be connected to the network to use units?

Only to interpret input like 5 kg. If you type `Quantity[5, "Kilograms"]` you don't need the network—except to deal with units like currencies whose values are always changing.

What can I do if my unit conversion gives me an exact fraction but I want a decimal number?

Use the function `N[...]` to find a decimal approximation. Or add a decimal point to a number in your input. We'll talk more about this in Section 23.

Why don't I get the same result for the currency conversion example?

Because, without doubt, the conversion rates have changed.

Tech Notes

- Degree isn't a function; like Red, Green, etc. it's a *symbol*. We'll talk more about these later.
 - AnglePath implements "turtle graphics" familiar from languages like Logo and Scratch.
-

More to Explore

Guide to Units in the Wolfram Language (wolfr.am/eiwl-17-more)

18 | Geocomputation

The Wolfram Language has extensive built-in knowledge of geography. For example, it knows where New York City is, and it can compute how far it is from there to Los Angeles.

Compute the distance between the centers of New York and Los Angeles:

```
In[1]:= GeoDistance[new york, los angeles]
```

```
Out[1]= 2432.07 mi
```

You can also plot locations on a map, using `GeoListPlot`.

Plot New York and Los Angeles on a map:

```
In[2]:= GeoListPlot[{new york, los angeles}]
```



Plot countries on a map:

```
In[3]:= GeoListPlot[{iceland, france, italy}]
```



You can do things at a much smaller scale too.

Plot two famous locations in Paris:

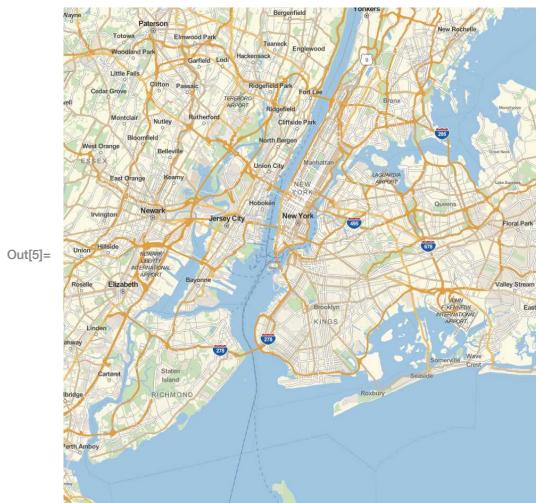
```
In[4]:= GeoListPlot[{Eiffel Tower, Louvre}]
```



`GeoListPlot` is the analog for geography of `ListPlot`. `GeoGraphics` is the analog of `Graphics`.

Generate a map of New York City:

```
In[5]:= GeoGraphics[New York City]
```



GeoPath represents a path on the surface of the Earth.

Show the shortest path from New York to Tokyo:

```
In[6]:= GeoGraphics[GeoPath[{New York, Tokyo}]]
```

Out[6]=



Styling works just like in other graphics:

```
In[7]:= GeoGraphics[Style[GeoPath[{New York, Tokyo}], Thick, Red]]
```

Out[7]=



GeoDisk is the analog of Disk; you specify its center and its radius.

Show a 1-mile-radius disk around the Eiffel Tower:

```
In[8]:= GeoGraphics[GeoDisk[eiffel tower], 1 mile]]
```

Out[8]=



Generate a table of maps, with disks whose sizes go up as powers of 10:

```
In[9]:= Table[GeoGraphics[GeoDisk[eiffel tower, 1 mile * 10^n]], {n, 0, 4}]
```



`GeoPosition` gives a position on the Earth. The numbers it contains are longitude and latitude—the standard coordinates on the surface of the Earth.

Find the geo position of the Eiffel Tower:

```
In[10]:= GeoPosition[eiffel tower]
```

```
Out[10]= GeoPosition[{48.8583, 2.29444}]
```

Draw a 4000-mile-radius disk around 0 latitude, 0 longitude:

```
In[11]:= GeoGraphics[GeoDisk[GeoPosition[{0, 0}], 4000 miles]]
```



Notice that the disk isn't quite circular. That's because we have to use a *projection* to draw the surface of the Earth on a flat map. It's one of the many subtleties in doing geocomputation.

`GeoNearest` finds what's nearest to a given place on the Earth. You tell it what type of thing to look for, and how many you want it to find.

Find the nearest 5 countries to 0 longitude, 0 latitude:

```
In[12]:= GeoNearest["Country", GeoPosition[{0, 0}], 5]
```

```
Out[12]= {Ghana, Ivory Coast, Togo, Benin, São Tomé and Príncipe}
```

Find the nearest cities instead:

```
In[13]:= GeoNearest["City", GeoPosition[{0, 0}], 5]
```

```
Out[13]= {Takoradi, Sekondi, Cape Coast, Elmina, Shama}
```

It's often nice to know your own geo position. So long as your computer, phone, etc. knows, Here will give you that.

Find where your computer (or phone, etc.) thinks it is:

```
In[14]:= Here
```

```
Out[14]= GeoPosition[{40.11, -88.24}]
```

You can do computations with Here.

Compute how far it is to the Eiffel Tower:

```
In[15]:= GeoDistance[Here, eiffel tower ]
```

```
Out[15]= 4245.54 mi
```

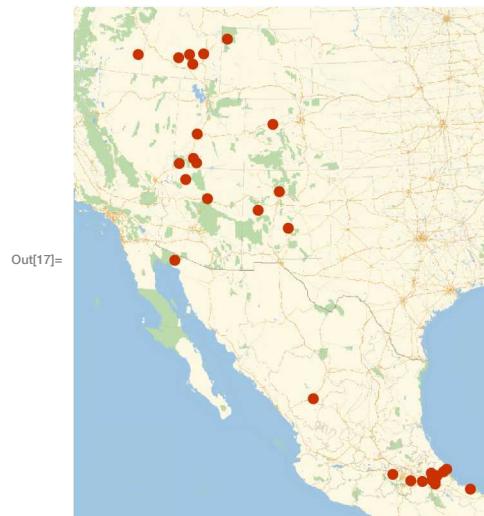
Find the nearest 5 volcanoes to my position:

```
In[16]:= GeoNearest["Volcano", Here, 5]
```

```
Out[16]= {Dotsero, Valles Caldera, Carrizozo, Zuni-Bandera, Yellowstone}
```

Plot volcanoes on a map:

```
In[17]:= GeoListPlot[GeoNearest["Volcano", Here, 30]]
```



Vocabulary

<code>GeoDistance[entity₁, entity₂]</code>	geo distance between entities
<code>GeoListPlot[{entity₁, entity₂, ...}]</code>	plot a list of entities on a map
<code>GeoGraphics[...]</code>	map constructed from primitives
<code>GeoPath[{entity₁, entity₂}]</code>	path between entities
<code>GeoDisk[entity, r]</code>	disk with radius r around an entity
<code>Here</code>	where your computer, phone, etc. thinks it is
<code>GeoPosition[entity]</code>	geo position of an entity
<code>GeoNearest["type", location, n]</code>	nearest n objects of a certain type to a location

Exercises

- 18.1** Find the distance from New York to London.
- 18.2** Divide the distance from New York to London by the distance from New York to San Francisco.
- 18.3** Find the distance from Sydney to Moscow in kilometers.
- 18.4** Generate a map of the United States.
- 18.5** Plot on a map Brazil, Russia, India and China.
- 18.6** Plot on a map the path from New York City to Beijing.
- 18.7** Plot a disk centered on the Great Pyramid, with radius 10 miles.
- 18.8** Plot a disk centered on New York with a radius large enough to just reach San Francisco.
- 18.9** Find the nearest 5 countries to the North Pole (`GeoPosition["NorthPole"]`).
- 18.10** Find the flags of the 3 countries nearest to latitude 45°, longitude 0°.
- 18.11** Plot the 25 volcanoes closest to Rome.
- 18.12** Find the difference in latitude between New York and Los Angeles.

Q&A

Can I get different projections for maps?

Yes. Just use the `GeoProjection` option. There are more than 300 built-in projections to choose from. The default projection used in any particular case depends on the scale and location of the map.

How detailed are the maps in the Wolfram Language?

They go down to the level of individual streets. Most streets in the world are included.

How does the Wolfram Language find my geo position?

It uses the function `FindGeoLocation`. On a mobile device, this will normally ask for your GPS position. On a computer, it'll normally try to deduce your location from your internet address—this won't always work correctly. You can always explicitly set your geo location by assigning a value to `$GeoLocation`.

How can I specify the range of a map?

Use the option `GeoRange → distance` (e.g. `GeoRange → 5 miles`) or `GeoRange → place` (e.g. `GeoRange → Europe`), as discussed in Section 20.

Can the Wolfram Language compute driving directions?

Yes. Use `TravelDirections`. `GeoDistance` gives the direct shortest path; `TravelDistance` gives the path following roads, etc. `TravelTime` gives estimated travel time.

Is the Wolfram Language restricted to maps of the Earth?

No. For example, the moon and Mars also work. Use the option `GeoModel → "Moon"`, etc.

Tech Notes

- You need to be connected to the network to use maps in the Wolfram Language.
- Instead of entering `nyc`, `LA` you can just enter `nyc, LA` and get the same result.
- If you give `GeoDistance` extended regions (like countries), it'll compute the shortest distance between any points in the regions.
- `GeoPosition` uses numerical values of longitude and latitude, not 35 Degree etc.
- `GeoPosition` normally uses decimal degrees. Use `FromDMS` to convert from degrees-minutes-seconds.
- To color regions on a map by values, use `GeoRegionValuePlot`.

More to Explore

[Guide to Maps & Cartography in the Wolfram Language \(wolfr.am/eiwl-18-more\)](http://wolfr.am/eiwl-18-more)

[Guide to Geographic Data & Entities in the Wolfram Language \(wolfr.am/eiwl-18-more2\)](http://wolfr.am/eiwl-18-more2)

19 | Dates and Times

In the Wolfram Language, Now gives your current date and time.

Get the current date and time (as of when I wrote this!):

In[1]:= Now

Out[1]=  Wed 23 Sep 2015 20:23:58 GMT-4.

You can do computations on this, for example adding a week.

Add a week to the current date and time:

In[2]:= Now + 1 week

Out[2]=  Wed 30 Sep 2015 20:24:04 GMT-4.

Use **Ctrl** + = to enter a date in any standard format.

Enter a date:

In[3]:= June 23, 1988

Out[3]=  Thu 23 Jun 1988

InputForm shows that internally a date is a **DateObject**[...]. You can always enter dates directly in this form.

Find the internal representation of a date:

In[4]:= InputForm[June 23, 1988]

Out[4]= DateObject[{1988, 6, 23}]

You can do arithmetic with dates, say, subtracting them to find how far apart they are.

Subtract two dates:

In[5]:= Now - June 23, 1988

Out[5]= 9953.85 days

Convert the date difference to years:

In[6]:= UnitConvert[Now - June 23, 1988, "Years"]

Out[6]= 27.2708 yr

DayRange is the analog of **Range** for dates:

Give a list of the days spanning from yesterday to tomorrow:

In[7]:= **DayRange[Yesterday, Tomorrow]**

Out[7]= Tue 22 Sep 2015 , Wed 23 Sep 2015 , Thu 24 Sep 2015 }

DayName finds the day of the week for a particular date.

Compute the day of the week 45 days from now:

In[8]:= **DayName[Today + 45 days]**

Out[8]= Saturday

Once you know a date, there are lots of things you can compute. For example, **MoonPhase** gives the phase of the moon (or, more accurately, the fraction of the Moon that is illuminated when seen from the Earth).

Compute the phase of the moon now:

In[9]:= **MoonPhase[Now]**

Out[9]= 0.7719

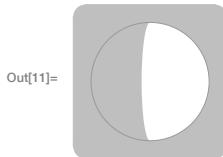
Compute the phase of the moon on a certain date:

In[10]:= **MoonPhase[june 23, 1988]**

Out[10]= 0.5716

Generate an icon for the phase of the moon:

In[11]:= **MoonPhase[june 23, 1988, "Icon"]**



If you know both the date and a location on Earth, you can work out when the sun will rise and set.

Compute when sunset will be today at my current location:

In[12]:= **Sunset[Here, Today]**

Out[12]= Wed 23 Sep 2015 19:50 GMT-4.

Successive sunrises are not exactly 1 day (24 hours) apart:

In[13]:= **Sunrise[Here, Tomorrow] – Sunrise[Here, Today]**

Out[13]= 1.00069 days

Time zones are one of many subtleties. LocalTime gives the time in the time zone of a particular location.

Find the local time now in New York City:

In[14]:= **LocalTime[New York]**

Out[14]=  **Wed 23 Sep 2015 20:25:43 GMT–4.**

Find the local time now in London:

In[15]:= **LocalTime[London]**

Out[15]=  **Thu 24 Sep 2015 01:25:46 GMT+1.**

Among the many areas where the Wolfram Language has extensive data is weather. The function AirTemperatureData uses this data to give the historical air temperature at a particular time and place.

Find the air temperature here at 6 pm yesterday:

In[16]:= **AirTemperatureData[Here, 6 pm yesterday]**

Out[16]= 80.6 °F

If you provide a pair of dates, AirTemperatureData computes a *time series* of estimated temperatures between those dates.

Give a time series of air temperature measurements from a week ago until now:

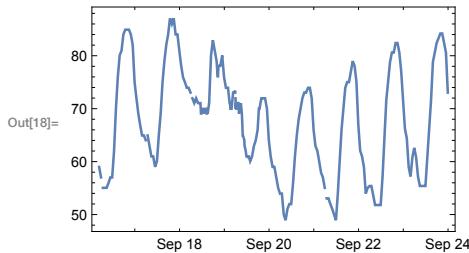
In[17]:= **AirTemperatureData[Here, {1 week ago , Now}]**

Out[17]= **TimeSeries[ Time: 16 Sep 2015 to 23 Sep 2015 Data points: 237]**

`DateListPlot` is the analog of `ListPlot` for time series, where each value occurs at a particular date.

Plot the list of air temperature measurements:

```
In[18]:= DateListPlot[AirTemperatureData[Here, {1 week ago}, Now]]
```



The plot shows that, not surprisingly, the temperature is higher during the day than at night.

Vocabulary

<code>Now</code>	current date and time
<code>Today</code>	date object for today
<code>Tomorrow</code>	date object for tomorrow
<code>Yesterday</code>	date object for yesterday
<code>DayRange[date₁, date₂]</code>	list of dates from <i>date₁</i> to <i>date₂</i>
<code>DayName[date]</code>	day of the week of <i>date</i>
<code>MoonPhase[date]</code>	moon phase on <i>date</i>
<code>Sunrise[date, location]</code>	time of sunrise on <i>date</i> at <i>location</i>
<code>Sunset[date, location]</code>	time of sunset on <i>date</i> at <i>location</i>
<code>LocalTime[location]</code>	current time at <i>location</i>
<code>AirTemperatureData[location, time]</code>	air temperature at <i>time</i> at <i>location</i>
<code>AirTemperatureData[location, {time₁, time₂}]</code>	time series of air temperatures from <i>time₁</i> to <i>time₂</i> at <i>location</i>
<code>DateListPlot[timeseries]</code>	plot a time series

Exercises

- 19.1** Compute how many days have elapsed since January 1, 1900.
- 19.2** Compute what day of the week January 1, 2000 was.
- 19.3** Find the date a hundred thousand days ago.
- 19.4** Find the local time in Delhi.
- 19.5** Find the length of daylight today by subtracting today's sunrise from today's sunset.
- 19.6** Generate an icon for the current phase of the moon.
- 19.7** Make a list of the numerical phase of the moon for each of the next 10 days.
- 19.8** Generate a list of icons for the moon phases from today until 10 days from now.
- 19.9** Compute the time today between sunrise in New York City and in London.
- 19.10** Find the air temperature at the Eiffel Tower at noon yesterday.
- 19.11** Plot the temperature at the Eiffel Tower over the past week.
- 19.12** Find the difference in air temperatures between Los Angeles and New York City now.

Q&A

How can I get a date as a string?

Use `DateString[date]`. There are many options for the format of the string. For example, `DateString[date, "DateShort"]` uses short day and month names.

How can I extract the month or some other element from a date?

Use `DatePicker`. DateValue[date, "Month"]` gives the month number, `DatePicker[date, "MonthName"]` gives the month name, etc.

How far in the past can dates be in the Wolfram Language?

As far as you want. The Wolfram Language knows about historical calendar systems, and the history of time zones. It also has the data to accurately compute sunrise, etc. going back at least 1000 years.

Where does the Wolfram Language get air temperature data from?

The worldwide network of weather stations, located at airports and other places. If you've got your own air temperature measuring device, you can connect it to the Wolfram Language through the Wolfram Data Drop (see Section 43).

What is a time series?

It's a way of specifying the values of something at a series of times. You can enter a time series in the Wolfram Language as `TimeSeries[{{time1, value1}, {time2, value2}, ...}]`. The Wolfram Language lets you do arithmetic and many other operations with time series.

What does DateListPlot do?

It plots values against times or dates. The values can be given in a `TimeSeries[...]` or in a list of the form `{{time1, value1}, {time2, value2}, ...}`.

Tech Notes

- The Wolfram Language decides whether to interpret a date like 8/10/15 as month/day/year or day/month/year based on what country you're in. You can pick the other interpretation if you want.
- **Monday**, etc. are *symbols* with intrinsic meaning, not strings.
- There are many subtleties in handling dates. An example is deciding when a particular day starts, accounting for time zones, etc. `DateObject` has options to address such issues.

More to Explore

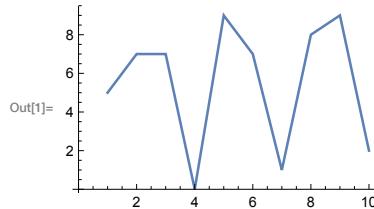
[Guide to Dates & Times in the Wolfram Language](#) (wolfr.am/eiwl-19-more)

20 | Options

Many functions in the Wolfram Language have *options* that determine the details of how they work. For example, in making a plot, you can use `PlotTheme → "Web"` to use a web-oriented visual theme. On a keyboard, the → is automatically formed if you type → (i.e. – followed by >).

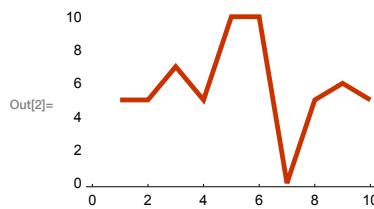
A standard plot, with no options given:

```
In[1]:= ListLinePlot[RandomInteger[10, 10]]
```



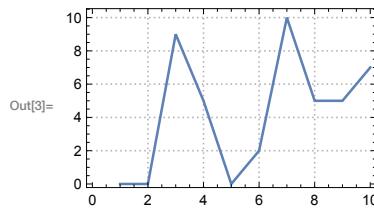
A plot with the `PlotTheme` option given as "Web":

```
In[2]:= ListLinePlot[RandomInteger[10, 10], PlotTheme → "Web"]
```



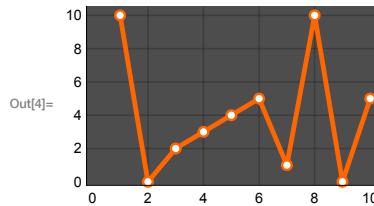
A plot with the `PlotTheme` option given as "Detailed":

```
In[3]:= ListLinePlot[RandomInteger[10, 10], PlotTheme → "Detailed"]
```



A plot with the `PlotTheme` option given as "Marketing":

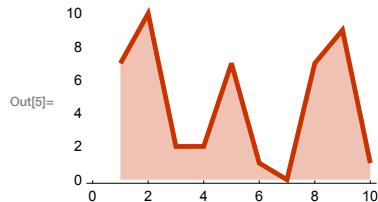
```
In[4]:= ListLinePlot[RandomInteger[10, 10], PlotTheme → "Marketing"]
```



You can add more options. For example, `Filling` specifies what filling to add to a plot.

Fill the plot to the axis:

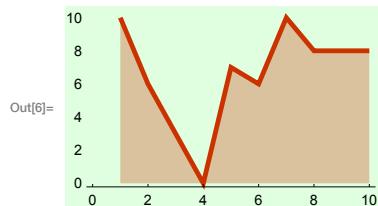
```
In[5]:= ListLinePlot[RandomInteger[10, 10], PlotTheme -> "Web", Filling -> Axis]
```



Background lets you specify a background color.

Also include an option for background color:

```
In[6]:= ListLinePlot[RandomInteger[10, 10],
  PlotTheme -> "Web", Filling -> Axis, Background -> LightGreen]
```

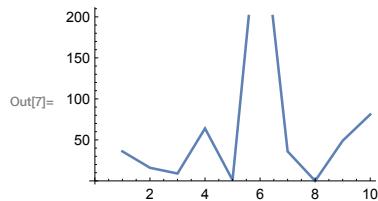


If you don't mention a particular option, the Wolfram Language will use a pre-defined default for that option. Most often that default is `Automatic`, which means that the language will automatically determine what to do.

One option that's often useful for graphics is `PlotRange`, which specifies what range of values to include in a plot. With the default `PlotRange -> Automatic`, the system will try to automatically show the “interesting” part of the plot. `PlotRange -> All` shows all values.

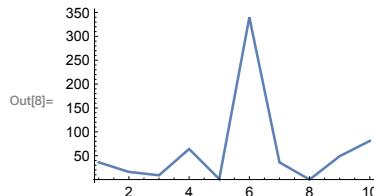
With default options all but one “outlier” value are displayed:

```
In[7]:= ListLinePlot[{36, 16, 9, 64, 1, 340, 36, 0, 49, 81}]
```



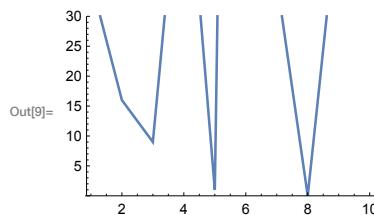
`PlotRange → All` says to include all points:

```
In[8]:= ListLinePlot[{36, 16, 9, 64, 1, 340, 36, 0, 49, 81}, PlotRange → All]
```



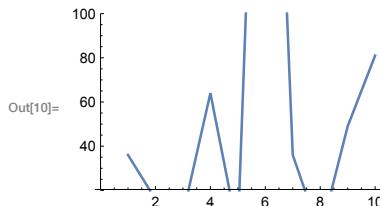
`PlotRange → 30` specifies to show values up to 30:

```
In[9]:= ListLinePlot[{36, 16, 9, 64, 1, 340, 36, 0, 49, 81}, PlotRange → 30]
```



`PlotRange → {20, 100}` specifies to show values between 20 and 100:

```
In[10]:= ListLinePlot[{36, 16, 9, 64, 1, 340, 36, 0, 49, 81}, PlotRange → {20, 100}]
```



You can specify ranges for all types of graphics. In `GeoListPlot` and `GeoGraphics` you can use the option `GeoRange` to specify what part of the world to include in a plot.

By default, a geo plot of France pretty much includes only France:

```
In[11]:= GeoListPlot[france]
```



This requests a range that includes all of Europe:

```
In[12]:= GeoListPlot[france, GeoRange -> europe]
```



Out[12]=

GeoRange → All specifies to use the whole world:

```
In[13]:= GeoListPlot[france, GeoRange -> All]
```



Out[13]=

There are many other options for `GeoListPlot`. For example `GeoBackground` specifies what kind of background should be used. `GeoLabels` adds labels.

Use a relief map as the background:

```
In[14]:= GeoListPlot[france, GeoRange -> europe, GeoBackground -> "ReliefMap"]
```



Out[14]=

Automatically add labels for geo objects:

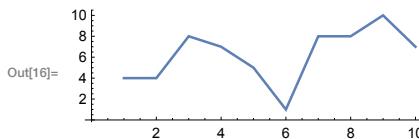
```
In[15]:= GeoListPlot[{paris, new york, sydney}, GeoLabels → Automatic]
```



The function `ListLinePlot` has 57 different options you can set; `GeoListPlot` has 54. Some options are common to all graphics functions. For example, `AspectRatio` determines the overall shape of graphics, specifying the ratio of height to width.

With an aspect ratio of 1/3, the plot is 3 times wider than it is tall:

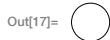
```
In[16]:= ListLinePlot[RandomInteger[10, 10], AspectRatio → 1/3]
```



The option `ImageSize` specifies the overall size of graphics.

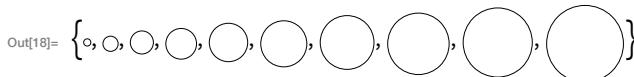
Draw a circle with a “tiny” overall image size:

```
In[17]:= Graphics[Circle[], ImageSize → Tiny]
```



Draw circles with specific image sizes between 5 and 50 pixels:

```
In[18]:= Table[Graphics[Circle[], ImageSize → n], {n, 5, 50, 5}]
```



It’s not just `Graphics` that allows options. Lots of other functions do too. An example is `Style`, which supports many options.

Set an option to use Chalkboard font to style text:

```
In[19]:= Style["text in a different font", 20, FontFamily → "Chalkboard"]
```

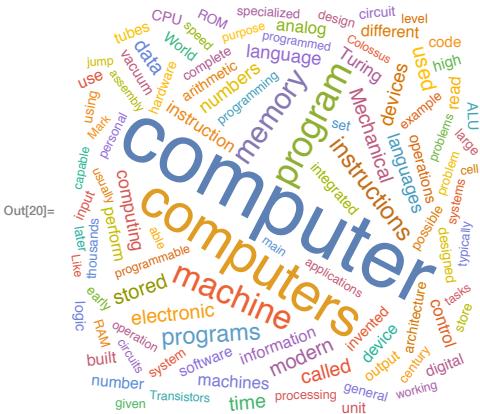
Out[19]=

text in a different font

It's quite common for options to describe details of output, for example in WordCloud.

Create a word cloud with random word orientations:

```
In[20]:= WordCloud[DeleteStopwords[WikipediaData["computer"]],  
WordOrientation → "Random"]
```



Grid has many options. The Frame option controls whether and how a frame is drawn.

Create a multiplication table with a frame around each entry:

```
In[21]:= Grid[Table[i*j, {i, 5}, {j, 5}], Frame → All]
```

Out[21]=

1	2	3	4	5
2	4	6	8	10
3	6	9	12	15
4	8	12	16	20
5	10	15	20	25

Like Graphics, Grid has a Background option:

```
In[22]:= Grid[Table[i*j, {i, 5}, {j, 5}], Frame → All, Background → LightYellow]
```

Out[22]=

1	2	3	4	5
2	4	6	8	10
3	6	9	12	15
4	8	12	16	20
5	10	15	20	25

Vocabulary

PlotTheme	theme for a plot (e.g. “Web”, “Detailed”, etc.)
Filling	filling to add to a plot (Axis, Bottom, etc.)
PlotRange	range of values to include in a plot (All, etc.)
GeoRange	geo range to include (All, specific country, etc.)
GeoBackground	background map (“ReliefMap”, “OutlineMap”, etc.)
GeoLabels	labels to add to a map (e.g. Automatic)
Background	background color
AspectRatio	ratio of height to width
ImageSize	size in pixels
Frame	whether to include a frame (True, All, etc.)
FontFamily	family of font to use (e.g. “Helvetica”)
WordOrientation	how to orient words in a word cloud

Exercises

- 20.1** Create a list plot of Range[10] themed for the web.
- 20.2** Create a list plot of Range[10] with filling to the axis.
- 20.3** Create a list plot of Range[10] with a yellow background.
- 20.4** Create a map of the world with Australia highlighted.
- 20.5** Create a map of the Indian Ocean with Madagascar highlighted.
- 20.6** Use GeoGraphics to create a map of South America showing topography (relief map):
- 20.7** Make a map of Europe with France, Finland and Greece highlighted and labeled.
- 20.8** Make a 12×12 multiplication table as a grid with white type on a black background.
- 20.9** Make a list of 100 disks with random integer image sizes up 40.
- 20.10** Make a list of pictures of regular pentagons with image size 30 and aspect ratios from 1 to 10.
- 20.11** Make a Manipulate that varies the size of a circle between 5 and 500.
- 20.12** Create a framed 10×10 grid of random colors.
- 20.13** Make a line plot of the lengths of Roman numerals up to 100, with a plot range that would be sufficient for all numerals up to 1000.

Q&A**How can I get a list of the options for a function?**

Look at the documentation. Or use for example `Options[WordCloud]`. Also, whenever you start typing the name of an option, you'll see a menu of possible completions.

How do I find out the possible settings for an option?

Look at the documentation for that option. Also, when you type →, you'll typically get a menu of possible common settings.

What is $opt \rightarrow value$ internally?

It's `Rule[opt, value]`. Rules are used in lots of places in the Wolfram Language. $a \rightarrow b$ is usually read aloud as “ a goes to b ” or “ a arrow b ”.

When are values of options given as strings?

Only a small set of standard option settings (such as `Automatic`, `None` and `All`) are not strings. Specialized settings for particular options are normally strings.

Can one reset the default for an option?

Yes, using `SetOptions`. Though you have to be careful not to forget that you've done it.

Tech Notes

- Many options can be set to be pure functions (see Section 26). It's important to put parentheses in the correct place, as in `ColorFunction → (Hue[#/4] &)`, or you won't get the correct meaning.
- `$FontFamilies` gives a list of possible setting for `FontFamily`.

More to Explore

[Guide to Graphics Options in the Wolfram Language](#) (wolfr.am/eiwl-20-more)

[Guide to Formatting Options in the Wolfram Language](#) (wolfr.am/eiwl-20-more2)

21 | Graphs and Networks

A *graph* is a way of showing connections between things—say, how web pages are linked, or how people form a social network.

Let's start with a very simple graph, in which 1 connects to 2, 2 to 3, and 3 to 4. Each of the connections is represented by \rightarrow (typed as \rightarrow).

A very simple graph of connections:

```
In[1]:= Graph[{1 → 2, 2 → 3, 3 → 4}]
```

```
Out[1]=
```

Automatically label all the “vertices”:

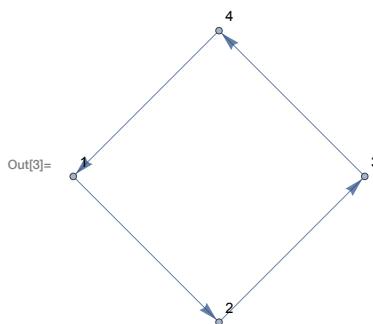
```
In[2]:= Graph[{1 → 2, 2 → 3, 3 → 4}, VertexLabels → All]
```

```
Out[2]=
```

Let's add one more connection: to connect 4 to 1. Now we have a loop.

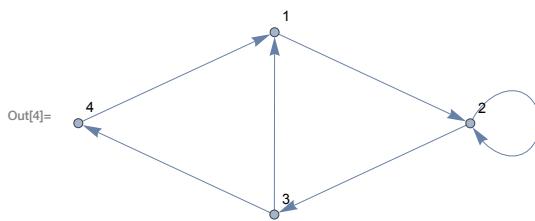
Add another connection, forming a loop:

```
In[3]:= Graph[{1 → 2, 2 → 3, 3 → 4, 4 → 1}, VertexLabels → All]
```



Add two more connections, including one connecting 2 right back to 2:

```
In[4]:= Graph[{1 → 2, 2 → 3, 3 → 4, 4 → 1, 3 → 1, 2 → 2}, VertexLabels → All]
```

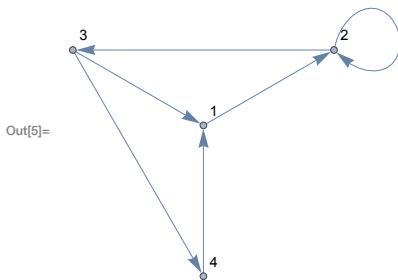


As we add connections, the Wolfram Language chooses to place the vertices or nodes of the graph differently. All that really matters for the meaning, however, is how the vertices are connected. And if you don't specify otherwise, the Wolfram Language will try to lay the graph out so it's as untangled and easy to understand as possible.

There are options, though, to specify other layouts. Here's an example. It's the same graph as before, with the same connections, but the vertices are laid out differently.

A different layout of the same graph (check by tracing the connections):

```
In[5]:= Graph[{1 → 2, 2 → 3, 3 → 4, 4 → 1, 3 → 1, 2 → 2},
  VertexLabels → All, GraphLayout → "RadialDrawing"]
```



You can do computations on the graph, say finding the shortest path that gets from 4 to 2, always following the arrows.

The shortest path from 4 to 2 on the graph goes through 1:

```
In[6]:= FindShortestPath[Graph[{1 → 2, 2 → 3, 3 → 4, 4 → 1, 3 → 1, 2 → 2}], 4, 2]
Out[6]= {4, 1, 2}
```

Now let's make another graph. This time let's have 3 nodes, and let's have a connection between every one of them.

Start by making an array of all possible connections between 3 objects:

```
In[7]:= Table[i → j, {i, 3}, {j, 3}]
Out[7]= {{1 → 1, 1 → 2, 1 → 3}, {2 → 1, 2 → 2, 2 → 3}, {3 → 1, 3 → 2, 3 → 3}}
```

The result here is a list of lists. But what `Graph` needs is just a single list of connections. We can get that by using `Flatten` to “flatten” out the sublists.

`Flatten` “flattens out” all sublists, wherever they appear:

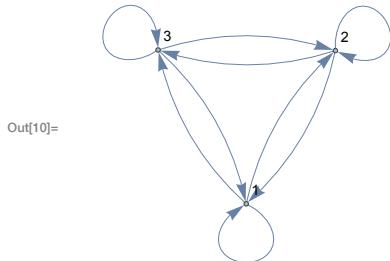
```
In[8]:= Flatten[{{a, b}, 1, 2, 3, {x, y, {z}}}]
Out[8]= {a, b, 1, 2, 3, x, y, z}
```

Get a “flattened” list of connections from the array:

```
In[9]:= Flatten[Table[i → j, {i, 3}, {j, 3}]]
Out[9]= {1 → 1, 1 → 2, 1 → 3, 2 → 1, 2 → 2, 2 → 3, 3 → 1, 3 → 2, 3 → 3}
```

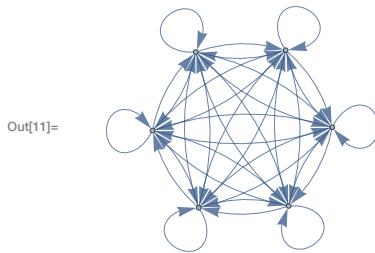
Show the graph of these connections:

```
In[10]:= Graph[Flatten[Table[i → j, {i, 3}, {j, 3}]], VertexLabels → All]
```



Generate the completely connected graph with 6 nodes:

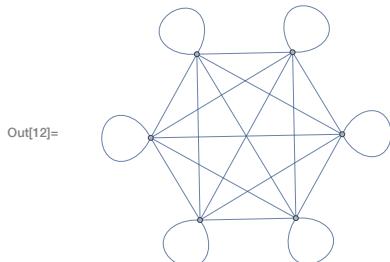
```
In[11]:= Graph[Flatten[Table[i → j, {i, 6}, {j, 6}]]]
```



Sometimes the “direction” of a connection doesn’t matter, so we can drop the arrows.

The “undirected” version of the graph:

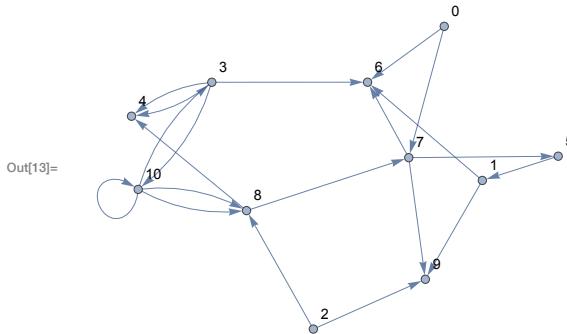
```
In[12]:= UndirectedGraph[Flatten[Table[i → j, {i, 6}, {j, 6}]]]
```



Now let's make a graph with random connections. Here is an example with 20 connections between randomly chosen nodes.

Make a graph with 20 connections between randomly chosen nodes numbered from 0 to 10:

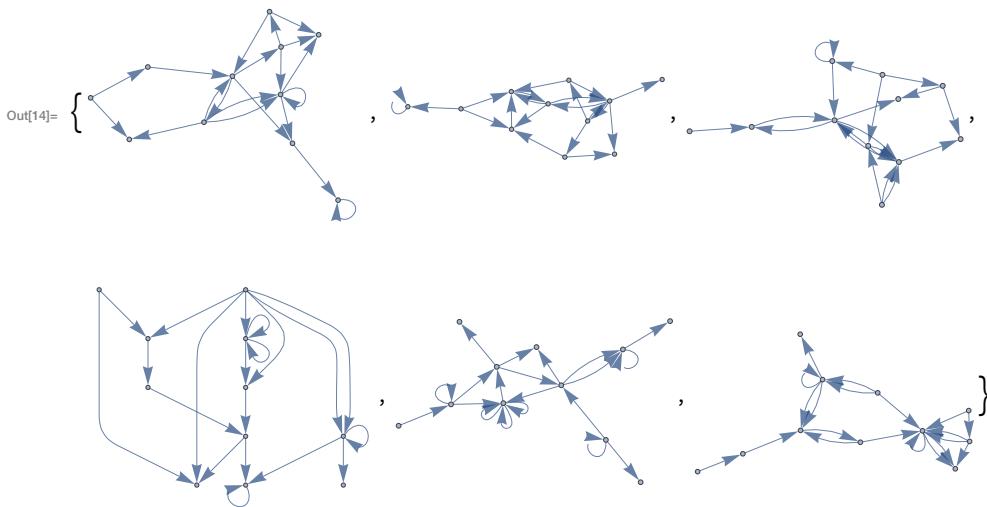
```
In[13]:= Graph[Table[RandomInteger[10] → RandomInteger[10], 20], VertexLabels → All]
```



You'll get a different graph if you generate different random numbers. Here are 6 graphs.

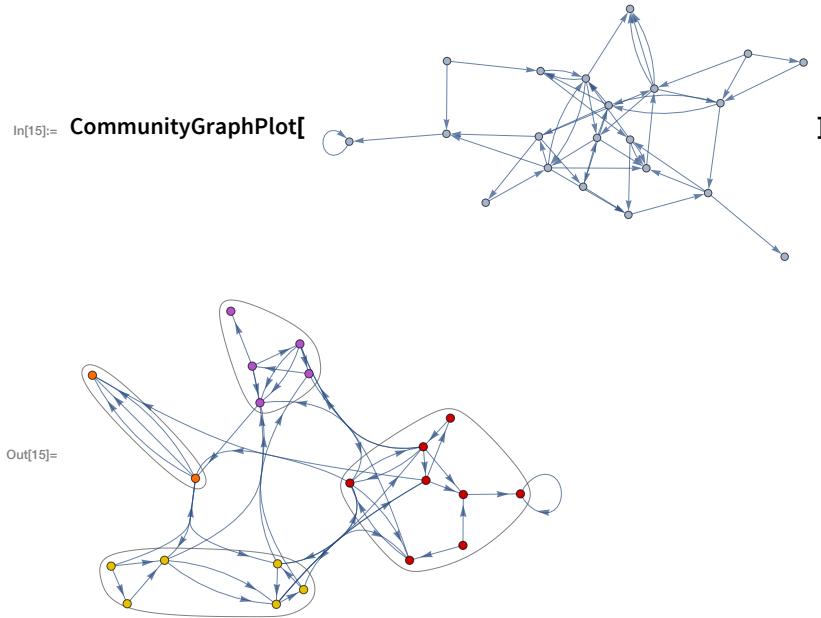
Six randomly generated graphs:

```
In[14]:= Table[Graph[Table[RandomInteger[10] → RandomInteger[10], 20]], 6]
```



There's lots of analysis that can be done on graphs. One example is to break a graph into "communities"—clumps of nodes that are more connected to each other than to the rest of the graph. Let's do that for a random graph.

Make a plot that collects "communities" of nodes together:



The result is a graph with the exact same connections as the original, but with the nodes arranged to illustrate the "community structure" of the graph.

Vocabulary

Graph [$\{i \rightarrow j, \dots\}$]	a graph or network of connections
UndirectedGraph [$\{i \rightarrow j, \dots\}$]	a graph with no directions to connections
VertexLabels	an option for what vertex labels to include (e.g. All)
FindShortestPath [<i>graph</i> , <i>a</i> , <i>b</i>]	find the shortest path from one node to another
CommunityGraphPlot [<i>list</i>]	display a graph arranged into "communities"
Flatten [<i>list</i>]	flatten out sublists in a list

Exercises

- 21.1** Make a graph consisting of a loop of 3 nodes.
- 21.2** Make a graph with 4 nodes in which every node is connected.
- 21.3** Make a table of undirected graphs with between 2 and 10 nodes in which every node is connected.
- 21.4** Use `Table` and `Flatten` to get `{1, 2, 1, 2, 1, 2}`.
- 21.5** Make a line plot of the result of concatenating all digits of all integers from 1 to 100 (i.e. ..., 8, 9, 1, 0, 1, 1, 1, 2, ...).
- 21.6** Make a graph with 50 nodes, in which node i connects to node $i+1$.
- 21.7** Make a graph with 4 nodes, in which each connection connects i to `Max[i, j]`.
- 21.8** Make a graph in which each connection connects i to $j-i$, where i and j both range from 1 to 5.
- 21.9** Generate a graph with 100 nodes, each connecting to one randomly chosen node.
- 21.10** Generate a graph with 100 nodes, each connecting to two randomly chosen nodes.
- 21.11** For the graph $\{1 \rightarrow 2, 2 \rightarrow 3, 3 \rightarrow 4, 4 \rightarrow 1, 3 \rightarrow 1, 2 \rightarrow 2\}$, make a grid giving the shortest paths between every pair of nodes, with the start node as row and end node as column.

Q&A

What's the difference between a "graph" and a "network"?

There's no difference. They're just different words for the same thing, though "graph" tends to be more common in math and other formal areas, and "network" more common in more applied areas.

What are the vertices and edges of a graph?

Vertices are the points, or nodes, of a graph. Edges are the connections. Because graphs have arisen in so many different places, there are quite a few different names used for the same thing.

How is $i \rightarrow j$ understood?

It's `Rule[i, j]`. Rules are used in lots of places in the Wolfram Language—such as giving settings for options.

Can I compute my Facebook friend graph?

Yes. Use `SocialMediaData["Facebook", "FriendNetwork"]`. Note, though, that only your friends who've opted in through Facebook will be included. (See Section 44.)

How big a graph can the Wolfram Language handle?

It's mostly limited by the amount of memory in your computer. Graphs with tens or hundreds of thousands of nodes are not a problem.

Can I specify properties of nodes and edges?

Yes. You can give `Graph` lists of nodes and edges that include things like `Property[node, VertexStyle → Red]` or `Property[edge, EdgeWeight → 20]`. You can also give overall options to `Graph`.

Tech Notes

- Graphs, like strings, images, graphics etc., are first-class objects in the Wolfram Language.
- You can enter undirected edges in a graph using `<->`, which displays as \longleftrightarrow .
- `CompleteGraph[n]` gives the completely connected graph with n nodes. Among other kinds of special graphs are `KaryTree`, `ButterflyGraph`, `HypercubeGraph`, etc.
- There are lots of ways to make random graphs (random connections, random numbers of connections, scale-free networks, etc.). `RandomGraph[{100, 200}]` makes a random graph with 100 nodes and 200 edges.
- `AdjacencyMatrix[graph]` gives the adjacency matrix for a graph. `AdjacencyGraph[matrix]` constructs a graph from an adjacency matrix.

More to Explore

Guide to Graphs and Networks in the Wolfram Language (wolfr.am/eiwl-21-more)

22 | Machine Learning

So far in this book, when we've wanted the Wolfram Language to do something, we've written code to tell it exactly what to do. But the Wolfram Language is also set up to be able to learn what to do just by looking at examples, using the idea of *machine learning*.

We'll talk about how to train the language yourself. But first let's look at some built-in functions that have already been trained on huge numbers of examples.

`LanguageIdentify` takes pieces of text, and identifies what human language they're in.

Identify the language each phrase is in:

```
In[1]:= LanguageIdentify[{ "thank you", "merci", "dar las gracias", "感謝", "благодарить"}]
Out[1]= { English, French, Spanish, Chinese, Russian }
```

The Wolfram Language can also do the considerably more difficult “artificial intelligence” task of identifying what an image is of.

Identify what an image is of:

```
In[2]:= ImageIdentify[

]
Out[2]= cheetah
```

There's a general function `Classify`, which has been taught various kinds of classification. One example is classifying the “sentiment” of text.

Upbeat text is classified as having positive sentiment:

```
In[3]:= Classify["Sentiment", "I'm so excited to be programming"]
Out[3]= Positive
```

Downbeat text is classified as having negative sentiment:

```
In[4]:= Classify["Sentiment", "math can be really hard"]
Out[4]= Negative
```

You can also train `Classify` yourself. Here's a simple example of classifying handwritten digits as 0 or 1. You give `Classify` a collection of training examples, followed by a particular handwritten digit. Then it'll tell you whether the digit you give is a 0 or 1.

With training examples, `Classify` correctly identifies a handwritten 0:

```
In[5]:= Classify[{0 → 0, 1 → 1, 0 → 0, 1 → 1, 1 → 1, 0 → 0, 0 → 0, 1 → 1, 1 → 1,
0 → 0, 0 → 0, 0 → 0, 1 → 1, 0 → 0, 1 → 1, 0 → 0, 1 → 1, 1 → 1, 1 → 1}, 0]
```

```
Out[5]= 0
```

To get some sense of how this works—and because it's useful in its own right—let's talk about the function `Nearest`, that finds what element in a list is nearest to what you supply.

Find what number in the list is nearest to 22:

```
In[6]:= Nearest[{10, 20, 30, 40, 50, 60, 70, 80}, 22]
```

```
Out[6]= {20}
```

Find the nearest three numbers:

```
In[7]:= Nearest[{10, 20, 30, 40, 50, 60, 70, 80}, 22, 3]
```

```
Out[7]= {20, 30, 10}
```

`Nearest` can find nearest colors as well.

Find the 3 colors in the list that are nearest to the color you give:

```
In[8]:= Nearest[{Red, Orange, Yellow, LightGreen, Green, DarkGreen, Cyan, LightBlue, Blue, DarkBlue, Purple, Magenta}, Yellow, 3]
```

```
Out[8]= {Yellow, LightGreen, Green}
```

It also works on words.

Find the 10 words nearest to "good" in the list of words:

```
In[9]:= Nearest[WordList[], "good", 10]
```

```
Out[9]= {"good", "food", "goad", "god", "gold", "goo", "goody", "goof", "goon", "goop"}
```

There's a notion of nearness for images too. And though it's far from the whole story, this is effectively part of what `ImageIdentify` is using.

Something that's again related is recognizing text (*optical character recognition* or OCR). Let's make a piece of text that's blurred.

Create an image of the word "hello", then blur it:

```
In[10]:= Blur[Rasterize[Style["hello", 30]], 3]
```

```
Out[10]= 
```

TextRecognize can recognize the original text string in this.

Recognize text in the image:

```
In[11]:= TextRecognize[
```

```
Out[11]= hello
```

One more example involves taking a collection of things—say colors—and finding clusters of similar ones. This can be achieved using FindClusters.

Collect "clusters" of similar colors into separate lists:

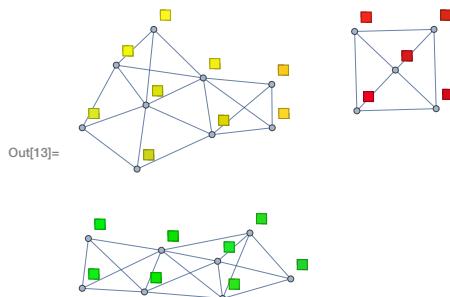
```
In[12]:= FindClusters[{, , , , , , , , , , , , , , , , , , , }]
```

```
Out[12]= {{, , , , , , , , , , , , , , , , , , , }, {, , , , , , , , , }, {, }]
```

You can get a different view by connecting each color to the three most similar colors in the list, then making a graph out of the connections. In the particular example here, there end up being three disconnected subgraphs.

Create a graph of connections based on nearness in "color space":

```
In[13]:= NearestNeighborGraph[{, , , , , , , , , , , , , , , , , , , , , , , , , , , , , }, 3, VertexLabels → All]
```



Vocabulary

<code>LanguageIdentify[<i>text</i>]</code>	identify what human language text is in
<code>ImageIdentify[<i>image</i>]</code>	identify what an image is of
<code>TextRecognize[<i>text</i>]</code>	recognize text from an image (OCR)
<code>Classify[<i>training</i>, <i>data</i>]</code>	classify data on the basis of training examples
<code>Nearest[<i>list</i>, <i>item</i>]</code>	find what element of <i>list</i> is nearest to <i>item</i>
<code>FindClusters[<i>list</i>]</code>	find clusters of similar items
<code>NearestNeighborGraph[<i>list</i>, <i>n</i>]</code>	connect elements of <i>list</i> to their <i>n</i> nearest neighbors

Exercises

- 22.1** Identify what language the word “ajatella” comes from.
- 22.2** Use `Ctrl + =` to get an image of a tiger, then apply `ImageIdentify`.
- 22.3** Make a table of image identifications for an image of a tiger, blurred by an amount from 1 to 5.
- 22.4** Classify the sentiment of “I’m so happy to be here”.
- 22.5** Find the 10 words in `WordList[]` that are nearest to “happy”.
- 22.6** Generate 20 random numbers up to 1000 and find which 3 are nearest to 100.
- 22.7** Generate a list of 10 random colors, and find which 5 are closest to Red.
- 22.8** Of the first 100 squares, find the one nearest to 2000.
- 22.9** Find the 3 European flags nearest to the flag of Brazil.
- 22.10** Make a graph of the 2 nearest neighbors of each color in `Table[Hue[h], {h, 0, 1, .05}]`.
- 22.11** Generate a list of 100 random numbers from 0 to 100, and make a graph of the 2 nearest neighbors of each one.
- 22.12** Collect the flags of Asia into clusters of similar flags.
- 22.13** Make raster images of the letters of the alphabet at size 20, then make a graph of the 2 nearest neighbors of each one.
- 22.14** Generate a table of the results of using `TextRecognize` on "hello" rasterized at size 50 and then blurred by between 1 and 10.

Q&A

How do Wolfram Language functions for machine learning work?

They use a range of state-of-the-art methods, and have meta-algorithms for picking between methods based on the particular problems they're given.

Doesn't `ImageIdentify` just need a dictionary?

No. Specific words can occur in several languages. One needs to learn from examples how to deduce the language from combinations of words.

Can `ImageIdentify` get the wrong answer?

Yet. It can make mistakes, much like a human. This happens particularly if it's asked about an object that's in an unusual configuration or environment.

Can I ask `ImageIdentify` the probabilities it assigns to different identifications?

Yes. To find the probabilities for the top 10 identifications in all categories use `ImageIdentify[image, All, 10, "Probability"]`.

How many examples does `Classify` typically need to work well?

If the general area (like everyday images) is one it already knows well, then as few as a hundred. But in areas that are new, it can take many millions of examples to achieve good results.

How does `Nearest` figure out a distance between colors?

It uses the function `ColorDistance`, which is based on a model of human color vision.

How does `Nearest` determine nearby words?

By looking at those at the smallest `EditDistance`, that is, reached by the smallest number of single-letter insertions, deletions and substitutions.

Can a single graph have several disconnected parts?

Absolutely. An example is the last graph in this section.

Tech Notes

- `BarcodeImage` and `BarcodeRecognize` work with bar codes and QR codes instead of pure text.
- `ImageIdentify` is the core of what the imageidentify.com website does.
- `ImageIdentify` uses methods that are inspired by idealized models of neural networks in the brain.
- If you just give `Classify` training examples, it'll produce a `ClassifierFunction` that can later be applied to many different pieces of data. This is pretty much always how `Classify` is used in practice.
- `Classify` automatically picks between methods such as *logistic regression*, *naive Bayes*, *random forests*, *support vector machines* and *neural networks*.
- `FindClusters` does *unsupervised machine learning*, where the computer just looks at data without being told anything about it. `Classify` does *supervised machine learning*, being given a set of training examples.

More to Explore

Guide to Machine Learning in the Wolfram Language (wolfr.am/eiwl-22-more)

23 | More about Numbers

When you do a computation with whole numbers, the Wolfram Language gives you an exact answer. It does the same with exact fractions.

Adding $1/2+1/3$ gives an exact answer as a fraction:

```
In[1]:= 1/2+1/3
```

```
Out[1]= 5/6
```

Often you'll just want a numerical or decimal approximation. You can get that using the function `N` (for “numerical”).

Get an approximate numerical answer:

```
In[2]:= N[1/2+1/3]
```

```
Out[2]= 0.833333
```

If there's any decimal number in your input, the Wolfram Language will automatically give you an approximate answer.

The presence of a decimal number makes the result be approximate:

```
In[3]:= 1.8/2+1/3
```

```
Out[3]= 1.23333
```

It's enough just to have a decimal point at the end of a number:

```
In[4]:= 1/2.+1/3
```

```
Out[4]= 0.833333
```

The Wolfram Language can handle numbers of any size, at least so long as they fit in your computer's memory.

Here's 2 raised to the power 1000:

```
In[5]:= 2^1000
```

```
Out[5]= 10715086071862673209484250490600018105614048117055336074437503883703510`.  
511249361224931983788156958581275946729175531468251871452856923140435`.  
984577574698574803934567774824230985421074605062371141877954182153046`.  
474983581941267398767559165543946077062914571196477686542167660429831`.  
652624386837205668069376
```

Get a numerical approximation:

```
In[6]:= N[2^1000]
```

```
Out[6]= 1.07151×10301
```

This approximate form is given in *scientific notation*. If you need to enter scientific notation, you can do it with `*^`.

Enter a number in scientific notation:

```
In[7]:= 2.7*^6
```

```
Out[7]= 2.7 × 106
```

Commonly used numbers like π (pi) are built into the Wolfram Language.

Get a numerical approximation to π :

```
In[8]:= N[Pi]
```

```
Out[8]= 3.14159
```

The Wolfram Language can compute to *arbitrary precision*, so for example it can find millions of digits of π if you want them.

Compute 250 digits of π :

```
In[9]:= N[Pi, 250]
```

```
Out[9]= 3.141592653589793238462643383279502884197169399375105820974944592307816406286`.  
208998628034825342117067982148086513282306647093844609550582231725359408128`.  
481117450284102701938521105559644622948954930381964428810975665933446128475`.  
648233786783165271201909
```

There are many functions in the Wolfram Language that handle *integers* (whole numbers). There are also many functions that handle *real numbers*—approximate numbers with decimals. An example is `RandomReal`, which gives random real numbers.

Generate a random real number in the range 0 to 10:

```
In[10]:= RandomReal[10]
```

```
Out[10]= 2.08658
```

Generate 5 random real numbers:

```
In[11]:= Table[RandomReal[10], 5]
```

```
Out[11]= {4.15071, 4.81048, 8.82945, 9.84995, 9.08313}
```

An alternative way to ask for 5 random real numbers:

```
In[12]:= RandomReal[10, 5]
```

```
Out[12]= {6.47318, 3.29181, 3.57615, 8.11204, 3.38286}
```

Random real numbers in the range 20 to 30:

```
In[13]:= RandomReal[{20, 30}, 5]
Out[13]= {24.1202, 20.1288, 20.393, 25.6455, 20.9268}
```

The Wolfram Language has a huge range of mathematical functions built in, from basic to very sophisticated.

Find the 100th prime number:

```
In[14]:= Prime[100]
Out[14]= 541
```

Find the millionth prime number:

```
In[15]:= Prime[1000000]
Out[15]= 15485863
```

Plot the first 50 primes:

```
In[16]:= ListPlot[Table[Prime[n], {n, 50}]]
```

Out[16]=

n	Prime[n]
1	2
2	3
3	5
4	7
5	11
6	13
7	17
8	19
9	23
10	29
11	31
12	37
13	41
14	43
15	47
16	53
17	59
18	61
19	67
20	71
21	73
22	79
23	83
24	89
25	97
26	101
27	103
28	107
29	109
30	113
31	127
32	131
33	137
34	139
35	149
36	151
37	157
38	163
39	173
40	179
41	181
42	191
43	193
44	197
45	199
46	211
47	223
48	227
49	229
50	233

Three functions common in many practical situations are `Sqrt` (square root), `Log10` (logarithm to base 10) and `Log` (natural logarithm).

The square root of 16 is 4:

```
In[17]:= Sqrt[16]
Out[17]= 4
```

If you don't ask for a numerical approximation, you'll get an exact formula:

```
In[18]:= Sqrt[200]
Out[18]= 10 Sqrt[2]
```

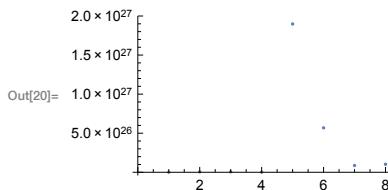
`N` gives a numerical approximation:

```
In[19]:= N[Sqrt[200]]
Out[19]= 14.1421
```

Logarithms are often useful when you're dealing with numbers that have a wide range of sizes. Let's plot the masses of the planets. With `ListPlot` one can't tell anything about the planets before Jupiter. But `ListLogPlot` shows the relative sizes much more clearly.

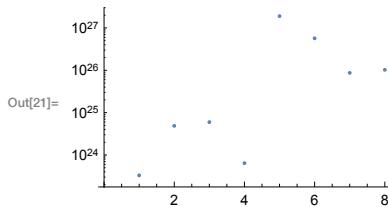
Make an ordinary `ListPlot` of the masses of the planets:

```
In[20]:= ListPlot[planets {"Mass"}]
```



Make a log plot:

```
In[21]:= ListLogPlot[planets {"Mass"}]
```



There are a few more functions that show up frequently in general programming. First, there's the almost trivial function `Abs`, that finds the absolute value, or positive part, of a number.

`Abs` effectively just drops minus signs:

```
In[22]:= {Abs[3], Abs[-3]}
```

```
Out[22]= {3, 3}
```

Next there's `Round`, which rounds to the nearest whole number.

`Round` rounds to the nearest whole number:

```
In[23]:= {Round[3.2], Round[3.4], Round[3.6], Round[3.9]}
```

```
Out[23]= {3, 3, 4, 4}
```

Another function that's very useful is `Mod`. Let's say you're counting up minutes in an hour. When you reach 60, you'll want to start again from 0. That's what `Mod` lets you do.

Compute a sequence of numbers mod 60:

```
In[24]:= {Mod[50, 60], Mod[55, 60], Mod[60, 60], Mod[65, 60], Mod[70, 60]}
```

```
Out[24]= {50, 55, 0, 5, 10}
```

Vocabulary

<code>N[expr]</code>	numerical approximation
<code>Pi</code>	the number π ($\pi \approx 3.14$)
<code>Sqrt[x]</code>	square root
<code>Log10[x]</code>	logarithm to base 10
<code>Log[x]</code>	natural logarithm (\ln)
<code>Abs[x]</code>	absolute value (drop minus signs)
<code>Round[x]</code>	round to nearest integer
<code>Prime[n]</code>	n^{th} prime number
<code>Mod[x, n]</code>	modulo ("clock arithmetic")
<code>RandomReal[max]</code>	random real number between 0 and \max
<code>RandomReal[max, n]</code>	list of n random real numbers
<code>ListLogPlot[data]</code>	plot on a logarithmic scale

Exercises

- 23.1 Find $\sqrt{2}$ to 500-digit precision.
- 23.2 Generate 10 random real numbers between 0 and 1.
- 23.3 Make a plot of 200 points with random real x and y coordinates between 0 and 1.
- 23.4 Create a random walk using `AnglePath` and 1000 random real numbers between 0 and 2π .
- 23.5 Make a table of `Mod[n^2, 10]` for n from 0 to 30.
- 23.6 Make a line plot of `Mod[n^n, 10]` for n from 1 to 100.
- 23.7 Make a table of the first 10 powers of π , rounded to integers.
- 23.8 Make a graph by connecting n with `Mod[n^2, 100]` for n from 0 to 99.
- 23.9 Generate graphics of 50 circles with random real coordinates 0 to 10, random real radii from 0 to 2, and random colors.

- 23.10** Make a plot of the n^{th} prime divided by $n \cdot \log(n)$, for n from 2 to 1000.
- 23.11** Make a line plot of the differences between successive primes up to 100.
- 23.12** Generate a sequence of 20 middle C notes with random durations between 0 and 0.5 seconds.
- 23.13** Make an array plot of $\text{Mod}[i, j]$ for i and j up to 50.
- 23.14** Make a list for n from 2 to 10 of array plots for x and y up to 50 of $x^y \bmod n$.

Q&A

What are examples of mathematical functions in the Wolfram Language?

From standard school math, ones like Sin, Cos, ArcTan, Exp, as well as GCD, Factorial, Fibonacci. From physics and engineering and higher math, ones like Gamma (“gamma function”), BesselJ (“Bessel function”), EllipticK (“elliptic integral”), Zeta (“Riemann zeta function”), PrimePi, EulerPhi. From statistics, ones like Erf, NormalDistribution, ChiSquareDistribution. Hundreds of functions altogether.

What is the precision of a number?

It’s the total number of decimal digits quoted in the number. $N[100/3, 5]$ gives 33.333, which has 5 digits of precision. The number $100/3$ is exact; $N[100/3, 5]$ approximates it to 5-digit precision.

What does the ` at the end of each line in a long number mean?

It’s there to show that the number continues onto the next line—like a hyphen in text.

Can I work with numbers in bases other than 10?

Yes. Enter a number in base 16 as 16^{ffa5} . Find digits using IntegerDigits[655, 16].

Can the Wolfram Language handle complex numbers?

Of course. The symbol I (capital “i”) represents the square root of -1 .

Why does $N[1.5/7, 100]$ not give me a 100-digit result?

Because 1.5 is an approximate number with much less than 100-digit precision. $N[15/70, 100]$ will for example give a 100-digit-precision number.

Tech Notes

- The Wolfram Language does “arbitrary-precision computation”, meaning that it can keep as many digits in a number as you want.
- When you generate a number with a certain precision using N, the Wolfram Language will automatically keep track of how that precision is affected by computations—so you don’t have to do your own numerical analysis of roundoff errors.
- If you type a number like 1.5, it’s assumed to be at the native “machine precision” of numbers on your computer (usually about 16 digits). Use 1.5`100 to specify 100-digit precision.
- PrimeQ tests if a number is prime (see Section 28). FactorInteger finds the factors of an integer.

- `RandomReal` can give numbers that aren't just uniformly distributed. For example, `RandomReal[NormalDistribution[]]` gives normally distributed numbers.
- `Round` rounds to the nearest integer (up or down); `Floor` always rounds down; `Ceiling` always rounds up.
- `RealDigits` is the analog of `IntegerDigits` for real numbers.

More to Explore

[Guide to Numbers in the Wolfram Language](#) (wolfr.am/eiwl-23-more)

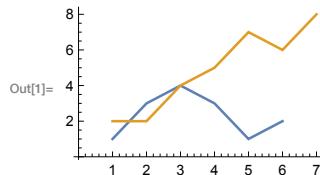
[Guide to Mathematical Functions in the Wolfram Language](#) (wolfr.am/eiwl-23-more2)

24 | More Forms of Visualization

We've seen how to plot lists of data with `ListPlot` and `ListLinePlot`. If you want to plot several sets of data at the same time, you just have to give them in a list.

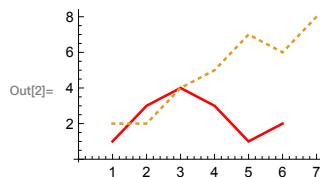
Plot two sets of data at the same time:

```
In[1]:= ListLinePlot[{{1, 3, 4, 3, 1, 2}, {2, 2, 4, 5, 7, 6, 8}}]
```



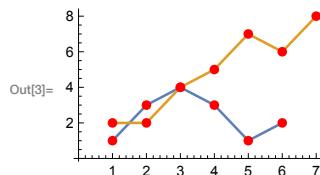
The `PlotStyle` option lets you specify the style for each set of data:

```
In[2]:= ListLinePlot[{{1, 3, 4, 3, 1, 2}, {2, 2, 4, 5, 7, 6, 8}}, PlotStyle -> {Red, Dotted}]
```



The `Mesh` option lets you show the actual data points too:

```
In[3]:= ListLinePlot[{{1, 3, 4, 3, 1, 2}, {2, 2, 4, 5, 7, 6, 8}}, Mesh -> All, MeshStyle -> Red]
```



As well as looking at the sequence of values in a list, it's also very common to want to see how often different values occur. You can do this with `Histogram`.

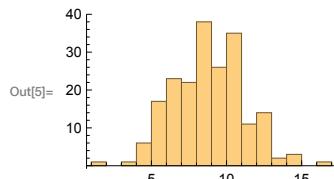
Here are the lengths of the first 30 common English words:

```
In[4]:= StringLength[Take[WordList[], 30]]
```

```
Out[4]= {1, 3, 8, 5, 6, 5, 7, 7, 9, 11, 5, 9, 5, 7, 9, 5, 9, 8, 4, 6, 5, 5, 10, 11, 12, 8, 10, 7, 9, 6}
```

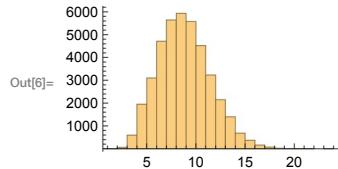
The histogram shows how often each length occurs among the first 200 words:

```
In[5]:= Histogram[StringLength[Take[WordList[], 200]]]
```



Including all the words gives a smoother result:

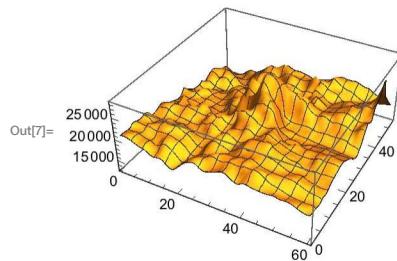
```
In[6]:= Histogram[StringLength[WordList[]]]
```



Sometimes you'll have data you want to visualize in 3D. For example, `GeoElevationData` can give an array of height values. `ListPlot3D` makes a 3D plot.

Find an array of height values around Mount Everest, and plot it in 3D:

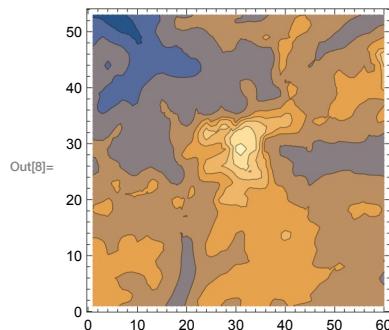
```
In[7]:= ListPlot3D[GeoElevationData[GeoDisk[ mount everest , 10 miles ]]]
```



An alternative visualization is a *contour plot*, in which one's effectively looking from above, and drawing contour lines at evenly spaced heights.

Make a contour plot, in which successive ranges of heights are separated by contour lines:

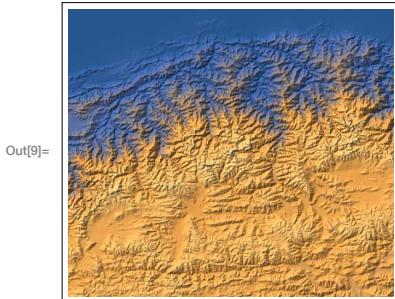
```
In[8]:= ListContourPlot[GeoElevationData[GeoDisk[ mount everest , 10 miles ]]]
```



When one's dealing with large amounts of data, it's often better to use a simpler visualization, such as a *relief plot*, that essentially just colors according to height.

Make a relief plot of the topography 100 miles around Mount Everest:

```
In[9]:= ReliefPlot[GeoElevationData[GeoDisk[{"mount everest"}, 100 miles]]]
```



Vocabulary

<code>ListLinePlot[{list₁, list₂, ...}]</code>	plot several lists together
<code>Histogram[list]</code>	make a histogram
<code>ListPlot3D[array]</code>	plot an array of heights in 3D
<code>ListContourPlot[array]</code>	plot contours for an array of heights
<code>ReliefPlot[array]</code>	make a relief plot
<code>GeoElevationData[region]</code>	array of geo elevations for a region
<code>PlotStyle</code>	option for styling each set of data
<code>Mesh</code>	whether to have a mesh of points or lines
<code>MeshStyle</code>	option for styling a mesh

Exercises

- 24.1** Make a plot with lines joining the squares, the cubes and the 4th powers of integers up to 10.
- 24.2** Make a plot of the first 20 primes, joined by a line, filled to the axis, and with a red dot at each prime.
- 24.3** Make a 3D plot of the topography for 20 miles around Mount Fuji.
- 24.4** Make a relief plot of the topography for 100 miles around Mount Fuji.
- 24.5** Make a 3D plot of heights generated from `Mod[i, j]` with `i` and `j` going up to 100.
- 24.6** Make a histogram of the differences between successive primes for the first 10000 primes.
- 24.7** Make a histogram of the first digits of squares of integers up to 10000 (illustrating *Benford's law*).

- 24.8** Make a histogram of the length of Roman numerals up to 1000.
- 24.9** Make a histogram of sentence lengths in the Wikipedia article on computers.
- 24.10** Make a list of histograms of 10000 instances of totals of n random reals up to 100, with n going from 1 to 5 (illustrating the central limit theorem).
- 24.11** Generate a 3D list plot using the image data from a binarized size-200 letter "W" as heights.

Q&A

What other kinds of visualizations are there?

Lots. Like `ListStepPlot` and `ListStreamPlot`, or `BubbleChart` and `BarChart3D`, or `SmoothHistogram` and `BoxWhiskerChart`, or `AngularGauge` and `VerticalGauge`.

How do I combine plots I've generated separately?

Use `Show` to combine them on common axes. Use `GraphicsGrid` etc. (see Section 37) to put them side by side.

How can I specify the bins to use in a histogram?

`Histogram[list, n]` uses n bins. `Histogram[list, {xmin, xmax, dx}]` uses bins from $xmin$ to $xmax$ in steps dx .

What's the difference between a bar chart and a histogram?

A bar chart is a direct representation of data; a histogram represents the frequency with which data occurs. In a bar chart, the height of each bar gives the value of a single piece of data. In a histogram, the height of each bar gives the total number of pieces of data that occur within the x range of the bar.

How can I draw contour lines on a 3D topography plot?

Use `MeshFunctions → (#3 &)`. The `(#3 &)` is a *pure function* (see Section 26) that uses the third (z) coordinate to make a mesh.

How do I tell the height of each contour line?

Use `ContourLabels → All` to label all the contour lines.

Tech Notes

- The Wolfram Language makes many automatic choices for visualization functions. You can override these choices using options.
- An alternative to using `PlotStyle` is to insert `Style` directly into the data given to functions like `ListLinePlot`.

More to Explore

Guide to Data Visualization in the Wolfram Language (wolfr.am/eiwl-24-more)

25 | Ways to Apply Functions

When you write $f[x]$ it means “apply the function f to x ”. An alternative way to write the same thing in the Wolfram Language is $f@x$.

$f@x$ is the same as $f[x]$:

```
In[1]:= f@x
```

```
Out[1]:= f[x]
```

It's often convenient to write out chains of functions using $@$:

```
In[2]:= f@g@h@x
```

```
Out[2]:= f[g[h[x]]]
```

Avoiding the brackets can make code easier to type, and read:

```
In[3]:= ColorNegate@EdgeDetect@
```



```
Out[3]=
```



There's a third way to write $f[x]$ in the Wolfram Language: as an “afterthought”, in the form $x // f$.

Apply f “as an afterthought” to x :

```
In[4]:= x // f
```

```
Out[4]:= f[x]
```

You can have a sequence of “afterthoughts”:

```
In[5]:= x // f // g // h
```

```
Out[5]:= h[g[f[x]]]
```

The functions here read in the order they are applied:

In[6]:=  // EdgeDetect // ColorNegate

Out[6]= 

A particularly common use of `//` is to apply `N` (for numerical evaluation) “as an afterthought”.

Apply numerical evaluation “as an afterthought”:

In[7]:= `2 Pi^3+1 // N`

Out[7]= 63.0126

In working with the Wolfram Language, a powerful notation that one ends up using all the time is `/@`, which means “apply to each element”.

Apply `f` to each element in a list:

In[8]:= `f/@{1, 2, 3}`

Out[8]= `{f[1], f[2], f[3]}`

`f` usually would just get applied to the whole list:

In[9]:= `f[{1, 2, 3}]`

Out[9]= `f[{1, 2, 3}]`

`Framed` is a function that displays a frame around something.

Display `x` framed:

In[10]:= `Framed[x]`

Out[10]= 

Applying `Framed` to a list just puts a frame around the whole list.

Apply `Framed` to a whole list:

In[11]:= `Framed[{x, y, z}]`

Out[11]= 

`@` does exactly the same thing:

```
In[12]:= Framed@{x, y, z}
```

```
Out[12]= {x, y, z}
```

Now use `/@` to apply `Framed` to each element in the list:

```
In[13]:= Framed/@{x, y, z}
```

```
Out[13]= {x, y, z}
```

The same thing works with any other function. For example, apply the function `Hue` separately to each number in a list.

`/@` applies `Hue` separately to each number in the list:

```
In[14]:= Hue/@{0.1, 0.2, 0.3, 0.4}
```

```
Out[14]= {■, ■, ■, ■}
```

Here's what the `/@` is doing:

```
In[15]:= {Hue[0.1], Hue[0.2], Hue[0.3], Hue[0.4]}
```

```
Out[15]= {■, ■, ■, ■}
```

It's the same story with `Range`, though now the output is a list of lists.

`/@` applies `Range` separately to each number, producing a list of lists:

```
In[16]:= Range/@{3, 2, 5, 6, 7}
```

```
Out[16]= {{1, 2, 3}, {1, 2}, {1, 2, 3, 4, 5}, {1, 2, 3, 4, 5, 6}, {1, 2, 3, 4, 5, 6, 7}}
```

Here's the equivalent, all written out:

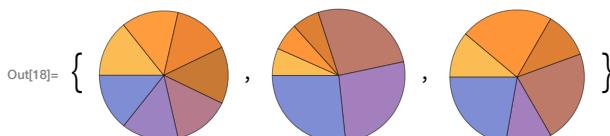
```
In[17]:= {Range[3], Range[2], Range[5], Range[6], Range[7]}
```

```
Out[17]= {{1, 2, 3}, {1, 2}, {1, 2, 3, 4, 5}, {1, 2, 3, 4, 5, 6}, {1, 2, 3, 4, 5, 6, 7}}
```

Given a list of lists, `/@` is what one needs to do an operation separately to each sublist.

Apply `PieChart` separately to each list in a list of lists:

```
In[18]:= PieChart/@{{1, 1, 1, 1, 1, 1}, {1, 1, 1, 4, 4, 4}, {1, 2, 1, 2, 1, 2}}
```



You can use exactly the same idea with lots of other functions.

Apply `Length` to each element, getting the length of each sublist:

```
In[19]:= Length/@{{a,a},{a,a,b,c},{a,a,b,b,b},{a,a,a},{c,c}}
```

```
Out[19]= {2,4,6,3,2}
```

Applying `Length` to the whole list just gives the total number of sublists:

```
In[20]:= Length@{{a,a},{a,a,b,c},{a,a,b,b,b},{a,a,a},{c,c}}
```

```
Out[20]= 5
```

Apply `Reverse` to each element, getting three different reversed lists:

```
In[21]:= Reverse/@{{a,b,c},{x,y,z},{1,2,3,4}}
```

```
Out[21]= {{c,b,a},{z,y,x},{4,3,2,1}}
```

Apply `Reverse` to the whole list, reversing its elements:

```
In[22]:= Reverse@{{a,b,c},{x,y,z},{1,2,3,4}}
```

```
Out[22]= {{1,2,3,4},{x,y,z},{a,b,c}}
```

As always, the form with brackets is exactly equivalent:

```
In[23]:= Reverse[{{a,b,c},{x,y,z},{1,2,3,4}}]
```

```
Out[23]= {{1,2,3,4},{x,y,z},{a,b,c}}
```

Some calculational functions are *listable*, which means they automatically apply themselves to elements in a list.

`N` is listable, so you don't have to use `/@` to get it applied to each element in a list:

```
In[24]:= N[{1/3,1/4,1/5,1/6}]
```

```
Out[24]= {0.333333,0.25,0.2,0.166667}
```

The same is true with `Prime`:

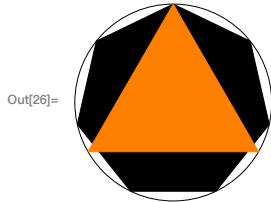
```
In[25]:= Prime[{10,100,1000,10000}]
```

```
Out[25]= {29,541,7919,104729}
```

A function like `Graphics` definitely isn't listable.

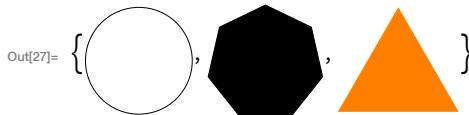
This makes a single graphic with three objects in it:

```
In[26]:= Graphics[{Circle[], RegularPolygon[7], Style[RegularPolygon[3], Orange]}]
```



This gives three separate graphics, with `Graphics` applied to each object:

```
In[27]:= Graphics/@{Circle[], RegularPolygon[7], Style[RegularPolygon[3], Orange]}
```



When you enter `f /@ {1, 2, 3}`, the Wolfram Language interprets it as `Map[f, {1, 2, 3}]`.
`f /@ x` is usually read as “map *f* over *x*”.

The internal interpretation of `f /@ {1, 2, 3}`:

```
In[28]:= Map[f, {1, 2, 3}]
```

```
Out[28]= {f[1], f[2], f[3]}
```

Vocabulary

<code>f @ x</code>	equivalent to <code>f[x]</code>
<code>x // f</code>	equivalent to <code>f[x]</code>
<code>f /@ {a, b, c}</code>	apply <i>f</i> separately to each element of the list
<code>Map[f, {a, b, c}]</code>	alternative form of <code>/@</code>
<code>Framed[expr]</code>	put a frame around something

Exercises

25.1 Use `/@` and `Range` to reproduce the result of `Table[f[n], {n, 5}]`.

25.2 Use `/@` twice to generate `Table[f[g[n]], {n, 10}]`.

25.3 Use `//` to create `a[b[c[d[x]]]]`.

25.4 Make a list of letters of the alphabet, with a frame around each one.

- 25.5** Color negate an image of each planet, giving a list of the results.
- 25.6** Use `/@` to draw separate maps of each country in the G5.
- 25.7** Binarize each flag in Europe, and make an image collage of the result.
- 25.8** Find a list of the dominant colors in images of the planets, putting the results for each planet in a column.
- 25.9** Find the total of the letter numbers given by `LetterNumber` for the letters in the word “wolfram”.

Q&A

Why not always use `f@x` instead of `f[x]`?

`f@x` is a fine equivalent to `f[x]`, but the equivalent of `f[1+1]` is `f@(1+1)`, and in that case, `f[1+1]` is shorter and easier to understand.

Why is `/@` called Map?

It comes from math. Given a set $\{1, 2, 3\}$, $f/@\{1, 2, 3\}$ can be thought of as mapping of this set to another one.

How does one say “//” and “`/@`”

Typically “slash slash” and “slash at”.

When do I need to use parentheses with `@`, `//` and `/@`?

It's determined by the *precedence* or *binding* of different operators. `@` binds tighter than `+`, so `f@1+1` means `f[1]+1` not `f@(1+1)` or `f[1+1]`. `//` binds looser than `+`, so `1/2+1/3//N` means $(1/2+1/3)/N$. In a notebook you can find how things are grouped by repeated clicking.

Tech Notes

- Quite a few functions are “listable”, so they automatically map themselves over lists.
- Range is listable, so `Range[{3, 4, 5}]` is the same as `Range/@{3, 4, 5}`.

More to Explore

Guide to Functional Programming in the Wolfram Language (wolfr.am/eiwl-25-more)

26 | Pure Anonymous Functions

After all the examples we've seen of the Wolfram Language, we're now ready to go to a slightly higher level of abstraction, and tackle the very important concept of *pure functions* (also known as *pure anonymous functions*).

Using pure functions will let us unlock a new level of power in the Wolfram Language, and also let us redo some of the things we've done before in a simpler and more elegant way.

Let's start with a simple example. Say we've got a list of images, and we want to apply `Blur` to each of them. That's easy to do with `/@`.

Apply `Blur` to each image in the list:

```
In[1]:= Blur/@{, , }

Out[1]= {, , 
```

But now let's say we want to include the parameter 5 in `Blur`. How can we do that? The answer is to use a pure function.

Include a parameter by introducing a pure function:

```
In[2]:= Blur[#, 5] & /@{, , }

Out[2]= {, , 
```

The original blur written as a pure function:

```
In[3]:= Blur[##] & /@{, , }

Out[3]= {, , 
```

The `##` is a “slot” into which each element is put. The `&` says that what comes before it is a pure function.

Here's the equivalent of `Blur[#, 5] & /@ ...` expanded out:

```
In[4]:= {Blur[, 5], Blur[, 5], Blur[, 5]}

Out[4]= {, , 
```

Let's look at some other examples. Every time, the slot indicates where to put each element when the pure function is applied.

Rotate each string by 90°:

```
In[5]:= Rotate[#, 90 Degree] & /@ {"one", "two", "three"}
```

```
Out[5]= {one, two, three}
```

Take a string and rotate it different amounts:

```
In[6]:= Rotate["hello", #] & /@ {30 °, 90 °, 180 °, 220 °}
```

```
Out[6]= {hello, heillo, oillo, hillo}
```

Show text in a list of different colors:

```
In[7]:= Style["hello", 20, #] & /@ {Red, Orange, Blue, Purple}
```

```
Out[7]= {hello, hello, hello, hello}
```

Make circles different sizes:

```
In[8]:= Graphics[Circle[], ImageSize -> #] & /@ {20, 40, 30, 50, 10}
```

```
Out[8]= {○, ○, ○, ○, ○}
```

Show framed columns of a color and its negation:

```
In[9]:= Framed[Column[{#, ColorNegate[#]}]] & /@ {Red, Green, Blue, Purple, Orange}
```

```
Out[9]= {█, █, █, █, █}
```

Compute the lengths of three Wikipedia articles:

```
In[10]:= StringLength[WikipediaData[#]] & /@ {"apple", "peach", "pear"}
```

```
Out[10]= {31045, 24153, 11115}
```

Pair topics with results:

```
In[11]:= {#, StringLength[WikipediaData[#]]} & /@ {"apple", "peach", "pear"}
```

```
Out[11]= {{apple, 31045}, {peach, 24153}, {pear, 11115}}
```

Make a grid of everything:

```
In[12]:= Grid[{\#\!, StringLength[WikipediaData[\#\!]}] & /@ {"apple", "peach", "pear"}]
apple 31045
peach 24153
pear 11115
```

This makes a list of digits, then maps a pure function over it:

```
In[13]:= Style[\#\!, Hue[\#\!/10], 5*\#\!] & /@ IntegerDigits[2^100]
Out[13]= {, 2, 6, 7, 6, 5, , 6, , , 2, 8, 2, 9, 4, , 4, 9, 6, 7, , , 3, 2, , 5, 3, 7, 6}
```

Here's what the pure function would do if mapped over {6, 8, 9}:

```
In[14]:= {Style[6, Hue[6/10], 5*6], Style[8, Hue[8/10], 5*8], Style[9, Hue[9/10], 5*9]}
Out[14]= {6, 8, 9}
```

Now that we've seen some examples of pure functions in action, let's look more abstractly at what's going on.

This maps an abstract pure function over a list:

```
In[15]:= f[\#\!, x] & /@ {a, b, c, d, e}
Out[15]= {f[a, x], f[b, x], f[c, x], f[d, x], f[e, x]}
```

Here's the minimal example:

```
In[16]:= f[\#\!] & /@ {a, b, c, d, e}
Out[16]= {f[a], f[b], f[c], f[d], f[e]}
```

It's equivalent to:

```
In[17]:= f /@ {a, b, c, d, e}
Out[17]= {f[a], f[b], f[c], f[d], f[e]}
```

We can put slots wherever we want in the pure function, as many times as we want. All the slots will get filled with whatever the pure function is applied to.

Apply a slightly more complicated pure function:

```
In[18]:= f[\#\!, {x, \#\!}, {\#\!, \#\!}] & /@ {a, b, c}
Out[18]= {f[a, {x, a}, {a, a}], f[b, {x, b}, {b, b}], f[c, {x, c}, {c, c}]}
```

It's easier to read in a column:

```
In[19]:= f[#, {x, #}, {#, #}] & /@ {a, b, c} // Column
Out[19]= 

|                      |
|----------------------|
| f[a, {x, a}, {a, a}] |
| f[b, {x, b}, {b, b}] |
| f[c, {x, c}, {c, c}] |


```

OK, now we're ready to finally discuss how pure functions really work. When we write $f[x]$, we're applying the function f to x . Often we'll use a specific named function instead of f , say Blur , so we have $\text{Blur}[x]$, etc.

But the point is that we can also replace f with a pure function. Then whatever we apply this to will be used to fill the slot in the pure function.

Apply a pure function to x , so the $\#$ slot gets filled with x :

```
In[20]:= f[#, a] & [x]
Out[20]= f[x, a]
```

An equivalent form, written with $@$ instead of $[...]$:

```
In[21]:= f[#, a] & @ x
Out[21]= f[x, a]
```

So now we can see what $/@$ is doing: it's just applying the pure function to each element in the list.

```
In[22]:= f[#, a] & /@ {x, y, z}
Out[22]= {f[x, a], f[y, a], f[z, a]}
```

The same thing, written out more explicitly:

```
In[23]:= {f[#, a] & @ x, f[#, a] & @ y, f[#, a] & @ z}
Out[23]= {f[x, a], f[y, a], f[z, a]}
```

Why is this useful? First of all, because it's the foundation for all the things pure functions do with $/@$. But it's actually also often useful on its own, for example as a way to avoid having to repeat things.

Here's an example of a pure function involving three occurrences of $\#$.

Apply a pure function to $\text{Blend}[\{\text{Red}, \text{Yellow}\}]$:

```
In[24]:= Column[{#, ColorNegate[#, #]} & [Blend[{Red, Yellow}]]]
Out[24]= 

|        |
|--------|
| Orange |
| Blue   |
| Orange |


```

This is what it looks like without the pure function:

```
In[25]:= Column[{Blend[{Red, Yellow}], ColorNegate[Blend[{Red, Yellow}]], Blend[{Red, Yellow}]}]
```



```
Out[25]=
```

In the Wolfram Language, a pure function works just like anything else. On its own, though, it doesn't do anything.

Enter a pure function on its own and it'll come back unchanged:

```
In[26]:= f[#, 2] &
```

```
Out[26]= f[#, 2] &
```

Give it to the function `Map` (`/@`), though, and it'll be used to do a computation.

`Map` uses the pure function to do a computation:

```
In[27]:= Map[f[#, 2] &, {a, b, c, d, e}]
```

```
Out[27]= {f[a, 2], f[b, 2], f[c, 2], f[d, 2], f[e, 2]}
```

Over the course of the next few sections, we'll see more and more uses of pure functions.

Vocabulary

`code &` a pure function

`#` slot in a pure function

Exercises

- 26.1 Use `Range` and a pure function to create a list of the first 20 squares.
- 26.2 Make a list of the result of blending yellow, green and blue with red.
- 26.3 Generate a list of framed columns containing the uppercase and lowercase versions of each letter of the alphabet.
- 26.4 Make a list of letters of the alphabet, in random colors, with frames having random background colors.
- 26.5 Make a table of G5 countries, together with their flags, and arrange the result in a fully framed grid.
- 26.6 Make a list of word clouds (with stopwords deleted) for the Wikipedia articles about apple, peach and pear.
- 26.7 Make a list of histograms of the word lengths in Wikipedia articles on apple, peach and pear.
- 26.8 Make a list of maps of Central America, highlighting each country in turn.

Q&A**Why are they called “pure functions”?**

Because all they do is serve as functions that can be applied to arguments. They're also sometimes called *anonymous functions*, because, unlike say `Blur`, they're not referred to by a name. Here I'm calling them “pure anonymous functions” to communicate both meanings.

Why does one need the &?

The `&` (*ampersand*) indicates that what comes before it is the “body” of a pure function, not the name of a function. `f/@{1, 2, 3}` gives `{f[1], f[2], f[3]}`, but `f &/@{1, 2, 3}` gives `{f, f, f}`.

What is `f[#, 1]` & interpreted as?

`Function[f[#, 1]]`. The `Function` here is sometimes called the “function function”.

Tech Notes

- Pure functions are a characteristic feature of *functional programming*. They're often called *lambda expressions*, after their use in mathematical logic in the 1930s. Confusingly, the term “pure function” sometimes just means a function that has no *side effects* (i.e. assigns no values to variables, etc.)
 - `Table[f[x], {x, {a, b, c}}]` actually does the same as `f/@{a, b, c}`. It's sometimes useful, particularly if one doesn't want to have to explain pure functions.
 - Be careful if you have multiple nested &'s in an expression! Sometimes you may have to insert parentheses. And sometimes you may have to use `Function` with a named variable, as in `Function[x, x^2]` rather than `#^2 &`, to avoid conflicts between uses of `#` in different functions.
 - It sometimes makes for good-looking code to write `Function[x, x^2]` as `x \[Function] x^2`. The `\[Function]` can be typed as `\[Function]` or `Esc fn Esc`.
 - Options can often be pure functions. It's important to put parentheses around the whole pure function, as in `ColorFunction \[Function] (Hue[#/4] &)`, or it won't be interpreted as you expect.
-

More to Explore

[Guide to Functional Programming in the Wolfram Language \(wolfr.am/eiwl-26-more\)](http://wolfr.am/eiwl-26-more)

27 | Applying Functions Repeatedly

$f[x]$ applies f to x . $f[f[x]]$ applies f to $f[x]$, or effectively nests the application of f . It's common to want to repeat or nest a function.

This makes a list of the results of nesting f up to 4 times:

In[1]:= **NestList[f, x, 4]**

Out[1]= $\{x, f[x], f[f[x]], f[f[f[x]]], f[f[f[f[x]]]]\}$

Using **Framed** as the function makes it a little more obvious what's going on:

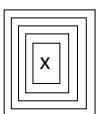
In[2]:= **NestList[Framed, x, 5]**

Out[2]= $\{x, \boxed{x}, \boxed{\boxed{x}}, \boxed{\boxed{\boxed{x}}}, \boxed{\boxed{\boxed{\boxed{x}}}}, \boxed{\boxed{\boxed{\boxed{\boxed{x}}}}}\}$

If you want to see a list of the results of successive nestings, use **NestList**. If you only want to see the final result, use **Nest**.

This gives the final result of 5 levels of nesting:

In[3]:= **Nest[Framed, x, 5]**

Out[3]= 

Nestedly apply **EdgeDetect** to an image, first finding edges, then edges of edges, and so on.

Nestedly do edge detection on an image:

In[4]:= **NestList[EdgeDetect, , 6]**

Out[4]= $\{\text{red circle}, \text{black circle}, \text{circle with white border}, \text{circle with black border}, \text{circle with white border}, \text{circle with black border}, \text{circle with white border}\}$

Use a pure function to both edge-detect and color-negate at each step:

In[5]:= **NestList[ColorNegate[EdgeDetect[#]] &, , 6]**

Out[5]= $\{\text{red circle}, \text{white circle}, \text{circle with black border}, \text{circle with white border}, \text{circle with black border}, \text{circle with white border}, \text{circle with black border}\}$

Start with red, and nestedly blend with yellow, getting more and more yellow.

Add another yellow into the blend at each step:

```
In[6]:= NestList[Blend[{#, Yellow}] &, Red, 20]
Out[6]= {Red, Orange, Yellow, Yellow}
```

If you successively apply a function that adds 1, you just get successive integers.

Nestedly add 1, getting successive numbers:

```
In[7]:= NestList[#+1 &, 1, 15]
Out[7]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16}
```

Nestedly multiply by 2, getting powers of 2.

The result doubles each time, giving a list of powers of 2:

```
In[8]:= NestList[2*# &, 1, 15]
Out[8]= {1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192, 16384, 32768}
```

Nested squaring very soon leads to big numbers:

```
In[9]:= NestList[#^2 &, 2, 6]
Out[9]= {2, 4, 16, 256, 65536, 4294967296, 18446744073709551616}
```

You can make nested square roots too.

Nestedly apply square root:

```
In[10]:= NestList[Sqrt[1+#] &, 1, 5]
Out[10]= {1, Sqrt[2], Sqrt[1+Sqrt[2]], Sqrt[1+Sqrt[1+Sqrt[2]]], 
          Sqrt[1+Sqrt[1+Sqrt[1+Sqrt[2]]]], Sqrt[1+Sqrt[1+Sqrt[1+Sqrt[1+Sqrt[2]]]]]}
```

The decimal version of the result converges quickly (to the *golden ratio*):

```
In[11]:= NestList[Sqrt[1+#] &, 1, 10] // N
Out[11]= {1., 1.41421, 1.55377, 1.59805, 1.61185, 1.61612, 1.61744, 1.61785, 1.61798, 1.61802, 1.61803}
```

`RandomChoice` randomly chooses from a list. You can use it to create a pure function that, say, randomly either adds `+1` or `-1`.

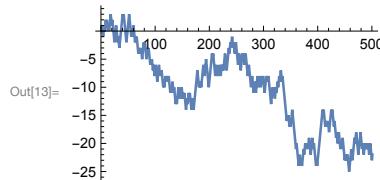
Randomly add or subtract 1 at each step, starting from 0:

```
In[12]:= NestList[#+RandomChoice[{+1,-1}] &, 0, 20]
```

```
Out[12]= {0, 1, 0, -1, -2, -3, -4, -5, -6, -5, -6, -5, -4, -5, -4, -3, -4, -3, -2, -1, -2}
```

This generates 500 steps in a “random walk”:

```
In[13]:= ListLinePlot[NestList[#+RandomChoice[{+1,-1}] &, 0, 500]]
```



So far, we’ve used `NestList` *iteratively*—effectively to perform a chain of applications of a particular function. But you can also use it for *recursion*, in which the very pattern of applications of the function is itself nested.

This does a chain of applications of the function `f`:

```
In[14]:= NestList[f[##] &, x, 3]
```

```
Out[14]= {x, f[x], f[f[x]], f[f[f[x]]]}
```

The pattern of applications of `f` is more complicated here:

```
In[15]:= NestList[f[##, ##] &, x, 3]
```

```
Out[15]= {x, f[x, x], f[f[x, x], f[x, x]], f[f[f[x, x], f[x, x]], f[f[x, x], f[x, x]]]}
```

Adding frames makes it a little easier to see what’s going on:

```
In[16]:= NestList[Framed[f[##, ##]] &, x, 3]
```

```
Out[16]= {x, f[x, x], ff[x, x], f[x, x], fff[x, x], f[x, x], fff[x, x], f[x, x]} }
```

Putting everything in columns shows the nested pattern of function applications.

The nested boxes are recursively combined in twos at each level:

```
In[17]:= NestList[Framed[Column[{#, #}]] &, x, 3]
```

```
Out[17]= {x, {x, x}, {{x, x}, {x, x}}, {{{x, x}, {x, x}}, {{{x, x}, {x, x}}}}
```

This gives a sequence of recursively nested grids:

```
In[18]:= NestList[Framed[Grid[{{#, #}, {#, #}}]] &, x, 3]
```

Out[18]=

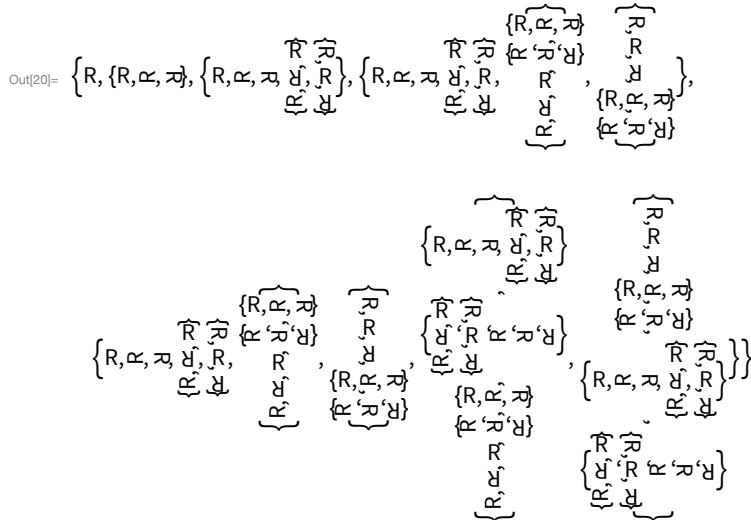
$$\left\{ \mathbf{x}, \begin{array}{|c|c|} \hline x & x \\ \hline x & x \\ \hline \end{array}, \begin{array}{|c|c|} \hline x & x \\ \hline x & x \\ \hline \end{array}, \dots \right\}$$

This forms the beginning of a fractal structure:

```
In[19]:= NestList[Framed[Grid[{{0, #}, {#, #}}]] &, x, 3]
```

It's easy to get some pretty ornate recursive structures:

```
In[20]:= NestList[Flatten[{#, Rotate[#, 90 °], Rotate[#, 270 °]}] &, "R", 4]
```



Not all results from recursion are so complicated. Here's an example that successively adds two shifted copies of a list together, as in $\{0, 1, 2, 1\} + \{1, 2, 1, 0\}$.

Prepend and append 0 to a list, then add together:

```
In[21]:= NestList[Join[{0}, #] + Join[#, {0}] &, {1}, 5]
```

```
Out[21]= {{1}, {1, 1}, {1, 2, 1}, {1, 3, 3, 1}, {1, 4, 6, 4, 1}, {1, 5, 10, 10, 5, 1}}
```

If you put the result in a grid, it forms *Pascal's triangle of binomial coefficients*:

```
In[22]:= NestList[Join[{0}, #] + Join[#, {0}] &, {1}, 8] // Grid
```

```
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
```

Here's another example of recursion with `NestList`.

Form a recursive structure with two functions `f` and `g`:

In[23]:= `NestList[{f[#], g[#]} &, x, 3]`

Out[23]= $\{x, \{f[x], g[x]\}, \{f[f[x], g[x]], g[\{f[x], g[x]\}]\}, \{f[\{f[f[x], g[x]]\}, g[\{f[x], g[x]\}]], g[\{f[\{f[x], g[x]\}], g[\{f[x], g[x]\}]\}]\}$

Even if things are arranged in columns, it's still quite difficult to understand the structure that's been created.

Arrange the recursive structure in columns:

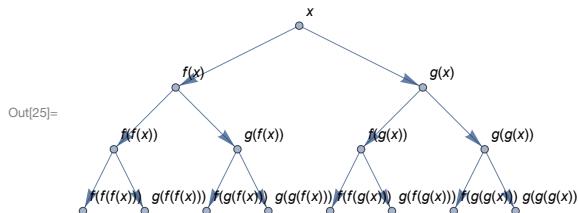
In[24]:= `NestList[Column[{f[#], g[#]}] &, x, 3]`

Out[24]= $\left\{x, \begin{array}{c} f[x] \\ g[x] \end{array}, \begin{array}{c} f[f[x]] \\ g[g[x]] \end{array}, \begin{array}{c} f[g[x]] \\ g[f[x]] \end{array}, \begin{array}{c} f[f[g[x]]] \\ g[g[f[x]]] \end{array}, \begin{array}{c} f[g[f[x]]] \\ g[g[g[x]]] \end{array} \right\}$

`NestGraph` is basically like `NestList`, except that it makes a graph rather than a list. It repeatedly applies a function to determine what nodes a particular node should connect to. In this case, it produces a tree of nodes, making it clearer what's going on.

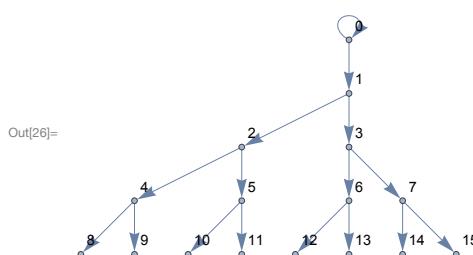
Start from `x`, then repeatedly connect to the list of nodes obtained by applying the function:

In[25]:= `NestGraph[{f[#], g[#]} &, x, 3, VertexLabels -> All]`



Repeatedly apply a numerical function to form another tree structure:

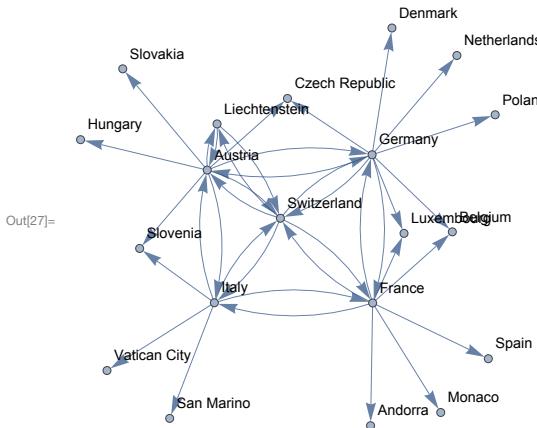
In[26]:= `NestGraph[{2 #, 2 # + 1} &, 0, 4, VertexLabels -> All]`



You can use `NestGraph` to effectively “crawl” outward creating a network. As an example, we can repeatedly apply a function that for any country gives a list of countries that border it. The result is a network that connects bordering countries, here starting with Switzerland.

“Crawl” outward 2 steps from Switzerland, connecting each country to all those that border it:

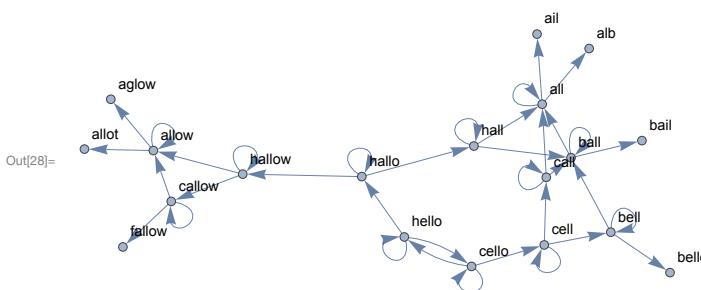
In[27]:= `NestGraph[#[{"BorderingCountries"}] &, "switzerland", 2, VertexLabels -> All]`



As another example, start from the word “hello” and successively connect every word to 3 words in the list of common words that `Nearest` considers nearest to it.

Create a network of nearby words with respect to 1-letter changes:

In[28]:= `NestGraph[Nearest[WordList[], #, 3] &, "hello", 4, VertexLabels -> All]`



Vocabulary

`NestList[f, x, n]` make a list of applying f to x up to n times

`Nest[f, x, n]` give the result of applying f to x exactly n times

`NestGraph[f, x, n]` make a graph by nestedly applying f starting with x

Exercises

- 27.1** Make a list of the results of nesting `Blur` up to 10 times, starting with a rasterized size-30 "X".
- 27.2** Make a list of nesting a frame with a random background color up to 10 times starting with `x`.
- 27.3** Start with a size-50 "A", then make a list of nestedly applying a frame and a random rotation 5 times.
- 27.4** Make a line plot of 100 iterations of the *logistic map iteration* $4 \#(1 - \#)$ &, starting from 0.2.
- 27.5** Find the numerical value of the result from 30 iterations of $1 + 1/\#$ & starting from 1.
- 27.6** Create a list of the first 10 powers of 3 (starting at 0) by nested multiplication.
- 27.7** Make a list of the result of nesting the (*Newton's method*) function $(\# + 2/\#)/2$ & up to 5 times starting from 1.0, and then subtract $\sqrt{2}$ from all the results.
- 27.8** Make graphics of a 1000-step 2D random walk which starts at $\{0, 0\}$, and in which at each step a pair of random numbers between -1 and $+1$ are added to the coordinates.
- 27.9** Make an array plot of 50 steps of Pascal's triangle modulo 2 by starting from $\{1\}$ and nestedly joining $\{0\}$ at the beginning and at the end, and adding these results together modulo 2.
- 27.10** Generate a graph by starting from 0, then nestedly 10 times connecting each node with value n to ones with values $n+1$ and $2n$.
- 27.11** Generate a graph obtained by nestedly finding bordering countries starting from the United States, and going 4 iterations.

Q&A

What's the difference between iteration and recursion?

If one does one thing repeatedly, it's iteration. When one takes the result of an operation and applies the same operation to it wherever it is, that's recursion. It's slightly confusing, because simple cases of recursion are just iteration. `NestList` always does recursion, but if only one slot appears in the function, the recursion can be "unrolled" into iteration.

What is the relation between nesting, recursion and fractals?

They are very closely related. The definitions aren't precise, but fractals are basically geometrical forms that exhibit some type of nested or recursive structure.

What is Pascal's triangle?

It's a very common structure discussed in elementary mathematics. Its definition is very close to the Wolfram Language code here: at each row each number is computed as the sum of the numbers directly above it and above it to its right. Each row gives the coefficients in the expansion of $(1+x)^n$.

Is `NestGraph` like a web crawler?

Conceptually yes. One can think of it as doing the analog of starting from a webpage and then visiting links from that page, and continuing recursively with that process. We'll see an example of this in Section 44.

Why do some but not all countries have arrows both ways in the bordering countries graph?

If `NestGraph` was run for enough steps, all countries would have arrows both ways, since if A borders B, then B borders A. But here we're stopping after just 2 steps, so many of the reverse connections haven't been reached.

Why use NestList for something like NestList[2*#, &, 1, 15]?

You don't need to. You can just use Power, as in Table[2^n , {n, 0, 15}]. But it's nice to see the sequence Plus, Times, Power arise from successive nesting (e.g. NestList[$2 + \#^2$ &, 0, 15] is Table[2^n , {n, 0, 15}]).

Is there a way to keep applying a function until nothing is changing?

Yes. Use FixedPoint or FixedPointList (see Section 41).

Tech Note

- The nearest words example can be made much more efficient by first computing a NearestFunction, then using this repeatedly, rather than computing Nearest from scratch for every word. This example is also closely related to NearestNeighborGraph, discussed in Section 22.

More to Explore

Guide to Functional Iteration in the Wolfram Language (wolfr.am/eiwl-27-more)

28 | Tests and Conditionals

Is $2+2$ equal to 4? Let's ask the Wolfram Language.

Test whether $2+2$ is equal to 4:

```
In[1]:= 2 + 2 == 4
```

```
Out[1]= True
```

Not surprisingly, testing whether $2+2$ is equal to 4 gives `True`.

We can also test whether 2×2 is greater than 5. We do that using `>`.

Test whether 2×2 is greater than 5:

```
In[2]:= 2 * 2 > 5
```

```
Out[2]= False
```

The function `If` lets you choose to give one result if a test is `True`, and another if it's `False`.

Since the test gives `True`, the result of the `If` is `x`:

```
In[3]:= If[2 + 2 == 4, x, y]
```

```
Out[3]= x
```

By using a pure function with `/@`, we can apply an `If` to every element of a list.

If an element is less than 4, make it `x`, otherwise make it `y`:

```
In[4]:= If[#, < 4, x, y] & /@ {1, 2, 3, 4, 5, 6, 7}
```

```
Out[4]= {x, x, x, y, y, y, y}
```

You can also test for less than or equal using `\leq` , which is typed as `\leq` .

If an element is less than or equal to 4, make it `x`; otherwise, make it `y`:

```
In[5]:= If[#,  $\leq$  4, x, y] & /@ {1, 2, 3, 4, 5, 6, 7}
```

```
Out[5]= {x, x, x, x, y, y, y}
```

This makes an element `x` only if it is equal to 4:

```
In[6]:= If[#, == 4, x, y] & /@ {1, 2, 3, 4, 5, 6, 7}
```

```
Out[6]= {y, y, y, x, y, y, y}
```

You can test whether two things are not equal using \neq , which is typed as `!=`.

If an element is not equal to 4, make it `x`; otherwise, make it `y`:

```
In[7]:= If[\# \neq 4, x, y] & /@ {1, 2, 3, 4, 5, 6, 7}
Out[7]= {x, x, x, y, x, x, x}
```

It's often useful to select elements in a list that satisfy a test. You can do this by using `Select`, and giving your test as a pure function.

Select elements in the list that are greater than 3:

```
In[8]:= Select[{1, 2, 3, 4, 5, 6, 7}, \# > 3 &]
Out[8]= {4, 5, 6, 7}
```

Select elements that are between 2 and 5:

```
In[9]:= Select[{1, 2, 3, 4, 5, 6, 7}, 2 \leq \# \leq 5 &]
Out[9]= {2, 3, 4, 5}
```

Beyond size comparisons like `<`, `>` and `==`, the Wolfram Language includes many other kinds of tests. Examples are `EvenQ` and `OddQ`, which test whether numbers are even or odd. (The “Q” indicates that the functions are asking a question.)

4 is an even number:

```
In[10]:= EvenQ[4]
Out[10]= True
```

Select even numbers from the list:

```
In[11]:= Select[{1, 2, 3, 4, 5, 6, 7, 8, 9}, EvenQ[\#] &]
Out[11]= {2, 4, 6, 8}
```

In this case, we don't need the explicit pure function:

```
In[12]:= Select[{1, 2, 3, 4, 5, 6, 7, 8, 9}, EvenQ]
Out[12]= {2, 4, 6, 8}
```

`IntegerQ` tests whether something is an integer; `PrimeQ` tests whether a number is prime.

Select prime numbers:

```
In[13]:= Select[{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, PrimeQ]
Out[13]= {2, 3, 5, 7}
```

Sometimes we need to combine tests. `&&` represents “and”, `||` represents “or” and `!` represents “not”.

Select elements of the list that are both even and greater than 2:

```
In[14]:= Select[{1, 2, 3, 4, 5, 6, 7}, EvenQ[#]&&# > 2 &]
```

```
Out[14]= {4, 6}
```

Select elements that are either even or greater than 4:

```
In[15]:= Select[{1, 2, 3, 4, 5, 6, 7}, EvenQ[#]||# > 4 &]
```

```
Out[15]= {2, 4, 5, 6, 7}
```

Select elements that are not either even or greater than 4:

```
In[16]:= Select[{1, 2, 3, 4, 5, 6, 7}, !(EvenQ[#]||# > 4) &]
```

```
Out[16]= {1, 3}
```

There are many other “Q functions” that ask various kinds of questions. `LetterQ` tests whether a string consists of letters.

The space between letters isn’t a letter; nor is “!”:

```
In[17]:= {LetterQ["a"], LetterQ["bc"], LetterQ["a b"], LetterQ["!"]}
```

```
Out[17]= {True, True, False, False}
```

Turn a string into a list of characters, then test which are letters:

```
In[18]:= LetterQ/@Characters["30 is the best!"]
```

```
Out[18]= {False, False, False, True, True, False, True, True, True, True, True, True, False}
```

Select the characters that are letters:

```
In[19]:= Select[Characters["30 is the best!"], LetterQ]
```

```
Out[19]= {i, s, t, h, e, b, e, s, t}
```

Select letters that appear after position 10 in the alphabet:

```
In[20]:= Select[Characters["30 is the best!"], LetterQ[#]&&LetterNumber[#] > 10 &]
```

```
Out[20]= {s, t, s, t}
```

You can use `Select` to find words in English that are *palindromes*, meaning that they are the same if you reverse them.

```
In[21]:= Select[WordList[], StringReverse[##] == ## &]
```

```
Out[21]= {a, aha, bib, bob, boob, civic, dad, deed, dud, ere, eve, ewe, eye, gag, gig, huh, kayak,
kook, level, ma'am, madam, minim, mom, mum, nan, non, noon, nun, oho, pap,
peep, pep, pip, poop, pop, pup, radar, refer, rotor, sis, tat, tenet, toot, tot, tut, wow}
```

`MemberQ` tests whether something appears as an element, or member, of a list.

5 appears in the list {1, 3, 5, 7}:

```
In[22]:= MemberQ[{1, 3, 5, 7}, 5]
```

```
Out[22]= True
```

Select numbers in the range 1 to 100 whose digit sequences contain 2:

```
In[23]:= Select[Range[100], MemberQ[IntegerDigits[##], 2] &]
```

```
Out[23]= {2, 12, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 42, 52, 62, 72, 82, 92}
```

`ImageInstanceQ` is a machine-learning-based function that tests whether an image is an instance of a particular kind of thing, like a cat.

Test if an image is of a cat:

```
In[24]:= ImageInstanceQ[, "cat"]
```

```
Out[24]= True
```

Select images of cats:

```
In[25]:= Select[{, , , , , ,
```



```
ImageInstanceQ[#, "cat"] &]
```

```
Out[25]= {, }
```

Here's a geographic example of Select: find which cities in a list are less than 3000 miles from San Francisco.

Select cities whose distance from San Francisco is less than 3000 miles:

```
In[26]:= Select[{london , nyc , tokyo , chicago },  
       GeoDistance[#, san francisco ] < 3000 miles &]  
  
Out[26]= {New York City , Chicago }
```

Vocabulary

$a == b$	test for equality
$a < b$	test whether less
$a > b$	test whether greater
$a \leq b$	test whether less or equal
$a \geq b$	test whether greater or equal
If [<i>test</i> , <i>u</i> , <i>v</i>]	give <i>u</i> if <i>test</i> is True and <i>v</i> if False
Select [<i>list</i> , <i>f</i>]	select elements that pass a test
EvenQ [<i>x</i>]	test whether even
OddQ [<i>x</i>]	test whether odd
IntegerQ [<i>x</i>]	test whether an integer
PrimeQ [<i>x</i>]	test whether a prime number
LetterQ [<i>string</i>]	test whether there are only letters
MemberQ [<i>list</i> , <i>x</i>]	test whether <i>x</i> is a member of <i>list</i>
ImageInstanceQ [<i>image</i> , <i>category</i>]	test whether <i>image</i> is an instance of <i>category</i>

Exercises

- 28.1** Test whether 123^321 is greater than 456^123 .
- 28.2** Get a list of numbers up to 100 whose digits add up to less than 5.
- 28.3** Make a list of the first 20 integers, with prime numbers styled red.
- 28.4** Find words in **WordList[]** that both begin and end with the letter “p”.
- 28.5** Make a list of the first 100 primes, keeping only ones whose last digit is less than 3.
- 28.6** Find Roman numerals up to 100 that do not contain “I”.
- 28.7** Get a list of Roman numerals up to 1000 that are palindromes.

- 28.8** Find names of integers up to 100 that begin and end with the same letter:
 - 28.9** Get a list of words longer than 15 characters from the Wikipedia article on words.
 - 28.10** Starting from 1000, divide by 2 if the number is even, and compute $3\# + 1$ & if the number is odd; do this repeatedly 200 times (*Collatz problem*).
 - 28.11** Make a word cloud of 5-letter words in the Wikipedia article on computers.
 - 28.12** Find words in `WordList[]` whose first 3 letters are the same as their last 3 read backward, but where the whole string is not a palindrome.
 - 28.13** Find all 10-letter words in `WordList[]` for which the total of `LetterNumber` values is 100.
-

Q&A

Why does one test equality with `==` not `=`?

Because `=` means something else in the Wolfram Language. You'll get very strange results if you use `=` instead of `==` by mistake. (`=` is for assigning values of variables.) To avoid possible confusion, `==` is often read as "double equals".

Why is "and" written as `&&`, not `&`?

Because `&` means other things in the Wolfram Language. For example it's what ends a pure function.

How are `==`, `>`, `&&`, etc. interpreted?

`==` is Equal, `≠` (`!=`) is Unequal, `>` is Greater, `≥` is GreaterEqual, `<` is Less, `&&` is And, `||` is Or, and `!` is Not.

When do I need to use parentheses with `&&`, `||`, etc.?

There's an order of operations that's a direct analog of arithmetic. `&&` is like `x`, `||` is like `+`, and `!` is like `-`. So `! p && q` means "(not `p`) and `q`"; `! (p && q)` means "not (`p` and `q`)".

What's special about "Q" functions?

They ask a question that normally has an answer of True or False.

What are some other "Q" functions?

`NumberQ`, `StringContainsQ`, `BusinessDayQ` and `ConnectedGraphQ` are a few.

Is there a better way to find real-world entities with certain properties than using Select?

Yes. You can do things like `Entity["Country", "Population" → GreaterThan[1×10^7 people]]` to find "implicit entities", then use `EntityList` to get explicit lists of entities.

Tech Notes

- True and False are typically called *Booleans* in computer science, after George Boole from the mid-1800s. Expressions with `&&`, `||`, etc. are often called *Boolean expressions*.
- In the Wolfram Language, True and False are *symbols*, and are not represented by 1 and 0 as in many other computer languages.
- If is often called a *conditional*. In `If[test, then, else]`, the *then* and *else* aren't computed unless the *test* says their condition is met.
- `PalindromeQ` directly tests if a string is a palindrome.
- In the Wolfram Language, x is a *symbol* (see Section 33) that could represent anything, so `x == 1` is just an *equation*, that isn't immediately True or False. `x === 1` ("triple equals") tests whether x is *the same* as 1, and since it isn't, it gives False.

More to Explore

[Guide to Functions That Apply Tests in the Wolfram Language](#) (wolfr.am/eiwl-28-more)

[Guide to Boolean Computation in the Wolfram Language](#) (wolfr.am/eiwl-28-more2)

29 | More about Pure Functions

We've seen how to make lists and arrays with `Table`. There's also a way to do it using pure functions—with `Array`.

Generate a 10-element array with an abstract function `f`:

```
In[1]:= Array[f, 10]
Out[1]= {f[1], f[2], f[3], f[4], f[5], f[6], f[7], f[8], f[9], f[10]}
```

Use a pure function to generate a list of the first 10 squares:

```
In[2]:= Array[#^2 &, 10]
Out[2]= {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
```

`Table` can give the same result, though you have to introduce the variable `n`:

```
In[3]:= Table[n ^ 2, {n, 10}]
Out[3]= {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
```

`Array[f, 4]` makes a single list of 4 elements. `Array[f, {3, 4}]` makes a 3×4 array.

Make a list of length 3, each of whose elements is a list of length 4:

```
In[4]:= Array[f, {3, 4}]
Out[4]= {{f[1, 1], f[1, 2], f[1, 3], f[1, 4]}, {f[2, 1], f[2, 2], f[2, 3], f[2, 4]}, {f[3, 1], f[3, 2], f[3, 3], f[3, 4]}}
```

Display it in a grid:

```
In[5]:= Array[f, {3, 4}] // Grid
Out[5]= 

|         |         |         |         |
|---------|---------|---------|---------|
| f[1, 1] | f[1, 2] | f[1, 3] | f[1, 4] |
| f[2, 1] | f[2, 2] | f[2, 3] | f[2, 4] |
| f[3, 1] | f[3, 2] | f[3, 3] | f[3, 4] |


```

If the function is `Times`, `Array` makes a multiplication table:

```
In[6]:= Grid[Array[Times, {5, 5}]]
Out[6]= 

|   |    |    |    |    |
|---|----|----|----|----|
| 1 | 2  | 3  | 4  | 5  |
| 2 | 4  | 6  | 8  | 10 |
| 3 | 6  | 9  | 12 | 15 |
| 4 | 8  | 12 | 16 | 20 |
| 5 | 10 | 15 | 20 | 25 |


```

What if we want to use a pure function in place of `Times`? When we compute `Times[3, 4]`, we say that `Times` is applied to two *arguments*. (In `Times[3, 4, 5]`, `Times` is applied to three arguments, etc.) In a pure function, `#1` represents the first argument, `#2` the second argument, and so on.

#1 represents the first argument, **#2** the second argument:

```
In[7]:= f[#1, #2] &[55, 66]
```

```
Out[7]= f[55, 66]
```

#1 always picks out the first argument, and **#2** the second argument:

```
In[8]:= f[#2, #1, {#2, #2, #1}] &[55, 66]
```

```
Out[8]= f[66, 55, {66, 66, 55}]
```

Now we can use **#1** and **#2** inside a function in `Array`.

Use a pure function to make a multiplication table:

```
In[9]:= Array[#1 * #2 &, {5, 5}] // Grid
```

```
1 2 3 4 5
2 4 6 8 10
Out[9]= 3 6 9 12 15
4 8 12 16 20
5 10 15 20 25
```

Use a different pure function that puts in `x` whenever the numbers are equal:

```
In[10]:= Array[If[#1 == #2, x, #1 * #2] &, {5, 5}] // Grid
```

```
x 2 3 4 5
2 x 6 8 10
Out[10]= 3 6 x 12 15
4 8 12 x 20
5 10 15 20 x
```

Here's the equivalent computation with `Table`:

```
In[11]:= Table[If[i == j, x, i * j], {i, 5}, {j, 5}] // Grid
```

```
x 2 3 4 5
2 x 6 8 10
Out[11]= 3 6 x 12 15
4 8 12 x 20
5 10 15 20 x
```

Now that we've discussed pure functions with more than one argument, we're in a position to talk about `FoldList`. You can think of `FoldList` as a 2-argument generalization of `NestList`.

`NestList` takes a single function, say `f`, and successively nests it:

```
In[12]:= NestList[f, x, 5]
```

```
Out[12]= {x, f[x], f[f[x]], f[f[f[x]]], f[f[f[f[x]]]]}
```

It's easier to understand when the function we use is `Framed`:

```
In[13]:= NestList[Framed, x, 5]
```

```
Out[13]= {x, , , , , }
```

`NestList` just keeps on applying a function to whatever result it got before. `FoldList` does the same, except that it also “folds in” a new element each time.

Here's `FoldList` with an abstract function `f`:

```
In[14]:= FoldList[f, x, {1, 2, 3, 4, 5}]
```

```
Out[14]= {x, f[x, 1], f[f[x, 1], 2], f[f[f[x, 1], 2], 3], f[f[f[f[x, 1], 2], 3], 4], f[f[f[f[f[x, 1], 2], 3], 4], 5]}
```

Including `Framed` makes it a little easier to see what's going on:

```
In[15]:= FoldList[Framed[f[#1, #2]] &, x, {1, 2, 3, 4, 5}]
```

```
Out[15]= {x, , , , , }
```

At first, this may look complicated and obscure, and it might seem hard to imagine how it could be useful. But actually, it's very useful, and surprisingly common in real programs.

`FoldList` is good for progressively accumulating things. Let's start with a simple case: progressively adding up numbers.

At each step `FoldList` folds in another element (`#2`), adding it to the result so far (`#1`):

```
In[16]:= FoldList[#1 + #2 &, 0, {1, 1, 1, 2, 0, 0}]
```

```
Out[16]= {0, 1, 2, 3, 5, 5, 5}
```

Here's the computation it's doing:

```
In[17]:= {0, 0+1, (0+1)+1, ((0+1)+1)+1, (((0+1)+1)+1)+2, (((((0+1)+1)+1)+2)+0, ((((0+1)+1)+1)+2)+0)+0}
```

```
Out[17]= {0, 1, 2, 3, 5, 5, 5}
```

Or equivalently:

```
In[18]:= {0, 0+1, 0+1+1, 0+1+1+1, 0+1+1+1+2, 0+1+1+1+2+0, 0+1+1+1+2+0+0}
```

```
Out[18]= {0, 1, 2, 3, 5, 5, 5}
```

It may be easier to see what's going on with symbols:

```
In[19]:= FoldList[##1+##2 &, 0, {a, b, c, d, e}]  
Out[19]= {0, a, a+b, a+b+c, a+b+c+d, a+b+c+d+e}
```

Of course, this case is simple enough that you don't actually need a pure function:

```
In[20]:= FoldList[Plus, 0, {a, b, c, d, e}]  
Out[20]= {0, a, a+b, a+b+c, a+b+c+d, a+b+c+d+e}
```

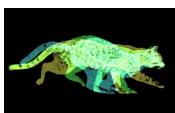
A classic use of `FoldList` is to successively “fold in” digits to reconstruct a number from its list of digits.

Successively construct a number from its digits, starting the folding process with its first digit:

```
In[21]:= FoldList[10 #1+#2 &, {8, 7, 6, 1, 2, 3, 9, 8, 7}]  
Out[21]= {8, 87, 876, 8761, 87612, 876123, 8761239, 87612398, 876123987}
```

Finally, let's use `FoldList` to “fold in” progressive images from a list, at each point adding them with `ImageAdd` to the image obtained so far.

Progressively “fold in” images from a list, combining them with `ImageAdd`:

```
In[22]:= FoldList[ImageAdd, {, , , , }]  
Out[22]= {, , , , }
```

The concept of `FoldList` is not at first the easiest to understand. But when you've got it, you've learned an extremely powerful *functional programming* technique that's an example of the kind of elegant abstraction possible in the Wolfram Language.

Vocabulary

Array[f, n] make an array by applying a function

Array[f, {m, n}] make an array in 2D

FoldList[f, x, list] successively apply a function, folding in elements of a list

Exercises

- 29.1** Use Prime and Array to generate a list of the first 100 primes.
- 29.2** Use Prime and Array to find successive differences between the first 100 primes.
- 29.3** Use Array and Grid to make a 10 by 10 addition table.
- 29.4** Use FoldList, Times and Range to successively multiply numbers up to 10 (making factorials).
- 29.5** Use FoldList and Array to compute the successive products of the first 10 primes.
- 29.6** Use FoldList to successively ImageAdd regular polygons with between 3 and 8 sides, and with opacity 0.2.

Q&A

What does # alone mean?

It's equivalent to `#1`—a slot to be filled from the first argument of a function.

How does one say #1?

Either “slot 1” (reflecting its role in a pure function), or “hash 1” (reflecting how it's written—the “#” is usually called “hash”).

Can one name arguments to a pure function?

Yes. It's done with `Function[{x, y}, x+y]`, etc. It's sometimes nice to do this for code readability, and it's sometimes necessary when pure functions are nested.

Can Array make more deeply nested structures?

Yes—as deep as you want. Lists of lists of lists... for any number of levels.

What is functional programming?

It's programming where everything is based on evaluating functions and combinations of functions. It's actually the only style of programming we've seen so far in this book. In Section 38 we'll see *procedural programming*, which is about going through a procedure and progressively changing values of variables.

Tech Notes

- `Fold` gives the last element of `FoldList`, just like `Nest` gives the last element of `NestList`.
- `FromDigits` reconstructs numbers from lists of digits, effectively doing what we used `FoldList` for above.
- `Accumulate[list]` is `FoldList[Plus, list]`.
- `Array` and `FoldList`, like `NestList`, are examples of what are called *higher-order functions*, that take functions as input. (In mathematics, they're also known as *functionals* or *functors*.)
- You can set up pure functions that take several arguments at a time using `##`, etc.
- You can animate a list of images using `ListAnimate`, and show the list in 3D using `Image3D`.

More to Explore

Guide to Functional Programming in the Wolfram Language (wolfr.am/eiwl-29-more)

30 | Rearranging Lists

It's common for lists that come out of one computation to have to be rearranged before going into another computation. For example, one might have a list of pairs, and need to convert it into a pair of lists, or vice versa.

Transpose a list of pairs so it becomes a pair of lists:

```
In[1]:= Transpose[{{1, 2}, {3, 4}, {5, 6}, {7, 8}, {9, 10}}]
Out[1]= {{1, 3, 5, 7, 9}, {2, 4, 6, 8, 10}}
```

Transpose the pair of lists back to a list of pairs:

```
In[2]:= Transpose[{{1, 3, 5, 7, 9}, {2, 4, 6, 8, 10}}]
Out[2]= {{1, 2}, {3, 4}, {5, 6}, {7, 8}, {9, 10}}
```

Thread is a closely related operation, often useful for generating input to Graph.

“Thread” → across the elements of two lists:

```
In[3]:= Thread[{1, 3, 5, 7, 9} → {2, 4, 6, 8, 10}]
Out[3]= {1 → 2, 3 → 4, 5 → 6, 7 → 8, 9 → 10}
```

Partition takes a list, and partitions it into blocks of a specified size.

Partition a 12-element list into blocks of size 3:

```
In[4]:= Partition[Range[12], 3]
Out[4]= {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}, {10, 11, 12}}
```

Partition a list of characters to display them in a grid:

```
In[5]:= Grid[Partition[Characters["An array of text made in the Wolfram Language"], 9],
Frame → All]
```

A	n		a	r	r	a	y	
o	f		t	e	x	t		m
a	d	e	i	n		t	h	
e		W	o	l	f	r	a	m
	L	a	n	g	u	a	g	e

If you don't tell it otherwise, Partition breaks a list up into non-overlapping blocks. But you can also tell it to break the list into blocks that have some specified offset.

Partition a list into blocks of size 3 with offset 1:

```
In[6]:= Partition[Range[10], 3, 1]
Out[6]= {{1, 2, 3}, {2, 3, 4}, {3, 4, 5}, {4, 5, 6}, {5, 6, 7}, {6, 7, 8}, {7, 8, 9}, {8, 9, 10}}
```

Partition a list of characters into blocks with an offset of 1:

```
In[7]:= Grid[Partition[Characters["Wolfram Language"], 12, 1], Frame -> All]
```

Out[7]=	<table border="1"> <tbody> <tr><td>W</td><td>o</td><td>l</td><td>f</td><td>r</td><td>a</td><td>m</td><td></td><td>L</td><td>a</td><td>n</td><td>g</td></tr> <tr><td>o</td><td>l</td><td>f</td><td>r</td><td>a</td><td>m</td><td></td><td>L</td><td>a</td><td>n</td><td>g</td><td>u</td></tr> <tr><td>l</td><td>f</td><td>r</td><td>a</td><td>m</td><td></td><td>L</td><td>a</td><td>n</td><td>g</td><td>u</td><td>a</td></tr> <tr><td>f</td><td>r</td><td>a</td><td>m</td><td></td><td>L</td><td>a</td><td>n</td><td>g</td><td>u</td><td>a</td><td>g</td></tr> <tr><td>r</td><td>a</td><td>m</td><td></td><td>L</td><td>a</td><td>n</td><td>g</td><td>u</td><td>a</td><td>g</td><td>e</td></tr> </tbody> </table>	W	o	l	f	r	a	m		L	a	n	g	o	l	f	r	a	m		L	a	n	g	u	l	f	r	a	m		L	a	n	g	u	a	f	r	a	m		L	a	n	g	u	a	g	r	a	m		L	a	n	g	u	a	g	e
W	o	l	f	r	a	m		L	a	n	g																																																		
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l	f	r	a	m		L	a	n	g	u	a																																																		
f	r	a	m		L	a	n	g	u	a	g																																																		
r	a	m		L	a	n	g	u	a	g	e																																																		

Use an offset of 2 instead:

```
In[8]:= Grid[Partition[Characters["Wolfram Language"], 12, 2], Frame -> All]
```

Out[8]=	<table border="1"> <tbody> <tr><td>W</td><td>o</td><td>l</td><td>f</td><td>r</td><td>a</td><td>m</td><td></td><td>L</td><td>a</td><td>n</td><td>g</td></tr> <tr><td>l</td><td>f</td><td>r</td><td>a</td><td>m</td><td></td><td>L</td><td>a</td><td>n</td><td>g</td><td>u</td><td>a</td></tr> <tr><td>r</td><td>a</td><td>m</td><td></td><td>L</td><td>a</td><td>n</td><td>g</td><td>u</td><td>a</td><td>g</td><td>e</td></tr> </tbody> </table>	W	o	l	f	r	a	m		L	a	n	g	l	f	r	a	m		L	a	n	g	u	a	r	a	m		L	a	n	g	u	a	g	e
W	o	l	f	r	a	m		L	a	n	g																										
l	f	r	a	m		L	a	n	g	u	a																										
r	a	m		L	a	n	g	u	a	g	e																										

Partition takes a list and breaks it into sublists. Flatten “flattens out” sublists.

Make a list of lists from digits of successive integers:

```
In[9]:= IntegerDigits/@Range[20]
```

```
Out[9]= {{1}, {2}, {3}, {4}, {5}, {6}, {7}, {8}, {9}, {1, 0}, {1, 1}, {1, 2}, {1, 3}, {1, 4}, {1, 5}, {1, 6}, {1, 7}, {1, 8}, {1, 9}, {2, 0}}
```

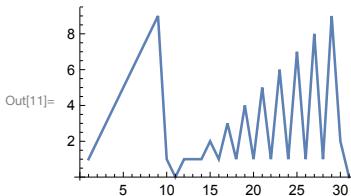
Make a flattened version:

```
In[10]:= Flatten[IntegerDigits/@Range[20]]
```

```
Out[10]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 0, 1, 1, 1, 2, 1, 3, 1, 4, 1, 5, 1, 6, 1, 7, 1, 8, 1, 9, 2, 0}
```

Make a plot from the sequence of digits:

```
In[11]:= ListLinePlot[Flatten[IntegerDigits/@Range[20]]]
```



Flatten will normally flatten out all levels of lists. But quite often you only want to flatten, say, one level of lists. This makes a 4×4 table in which each element is itself a list.

Make a list of lists of lists:

```
In[12]:= Table[IntegerDigits[i^j], {i, 4}, {j, 4}]
```

```
Out[12]= {{{1}, {1}, {1}, {1}}, {{2}, {4}, {8}, {16}}, {{3}, {9}, {27}, {81}}, {{4}, {16}, {64}, {256}}}
```

Flatten everything out:

```
In[13]:= Flatten[Table[IntegerDigits[i^j], {i, 4}, {j, 4}]]  
Out[13]= {1, 1, 1, 1, 2, 4, 8, 1, 6, 3, 9, 2, 7, 8, 1, 4, 1, 6, 6, 4, 2, 5, 6}
```

Flatten out only one level of list:

```
In[14]:= Flatten[Table[IntegerDigits[i^j], {i, 4}, {j, 4}], 1]  
Out[14]= {{1}, {1}, {1}, {1}, {2}, {4}, {8}, {1, 6}, {3}, {9}, {2, 7}, {8, 1}, {4}, {1, 6}, {6, 4}, {2, 5, 6}}
```

`ArrayFlatten` is a generalization of `Flatten`, which takes arrays of arrays, and flattens them into individual arrays.

This generates a deeply nested structure that's hard to understand:

```
In[15]:= NestList[{{#, 0}, {#, #}} &, {{1}}, 2]  
Out[15]= {{{1}}, {{{{1}}, 0}, {{{1}}, {{1}}}},  
 {{{{1}}, 0}, {{1}}, {{1}}}, 0}, {{{{1}}, 0}, {{{1}}, {{1}}}, {{1}}, {{1}}}, 0}, {{{{1}}, 0}, {{{1}}, {{1}}}}}
```

`ArrayFlatten` makes a structure that's a little easier to understand:

```
In[16]:= NestList[ArrayFlatten[{{#, 0}, {#, #}}] &, {{1}}, 2]  
Out[16]= {{1}, {1, 0}, {1, 1}, {{1, 0, 0, 0}, {1, 1, 0, 0}, {1, 0, 1, 0}, {1, 1, 1, 1}}}
```

With `ArrayPlot`, it's considerably easier to see what's going on:

```
In[17]:= ArrayPlot /@ NestList[ArrayFlatten[{{#, 0}, {#, #}}] &, {{1}}, 4]  
Out[17]= {
```

Generate a *fractal Sierpinski pattern* with 8 levels of nesting:

```
In[18]:= ArrayPlot[Nest[ArrayFlatten[{{#, 0}, {#, #}}] &, {{1}}, 8]]  
Out[18]=
```

There are many other ways to rearrange lists. For example, `Split` splits a list into runs of identical elements.

Split a list into sequences of successive identical elements:

```
In[19]:= Split[{1, 1, 1, 2, 2, 1, 1, 3, 1, 1, 1, 2}]
Out[19]= {{1, 1, 1}, {2, 2}, {1, 1}, {3}, {1, 1, 1}, {2}}
```

`Gather`, on the other hand, gathers identical elements together, wherever they appear.

Gather identical elements together in lists:

```
In[20]:= Gather[{1, 1, 1, 2, 2, 1, 1, 3, 1, 1, 1, 2}]
Out[20]= {{1, 1, 1, 1, 1, 1, 1}, {2, 2, 2}, {3}}
```

`GatherBy` gathers elements according to the result of applying a function to them.

Here it's using `LetterQ`, so that it gathers separately letters and non-letters.

Gather characters according to whether they are letters or not:

```
In[21]:= GatherBy[Characters["It's true that 2+2 is equal to 4!"], LetterQ]
Out[21]= {{l, t, s, t, r, u, e, t, h, a, t, i, s, e, q, u, a, l, t, o}, {' ', , , 2, +, 2, , , , 4, !}}
```

`SortBy` sorts according to the result of applying a function.

`Sort` normally sorts shorter lists before longer ones:

```
In[22]:= Sort[Table[IntegerDigits[2^n], {n, 10}]]
Out[22]= {{2}, {4}, {8}, {1, 6}, {3, 2}, {6, 4}, {1, 2, 8}, {2, 5, 6}, {5, 1, 2}, {1, 0, 2, 4}}
```

Here `SortBy` is told to sort by the first element in each list:

```
In[23]:= SortBy[Table[IntegerDigits[2^n], {n, 10}], First]
Out[23]= {{1, 6}, {1, 2, 8}, {1, 0, 2, 4}, {2}, {2, 5, 6}, {3, 2}, {4}, {5, 1, 2}, {6, 4}, {8}}
```

`Sort` sorts a list in order. `Union` also removes any repeated elements.

Find all the distinct elements that appear:

```
In[24]:= Union[{1, 9, 5, 3, 1, 4, 3, 1, 3, 5, 3, 9}]
Out[24]= {1, 3, 4, 5, 9}
```

You can use `Union` to find the “union” of elements that appear in any of several lists.

Get a list of all elements that appear in any of the lists:

```
In[25]:= Union[{2, 1, 3, 7, 9}, {4, 5, 1, 2, 3, 3}, {3, 1, 2, 8, 5}]
Out[25]= {1, 2, 3, 4, 5, 7, 8, 9}
```

Find which elements are common to all the lists:

```
In[26]:= Intersection[{2, 1, 3, 7, 9}, {4, 5, 1, 2, 3, 3}, {3, 1, 2, 8}]
```

```
Out[26]= {1, 2, 3}
```

Find which elements are in the first list but not the second one:

```
In[27]:= Complement[{4, 5, 1, 2, 3, 3}, {3, 1, 2, 8}]
```

```
Out[27]= {4, 5}
```

Find letters that appear in any of English, Swedish and Turkish:

```
In[28]:= Union[Alphabet["English"], Alphabet["Swedish"], Alphabet["Turkish"]]
```

```
Out[28]= {g, s, a, å, ä, b, c, ç, d, e, f, g, h, i, l, j, k, l, m, n, o, ö, p, q, r, s, t, u, ü, v, w, x, y, z}
```

Letters that appear in Swedish but not English:

```
In[29]:= Complement[Alphabet["Swedish"], Alphabet["English"]]
```

```
Out[29]= {å, ä, ö}
```

Another of the many functions you can apply to lists is **Riffle**, which inserts things between successive elements of a list.

Riffle *x* in between the elements of a list:

```
In[30]:= Riffle[{1, 2, 3, 4, 5}, x]
```

```
Out[30]= {1, x, 2, x, 3, x, 4, x, 5}
```

Riffle *--* into a list of characters:

```
In[31]:= Riffle[Characters["WOLFRAM"], "--"]
```

```
Out[31]= {W, --, O, --, L, --, F, --, R, --, A, --, M}
```

Join everything together in a single string:

```
In[32]:= StringJoin[Riffle[Characters["WOLFRAM"], "--"]]
```

```
Out[32]= W--O--L--F--R--A--M
```

Functions like **Partition** let you take a list and break it into sublists. Sometimes you'll instead just want to start from a collection of possible elements, and form lists from them.

Permutations gives all possible orderings, or *permutations*, of a list.

Generate a list of the $3! = 3 \times 2 \times 1 = 6$ possible orderings of 3 elements:

```
In[33]:= Permutations[{Red, Green, Blue}]
```

```
Out[33]= {{Red, Green, Blue}, {Red, Blue, Green}, {Green, Red, Blue}, {Green, Blue, Red}, {Blue, Red, Green}, {Blue, Green, Red}}
```

Generate all $2^3 = 8$ possible subsets of a list of 3 elements:

```
In[34]:= Subsets[{Red, Green, Blue}]
```

```
Out[34]= {{}, {Red}, {Green}, {Blue}, {Red, Green}, {Red, Blue}, {Green, Blue}, {Red, Green, Blue}}
```

Tuples takes a list of elements, and generates all possible combinations of a given number of those elements.

Generate a list of all possible triples of red and green:

```
In[35]:= Tuples[{Red, Green}, 3]
```

```
Out[35]= {{Red, Red, Red}, {Red, Red, Green}, {Red, Green, Red}, {Red, Green, Green}, {Green, Red, Red}, {Green, Red, Green}, {Green, Green, Red}, {Green, Green, Green}}
```

RandomChoice lets you make a random choice from a list of elements.

Make a single random choice from a list:

```
In[36]:= RandomChoice[{Red, Green, Blue}]
```

```
Out[36]= Green
```

Make a list of 20 random choices:

```
In[37]:= RandomChoice[{Red, Green, Blue}, 20]
```

```
Out[37]= {Blue, Green, Red, Green, Blue, Red, Blue, Green, Blue, Red, Green, Blue, Red, Blue, Green, Blue, Red, Blue, Green, Blue}
```

Make 5 lists of 3 random choices:

```
In[38]:= RandomChoice[{Red, Green, Blue}, {5, 3}]
```

```
Out[38]= {{Blue, Green, Blue}, {Blue, Red, Red}, {Red, Red, Red}, {Green, Red, Blue}, {Blue, Red, Red}}
```

RandomSample picks a random sample of elements from a list, never picking any particular element more than once.

Pick 20 elements from the range 1 to 100, never picking any number more than once:

```
In[39]:= RandomSample[Range[100], 20]
```

```
Out[39]= {82, 3, 93, 92, 39, 45, 63, 32, 79, 75, 34, 1, 11, 59, 98, 67, 38, 44, 28, 76}
```

If you don't say how many elements to pick you get a random ordering of the whole list:

```
In[40]:= RandomSample[Range[10]]
```

```
Out[40]= {2, 5, 1, 7, 4, 6, 3, 8, 9, 10}
```

Vocabulary

Transpose [<i>list</i>]	transpose inner and outer lists
Thread [<i>list</i> ₁ → <i>list</i> ₂]	thread across elements of lists
Partition [<i>list</i> , <i>n</i>]	partition into blocks of size <i>n</i>
Flatten [<i>list</i>]	flatten out all sublists
Flatten [<i>list</i> , <i>k</i>]	flatten out <i>k</i> levels of sublists
ArrayFlatten [<i>list</i>]	flatten arrays of arrays
Split [<i>list</i>]	split into runs of identical elements
Gather [<i>list</i>]	gather identical elements into lists
GatherBy [<i>list</i> , <i>f</i>]	gather according to results of applying <i>f</i>
SortBy [<i>list</i> , <i>f</i>]	sort according to results of applying <i>f</i>
Riffle [<i>list</i> , <i>x</i>]	riffle <i>x</i> between elements of <i>list</i>
Union [<i>list</i>]	distinct elements in <i>list</i>
Union [<i>list</i> ₁ , <i>list</i> ₂ , ...]	elements that appear in any of the lists
Intersection [<i>list</i> ₁ , <i>list</i> ₂ , ...]	elements that appear in all the lists
Complement [<i>list</i> ₁ , <i>list</i> ₂]	elements that appear in <i>list</i> ₁ but not <i>list</i> ₂
Permutations [<i>list</i>]	all possible permutations (orderings)
Subsets [<i>list</i>]	all possible subsets
Tuples [<i>list</i> , <i>n</i>]	all possible combinations of <i>n</i> elements
RandomChoice [<i>list</i>]	random choice from <i>list</i>
RandomChoice [<i>list</i> , <i>n</i>]	<i>n</i> random choices
RandomSample [<i>list</i> , <i>n</i>]	<i>n</i> random non-repeating samples
RandomSample [<i>list</i>]	random ordering of a list

Exercises

- 30.1 Use `Thread` to make a list of rules with each letter of the alphabet going to its position in the alphabet.
- 30.2 Make a 4×6 grid of the first 24 letters of the alphabet.
- 30.3 Make a grid of the digits in 2^{1000} , with 50 digits per row, and put frames around everything.
- 30.4 Make a grid of the first 400 characters in the Wikipedia article for “computers”, with 20 characters per row, and frames around everything.
- 30.5 Make a line plot of the flattened list of the digits from the numbers from 0 to 200 (*Champernowne sequence*).
- 30.6 Make 4 steps in the “*Menger sponge*” analog of the fractal Sierpinski pattern from the text, but with a “kernel” of the form $\{\{\#, \#, \#\}, \{\#, 0, \#\}, \{\#, \#, \#\}\}$.
- 30.7 Find *Pythagorean triples* involving only integers by selecting $\{x, y, \text{Sqrt}[x^2 + y^2]\}$ with x and y up to 20.
- 30.8 Find the lengths of the longest sequences of identical digits in 2^n for n up to 100.
- 30.9 Take the names of integers up to 100 and gather them into sublists according to their first letters.
- 30.10 Sort the first 50 words in `WordList[]` by their last letters.
- 30.11 Make a list of the first 20 squares, sorted by their first digits.
- 30.12 Sort integers up to 20 by the length of their names in English.
- 30.13 Get a random sample of 20 words from `WordList[]`, and gather them into sublists by length.
- 30.14 Find letters that appear in Ukrainian but not Russian.
- 30.15 Use `Intersection` to find numbers that appear both among the first 100 squares and cubes.
- 30.16 Find the list of countries that are in both NATO and the G8.
- 30.17 Make a grid in which all possible permutations of the numbers 1 through 4 appear as successive columns.
- 30.18 Make a list of all distinct strings that can be obtained by permuting the characters in “hello”.
- 30.19 Make an array plot of the sequence of possible 5-tuples of 0 and 1.
- 30.20 Generate a list of 10 random sequences of 5 letters.
- 30.21 Find a simpler form for `Flatten[Table[{i, j, k}, {i, 2}, {j, 2}, {k, 2}], 2]`.

Q&A**What does Partition do if the blocks don't fit perfectly?**

If it's not told otherwise, it'll only include complete blocks, so it'll just drop elements that appear only in incomplete blocks. You can tell it to do something different if you want.

Tech Note

- `DeleteDuplicates[list]` does the same as `Union[list]`, except it doesn't reorder elements.

More to Explore

Guide to Rearranging & Restructuring Lists in the Wolfram Language (wolfr.am/eiwl-30-more)

31 | Parts of Lists

Part lets you pick out an element of a list.

Pick out element 2 from a list:

```
In[1]:= Part[{a, b, c, d, e, f, g}, 2]
```

```
Out[1]= b
```

`[[...]]` is an alternative notation.

Use `[[2]]` to pick out element 2 from a list:

```
In[2]:= {a, b, c, d, e, f, g}[[2]]
```

```
Out[2]= b
```

Negative part numbers count from the end of a list:

```
In[3]:= {a, b, c, d, e, f, g}[[{-2}]]
```

```
Out[3]= f
```

You can ask for a list of parts by giving a list of part numbers.

Pick out parts 2, 4 and 5:

```
In[4]:= {a, b, c, d, e, f, g}[[{2, 4, 5}]]
```

```
Out[4]= {b, d, e}
```

`;;` lets you ask for a *span* or sequence of parts.

Pick out parts 2 through 5:

```
In[5]:= {a, b, c, d, e, f, g}[[2 ;; 5]]
```

```
Out[5]= {b, c, d, e}
```

Take the first 4 elements from a list:

```
In[6]:= Take[{a, b, c, d, e, f, g}, 4]
```

```
Out[6]= {a, b, c, d}
```

Take the last 2 elements from a list:

```
In[7]:= Take[{a, b, c, d, e, f, g}, -2]
```

```
Out[7]= {f, g}
```

Drop the last 2 elements from a list:

```
In[8]:= Drop[{a, b, c, d, e, f, g}, -2]
```

```
Out[8]= {a, b, c, d, e}
```

Let's now talk about lists of lists, or arrays. Each sublist acts as a *row* in the array.

```
In[9]:= {{a, b, c}, {d, e, f}, {g, h, i}} // Grid
```

```
a b c  
Out[9]= d e f  
g h i
```

Pick out the second sublist, corresponding to the second row, in the array:

```
In[10]:= {{a, b, c}, {d, e, f}, {g, h, i}}[[2]]
```

```
Out[10]= {d, e, f}
```

This picks out element 1 on row 2:

```
In[11]:= {{a, b, c}, {d, e, f}, {g, h, i}}[[2, 1]]
```

```
Out[11]= d
```

It's also often useful to pick out *columns* in an array.

Pick out the first column, by getting element 1 on all rows:

```
In[12]:= {{a, b, c}, {d, e, f}, {g, h, i}}[[All, 1]]
```

```
Out[12]= {a, d, g}
```

The function **Position** finds the list of positions at which something appears.

Here there's only one d, and it appears at position 2, 1:

```
In[13]:= Position[{{a, b, c}, {d, e, f}, {g, h, i}}, d]
```

```
Out[13]= {{2, 1}}
```

This gives a list of all positions at which x appears:

```
In[14]:= Position[{{x, y, x}, {y, y, x}, {x, y, y}, {x, x, y}}, x]
```

```
Out[14]= {{1, 1}, {1, 3}, {2, 3}, {3, 1}, {4, 1}, {4, 2}}
```

The positions at which "a" occurs in a list of characters:

```
In[15]:= Position[Characters["The Wolfram Language"], "a"]
```

```
Out[15]= {{10}, {14}, {18}}
```

Find the positions at which 0 occurs in the digit sequence of 2^{500} :

```
In[16]:= Flatten[Position[IntegerDigits[2^500], 0]]
```

```
Out[16]= {7, 9, 19, 20, 44, 47, 50, 65, 75, 88, 89, 96, 103, 115, 116, 119, 120, 137}
```

The function `ReplacePart` lets you replace parts of a list:

Replace part 3 with x:

```
In[17]:= ReplacePart[{a, b, c, d, e, f, g}, 3 → x]
```

```
Out[17]= {a, b, x, d, e, f, g}
```

Replace two parts:

```
In[18]:= ReplacePart[{a, b, c, d, e, f, g}, {3 → x, 5 → y}]
```

```
Out[18]= {a, b, x, d, y, f, g}
```

Replace 5 randomly chosen parts with “—”:

```
In[19]:= ReplacePart[Characters["The Wolfram Language"], Table[RandomInteger[20] → "—", 5]]
```

```
Out[19]= {T, h, e, , W, —, l, f, r, a, m, —, —, —, n, g, u, a, g, —}
```

Sometimes one wants particular parts of a list to just disappear. One can do this by replacing them with `Nothing`.

`Nothing` just disappears:

```
In[20]:= {1, 2, Nothing, 4, 5, Nothing}
```

```
Out[20]= {1, 2, 4, 5}
```

Replace parts 1 and 3 with `Nothing`:

```
In[21]:= ReplacePart[{a, b, c, d, e, f, g}, {1 → Nothing, 3 → Nothing}]
```

```
Out[21]= {b, d, e, f, g}
```

Take 50 random words, dropping ones longer than 5 characters, and reversing others:

```
In[22]:= If[StringLength[\#] > 5, Nothing, StringReverse[\#]] &/@ RandomSample[WordList[], 50]
```

```
Out[22]= {yllud, yciuj, poons, tsioh}
```

`Take` takes a specified number of elements in a list based on their position. `TakeLargest` and `TakeSmallest` take elements based on their size.

Take the 5 largest elements from a list:

```
In[23]:= TakeLargest[Range[20], 5]
```

```
Out[23]= {20, 19, 18, 17, 16}
```

`TakeLargestBy` and `TakeSmallestBy` take elements based on applying a function.

From the first 100 Roman numerals take the 5 that have the largest string length:

```
In[24]:= TakeLargestBy[Array[RomanNumeral, 100], StringLength, 5]
```

```
Out[24]= {LXXXVIII, LXXXIII, XXXVIII, LXXVIII, LXXXVII}
```

Vocabulary

<code>Part[list, n]</code>	part n of a list
<code>list[[n]]</code>	short notation for part n of a list
<code>list[[{n1, n2, ...}]]</code>	list of parts n_1, n_2, \dots
<code>list[[n1;;n2]]</code>	span (sequence) of parts n_1 through n_2
<code>list[[m, n]]</code>	element from row m , column n of an array
<code>list[[All, n]]</code>	all elements in column n
<code>Take[list, n]</code>	take the first n elements of a list
<code>TakeLargest[list, n]</code>	take the largest n elements of a list
<code>TakeSmallest[list, n]</code>	take the smallest n elements of a list
<code>TakeLargestBy[list, f, n]</code>	take elements largest by applying f
<code>TakeSmallestBy[list, f, n]</code>	take elements smallest by applying f
<code>Position[list, x]</code>	all positions of x in <i>list</i>
<code>ReplacePart[list, n → x]</code>	replace part n of <i>list</i> with x
<code>Nothing</code>	a list element that is automatically removed

Exercises

- 31.1** Find the last 5 digits in 2^{1000} .
- 31.2** Pick out letters 10 through 20 in the alphabet.
- 31.3** Make a list of the letters at even-numbered positions in the alphabet.
- 31.4** Make a line plot of the second to last digit in the first 100 powers of 12.
- 31.5** Join lists of the first 20 squares and cubes, and get the 10 smallest elements of the combined list.
- 31.6** Find the positions of the word “software” in the Wikipedia entry for “computers”.
- 31.7** Make a histogram of where the letter “e” occurs in the words in `WordList[]`.
- 31.8** Make a list of the first 100 cubes, with every one whose position is a square replaced by Red.
- 31.9** Make a list of the first 100 primes, dropping ones whose first digit is less than 5.
- 31.10** Make a grid starting with `Range[10]`, then at each of 9 steps randomly removing another element.
- 31.11** Find the longest 10 words in `WordList[]`.
- 31.12** Find the 5 longest integer names for integers up to 100.
- 31.13** Find the 5 English names for integers up to 100 that have the largest number of “e”s in them.

Q&A**How does one read $list[[n]]$ out loud?**

Usually “*list* part *n*” or “*list* sub *n*”. The second form (with “sub” short for “subscript”) comes from thinking about math and extracting components of vectors.

What happens if one asks for a part of a list that doesn’t exist?

One gets a message, and the original computation is returned undone.

Can I just get the first position at which something appears in a list?

Yes. Use `FirstPosition`.

Tech Notes

- `First` and `Last` are equivalent to `[[1]]` and `[[[-1]]`.
- In specifying parts, `1 ;; -1` is equivalent to `All`.

More to Explore

Guide to Parts of Lists in the Wolfram Language (wolfr.am/eiwl-31-more)

32 | Patterns

Patterns are a fundamental concept in the Wolfram Language. The pattern `_` (read “blank”) stands for anything.

`Cases` finds all cases in a list that match whatever pattern you specify.

Find all cases of sublists of two elements, where the elements can each be anything:

```
In[1]:= Cases[{{1, Red}, {1, Blue}, {1, Red, Blue}, {2, Green, Blue}, {2, Blue}}, {_,_}]  
Out[1]= {{1, Red}, {1, Blue}, {2, Blue}}
```

Find cases of a list consisting of the element 1 followed by anything:

```
In[2]:= Cases[{{1, Red}, {1, Blue}, {1, Red, Blue}, {2, Green, Blue}, {2, Blue}}, {1, _}]  
Out[2]= {{1, Red}, {1, Blue}}
```

In a pattern, `1 | 2` indicates “either 1 or 2”.

Find cases of either 1 or 2, followed by anything:

```
In[3]:= Cases[{{1, Red}, {1, Blue}, {1, Red, Blue}, {2, Green, Blue}, {2, Blue}}, {1 | 2, _}]  
Out[3]= {{1, Red}, {1, Blue}, {2, Blue}}
```

The notation `__` (“double blank”) indicates any sequence of things.

Find cases of any sequence of things, followed by blue:

```
In[4]:= Cases[{{1, Red}, {3, Blue}, {1, Red, Blue}, {2, Green, Blue}, {2, Blue}}, {___, Blue}]  
Out[4]= {{3, Blue}, {1, Red, Blue}, {2, Green, Blue}, {2, Blue}}
```

Find cases that start with 1 or 2, then have any sequence, then end with blue:

```
In[5]:= Cases[{{1, Red}, {3, Blue}, {1, Red, Blue}, {2, Green, Blue}, {2, Blue}}, {1 | 2, ___, Blue}]  
Out[5]= {{1, Red, Blue}, {2, Green, Blue}}
```

`Cases` can pick out anything from a list, not just lists that appear:

```
In[6]:= Cases[{f[Red], g[Green, Blue], {1, 2, 3}, f[Blue], f[3, 4]}, f[___]]  
Out[6]= {f[Red], f[Blue], f[3, 4]}
```

One of the many uses of patterns is to define replacements. `/.` (“slash dot”) performs a replacement.

Replace `b` with `Red` in a list:

```
In[7]:= {a, b, a, a, b, b, a, b} /. b → Red  
Out[7]= {a, Red, a, a, Red, Red, a, Red}
```

Replace all occurrences of 2-element lists that begin with 1:

```
In[8]:= {{1, Red}, {1, Blue}, {1, Red, Blue}, {2, Green, Blue}, {2, Blue}} /. {1, _} → x
Out[8]= {x, x, {1, Red, Blue}, {2, Green, Blue}, {2, Blue}}
```

You can have a list of replacements to use:

```
In[9]:= {{1, Red}, {1, Blue}, {1, Red, Blue}, {2, Green, Blue}, {2, Blue}} /. {{1, _} → x, {_ _, Blue} → y}
Out[9]= {x, x, y, y, y}
```

The “blank” pattern $_$ matches absolutely anything. This means, for example, that ${_, _}$ matches any list of two elements. But what if you want to insist that the two elements be the same? You can do that using a pattern like $\{x_, x_ \}$.

$\{_, _\}$ matches any list of two elements, whether the elements are the same or not:

```
In[10]:= Cases[{{Yellow, Yellow, Yellow}, {Yellow, Yellow}, {Yellow, Orange}, {Yellow, Purple}, {Orange, Yellow}, {Orange, Orange}, {Purple, Yellow}, {Orange, Orange}}, {_, _}]
Out[10]= {{Yellow, Yellow}, {Yellow, Orange}, {Yellow, Purple}, {Orange, Yellow}, {Orange, Orange}}
```

$\{x_, x_ \}$ matches only lists of two identical elements:

```
In[11]:= Cases[{{Yellow, Yellow, Yellow}, {Yellow, Yellow}, {Yellow, Orange}, {Yellow, Purple}, {Orange, Yellow}, {Orange, Orange}, {Purple, Yellow}, {Orange, Orange}}, {x_, x_}]
Out[11]= {{Yellow, Yellow}, {Orange, Orange}}
```

$x_$ is an example of a *named pattern*. Named patterns are especially important in replacements, because they give one way to make use of parts of what one’s replacing.

Use the named pattern $x_$ in a replacement:

```
In[12]:= {{1, Red}, {1, Blue}, {1, Red, Blue}, {2, Green, Blue}, {2, Blue}} /. {1, x_} → {x, x, x, Orange}
Out[12]= {{Red, Red, Red, Orange}, {Blue, Blue, Blue, Orange}, {1, Red, Blue}, {2, Green, Blue}, {2, Blue}}
```

The form $a \rightarrow b$ is usually called a *rule*. If $x_$ appears on the left-hand side of a rule, then whatever the $x_$ matches can be referred to on the right-hand side as x .

Use x in the right-hand side of the rule to refer to what $x_$ matches:

```
In[13]:= {f[1], g[2], f[2], f[6], g[3]} /. f[x_] → x + 10
Out[13]= {11, g[2], 12, 16, g[3]}
```

You can use rules inside `Cases` as well.

Pick out elements in the list that match $f[x_]$, and give the result of replacing them by $x + 10$:

```
In[14]:= Cases[{f[1], g[2], f[2], f[6], g[3]}, f[x_] → x + 10]
Out[14]= {11, 12, 16}
```

Later on, we’ll see how named patterns are crucial to defining your own functions in the Wolfram Language.

Vocabulary

<code>-</code>	pattern standing for anything (“blank”)
<code>--</code>	pattern standing for any sequence (“double blank”)
<code>x_</code>	pattern named <i>x</i>
<code>a b</code>	pattern matching <i>a</i> or <i>b</i>
<code>Cases[list, pattern]</code>	find cases of a pattern in a list
<code>lhs → rhs</code>	rule for transforming <i>lhs</i> into <i>rhs</i>
<code>expr /. lhs → rhs</code>	replace <i>lhs</i> by <i>rhs</i> in <i>expr</i>

Exercises

- 32.1** Find lists beginning with 1 and ending with 9 in `IntegerDigits[Range[1000]]`.
- 32.2** Find lists of three identical elements in `IntegerDigits[Range[1000]]`.
- 32.3** In the digit lists for the first 1000 squares, find those that begin with 9 and end with 0 or 1.
- 32.4** In `IntegerDigits[Range[1000]]` replace all 0's by Gray and all 9's by Orange.
- 32.5** Make a list of the digits of 2^{1000} , replacing all zeros by Red.
- 32.6** Remove the vowels a, e, i, o and u from the list of characters in “The Wolfram Language”.
- 32.7** Find a simpler form for `Select[IntegerDigits[2^1000], # == 0 || # == 1 &]`.
- 32.8** Find a simpler form for `Select[IntegerDigits[Range[100, 999]], First[#] == Last[#] &]`.

Q&A

Do the names of pattern variables (`x_` etc.) matter?

No. They just have to be consistent inside a given pattern. Different patterns can reuse the same name for different purposes, and the name can also appear outside the pattern.

What else can be used to define patterns?

We'll discuss several more things in Section 41.

What's the difference between `|` and `||`?

`p | q` is a pattern construct, that matches either `p` or `q`. `p || q` is a logic construct, that tests whether `p` or `q` is True.

How is `/.` interpreted?

It's the function `ReplaceAll`. `Replace` tries to replace a whole expression. `ReplaceList` gives a list of results from all possible ways to match a particular pattern.

If `/.` has several replacements, which one will it use?

It uses the first one that applies. If replacements apply to multiple levels of an expression, `/.` will use it on the outermost level.

Tech Notes

- Patterns for strings are discussed in Section 42.
- Pattern matching in the Wolfram Language takes account of facts such as the equivalence of $x+y$ and $y+x$, or $x+(y+z)$ and $(x+y)+z$. See Section 41.
- In writing $lhs \rightarrow rhs$, the *lhs* stands for “left-hand side” and the *rhs* for “right-hand side”.
- Patterns are *scoping constructs*, in the sense that they localize names like the *x* in *x_* to just the scope of the pattern.
- In the rare case that */.* is followed by a digit (like 0), you need to leave a space to avoid confusion with division.

More to Explore

Guide to Patterns in the Wolfram Language (wolfr.am/eiwl-32-more)

33 | Expressions and Their Structure

We've now seen all sorts of things that exist in the Wolfram Language: lists, graphics, pure functions and much more. And now we're ready to discuss a very fundamental fact about the Wolfram Language: that each of these things—and in fact everything the language deals with—is ultimately constructed in the same basic kind of way. Everything is what's called a *symbolic expression*.

Symbolic expressions are a very general way to represent structure, potentially with meaning associated with that structure. `f[x, y]` is a simple example of a symbolic expression. On its own, this symbolic expression doesn't have any particular meaning attached, and if you type it into the Wolfram Language, it'll just come back unchanged.

`f[x, y]` is a symbolic expression with no particular meaning attached:

```
In[1]:= f[x, y]
```

```
Out[1]= f[x, y]
```

`{x, y, z}` is another symbolic expression. Internally, it's `List[x, y, z]`, but it's displayed as `{x, y, z}`.

The symbolic expression `List[x, y, z]` displays as `{x, y, z}`:

```
In[2]:= List[x, y, z]
```

```
Out[2]= {x, y, z}
```

Symbolic expressions are often nested:

```
In[3]:= List[List[a, b], List[c, d]]
```

```
Out[3]= {{a, b}, {c, d}}
```

`FullForm` shows you the internal form of any symbolic expression.

```
In[4]:= FullForm[{{a, b}, {c, d}}]
```

```
Out[4]= List[List[a, b], List[c, d]]
```

`Graphics[Circle[{0, 0}]]` is another symbolic expression, that just happens to display as a picture of a circle. `FullForm` shows its internal structure.

This symbolic expression displays as a circle:

```
In[5]:= Graphics[Circle[{0, 0}]]
```



`FullForm` shows its underlying symbolic expression structure:

```
In[6]:= FullForm[]
```

```
Out[6]= Graphics[Circle[List[0, 0]]]
```

Symbolic expressions often don't just display in special ways, they actually *evaluate* to give results.

A symbolic expression that evaluates to give a result:

```
In[7]:= Plus[2, 2]
```

```
Out[7]= 4
```

The elements of the list evaluate, but the list itself stays symbolic:

```
In[8]:= {Plus[3, 3], Times[3, 3], Power[3, 3]}
```

```
Out[8]= {6, 9, 27}
```

Here's the symbolic expression structure of the list:

```
In[9]:= {Plus[3, 3], Times[3, 3], Power[3, 3]} // FullForm
```

```
Out[9]= List[6, 9, 27]
```

This is just a symbolic expression that happens to evaluate:

```
In[10]:= Blur[, 5]
```

```
Out[10]= 
```

You could write it like this:

```
In[11]:= Blur[Graphics[Circle[{0, 0}]], 5]
```

```
Out[11]= 
```

What are the ultimate building blocks of symbolic expressions? They're called *atoms* (after the building blocks of physical materials). The main kinds of atoms are numbers, strings and *symbols*.

Things like `x`, `y`, `f`, `Plus`, `Graphics` and `Table` are all symbols. Every symbol has a unique name. Sometimes it'll also have a meaning attached. Sometimes it'll be associated with evaluation. Sometimes it'll just be part of defining a structure that other functions can use. But it doesn't have to have any of those things; it just has to have a name.

A crucial defining feature of the Wolfram Language is that it can handle symbols purely as symbols—“symbolically”—without them having to evaluate, say to numbers.

In the Wolfram Language, x can just be x , without having to evaluate to anything:

In[12]:= x

Out[12]= x

x does not evaluate, but the addition is still done, here according to the laws of algebra:

In[13]:= $x+x+x+2y+y+x$

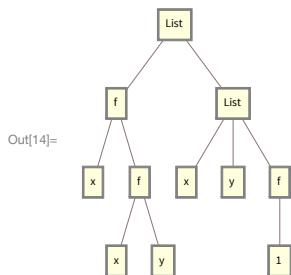
Out[13]= $4x+3y$

Given symbols like x , y and f , one can build up an infinite number of expressions from them. There's $f[x]$, and $f[y]$, and $f[x, y]$. Then there's $f[f[x]]$ or $f[x, f[x, y]]$, or, for that matter, $x[x][y, f[x]]$ or whatever.

In general, each expression corresponds to a tree, whose ultimate “leaves” are atoms. You can display an expression as a tree using `TreeForm`.

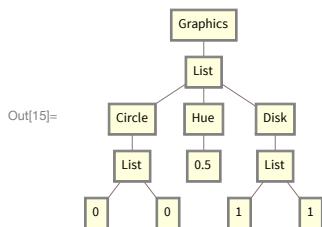
An expression shown in tree form:

In[14]:= `TreeForm[{f[x], f[x, y]}, {x, y, f[1]}]`



Here's a graphics expression shown in tree form:

In[15]:= `TreeForm[Graphics[{Circle[{0, 0}], Hue[0.5], Disk[{1, 1}]}]]`



Because expressions ultimately have a very uniform structure, operations in the Wolfram Language operate in a very uniform way on them.

For example, any expression has parts, just like a list, and you can extract them using `[[...]]`.

This is equivalent to `{x, y, z}[[2]]`, which extracts the second element in a list:

```
In[16]:= List[x, y, z][[2]]
```

```
Out[16]= y
```

Extracting parts works exactly the same way for this expression:

```
In[17]:= f[x, y, z][[2]]
```

```
Out[17]= y
```

This extracts the coordinates of the center of the circle:

```
In[18]:= Graphics[Circle[{0, 0}]][[1, 1]]
```

```
Out[18]= {0, 0}
```

This works exactly the same:

```
In[19]:= Circle[[1, 1]]
```

```
Out[19]= {0, 0}
```

In `f[x, y]`, `f` is called the *head* of the expression. `x` and `y` are called *arguments*.

The function `Head` extracts the head of an expression.

The head of a list is `List`:

```
In[20]:= Head[{x, y, z}]
```

```
Out[20]= List
```

Every part of an expression has a head, even its atoms.

The head of an integer is `Integer`:

```
In[21]:= Head[1234]
```

```
Out[21]= Integer
```

The head of an approximate real number is `Real`:

```
In[22]:= Head[12.45]
```

```
Out[22]= Real
```

The head of a string is `String`:

```
In[23]:= Head["hello"]
```

```
Out[23]= String
```

Even symbols have a head: `Symbol`.

```
In[24]:= Head[x]
```

```
Out[24]= Symbol
```

In patterns, you can ask to match expressions with particular heads. `_Integer` represents any integer, `_String` any string, and so on.

`_Integer` is a pattern that matches only objects with head `Integer`:

```
In[25]:= Cases[{x, y, 3, 4, z, 6, 7}, _Integer]
```

```
Out[25]= {3, 4, 6, 7}
```

Named patterns can have specified heads too:

```
In[26]:= Cases[{99, x, y, z, 101, 102}, n_Integer → {n, n}]
```

```
Out[26]= {{99, 99}, {101, 101}, {102, 102}}
```

In using the Wolfram Language, most of the heads you'll see are symbols. But there are important cases where there are more complicated heads. One such case is pure functions—where when you apply a pure function, the pure function appears as the head.

Here is the full form of a pure function (# is `Slot[1]`):

```
In[27]:= FullForm[#^2 &]
```

```
Out[27]= Function[Power[Slot[1], 2]]
```

When you apply the pure function, it appears as a head:

```
In[28]:= Function[Power[Slot[1], 2]] [1000]
```

```
Out[28]= 1000000
```

As you become more sophisticated in Wolfram Language programming, you'll encounter more and more examples of complicated heads. In fact, many functions that we've already discussed have *operator forms* where they appear as heads—and using them in this way leads to very powerful and elegant styles of programming.

`Select` appears as a head here:

```
In[29]:= Select[# > 4 &][{1, 2.2, 3, 4.5, 5, 6, 7.5, 8}]
```

```
Out[29]= {4.5, 5, 6, 7.5, 8}
```

Both `Cases` and `Select` appear as heads here:

```
In[30]:= Cases[_Integer]@Select[# > 4 &]@{1, 2.2, 3, 4.5, 5, 6, 7.5, 8}
```

```
Out[30]= {5, 6, 8}
```

All the basic structural operations that we have seen for lists work exactly the same for arbitrary expressions.

`Length` does not care what the head of an expression is; it just counts arguments:

```
In[31]:= Length[f[x, y, z]]
```

```
Out[31]= 3
```

`/@` does not care about the head of an expression either; it just applies a function to the arguments:

```
In[32]:= f/@g[x, y, z]
```

```
Out[32]= g[f[x], f[y], f[z]]
```

Since there are lots of functions that generate lists, it's often convenient to build up structures as lists even if eventually one needs to replace the lists with other functions.

`@@` effectively replaces the head of the list with `f`:

```
In[33]:= f@@@{x, y, z}
```

```
Out[33]= f[x, y, z]
```

This yields `Plus[1, 1, 1, 1]`, which then evaluates:

```
In[34]:= Plus@@@{1, 1, 1, 1}
```

```
Out[34]= 4
```

This turns a list into a rule:

```
In[35]:= #1 → #2 & @@{x, y}
```

```
Out[35]= x → y
```

Here's a simpler alternative, without the explicit pure function:

```
In[36]:= Rule@@@{x, y}
```

```
Out[36]= x → y
```

A surprisingly common situation is to have a list of lists, and to want to replace the inner lists with some function. It's possible to do this with `@@` and `/@`. But `@@@` provides a convenient direct way to do it.

Replace the inner lists with `f`:

```
In[37]:= f@@@@{{1, 2, 3}, {4, 5, 6}}
```

```
Out[37]= {f[1, 2, 3], f[4, 5, 6]}
```

Turn the inner lists into rules:

```
In[38]:= Rule@@@{{1, 10}, {2, 20}, {3, 30}}
```

```
Out[38]= {1 → 10, 2 → 20, 3 → 30}
```

Here's an example of how `@@@` can help construct a graph from a list of pairs.

This generates a list of pairs of characters:

```
In[39]:= Partition[Characters["antidisestablishmentarianism"], 2, 1]
```

```
Out[39]= {{a, n}, {n, t}, {t, i}, {i, d}, {d, i}, {i, s}, {s, e}, {e, s}, {s, t}, {t, a}, {a, b}, {b, l}, {l, i}, {i, s}, {s, h}, {h, m}, {m, e}, {e, n}, {n, t}, {t, a}, {a, r}, {r, i}, {i, a}, {a, n}, {n, i}, {i, s}, {s, m}}
```

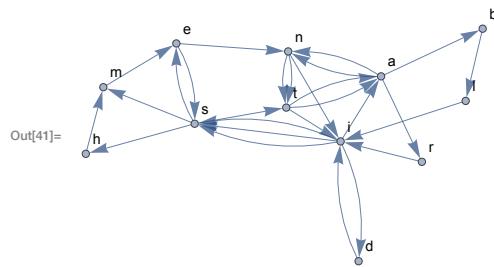
Turn this into a list of rules:

```
In[40]:= Rule@@@Partition[Characters["antidisestablishmentarianism"], 2, 1]
```

```
Out[40]= {a → n, n → t, t → i, i → d, d → i, i → s, s → e, e → s, s → t, t → a, a → b, b → l, l → i, i → s, s → h, h → m, m → e, e → n, n → t, t → a, a → r, r → i, i → a, a → n, n → i, i → s, s → m}
```

Form a *transition graph* showing how letters follow each other:

```
In[41]:= Graph[Rule@@@Partition[Characters["antidisestablishmentarianism"], 2, 1], VertexLabels → All]
```



Vocabulary

FullForm [<i>expr</i>]	show full internal form
---------------------------------	-------------------------

TreeForm [<i>expr</i>]	show tree structure
---------------------------------	---------------------

Head [<i>expr</i>]	extract the head of an expression
-----------------------------	-----------------------------------

_head	match any expression with a particular head
--------------	---

f @@ list	replace the head of <i>expr</i> with <i>f</i>
------------------	---

f @@@ {list₁, list₂, ...}	replace heads of list ₁ , list ₂ , ... with <i>f</i>
--	--

Exercises

- 33.1** Find the head of the output from `ListPlot`.
- 33.2** Use `@@` to compute the result of multiplying together integers up to 100.
- 33.3** Use `@@@` and `Tuples` to generate `{f[a, a], f[a, b], f[b, a], f[b, b]}`.
- 33.4** Make a list of tree forms for the results of 4 successive applications of `#^#` & starting from `x`.
- 33.5** Find the unique cases where $i^2/(j^2+1)$ is an integer, with `i` and `j` going up to 20.
- 33.6** Create a graph that connects successive pairs of numbers in `Table[Mod[n^2+n, 100], {n, 100}]`.
- 33.7** Generate a graph showing which word can follow which in the first 200 words of the Wikipedia article on computers.
- 33.8** Find a simpler form for `f@@@# & /@ {{1, 2}, {7, 2}, {5, 4}}`.

Q&A

How are `@@` and `@@@` interpreted?

`f@@@expr` is `Apply[f, expr]`. `f@@@@expr` is `Apply[f, expr, {1}]`. They're usually just read as "double at" and "triple at".

Are all expressions in the Wolfram Language trees?

At a structural level, yes. When there are variables with values assigned (see Section 38), though, they can behave more like directed graphs. And of course one can use `Graph` to represent any graph as an expression in the Wolfram Language.

Tech Notes

- The basic concept of symbolic languages comes directly from work in mathematical logic stretching back to the 1930s and before, but other than in the Wolfram Language it's been very rare for it to be implemented in practice.
- Wolfram Language expressions are a bit like XML expressions (and can be converted to and from them). But unlike XML expressions, Wolfram Language expressions can evaluate so that they automatically change their structure.
- Things like `Select[f]` that are set up to be applied to expressions are called *operator forms*, by analogy with operators in mathematics. Using `Select[f][expr]` instead of `Select[expr, f]` is often called *currying*, after a logician named Haskell Curry.
- Symbols like `x` can be used to represent algebraic variables or "unknowns". This is central to doing many kinds of mathematics in the Wolfram Language.
- `LeafCount` gives the total number of atoms at the leaves of an expression tree. `ByteCount` gives the number of bytes needed to store the expression.

More to Explore

[Guide to Expressions in the Wolfram Language](http://wolfr.am/eiwl-33-more) (`wolfr.am/eiwl-33-more`)

34 | Associations

Associations are a kind of generalization of lists, in which every element has a key as well as a value. `Counts` is a typical function that produces an association.

`Counts` gives an association that reports how many times each different element occurs:

```
In[1]:= Counts[{a, a, b, c, a, a, b, c, c, a, a}]
Out[1]= <| a -> 6, b -> 2, c -> 3 |>
```

You can get the value associated with a particular key using [...].

Find the value associated with `c` in the association:

```
In[2]:= <| a -> 6, b -> 2, c -> 3 |> [c]
Out[2]= 3
```

Operations that work on lists typically also work on associations—but apply themselves only to the values, not the keys.

This adds 500 to each value in the association:

```
In[3]:= <| a -> 6, b -> 2, c -> 3 |> +500
Out[3]= <| a -> 506, b -> 502, c -> 503 |>
```

`/@` applies a function to each value in the association:

```
In[4]:= f/@ <| a -> 6, b -> 2, c -> 3 |>
Out[4]= <| a -> f[6], b -> f[2], c -> f[3] |>
```

`Total` gives the total of the values:

```
In[5]:= Total[ <| a -> 6, b -> 2, c -> 3 |> ]
Out[5]= 11
```

`Sort` operates on the values:

```
In[6]:= Sort[ <| a -> 6, b -> 2, c -> 3 |> ]
Out[6]= <| b -> 2, c -> 3, a -> 6 |>
```

`KeySort` operates on the keys:

```
In[7]:= KeySort[ <| c -> 1, b -> 2, a -> 4 |> ]
Out[7]= <| a -> 4, b -> 2, c -> 1 |>
```

The functions **Keys** and **Values** extract the keys and values of an association.

Get the list of keys in the association:

```
In[8]:= Keys[ <| a → 6, b → 2, c → 3 |> ]
```

```
Out[8]= {a, b, c}
```

Get the list of values in the association:

```
In[9]:= Values[ <| a → 6, b → 2, c → 3 |> ]
```

```
Out[9]= {6, 2, 3}
```

Normal turns an association into a normal list of rules. **Association** makes an association from a list of rules.

```
In[10]:= Normal[ <| a → 6, b → 2, c → 3 |> ]
```

```
Out[10]= {a → 6, b → 2, c → 3}
```

```
In[11]:= Association[{a → 6, b → 2, c → 3}]
```

```
Out[11]= <| a → 6, b → 2, c → 3 |>
```

LetterCounts counts how many times letters occur in a string.

Count how many times each letter occurs in the Wikipedia article on computers:

```
In[12]:= LetterCounts[WikipediaData["computers"]]
```

```
Out[12]= <| e → 4833, t → 3528, a → 3207, o → 3059, r → 2907, i → 2818, n → 2747, s → 2475, c → 1800,
l → 1673, m → 1494, h → 1473, u → 1357, d → 1329, p → 1153, g → 818, f → 766,
y → 594, b → 545, w → 456, v → 391, k → 174, T → 150, A → 110, I → 101, C → 84,
M → 82, x → 77, S → 68, P → 64, q → 58, U → 55, B → 45, H → 43, E → 42, R → 41,
L → 41, z → 38, O → 38, D → 37, W → 30, N → 29, F → 28, j → 25, G → 23, J → 17,
K → 14, V → 10, Z → 8, Q → 4, ū → 4, ī → 2, ā → 2, Y → 1, é → 1, á → 1 |>
```

KeyTake picks out elements of an association that appear in a list of keys you specify. Here we're taking the elements whose keys are letters in the (lowercase) alphabet.

Take only those elements in the association whose keys appear as letters in the alphabet:

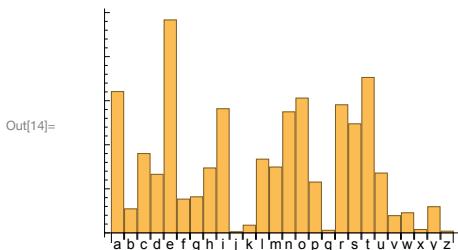
```
In[13]:= KeyTake[LetterCounts[WikipediaData["computers"]], Alphabet[]]
```

```
Out[13]= <| a → 3207, b → 545, c → 1800, d → 1329, e → 4833, f → 766, g → 818, h → 1473,
i → 2818, j → 25, k → 174, l → 1673, m → 1494, n → 2747, o → 3059, p → 1153, q → 58,
r → 2907, s → 2475, t → 3528, u → 1357, v → 391, w → 456, x → 77, y → 594, z → 38 |>
```

`BarChart` plots the values in an association. With the option `ChartLabels → Automatic`, it uses the keys as labels.

Make a bar chart of how many times each letter appears; “e” is the most common:

```
In[14]:= BarChart[KeyTake[LetterCounts[WikipediaData["computers"]], Alphabet[]],  
ChartLabels → Automatic]
```



Here’s a direct way to apply a pure function to an association:

Apply a pure function to an association:

```
In[15]:= f[#[{"apples"}, #[{"oranges"}] &[ <| "apples" → 10, "oranges" → 12, "pears" → 4 |> ]  
Out[15]= f[10, 12]
```

It’s very common to have keys that are strings, and the Wolfram Language has a special way to handle these when it comes to pure functions: you can just use `#key` to refer to an element whose key is “key”.

Use the simpler notation for association elements whose keys are strings:

```
In[16]:= f[##apples, ##oranges] &[ <| "apples" → 10, "oranges" → 12, "pears" → 4 |> ]  
Out[16]= f[10, 12]
```

As a more realistic example, apply a pure function that extracts the value for “e” from the letter counts, and divides by the total. The `N` gives the result as a decimal.

Compute the fraction of letters in the “computers” article that are “e”:

```
In[17]:= #e/Total[##] & @LetterCounts[WikipediaData["computers"]] // N  
Out[17]= 0.11795
```

Vocabulary

<code>< key₁ → value₁, key₂ → value₂, ... ></code>	an association
<code>Association[rules]</code>	turn a list of rules into an association
<code>assoc[key]</code>	extract an element of an association
<code>Keys[assoc]</code>	list of keys in an association
<code>Values[assoc]</code>	list of values in an association
<code>Normal[assoc]</code>	turn an association into a list of rules
<code>KeySort[assoc]</code>	sort an association by its keys
<code>KeyTake[assoc, keys]</code>	take elements with particular keys
<code>#key</code>	function slot for an element with key “key”
<code>Counts[list]</code>	an association with counts of distinct elements
<code>LetterCounts[string]</code>	an association with counts of distinct letters

Exercises

- 34.1** Make a list, in order, of the number of times each of the digits 0 through 9 occurs in 3^{100} .
- 34.2** Make a labeled bar chart of the number of times each of the digits 0 through 9 occurs in 2^{1000} .
- 34.3** Make a labeled bar chart of the number of times each possible first letter occurs in words from `WordList[]`.
- 34.4** Make an association giving the 5 most common first letters of words in `WordList[]` and their counts.
- 34.5** Find the numerical ratio of the number of occurrences of “q” and “u” in the Wikipedia entry for computers.
- 34.6** Find the 10 most common words in `ExampleData[{"Text", "AliceInWonderland"}]`.

Q&A

Why are associations called “associations”?

Because they *associate* values with keys. Other names used for the same concept are associative arrays, dictionaries, hashmaps, structs, key-value maps and symbolically indexed lists.

Can an association have several elements with the same key?

No. Keys in an association are always unique.

What happens if I ask for a key that’s not in an association?

Normally you get `Missing[...]`. But if you use `Lookup` to look up the key, you can specify what to return if the key is absent.

How can I do operations on the keys of an association?

Use KeyMap, or use functions like KeySelect and KeyDrop. AssociationMap creates an association by mapping a function over keys.

How can I combine several associations into one?

Use Merge. You have to give a function to say what to do if the same key occurs in multiple associations.

Can one use [...] to extract part of an association, like one extracts part of a list?

Yes, if you explicitly say `assoc[[Key[key]]].assoc[[2]]`, for example, will extract the second element of `assoc`, whatever key it has. `assoc[[key]]` is a special case that works the same as `assoc[key]`.

What happens in pure functions if the keys in an association aren't strings?

You can't use `#key` anymore; you have to explicitly use `#key`.

Tech Notes

- Most functions effectively operate on associations as if they were operating on lists of their values. Functions that thread themselves over lists typically do the same over associations.
- Associations are like tables in a relational database. JoinAcross does the analog of a database join.

More to Explore

Guide to Associations in the Wolfram Language (wolfr.am/eiwl-34-more)

35 | Natural Language Understanding

We saw earlier how to use `Ctrl + =` to enter natural language input. Now we're going to talk about how to set up functions that understand natural language.

`Interpreter` is the key to much of this. You tell `Interpreter` what type of thing you want to get, and it will take any string you provide, and try to interpret it that way.

Interpret the string "nyc" as a city:

```
In[1]:= Interpreter["City"]["nyc"]
```

```
Out[1]= New York City
```

"The big apple" is a nickname for New York City:

```
In[2]:= Interpreter["City"]["the big apple"]
```

```
Out[2]= New York City
```

Interpret the string "hot pink" as a color:

```
In[3]:= Interpreter["Color"]["hot pink"]
```

```
Out[3]=
```



`Interpreter` converts natural language to Wolfram Language expressions that you can compute with. Here's an example involving currency amounts.

Interpret various currency amounts:

```
In[4]:= Interpreter["CurrencyAmount"][
  {"4.25 dollars", "34 russian rubles", "5 euros", "85 cents"}]
```

```
Out[4]= { $4.25, py634, €5, 85¢ }
```

Compute the total, doing conversions at current exchange rates:

```
In[5]:= Total[{ $4.25, py634, €5, 85¢ }]
```

```
Out[5]= $11.07
```

Here's another example, involving locations.

`Interpreter` gives the geo location of the White House:

```
In[6]:= Interpreter["Location"]["White House"]
```

```
Out[6]= GeoPosition[{38.8977, -77.0366}]
```

It can also work from a street address:

```
In[7]:= Interpreter["Location"]["1600 Pennsylvania Avenue, Washington, DC"]
```

```
Out[7]= GeoPosition[{38.8977, -77.0366}]
```

Interpreter handles many hundreds of different types of objects.

Interpret names of universities (which “U of I” is picked depends on geo location):

```
In[8]:= Interpreter["University"][{"Harvard", "Stanford", "U of I"}]
```

```
Out[8]= {Harvard University, Stanford University,  
University of Illinois at Urbana-Champaign}
```

Interpret names of chemicals:

```
In[9]:= Interpreter["Chemical"][{"H2O", "aspirin", "CO2", "wolfram"}]
```

```
Out[9]= {water, aspirin, carbon dioxide, tungsten}
```

Interpret names of animals, then get images of them:

```
In[10]:= EntityValue[Interpreter["Animal"][{"cheetah", "tiger", "elephant"}], "Image"]
```

```
Out[10]= {, , }
```

Interpreter interprets whole strings. TextCases, on the other hand, tries to pick out instances of what you request from a string.

Pick out the nouns in a piece of text:

```
In[11]:= TextCases["A sentence is a linguistic construct", "Noun"]
```

```
Out[11]= {sentence, construct}
```

Pick out currency amounts:

```
In[12]:= TextCases["Choose between $5, €5 and ¥5000", "CurrencyAmount"]
```

```
Out[12]= {$5, €5, ¥5000}
```

You can use `TextCases` to pick out particular kinds of things from a piece of text. Here we pick out instances of country names in a Wikipedia article.

Generate a word cloud of country names from the Wikipedia article on the EU:

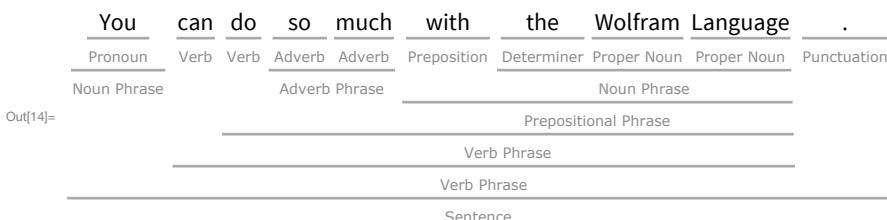
```
In[13]:= WordCloud[TextCases[WikipediaData["EU"], "Country"]]
```



TextStructure shows you the whole structure of a piece of text.

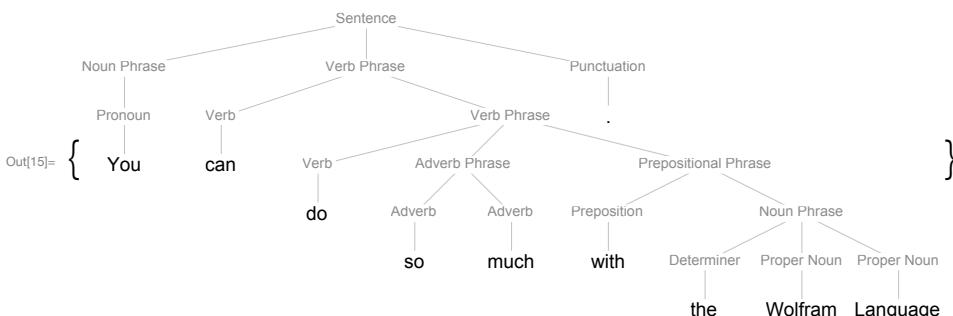
Find how a sentence of English can be parsed into grammatical units:

```
In[14]:= TextStructure["You can do so much with the Wolfram Language."]
```



An alternative representation, as a graph:

In[15]:= TextStructure["You can do so much with the Wolfram Language.", "ConstituentGraphs"]



Vocabulary

<code>Interpreter["type"]</code>	specify a function to interpret natural language
<code>TextCases["text", "type"]</code>	find cases of a given type of object in <i>text</i>
<code>TextStructure["text"]</code>	find the grammatical structure of <i>text</i>

Exercises

- 35.1** Use `Interpreter` to find the location of the Eiffel Tower.
- 35.2** Use `Interpreter` to find a university referred to as “U of T”.
- 35.3** Use `Interpreter` to find the chemicals referred to as C2H4, C2H6 and C3H8.
- 35.4** Use `Interpreter` to interpret the date “20140108”.
- 35.5** Find universities that can be referred to as “U of X”, where x is any letter of the alphabet.
- 35.6** Find which US state capital names can be interpreted as movie titles (use `CommonName` to get the string versions of entity names).
- 35.7** Find cities that can be referred to by permutations of the letters a, i, l and m.
- 35.8** Make a word cloud of country names in the Wikipedia article on “gunpowder”.
- 35.9** Find all nouns in “She sells seashells by the sea shore.”
- 35.10** Use `TextCases` to find the number of nouns, verbs and adjectives in the first 1000 characters of the Wikipedia article on computers.
- 35.11** Find the grammatical structure of the first sentence of the Wikipedia article about computers.
- 35.12** Find the 10 most common nouns in `ExampleData[{"Text", "AliceInWonderland"}]`.
- 35.13** Make a community graph plot of the graph representation of the text structure of the first sentence of the Wikipedia article about language.

Q&A

What possible types of interpreters are there?

It's a long list. Check out the documentation, or evaluate `$InterpreterTypes` to see the list.

Does Interpreter need a network connection?

In simple cases, such as dates or basic currency, no. But for full natural-language input, yes.

When I say “4 dollars”, how does it know if I want US dollars or something else?

It uses what it knows of your geo location to tell what kind of dollars you're likely to mean.

Can Interpreter deal with arbitrary natural language?

If something can be expressed in the Wolfram Language, then `Interpreter` should be able to interpret it. `Interpreter["SemanticExpression"]` takes any input, and tries to understand its meaning so as to get a Wolfram Language expression that captures it. What it's doing is essentially the first stage of what `Wolfram|Alpha` does.

Can I add my own interpreters?

Yes. `GrammarRules` lets you build up your own grammar, making use of whatever existing interpreters you want.

What about languages other than English?

`Interpreter` mostly works only with English. But for example `WordTranslation` translates words to and from many languages.

Can I find the meaning of a word?

`WordDefinition` gives dictionary definitions.

Tech Notes

- TextStructure requires complete grammatical text, but Interpreter uses many different techniques to also work with fragments of text.
 - When you use `Ctrl + =` you can resolve ambiguous input interactively. With Interpreter you have to do it programmatically, using the option `AmbiguityFunction`.
-

More to Explore

Guide to Natural Language Interpreters in the Wolfram Language (wolfr.am/eiwl-35-more)

36 | Creating Websites and Apps

The Wolfram Language makes it easy to put anything you create onto the web.

Create some graphics:

```
In[1]:= GeoGraphics[GeoRange → All, GeoProjection → "Albers"]
```

Out[1]=

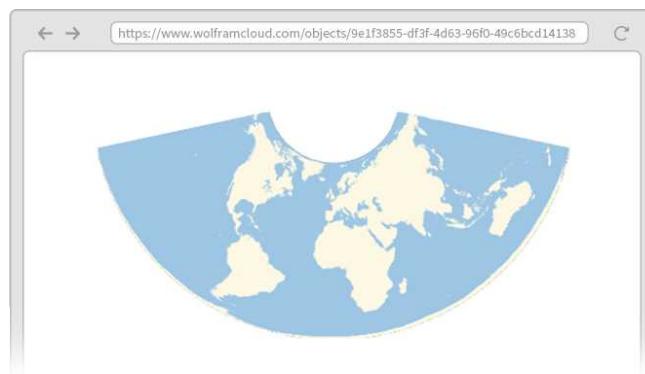


Deploy it to the cloud:

```
In[2]:= CloudDeploy[GeoGraphics[GeoRange → All, GeoProjection → "Albers"]]
```

```
Out[2]= CloudObject[https://www.wolframcloud.com/objects/9e1f3855-df3f-4d63-96f0-49c6bcd14138]
```

If you don't tell it a location, `CloudDeploy` will set up a new web page, with a unique address. If you go to that web page, you'll find your graphics there.



If you want the whole world to be able to see what you created, you can do that too (at least so long as you've got appropriate access to the Wolfram Cloud).

Deploy to the Wolfram Cloud, giving everyone permission to see what you've made:

```
In[3]:= CloudDeploy[Graphics[{Red, Disk[]}], Permissions → "Public"]
```

```
Out[3]= CloudObject[https://www.wolframcloud.com/objects/b76ab315-ee3a-4400-bed8-66c3c9b07c22]
```

Anyone who has the web address (URL) can now go there to see what you've made. URLShorten makes a short URL that's easier to tell to people.

Make a short URL for the web page you've created:

```
In[4]:= URLShorten[CloudDeploy[Graphics[{Red, Disk[]}], Permissions -> "Public"]]
```

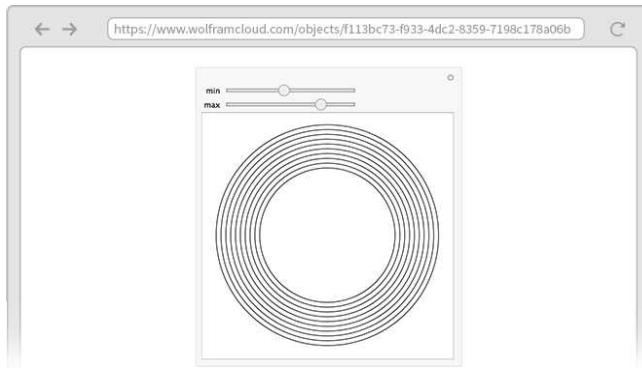
```
Out[4]= https://wolfr.am/7vm~o2zC
```

You can deploy active content, like Manipulate, to the web too.

```
In[5]:= CloudDeploy[Manipulate[
  Graphics[Table[Circle[{0, 0}, r], {r, min, max}]], {min, 1, 30, 1}, {max, 1, 30, 1}]]
```

```
Out[5]= CloudObject[https://www.wolframcloud.com/objects/f113bc73-f933-4dc2-8359-7198c178a06b]
```

Now you'll get a web page with active sliders and so on. You'll be able to use them in any standard web browser, though because they have to communicate over the internet, they'll run slower than if they were directly on your computer.



CloudDeploy normally works by first doing whatever computation it can, and then putting the result on the web. This means, for example, that CloudDeploy[Now] will make a web page that always just shows the time when it was deployed. If instead you want to make a web page that gives the current time every time the page is requested, you can do this with CloudDeploy[Delayed[Now]].

Use Delayed to make a clock that's regenerated every time the web page is requested:

```
In[6]:= CloudDeploy[Delayed[ClockGauge[Now]]]
```

```
Out[6]= CloudObject[https://www.wolframcloud.com/objects/94aaaf4ad-daea-4fe5-a50f-97b146a8b6ff]
```

Now every time you go to the web page, it'll regenerate the clock in the Wolfram Cloud, and put a new version onto the web page.



You can create a “real-time dashboard” by specifying an update interval.

Set up the web page you create to automatically update itself every 2 seconds:

```
In[7]:= CloudDeploy[Delayed[ClockGauge[Now], UpdateInterval → 2]]
```

```
Out[7]= CloudObject[https://www.wolframcloud.com/objects/88e8fb8a-6d50-4474-b52a-6458a9aacca1]
```



We've talked about things in terms of web pages. But actually, all of this works on mobile too, where you're for example viewing things through the Wolfram Cloud app.

But what about making your own apps on the web or on mobile? In the Wolfram Language it's for example easy to set up a form-based app.

The basic idea is to set up a `FormFunction` that defines both the structure of the form, and the action that's performed when the form is submitted.

Let's set up a form with one field labeled `name` that expects the name of an animal, then generates an image of that animal—and then deploys this to the cloud.

Set up a form-based app with a single input field for entering the name of an animal:

```
In[8]:= CloudDeploy[FormFunction[{"name" → "Animal"}, #&name["Image"] &]]
```

```
Out[8]= CloudObject[https://www.wolframcloud.com/objects/6925826b-776e-429a-bb0a-629be4594f35]
```

Now if you go to that web address, you'll see a form:



Submit the form and you'll get back a picture of a tiger:



You can have your form generate anything, including, for example, an active Manipulate.

Generate a Manipulate from a form:

```
In[9]:= CloudDeploy[FormFunction[{"name" → "Animal"},  
Manipulate[Rotate[#, name["Image"], θ], {θ, 0, 360 °}] &]  
Out[9]= CloudObject[https://www.wolframcloud.com/objects/0870f086-37b1-4e3c-b078-510b9e95938b]
```



You can set up a form with any number of fields. For each field you say what type of input it should accept using the same specifications as Interpreter.

Deploy a form that accepts two numbers:

```
In[10]:= CloudDeploy[FormFunction[{"x" → "Number", "y" → "Number"}, #x + #y &]  
Out[10]= CloudObject[https://www.wolframcloud.com/objects/464eeeff-c7a0-4f93-b132-6721302a6048]
```



If you try to give this form an input that isn't a number, you'll get an error:

You can have fields that expect strings ("String") or integers ("Integer") or dates ("Date") or hundreds of other kinds of things.

When you ask for “real-world” types of input, like "Animal" or "City", CloudDeploy automatically sets up *smart fields* in your form, indicated by , that use natural language understanding to interpret what's entered in them. But for more-abstract types of input, like numbers, you can for example choose between "Number", "SemanticNumber" and "ComputedNumber".

"Number" only allows explicit numbers, like 71. "SemanticNumber" also allows numbers in natural language, like “seventy-one”. "ComputedNumber" also allows numbers that have to be computed, like “20th prime number”.

Allow numbers specified by natural language:

```
In[11]:= CloudDeploy[
  FormFunction[{"x" → "SemanticNumber", "y" → "SemanticNumber"}, #x + #y &]]
Out[11]= CloudObject[https://www.wolframcloud.com/objects/662dc9bd-89ff-4c58-85c9-43ae1276082b]
```

“Seventy-one” works as a semantic number; to find the prime requires a computed number:

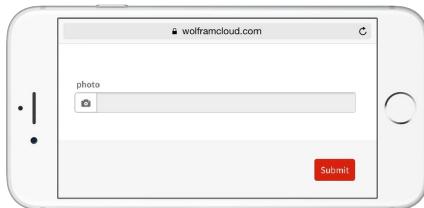
If you specify a type of input like "Image", you'll get special controls for acquiring the image—like direct access to the camera or photo gallery on a mobile device.

Deploy an edge-detection app for images:

```
In[12]:= CloudDeploy[FormFunction[{"photo" → "Image"}, EdgeDetect[##photo] &]]
```

```
Out[12]= CloudObject[https://www.wolframcloud.com/objects/727c12b9-6e42-496f-aa1d-0c5630c0fc5c]
```

On a mobile device, you can get the image from the camera:



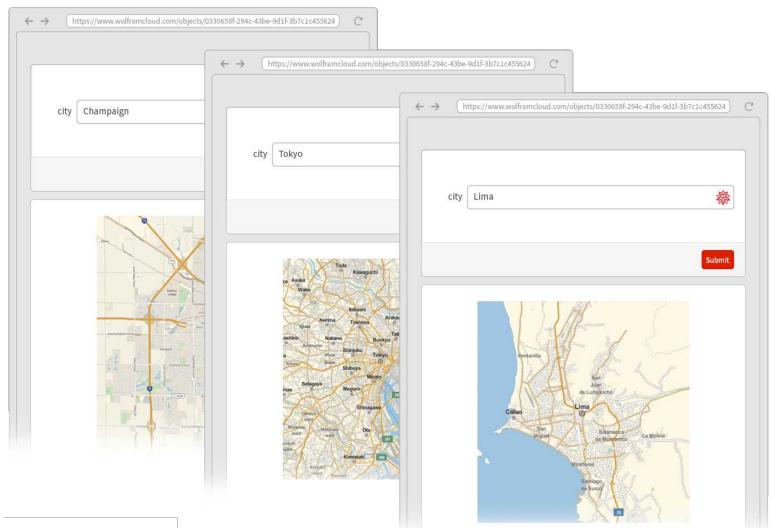
FormFunction lets you set up “one-shot” forms. You fill in a form, press Submit, then get a result. If you want to get another result, you have to go back to the form, and then submit it again. FormPage lets you set up pages that always include the form along with the result—like, for example, Wolfram|Alpha or a search engine would.

Create a form page that shows a map of a city:

```
In[13]:= CloudDeploy[FormPage[{"city" → "City"}, GeoGraphics[##city] &]]  
Out[13]= CloudObject[https://www.wolframcloud.com/objects/0330658f-294c-43be-9d1f-3b7c1c455624]
```



You can change the field and submit again to get a new result:



Vocabulary

CloudDeploy[<i>expr</i>]	deploy to the cloud
Delayed[<i>expr</i>]	computation delayed until it is requested
FormFunction[<i>form</i>, <i>function</i>]	representation of a deployable form
FormPage[<i>form</i>, <i>function</i>]	representation of a deployable form+result page
URLShorten[<i>url</i>]	make a short version of a web URL

Exercises

- 36.1** Deploy a map of your current location to the web.
- 36.2** Deploy to the web a map that shows the current inferred location of the user.
- 36.3** Create a website that displays a new random number up to 1000 at size 100 every time it is visited.
- 36.4** Deploy a form on the web that takes a number x and returns x^x .
- 36.5** Deploy a form on the web that takes numbers x and y and computes x^y .
- 36.6** Deploy a form on the web that takes the topic of a Wikipedia page and gives a word cloud for the page (with stopwords removed).
- 36.7** Deploy a form page on the web that takes a string and repeatedly gives a reversed version at size 50.
- 36.8** Deploy a form page on the web that takes an integer n and repeatedly generates a picture of a polygon with random color and n sizes.
- 36.9** Deploy a form page that takes a location and a number n and repeatedly gives a map of the n nearest volcanoes to the location.

Q&A

Why are the web addresses so long?

They're UUIDs (*universally unique identifiers*) that are long enough that with overwhelming probability no two identical ones will ever be created in the history of the universe.

How do I deploy to the cloud in a particular format?

Just ask for the format using `ExportForm` (or by giving the format inside `FormFunction`). Popular formats include "GIF", "PNG", "JPEG", "SVG", "PDF", "HTMLFragment".

How can I embed a web page generated by CloudDeploy?

Use `EmbedCode` to generate the necessary HTML.

How do I specify a label for a field in a form?

Just say e.g. `{"s", "Enter a string here"} → "String"`. By default, the label shown for a field is just the name you use for the variable corresponding to the field. You can use any label you want for a field, including graphics, etc.

How do I specify initial or default values for a field in a form?

Say e.g. `"n" → "Integer" → 100`.

How do I restrict the values that can be entered in a particular field of a form?

Use `Restricted`. For example, `Restricted["Number", {0, 10}]` specifies a number between 0 and 10. `Restricted["Location", italy]` specifies a location in Italy.

How do I specify the look of a form?

To start with, use options like `FormTheme`. For more control, you can put a `FormObject` inside a `FormFunction`, and give very detailed instructions. You can include any headers or text or styling that can appear in a notebook.

Can forms have checkboxes, sliders, etc.?

Yes. They can use the same controls as `Manipulate`, including checkboxes, radio buttons, popup menus, sliders, color pickers, etc.

Can I make extensible and multipage forms?

Yes. Field specifications can include constructs like `RepeatingElement` and `CompoundElement`. And forms can consist of lists of pages, including ones that are generated on the fly.

When I deploy to the cloud, where are the computations actually done?

In the cloud . Or in practice: on computers at centralized locations around the world.

How can I create a mobile app?

When you deploy to the cloud, you immediately have a mobile app that you can access from within the Wolfram Cloud app. You can specify a custom icon for your app using `IconRules`.

What about APIs?

`APIFunction` works very much like `FormFunction`, except it creates a web API that you can call by giving parameters in a URL query. `EmbedCode` lets you take an `APIFunction`, and generate code to call the API from many external programming languages and environments.

Tech Notes

- 3D graphics are by default deployed to be rotatable and zoomable.
- `AutoRefreshed` makes a web page be automatically refreshed on a schedule, so it's ready if you ask for it.
- You can give detailed permissions for who can access something you deploy to the cloud, and what they can do with it. Setting `$Permissions` specifies a default choice of permissions.

More to Explore

[Guide to Creating Web Sites and Apps in the Wolfram Language](#) (wolfr.am/eiwl-36-more)

37 | Layout and Display

Earlier we saw how to use `Framed` to add a frame when one displays something.

Generate a number and put a frame around it:

```
In[1]:= Framed[2^100]
```

```
Out[1]= 1267650600228229401496703205376
```

You can give options to `Framed`.

Specify a background color and a frame style:

```
In[2]:= Framed[2^100, Background -> LightYellow, FrameStyle -> LightGray]
```

```
Out[2]= 1267650600228229401496703205376
```

`Labeled` lets you make things be labeled.

Add a label to the framed number:

```
In[3]:= Labeled[Framed[2^100], "a big number"]
```

```
Out[3]= 1267650600228229401496703205376
```

a big number

This adds a label to a number styled with a yellow background:

```
In[4]:= Labeled[Style[2^100, Background -> Yellow], "a big number"]
```

```
Out[4]= 1267650600228229401496703205376
```

a big number

This adds styling to the label:

```
In[5]:= Labeled[Style[2^100, Background -> Yellow], Style["a big number", Italic, Orange]]
```

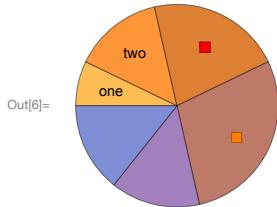
```
Out[5]= 1267650600228229401496703205376
```

a big number

You can use `Labeled` in graphics as well.

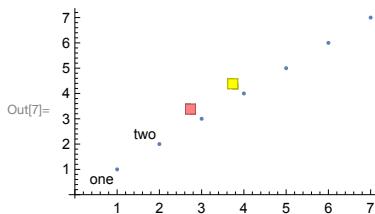
Make a pie chart in which some wedges are labeled:

```
In[6]:= PieChart[
  {Labeled[1, "one"], Labeled[2, "two"], Labeled[3, Red], Labeled[4, Orange], 2, 2}]
```



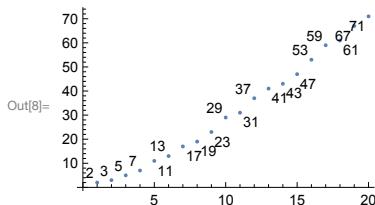
Plot labeled points:

```
In[7]:= ListPlot[
  {Labeled[1, "one"], Labeled[2, "two"], Labeled[3, Pink], Labeled[4, Yellow], 5, 6, 7}]
```



Plot all the first twenty primes labeled with their values:

```
In[8]:= ListPlot[Table[Labeled[Prime[n], Prime[n]], {n, 20}]]
```



There are all sorts of ways to annotate graphics. `Style` directly inserts styling.

`Tooltip` generates interactive tooltips. `Legended` puts labels into a legend on the side.

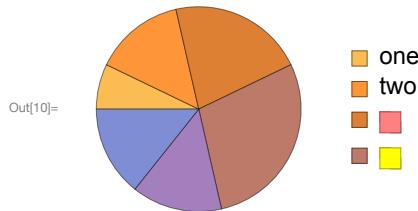
Specify styles for the first three pie wedges:

```
In[9]:= PieChart[{Style[3, Red], Style[2, Green], Style[1, Yellow], 1, 2}]
```



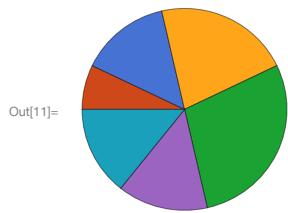
Set up words and colors as legends for pie wedges:

```
In[10]:= PieChart[{Legended[1, "one"], Legended[2, "two"],
Legended[3, Pink], Legended[4, Yellow], 2, 2}]
```



The default plot theme for the web is more brightly colored:

```
In[11]:= PieChart[{1, 2, 3, 4, 2, 2}, PlotTheme -> "Web"]
```



It's common to want to combine different objects for presentation. Row, Column and Grid are convenient ways to do this.

Display a list of objects in a row:

```
In[12]:= Row[{Yellow, Pink, Cyan}]
```

Out[12]=

Display objects in a column:

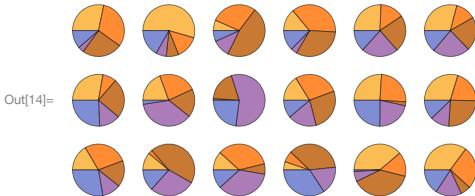
```
In[13]:= Column[{Yellow, Pink, Cyan}]
```

Out[13]=

Use GraphicsRow, GraphicsColumn and GraphicsGrid to arrange objects to fit in a certain overall size.

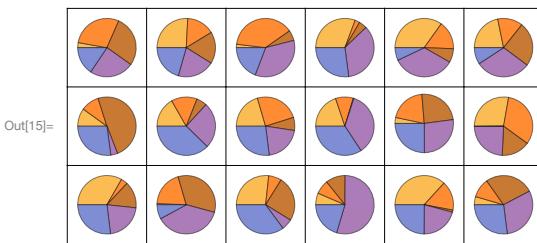
Generate an array of random pie charts, sized to fit:

```
In[14]:= GraphicsGrid[Table[PieChart[RandomReal[10, 5]], 3, 6]]
```



Do it with a frame around everything:

```
In[15]:= GraphicsGrid[Table[PieChart[RandomReal[10, 5]], 3, 6], Frame -> All]
```



Vocabulary

Framed [<i>expr</i>]	add a frame
Labeled [<i>expr</i> , <i>lab</i>]	add a label
Tooltip [<i>expr</i> , <i>lab</i>]	add an interactive tooltip
Legended [<i>expr</i> , <i>lab</i>]	add a legend
Row [{ <i>expr</i> ₁ , <i>expr</i> ₂ , ...}]	arrange in a row
Column [{ <i>expr</i> ₁ , <i>expr</i> ₂ , ...}]	arrange in a column
GraphicsRow [{ <i>expr</i> ₁ , <i>expr</i> ₂ , ...}]	arrange in a resizable row
GraphicsColumn [{ <i>expr</i> ₁ , <i>expr</i> ₂ , ...}]	arrange in a resizable column
GraphicsGrid [<i>array</i>]	arrange in a resizable grid

Exercises

- 37.1** Make a list of numbers up to 100, with even numbers on yellow and odd numbers on light gray.
- 37.2** Make a list of numbers up to 100, with primes framed.
- 37.3** Make a list of numbers up to 100, with primes framed and labeled in light gray with their values modulo 4.
- 37.4** Create a 3×6 `GraphicsGrid` of randomly colored disks.
- 37.5** Make a pie chart of the GDPs of the countries in the G5, labeling each wedge.
- 37.6** Make a pie chart of the populations of the countries in the G5, giving a legend for each wedge.
- 37.7** Make a 5×5 `GraphicsGrid` of pie charts that give the relative frequencies of digits in 2^n with n starting at 1.
- 37.8** Make a graphics row of word clouds for Wikipedia articles on the G5 countries.

Q&A

Can I get rounded corners with `Framed`?

Yes. Use an option like `RoundingRadius → 0.2`.

What kinds of things can be in a label?

Anything you want. A label can be text or a graphic or, for that matter, a whole notebook.

Can I use `Labeled` to put labels in places other than at the bottom?

Yes. Use e.g. `Labeled[expr, label, Left]` or `Labeled[expr, label, Right]`.

How do I determine where a legend goes?

Use `Placed`.

Can visualization be animated or dynamic?

Yes. `ListAnimate` creates an animation. Constructs from `Tooltip` to `Manipulate` can be used to set up dynamic visualizations.

Tech Notes

- The Wolfram Language tries to place labels where they won't interfere with the data that's being plotted.
- You can resize any graphic using `Show[graphic, ImageSize → width]` or `Show[graphic, ImageSize → {width, height}]`. `ImageSize → Tiny`, etc. also work.

More to Explore

Guide to Labeling & Annotation in the Wolfram Language (wolfr.am/eiwl-37-more)

38 | Assigning Names to Things

Particularly when you're doing quick experiments in the Wolfram Language, it's often convenient to use % to refer to the result of your most recent computation.

Do a simple computation:

```
In[1]:= Range[10]
```

```
Out[1]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

% gives the result of the previous computation:

```
In[2]:= %
```

```
Out[2]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

This squares the most recent result:

```
In[3]:= %^2
```

```
Out[3]= {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
```

If you expect to refer to a result later, you can assign it a name. For example, you can say `thing = Range[10]` to assign `thing` as a name for the result of `Range[10]`.

Assign `thing` to be the result of `Range[10]`:

```
In[4]:= thing = Range[10]
```

```
Out[4]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

Whenever `thing` appears, it'll be replaced by the result of `Range[10]`:

```
In[5]:= thing
```

```
Out[5]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

Square the value of `thing`:

```
In[6]:= thing^2
```

```
Out[6]= {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
```

You can assign a name to the result of a computation even if you never display the result. End your input with ; (semicolon) to avoid seeing the result.

Assign a name to a list of a million elements, but don't display the list:

```
In[7]:= millionlist = Range[1 000 000];
```

Find the total of all the elements in the list:

```
In[8]:= Total[millionlist]
```

```
Out[8]= 500 000 500 000
```

When you assign a name to something, the name will stay until you explicitly clear it.

Assign x the value 42:

```
In[9]:= x = 42
```

```
Out[9]= 42
```

You might have thought this would be $\{x, y, z\}$ —but x has value 42:

```
In[10]:= {x, y, z}
```

```
Out[10]= {42, y, z}
```

If you want to clear assignments, use `Clear`.

Clear any assignment for x :

```
In[11]:= Clear[x]
```

Now x , like y and z , isn't replaced:

```
In[12]:= {x, y, z}
```

```
Out[12]= {x, y, z}
```

Assigning a *global value* for x , like $x = 42$, is potentially a big deal, because it can affect everything you do in your session (at least until you clear x). Something that's much safer—and extremely useful—is just to assign a temporary, local value to x , inside a *module*.

This locally sets the value of x to be `Range[10]` inside the `Module`:

```
In[13]:= Module[{x = Range[10]}, x^2]
```

```
Out[13]= {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
```

Outside the module, x still has no value assigned:

```
In[14]:= x
```

```
Out[14]= x
```

You can have as many *local variables* inside a module as you want.

Define local variables x and n , then compute x^n using the values you've assigned:

```
In[15]:= Module[{x = Range[10], n = 2}, x^n]
```

```
Out[15]= {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
```

In the *functional style* of programming that we've used in most of this book, you carry out sequences of operations by applying sequences of functions. This is a very powerful and direct style of programming that's uniquely suited to the Wolfram Language.

But once one's defining variables, one can use a different style, in which one doesn't feed results directly from one function to another, but instead assigns them as values of variables to be used later. This kind of *procedural programming* is what's been used in lower-level languages since the very earliest days of computing.

It's still useful in the Wolfram Language, though in many ways it's eclipsed by functional programming, as well as by the *pattern-based* style of programming that we'll discuss later.

To specify sequences of actions in the Wolfram Language one just separates them by semicolons (;). (Putting a semicolon at the end is like specifying an empty final action, which is why this has the effect of not displaying a result.)

Do a sequence of operations; the result is what the last operation produces:

```
In[16]:= x = Range[10]; y = x^2; y = y + 10 000
```

```
Out[16]= {10 001, 10 004, 10 009, 10 016, 10 025, 10 036, 10 049, 10 064, 10 081, 10 100}
```

This defined global values for x and y ; don't forget to clear them:

```
In[17]:= Clear[x, y]
```

You can use semicolons to do sequences of operations inside `Module`.

This does a sequence of operations, with x and y maintained as local variables:

```
In[18]:= Module[{x, y}, x = Range[10]; y = x^2; y = y + 10 000]
```

```
Out[18]= {10 001, 10 004, 10 009, 10 016, 10 025, 10 036, 10 049, 10 064, 10 081, 10 100}
```

You can mix local variables that do and don't have initial values:

```
In[19]:= Module[{x, y, n = 2}, x = Range[10]; y = x^n; y = y + 10 000]
```

```
Out[19]= {10 001, 10 004, 10 009, 10 016, 10 025, 10 036, 10 049, 10 064, 10 081, 10 100}
```

You can nest modules—which is useful if you're building large programs where you want to isolate different parts of your code.

Vocabulary

<code>%</code>	the most recent computation result
<code>x = value</code>	assign a value
<code>Clear[x]</code>	clear a value
<code>Module[{x = value}, ...]</code>	set up a temporary variable
<code>expr ;</code>	do a computation, but don't display its result
<code>expr₁; expr₂; ...</code>	do a sequence of computations

Exercises

- 38.1** Use `Module` to compute x^2+x where x is `Range[10]`.
- 38.2** Use `Module` to generate a list of 10 random integers up to 100, then make a column giving the original list, and the results of applying `Sort`, `Max` and `Total` to it.
- 38.3** Use `Module` to generate an image collage from a picture of a giraffe, and the results of applying `Blur`, `EdgeDetect` and `ColorNegate` to it.
- 38.4** Inside a `Module`, let $r = Range[10]$, then make a line plot of r joined with the reverse of r joined with r joined with the reverse of r .
- 38.5** Find a simpler form for `{Range[10]+1, Range[10]-1, Reverse[Range[10]]}`.
- 38.6** Find a simpler form for `Module[{u = 10}, Join[{u}, Table[u = Mod[17 u + 2, 11], 20]]]`.
- 38.7** Generate 10 random strings made of 5 letters, in which consonants (non-vowels) alternate with vowels (aeiou).

Q&A

How come we're at Section 38, and only now introducing variable assignments?

Because, as we've seen, in the Wolfram Language one can go a very long way without introducing them. And it tends to be a lot cleaner to program without them.

Can I assign a name to anything?

Yes. Graphics, arrays, images, pure functions, whatever.

How does one read `x = 4` out loud?

" x equals 4", or, more rarely, "assign x to 4", or "give x the value 4".

What are good principles for global names?

Use names that are specific and explicit. Don't worry if they're long; they'll get autocompleted when you type. For "informal" names, start with lowercase letters. For more carefully thought out names, consider using capitalization like built-in functions do.

% gives the previous result. What about the result before that, etc.?

`%%` gives the next to last, `%%%` gives the next-to-next-to-last, etc. `% n` gives the result on line *n* (i.e. the result labeled `Out[n]`).

Can I assign to several variables at once?

Yes. `x = y = 6` assigns both `x` and `y` to 6. `{x, y} = {3, 4}` assigns `x` to 3 and `y` to 4. `{x, y} = {y, x}` swaps the values of `x` and `y`.

What happens if a variable escapes from a Module without having been assigned a value?

Try it! You'll find you'll get a new variable that's been given a unique name.

What about other procedural programming constructs, like Do and For?

The Wolfram Language has those. `Do` is sometimes worth using, particularly when your objective is side effects, like assigning variables or exporting data. `For` is almost always a bad idea, and can almost always be replaced by much cleaner code using constructs such as `Table`.

Tech Notes

- The result of `x = 2 + 2` is just 4, but as a *side effect* an assignment is made for `x`.
- In pure functional programming, the only effect of any computation is to give the result. As soon as you're assigning values, there's effectively a hidden state that's set up by those assignments.
- `x=.` is an alternative to `Clear[x]`.
- `Module` does *lexical scoping*, which means it effectively localizes the names of variables. `Block` does *dynamic scoping*, which means it localizes the values, but not the names, of variables. Both are useful in different circumstances. `Table` effectively uses `Block`.
- `x ++` is equivalent to `x = x + 1`. `x += n` is equivalent to `x = x + n`. `AppendTo[x, n]` is equivalent to `x = Append[x, n]` or `x = Join[x, {n}]`.

39 | Immediate and Delayed Values

There are two ways to assign a value to something in the Wolfram Language: *immediate assignment* (`=`), and *delayed assignment* (`:=`).

In immediate assignment, the value is computed immediately when the assignment is done, and is never recomputed. In delayed assignment, the computation of the value is delayed, and is done every time the value is requested.

As a simple example, consider the difference between `value = RandomColor[]` and `value := RandomColor[]`.

In immediate assignment (`=`), a random color is immediately generated:

```
In[1]:= value = RandomColor[]
```

```
Out[1]= 
```

Every time you ask for `value`, you get the same random color:

```
In[2]:= value
```

```
Out[2]= 
```

In delayed assignment (`:=`), no random color is immediately generated:

```
In[3]:= value := RandomColor[]
```

Each time you ask for `value`, `RandomColor[]` is computed, and a new color is generated:

```
In[4]:= value
```

```
Out[4]= 
```

The color will typically be different every time:

```
In[5]:= value
```

```
Out[5]= 
```

It's very common to use `:=` if something isn't ready yet when you're defining a value.

You can make a delayed assignment for `circles` even though `n` doesn't yet have a value:

```
In[6]:= circles := Graphics[Table[Circle[{x, 0}, x/2], {x, n}]]
```

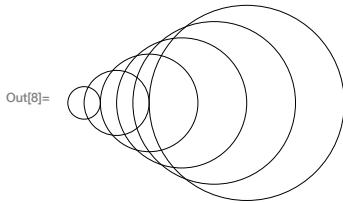
Give `n` a value:

```
In[7]:= n = 6
```

```
Out[7]= 6
```

Now you can ask for circles:

In[8]:= **circles**



The idea of delayed assignment is directly analogous to the `Delayed` construct we discussed for deploying web pages. In delayed assignment we don't compute a value until we need it. Similarly, when we use `CloudDeploy` with `Delayed` we don't compute the content of a web page until someone asks for it.

There's a notion of delayed rules too. $x \rightarrow rhs$ computes rhs immediately. But in a delayed rule $x \Rightarrow rhs$ (typed `:>`), rhs is instead recomputed every time it's requested.

This is an immediate rule, where a specific value for `RandomReal[]` is immediately computed:

In[9]:= **$x \rightarrow \text{RandomReal}[]$**

Out[9]= $x \rightarrow 0.522293$

You can replace four x 's, but they'll all be the same:

In[10]:= **{ x, x, x, x } /. $x \rightarrow \text{RandomReal}[]$**

Out[10]= {0.821639, 0.821639, 0.821639, 0.821639}

This is a delayed rule, where the computation of `RandomReal[]` is delayed:

In[11]:= **$x \Rightarrow \text{RandomReal}[]$**

Out[11]= $x \Rightarrow \text{RandomReal}[]$

`RandomReal[]` is computed separately when each x is replaced, giving four different values:

In[12]:= **{ x, x, x, x } /. $x \Rightarrow \text{RandomReal}[]$**

Out[12]= {0.536115, 0.84214, 0.242933, 0.514131}

Vocabulary

$x := value$ delayed assignment, evaluated every time x is requested

$x \Rightarrow value$ delayed rule, evaluated every time x is encountered (typed `:>`)

Exercises

- 39.1** Replace x in $\{x, x+1, x+2, x^2\}$ by the same random integer up to 100.
- 39.2** Replace each x in $\{x, x+1, x+2, x^2\}$ by a separately chosen random integer up to 100.
-

Q&A

Why not always use `:=`?

Because you don't want to have to recompute things unless it's necessary. It's more efficient to just compute something once, then use the result over and over again.

How does one read `:=` and `:>` out loud?

`:=` is usually just “colon equals”, though sometimes “delayed assignment”. `:>` is usually “colon greater”, though sometimes “delayed rule”.

What happens if I do $x = x + 1$, with x not having a value?

You'll start an infinite loop that'll eventually get cut off by the system. $x = \{x\}$ is the same story.

What's the significance of inputs being labeled `In[n]:=`, and outputs `Out[n]=?`

It indicates that inputs are assigned to `In[n]` and outputs to `Out[n]`. The `:=` for input means the assignment is delayed, so that if you ask for `In[n]` the result will be recomputed.

Tech Notes

- In the Wolfram Language, the process of computing results is often called *evaluation*, because it involves finding values of things.
- The Wolfram Language has many ways of controlling evaluation. An example is the function `Hold`, which maintains an expression in “held” form until it is “released”.
- The internal form of $x = y$ is `Set[x, y]`. $x := y$ is `SetDelayed[x, y]`. $x \rightarrow y$ is `RuleDelayed[x, y]`.

40 | Defining Your Own Functions

As we've seen in this book, there's a huge amount that can be done with functions that are already built into the Wolfram Language. But you can go even further if you define your own functions too. And the Wolfram Language has a very flexible way of letting you do this.

Let's start with a typical, simple example of a function definition.

This defines a function `pinks` that takes any argument:

```
In[1]:= pinks[n_] := Table[Pink, n]
```

This uses the function definition:

```
In[2]:= pinks[5]
```

```
Out[2]= {Pink, Pink, Pink, Pink, Pink}
```

```
In[3]:= pinks[10]
```

```
Out[3]= {Pink, Pink, Pink, Pink, Pink, Pink, Pink, Pink, Pink, Pink}
```

How does this function definition work? The idea is that the `:=` defines a value for the pattern `pinks[n_]`. When you ask for `pinks[5]`, this matches the `pinks[n_]` pattern, and the value you've defined for this is used.

But this is just the beginning of the story of function definition in the Wolfram Language. Because in the Wolfram Language, you can give a definition for anything.

Here's a list of expressions:

```
In[4]:= {f[Red], f[Yellow], f[Green], f[Orange], f[Magenta]}
```

```
Out[4]= {f[Red], f[Yellow], f[Green], f[Orange], f[Magenta]}
```

Define values for `f[Red]` and `f[Green]`:

```
In[5]:= f[Red] = 1000; f[Green] = 2000;
```

Now `f[Red]` and `f[Green]` are replaced by the values defined; the other cases are left unchanged:

```
In[6]:= {f[Red], f[Yellow], f[Green], f[Orange], f[Blue]}
```

```
Out[6]= {1000, f[Yellow], 2000, f[Orange], f[Blue]}
```

Now let's add a definition for the pattern `f[x_]`. The Wolfram Language will use this whenever the special definitions for `f[Red]` and `f[Green]` don't apply.

Define a value for `f` with any argument:

```
In[7]:= f[x_] := Framed[Column[{x, ColorNegate[x]}]]
```

If the special cases don't apply, the general definition is used:

```
In[8]:= {f[Red], f[Yellow], f[Green], f[Orange], f[Blue]}
```

```
Out[8]= {1000, , 2000, , }
```

Clear definitions for `f` to make sure nothing gets confused later:

```
In[9]:= Clear[f]
```

As another example, let's do the classic computer science exercise of defining the factorial function. Start by saying that `factorial[1] = 1`. Then define how to compute `factorial[n_]` recursively in terms of another instance of factorial.

Give a recursive definition of factorial:

```
In[10]:= factorial[1] = 1; factorial[n_Integer] := n*factorial[n-1]
```

Ask for `factorial[50]`:

```
In[11]:= factorial[50]
```

```
Out[11]= 30 414 093 201 713 378 043 612 608 166 064 768 844 377 641 568 960 512 000 000 000 000
```

There's also a built-in factorial function, which gives the same result:

```
In[12]:= 50!
```

```
Out[12]= 30 414 093 201 713 378 043 612 608 166 064 768 844 377 641 568 960 512 000 000 000 000
```

Instead of having definitions for `factorial[1]` and `factorial[n_]` we could have had a single definition and used If. But having separate definitions for each case tends to make things much easier to read and understand.

An alternative definition using If:

```
In[13]:= factorial[n_Integer] := If[n == 1, 1, n*factorial[n-1]]
```

It's nice to be able to break out special cases, but the real power of being able to make definitions for anything comes when one goes beyond simple *function[arguments]* cases.

As a simple example, consider making a definition for `plusminus[{x_, y_}]`.

Define a value for a pattern:

```
In[14]:= plusminus[{x_, y_}] := {x+y, x-y}
```

Use the definition:

```
In[15]:= plusminus[{4, 1}]
```

```
Out[15]= {5, 3}
```

A much less elegant form based on a traditional *function[argument]* definition:

```
In[16]:= plusminus[v_] := {v[[1]] + v[[2]], v[[1]] - v[[2]]}
```

It's very common to want to define a function that applies only to objects with a certain structure. This is easy to do with patterns. Here's an example.

A list with some Framed objects:

```
In[17]:= {a, Framed[b], c, Framed[{d, e}], 100}
```

```
Out[17]= {a, b, c, {d, e}, 100}
```

Define a function that applies only to framed objects:

```
In[18]:= highlight[Framed[x_]] := Style[Labeled[x, "+"], 20, Background → LightYellow]
```

Apply highlight to each element of a list; it knows what to do when it's given something framed:

```
In[19]:= highlight/@{a, Framed[b], c, Framed[{10, 20}], 100}
```

```
Out[19]= {highlight[a], b, highlight[c], {10, 20}, highlight[100]}
```

+ +

This definition applies to anything with head List:

```
In[20]:= highlight[list_List] := highlight/@list
```

Now you no longer have to use /@:

```
In[21]:= highlight[{a, Framed[b], c, Framed[{10, 20}], 100}]
```

```
Out[21]= {highlight[a], b, highlight[c], {10, 20}, highlight[100]}
```

+ +

Give a general case, to use if none of the special cases apply:

```
In[22]:= highlight[x_] := Style[Rotate[x, -30 Degree], 20, Orange]
```

This uses the special cases when it can, then the general case when nothing else applies:

```
In[23]:= highlight[{a, Framed[b], c, Framed[{10, 20}], 100}]
```

```
Out[23]= {a, b, c, {10, 20}, 100}
```

+ +

Exercises

Note: These exercises involve defining functions. Remember to use `Clear` to get rid of definitions once you're finished with each exercise.

- 40.1** Define a function `f` that computes the square of its argument.
- 40.2** Define a function `poly` that takes an integer, and makes a picture of an orange regular polygon with that number of sides.
- 40.3** Define a function `f` that takes a list of two elements and puts them in reverse order.
- 40.4** Create a function `f` that takes two arguments and gives the result of multiplying them and dividing by their sum.
- 40.5** Define a function `f` that takes a list of two elements and returns a list of their sum, difference and ratio.
- 40.6** Define a function `evenodd` that gives `Black` if its argument is even and `White` otherwise, but gives `Red` if its argument is 0.
- 40.7** Define a function `f` of three arguments where the second two arguments are added if the first argument is 1, multiplied if it's 2, and raised to a power if it's 3.
- 40.8** Define a Fibonacci function `f` with `f[0]` and `f[1]` both being 1, and `f[n]` for integer `n` being the sum of `f[n - 1]` and `f[n - 2]`.
- 40.9** Create a function `animal` that takes a string, and gives a picture of an animal with that name.
- 40.10** Define a function `nearwords` that takes a string and an integer `n`, and gives the `n` words in `WordList[]` that are nearest to a given string.

Q&A

What kind of a pattern can be used in a function definition?

Absolutely any pattern you want. Even one where the head is itself a pattern.

How can I see the definitions made for a particular function?

Use `?f` to see the definitions for `f`.

How do I overwrite an existing function definition?

Just make a new definition for the same pattern. Use `Clear` to remove all definitions.

How are different definitions for a particular function sorted?

Typically from most specific to least specific. If there are definitions that can't be ordered by specificity, definitions made later are put later. When definitions are used, the earlier ones are tried first.

Can I redefine built-in functions like Max or Plus?

Usually yes. First, though, you often have to say e.g. `Unprotect[Max]`. Then definitions you add will be used in preference to built-in ones. Some functions, like `Plus`, are so fundamental that the system locks them in a protected state. Even in this case, though, you can make "upvalue" definitions that are associated with particular structures of arguments.

Can I do object-oriented programming in the Wolfram Language?

A symbolic generalization of object-oriented programming, yes. Given an object "type" `t`, one wants to make definitions e.g. for `f[t...]` and `g[t...]`. One can associate such definitions with `t` by saying `t /: f[t...] = ...` In the Wolfram Language, this is called *defining an upvalue for t*.

Can I use = instead of := for function definitions?

Sometimes, `f[n_] = n^2` will work fine, because the right-hand side doesn't evaluate when you make the assignment. `f[n_] = Now` and `f[n_] := Now` will give different results. And in many cases the right-hand side can't be meaningfully evaluated until specific arguments are given.

How can I share function definitions with other people?

Just send the code! A convenient way to do this through the cloud is to use `CloudSave` and `CloudGet`, as discussed in Section 43.

Tech Notes

- Many low-level languages require functions to have particular *static types* of arguments (e.g. integers, reals, strings). Some languages allow *dynamic typing*, with arguments allowed to have any of a certain set of types. The Wolfram Language generalizes this by allowing arguments to be defined by arbitrary symbolic structures.
- Having a pattern like `{x_, y_}` in a function definition allows immediate and convenient *destructuring* of the function argument.
- Definitions can be associated with the head of a function (“downvalues”), with the heads of its arguments (“upvalues”), or with the head of the head, etc. (“subvalues”). Upvalues are effectively a generalization of *methods* in object-oriented languages.
- `f = (#^2 &)` and `f[n_] := n^2` are two ways of defining a function, that for example give the same results for `f[10]`. Pure function definitions tend to be easier to combine with each other, but much coarser in their handling of argument structures.

More to Explore

[Guide to Defining Functions in the Wolfram Language](#) (wolfr.am/eiwl-40-more)

41 | More about Patterns

Within the Wolfram Language, there's a whole sublanguage of patterns. We've already seen some of its important elements.

$_$ ("blank") stands for anything. $x_$ (" x blank") stands for anything, but calls it x . $_h$ stands for anything with head h . And x_h stands for anything with head h , and calls it x .

Define a function whose argument is an integer named n :

```
In[1]:= digitback[n_Integer] := Framed[Reverse[IntegerDigits[n]]]
```

The function evaluates whenever the argument is an integer:

```
In[2]:= {digitback[1234], digitback[6712], digitback[x], digitback[{4, 3, 2}], digitback[2^32]}

Out[2]= {{4, 3, 2, 1}, {2, 1, 7, 6}, digitback[x], digitback[{4, 3, 2}], {6, 9, 2, 7, 6, 9, 4, 9, 2, 4}}
```

Sometimes you may want to put a condition on a pattern. You can do this with $/;$ ("slash semi"). $n_Integer /; n > 0$ means any integer that is greater than 0.

Give a definition which only applies when $n > 0$:

```
In[3]:= pdigitback[n_Integer /; n > 0] := Framed[Reverse[IntegerDigits[n]]]
```

The definition doesn't apply to negative numbers:

```
In[4]:= {pdigitback[1234], pdigitback[-1234], pdigitback[x], pdigitback[2^40]}

Out[4]= {{4, 3, 2, 1}, pdigitback[-1234], pdigitback[x], {6, 7, 7, 7, 2, 6, 1, 1, 5, 9, 9, 0, 1}}
```

The $/;$ can go anywhere—even at the end of the whole definition.

Define different cases of the check function:

```
In[5]:= check[x_, y_] := Red /; x > y
```

```
In[6]:= check[x_, y_] := Green /; x ≤ y
```

Some examples of the check function:

```
In[7]:= {check[1, 2], check[2, 1], check[3, 4], check[50, 60], check[60, 50]}
```

```
Out[7]= {Green, Red, Green, Green, Red}
```

$_{--}$ ("double blank") stands for any sequence of one or more arguments.

$_{---$ ("triple blank") stands for zero or more.

Define a function that looks for black and white (in that order) in a list.

The pattern matches black followed by white, with any elements before, between and after them:

```
In[8]:= blackwhite[{_ ___, Black, m ___, White, ___}] := {m, m, m}
```

Pick out the (smallest) sequence between a black and a white:

```
In[9]:= blackwhite[{■, ■, ■, ■, ■, ■, ■, ■, ■}]
```



```
Out[9]= {■, ■, ■, ■, ■, ■, ■, ■, ■}
```

By default, `__` and `___` pick the shortest matches that work. You can use `Longest` to make them pick the longest instead.

Specify that the sequence between black and white should be as long as possible:

```
In[10]:= blackwhitex[{___, Black, Longest[m___], White, ___}]:= {m, m, m}
```

Now `m` grabs elements all the way to the last whitespace.

```
In[11]:= blackwhitex[{█, █, █, █, █, █, █, █, █}]
```

Out[11]= {█, █, █, █, █, █, █, █, █, █, █, █, █, █, █, █}

`x | y | z` matches `x`, `y` or `z`. `x..` matches any number of repetitions of `x`.

`bwcut` effectively cuts out the longest run of black and white:

```
In[12]:= bwcut[{a___, Longest[(Black | White)..], b___}] := {{a}, Red, {b}}
```

In[13]:= **bwcut**[{, , , , , , , }]

```
Out[13]= {{\textcolor{red}{\text{pink}}}, \textcolor{red}{\text{red}}, {\textcolor{yellow}{\text{yellow}}}}
```

The pattern `x_` is actually short for `x:_`, which means “match anything (i.e. `_`) and name the result `x`”. You can use notations like `x_` for more complicated patterns too.

Set up a pattern named `m` that matches a list of two pairs.

`grid22[m:{{}}]:=Grid[m, Frame -> All]`

```
In[15]:= {grid22[{{a, b}, {c, d}}], grid22[{{12, 34}, {56, 78}}],  
grid22[{{123, 456}}] grid22[{{1, 2, 3}, {4, 5, 6}}]}
```

```
Out[15]= { {{a, b}, {c, d}}, {{12, 34}, {56, 78}}, grid22[{123, 456}], grid22[{{1, 2, 3}, {4, 5, 6}}] }
```

Name the sequence of black and white, so it can be used in the result:

```
In[16]:= bwcut[{a, r}, Longest[Black | White]..], b } := {{a}, Framed[Length[{r}], {b}]}
```

```
In[17]:= bwcut[{, , , , , , , }]
```

Out[17]= $\left\{ \{\textcolor{red}{\boxed{1}}, \textcolor{red}{\boxed{2}}\}, \boxed{5}, \{\textcolor{yellow}{\boxed{3}}\} \right\}$

As a final example, let's use patterns to implement the classic computer science algorithm of sorting a list by repeatedly swapping pairs of successive elements that are found to be out of order. It's easy to write each step in the algorithm as a replacement for a pattern.

Replace the first elements one finds out of order by ones that are in order:

```
In[18]:= {5, 4, 1, 3, 2} /. {a___, b_, c_, d___} /; b > c → {a, c, b, d}
```

Out[18]= {4, 5, 1, 3, 2}

Do the same operation 10 times, eventually sorting the list completely:

```
In[19]:= NestList[(# /. {x___, b_, a_, y___} /; b > a → {x, a, b, y}) &, {4, 5, 1, 3, 2}, 10]
Out[19]= {{4, 5, 1, 3, 2}, {4, 1, 5, 3, 2}, {1, 4, 5, 3, 2}, {1, 4, 3, 5, 2}, {1, 3, 4, 5, 2},
{1, 3, 4, 2, 5}, {1, 3, 2, 4, 5}, {1, 2, 3, 4, 5}, {1, 2, 3, 4, 5}, {1, 2, 3, 4, 5}}
```

At the beginning, we won't know how long it'll take to finish sorting a particular list. So the best thing is to use `FixedPointList`, which is like `NestList`, except that you don't have to tell it a specific number of steps, and instead it just goes on until the result reaches a *fixed point*, where nothing more is changing.

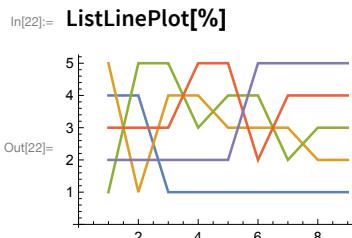
Do the operation until a fixed point is reached:

```
In[20]:= FixedPointList[(# /. {x___, b_, a_, y___} /; b > a → {x, a, b, y}) &, {4, 5, 1, 3, 2}]
Out[20]= {{4, 5, 1, 3, 2}, {4, 1, 5, 3, 2}, {1, 4, 5, 3, 2}, {1, 4, 3, 5, 2},
{1, 3, 4, 5, 2}, {1, 3, 4, 2, 5}, {1, 3, 2, 4, 5}, {1, 2, 3, 4, 5}, {1, 2, 3, 4, 5}}
```

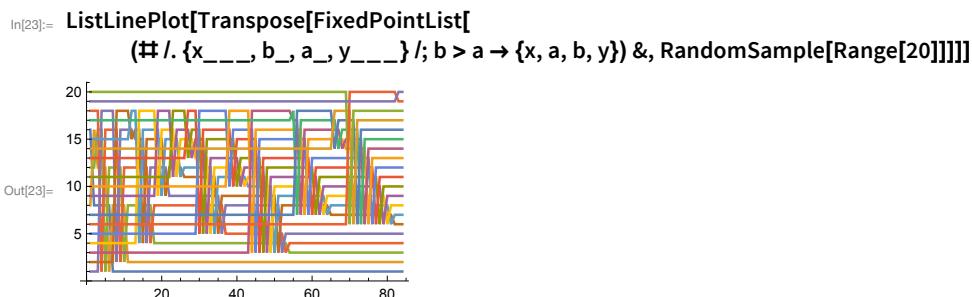
Transpose to find the list of elements appearing first, second, etc. at successive steps:

```
In[21]:= Transpose[%]
Out[21]= {{4, 4, 1, 1, 1, 1, 1, 1}, {5, 1, 4, 4, 3, 3, 3, 2, 2},
{1, 5, 5, 3, 4, 4, 2, 3, 3}, {3, 3, 3, 5, 5, 2, 4, 4, 4}, {2, 2, 2, 2, 2, 5, 5, 5}}
```

`ListLinePlot` plots each list in a different color, showing how the sorting process proceeds:



Here's the result for sorting a random length-20 list:



Vocabulary

<code>patt1; cond</code>	a pattern that matches if a condition is met
<code>---</code>	a pattern for any sequence of zero or more elements (“triple blank”)
<code>patt ..</code>	a pattern for one or more repeats of <i>patt</i>
<code>Longest[patt]</code>	a pattern that picks out the longest sequence that matches
<code>FixedPointList[f, x]</code>	keep nesting <i>f</i> until the result no longer changes

Exercises

- 41.1** Find the list of digits for squares of numbers less than 100 that contain successive repeated digits.
- 41.2** In the first 100 Roman numerals, find those containing L, I and X in that order.
- 41.3** Define a function *f* that tests whether a list of integers is the same as its reverse.
- 41.4** Get a list of pairs of successive words in the Wikipedia article on alliteration that have identical first letters.
- 41.5** Use Grid to show the sorting process in this section for {4, 5, 1, 3, 2}, with successive steps going down the page.
- 41.6** Use ArrayPlot to show the sorting process in this section for a list of length 50, with successive steps going across the page.
- 41.7** Start with 1.0, then repeatedly apply the “Newton’s method” function $(\# + 2/\#)/2$ & until the result no longer changes.
- 41.8** Implement Euclid’s algorithm for GCD in which {a, b} is repeatedly replaced by {b, Mod[a, b]} until b is 0, and apply the algorithm to 12345, 54321.
- 41.9** Define *combinators* using the rules $s[x_][y_][z_] \rightarrow x[z][y[z]]$, $k[x_][y_] \rightarrow x$, then generate a list by starting with $s[s][k][s[s][s]][s]$ and applying these rules until nothing changes.
- 41.10** Remove all trailing 0’s from the digit list for 100!.
- 41.11** Start from {1, 0} then for 200 steps repeatedly remove the first 2 elements, and append {0, 1} if the first element is 1 and {1, 0, 0} if it is 0, and get a list of the lengths of the sequences produced (*tag system*).
- 41.12** Start from {0, 0} then for 200 steps repeatedly remove the first 2 elements, and append {2, 1} if the first element is 0, {0} if the first element is 1, and {0, 2, 1, 2} if it is 2, and make a line plot of the lengths of the sequences produced (*tag system*).

Q&A**What are other pattern constructs in the Wolfram Language?**

`Except[patt]` matches anything except `patt`. `PatternSequence[patt]` matches a sequence of arguments in a function. `OrderlessPatternSequence[patt]` matches them in any order. `f[x_:v]` defines `v` as a default value, so `f[]` is matched, with `x` being `v`.

How can one see all the ways a pattern could match a particular expression?

Use `ReplaceList`. `Replace` gives the first match; `ReplaceList` gives a list of all of them.

What does `FixedPointList` do if there's no fixed point?

It'll eventually stop. There's an option that tells it how far to go. `FixedPointList[f, x, n]` stops after at most `n` steps.

Tech Notes

- In a repeating pattern `patt ..`, don't forget to leave a space in e.g. `0 ..` to avoid confusion with decimal numbers.
- Functions can have attributes that affect how pattern matching works. For example, `Plus` has attributes `Flat` and `Orderless`. `Flat` means that `b + c` can be pulled out of `a + b + c + d`. `Orderless` means that elements can be reordered, so `a + c` can be pulled out. (`Flat` is like the mathematical property of *associativity*; `Orderless` like *commutativity*.)
- The algorithm for sorting shown is usually called *bubble sort*. For a list of length `n`, it'll typically take about n^2 steps. The built-in Wolfram Language function `Sort` is much faster, and takes only a little over `n` steps.

More to Explore

[Guide to Patterns in the Wolfram Language \(wolfr.am/eiwl-41-more\)](http://wolfr.am/eiwl-41-more)

42 | String Patterns and Templates

String patterns work very much like other patterns in the Wolfram Language, except that they operate on sequences of characters in strings rather than parts of expressions. In a string pattern, you can combine pattern constructs like `_` with strings like "abc" using `~~`.

This picks out all instances of `+` followed by a single character:

```
In[1]:= StringCases["+string +patterns are +quite +easy", "+" ~~ _]
Out[1]= {+s, +p, +q, +e}
```

This picks out three characters after each `+`:

```
In[2]:= StringCases["+string +patterns are +quite +easy", "+" ~~ _ ~~ _ ~~ _]
Out[2]= {+str, +pat, +qui, +eas}
```

Use the name `x` for the character after each `+`, and return that character framed:

```
In[3]:= StringCases["+string +patterns are +quite +easy", "+" ~~ x_ → Framed[x]]
Out[3]= {Framed[s], Framed[p], Framed[q], Framed[e]}
```

In a string pattern, `_` stands for any single character. `__` ("double blank") stands for any sequence of one or more characters, and `___` ("triple blank") stands for any sequence of zero or more characters. `__` and `___` will normally grab as much of the string as they can.

Pick out the sequence of characters between `[` and `]`:

```
In[4]:= StringCases["the [important] word", "[" ~~ x_ _ ~~ "]" → Framed[x]]
Out[4]= {Framed[important]}
```

`__` normally matches as long a sequence of characters as it can:

```
In[5]:= StringCases["now [several] important [words]", "[" ~~ x_ _ ~~ "]" → Framed[x]]
Out[5]= {Framed[several] important [words]}
```

`Shortest` forces the shortest match:

```
In[6]:= StringCases["now [several] important [words]", 
  "[" ~~ Shortest[x_ _] ~~ "]" → Framed[x]]
Out[6]= {Framed[several], Framed[words]}
```

`StringCases` picks out cases of a particular pattern in a string. `StringReplace` makes replacements.

Make replacements for characters in the string:

```
In[7]:= StringReplace["now [several] important [words]", {"[" → "<<", "]" → ">>"]}
Out[7]= now <<several>> important <<words>>
```

Make replacements for patterns, using \rightarrow to compute `ToUpperCase` in each case:

```
In[8]:= StringReplace["now [several] important [words]",  
  "[" ~~ Shortest[x__] ~~ "]"  $\rightarrow$  ToUpperCase[x]]
```

```
Out[8]= now SEVERAL important WORDS
```

Use `NestList` to apply a string replacement repeatedly:

```
In[9]:= NestList[StringReplace[#, {"A"  $\rightarrow$  "AB", "B"  $\rightarrow$  "BA"}] &, "A", 5]
```

```
Out[9]= {A, AB, ABBA, ABBABAAB, ABBABAABBAABABBA, ABBABAABBAABBBABAABABBAABBABAAB}
```

`StringMatchQ` tests whether a string matches a pattern.

Select common words that match the pattern of beginning with a and ending with b:

```
In[10]:= Select[WordList[], StringMatchQ[#, "a" ~~ __ ~~ "b"] &]
```

```
Out[10]= {absorb, adsorb, adverb, alb, aplomb}
```

You can use | and .. in string patterns just like in ordinary patterns.

Pick out any sequence of A or B repeated:

```
In[11]:= StringCases["the AAA and the BBB and the ABABBBABABABA", ("A" | "B")..]
```

```
Out[11]= {AAA, BBB, ABABBBABABABA}
```

In a string pattern, `LetterCharacter` stands for any letter character, `DigitCharacter` for any digit character, and `Whitespace` for any sequence of “white” characters such as spaces.

Pick out sequences of digit characters:

```
In[12]:= StringCases["12 and 123 and 4567 and 0x456", DigitCharacter..]
```

```
Out[12]= {12, 123, 4567, 0, 456}
```

Pick out sequences of digit characters “flanked” by whitespace:

```
In[13]:= StringCases["12 and 123 and 4567 and 0x456",  
  Whitespace ~~ DigitCharacter.. ~~ Whitespace]
```

```
Out[13]= {123, 4567}
```

It's common in practice to want to go back and forth between strings and lists.

You can split a string into a list of pieces using `StringSplit`.

Split a string into a list of pieces, by default breaking at spaces:

```
In[14]:= StringSplit["a string to split"]
```

```
Out[14]= {a, string, to, split}
```

This uses a string pattern to decide where to split:

```
In[15]:= StringSplit["you+can+split--at+any--delimiter", "+" | "--"]
```

```
Out[15]= {you, can, split, at, any, delimiter}
```

Within strings, there's a special newline character which indicates where the string should break onto a new line. The newline character is represented within strings as `\n`.

Split at newlines:

```
In[16]:= StringSplit["first line
second line
third line", "\n"]
Out[16]= {first line, second line, third line}
```

`StringJoin` joins any list of strings together. In practice, though, one often wants to insert something between the strings before joining them. `StringRiffle` does this.

Join strings, riffling the string "___" in between them:

```
In[17]:= StringRiffle[{"a", "list", "of", "strings"}, "___"]
Out[17]= a___list___of___strings
```

In assembling strings, one often wants to turn arbitrary Wolfram Language expressions into strings. One can do this using `TextString`.

`TextString` turns numbers and other Wolfram Language expressions into strings:

```
In[18]:= StringJoin["two to the ", TextString[50], " is ", TextString[2^50]]
Out[18]= two to the 50 is 1125899906842624
```

A more convenient way to create strings from expressions is to use *string templates*. String templates work like pure functions in that they have slots into which arguments can be inserted.

In a string template each `` is a slot for a successive argument:

```
In[19]:= StringTemplate["first `` then ``"][[100, 200]
Out[19]= first 100 then 200
```

Named slots pick elements from an association:

```
In[20]:= StringTemplate["first: `a`; second `b`; first again `a`"][
  <| "a" → "AAAA", "b" → "BB BBB" |>]
Out[20]= first: AAAA; second BB BBB; first again AAAA
```

You can insert any expression within a string template by enclosing it with `<* ... *>`. The value of the expression is computed when the template is applied.

Evaluate the `<* ... *>` when the template is applied; no arguments are needed:

```
In[21]:= StringTemplate["2 to the 50 is <* 2^50 *>"][]
Out[21]= 2 to the 50 is 1125899906842624
```

Use slots in the template ('` is the backquote character):

```
In[22]:= StringTemplate["`1` to the `2` is <* `1`^`2` *>"][[2, 50]]
```

```
Out[22]= 2 to the 50 is 1125899906842624
```

The expression in the template is evaluated when the template is applied:

```
In[23]:= StringTemplate["the time now is <* Now *>"][]
```

```
Out[23]= the time now is Wed 16 Sep 2015 16:50:43
```

Vocabulary

<i>patt₁ ~~ patt₂</i>	sequence of string patterns
Shortest[patt]	shortest sequence that matches
StringCases[string, patt]	cases within a string matching a pattern
StringReplace[string, patt → val]	replace a pattern within a string
StringMatchQ[string, patt]	test whether a string matches a pattern
LetterCharacter	pattern construct matching a letter
DigitCharacter	pattern construct matching a digit
Whitespace	pattern construct matching spaces etc.
\n	newline character
StringSplit[string]	split a string into a list of pieces
StringJoin[{string₁, string₂, ...}]	join strings together
StringRiffle[{string₁, string₂, ...}, m]	join strings, inserting <i>m</i> between them
TextString[expr]	make a text string out of anything
StringTemplate[string]	create a string template to apply
` `	slot in a string template
<*...*>	expression to evaluate in a string template

Exercises

- 42.1** Replace each space in "1 2 3 4" with "---".
- 42.2** Get a sorted list of all sequences of 4 digits (representing possible dates) in the Wikipedia article on computers.
- 42.3** Extract “headings” in the Wikipedia article about computers, as indicated by strings starting and ending with “===”.
- 42.4** Use a string template to make a grid of results of the form $i+j = \dots$ for i and j up to 9.
- 42.5** Find names of integers below 50 that have an “i” somewhere before an “e”.
- 42.6** Make any 2-letter word uppercase in the first sentence from the Wikipedia article on computers.
- 42.7** Make a labeled bar chart of the number of countries whose `TextString` names start with each possible letter.
- 42.8** Find simpler code for
`Grid[Table[StringJoin[TextString[i], "^", TextString[j]], "=", TextString[i^j]], {i, 5}, {j, 5}]]`.

Q&A

How should one read ~~ out loud?

It's usually read “tilde tilde”. The underlying function is `StringExpression`.

Can I write rules for understanding natural language?

Yes, but we didn't cover that here. The key function is `GrammarRules`.

Tech Notes

- There's a correspondence between patterns for strings, and patterns for sequences in lists. `SequenceCases` is the analog for lists of `StringCases` for strings.
- The option `Overlaps` specifies whether or not to allow overlaps in string matching. Different functions have different defaults.
- String patterns by default match longest sequences, so you need to specify `Shortest` if you want it. Expression patterns by default match shortest sequences.
- Among string pattern constructs are `Whitespace`, `NumberString`, `WordBoundary`, `StartOfLine`, `EndOfLine`, `StartOfString` and `EndOfString`.
- Anywhere in a Wolfram Language symbolic string pattern, you can use `RegularExpression` to include regular expression syntaxes like `x*` and `[abc][def]`.
- You can compare strings using operations like `SequenceAlignment`. This is particularly useful in bioinformatics.
- `FileTemplate`, `XMLTemplate` and `NotebookTemplate` let you do the analog of `StringTemplate` for files, XML (and HTML) documents, and notebooks.
- The Wolfram Language includes the function `TextSearch`, for searching text in large collections of files.

More to Explore

Guide to String Patterns in the Wolfram Language (wolfr.am/eiwl-42-more)

43 | Storing Things

The Wolfram Language makes it easy to store things either in the Wolfram Cloud, or locally on your computer system. Let's talk about the Wolfram Cloud first.

In the Wolfram Cloud everything is a *cloud object*, specified by a UUID (universally unique identifier).

Cloud objects are immediately assigned a UUID:

```
In[1]:= CloudObject[]
```

```
Out[1]= CloudObject[https://www.wolframcloud.com/objects/388b0fd0-7769-42e4-a992-7d1b9985fe55]
```

As soon as you create a cloud object, it's assigned a long randomly generated UUID. The great thing about UUIDs is that one can assume that there'll never be two the same generated. (There are 300 trillion trillion trillion possible Wolfram UUIDs.)

Put a Wolfram Language expression into the cloud:

```
In[2]:= CloudPut[{, 

```

```
Out[2]= CloudObject[https://www.wolframcloud.com/objects/715b04e7-e589-4ebb-8b88-dde32fe0718b]
```

Get the expression back from the cloud:

```
In[3]:= CloudGet[%]
```

```
Out[3]= {, 
```

If you've made definitions using = and :=, you can save these using CloudSave. (If your definitions depend on other definitions, these will get saved too.) You can load your definitions into a new session using CloudGet.

Make a definition:

```
In[4]:= colorlist[n_Integer] := RandomColor[n]
```

Save it in the cloud to be retrieved later using CloudGet:

```
In[5]:= CloudSave[colorlist]
```

```
Out[5]= CloudObject[https://www.wolframcloud.com/objects/b274c11e-88c2-44d9-b805-599dbf7f898e]
```

CloudPut lets you store single Wolfram Language expressions. But what if you want to progressively accumulate expressions—coming either from within the Wolfram Language, or, say, from an outside device or sensor?

The Wolfram Data Drop lets you do exactly this. You start by creating a databin. You can do this in the Wolfram Language using `CreateDatabin`.

Create a databin:

```
In[6]:= bin = CreateDatabin[]
```

```
Out[6]= Databin[   Short ID: 6nzQYo8x ]  
Entry count: 0
```

You can add data to this databin from all sorts of outside devices and services—as well as using the `DatabinAdd` function in the Wolfram Language.

Add an entry to a databin:

```
In[7]:= DatabinAdd[bin, {1, 2, 3, 4}]
```

```
Out[7]= Databin[   Short ID: 6nzQYo8x ]  
Entry count: 1
```

Here's a databin that's accumulated data from a little sensor on my desk. `DateListPlot` plots the time series of data.

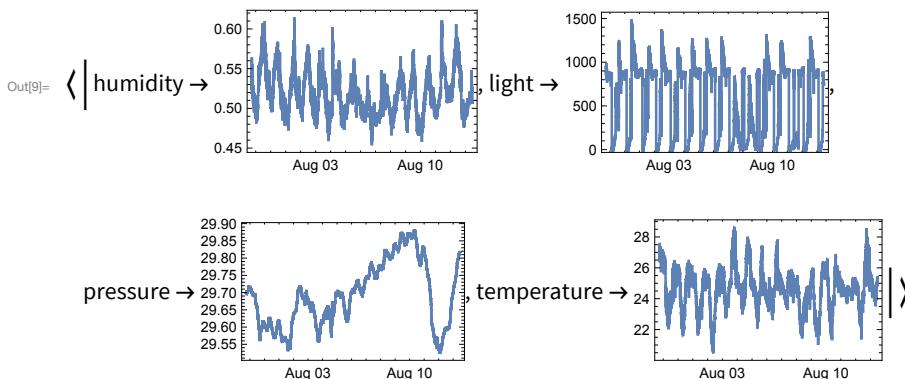
Use a short ID to reference a databin connected to sensors on my desk:

```
In[8]:= Databin["7m3ujLVf"]
```

```
Out[8]= Databin[   Name: Desktop sensors ]  
Entry count: 42282
```

Plot time series from the databin:

```
In[9]:= DateListPlot[Databin["7m3ujLVf"]]
```



Wolfram Data Drop, like `CloudPut` and `CloudSave`, saves data into the cloud. But particularly if you're not connected to the cloud, you may instead want to store things on a local computer. If you figure out where in your filesystem you want the files to go, you can use `Put`, `Save` and `Get` to store and retrieve them.

It's also possible to get the Wolfram Language to automatically figure out an "anonymous" local location. You can do this with `LocalObject`.

Generate an "anonymous" location for `Put`, `Save`, etc.:

```
In[10]:= LocalObject[]
```

```
Out[10]= LocalObject[file:///home/Wolfram/Objects/365e034d-9830-4842-8681-75d3714b3d19]
```

Put an image to the location:

```
In[11]:= Put[, %]
```

```
Out[11]= LocalObject[file:///home/Wolfram/Objects/365e034d-9830-4842-8681-75d3714b3d19]
```

Get the image back:

```
In[12]:= Get[%]
```

```
Out[12]= 
```

Vocabulary

<code>CloudObject[]</code>	create a cloud object
<code>CloudPut[expr]</code>	put into the cloud
<code>CloudGet[obj]</code>	get from the cloud
<code>CloudSave[s]</code>	save definitions to the cloud
<code>CreateDatabin[]</code>	create a new databin
<code>DatabinAdd[obj, value]</code>	add something to a databin
<code>DateListPlot[data]</code>	make a date list plot
<code>LocalObject[]</code>	create a local object
<code>Put[expr, obj]</code>	put into a local object
<code>Get[obj]</code>	get from a local object

Q&A**What do the letters and numbers in UUIDs mean?**

They're digits in hexadecimal (base 16); the letters "a" through "f" are the hex digits 10 through 15. Each UUID has 32 hex digits, corresponding to $16^{32} \approx 3 \times 10^{38}$ possibilities.

How does the number of possible UUIDs compare to other things?

It's about the number of atoms in a cubic kilometer of water, or 100 trillion times the number of stars in the universe. If every one of 50 billion computers on Earth generated a UUID at 10 GHz, it'd take the age of the universe to run out of UUIDs.

Can I specify a file name in CloudObject, CloudPut, etc.?

Yes. And the file name will be taken to be relative to your home directory in the cloud. The file will also get a URL, that includes the base of the cloud you're using, and your user ID.

Can I work with databins without using the Wolfram Language?

Yes. You can use the web and many other systems to create and add to databins.

Tech Notes

- When you work in the cloud, Wolfram Notebook documents are automatically stored after every change you make—unless you say not to do this.
- You can save large objects more efficiently with `DumpSave` than `Save`, but then the file you create is binary, not text.
- Databins can have *data signatures*, that specify how data in them should be interpreted, for example in terms of units, date formats, etc.

More to Explore

[Guide to Files in the Wolfram Language](#) (wolfr.am/eiwl-43-more)

[Wolfram Data Drop](#) (wolfr.am/eiwl-43-more2)

44 | Importing and Exporting

Everything we've done so far in this book has been done entirely within the Wolfram Language and the Wolfram Knowledgebase. But sometimes you need to get things from the outside. Needless to say, they often won't be as clean and organized as what we're used to inside the Wolfram Language.

As a first example, let's import the text from the front page of the United Nations website. We can do this using the function `Import`.

Import a text version of the front page of the UN website:

```
In[1]:= Import["http://un.org"]

Out[1]= It's earth hour at UN Headquarters in New York – Switch your computer off!
          مرحبًا
          欢迎光临
          Welcome
          Bienvenue
          Добро пожаловать
          Bienvenidos

          عالمك إنها – المتحدة الأمم
          联合国，您的世界！
          United Nations – It's your world!
          Nations Unies – C'est votre monde!
          Организация Объединенных Наций – это ваш мир!
          Las Naciones Unidas son su mundo
```

The result is a string, with some blank lines—and (at least as retrieved here) a strange message at the top. Let's start by splitting the string at newlines.

Split at newlines to get a list of strings:

```
In[2]:= StringSplit[Import["http://un.org"], "\n"]

Out[2]= {It's earth hour at UN Headquarters in New York – Switch your computer off! ,
          مرحبًا ، 欢迎光临 , Welcome , Bienvenue , Добро пожаловать ,
          Bienvenidos , عالمك إنها – المتحدة الأمم ، 联合国，您的世界！ ,
          United Nations – It's your world! , Nations Unies – C'est votre monde! ,
          Организация Объединенных Наций – это ваш мир! ,
          Las Naciones Unidas son su mundo}
```

Identify the language for each string (blank lines are considered English):

```
In[3]:= LanguageIdentify[StringSplit[Import["http://un.org"], "\n"]]

Out[3]= {English , English , English , English , Arabic ,
          Chinese , English , French , Russian , Spanish , English ,
          Arabic , Chinese , English , French , Russian , Spanish }
```

`Import` lets you import a wide variety of different elements. "Hyperlinks" gets hyperlinks that appear on a web page; "Images" gets images.

Get a list of the hyperlinks on the front of the UN website:

```
In[4]:= Import["http://un.org", "Hyperlinks"]
```

```
Out[4]= {"/www.un.org/ar/index.html", "/www.un.org/zh/index.html", "/www.un.org/en/index.html",
"/www.un.org/fr/index.html", "/www.un.org/ru/index.html", "/www.un.org/es/index.html"}
```

Get a list of the images on the front page of Wikipedia:

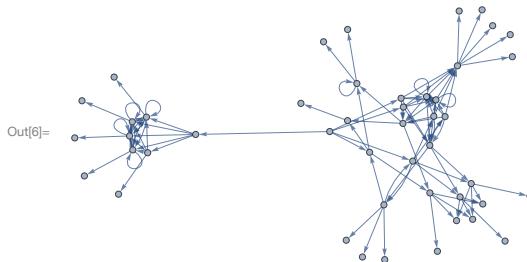
```
In[5]:= Import["http://wikipedia.org", "Images"]
```

```
Out[5]= {WIKIPEDIA, , , , , , , , , , , , , }
```

As a more sophisticated example, here's a graph of the hyperlinks in part of my website. To keep it manageable, I've taken just the first 5 hyperlinks at each level, and gone only 3 levels.

Compute part of the hyperlink graph for my website:

```
In[6]:= NestGraph[Take[Import[#, "Hyperlinks"], 5] &, "http://stephenwolfram.com", 3]
```



The Wolfram Language can import hundreds of formats—including spreadsheets, images, sounds, geometry, databases, log files and more. `Import` will automatically look at the file extension (.png, .xls, etc.) to determine what to do.

Import a picture from my website:

```
In[7]:= Import["http://www.stephenwolfram.com/images/stephen-wolfram-portrait.png"]
```



The Wolfram Language recognizes me!

```
In[8]:= Classify["NotablePerson", %]
```

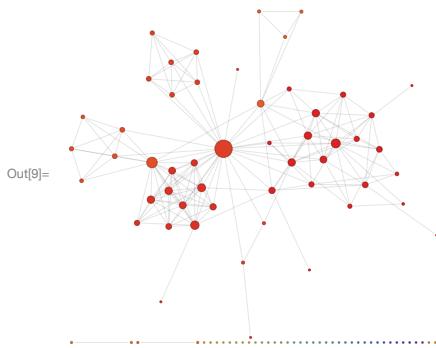
```
Out[8]= Stephen Wolfram
```

As well as importing from the web, `Import` can also import from your own files, stored in your computer system or in the Wolfram Cloud.

The Wolfram Language lets you not only deal with web pages and files, but also with *services* or *APIs*. For example, `SocialMediaData` lets you get data from social media services—at least once you've authorized them to send the data.

Find the network of my Facebook friends who give access to their connection data:

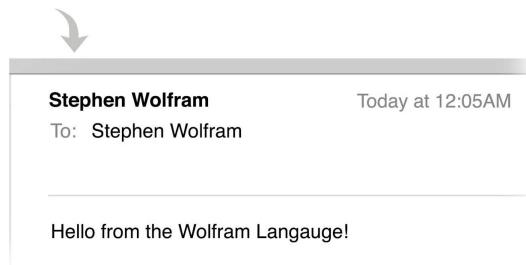
```
In[9]:= SocialMediaData["Facebook", "FriendNetwork"]
```



You can also send things from the Wolfram Language, for example by email. `SendMail` does this.

Send oneself a message by email:

```
In[10]:= SendMail["Hello from the Wolfram Language!"]
```



Send email to a test account with subject “Wolf” and a picture of a wolf attached:

```
In[11]:= SendMail["test@wolfram.com", {"Wolf", "Here's a wolf...", }]
```



If you want to interact with external programs and services, you’ll often have to *export* things from the Wolfram Language.

Export a graphic of a circle to the cloud in PDF format:

```
In[12]:= CloudExport[Graphics[Circle[]], "PDF"]
```

```
Out[12]= CloudObject[https://www.wolframcloud.com/objects/6d93f2de-6597-4d9f-9edb-7cdc342571b8]
```

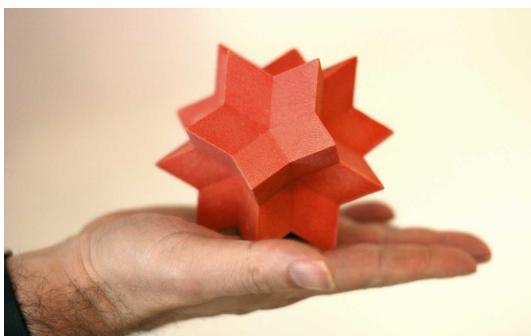
You can also export to local files using `Export`.

Export 3D geometry in a format suitable for 3D printing:

```
In[13]:= Export["spikey.stl", rhombic hexecontahedron ["Image"]]
```

```
Out[13]= spikey.stl
```

Here’s the result of 3D printing from the `spikey.stl` file:



Vocabulary

Import [<i>loc</i>]	import from an external location
SocialMediaData [...]	get data from social media networks
SendMail [<i>expr</i>]	send email
CloudExport [<i>expr, format</i>]	export in a certain format to the cloud
Export [<i>file, expr</i>]	export to a file

Exercises

- 44.1** Import the images from <http://google.com>.
- 44.2** Make an image collage of disks with the dominant colors from images on <http://google.com>.
- 44.3** Make a word cloud of the text on <http://bbc.co.uk>.
- 44.4** Make an image collage of the images on <http://whitehouse.gov>.
- 44.5** Use `ImageInstanceQ` to find pictures on <https://en.wikipedia.org/wiki/Ostrich> that are of birds.
- 44.6** Use `TextCases` with "Country" to find instances of country names on <http://nato.int>, and make a word cloud of them.
- 44.7** Find the number of links on <https://en.wikipedia.org>.
- 44.8** Send yourself mail with a map of your current location.
- 44.9** Send yourself mail with an icon of the current phase of the moon.

Q&A

Why doesn't Import retrieve elements that I see when I visit a web page in a browser?

Probably because they're not directly present in the HTML source of the web page, which is what Import looks at. They're probably being dynamically added using JavaScript.

What formats can Import handle?

See the list at wolfr.am/ref-importexport or evaluate `$ImportFormats`.

How do Import and Export figure out what format to use?

You can explicitly tell them. Or they can determine it from file extensions, like `.gif` or `.mbox`.

What is an API?

An application program interface. It's an interface that a program exposes to other programs, rather than to humans. The Wolfram Language has several APIs, and lets you create your own, using `APIFunction`.

How do I authorize a connection to an external account of mine?

When you use `SocialMediaData` or `ServiceConnect`, you'll typically be prompted to authorize the Wolfram Connection app for that particular service.

Tech Notes

- `ImportString` lets you “import” from a string rather than from an external file or URL. `ExportString` “exports” to a string.
- `SendMail` uses either mail server preferences you set up, or a proxy in the Wolfram Cloud.
- The Wolfram Language supports many external services. Typically it uses mechanisms like OAuth to authenticate them.
- Another way to get (and send) data is through direct connection from your computer to a sensor, Arduino, etc. The Wolfram Language has a whole framework for dealing with such things, including functions such as `DeviceReadTimeSeries`.

More to Explore

Guide to Importing and Exporting in the Wolfram Language (wolfr.am/eiwl-44-more)

45 | Datasets

Especially in larger organizations, computing often centers around dealing with large amounts of structured data. The Wolfram Language has a very powerful way to deal with structured data, using what it calls *datasets*.

A simple example of a dataset is formed from an association of associations.

Create a simple dataset that can be viewed as having 2 rows and 3 columns:

```
In[1]:= data = Dataset[  
  <|"a" -> <|"x" -> 1, "y" -> 2, "z" -> 3|>, "b" -> <|"x" -> 5, "y" -> 10, "z" -> 7|>|>]  
  
Out[1]= 

|   | x | y  | z |
|---|---|----|---|
| a | 1 | 2  | 3 |
| b | 5 | 10 | 7 |

  
2 levels | 2 rows
```

The Wolfram Language displays most datasets in tabular form. You can extract parts from datasets just like you would from associations.

Get the element from “row b” and “column z”:

```
In[2]:= data["b", "z"]  
Out[2]= 7
```

You can first extract the whole “b row”, then get the “z” element of the result:

```
In[3]:= data["b"]["z"]  
Out[3]= 7
```

You can also just get the whole “b row” of the dataset. The result is a new dataset, which for ease of reading happens to be displayed in this case as a column.

Generate a new dataset from the “b row” of the original dataset:

```
In[4]:= data["b"]  
  
Out[4]= 

|   |    |
|---|----|
| x | 5  |
| y | 10 |
| z | 7  |

  
1 level | 3 elements
```

Here is the dataset that corresponds to the “z column” for all “rows”.

Generate a dataset consisting of the “z column” for all rows:

```
In[5]:= data[All, "z"]  
  
Out[5]= 

|   |   |
|---|---|
| a | 3 |
| b | 7 |

  
1 level | 2 elements
```

Extracting parts of datasets is just the beginning. Anywhere you can ask for a part you can also give a function that will be applied to all parts at that level.

Get totals for each row by applying **Total** to all columns for all the rows:

```
In[6]:= data[All, Total]
```

a	6
b	22
1 level 2 elements	

If we use **f** instead of **Total**, we can see what's going on: the function is being applied to each of the "row" associations.

Apply the function **f** to each row:

```
In[7]:= data[All, f]
```

a	$f[x \rightarrow 1, y \rightarrow 2, z \rightarrow 3]$
b	$f[x \rightarrow 5, y \rightarrow 10, z \rightarrow 7]$
1 level 2 elements	

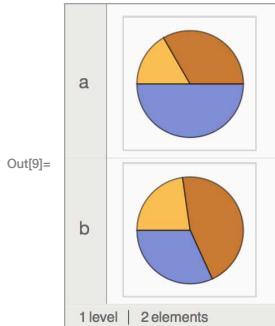
Apply a function that adds the **x** and **z** elements of each association:

```
In[8]:= data[All, #x + #z &]
```

a	4
b	12
1 level 2 elements	

You can use any function; here's **PieChart**:

```
In[9]:= data[All, PieChart]
```



You can give a function to apply to all rows too.

This extracts the value of each "z column", then applies **f** to the association of results:

```
In[10]:= data[f, "z"]
```

```
Out[10]=  $f[a \rightarrow 3, b \rightarrow 7]$ 
```

Apply **f** to the totals of all columns:

```
In[11]:= data[f, Total]
```

```
Out[11]=  $f[a \rightarrow 6, b \rightarrow 22]$ 
```

Find the maximum of these totals:

```
In[12]:= data[Max, Total]
```

```
Out[12]= 22
```

You can always “chain” queries, for example first finding the totals for all rows, then picking out the result for the “b row”.

Find totals for all rows, then pick out the total for the “b row”:

```
In[13]:= data[All, Total]["b"]
```

```
Out[13]= 22
```

It’s equivalent to this:

```
In[14]:= data["b", Total]
```

```
Out[14]= 22
```

Particularly when one’s dealing with large datasets, it’s common to want to select parts based on a criterion. The *operator form* of Select provides a very convenient way to do this.

Select numbers greater than 5 from a list:

```
In[15]:= Select[{1, 3, 6, 8, 2, 5, 9, 7}, # > 5 &]
```

```
Out[15]= {6, 8, 9, 7}
```

Another way to get the same answer, using the operator form of Select:

```
In[16]:= Select[# > 5 &][{1, 3, 6, 8, 2, 5, 9, 7}]
```

```
Out[16]= {6, 8, 9, 7}
```

The operator form of Select is a function which can be applied to actually perform the Select operation.

Make a dataset by selecting only rows whose “z column” is greater than 5:

```
In[17]:= data[Select[#z > 5 &]]
```

	x	y	z
b	5	10	7
2 levels 1 rows			

For each row, select columns whose values are greater than 5, leaving a ragged structure:

```
In[18]:= data[All, Select[# > 5 &]]
```

a	< >
b	< y → 10, z → 7 >
2 levels 2 rows	

Normal turns the dataset into an ordinary association of associations:

```
In[19]:= Normal[%]
```

```
Out[19]= <| a -> <| |>, b -> <| y -> 10, z -> 7 |> |>
```

Many Wolfram Language functions have operator forms.

Sort according to the values of a function applied to each element:

```
In[20]:= SortBy[{1, 3, 6, 8, 2, 5, 9, 7}, If[EvenQ[#], #, 10 + #] &]
```

```
Out[20]= {2, 6, 8, 1, 3, 5, 7, 9}
```

SortBy has an operator form:

```
In[21]:= SortBy[If[EvenQ[#], #, 10 + #] &][{1, 3, 6, 8, 2, 5, 9, 7}]
```

```
Out[21]= {2, 6, 8, 1, 3, 5, 7, 9}
```

Sort rows according to the value of the difference of the x and y columns:

```
In[22]:= data[SortBy[##x - ##y &]]
```

	x	y	z
b	5	10	7
a	1	2	3
2 levels 2 rows			

Sort the rows, and find the total of all columns:

```
In[23]:= data[SortBy[##x - ##y &], Total]
```

	22
a	6
1 level 2 elements	

Sometimes you want to apply a function to each element in the dataset.

Apply f to each element in the dataset:

```
In[24]:= data[All, All, f]
```

	x	y	z
a	f[1]	f[2]	f[3]
b	f[5]	f[10]	f[7]
2 levels 2 rows			

Square each element before sorting and totaling:

```
In[25]:= data[SortBy[##x - ##y &], Total, #^2 &]
```

	174
a	14
1 level 2 elements	

Datasets can contain arbitrary mixtures of lists and associations. Here's a dataset that can be thought of as a *list of records* with named *fields*.

A dataset formed from a list of associations:

```
In[26]:= Dataset[{<| "x" → 2, "y" → 4, "z" → 6 |>, <| "x" → 11, "y" → 7, "z" → 1 |>}]
```

	x	y	z
Out[26]=	2	4	6
	11	7	1

2 levels | 2 rows

It's OK for entries to be missing:

```
In[27]:= Dataset[{<| "x" → 2, "y" → 4, "z" → 6 |>, <| "x" → 11, "y" → 7 |>}]
```

	x	y	z
Out[27]=	2	4	6
	11	7	KeyAbsent

2 levels | 2 rows

Now that we've seen some simple examples, it's time to look at something slightly more realistic. Let's import a dataset giving properties of planets and moons. The dataset has a hierarchical structure, with each planet having a mass and radius of its own, and then also having a collection of moons, each of which have their own properties. This general structure is extremely common in practice (think students and grades, customers and orders, etc.).

Get a hierarchical dataset of planets and moons from the cloud:

```
In[28]:= planets = CloudGet["http://wolfr.am/7FxLgPm5"]
```

	Mass	Radius	Moons
Mercury	$0.0552734 M_{\oplus}$	2439.7 km	< >
Venus	$0.814996 M_{\oplus}$	6051.9 km	< >
Earth	$1.000000 M_{\oplus}$	6367.4447 km	< Moon → < Mass → $0.012300 M_{\oplus}$, Radius → 1737.5 km > >
Mars	$0.107447 M_{\oplus}$	3386. km	< Phobos → < Mass → $1.796 \times 10^{-9} M_{\oplus}$, Radius → 11.1 km >, Deimos → < Mass → $1.067 \times 10^{-10} M_{\oplus}$, Radius → 23.9 km > >
Jupiter	$317.828 M_{\oplus}$	69173. km	< Ganymede → < Mass → $0.024807 M_{\oplus}$, Radius → 2631.2 km >, Callisto → < Mass → $0.000102 M_{\oplus}$, Radius → 4744. km > >
Saturn	$95.1608 M_{\oplus}$	57316. km	< Titan → < Mass → $0.022525 M_{\oplus}$, Radius → 2575.5 km >, Rhea → < Mass → $0.00015903 M_{\oplus}$, Radius → 1524. km >, Iapetus → < Mass → $0.000102 M_{\oplus}$, Radius → 1433. km > >
Uranus	$14.5357 M_{\oplus}$	25266. km	< Titania → < Mass → $0.0005903 M_{\oplus}$, Radius → 788.9 km >, Oberon → < Mass → $0.00015903 M_{\oplus}$, Radius → 1353.4 km >, Proteus → < Mass → $0.000102 M_{\oplus}$, Radius → 1062. km > >
Neptune	$17.1478 M_{\oplus}$	24553. km	< Triton → < Mass → $0.0035823 M_{\oplus}$, Radius → 1353.4 km >, Proteus → < Mass → $0.00015903 M_{\oplus}$, Radius → 1062. km > >

4 levels | 8 rows

Find the radii of all the planets:

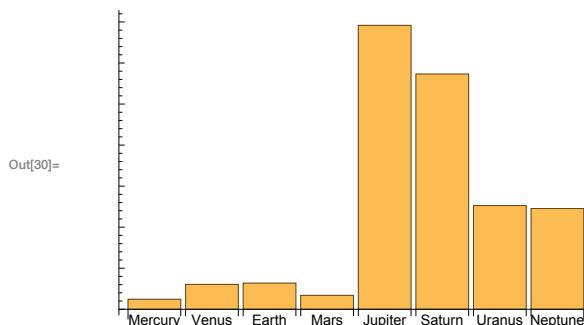
```
In[29]:= planets[All, "Radius"]
```

Mercury	2439.7 km
Venus	6051.9 km
Earth	6367.4447 km
Mars	3386. km
Jupiter	69173. km
Saturn	57316. km
Uranus	25266. km
Neptune	24553. km

Out[29]= 1 level | 8 elements

Make a bar chart of planet radii:

```
In[30]:= BarChart[planets[All, "Radius"], ChartLabels → Automatic]
```



If we ask about the moons of Mars, we get a dataset, which we can then query further.

Get a dataset about the moons of Mars:

```
In[31]:= planets["Mars", "Moons"]
```

	Mass	Radius
	Phobos	$1.796 \times 10^{-9} M_{\oplus}$ 11.1 km
Out[31]=	Deimos	$2.5 \times 10^{-10} M_{\oplus}$ 6.2 km

2 levels | 2 rows

“Drill down” to make a table of radii of all the moons of Mars:

```
In[32]:= planets["Mars", "Moons", All, "Radius"]
```

Phobos	11.1 km
Deimos	6.2 km

Out[32]= 1 level | 2 elements

We can do computations about the moons of all planets. First, let's just find out how many moons are listed for each planet.

Make a dataset of the number of moons listed for each planet:

In[33]:= **planets[All, "Moons", Length]**

Mercury	0
Venus	0
Earth	1
Mars	2
Jupiter	38
Saturn	33
Uranus	21
Neptune	8

1 level | 8 elements

Find the total mass of all moons for each planet:

In[34]:= **planets[All, "Moons", Total, "Mass"]**

Mercury	$0 M_{\oplus}$
Venus	$0 M_{\oplus}$
Earth	$0.012300 M_{\oplus}$
Mars	$2.04 \times 10^{-9} M_{\oplus}$
Jupiter	$0.065807 M_{\oplus}$
Saturn	$0.023527 M_{\oplus}$
Uranus	$0.001531 M_{\oplus}$
Neptune	$0.003598 M_{\oplus}$

1 level | 8 elements

Get the same result, but only for planets with more than 10 moons:

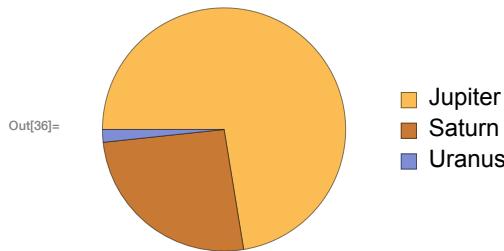
In[35]:= **planets[Select[Length[#Moons] > 10 &], "Moons", Total, "Mass"]**

Jupiter	$0.065807 M_{\oplus}$
Saturn	$0.023527 M_{\oplus}$
Uranus	$0.001531 M_{\oplus}$

1 level | 3 elements

Make a pie chart of the result:

```
In[36]:= PieChart[% , ChartLegends → Automatic]
```



Get a dataset with moons that are more than 1% of the mass of the Earth.

For all moons, select ones whose mass is greater than 0.01 times the mass of the Earth:

```
In[37]:= planets[All, "Moons", Select[#Mass > .01 earth masses &]]
```

Mercury	$\langle \rangle$
Venus	$\langle \rangle$
Earth	$\langle \text{Moon} \rightarrow \langle \text{Mass} \rightarrow 0.012300 M_{\oplus}, \text{Radius} \rightarrow 1737.5 \text{ km} \rangle \rangle$
Mars	$\langle \rangle$
Jupiter	$\langle \text{Ganymede} \rightarrow \langle \text{Mass} \rightarrow 0.024807 M_{\oplus}, \text{Radius} \rightarrow 2631.2 \text{ km} \rangle, \text{Callisto} \rightarrow \langle \text{Mass} \rightarrow 0.018011 M_{\oplus}, \text{Radius} \rightarrow 2474.8 \text{ km} \rangle \rangle$
Saturn	$\langle \text{Titan} \rightarrow \langle \text{Mass} \rightarrow 0.022525 M_{\oplus}, \text{Radius} \rightarrow 2575.5 \text{ km} \rangle \rangle$
Uranus	$\langle \rangle$
Neptune	$\langle \rangle$

3 levels | 8 rows

Get the list of keys (i.e. moon names) in the resulting association for each planet:

```
In[38]:= planets[All, "Moons", Select[#Mass > .01 earth masses &]][All, Keys]
```

Mercury	{}
Venus	{}
Earth	{Moon}
Mars	{}
Jupiter	{Ganymede, Callisto, Io}
Saturn	{Titan}
Uranus	{}
Neptune	{}

2 levels | 8 rows

Get the underlying association:

In[39]:= **Normal[%]**

Out[39]= $\langle | \text{Mercury} \rightarrow \{\}, \text{Venus} \rightarrow \{\}, \text{Earth} \rightarrow \{\text{Moon}\}, \text{Mars} \rightarrow \{\},$
 $\text{Jupiter} \rightarrow \{\text{Ganymede, Callisto, Io}\}, \text{Saturn} \rightarrow \{\text{Titan}\}, \text{Uranus} \rightarrow \{\}, \text{Neptune} \rightarrow \{\} | \rangle$

Catenate the lists for all keys:

In[40]:= **Catenate[%]**

Out[40]= {Moon, Ganymede, Callisto, Io, Titan}

Here's the whole computation in one line:

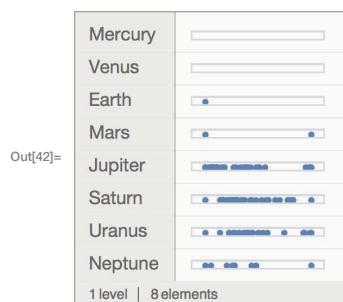
In[41]:= **planets[All, "Moons", Select[#Mass > .01 earth masses &]] // Catenate, Keys] // Normal**

Out[41]= {Moon, Ganymede, Callisto, Io, Titan}

Here's one more example, where we find the logarithm of the mass of each moon, then make a number line plot of these values for each planet.

Make number line plots of the logarithms of masses for moons of each planet:

In[42]:= **planets[All, "Moons", NumberLinePlot[Values[#] &, Log[#Mass/1 earth mass]] &]**



As a final example, let's make a word cloud of names of moons, sized according to the masses of the moons. To do this, we need a single association that associates the name of each moon with its mass.

When given an association, WordCloud determines sizes from values in the association:

In[43]:= **WordCloud[<| "A" → 5, "B" → 4, "C" → 3, "D" → 2, "E" → 1 |>]**



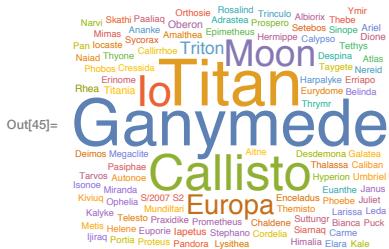
The function Association combines associations:

In[44]:= **Association[<| "a" → 1, "b" → 2 |>, <| "c" → 3 |>]**

Out[44]= $\langle | a \rightarrow 1, b \rightarrow 2, c \rightarrow 3 | \rangle$

Generate the word cloud of moon masses:

```
In[45]:= planets[WordCloud[Association[Values[#]]] &, "Moons", All, "Mass"]
```



For what it does, the code here is surprisingly simple. But we can make it slightly more streamlined by using `@*` or `/*`.

We've seen before that we can write something like `f[g[x]]` as `f@g@x` or `x//g//f`.

We can also write it `f[g[#]] &[x]`. But what about `f[g[#]] &?` Is there a short way to write this? The answer is that there is, in terms of the *function composition operators* `@*` and `/*`.

`f@*g@*h` represents a composition of functions to be applied right-to-left:

```
In[46]:= (f@*g@*h)[x]
```

```
Out[46]= f[g[h[x]]]
```

`h /* g /* f` represents a composition of functions to be applied left-to-right:

```
In[47]:= (h /* g /* f)[x]
```

```
Out[47]= f[g[h[x]]]
```

Here's the previous code rewritten using composition `@*`:

```
In[48]:= planets[WordCloud@*Association@*Values, "Moons", All, "Mass"]
```



And using right composition /*:

```
In[49]:= planets[Values /* Association /* WordCloud, "Moons", All, "Mass"]
```



Vocabulary

Dataset [<i>data</i>]	a dataset
Normal [<i>dataset</i>]	convert a dataset to normal lists and associations
Catenate [{ <i>assoc</i> ₁ , ...}]	catenate associations, combining their elements
<i>f</i> @* <i>g</i>	composition of functions (<i>f</i> [<i>g</i> [<i>x</i>]] when applied to <i>x</i>)
<i>f</i> /* <i>g</i>	right composition (<i>g</i> [<i>f</i> [<i>x</i>]] when applied to <i>x</i>)

Exercises

Note: These exercises use the dataset `planets = CloudGet["http://wolfr.am/7FxLgPm5"]`.

- 45.1** Make a word cloud of the planets, with weights determined by their number of moons.
 - 45.2** Make a bar chart of the number of moons for each planet.
 - 45.3** Make a dataset of the masses of the planets, sorted by their number of moons.
 - 45.4** Make a dataset of planets and the mass of each one's most massive moon.
 - 45.5** Make a dataset of masses of planets, where the planets are sorted by the largest mass of their moons.
 - 45.6** Make a dataset of the median mass of all moons for each planet.
 - 45.7** For each planet, make a list of moons larger in mass than 0.0001 Earth masses.
 - 45.8** Make a word cloud of countries in Central America, with the names of countries proportional to the lengths of the Wikipedia article about them.

Q&A

What kinds of data can datasets contain?

Any kinds. Not just numbers and text but also images, graphs and lots more. There's no need for all elements of a particular row or column to be the same type.

Can I turn spreadsheets into datasets?

Yes. `SemanticImport` is often a good way to do it.

What are databases and how do they relate to Dataset?

Databases are a traditional way to store structured data in a computer system. Databases are often set up to allow both reading and writing of data. `Dataset` is a way to represent data that might be stored in a database so that it's easy to manipulate with the Wolfram Language.

How does data in Dataset compare to data in an SQL (relational) database?

SQL databases are strictly based on tables of data arranged in rows and columns of particular types, with additional data linked in through "foreign keys". `Dataset` can have any mixture of types of data, with any number of levels of nesting, and any hierarchical structure, somewhat more analogous to a NoSQL database, but with additional operations made possible by the symbolic nature of the language.

Tech Notes

- `Dataset` supports a new kind of *symbolic database structure* which generalizes both *relational* and *hierarchical databases*.
- `Dataset` has many additional mechanisms and capabilities that we haven't discussed.
- Everything that can be done with queries on datasets can also be done by using functions like `Map` and `Apply` on underlying lists and association—but it's typically much simpler with dataset queries.
- You can connect the Wolfram Language directly to SQL databases—and do queries with SQL syntax—using `DatabaseLink`.

More to Explore

Guide to Computation with Structured Datasets in the Wolfram Language (wolfr.am/eiwl-45-more)

46 | Writing Good Code

Writing good code is in many ways like writing good prose: you need to have your thoughts clear, and express them well. When you first start writing code, you'll most likely think about what your code does in English or whatever natural language you use. But as you become fluent in the Wolfram Language you'll start thinking directly in code, and it'll be faster for you to type a program than to describe what it does.

My goal as a language designer has been to make it as easy as possible to express things in the Wolfram Language. The functions in the Wolfram Language are much like the words in a natural language, and I've worked hard to choose them well.

Functions like `Table` or `NestList` or `FoldList` exist in the Wolfram Language because they express common things one wants to do. As in natural language, there are always many ways one can in principle express something. But good code involves finding the most direct and simple way.

To create a table of the first 10 squares in the Wolfram Language, there's an obvious good piece of code that just uses the function `Table`.

Simple and good Wolfram Language code for making a table of the first 10 squares:

```
In[1]:= Table[n^2, {n, 10}]  
Out[1]= {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
```

Why would anyone write anything else? A common issue is not thinking about the “whole table”, but instead thinking about the steps in building it. In the early days of computing, computers needed all the help they could get, and there was no choice but to give code that described every step to take.

A much worse piece of code that builds up the table step by step:

```
In[2]:= Module[{list, i}, list = {}; For[i = 1, i ≤ 10, i++, list = Append[list, i^2]]; list]  
Out[2]= {1, 4, 9, 16, 25, 36, 49, 64, 81, 100}
```

But the point of the Wolfram Language is to let one express things at a higher level—and to create code that as directly as possible captures the concept of what one wants to do. Once one knows the language, it's vastly more efficient to operate at this level. And it leads to code that's easier for both computers and humans to understand.

In writing good code, it's important to ask frequently, “What's the big picture of what this code is trying to do?” Often you'll start off understanding only some part, and writing code just for that. But then you'll end up extending it, and adding more and more pieces to your code. But if you think about the big picture you may suddenly realize that there's some more powerful function—like a `Fold`—that you can use to make your code nice and simple again.

Make code to convert *{hundreds, tens, ones}* digits to a single integer:

```
In[3]:= fromdigits[{h_, t_, o_}] := 100 h + 10 t + o
```

Run the code:

```
In[4]:= fromdigits[{5, 6, 1}]
```

```
Out[4]= 561
```

Write a generalization to a list of any length, using Table:

```
In[5]:= fromdigits[list_List] := Total[Table[10^(Length[list]-i)*list[[i]], {i, Length[list]}]]
```

The new code works:

```
In[6]:= fromdigits[{5, 6, 1, 7, 8}]
```

```
Out[6]= 56178
```

Simplify the code by multiplying the whole list of powers of 10 at the same time:

```
In[7]:= fromdigits[list_List] := Total[10^Reverse[Range[Length[list]] - 1]*list]
```

Try a different, recursive, approach, after first clearing the earlier definitions:

```
In[8]:= Clear[fromdigits]
```

```
In[9]:= fromdigits[{k_}] := k
```

```
In[10]:= fromdigits[{digits___, k_}] := 10*fromdigits[{digits}]+k
```

The new approach works too:

```
In[11]:= fromdigits[{5, 6, 1, 7, 8}]
```

```
Out[11]= 56178
```

But then you realize: it's actually all just a Fold!

```
In[12]:= Clear[fromdigits]
```

```
In[13]:= fromdigits[list_] := Fold[10##1+##2 &, list]
```

```
In[14]:= fromdigits[{5, 6, 1, 7, 8}]
```

```
Out[14]= 56178
```

Of course, there's a built-in function that does it too:

```
In[15]:= FromDigits[{5, 6, 1, 7, 8}]
```

```
Out[15]= 56178
```

Why is it good for code to be simple? First, because it's more likely to be correct. It's much easier for a mistake to hide in a complicated piece of code than a simple one. Code that's simple is also usually more general, so it'll tend to cover even cases you didn't think of, and avoid you having to write more code. And finally, simple

code tends to be much easier to read and understand. (Simpler isn't always the same as shorter, and in fact short “code poetry” can get hard to understand.)

An overly short version of `fromdigits`, that's starting to be difficult to understand:

```
In[16]:= fromdigits = Fold[{10, 1}.{##} &, ##] &;
```

It still works though:

```
In[17]:= fromdigits[{5, 6, 1, 7, 8}]
```

```
Out[17]= 56178
```

If what you're trying to do is complicated, then your code may inevitably need to be complicated. Good code, though, is broken up into functions and definitions that are each as simple and self-contained as possible. Even in very large Wolfram Language programs, there may be no individual definitions longer than a handful of lines.

Here's a single definition that combines several cases:

```
In[18]:= fib[n_] := If[! IntegerQ[n] || n < 1, "Error", If[n == 1 || n == 2, 1, fib[n - 1] + fib[n - 2]]]
```

It's much better to break it up into several simpler definitions:

```
In[19]:= fib[1] = fib[2] = 1;
```

```
In[20]:= fib[n_Integer] := fib[n - 1] + fib[n - 2]
```

A very important aspect of writing good code is choosing good names for your functions. For the built-in functions of the Wolfram Language, I've made a tremendous effort over the course of decades to pick names well—and to capture in their short names the essence of what the functions do, and how to think about them.

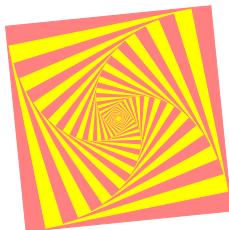
When you're writing code, it's common to first define a new function because you need it in some very specific context. But it's almost always worth trying to give it a name that you'll understand even outside that context. And if you can't find a good name, it's often a sign that it's not quite the right function to define in the first place.

A sign of a good function name is that when you read it in a piece of code, you immediately know what the code does. And indeed, it's an important feature of the Wolfram Language that it's typically easier to read and understand well-written code directly than from any kind of textual description of it.

How would one describe this in plain English?

```
In[21]:= Graphics[  
  {White, Riffle[NestList[Scale[Rotate[#, 0.1], 0.9] &, Rectangle[], 40], {Pink, Yellow}]}]
```

Out[21]=



When you write Wolfram Language code, you'll sometimes have to choose between using a single rare built-in function that happens to do exactly what you want—and building up the same functionality from several more common functions. In this book, I've sometimes chosen to avoid rare functions so as to minimize vocabulary. But the best code tends to use single functions whenever it can—because the name of the function explains the intent of the code in a way that individual pieces cannot.

Use a small piece of code to reverse the digits in an integer:

```
In[22]:= FromDigits[Reverse[IntegerDigits[123 456]]]
```

Out[22]= 654 321

Using a single built-in function explains the intent more clearly:

```
In[23]:= IntegerReverse[123 456]
```

Out[23]= 654 321

Good code needs to be correct and easy to understand. But it also needs to run efficiently. And in the Wolfram Language, simpler code is typically better here too—because by explaining your intent more clearly, it makes it easier for the Wolfram Language to optimize how the computations you need are done internally.

With every new version, the Wolfram Language does better at automatically figuring out how to make your code run fast. But you can always help by structuring your algorithms well.

Timing gives the result of a computation, together with its timing (in seconds):

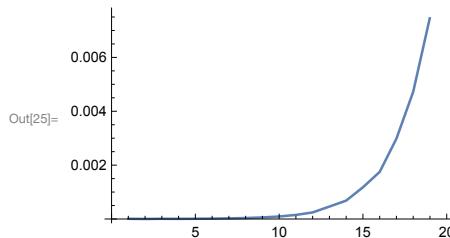
```
In[24]:= Timing[fib[20]]
```

Out[24]= {0.021843, 6765}

Plot the time to compute $\text{fib}[n]$ according to the definitions above.

With the definitions of `fib` above, the time grows very rapidly:

```
In[25]:= ListLinePlot[Table[First[Timing[fib[n]]], {n, 20}]]
```



The algorithm we used happens to do an exponential amount of unnecessary work recomputing what it's already computed before. We can avoid this by making the definition for `fib[n_]` always do an assignment for `fib[n]`, so it stores the result of each intermediate computation.

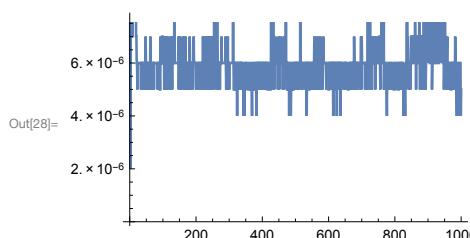
Redefine the `fib` function to remember every value it computes:

```
In[26]:= fib[1] = fib[2] = 1;
```

```
In[27]:= fib[n_Integer] := fib[n] = fib[n - 1] + fib[n - 2]
```

Now even up to 1000 each new value takes only microseconds to compute:

```
In[28]:= ListLinePlot[Table[First[Timing[fib[n]]], {n, 1000}]]
```



Vocabulary

`FromDigits[list]` assemble an integer from its digits

`IntegerReverse[n]` reverse the digits in an integer

`Timing[expr]` do a computation, timing how long it takes

Exercises

- 46.1** Find a simpler form for `Module[{a, i}, a = 0; For[i = 1, i ≤ 1000, i++, a = i * (i + 1) + a]; a]`.
- 46.2** Find a simpler form for `Module[{a, i}, a = x; For[i = 1, i ≤ 10, i++, a = 1 / (1 + a)]; a]`.
- 46.3** Find a simpler form for
`Module[{i, j, a}, a = {}; For[i = 1, i ≤ 10, i++, For[j = 1, j ≤ 10, j++, a = Join[a, {i, j}]]]; a]`.
- 46.4** Make a line plot of the timing for computing n^n for n up to 10000.
- 46.5** Make a line plot of the timing for `Sort` to sort `Range[n]` from a random order, for n up to 200.

Q&A

What does `i++` mean?

It's a short notation for `i = i + 1`. It's the same notation that C and many other low-level computer languages use for this increment operation.

What does the `For` function do?

It's a direct analog of the `for (...)` statement in C. `For[start, test, step, body]` first executes `start`, then checks `test`, then executes `step`, then `body`. It does this repeatedly until `test` no longer gives `True`.

Why can shortened pieces of code be hard to understand?

The most common issue is that variables and sometimes even functions have been factored out, so there are fewer names to read that might give clues about what the code is supposed to do.

What's the best IDE for authoring Wolfram Language code?

For everyday programming, Wolfram Notebooks are best. Make sure to add sections, text, and examples right alongside your code. For large multi-developer software projects, Wolfram Workbench provides an Eclipse-based IDE.

What does `Timing` actually measure?

It measures the CPU time spent in the Wolfram Language actually computing your result. It doesn't include time to display the result. Nor does it include time spent on external operations, like fetching data from the cloud. If you want the absolute "wall clock" time, use `AbsoluteTiming`.

How can I get more accurate timings for code that runs fast?

Use `RepeatedTiming`, which runs code many times and averages the timings it gets. (This won't work if the code is modifying itself, like in the last definition of `fib` above.)

What are some tricks for speeding up code?

Beyond keeping the code simple, one thing is not to recompute anything you don't have to. Also, if you're dealing with lots of numbers, it may make sense to use `N` to force the numbers to be approximate. For some internal algorithms you can pick your `PerformanceGoal`, typically trading off speed and accuracy. There are also functions like `Compile` that force more of the work associated with optimization to be done up front, rather than during a computation.

Tech Notes

- Complicated behavior can arise even from extremely simple code: that's what my 1280-page book *A New Kind of Science* is about. A good example is `CellularAutomaton[30, {{1}, 0}]`.
- The `fib` function is computing `Fibonacci[n]`. The original definition always recurses down a whole tree of $O(1.618^n)$ values.
- Remembering values that a function has computed before is sometimes called memoization, sometimes dynamic programming, and sometimes just caching.
- The function `IntegerReverse` is new in Version 10.3.
- For large programs, the Wolfram Language has a framework for isolating functions into *contexts* and *packages*.
- `If[#1 > 2, 2 #0[#1 - #0[#1 - 2]], 1] & /@ Range[50]` is an example of short code that's seriously challenging to understand...

47 | Debugging Your Code

Even the most experienced programmers have to spend time debugging their code. It's an inevitable part of programming. But in the Wolfram Language it's particularly easy, especially if you follow a few principles.

The first and most important principle is to try out any piece of code you write. Because the Wolfram Language is interactive—and symbolic—you can always instantly do this. So even if you just make a tiny change, run your code again, and see if your test examples still work. If they don't, fix the code before you go on.

The Wolfram Language can sometimes tell as soon as you type something in that it's likely to be wrong—and then it'll color it red.

A piece of code with various problems, indicated with red:

```
WordCloud[Nest[Join[#, Length[.] + Reverse[#, 1, 2]] &, {0}, m], Spacings -> 0]
```

Once you run a piece of code, the Wolfram Language can also often tell if things have gone obviously wrong—in which case it'll display a message. The code here ends up, for example, asking for the first element of a length-0 list.

Something is obviously wrong when `First` is applied to the {} that's produced by `Cases`:

```
In[1]:= First[Cases[{1, 2, 3, 4}, 777]]
First::nofirst: {} has zero length and no first element. >
Out[1]= First[{}]
```

Sometimes the Wolfram Language may not know what to do with something, but it may not be sure there's a problem. In such cases, it'll just leave the input unchanged and return it in a symbolic form that can later get a definite value.

Without values for `a`, `b` and `c`, the Wolfram Language just returns this unchanged:

```
In[2]:= Graph[{a, b, c}]
Out[2]= Graph[{a, b, c}]
```

If you generate graphics with symbolic pieces that can't be rendered, you'll get a pink box:

```
In[3]:= Graphics[{Circle[{0, 0}], Disk[{a, b}]}]
```



As you build up functions, it's quite common to have fragments of code that you want to test without finishing the function. You can do this by setting values of variables using `With`—which works like `Module`, except it doesn't allow values to be reset.

Use `With` to temporarily set $m = 4$ to test a fragment of code:

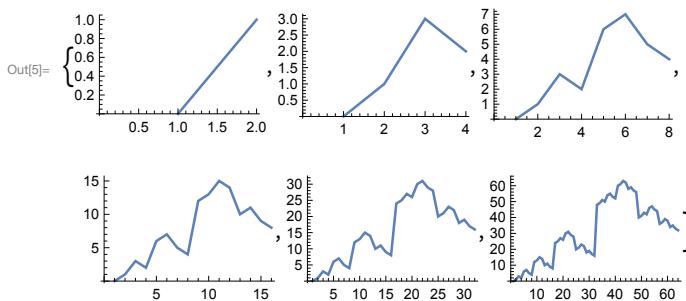
```
In[4]:= With[{m = 4}, Nest[Join[#, Length[#] + Reverse[#]] &, {0}, m]]
```

```
Out[4]= {0, 1, 3, 2, 6, 7, 5, 4, 12, 13, 15, 14, 10, 11, 9, 8}
```

When debugging takes a long time it's usually because one's made the wrong assumption about what one's code is doing. And in my experience, the best way to overcome this is just to systematically analyze how the code behaves, making tables of results, generating visualizations, testing out assertions, and so on.

Make plots to see what the code is doing:

```
In[5]:= ListLinePlot /@ Table[Nest[Join[#, Length[#] + Reverse[#]] &, {0}, m], {m, 6}]
```



If this code is working correctly, the result should contain every number from 0 to $2^m - 1$:

```
In[6]:= Sort[With[{m = 4}, Nest[Join[#, Length[#] + Reverse[#]] &, {0}, m]]]
```

```
Out[6]= {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}
```

Systematically check this up to $m = 10$:

```
In[7]:= Table[
  Sort[Nest[Join[#, Length[#] + Reverse[#]] &, {0}, m]] == Range[0, 2^m - 1], {m, 10}]
Out[7]= {True, True, True, True, True, True, True, True, True, True}
```

Sometimes it's not sufficient just to see the final result of a piece of code; you need to see what's going on inside as well. You can insert the function `Echo` anywhere to print intermediate results or values from your code.

`Echo` prints values, but does not interfere with the result:

```
In[8]:= Table[Echo[n]^2, {n, 3}]
```

```
» 1
» 2
» 3
```

```
Out[8]= {1, 4, 9}
```

If you're running a long computation, you can monitor its progress using `Monitor`.

Continuously show (with a frame) the value of `n` reached so far:

```
In[9]:= Monitor[Table[PrimeQ[2^2^n+1], {n, 15}], Framed[n]]
```

```
11 12 13 14 15
```

```
Out[9]= {True, True, True, True, False, False, False, False, False, False, False, False, False, False}
```

`Echo` and `Monitor` just display things. If you want to actually capture intermediate results, you can do it using `Sow` and `Reap`.

`Reap` gives the final result together with a list of everything that was sown by `Sow`:

```
In[10]:= Reap[Total[Table[Sow[n], {n, 5}]]]
```

```
Out[10]= {15, {{1, 2, 3, 4, 5}}}
```

This sows successive values of `Length[#]`, and reaps the results:

```
In[11]:= Last[Reap[Nest[Join[#, Sow[Length[#]]+Reverse[#]] &, {0}, 10]]]
```

```
Out[11]= {{1, 2, 4, 8, 16, 32, 64, 128, 256, 512}}
```

Vocabulary

<code>With[{<i>x</i> = <i>value</i>}, <i>expr</i>]</code>	compute <i>expr</i> with <i>x</i> replaced by <i>value</i>
---	--

<code>Echo[<i>expr</i>]</code>	display and return the value of <i>expr</i>
--------------------------------	---

<code>Monitor[<i>expr</i>, <i>obj</i>]</code>	continually display <i>obj</i> during a computation
---	---

<code>Sow[<i>expr</i>]</code>	sow the value of <i>expr</i> for subsequent reaping
-------------------------------	---

<code>Reap[<i>expr</i>]</code>	collect values sowed while <i>expr</i> is being computed
--------------------------------	--

Exercises

47.1 Correct the program `Counts[StringTake[#, 2] & /@ WordList[]]` for counting possible first two letters in words.

47.2 Use `Sow` and `Reap` to find intermediate values of `# 1` in `Fold[10 # 1 + # 2 &, {1, 2, 3, 4, 5}]`.

47.3 Use `Sow` and `Reap` to get a list of all cases where `#/2` is used in `Nest[If[EvenQ[#], #/2, 3 # + 1] &, 1000, 20]`.

Q&A**Can I cause a problem by just trying to run a piece of code?**

Not unless your code is for example set up to delete something. The Wolfram Language has protections against “obviously wrong” infinite loops and so on. If something is taking too long, you can always abort it. If you’re really concerned about resources, use `TimeConstrained` and `MemoryConstrained`.

Is there a most common type of bug in Wolfram Language code?

Not really. The design of the language makes all sorts of bugs that are common in other programming languages rare. For example, “off by one” errors are common in languages where you’re always explicitly manipulating loop variables, but are rare when you’re using Wolfram Language “whole-list” functions like `Table`.

If I can’t figure out what’s going on, is it worth just trying random things?

If you think you’re close, it’s often not crazy to try making small random changes to code to see what happens. Since in the Wolfram Language simpler code is usually more likely to be correct, it’s not uncommon to hit the code you want by a small amount of random searching.

Is there a way to do interactive step-by-step debugging in the Wolfram Language?

Yes (at least with a native desktop interface)—though it’s rarely used. Given the structure of the Wolfram Language, systematically capturing and analyzing intermediate results is almost always a better approach.

How can I tell what’s wrong when a piece of graphics is “pinked”?

Hover over it and you’ll see the underlying symbolic expression. Or press the `+` to print messages.

What is the code that makes the fractal-like graphics doing?

It’s making a *Gray code*—an ordering of integers so that only one binary digit changes at each step.

Tech Notes

- For large-scale software development, the Wolfram Language has a built-in framework for creating and running systematic tests, with functions like `VerificationTest` and `TestReport`.
- A major goal of good language design is to encourage people to write correct code.

More to Explore

[Guide to Debugging in the Wolfram Language](#) (wolfr.am/eiwl-47-more)

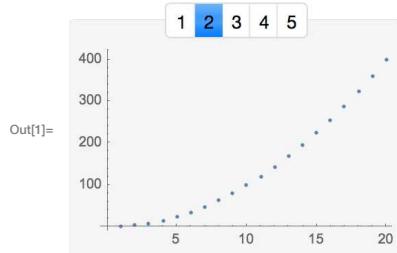
What We Haven't Discussed

There's a lot more to the Wolfram Language than we've been able to cover in this book. Here's a sampling of just a few of the many topics and areas that we've missed.

User Interface Construction

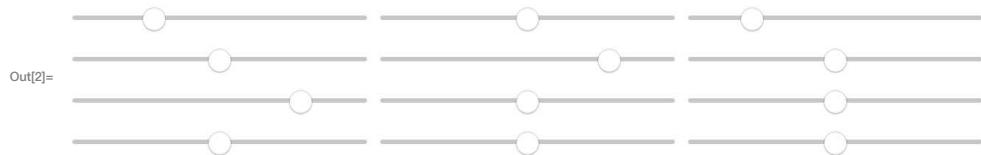
Set up a tabbed interface:

```
In[1]:= TabView[Table[ListPlot[Range[20]^n], {n, 5}]]
```



User interfaces are just another type of symbolic expression; make a grid of sliders:

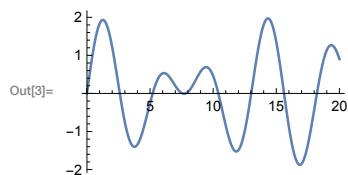
```
In[2]:= Grid[Table[Slider[], 4, 3]]
```



Function Visualization

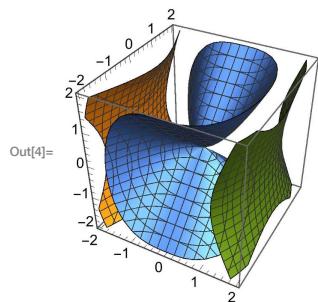
Plot a function:

```
In[3]:= Plot[Sin[x] + Sin[Sqrt[2] x], {x, 0, 20}]
```



3D contour plot:

```
In[4]:= ContourPlot3D[x^3 + y^2 - z^2, {x, -2, 2}, {y, -2, 2}, {z, -2, 2}]
```



Mathematical Computation

Do symbolic computations, with x as an algebraic variable:

```
In[5]:= Factor[x^10 - 1]
```

```
Out[5]= (-1+x)(1+x)(1-x+x^2-x^3+x^4)(1+x+x^2+x^3+x^4)
```

Get symbolic solutions to equations:

```
In[6]:= Solve[x^3 - 2x + 1 == 0, x]
```

```
Out[6]= {x → 1}, {x → 1/2 (-1 - √5)}, {x → 1/2 (-1 + √5)}
```

Do calculus symbolically:

```
In[7]:= Integrate[Sqrt[x + Sqrt[x]], x]
```

```
Out[7]= 1/12 √(√x + x) (-3 + 2 √x + 8x) + 1/8 Log[1 + 2 √x + 2 √(√x + x)]
```

Display results in traditional mathematical form:

```
In[8]:= Integrate[AiryAi[x], x] // TraditionalForm
```

$$\frac{x \left(\sqrt{3} x \Gamma\left(\frac{2}{3}\right)^2 {}_1F_2\left(\frac{2}{3}; \frac{4}{3}, \frac{5}{3}; \frac{x^3}{9}\right) - 3 \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right) {}_1F_2\left(\frac{1}{3}; \frac{2}{3}, \frac{4}{3}; \frac{x^3}{9}\right)\right)}{9 \times 3^{2/3} \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right) \Gamma\left(\frac{5}{3}\right)}$$

Use 2D notation for input:

```
In[9]:= Sum[Binomial[n, i] i!, {i, 0, n}]
```

```
Out[9]= π^(n/2)/(2^n n!)
```

Numerics

Minimize a function inside a spherical ball:

```
In[10]:= NMinimize[{x^4 + y^4 - z/(x+1), y > 0}, {x, y, z} ∈ Ball[]]
```

```
Out[10]= {-7.34516, {x → -0.971029, y → 0.0139884, z → 0.238555}}
```

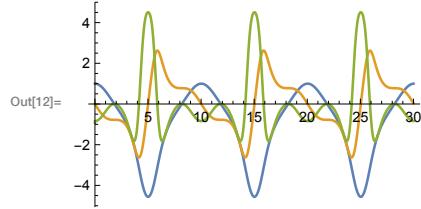
Solve a differential equation to get an approximate function:

```
In[11]:= NDSolve[{y''[x] + Sin[y[x]] y[x] == 0, y[0] == 1, y'[0] == 0}, y, {x, 0, 30}]
```

```
Out[11]= {y → InterpolatingFunction[ Domain: {{0., 30.}} ]}
```

Make a plot using the approximate function:

```
In[12]:= Plot[Evaluate[{y[x], y'[x], y''[x]} /. %], {x, 0, 30}]
```



Geometry

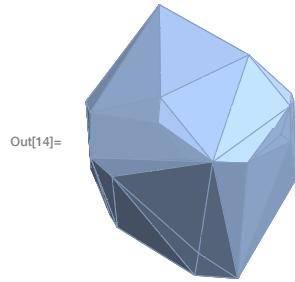
The area of a disk (filled circle) of radius r :

```
In[13]:= Area[Disk[{0, 0}, r]]
```

```
Out[13]=  $\pi r^2$ 
```

Make a shape by “shrinkwrapping” around 100 random points in 3D:

```
In[14]:= ConvexHullMesh[RandomReal[1, {100, 3}]]
```



Algorithms

Find the shortest tour of the capitals of Europe (*traveling salesman problem*):

```
In[15]:= With[{c = europe capital cities coordinates },
  GeoListPlot[c[[Last@FindShortestTour[c]]], Joined → True]]
```



Out[15]=

Factor a big number:

```
In[16]:= FactorInteger[2^255 - 1]
```

```
Out[16]= {{7, 1}, {31, 1}, {103, 1}, {151, 1}, {2143, 1}, {11119, 1}, {106591, 1}, {131071, 1}, {949111, 1}, {9520972806333758431, 1}, {5702451577639775545838643151, 1}}
```

Logic

Make a truth table:

```
In[17]:= BooleanTable[p || q && (p || !q), {p}, {q}] // Grid
```

```
Out[17]= True  True  
        False False
```

Find a minimal representation of a Boolean function:

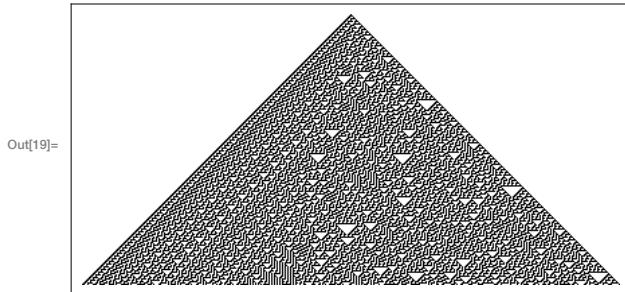
```
In[18]:= BooleanMinimize[BooleanCountingFunction[{2, 3}, {a, b, c, d}]] // TraditionalForm
```

```
Out[18]= (a ∧ b ∧ ¬d) ∨ (a ∧ ¬b ∧ c) ∨ (a ∧ ¬c ∧ d) ∨ (¬a ∧ b ∧ d) ∨ (b ∧ c ∧ ¬d) ∨ (¬b ∧ c ∧ d)
```

The Computational Universe

Run my favorite example of a very simple program with very complex behavior:

```
In[19]:= ArrayPlot[CellularAutomaton[30, {{1}, 0}, 200]]
```



Building APIs

Deploy a simple web API that finds the distance from a specified location:

```
In[20]:= CloudDeploy[APIFunction[{"loc" → "Location"}, GeoDistance[#, loc, Here] &]]
```

```
Out[20]= CloudObject[https://www.wolframcloud.com/objects/0850dc98-e7d7-4fa6-884b-642ce545d3c3]
```

Create embeddable code for an external Java program to call the API:

```
In[21]:= EmbedCode[%, "Java"]
```

Embeddable Code

Use the code below to call the Wolfram Cloud function from Java:

Code

Copy to Clipboard

```
import java.io.IOException;
public class WolframCloudCall {
    public static String call(String loc) throws IOException {
        URL _url = new URL("http://www.wolframcloud.com/objects/09302cdc-4d52-457e-86ae-b834bd9717ae");
        HttpURLConnection _conn = (HttpURLConnection) _url.openConnection();
        _conn.setRequestMethod("POST");
        _conn.setDoOutput(true);
        _conn.setDoInput(true);
        _conn.setUseCaches(false);
        _conn.setAllowUserInteraction(false);
        _conn.setRequestProperty("Content-Type", "application/x-www-form-urlencoded; charset=utf-8");
        _conn.setRequestProperty("User-Agent", "EmbedCode-Java/1.0");
        DataOutputStream _out = new DataOutputStream(_conn.getOutputStream());
        _out.writeBytes("loc");
        _out.writeBytes("=");
    }
}
```

Out[21]=

Document Generation

Documents are symbolic expressions, like everything else:

```
In[22]:= DocumentNotebook[
  {Style["A Circle", "Section"], Style["How to make a circle"], Graphics[Circle[]]}]
```

Out[22]=

Evaluation Control

Hold a computation unevaluated:

```
In[23]:= Hold[2+2 == 4]
```

```
Out[23]= Hold[2 + 2 == 4]
```

Release the hold:

```
In[24]:= ReleaseHold[%]
```

```
Out[24]= True
```

Systems-Level Operations

Run an external process (not allowed in the cloud!):

```
In[25]:= RunProcess["ps", "StandardOutput"]
```

```
Out[25]= PID TTY      TIME CMD
          374 ttys000  0:00.03 -tcsh
          40192 ttys000  0:00.66 ssh pi
          60521 ttys001  0:00.03 -tcsh
```

Encrypt anything:

```
In[26]:= Encrypt["sEcreTkey", "Read this if you can!"]
```

```
Out[26]= EncryptedObject[ data length: 32 bytes
                           IV length: 128 bits
                           original form: String]
```

Parallel Computation

I'm running on a 12-core machine:

```
In[27]:= $ProcessorCount
```

```
Out[27]= 12
```

Sequentially test a sequence of (big) numbers for primality, and find the total time taken:

```
In[28]:= Table[PrimeQ[2^Prime[n]-1], {n, 500}] // Counts // AbsoluteTiming
```

```
Out[28]= {4.15402, <| True → 18, False → 482 |>}
```

Doing the same thing in parallel takes considerably less time:

```
In[29]:= ParallelTable[PrimeQ[2^Prime[n]-1], {n, 500}] // Counts // AbsoluteTiming
```

```
Out[29]= {0.572106, <| True → 18, False → 482 |>}
```


Afterword: Being a Programmer

If you've understood what's in this book, and can do its exercises, then you can now consider yourself a Wolfram Language programmer! There'll always be more you can learn, but you're ready to start using what you know to do real programming.

What can you do? An amazing amount! In fact, there'll probably be something you want to program every day. With a traditional computer language it'd take too long to actually do it, though. But with the Wolfram Language—with all its built-in knowledge and automation—anyone who knows the language can write very useful programs even in a matter of minutes.

And this means you'll routinely be able to write programs for all sorts of things. Things you want to understand, things you want to create, things you want to do for other people. Sometimes you'll dash off a program, run it once, and never use it again. But much more often you'll end up using your program many times—and maybe progressively making it more and more sophisticated over time.

For everyday programming, it's normally best to write Wolfram Language programs directly in Wolfram Notebooks, that let you—like in this book—mix results and textual explanations along with code. After a while, you'll probably end up with lots of Wolfram Notebooks, that do lots of different things.

Quite often you'll just run your programs in those notebooks. But you'll also often want to deploy the programs to create websites, apps or whatever. And one of the great things about the Wolfram Language is that this is easy to do.

There's nothing to say that with a few well-chosen lines of Wolfram Language you might not be able to create something like a website that many people will want to use. But more often you'll find there are all sorts of extra details you want to cover, and you'll end up writing a significantly longer program to handle all of them.

In many ways, there's nothing different about a longer Wolfram Language program. Even if there are millions of lines of code (as, for example, in Wolfram | Alpha), they locally all look pretty much like the code in this book; just a lot more of it.

In any programming project, however, there are some new issues that come up when programs get larger. You need to be sure to maintain systematic tests (which in the Wolfram Language you can do using `VerificationTest`). You need to organize code into properly separated packages. And particularly if multiple programmers are involved, you need version control, code reviews, and other management structures.

But beyond that, you need good overall design and architecture. How will users—and programmers—understand your system? What structures will you use to represent whatever you're dealing with? How will different parts of your code interact? These are the kinds of things people in charge of building large software systems need to think about, and it can take considerable skill and experience to get them right.

But what if you're just getting started? What does it take to create the kinds of programs you need to do things? The first step in creating a program for something is to see how to think about the thing in computational terms.

It might be something where computers have long been used. It might be something that's only now conceivable for computers as a result of the Wolfram Language. Whatever it is, try to imagine a Wolfram Language function for doing it.

What input would the function get? What output would it generate? What might the function be called? Don't at first think about how you'd write the code. Just think about what the function should do. And only after you've understood that, start writing the code.

Look in this book and on the Wolfram Language website for examples that are similar to what you're trying to do. If you're lucky, you'll get everything you need. But maybe there'll be some part that it's just not clear how to achieve. And if that happens, it's often good to imagine how you'd explain what's needed to someone with infinite technical capability. Often just doing that will lead you to identify at least some definite tasks—which you can then start formulating in the Wolfram Language.

A great feature of the Wolfram Language is that it's always easy to do experiments with. You can try all sorts of things, and see what works. It's always worthwhile to be as systematic in your explorations as you can. And to recognize that in the Wolfram Language it's usually less about writing lots of code than about understanding how best to think about what you want in computational terms.

It's extremely satisfying to take an idea and turn it into a working program. It's also a powerful and valuable thing to be able to do. And with the Wolfram Language there are now remarkable opportunities to build programs far beyond anything possible before—and to make advances in many areas. And with what you've learned in this book, you should now be in a position to be part of that.

Answers to Exercises

Note: Almost all the exercises have many possible correct answers; what's listed here are merely sample correct answers.

1 | Starting Out: Elementary Arithmetic

```
1.1 1 + 2 + 3
1.2 1 + 2 + 3 + 4 + 5
1.3 1 * 2 * 3 * 4 * 5
1.4 5^2
1.5 3^4
1.6 10^12
1.7 2^2 2^2
1.8 (4 - 2) * (3 + 4)
1.9 120000 * 123
```

```
5.5 9 + Range[11]
5.6 Sort[Join[Range[5]^2, Range[5]^3]]
5.7 Length[IntegerDigits[2^128]]
5.8 First[IntegerDigits[2^32]]
5.9 Take[IntegerDigits[2^100], 10]
5.10 Max[IntegerDigits[2^20]]
5.11 Count[IntegerDigits[2^1000], 0]
5.12 Part[Sort[IntegerDigits[2^20]], 2]
5.13 ListLinePlot[IntegerDigits[2^128]]
5.14 Take[Drop[Range[100], 10], 10]
```

2 | Introducing Functions

```
2.1 Plus[7, 6, 5]
2.2 Times[2, Plus[3, 4]]
2.3 Max[6 * 8, 5 * 9]
2.4 RandomInteger[1000]
2.5 Max[10, RandomInteger[20]]
```

6 | Making Tables

```
6.1 Table[1000, 5]
6.2 Table[n^3, {n, 10, 20}]
6.3 NumberLinePlot[Table[n^2, {n, 20}]]
6.4 Range[2, 20, 2]
6.5 Table[n, {n, 10}]
6.6 BarChart[Table[n^2, {n, 10}]]
6.7 Table[IntegerDigits[n^2], {n, 10}]
6.8 ListLinePlot[Table[Length[IntegerDigits[n^2]], {n, 100}]]
6.9 Table[First[IntegerDigits[n^2]], {n, 20}]
6.10 ListLinePlot[Table[First[IntegerDigits[n^2]], {n, 100}]]
```

3 | First Look at Lists

```
3.1 Range[4]
3.2 Range[100]
3.3 Reverse[Range[4]]
3.4 Reverse[Range[50]]
3.5 Join[Range[4], Reverse[Range[4]]]
3.6 ListPlot[Join[Range[100], Reverse[Range[99]]]]
3.7 Range[RandomInteger[10]]
3.8 Range[10]
3.9 Range[5]
3.10 Join[Range[10], Range[10], Range[5]]
3.11 Join[Range[20], Reverse[Range[20]]]
```

7 | Colors and Styles

```
7.1 {Red, Yellow, Green}
7.2 Column[{Red, Yellow, Green}]
7.3 ColorNegate[Orange]
7.4 Table[Hue[h], {h, 0, 1, 0.02}]
7.5 Table[RGBColor[1, g, 1], {g, 0, 1, 0.05}]
7.6 Blend[{Pink, Yellow}]
7.7 Table[Blend[{Yellow, Hue[x]}], {x, 0, 1, .05}]
7.8 Table[Style[n, Hue[n]], {n, 0, 1, .1}]
7.9 Style[Purple, 100]
7.10 Table[Style[Red, x], {x, 10, 100, 10}]
7.11 Style[999, Red, 100]
7.12 Table[Style[n^2, n^2], {n, 10}]
7.13 Table[Part[{Red, Yellow, Green}, RandomInteger[2] + 1], 100]
7.14 Table[Style[Part[IntegerDigits[2^1000], n], 3 * Part[IntegerDigits[2^1000], n]], {n, 50}]
```

4 | Displaying Lists

```
4.1 BarChart[{1, 1, 2, 3, 5}]
4.2 PieChart[Range[10]]
4.3 BarChart[Reverse[Range[20]]]
4.4 Column[Range[5]]
4.5 NumberLinePlot[{1, 4, 9, 16, 25}]
4.6 {PieChart[{1}], PieChart[{1, 1}], PieChart[{1, 1, 1}]}
4.7 Column[{PieChart[{1}], PieChart[{1, 1}], PieChart[{1, 1, 1}]}]
```

8 | Basic Graphics Objects

```
8.1 Graphics[RegularPolygon[3]]
8.2 Graphics[Style[Circle[], Red]]
8.3 Graphics[Style[RegularPolygon[8], Red]]
8.4 Table[Graphics[Style[Disk[], Hue[h]]], {h, 0, 1, 0.1}]
```

5 | Operations on Lists

```
5.1 Reverse[Range[10]^2]
5.2 Total[Range[10]^2]
5.3 ListPlot[Range[10]^2]
5.4 Sort[Join[Range[4], Range[4]]]
```

```

8.5 Column[{Graphics[Style[RegularPolygon[3], Red]],
  Graphics[Style[RegularPolygon[3], Green]]}]
8.6 Table[Graphics[Style[RegularPolygon[n], Pink]], {n, 5, 10}]
8.7 Graphics3D[Style[Cylinder[], Purple]]
8.8 Graphics[Reverse[Table[Style[RegularPolygon[n],
  RandomColor[]], {n, 3, 8}]]]

```

9 | Interactive Manipulation

```

9.1 Manipulate[Range[n], {n, 0, 100}]
9.2 Manipulate[ListPlot[Range[n]], {n, 5, 50, 1}]
9.3 Manipulate[Column[Table[x, n], {n, 1, 10, 1}]}
9.4 Manipulate[Graphics[Style[Disk[], Hue[h]]], {h, 0, 1}]
9.5 Manipulate[Graphics[
  Style[Disk[], RGBColor[red, green, blue]]],
  {red, 0, 1}, {green, 0, 1}, {blue, 0, 1}]
9.6 Manipulate[IntegerDigits[n], {n, 1000, 9999, 1}]
9.7 Manipulate[Table[Hue[h], {h, 0, 1, 1/n}], {n, 5, 50, 1}]
9.8 Manipulate[Table[Graphics[Style[RegularPolygon[6],
  Hue[h]]], n], {n, 1, 10, 1}, {h, 0, 1}]
9.9 Manipulate[Graphics[Style[RegularPolygon[n], color]],
  {n, 5, 20, 1}, {color, {Red, Yellow, Blue}}]
9.10 Manipulate[PieChart[Table[1, n]], {n, 1, 10, 1}]
9.11 Manipulate[BarChart[IntegerDigits[n]], {n, 100, 999, 1}]
9.12 Manipulate[Table[RandomColor[], n], {n, 1, 50, 1}]
9.13 Manipulate[Column[Table[a^m, {m, n}],
  {n, 1, 10, 1}, {a, 1, 25, 1}]}
9.14 Manipulate[NumberLinePlot[
  Table[x^n, {x, 10}]], {n, 0, 5}]
9.15 Manipulate[Graphics3D[Style[Sphere[],
  RGBColor[n, 1 - n, 0]]], {n, 0, 1}]

```

10 | Images

```

10.1 ColorNegate[EdgeDetect[]]
10.2 Manipulate[Blur[, r], {r, 0, 20}]
10.3 Table[EdgeDetect[Blur[, n]], {n, 10}]
10.4 ImageCollage[{, Blur[],
  EdgeDetect[, Binarize[]]}
10.5 ImageAdd[Binarize[, ]]
10.6 Manipulate[EdgeDetect[Blur[, r]], {r, 0, 20}]
10.7 EdgeDetect[Graphics3D[Sphere[]]]
10.8 Manipulate[Blur[Graphics[Style[RegularPolygon[5,
  Purple]], r], {r, 0, 20}]]
10.9 ImageCollage[Table[Graphics[Style[Disk[],
  RandomColor[]]], 9]]
10.10 ImageCollage[Table[Graphics3D[Style[
  Sphere[], Hue[h]]], {h, 0, 1, 0.2}]]
10.11 Table[Blur[Graphics[Disk[]], n], {n, 0, 30, 5}]
10.12 ImageAdd[Graphics[Disk[]], ]
10.13 ImageAdd[Graphics[Style[
  RegularPolygon[8], Red]], ]
10.14 ImageAdd[, ColorNegate[EdgeDetect[]]]

```

11 | Strings and Text

```

11.1 StringJoin["Hello", "Hello"]
11.2 ToUpperCase[StringJoin[Alphabet[]]]
11.3 StringReverse[StringJoin[Alphabet[]]]
11.4 StringJoin[Table["AGCT", 100]]
11.5 StringTake[StringJoin[Alphabet[]], 6]
11.6 Column[Table[StringTake["this is about strings", n],
  {n, StringLength["this is about strings"]}]]
11.7 BarChart[StringLength[TextWords["A long time ago,
  in a galaxy far, far away"]]]
11.8 StringLength[WikipediaData["computer"]]
11.9 Length[TextWords[WikipediaData["computer"]]]
11.10 First[TextSentences[WikipediaData["strings"]]]
11.11 StringJoin[StringTake[TextSentences[
  WikipediaData["computers"]], 1]]
11.12 Max[StringLength[WordList[]]]
11.13 Count[StringTake[WordList[], 1], "q"]
11.14 ListLinePlot[Take[StringLength[WordList[]], 1000]]
11.15 WordCloud[Characters[StringJoin[WordList[]]]]
11.16 WordCloud[StringTake[StringReverse[WordList[]], 1]]
11.17 RomanNumeral[1959]
11.18 Max[StringLength[RomanNumeral[Range[2020]]]]
11.19 WordCloud[Table[StringTake[
  RomanNumeral[n], 1], {n, 100}]]
11.20 Length[Alphabet["Russian"]]
11.21 ToUpperCase[Alphabet["Greek"]]
11.22 BarChart[LetterNumber[Characters["wolfram"]]]
11.23 StringJoin[FromLetterNumber[Table[
  RandomInteger[25] + 1, 1000]]]
11.24 Table[StringJoin[FromLetterNumber[
  Table[RandomInteger[25] + 1, 5]], 100]
11.25 ColorNegate[Rasterize[Style["A", 200]]]
11.26 Manipulate[Style[FromLetterNumber[n, 100],
  {n, 1, Length[Alphabet[]]}, 1]]
11.27 Manipulate[ColorNegate[EdgeDetect[
  Rasterize[Style[c, 100]]]], {c, Alphabet[]}]
11.28 Manipulate[Blur[Rasterize[Style["A", 200]], r], {r, 0, 50}]

```

12 | Sound

```

12.1 Sound[{SoundNote[0], SoundNote[4], SoundNote[7]}]
12.2 Sound[SoundNote["A", 2, "Cello"]]
12.3 Sound[Table[SoundNote[n, 0.05], {n, 0, 48}]]
12.4 Sound[Reverse[Table[SoundNote[n], {n, 0, 12}]]]
12.5 Sound[Table[SoundNote[12 * n], {n, 0, 4}]]
12.6 Sound[Table[SoundNote[
  RandomInteger[12], 2, "Trumpet"], 10]]
12.7 Sound[Table[SoundNote[RandomInteger[12],
  RandomInteger[10]/10], 10]]
12.8 Sound[Table[SoundNote[Part[IntegerDigits[2^31],
  n], 1], {n, Length[IntegerDigits[2^31]]}]]
12.9 Sound[Table[SoundNote[Part[Characters["CABBAGE"],
  n], 3, "Guitar"], {n, 1, 7}]]
12.10 Sound[Table[SoundNote[Part[
  LetterNumber[Characters["wolfram"]], n], 1], {n, StringLength["wolfram"]}]]

```

13 | Arrays, or Lists of Lists

```

13.1 Grid[Table[i*j, {i, 12}, {j, 12}]]
13.2 Grid[Table[RomanNumeral[i*j], {i, 5}, {j, 5}]]
13.3 Grid[Table[RandomColor[], 10, 10]]
13.4 Grid[Table[Style[RandomInteger[10], RandomColor[]], 10, 10]]
13.5 Grid[Table[StringJoin[FromLetterNumber[{i, j}]], {i, 26}, {j, 26}]]
13.6 Grid[{{PieChart[{1, 4, 3, 5, 2}], NumberLinePlot[{1, 4, 3, 5, 2}], {ListLinePlot[{1, 4, 3, 5, 2}], BarChart[{1, 4, 3, 5, 2}]}}]
13.7 ArrayPlot[Table[Hue[i*j], {i, 0, 1, .05}, {j, 0, 1, .05}]]
13.8 ArrayPlot[Table[Hue[x/y], {x, 50}, {y, 50}]]
13.9 ArrayPlot[Table[StringLength[RomanNumeral[i*j]], {i, 100}, {j, 100}]]
```

14 | Coordinates and Graphics

```

14.1 Graphics[Table[Circle[{0, 0}, r], {r, 5}]]
14.2 Graphics[Table[Style[Circle[{0, 0}, r], RandomColor[]], {r, 10}]]
14.3 Graphics[Table[Circle[{x, y}], {x, 10}, {y, 10}]]
14.4 Graphics[Table[Point[{x, y}], {x, 10}, {y, 10}]]
14.5 Manipulate[Graphics[Table[Circle[{0, 0}, r], {r, n}], {n, 1, 20, 1}]]
14.6 Graphics3D[Table[Style[Sphere[Table[RandomInteger[10], 3]], RandomColor[]], 50]]
14.7 Graphics3D[Table[Style[Sphere[{x, y, z}, 1/2], RGBColor[{x/10, y/10, z/10}], {x, 10}, {y, 10}, {z, 10}]]
14.8 Manipulate[Graphics[Table[Circle[{a x, 0}, x], {x, 10}], {a, -2, 2}]]
14.9 Graphics[Table[RegularPolygon[{x, y}, 1/2, 6], {x, 5}, {y, 5}]]
14.10 Graphics3D[Line[Table[RandomInteger[50], 50, 3]]]
```

16 | Real-World Data

Note: = [] represents natural language input

```

16.1 = [flag of switzerland]
16.2 = [elephant]["Image"]
16.3 EntityValue[= [planets], "Mass"]
16.4 BarChart[EntityValue[= [planets], "Mass"]]
16.5 ImageCollage[EntityValue[= [planets], "Image"]]
16.6 EdgeDetect[= [china]["Flag"]]
16.7 = [empire state building]["Height"]
16.8 = [empire state building]["Height"] / [great pyramid]["Height"]
16.9 = [mount everest]["Elevation"] / [empire state building]["Height"]
16.10 DominantColors[= [starry night]["Image"]]
16.11 DominantColors[ImageCollage[EntityValue[= [countries in europe], "FlagImage"]]]
16.12 PieChart[= [countries in europe]["GDP"]]
16.13 ImageAdd[= [koala]["Image"], = [australia]["Flag"]]
```

17 | Units

Note: = [] represents natural language input

```

17.1 UnitConvert[= [4.5 lbs], "Kilograms"]
17.2 UnitConvert[= [60.25 mph], = [km / hr]]
17.3 UnitConvert[= [height of the eiffel tower], "Miles"]
17.4 = [height of mount everest] / = [height of the eiffel tower]
17.5 = [mass of earth] / = [mass of moon]
17.6 CurrencyConvert[= [2500 japanese yen], = [us dollars]]
17.7 UnitConvert[= [35 ounces + 1 / 4 ton + 45 lbs + 9 stone], = [kilograms]]
17.8 UnitConvert[= [planets]["DistanceFromEarth"], "LightMinutes"]
17.9 Rotate["hello", 180 °]
17.10 Table[Rotate[Style["A", 100], n Degree], {n, 0, 360, 30}]
17.11 Manipulate[Rotate[= [cat]["Image"], θ], {θ, 0 °, 180 °}]
17.12 Graphics[Line[AnglePath[Table[n Degree, {n, 0, 180}]]]]
17.13 Manipulate[Graphics[Line[AnglePath[Table[x, 100]}]], {x, 0, 360 °}]
17.14 Graphics[Line[AnglePath[30 ° * IntegerDigits[2^10 000]]]]
```

18 | Geocomputation

Note: = [] represents natural language input

```

18.1 GeoDistance[= [new york], = [london]]
18.2 GeoDistance[= [new york], = [london]] / GeoDistance[= [new york], = [san francisco]]
18.3 UnitConvert[GeoDistance[= [sydney], = [moscow]], = [km]]
18.4 GeoGraphics[= [united states]]
18.5 GeoListPlot[= [brazil], = [russia], = [india], = [china]]
18.6 GeoGraphics[GeoPath[{= [new york], = [beijing]}]]
18.7 GeoGraphics[GeoDisk[= [great pyramid], = [10 miles]]]
18.8 GeoGraphics[GeoDisk[= [new york], GeoDistance[= [new york city], = [san francisco]]]]
18.9 GeoNearest["Country", GeoPosition["NorthPole"], 5]
18.10 EntityValue[GeoNearest["Country", GeoPosition[{45, 0}], 3], "Flag"]
18.11 GeoListPlot[GeoNearest["Volcano", = [rome], 25]]
18.12 GeoPosition[= [new york]][{1, 1}] - GeoPosition[= [los angeles]][{1, 1}]
```

19 | Dates and Times

Note: = [] represents natural language input

```

19.1 Now - = [january 1, 1900]
19.2 DayName[= [january 1, 2000]]
19.3 Today - = [100 000 days]
19.4 LocalTime[= [delhi]]
19.5 Sunset[Here, Today] - Sunrise[Here, Today]
19.6 MoonPhase[Now, "Icon"]
19.7 Table[MoonPhase[Today + n = [days]], {n, 10}]
19.8 Table[MoonPhase[Today + n = [days], "Icon"], {n, 10}]
```

```

19.9 Sunrise[ = [new york city], Today] – Sunrise[
  = [london], Today]
19.10 AirTemperatureData[ = [eiffel tower],
  = [noon yesterday]]
19.11 DateListPlot[AirTemperatureData[ = [eiffel tower],
  {Now – = [1 week], Now}]]
19.12 AirTemperatureData[ = [los angeles]] –
  AirTemperatureData[ = [new york]]

```

20 | Options

Note: [=] represents natural language input

```

20.1 ListPlot[Range[10], PlotTheme → "Web"]
20.2 ListPlot[Range[10], Filling → Axis]
20.3 ListPlot[Range[10], Background → Yellow]
20.4 GeoListPlot[ = [australia], GeoRange → All]
20.5 GeoListPlot[ = [madagascar],
  GeoRange → = [indian ocean]]
20.6 GeoGraphics[ = [south america],
  GeoBackground → "ReliefMap"]
20.7 GeoListPlot[{ = [france], = [finland], = [greece]},
  GeoRange → = [europe], GeoLabels → Automatic]
20.8 Grid[Table[Style[i * j, White], {i, 12}, {j, 12}],
  Background → Black]
20.9 Table[Graphics[Disk[], ImageSize → RandomInteger[40]], 100]
20.10 Table[Graphics[RegularPolygon[5], ImageSize → 30,
  AspectRatio → n], {n, 1, 10}]
20.11 Manipulate[Graphics[Circle[], ImageSize → s],
  {s, 5, 500}]
20.12 Grid[Table[RandomColor[], 10, 10], Frame → All]
20.13 ListLinePlot[Table[StringLength[
  RomanNumeral[n]], {n, 100}], PlotRange → Max[
  Table[StringLength[RomanNumeral[n]], {n, 1000}]]]

```

21 | Graphs and Networks

```

21.1 Graph[{1 → 2, 2 → 3, 3 → 1}]
21.2 Graph[Flatten[Table[i → j, {i, 4}, {j, 4}]]]
21.3 Table[UndirectedGraph[Flatten[
  Table[i → j, {i, n}, {j, n}]]], {n, 2, 10}]
21.4 Flatten[Table[{1, 2}, 3]]
21.5 ListLinePlot[Flatten[Table[IntegerDigits[n], {n, 100}]]]
21.6 Graph[Table[i → i + 1, {i, 50}]]
21.7 Graph[Flatten[Table[i → Max[i, j], {i, 4}, {j, 4}]]]
21.8 Graph[Flatten[Table[i → j – i, {i, 5}, {j, 5}]]]
21.9 Graph[Table[i → RandomInteger[{1, 100}], {i, 100}]]
21.10 Graph[Flatten[Table[{i → RandomInteger[{1, 100}],
  i → RandomInteger[{1, 100}]], {i, 100}]]]
21.11 Grid[Table[FindShortestPath[
  Graph[{1 → 2, 2 → 3, 3 → 4, 4 → 1, 3 → 1, 2 → 2}],
  i, j], {i, 4}, {j, 4}]]

```

22 | Machine Learning

Note: [=] represents natural language input

```

22.1 LanguageIdentify["ajatella"]
22.2 ImageIdentify[ = [image of a tiger]]
22.3 Table[ImageIdentify[
  Blur[ = [image of a tiger], r]], {r, 5}]

```

```

22.4 Classify["Sentiment", "I'm so happy to be here"]
22.5 Nearest[WordList[], "happy", 10]
22.6 Nearest[RandomInteger[1000, 20], 100, 3]
22.7 Nearest[Table[RandomColor[], 100], Red, 5]
22.8 First[Nearest[Table[n^2, {n, 100}], 2000]]
22.9 Nearest[ = [european flags], = [flag of brazil], 3]
22.10 NearestNeighborGraph[Table[Hue[h], {h, 0, 1, .05}],
  2, VertexLabels → All]
22.11 NearestNeighborGraph[Table[
  RandomInteger[100], 40], 2, VertexLabels → All]
22.12 FindClusters[ = [flags of Asia]]
22.13 NearestNeighborGraph[Table[Rasterize[Style[
  FromLetterNumber[n], 20]], {n, 26}], 2,
  VertexLabels → All]
22.14 Table[TextRecognize[Blur[Rasterize[
  Style["hello", 50]], n]], {n, 10}]

```

23 | More about Numbers

```

23.1 N[Sqrt[2], 500]
23.2 RandomReal[1, 10]
23.3 ListPlot[Table[RandomReal[1, 2], 200]]
23.4 Graphics[Line[AnglePath[RandomReal[2 Pi, 1000]]]]
23.5 Table[Mod[n^2, 10], {n, 0, 30}]
23.6 ListLinePlot[Table[Mod[n^n, 10], {n, 100}]]
23.7 Table[Round[Pi^n], {n, 10}]
23.8 Graph[Table[n → Mod[n^2, 100], {n, 0, 99}]]
23.9 Graphics[Table[Style[Circle[RandomReal[10, 2],
  RandomReal[2]], RandomColor[], 50]]]
23.10 ListPlot[Table[Prime[n] / (n Log[n]), {n, 2, 1000}]]
23.11 ListLinePlot[Table[Prime[n + 1] – Prime[n], {n, 100}]]
23.12 Sound[Table[SoundNote["C", RandomReal[0.5]], 20]]
23.13 ArrayPlot[Table[Mod[i, j], {i, 50}, {j, 50}]]
23.14 Table[ArrayPlot[Table[
  Mod[x^y, n], {x, 50}, {y, 50}]], {n, 2, 20}]

```

24 | More Forms of Visualization

Note: [=] represents natural language input

```

24.1 ListLinePlot[Table[n^p, {p, 2, 4}, {n, 10}]]
24.2 ListLinePlot[Table[Prime[n], {n, 20}], Filling → Axis,
  Mesh → True, MeshStyle → Red]
24.3 ListPlot3D[GeoElevationData[GeoDisk[ = [mount fuji],
  = [20 miles]]]]
24.4 ReliefPlot[GeoElevationData[GeoDisk[ = [mount fuji],
  = [100 miles]]]]
24.5 ListPlot3D[Table[Mod[i, j], {i, 100}, {j, 100}]]
24.6 Histogram[Table[Prime[n + 1] – Prime[n], {n, 10000}]]
24.7 Histogram[Table[First[IntegerDigits[n^2]], {n, 10000}]]
24.8 Histogram[Table[StringLength[
  RomanNumeral[n]], {n, 1000}]]
24.9 Histogram[StringLength[TextSentences[
  WikipediaData["computers"]]]]
24.10 Table[Histogram[Table[Total[
  RandomReal[100, n]], 10000]], {n, 5}]
24.11 ListPlot3D[1 – ImageData[Binarize[
  Rasterize[Style["W", 200]]]]]

```

25 | Ways to Apply Functions

Note: [=] represents natural language input 

```
25.1 f/@Range[5]
25.2 f/@g/@Range[10]
25.3 x//d//c//b//a
25.4 Framed /@Alphabet[]
25.5 ColorNegate /@ EntityValue[= [planets], "Image"]
25.6 GeoGraphics /@ EntityList[= [countries in g5]]
25.7 ImageCollage[Binarize /@ = [flags of europe]]
25.8 Column /@ DominantColors /@ EntityValue[= [planets], "Image"]
25.9 Total[LetterNumber /@ Characters["wolfram"]]
```

26 | Pure Anonymous Functions

Note: [=] represents natural language input 

```
26.1 #^2 & /@ Range[20]
26.2 Blend[{#, Red}] & /@ {Yellow, Green, Blue}
26.3 Framed[Column[{ToUpperCase[#, #]}]] & /@ Alphabet[]
26.4 Framed[Style[#, RandomColor[]], Background -> RandomColor[]] & /@ Alphabet[]
26.5 Grid[{#, EntityValue[#, "Flag"]} ] & /@ EntityList[= [5 countries]], Frame -> All]
26.6 WordCloud[DeleteStopwords[WikipediaData[#]]] & /@ {"apple", "peach", "pear"}
26.7 Histogram[StringLength[TextWords[WikipediaData[#]]]] & /@ {"apple", "peach", "pear"}
26.8 GeoListPlot[{#, GeoRange -> [central america]}] & /@ EntityList[= [central america]]
```

27 | Applying Functions Repeatedly

Note: [=] represents natural language input 

```
27.1 NestList[Blur, Rasterize[Style["X", 30]], 10]
27.2 NestList[Framed[#, Background -> RandomColor[]] &, x, 10]
27.3 NestList[Rotate[Framed[#], RandomReal[{0, 360 °}]] &, Style["A", 50], 5]
27.4 ListLinePlot[NestList[4 # (1 - #) &, 0.2, 100]]
27.5 Nest[1 + 1/# &, 1, 30] // N
27.6 NestList[3 * # &, 1, 10]
27.7 NestList[(# + 2/#)/2 &, 1, 0.5] - Sqrt[2]
27.8 Graphics[Line[NestList[#+ RandomReal[{-1, 1}, 2] &, {0, 0}, 1000]]]
27.9 ArrayPlot[NestList[Mod[Join[{0}, #] + Join[{#}, {0}], 2] &, {1}, 50]]
27.10 NestGraph[{# + 1, 2 #} &, 0, 10]
27.11 NestGraph[{#| "BorderingCountries"} &, = [US], 4, VertexLabels -> All]
```

28 | Tests and Conditionals

```
28.1 123^321 > 456^123
28.2 Select[Range[100], Total[IntegerDigits[#]] < 5 &]
28.3 If[PrimeQ[#], Style[#, Red], #] & /@ Range[20]
28.4 Select[WordList[], StringTake[#, 1] == StringTake[StringReverse[#, 1], 1] == "p" &]
28.5 Select[Array[Prime, 100], Last[IntegerDigits[#]] < 3 &]
28.6 Select[RomanNumeral[Range[100]], ! MemberQ[Characters[#], {"I"}] &]
```

```
28.7 Select[RomanNumeral[Range[1000]], # == StringReverse[#] &]
28.8 Select[Table[IntegerName[n], {n, 100}], First[Characters[#]] == Last[Characters[#]] &]
28.9 Select[TextWords[WikipediaData["words"]], StringLength[#] > 15 &]
28.10 NestList[If[EvenQ[#], #/2, 3 # + 1] &, 1000, 200]
28.11 WordCloud[Select[TextWords[WikipediaData["computers"]], StringLength[#] == 5 &]]
28.12 Select[WordList[], StringLength[#] >= 3 && # != StringReverse[#] && StringTake[#, 3] == StringTake[StringReverse[#, 3]] &]
28.13 Select[Select[WordList[], StringLength[#] == 10 &], Total[LetterNumber /@ Characters[#]] == 100 &]
```

29 | More about Pure Functions

```
29.1 Array[Prime, 100]
29.2 Array[Prime[# + 1] - Prime[#] &, 99]
29.3 Grid[Array[Plus, {10, 10}]]
29.4 FoldList[Times, 1, Range[10]]
29.5 FoldList[Times, 1, Array[Prime, 10]]
29.6 FoldList[ImageAdd, Table[Graphics[Style[RegularPolygon[n], Opacity[.2]]], {n, 3, 8}]]]
```

30 | Rearranging Lists

Note: [=] represents natural language input 

```
30.1 Thread[Alphabet[] -> Range[Length[Alphabet[]]]]
30.2 Grid[Partition[Alphabet[], 6]]
30.3 Grid[Partition[IntegerDigits[2^1000], 50], Frame -> All]
30.4 Grid[Partition[Characters[StringTake[WikipediaData["computers"], 400]], 20], Frame -> All]
30.5 ListLinePlot[Flatten[IntegerDigits /@ Range[0, 2000]]]
30.6 ArrayPlot[ /@ NestList[ArrayFlatten[{#, #, #}, {#, 0, #}, {#, #, #}]] &, {{1}}, 4]
30.7 Select[Flatten[Table[{x, y, Sqrt[x^2 + y^2]}, {x, 20}, {y, 20}], 1], IntegerQ[Last[#]] &]
30.8 Table[Max[Length /@ Split[IntegerDigits[2^n]]], {n, 100}]
30.9 GatherBy[Array[IntegerName, 100], StringTake[#, 1] &]
30.10 SortBy[Take[WordList[], 50], StringTake[StringReverse[#], 1] &]
30.11 SortBy[Table[n^2, {n, 20}], First[IntegerDigits[#]] &]
30.12 SortBy[Range[20], IntegerName]
30.13 GatherBy[RandomSample[WordList[], 20], StringLength]
30.14 Complement[Alphabet["Ukrainian"], Alphabet["Russian"]]
30.15 Intersection[Range[100]^2, Range[100]^3]
30.16 Intersection[EntityList[= [nato]], EntityList[= [g8]]]
30.17 Grid[Transpose[Permutations[Range[4]]]]
30.18 Union[StringJoin /@ Permutations[Characters["hello"]]]
30.19 ArrayPlot[Tuples[{0, 1}, 5]]
30.20 Table[StringJoin[RandomChoice[Alphabet[], 5]], 10]
30.21 Tuples[{1, 2}, 3]
```

31 | Parts of Lists

```

31.1 Take[IntegerDigits[2^1000], -5]
31.2 Alphabet[][[10;; 20]]
31.3 Part[Alphabet[], Range[2, Length[Alphabet[]], 2]]
31.4 ListLinePlot[Table[IntegerDigits[12^n][[-2]], {n, 100}]]
31.5 TakeSmallest[Join[Table[n^2, {n, 20}], Table[n^3, {n, 20}], 10]
31.6 Flatten[Position[TextWords[WikipediaData["computers"]], "software"]]
31.7 Histogram[Flatten[Position[Characters[#], "e"] & /@ WordList[]]]
31.8 ReplacePart[Range[100]^3, Thread[Table[n^2, {n, 10}] → Red]]
31.9 If[First[IntegerDigits[#]] < 5, Nothing, #] & /@ Array[Prime, 100]
31.10 Grid[NestList[ReplacePart[#, RandomInteger[{1, Length[#]}] → Nothing] &, Range[10], 9]]
31.11 TakeLargestBy[WordList[], StringLength, 10]
31.12 TakeLargestBy[Array[IntegerName, 100], StringLength, 5]
31.13 TakeLargestBy[Array[IntegerName, 100], Count[Characters[#], "e"] &, 5]

```

32 | Patterns

```

32.1 Cases[IntegerDigits[Range[1000]], {1, ___, 9}]
32.2 Cases[IntegerDigits[Range[1000]], {x_, x_, x_}]
32.3 Cases[IntegerDigits[Range[1000]^2], {9, ___, 0 | 1}]
32.4 IntegerDigits[Range[100]] /. {0 → Gray, 9 → Orange}
32.5 IntegerDigits[2^1000] /. 0 → Red
32.6 Characters["The Wolfram Language"]/. {"a" | "e" | "i" | "o" | "u" → Nothing}
32.7 Cases[IntegerDigits[2^1000], 0 | 1]
32.8 Cases[IntegerDigits[Range[100, 999]], {x_, ___, x_}]

```

33 | Expressions and Their Structure

```

33.1 Head[ListPlot[Range[5]]]
33.2 Times @@ Range[100]
33.3 f @@@@ Tuples[{a, b}, 2]
33.4 TreeForm /@ NestList[##^# &, x, 4]
33.5 Union[Cases[Flatten[Table[i^2 / (j^2 + 1), {i, 20}, {j, 20}], _Integer]]
33.6 GraphRule @@@@ Partition[Table[Mod[n^2 + n, 100], {n, 100}], 2, 1]
33.7 GraphRule @@@@ Partition[TextWords[WikipediaData["computers"]], 200, 2, 1], VertexLabels → All]
33.8 f @@@@ {{1, 2}, {7, 2}, {5, 4}}

```

34 | Associations

```

34.1 Values[KeySort[Counts[IntegerDigits[3^1000]]]]
34.2 BarChart[KeySort[Counts[IntegerDigits[2^1000]]], ChartLabels → Automatic]
34.3 BarChart[Counts[StringTake[WordList[], 1]], ChartLabels → Automatic]
34.4 TakeLargest[Counts[StringTake[WordList[], 1]], 5]
34.5 #q /@ #u & @ LetterCounts[WikipediaData["computers"]] // N
34.6 Keys[TakeLargest[Counts[TextWords[ExampleData[{"Text", "AliceInWonderland"}]]], 10]]

```

35 | Natural Language Understanding

Note: = [] represents natural language input

```

35.1 Interpreter["Location"]["eiffel tower"]
35.2 Interpreter["University"]["U of T"]
35.3 Interpreter["Chemical"]["C2H4", "C2H6", "C3H8"]
35.4 Interpreter["Date"]["20140108"]
35.5 Cases[Interpreter["University"][[StringJoin["U of ", #] &] & @ ToUpperCase[Alphabet[]]], _Entity]
35.6 Cases[Interpreter["Movie"][[CommonName /@ = [us state capitals]], _Entity]
35.7 Cases[Interpreter["City"][[StringJoin /@ Permutations[{"l", "i", "m", "a"}]], _Entity]
35.8 WordCloud[TextCases[WikipediaData["gunpowder"], "Country"]]
35.9 TextCases["She sells seashells by the sea shore.", "Noun"]
35.10 Length[TextCases[StringTake[WikipediaData["computers"], 1000], #]] & /@ {"Noun", "Verb", "Adjective"}
35.11 TextStructure[First[TextSentences[WikipediaData["computers"]]]]
35.12 Keys[TakeLargest[Counts[TextCases[ExampleData[{"Text", "AliceInWonderland"}]], "Noun"]], 10]
35.13 CommunityGraphPlot[First[TextStructure[First[TextSentences[WikipediaData["language"]]]]], "ConstituentGraphs"]

```

36 | Creating Websites and Apps

```

36.1 CloudDeploy[GeoGraphics[]]
36.2 CloudDeploy[Delayed[GeoGraphics[]]]
36.3 CloudDeploy[Delayed[Style[RandomInteger[1000], 100]]]
36.4 CloudDeploy[FormFunction[{ "x" → "Number"}, #x^#x &]]
36.5 CloudDeploy[FormFunction[{ "x" → "Number", "y" → "Number"}, #x^#y &]]
36.6 CloudDeploy[FormFunction[{ "topic" → "String"}, WordCloud[DeleteStopwords[WikipediaData[topic]]] &]]
36.7 CloudDeploy[FormPage[{ "string" → "String"}, Style[StringReverse[ToString], 50] &]]
36.8 CloudDeploy[FormPage[{ "n" → "Integer"}, Graphics[Style[RegularPolygon[#[n], RandomColor[]]]] &]]
36.9 CloudDeploy[FormPage[{ "location" → "Location", "n" → "Integer"}, GeoListPlot[GeoNearest[{"Volcano"}, location, n]] &]]

```

37 | Layout and Display

Note: = [] represents natural language input

```

37.1 Style[#, Background → If[EvenQ[##], Yellow, LightGray]] & /@ Range[100]
37.2 If[PrimeQ[##], Framed[##], ##] & /@ Range[100]
37.3 If[PrimeQ[##], Labeled[Framed[##], Style[Mod[##, 4], LightGray]], ##] & /@ Range[100]
37.4 GraphicsGrid[Table[Graphics[Style[Disk[], RandomColor[]]], 3, 6]]
37.5 PieChart[Labeled[##["GDP"], ##] & /@ 

```

```

37.6 PieChart[Legended[#[{"Population"}], #]& /@ EntityList[{"g5 countries"}]]
37.7 GraphicsGrid[Partition[Table[PieChart[Counts[IntegerDigits[2^n]], {n, 25}], 5]]]
37.8 GraphicsRow[WordCloud[DeleteStopwords[WikipediaData[#]]]& /@ EntityList[{"g5 countries"}]]

```

38 | Assigning Names to Things

Note: [=] represents natural language input ↗

```

38.1 Module[{x = Range[10]}, x^2+x]
38.2 Module[{x = RandomInteger[100, 10]}, Column[{x, Sort[x], Max[x], Total[x]}]]
38.3 Module[{g == [picture of a giraffe]}, ImageCollage[{g, Blur[g], EdgeDetect[g], ColorNegate[g]}]]
38.4 Module[{r = Range[10]}, ListLinePlot[Join[r, Reverse[r], r, Reverse[r]]]]
38.5 Module[{x = Range[10]}, {x + 1, x - 1, Reverse[x]}]
38.6 NestList[Mod[17#+2, 11]&, 10, 20]
38.7 Table[StringJoin[Module[{v = Characters["aeiou"], c, c = Complement[Alphabet[], v]; RandomChoice /@ {c, v, c, v, c}], 10]

```

39 | Immediate and Delayed Values

```

39.1 {x, x+1, x+2, x^2}&.x → RandomInteger[100]
39.2 {x, x+1, x+2, x^2}&.x → RandomInteger[100]

```

40 | Defining Your Own Functions

```

40.1 ff[x_]:=x^2
40.2 poly[n_Integer]:=Graphics[Style[RegularPolygon[n], Orange]]
40.3 ff[{a_, b_}]:={b, a}
40.4 ff[x_, y_]:=(x*y)/(x+y)
40.5 ff[{a_, b_}]:={a+b, a-b, a/b}
40.6 evenodd[n_Integer]:=If[EvenQ[n], Black, White];
evenodd[0]=Red
40.7 ff[1, x_, y_]:=x+y; ff[2, x_, y_]:=x*y;
ff[3, x_, y_]:=x^y
40.8 ff[0]=ff[1]=1; ff[n_Integer]:=ff[n-1]+ff[n-2]
40.9 animal[s_String]:=Interpreter["Animal"][s]["Image"]
40.10 nearwords[s_String, n_Integer]:=Nearest[WordList[], s, n]

```

41 | More about Patterns

```

41.1 Cases[Table[IntegerDigits[n^2], {n, 100}],
{___, x, ___, ___}]
41.2 StringJoin /@ Cases[Array[Characters[RomanNumeral[#]]&, 100],
{___, "L", ___, "I", ___, "X", ___}]
41.3 ff[x:{_Integer..}]:=x==Reverse[x]
41.4 Cases[Partition[TextWords[WikipediaData["alliteration"]], 2, 1], {a_, b_}/;
StringTake[a, 1]==StringTake[b, 1]]
41.5 Grid[FixedPointList[#[#/.{x___, b_, a_, y___}]/;b>a→{x, a, b, y})&, {4, 5, 1, 3, 2}]]
41.6 ArrayPlot[Transpose[FixedPointList[#/.{x___, b_, a_, y___}]/;b>a→{x, a, b, y})&,
RandomSample[Range[50]]]]

```

```

41.7 FixedPointList[#[#+2/#]/2&, 1.0]
41.8 FixedPointList[#/.{a_, b_};b≠0→{b, Mod[a, b]}&, {12345, 54321}]
41.9 FixedPointList[#/.{s[x_][y_][z_]→x[z][y[z]], k[x_][y_]→x}&, s[s][k][s[s][s]][s][s]]
41.10 IntegerDigits[1001]&.{x___, 0..}→{x}
41.11 Length /@ NestList[#/.{1, _, x___}→{x, 0, 1}, {0, _, x___}→{x, 1, 0, 0}&, {1, 0}, 200]
41.12 ListLinePlot[Length /@ NestList[#/.{0, _, x___}→{x, 2, 1}, {1, _, x___}→{x, 0}, {2, _, x___}→{x, 0, 2, 1, 2}&, {0, 0}, 200]]

```

42 | String Patterns and Templates

Note: [=] represents natural language input ↗

```

42.1 StringReplace["1 2 3 4", " "→"---"]
42.2 Sort[StringCases[WikipediaData["computers"], DigitCharacter~~DigitCharacter~~DigitCharacter~~DigitCharacter]]
42.3 StringCases[WikipediaData["computers"], Shortest["=="~~x_~~"==""]→x]
42.4 Grid[Table[StringTemplate["`1`+`2`=`3`"][[i, j, i+j], {i, 9}, {j, 9}]]]
42.5 Select[Table[IntegerName[n], {n, 50}], StringMatchQ[#, ___~~"i"~~___~~"e"~~___]&]
42.6 StringReplace[First[TextSentences[WikipediaData["computers"]]], x:(Whitespace ~~ LetterCharacter ~~ LetterCharacter ~~ Whitespace)→UpperCase[x]]
42.7 BarChart[KeySort[Counts[StringTake[TextString /@ EntityList[{"countries"}], 1]]], ChartLabels→Automatic]
42.8 Table[StringTemplate["`1`+`2`=`3`"][[i, j, i+j], {i, 5}, {j, 5}]]//Grid

```

44 | Importing and Exporting

Note: [=] represents natural language input ↗

```

44.1 Import["http://google.com", "Images"]
44.2 ImageCollage[Graphics[Style[Disk[], #]]]& /@ (Union @@ DominantColors /@ Import["http://google.com", "Images"])
44.3 WordCloud[Import["http://bbc.co.uk"]]
44.4 ImageCollage[Import["http://whitehouse.gov", "Images"]]
44.5 Select[Import["https://en.wikipedia.org/wiki/Ostrich", "Images"], ImageGetInstanceQ[#, {bird}]&]
44.6 WordCloud[TextCases[Import["http://www.nato.int"], "Country"]]
44.7 Length[Import["https://en.wikipedia.org/", "Hyperlinks"]]
44.8 SendMail[GeoGraphics[Here]]
44.9 SendMail[MoonPhase[Now, "Icon"]]

```

45 | Datasets

Note: [=] represents natural language input ↗

```

45.1 WordCloud[Normal[planets[All, "Moons", Length]]]
45.2 BarChart[planets[All, "Moons", Length], ChartLabels→Automatic]
45.3 planets[SortBy[Length[Moons]&], "Mass"]

```

```

45.4 planets[All, "Moons", Max, "Mass"]
45.5 planets[All, "Moons", Total, "Mass"][[Sort]
45.6 planets[All, "Moons", Median, "Mass"]
45.7 planets[All, "Moons", Select[##>Mass>
    =[0.0001 earth mass]&]/* Keys]
45.8 WordCloud[Association[##>StringLength[
    WikipediaData[##]] &/@ EntityList[
    ={central america}]]]

```

46 | Writing Good Code

```

46.1 Total[Table[i*(i+1), {i, 1000}]]
46.2 Nest[1/(1+##)&, x, 10]
46.3 Flatten[Array[List, {10, 10}]]
46.4 ListLinePlot[Table[First[Timing[n^n]], {n, 10000}]]
46.5 ListLinePlot[Table[First[Timing[Sort[
    RandomSample[Range[n]]]]], {n, 200}]]

```

47 | Debugging Your Code

```

47.1 Counts[If[StringLength[##]>1, StringTake[##, 2],
    Nothing]&/@WordList[]]
47.2 First[Last[Reap[Fold[10 Sow[##1]+##2 &,
    {1, 2, 3, 4, 5}]]]]
47.3 Last[Reap[Nest[{If[EvenQ[##], Sow[##]/2, 3##+1]&,
    1000, 20}]]//First]

```

Index

- 3D graphics, 30, 68, 138
 - box around (**Boxed**), 71
 - in the cloud, 226
- 3D printing, 268
- 3D shapes, 68, 71
- 3n+1* problem, 168
- Abbreviations, for units, 90
- Aborting evaluation, 294
- Abs**, 132
- Absolute value (**Abs**), 132
- AbsoluteTiming**, 288, 301
- Access, to cloud deployments, 217
- Accumulate
 - in databin, 262
 - with **FoldList**, 173
- Accumulate**, 175
- Add to databin (**DatabinAdd**), 262
- Addition, of lists, 15
 - Plus**, 1
- AdjacencyGraph**, 121
- AdjacencyMatrix**, 121
- Afterthought (function
 - application as), 141
- Age group, target, xiv
- Age of universe, compared
 - to UUIDs, 264
- AI (artificial intelligence), 123
- Aircraft positions, 84
- AirTemperatureData**, 103
- AiryAi, 296
- Algebra, 296
- Algebraic variables, symbols as, 204
- Algorithms
 - built in, 298
 - for music, 54
 - for sorting, 250
- Alignment, of sequences, 259
- All (as option value), 109
- Alpha (Wolfram|Alpha), vii
- Alphabet**, 49, 181, 206
- Alphabet position
 - (**LetterNumber**), 49
- Alternation (in regular expressions), 259
- Alternatives
 - in patterns (|), 193
 - in string patterns (|), 256
- Ambiguity, 80, 214
- AmbiguityFunction**, 215
- Amusement parks, as entities, 81
- Anatomy, as example of data, 83
- And (&&), 165, 298
- Android deployment, 219
- AnglePath**, 89
- Angles, units for, 88
- AngularGauge**, 140
- Animals
 - interpreter for, 212
 - web app about, 220
- Animation, 175, 231
 - in **Manipulate**, 37
- Annotations
 - for code, 288
 - in plots, 228
- Anonymous functions, 147
- Anonymous local files, 263
- Answer, latest (%), 233
- APIFunction**, 226, 299
- APIs
 - building, 226, 299
 - external, 267, 269
- Append**, 237, 283
- AppendTo**, 237
- Apply**, 204
 - Applying functions, 141
- Approximate numbers, 129
- Apps, creating, 217
- Arbitrary precision, 130
- Architecture, of systems, 303
- ArcTan**, 134
- Arduino, 270
- Area**, 297
 - Arguments, of functions, 171, 200
- Arithmetic, 1
 - modulo, 133
 - on lists, 15
 - with dates, 101
 - with units, 87
- Array**, 171
 - Array subscript (**Part**), 16
- ArrayFlatten**, 179
- ArrayPlot**, 58, 299
- Arrays
 - lists as, 10
 - multidimensional, 57
 - of arrays, 179
- Arrow** (graphics primitive), 71
- Arrow (→), 107
- Artificial intelligence, 123
- ASCII (**ToCharacterCode**), 52
- AspectRatio**, 111
- Assignment
 - delayed, 239
 - immediate, 239
 - of names, 233
- Association**, 206
- AssociationMap**, 209
- Associations, 205
 - and string templates, 257
 - combining, 280
 - in datasets, 271
 - of associations, 271
- Associative arrays
 - (associations), 208
- Associativity, 253
- Assumptions, in debugging, 292
- At sign (@), 141
- Atoms
 - compared to UUIDs, 264
 - in expressions, 198
- Attributes, of functions, 253
- Audio, 53
- Authorization, of external services, 269
- Autocomplete menu, xiii, 75
- Automatic**, for options, 108
- AutoRefreshed**, 226
- Autoupdating website, 219
- Axes, 71
- Background**, 108
 - for **Framed**, 227
 - for **Grid**, 112
- Backquote (`), 134, 258
- Backslashes
 - for entering quotes, 52
 - in numbers, 134
- Backtick (`), 134, 258
- Backward
 - (**Reverse**), 8
 - (**StringReverse**), 45
- Ball**, 296
- BarChart**, 11, 80, 276
 - for associations, 207
 - versus **Histogram**, 140
- BarChart3D**, 140
- BarcodeImage**, 127
- BarcodeRecognize**, 127
- Base expansions
 - (**IntegerDigits**), 18, 134
- Bases, numbers in other, 18, 134
- BBC, word cloud from website of, 269
- Beep**, 56
- Beginning of list (**First**), 16
- Benford's law, 139
- BesselJ**, 134
- Bidirectional graph
 - (**UndirectedGraph**), 117
- Big numbers, 2
- Binaryize**, 41
- Binary digits (**IntegerDigits**), 18
- Binary dump format
 - (**DumpSave**), 264
- Bindings, for APIs
 - (**EmbedCode**), 226
- Binomial**, 296
- Binomial coefficients, 157, 296
- Bins, in histogram, 140
- Bioinformatics, 259
- Bitmaps, 49
- Black**, 25
 - Black and white (**Binaryize**), 41
- Blank (_), 193
- Blend**, 25, 154
- Block**, compared to **Module**, 237
- Block matrices (**ArrayFlatten**), 179
- Blocks, partitioning into, 177
- Blue**, 25
- Blur**, 39, 147
 - text recognition and, 125
- Bold**, 28
- Boolean computation, 298
- Boolean operations, 165

- BooleanCountingFunction**, 298
BooleanMinimize, 298
Booleans, 169
BooleanTable, 298
 Bordering countries, 79
 Borders, of graphics (**EdgeForm**), 71
 Bowtie polygon, 71
Boxed, 71
BoxWhiskerChart, 140
 Braces $\{\dots\}$, for lists, 10
 Brackets $\{[\dots]\}$, square, 3
 Brains, as inspiration for
 ImageIdentify, 127
 Break strings (**StringSplit**), 256
 Bubble sort, 253
BubbleChart, 140
 Bugs, 284, 291
 Built-in functions, 73
 redefining, 246
BusinessDayQ, 168
ButterflyGraph, 121
ByteCount, 204
- C language, 288
 C++, xi, 288
 Caching, in function
 definitions, 289
 Caffeine, 82
Cake chart (PieChart), 11
 Calculus, 296
 Calendar systems, 105
 Calling APIs, 226
 Camera
 in apps, 223
 picture from, 39
 Canonical form, for entities, 80
 Capital letters, for functions, 3–4
Capitalize (ToUpperCase), 45
 Capitals, of Europe, 298
 Car, directions by, 99
 Cartography, 93
Cases, 193, 201
 operator form of, 201
Catenate, 279
 Cats, testing for pictures of, 166
Ceiling, 135
CellularAutomaton, 289, 299
 Centimeters, 87
 Central limit theorem, 140
 Chaining, of dataset queries, 273
 Chains of functions, 141
 Chalkboard font, 111
 Champernowne sequence, 184
 Character codes, 52
 Characters, in string patterns, 255
Characters, 46, 165, 177, 188, 203
 Chart
 bar (**BarChart**), 11
 pie (**PieChart**), 11
ChartLabels, 207
 Checkboxes
 in forms, 225
 in **Manipulate**, 37
 Cheetah, image
 identification of, 123
- Chemicals, interpreter for, 212
ChiSquareDistribution, 134
 Choose elements (**Select**), 164
 Chords, musical, 55
ChromaticityPlot, 28
ChromaticityPlot3D, 28
Circle, 29, 148
 size of, 111
 with coordinates, 64
Circle graph (PieChart), 11
 Circle packing, 65
 Cities
 as entities, 93
 distance between, 93
 nearest, 97
 Classes of entities, 79
ClassifierFunction, 127
Classify, 123, 267
Clear, 234, 244
 Clock, on web, 219
 Clock arithmetic (**Mod**), 133
ClockGauge, 218
 Closest (**Nearest**), 124
 Cloud
 deployment to, 217
 interactivity on, 36, 221
 Manipulate on, 36, 221
 physical location of, 226
 Wolfram Knowledgebase in, 84
 Cloud base, 264
 CloudCDF, 221
 Cloud objects, 261
CloudDeploy, 217, 299
CloudExport, 268
CloudGet, 261, 275
CloudPut, 261
CloudSave, 261
 Cluster analysis, 125
 Code
 annotations to, 288
 debugging of, 291
 efficiency of, 286
 making random changes in, 294
 optimization of, 288
 overly short, 285, 289
 writing good, 283
 Code captions, xiii, 76
 Code generation (**EmbedCode**), 226
 Code reviews, 303
Collage (ImageCollage), 40
 Collatz problem, 168
 Collect elements (**Gather**), 180
 Collisions, of UUIDs, 261
 Colon $(:)$, in patterns, 250
 Colon-equals $(:=)$, 239
 Colon-greater $(:>)$, 240
 Color cube, 70
 Color pickers, in forms, 225
 Color spaces, 28
 nearness in, 125
 Color wheel, 28
ColorDistance, 127
ColorFunction, 114
ColorNegate, 25, 141, 148, 153
 for images, 39
- Colors, 25
 background (**Background**), 108
 blending (**Blend**), 25
 dominant (**DominantColors**), 41
 in graphics (**Style**), 30
 models of, 28
 names of, 28
 nearest, 124–125
 of text (**Style**), 26
 picking in **Manipulate**, 37
 random (**RandomColor**), 26
- Column**, 12, 229
 Column chart (**BarChart**), 11
 Columns
 in arrays, 188
 in datasets, 271
 Combinations (**Tuples**), 182
 Combinators, 252
 Combining
 associations (**Association**), 280
 colors (**Blend**), 25
 lists (**Join**), 8
 plots (**Show**), 140
 strings (**StringJoin**), 45
 Comments in code, 288
 Common elements
 (**Intersection**), 181
 Common words
 in English (**WordList**), 47
 in word cloud, 47, 279
 Communities, in graphs, 119
 Community, Wolfram, xv
CommunityGraphPlot, 119
 Commutativity, 253
Compile, 288
Complement, 181
 Complementary color
 (**ColorNegate**), 25
 Complete graph, 117
CompleteGraph, 121
 Complex numbers, 134
 Complexity, from simple
 programs, 289, 299
 Composite heads, 201
 Compositing images
 (**ImageAdd**), 42
 Composition operators, 280
 Compound expressions $(;)$, 235
CompoundElement, 226
 Computational geometry, 297
 Computational thinking, ix, 304
 Computational universe, 299
 Computed number fields, 222
 Computer algebra, 296
 Concatenate
 Catenate, 279
 Join, 8
 StringJoin, 45
 Conditionals, 163
 Conditions, on patterns $(/;)$, 249
Cone, 30
 Connected devices, data from,
 261
 Connected graph, 127
ConnectedGraphQ, 168

Connections, in graphs, 115
 Connectivity (internet)
 for **Interpreter**, 214
 for units, 90
 needed for data, 84
 Constituent graph, 213
 Constraints, on form fields, 225
 Contexts, 289
 Contour lines (**MeshFunctions**),
 140
 Contour plot, 138
ContourLabels, 140
ContourPlot3D, 295
Control + \equiv , 77, 211
 for dates, 101
 for units, 87
ControllerInformation, 37
 Conversions, between units, 87
ConvexHullMesh, 297
 Coordinates, 63
 on Earth's surface, 96
 Corners, of polygons, 67
 Corporate data, 271
Cos, 134
Count, 16
 Countries
 as entities, 77
 graph of bordering, 159
 maps of, 93
 nearest, 96
 picking out names of, 213
 Country plots
 (**GeoRegionValuePlot**), 99
CountryData, 84
Counts, 205, 301
 Courses, use of book in, xiv
 CPU time, 288
 Crawling, on network
 (**NestGraph**), 159
CreateDatabin, 262
 Creating functions, 243
 Cryptography, 300
Cuboid, 71
 Curly brackets $\{\dots\}$, for lists, 10
 Currency
 interpreter for, 211
 units of, 88
 Current location (**Here**), 97
 Current time, 101
CurrentImage, 39
 Currying, 204
 Customer data, 275
Cylinder, 30

 Dashboard
 autorefreshed, 226
 creating a web, 219
 Data
 hierarchical, 275
 importing, 265
 real-world, 77
 Data Drop, Wolfram, 261
 Data signatures, for databins, 264
 Database join (**JoinAcross**), 209
 DatabaseLink, 282

 Databases, 282
 connecting to external, 282
 importing, 266
DatabinAdd, 262
 Databins (in Wolfram
 Data Drop), 262
Dataset, 271
DateListPlot, 104, 262
DateObject, 101
 Dates, 101
 formats of, 101
DateString, 105
DateValue, 105
 Day of week (**DayName**), 102
DayName, 102
DayRange, 102
 DDMMYY (date format), 106
 Debugging, 291
 interactive, 294
 Decimal approximation, 129
 Decimal degrees, 99
 Decimal expansion
 (**IntegerDigits**), 16
 Decimal points, and approximate
 numbers, 129
 Default values, in patterns, 253
 Defining variables, 233
 Definitions
 of functions, 243
 of words (**WordDefinition**), 214
Defocus (**Blur**), 39
Degree, 88
 Degrees-minutes-seconds, 99
Delayed, 218
 as analog of := , 240
 Delayed assignment (:=), 239
 Delayed rules (\rightarrow), 240
DeleteMissing, 84
DeleteStopwords, 47
 Deleting, elements of lists, 189
 Delimiters, split strings at, 256
 Demonstrations Project, xv, 37
 Deployment (**CloudDeploy**), 217
 Desk, sensor on my, 262
 Destructuring, of function
 arguments, 247
 Device framework, 270
DeviceReadTimeSeries, 270
 Diagramming, sentence, 213
 Dictionaries (associations), 208
 Dictionary
 and **LanguagIdentify**, 127
 definitions
 (**WordDefinition**), 214
 list (**WordList**), 47
 Differential equation, 297
DigitCharacter, 256
 Digits
 as examples of coding
 styles, 284
 folding in, 174
 number of, 130
 of integers (**IntegerDigits**), 16
 Directed graph (**Graph**), 117
 Disambiguation, 80, 214

 Disappear (**Nothing**), 189
 Disconnected graph, 127
Disk, 29
 area of, 297
Display (**Echo**), 292
 Displaying lists, 11
 Distance, between cities, 93
 Divide string (**StringSplit**), 256
 Division (**Divide**), 1
 by zero, 2
 DMS, for geo positions, 99
Do, 237
 Document generation, 300
 Documentation, for Wolfram
 Language, xv, 73
DocumentNotebook, 300
 Dollars
 as units, 88
 interpreter for, 211
DominantColors, 41
Dotted, 137
 Double at (@@), 204
 Double blank (_), 193
 Double equals (==), 168
 Downvalues, 247
 Drawing
 from **EdgeDetect**, 41
 of graphs, 116
 Driving directions, 99
Drop, 17
DumpSave, 264
 Duplicates, remove (**Union**), 180
 Dynamic programming, 289
 Dynamic scoping, 237
 Dynamic typing, 247
 Dynamic visualization, 231

 Each element, apply to (/@), 142
 Earth surface path, 95
 Earthquakes, 84
Echo, 292
 Eclipse IDE, 288
EdgeDetect, 41, 49, 141, 153,
 223
EdgeForm, 71
 Edges, of graph, 120
EdgeWeight, 120
EditDistance, 127
 Efficiency, of code, 286
 Eiffel Tower, 81, 94
 Elements
 of lists (**Part**), 16
 test for (**MemberQ**), 166
 Elevation data (for Earth), 138
EllipticK (elliptic integral), 134
 Email, sending, 267
EmbedCode, 225, 299
 Embedding, of graphs, 116
EmitSound, 55
 Emitted light colors, 28
 Empty list, 18
Encrypt, 300
 End of list (**Last**), 16
EndOfLine, 259
EndOfString, 259

- English
 alphabet, 181
 as input language, 77
 computation on, 46
 lengths of words in, 137
 versus Wolfram Language, 286
 words in (**WordList**), 47
- Entities**, 77
 implicitly specified, 168
 in text (**TextCases**), 212
 searching for, 168
 types of, 80
- EntityInstance**, 84
- EntityList**, 79
- EntityProperties**, 81
- EntityValue**, 78
- Equality testing (**Equal**), 163
- Equations**
 solving (**Solve**), 296
 structure of (==), 169
- Erf**, 134
- Errors, input, 4, 291
- Escaping in strings, 52
- Euclid's algorithm, 252
- EulerPhi**, 134
- Europe, shortest tour of capitals, 298
- European
 dates, 106
 units, 90
- Euros, interpreter for, 211
- Evaluation**
 controlled, 300
 in Wolfram Language, 199, 241
- Even-odd rule, 71
- EvenQ**, 164
- Everest, Mount, 138
- Except**, 253
- Exp**, 134
- Experiments, in Wolfram
 Language, 304
- Exponential time, 287
- Export**, 268
- ExportString**, 270
- Expressions
 size of, 204
 symbolic, 197
- Extensible forms, 226
- External device, data from, 261
- External programs, 268, 300
- External services, 267
- Eyes, and color, 28
- Facebook social graph, 120, 267
- Factor**, 296
- Factorial function
 built-in (!), 134, 244
 defining a, 244
- FactorInteger**, 134, 298
- False**, 163
- Feet (unit), 87
- Fibonacci**, 134, 289
 as example of coding, 285
- Fields
 for forms, 221
 in datasets, 275
- File path, 263
- Files
 extensions for, 269
 importing from, 267
 local, 262
 names of, 264
 searching text, 259
- FileTemplate**, 259
- Filled disk (**Disk**), 29
- FilledCurve**, 71
- Filling**, 108
- Filter a list (**Select**), 164
- FindClusters**, 125
- FindGeoLocation**, 99
- FindShortestPath**, 116
- FindShortestTour**, 298
- Finishing input, xiii
- First**, 16
 applied incorrectly, 291
- First letter (**StringTake**), 45
- FirstPosition**, 191
- FixedPointList**, 251
- Flags, of countries, 77
- Flat**, as function attribute, 253
- Flat (b) notes, 55
- Flatten**, 116, 178, 188
- Floating point numbers, 134
- Floor**, 135
- Fluency, in Wolfram
 Language, viii, 283
- Fold**, 175, 283
- Folded polygons, 71
- FoldList**, 172
- Font size (**Style**), 27
- \$FontFamilies**, 114
- FontFamily**, 28, 111
- For**, 237, 283, 288
- For each (f@), 142
- Foreign exchange, 88
- Foreign keys, 282
- Foreign languages
 function names in, 76
WordTranslation, 214
- Form fields, restrictions on
 values in, 225
- Form-based apps, 219
- Formats
 cloud deployment, 225
 import, 269
- FormFunction**, 219
- FormObject**, 225
- FormPage**, 223
- Forms
 apps based on, 219
 extensible, 226
 multipage, 226
- FormTheme**, 225
- Forum (Wolfram Community), xv
- Fractals, 156, 179
- Fractions, exact, 129
- Frame**, 112, 230
- Framed**, 142, 148, 153, 155, 173, 227
 rounded corners for, 231
- Free-form input, xiii, 77, 211
- Frequency, of letters, 207
- Frequency plot (**Histogram**), 137
- Friend network, 120, 267
- FromCharacterCode**, 52
- FromDigits**, 18, 175, 284, 286
- FromDMS**, 99
- FromLetterNumber**, 49
- Fuji, Mount, 139
- FullForm**, 198
- Function function (**Function**), 152
- Function page, in documentation, 75
- Functional programming,
 152, 174–175, 237
 lack of assignments in, 235
- Functions**
 approximate, 297
 attributes of, 253
 built-in, 3, 73
 compared to math, 5
 composition of, 280
 defining your own, 243
 plotting, 295
 pure, 147
 reading out loud, 4
 repeated application of, 153
 saving to cloud of, 261
 templates for, 76
 translations of names for, 76
 ways to apply, 141
- Functors**, 175
- G5, 231
- Gamepads, for **Manipulate**, 37
- Gamma** (gamma function), 134
- Ganymede, 280
- Gather**, 180
- GatherBy**, 180
- Gaussian-distributed numbers
 (**RandomReal**), 135
- GCD**, 134
- GCD algorithm, 252
- Genomics, 259
 as string example, 51
- GeoBackground**, 110
- Geocomputation, 93
- Geodesic path, 95
- GeoDisk**, 95
- GeoDistance**, 93, 167
- GeoElevationData**, 138
- Geographic position
 (**GeoPosition**), 96
- GeoGraphics**, 217
 ranges in, 109
- geoIP, 99
- GeoLabels**, 110
- GeoListPlot**, 93
 ranges in, 109
- \$GeoLocation**, 99
- Geometry, 297
 as example of topic, 74
 export of, 268
 fractal, 160
 importing, 266
 transformations in, 71
- GeoModel**, 99
- GeoNearest**, 96

- GeoPath**, 95
- GeoPosition**, 96
- GeoProjection**, 98, 217
- GeoRange**, 99, 109
- GeoRegionValuePlot**, 99
- Get**, 262
 - Get from cloud (**CloudGet**), 261
 - Ghost image (**ColorNegate**), 39
 - GIF, 225, 269
 - GIS (geocomputation), 93
 - Giza Pyramid, 81
 - Global values, 234
 - Goes to (**Rule**), 114
 - Golden ratio, 154, 289
 - Good code, 283
 - GPS position, 99
 - Grades data, 275
 - GrammarRules**, 214, 259
 - Grammatical structure (**TextStructure**), 213
 - Graph**, 115
 - using **Thread** in constructing, 177
 - Graphics**
 - 3D, 30, 138
 - as not listable, 145
 - colors in (**Style**), 30
 - interactive, 34
 - labeling in, 228
 - lighting in, 31
 - objects in, 29
 - of data (**ListPlot**), 7
 - primitives, 66
 - resizing of, 231
 - GraphicsColumn**, 229
 - GraphicsGrid**, 229
 - GraphicsRow**, 229
 - GraphLayout**, 116
 - Graphs**
 - constructing from lists, 203
 - nested, 158
 - of nearest neighbors, 125
 - random (**RandomGraph**), 121
 - social, 267
 - Gray code, as example of program, 292, 294
 - GrayLevel**, 28
 - Great circle path, 95
 - Great Pyramid, 81
 - Greater than (**>**), 163
 - GreaterEqual** (**≥**), 168
 - GreaterThan**, 168
 - Greek alphabet, 51–52
 - Green**, 25
 - Grid**, 57, 149, 171, 177, 229
 - nested, 156
 - options for, 112
 - Gröbner bases, ix
 - Grouping** (**FindClusters**), 125
 - GUI construction, 295
 - Guide page, in documentation, 74
 - Gunpowder, 214
 - Handwritten digits, 124
 - Hash (#), 175
 - Hashmaps (associations), 208
 - Haskell Curry, 204
 - Head**, 200
 - Heads, associating values with, 247
 - Height map, 138
 - Height/width for plots, 111
 - Help, on functions, 76
 - Here**, 97
 - Hexadecimal, in UUIDs, 264
 - Hexagon (**RegularPolygon**), 29
 - Hidden state, 237
 - IDs, for **Manipulate**, 37
 - Hierarchical data, 275, 282
 - Higher-order functions, 175
 - Histogram**, 137
 - Historical calendars, 105
 - Hold**, 241, 300
 - Home page, for Wolfram Language, xv
 - Hot pink, 211
 - Hour, as unit, 87
 - HTML
 - exporting fragments, 225
 - importing from web, 269
 - templates (**XMLElement**), 259
 - Hue**, 26, 143
 - Human language
 - identification of, 123
 - input in, 77, 211
 - learning as comparison, viii
 - Humidity, from desk sensor, 262
 - HypercubeGraph**, 121
 - Hyperlinks
 - from my website, 266
 - importing, 266
 - network of, 115
 - i (square root of -1), 134
 - IconRules**, 226
 - Icons, for mobile apps, 226
 - IDE, for Wolfram Language, 288
 - Identity testing (**==**), 169
 - Idioms, in code, 286
 - If**, 163
 - in function definitions, 244
 - Illumination, in 3D graphics, 31
 - Image3D**, 61, 175
 - ImageAdd**, 42, 174
 - ImageCollage**, 40
 - ImageData**, 59
 - ImageIdentify**, 123
 - imageidentify.com, 127
 - ImageInstanceQ**, 166
 - Images, 39
 - adding (**ImageAdd**), 42
 - as arrays of numbers, 59
 - combining (**ImageCollage**), 40
 - getting from data, 61
 - importing from websites, 266
 - in forms, 223
 - nearness for, 124
 - of entities, 79
 - testing for objects in, 166
 - ImageSize**, 111, 231
 - Immediate assignment (=), 239
 - Immersion, for language learning, viii
 - Imperial units, 90
 - Implicit entities, 168
 - Import**, 265
 - \$ImportFormats**, 269
 - ImportString**, 270
 - In**, 241
 - Increment** (++), 237, 283, 288
 - Indices
 - found with **Position**, 188
 - in lists (**Part**), 16
 - in tables, 19
 - Infinite loops, 241
 - Infinity, 2
 - Input, terminating, xiii
 - Input fields
 - in **Manipulate**, 37
 - in web forms, 220
 - Input labels (**In**), 241
 - InputForm**, 46
 - for dates and times, 101
 - for entities, 80
 - for quantities, 87
 - Inserting in strings (**StringRiffle**), 257
 - Inset** (graphics primitive), 71
 - Instances, of entities (**EntityInstance**), 84
 - Instances (**Cases**), 193
 - Instant APIs, 226, 299
 - Instruments, musical, 54
 - IntegerDigits**, 16, 134, 149, 286
 - IntegerName**, 48
 - IntegerQ**, 164
 - IntegerReverse**, 286
 - Integers
 - big, 2
 - English names (**IntegerName**), 48
 - factoring, 298
 - random, 3
 - Integrate**, 296
 - Interactivity, 33
 - in 3D graphics, 30
 - of cloud graphics, 226
 - on cloud, 36
 - Interleaving
 - for lists (**Riffle**), 181
 - for strings (**StringRiffle**), 257
 - Intermediate results
 - capturing, 293
 - printing, 292
 - Internal form (**FullForm**), 198
 - International characters, 52
 - International versions, code
 - captions for, 76
 - Internet address, for geo location, 99
 - Internet connectivity
 - for data, 84
 - for maps, 99
 - InterpolatingFunction**, 297
 - Interpretations, multiple, 80
 - Interpreter**, 211
 - Interpreters, for web forms, 222
 - \$InterpreterTypes**, 214

- Intersection**, 181
iOS deployment, 219
IT organizations, 271
Italic, 28
Iteration, 155
 for tables (**Table**), 19
 versus recursion, 160
Iterator specification, 23
- Java**, xi
 code generation for, 299
JavaScript, xi
 importing from web, 269
Join, 8, 157, 237
JoinAcross, 209
 Joining points, 67
Joysticks, for **Manipulate**, 37
JPEG, 225
- KaryTree**, 121
Key-value maps (associations), 208
KeyDrop, 209
KeyMap, 209
Keys, in associations, 205–206
KeySelect, 209
KeySort, 205
KeyTake, 206
Kilograms, 88
Knowledge-based language, xi
Knowledgebase, 77
Koala, as example of entity, 81
- LABColor**, 28
Labeled, 227
Labels
 for any expressions, 227
 for bar charts, 207
 for contour lines, 140
 for form fields, 225
 in maps, 111
 of graphs, 115
 placement of, 231
Lambda expressions, 152
Language design, 283
Languagelidentify, 123, 265
Languages
 code captions in other, xiii
 translation between, 214
Large programs, xi, 289
Largest elements (TakeLargest), 189
Largest (Max), 3
Last, 16
Latest result (%), 233
Latin, viii
Latitude, 96
Layout
 of graphics, 227
 of graphs, 116
 of tabular data, 57, 229
LeafCount, 204
Leaves, in expressions, 199
Left composition, 280
Legended, 228
Legends, placement of, 231
- Length**
 of general expressions
 (**Length**), 202
 of lists (**Length**), 15, 144
 of strings (**StringLength**), 45
Less, 168
LetterCharacter, 256
LetterCounts, 206
LetterNumber, 49, 165
LetterQ, 165, 180
Letters, in strings (**Characters**), 46
Lexical scoping, 237
Lhs (left-hand side), 196
Light, from desk sensor, 262
LightGray, 227
Lighting, simulated in 3D
 graphics, 31
LightYellow, 227
Line, 66
Line drawing (from EdgeDetect), 41
Line numbering, in sessions, 237
Lines, terminating input, xiii
Link graph, 266
Listability, 15, 144, 146
 and associations, 209
ListAnimate, 175, 231
ListLinePlot, 11, 48, 63
 options for, 107
 with multiple datasets, 137
ListLogPlot, 132
ListPlay, 56
ListPlot, 7, 63
 analog for time series, 104
 labeled, 228
ListPlot3D, 138
Lists
 arithmetic on, 15
 as functions, 10
 as tables (**Table**), 19
 dropping sublists (**Drop**), 17
 filtering (**Select**), 164
 first look at, 7
 in datasets, 275
 internal structure of, 197
 joining (**Join**), 8
 length of (**Length**), 15
 membership in (**MemberQ**),
 166
 of rules, 177
 of rules and associations, 206
 operations on, 15
 parts of, 16, 187
 random choice from
 (**RandomChoice**), 155, 182
 sequences in, 259
 symbolic, 10
 taking sublists (**Take**), 17
 visualization of, 11
 zero-length, 18
ListStepPlot, 140
ListStreamPlot, 140
Load definitions (CloudGet), 261
Local files, 262
 importing from, 267
Local values, 234
- Local variables**, in modules, 235
Localized names, 237
Localized values, 237
LocalObject, 263
LocalTime, 103
Location, interpreter for, 211
Location (Here), 97
Locked functions, 246
Log, 279
Log files, importing, 266
Log plot (ListLogPlot), 132
Log10, 131
Logarithm
 base 10 (**Log10**), 131
 natural (**Log**), 131
Logic computation, 298
Logic operations, 165
Logistic regression, 127
Logo, and **AnglePath**, 91
London, as time zone example,
 103
Long computations, monitoring,
 293
Long output, suppressing, 233
Longer programs, 303
Longest, 250
Longitude, 96
Loop variable errors, 294
Loops
 for, 288
 in graphs, 115
 infinite, 241, 294
Los Angeles, 93
Louvre, 94
- Machine learning**, 123
Machine precision, 134
Magnitude (Abs), 132
Mail, sending, 267
Mail server, for **SendMail**, 270
Manipulate, 33
 deploying to web, 218
 generating in cloud, 221
Map (@), 145
Maps
 geographic, 93
 range of (**GeoRange**), 99
Market prices, 84
Marketing, plots themed for, 107
Mars, maps of, 99
Matches
 all possible, 253
 for patterns (**Cases**), 193
 for string patterns, 256
 longest, 250
 shortest, 250
Matching of brackets ([...]), 4
Math notation, 296
Math typesetting, 296
Mathematica, vii
Mathematical computation, 296
Mathematical functions, 134
 visualization of, 295
Mathematical logic, 152, 204
Mathematical operations, 1

Mathematics
 comparison to learning, viii
 notion of function in, 5
 notion of mapping in, 146
 prerequisites, ix, xiv
Matrices, lists of lists as, 61
Max, 3
 in **Dataset**, 273
MBOX, 269
 Meaning, of words, 214
MemberQ, 166
 Memoization, in function definitions, 289
 Memory, and big numbers, 2
MemoryConstrained, 294
 Menus
 in forms, 225
 in **Manipulate**, 37
 Mercury, as disambiguation example, 80
Merge, for associations, 209, 280
Mesh, 137
MeshFunctions, 140
 Meshing, 297
MeshStyle, 137
 Messages
 from code, 291
 sending mail, 267
 Meta-algorithms, 127
 Methods, symbolic generalization of, 247
 Middle C (note), 53
 MIDI, 55
Min, 4
 Minimization, of Boolean functions, 298
Minimize, 296
Minus (Subtract), 4
Missing, 84, 208
 Missing entries, in datasets, 275
 Mixing colors (**Blend**), 25
 ML (machine learning), 123
 MMDDYY (date format), 106
 Mobile apps, creation of, 219, 226
Mod, 133
 Modularity, in good code, 285
Module, 234
 compared to **With**, 291
Modulo (Mod), 133
 Molecule, as example of entity, 82
 Money, computations with, 88
Monitor, 293
 Month, from date, 105
 Moon
 maps of, 99
 phase of (**MoonPhase**), 102
MoonPhase, 102
 Moons, as examples of **Dataset**, 275
 Mosaic (**ImageCollage**), 40
Most, 18
 Mount Everest, 138
 Mount Fuji, 139
 Movie titles, 214
MovieData, 84
 Multicore system, 301
Multipage forms, 226
Multiple assignments, 237
Multiplication sign (\times), 2
Multiplication table, 58, 171
Multiplication (Times), 1
Music, 53
N, 129, 142, 288
 Naive Bayes machine learning, 127
 Named patterns, 194, 250
Naming
 of fields in datasets, 275
 of functions, 285
 of objects, 233
 principles of, 236
NATO, 184
 website of, 269
 Natural language, and language identification, 123
 Natural language input, xiii, 211
 Natural language understanding, 211
 Natural logarithm, 131
NDSolve, 297
Nearest, 124, 159
NearestFunction, 161
NearestNeighborGraph, 125, 161
 Negative part numbers, 187
 Negative sentiment, 123
 Neighbors, nearest (**Nearest**), 124
Nest, 153, 292
NestGraph, 158
 Nesting, of functions, 153
NestList, 153, 179, 251, 256
 versus **FoldList**, 172
 Net of polyhedron, 83
 Networks, 115
 Neural networks, 127
New Kind of Science, A, 289, 299
 New York City, 93–94, 211
 as entity, 80
 as time zone example, 103
 Newlines, 257
 split at (**StringSplit**), 256, 265
 Newton's method, 160, 252
 Night, temperatures at, 104
 NKS (*A New Kind of Science*), 289
 NLP (natural language processing), 212
NLU (natural language understanding), 211
 Nodes, of graph, 120
 Nonlinear optimization, 296
Normal
 for associations, 206
 on **Dataset**, 274
 Normal-distributed numbers
 (**RandomReal**), 135
NormalDistribution, 134
 NoSQL databases, 282
 Not (!), 165
 Notable person classifier, 267
 Notation, mathematical, 296
 Note, musical (**SoundNote**), 53
 Notebook generation, 300
Notebooks, xiii, 303
 as forms, 225
 as IDE, 288
 in cloud, 264
NotebookTemplate, 259
Nothing, 189
Nouns, 212
Now, 101, 218
 Null list, 18
 Number crunching, 296
 Number of occurrences (**Count**), 16
NumberLinePlot, 12, 279
NumberQ, 168
Numbers
 arithmetic with, 1
 big, 2, 129
 digits of, 16
 from digits (**FromDigits**), 18
 Gaussian distributed
 (**RandomReal**), 135
 in other bases, 18, 134
 list of, 7
 maximum size of, 2
 of letters (**LetterNumber**), 49
 precision of, 134
 random, 3, 130
 random sequences of, 22, 131
 range of, 7
 tests on, 164
NumberString, 259
 Numerical analysis, 134
 Numerical approximation, 129
 Numerics, 296
OAuth, 270
 Obfuscated code, 285, 289
 Object recognition
 (**ImageIdentify**), 123
 Object-oriented programming, 246
 Occurrences (**Count**), 16
 OCR (optical character recognition), 125
Octagon (RegularPolygon), 29
 Octave, 53
OddQ, 164
ODE, 297
 Off-by-one errors, 294
 Offsets, in partitioning, 177
 On-demand computation, 218, 239
 OOP (object-oriented programming), 246
Opacity, 69
 Operations, order of, 2
 Operator forms, 201, 204, 273
 Operators, in mathematics, 204
 Opportunities, with
 programming, 304
 Optical character recognition, 125
 Optimization
 numerical, 296
 of code, 288
 Options, 107
 autocompletion of, 114
 global setting of, 114
 pure functions as settings for, 114

- Options** (list of options), 114
Or
 Boolean (`|`), 165, 298
 for patterns (`|`), 193
Orange, 25
Order of operations, 2
Orderless, as function attribute, 253
OrderlessPatternSequence, 253
 Ostrich, as example of image import, 269
Out, 241
 Outliers, 108
 Output, suppressing, 233
 Output labels (**Out**), 237, 241
Overlaps, 259
 Overlaying plots (**Show**), 140

 Packages, 289, 303
 Paint colors, 28
 Pairs (**Tuples**), 182
PalindromeQ, 169
 Palindromes, in English, 166
 Parallel computation, 301
Parallelogram, as example function, 75
ParallelTable, 301
 Parentheses (`(...)`), 2
 and functional forms, 146
 in logic operations, 168
 Paris, 94
 Parsing (**TextStructure**), 213
Part, 16, 187
Partition, 177, 203
 Parts
 and associations, 209
 of datasets, 271
 of general expressions, 200
 replacing (**ReplacePart**), 189
 Pascal's triangle, 157, 160
 modulo two, 160
 Path
 in graph, 116
 traveling salesman, 298
 with angles (**AnglePath**), 89
 Pattern-based programming, 235
 Patterns, 193, 249
 and function definitions, 243
 conditions on (`/;`), 249
 for strings, 255
 named, 194
PatternSequence, 253
 Pause, in sound, 55
 PDF, 225, 268
Pentagon (**RegularPolygon**), 29
 Perception of color, 28
PerformanceGoal, 288
 Permanence, of names, 234
Permissions, 217
 in cloud, 226
\$Permissions, 226
Permutations, 182
 Persistence, of names, 234
 Phase of moon (**MoonPhase**), 102
 Phone, location of, 97

 Photo mosaic (**ImageCollage**), 40
 Photos
 CurrentImage, 39
 in apps, 223
 Phrase structure, 213
Pi (π), 130
 Piano, 53
Pictogram (**BarChart**), 11
PieChart, 11, 78, 143
 annotation of, 228
 in **Dataset**, 272
 labeled, 228
 Pigment colors, 28
 Pink, hot, 211
 Pink boxes, and graphics
 errors, 291, 294
 Pitch, of musical notes, 54
 Pixel array, 59
 Pixel size, of graphics
 (**ImageSize**), 111
 Place value (**IntegerDigits**), 16
Placed, 231
 Placement, of labels, 231
 Planets, 79
 as example of **Dataset**, 275
 plot of sizes of, 132
PLI (Programmable Linguistic Interface), 214
Plot, of arrays (**ArrayPlot**), 58
Plot, for functions, 295
PlotRange, 108
 Plots
 combining (**Show**), 140
 of colors, 28
 of lists (**ListPlot**), 7
 range of (**PlotRange**), 108
PlotStyle, 137
PlotTheme, 107, 229
Plus, 1, 198
 as a function, 3
 PNG, 225
 importing, 266
Point, 66
 Point size, of text, 27
 Points
 in graphics, 63
 on graph, 120
Polygon, regular
 (**RegularPolygon**), 29
Polygon, 66
 Polyhedron, net of, 83
PolyhedronData, 71
 Polynomial factoring, 296
 Popup menus, in forms, 225
Position, 188
 Positions (coordinates), 65
 Positive part (**Abs**), 132
 Positive sentiment, 123
 Post tag system, 252
 Postfix form (`//`), 141
 Pounds, as weight unit, 88
 Powers of ten, on map, 96
 Powers of two, 154
 Powers of numbers (**Power**), 1
 definition of, 2

 Precedence
 of arithmetic operations, 2
 of logic operations, 168
 Precision, of numbers, 130, 134
 Prefix form (@), 141
 Pressure, from desk sensor, 262
 Previous result (%), 233
 Primality testing (**PrimeQ**), 164
 Primary colors, 28
Prime, 131, 144, 228, 301
PrimePi, 134
PrimeQ, 134, 164, 293, 301
Print (**Echo**), 292
 Printout, 3D, 268
 Probabilities
 in **ImageIdentify**, 127
 of UUID collisions, 225
 Procedural programming, 235
 compared to functional, 175
\$ProcessorCount, 301
 Programmers
 being, 303
 fast intro for, vii
 Programming Lab, Wolfram, xiii–xv
 Programming languages, other, xi, xiv
 Projections, in maps, 96, 98
 Pronunciation, of functions, 4
 Properties, of entities, 77
Property, 120
 Prose, compared to code, 283
 Proxy, for **SendMail**, 270
 ps (external process), 300
 Pseudorandom numbers, 3
 Publishing, web page, 217
 Pure colors, 28
 Pure functions, 147, 171
 and parentheses, 152
 arrow form of, 152
 as composite heads, 201
 nesting of, 153
 versus named function
 definitions, 247
 with explicit variables, 152
Purple, 25
Put, 262
 Put into cloud (**CloudPut**), 261
 Pyramid, Great, 81
 Pythagorean triples, 184
 Python, xi

 Q functions, 164, 168
 QA (quality assurance), 294
 QR codes, 127
 Quadtree, 156
 Quality assurance, 294
Quantity, 87
 Queries, on datasets, 273
 Question functions, 164
 Quotes, to indicate strings, 45
 Quoting (evaluation control), 300

 Radians, 88
 Radio buttons, in forms, 225

- Radio stations, number in countries, 78
- Radius, of circle, 65
- Ragged arrays, 61
- Rainbow (color lists), 26
- Raising to powers (**Power**), 1
definition of, 2
- Random forests machine learning, 127
- Random graph, 118
- Random numbers as coordinates, 63
continuous, 130
grid of, 58
integer, 3, 22
seeding (**SeedRandom**), 36
- Random walk, 155
- RandomChoice**, 155, 182
- RandomColor**, 26, 239
- RandomEntity**, 84
- RandomGraph**, 121
- RandomInteger**, 3
sequences from, 22
- RandomReal**, 130, 240, 297
- RandomSample**, 182
- Range
of map, 99
of plots (**PlotRange**), 108
- Range**, 7
analog for dates, 102
listability of, 146
with negative numbers, 23
with strides, 21
- Rapid prototyping (3D printing), 268
- Rasterize**, 49, 125
data from, 59
- Readability, of code, 285
- Real numbers, 130
- Real-time display, 33
on web, 219
- Real-world data, 77
- RealDigits**, 135
- Reap**, 293
- Recognition
of barcodes, 127
of images (**ImageIdentify**), 123
of text (**TextRecognize**), 125
- Recomputation, 241
avoiding, 288
in Fibonacci algorithm, 287
- Records, in datasets, 275
- Recursion, 155
infinite, 241
versus iteration, 160
- Recursive definition, 244, 284
- Red**, 25, 95, 137
- Red input, 4, 291
- Red-green-blue colors
(**RGBColor**), 25
- Reflected light colors, 28
- RegularExpression**, 259
- RegularPolygon**, 29
and coordinates, 66
- Relational databases, 282
and associations, 209
- ReleaseHold**, 300
- Relief maps, 110, 139
- ReliefPlot**, 139
- Repeat, list elements (**Table**), 19
- RepeatedTiming**, 288
- RepeatingElement**, 226
- Repetition, of functions, 153
- Repetitions
in patterns (..), 250
in string patterns, 256
- ReplaceAll**, 195
- ReplaceList**, 195, 253
- Replacements, 193
in strings (**StringReplace**), 255
- ReplacePart**, 189
- Replicate, list elements (**Table**), 19
- Resizing graphics, 231
- Resolution, of images, 59
- Rest, musical, 55
- Rest**, 18
- RESTful APIs, 226
- Restricted**, 225
- Result, latest (%), 233
- Reversal, of string
(**StringReverse**), 45
- Reverse**, 8, 144, 286
- RGBColor**, 25
- Rhombic hexecontahedron, 83, 268
- rhs (right-hand side), 196
- Riffle**, 181
- Right composition, 280
- RomanNumeral**, 48, 189
- Rotate**, 71, 88, 148
- Rotation, of 3D graphics, 30
- Round**, 132
- Round down (**Floor**), 135
- Round up (**Ceiling**), 135
- Rounded corners, 231
- RoundingRadius**, 231
- Roundoff errors, 134
- Row**, for display, 229
- Rows
in arrays, 188
in datasets, 271
- Rubles, interpreter for, 211
- Rule 30 (cellular automaton), 289, 299
- Rule** (→), 107, 203
as used in options, 114
in **Graph**, 120
in replacements, 194
- RuleDelayed** (↔), 241
- Rules
delayed (↔), 240
lists of and associations, 206
- Run-length encoding (**Split**), 180
- Running time
for bubble sort, 253
for recursive definition, 287
- RunProcess**, 300
- Runs of elements (**Split**), 180
- Russian alphabet, 49
- Sameness testing (==), 169
- Sampled sound, 56
- San Francisco, 167
- Sandboxed code execution, 294
- Saturation, of color, 28
- Save**, 262
- Saving
in binary, 264
to cloud (**CloudSave**), 261
- Scale**, 71
- Scale-free network, 121
- Scientific notation, 130
- Scoping, of variables, 234
- Scoping constructs, patterns as, 196
- Scratch, and **AnglePath**, 91
- Search engine, building a web page like a, 223
- Searching, text files, 259
- Secondary colors, 28
- Sector graph (**PieChart**), 11
- See-through graphics (**Opacity**), 69
- SeedRandom**, 36
- Select**, 164
in datasets, 273
operator form of, 201, 273
- Semantic expression, as interpretation, 214
- Semantic number, as interpretation, 222
- SemanticImport**, 282
- Semantics, 211
- Semicolon (;), at end of line, 233
- Semitones, 53
- Sending mail (**SendMail**), 267
- SendMail**, 267
server for, 270
- Sensors
data from, 261
direct connection to, 270
- Sentence diagramming, 213
- Sentences
lengths of, 140
lists of (**TextSentences**), 47
- Sentiment analysis, 123
- Sequence of numbers (**Range**), 7
- SequenceAlignment**, 259
- SequenceCases**, 259
- Sequences
in lists, 259
of operations (;) 235
of parts, 187
- ServiceConnect**, 269
- Services, accessing external, 267
- Sessions, results in, 233
- Set** (=), 241
- Set complement (**Complement**), 181
- Set intersection (**Intersection**), 181
- Set union (**Union**), 180
- SetDelayed** (:=), 241
- SetOptions**, 114
- Setting names, 233
- Shade
of color (**Hue**), 28
of gray (**GrayLevel**), 28
- Shapes, of plots (**AspectRatio**), 111
- Sharp (#) notes, 55
- Shift** + **[Enter]**, xiii

- Short URL (URLShorten),** 218
Shortest, 255, 259
 Shortest path, in graph, 116
 Shrinkwrapping, 297
 SI units, 90
 Side effects, 152, 237
 Side labels, in plots (**Legended**), 228
 Sierpinski pattern, 179
Sigma (Sum), 296
 Simple programs, 299
 Simplicity of good code, 284
 Simplification, of Boolean functions, 298
Sin, 134, 295
Size
 of big numbers, 2
 of expressions, 204
 of graphics (**ImageSize**), 111
 of text, 27
SK combinators, 252
Sketch (from EdgeDetect), 41
Skull, 83
Slash at (/@), 142, 146
Slash dot (/.), 193
Slash semi (/;), 249
Slash slash (//), 146
Slider, 295
Sliders
 in forms, 225
 in **Manipulate**, 33
 on web, 218
Slot (#), 175
Slots
 in pure functions, 171
 in pure functions (#), 147
 in string templates, 257
Smallest elements
 (**TakeSmallest**), 189
Smallest value (Min), 4
Smart fields, 222
SmoothHistogram, 140
Social networks, 115
SocialMediaData, 267
Software engineering, 288, 303
Solid objects, 69
Solve, 296
Sonification, 53
Sort, 15, 180
 in associations, 205
 of characters, 46
 timing of, 288
SortBy, 180
 operator form of, 274
Sorting algorithm, 250
Sound, 53
 importing, 266
SoundNote, 53
Sow, 293
Spaces
 for multiplication, 2
 in string patterns, 256
Spanish captions, xiii
Spans, in lists, 187
SparseArray, 61
Speak, 56
Speed, of code, 286
Sphere, 30
 and coordinates, 68
Spikey
 3D printout of, 268
 net of, 83
Spiral, from AnglePath, 89
Split, 180
Split strings (StringSplit), 256
Spoken output (Speak), 56
Spreadsheets
 as source for datasets, 282
 importing, 266
SQL databases, 282
 connecting to, 282
Sqrt (square root), 131
Square brackets, 4
 for functions, 3
Square root (Sqrt), 131
 nested, 154
Squares
 generated with **Array**, 171
 of numbers (**Power**), 1
 table of, 20, 283
Squaring, nested, 154
Stacked output (Column), 12
Star
 for multiplication, 2
 in regular expressions, 259
Starry Night, The (painting), 82
Start of list (First), 16
StartOfLine, 259
StartOfString, 259
State capitals, 214
Static typing, 247
Stephen Wolfram
A New Kind of Science
 book by, 289
 as **CurrentImage** example, 39
 as example of entity, 82
 automatic recognition of, 267
 home page, xv
 importing from website of, 266
 stephenwolfram.com, 266
Steps
 debugging in, 294
 in lists, 21
 in procedural programs, 235
STL file, 268
Stock prices, 84
Stopwords, 47
Store in cloud (CloudPut), 261
Stream plot (ListStreamPlot),
 140
Street
 addresses, 212
 maps, 98
Stride, in lists, 21
String templates, 257
StringCases, 255
StringContainsQ, 168
StringJoin, 45, 257
StringLength, 45, 137, 148
StringMatchQ, 256
StringReplace, 255
StringReverse, 45, 166
StringRiffle, 257
Strings, 45
 dates as, 105
 exporting from
 (**ExportString**), 270
 from expressions, 257
 importing from
 (**ImportString**), 270
 joining (**StringJoin**), 45
 natural language
 interpretation of, 211
 overlaps in matching for, 259
 patterns for, 255
 quotes in (**InputForm**), 46
 to lists of letters (**Characters**), 46
 versus symbols, 52
StringSplit, 256, 265
StringTake, 45
StringTemplate, 258
Structs (associations), 208
Structured data, 271
Student data, 275
Style, 49, 95, 148
 for labels, 227
 in document generation, 300
 in plots, 228
 options in, 111
Styles, in forms, 225
Sub, for Part, 191
Subject line, for email, 268
Sublists
 creating, 177
 operations on, 259
Subscript (Part), 16
Subsets, 182
Subtraction (Subtract), 1, 4
Subvalues, 247
Successor function, 154
Suggestions, xiii
Sum, 296
Sum of elements (Total), 15
Sunrise, 103
Sunset, 102
Supervised machine learning, 127
Support vector machines, 127
Suppressing output, 233
SVG, 225
Swapping values, 237
Swatches, of color, 28
Swedish, alphabet, 181
Switzerland
 as example, 79
 graph of bordering countries,
 159
Symbolic
 database generalization, 282
 documents, 300
 expressions, 197
 generalization of types, 247
 images as, 44
 languages, 10, 13, 84, 204
 lists as example, 10
 mathematics, 296
 mixing of types in lists, 13
 representation of infinity, 2
 results, 291
 user interfaces, 295

Symbolically indexed lists
 (associations), 208
 Symbols, 198
 Syntax errors, 4, 291
 Systems operations, 300
Table, 19, 283
 and */@*, 152
 for lists of lists, 57
 in earlier versions, 23
 versus **Array**, 171
 Tabular data, 271
 Tabular output (**Grid**), 57
 Tag cloud (**WordCloud**), 47
 Tag system, 252
Take, 17, 187
 analog for keys (**KeyTake**), 206
TakeLargest, 189
TakeSmallest, 189
 Temperature
 AirTemperatureData, 103
 from desk sensor, 262
 Templates
 for built-in functions, 76
 string, 257
 Testing
 framework for, 294
 importance of, 291
 of larger programs, 303
TestReport, 294
 Tests, 163
 on form fields, 225
Tetrahedron, 71
 Text
 color of (**Style**), 26
 computation with, 45
 rotation of (**Rotate**), 88
 structure of, 213
Text (graphics primitive), 71
 Textbook math, 296
TextCases, 212
TextRecognize, 125
TextSearch, 259
TextSentences, 47
TextString, 257
TextWords, 46
 Theme
 of forms (**FormTheme**), 225
 of graphics (**PlotTheme**), 107
 Theorems, about Wolfram Language, 10
Thick, 95
Thread, 177
 Three-dimensional
 graphics, 30, 68, 138
 in the cloud, 226
 Three-dimensional printing, 268
 Tilde tilde (~~), 259
 Time series, 103
 Time zones, 103
TimeConstrained, 294
 Times, 101
Times, 1, 171
 as a function, 3
TimeSeries, 105
Timing, 286, 288
 Tint, 28
 Tiny, as **ImageSize**, 111
 ToCharacterCode, 52
 Today, 102
 Tokyo, 95
 Tomorrow, 102
 Tooltip, 228
 Topography, 138
Total, 15, 234, 284
 in associations, 205
 in **Dataset**, 272
 ToUpperCase, 45
 TraditionalForm, 296, 298
 Traffic lights, as color example, 27
 Training, in machine learning, 124
 Transition graph, 203
 Translate, 71
 Translation
 of function names, 76
 of words (**WordTranslation**), 214
 Transparency (**Opacity**), 69
 Transpose, 177, 251
 TravelDirections, 99
 TravelDistance, 99
 Traveling salesman problem, 298
 TravelTime, 99
 Tree
 as graph (**KaryTree**), 121
 from **NestGraph**, 158
 TreeForm, 199
 Trees, expressions as, 204
 Triangle (**RegularPolygon**), 31
 Triple at (@@@), 204
 Triple blank (_), 249
 Triple equals (==), 169
 Triples (**Tuples**), 182
 True, 163
 Truth table, 298
 TSP (traveling salesman problem), 298
 Tuples, 182
 Turkish, alphabet, 181
 Turning path (**AnglePath**), 89
 Turtle graphics (**AnglePath**), 91
 Tweet-a-Program, xv
 Types, in Wolfram Language, 246
 Typesetting, 296
 UI construction, 295
 UN website, 265
 UndirectedGraph, 117
 Unequal (\neq), 168
 Unevaluated results, 291
 Unicode, 52
 Union, 180
 Unique identifiers, 225
 Unit tests, 294
 UnitConvert, 87
 for dates, 101
 United Nations website, 265
 United States
 as entity, 77
 map of, 93
 Units, of measure, 87
 Universally unique identifiers, 225
 Universities, interpreter for, 212
 Unknowns, symbols as, 204
 Unprotect, 246
 Unravel (**Flatten**), 178
 Unsupervised machine learning, 127
 UpdateInterval, 219
 Upper case (**ToUpperCase**), 45
 Upvalues, 246–247
 URLShorten, 218
 US units, 90
 User functions, defining, 243
 User ID, 264
 User interface construction, 295
 UUIDs, 225, 261
 format of, 264
 number of, 264
 Validation, of form fields, 225
 Values, in associations, 206
 Van Gogh, painting as example, 82
 Vanish (**Nothing**), 189
 Variables
 global, 234
 in tables, 19
 local, 234
 names of, 20, 23
 scoping of, 234
 Vector plot (**ListStreamPlot**), 140
 Vectors, lists as, 61
 VerificationTest, 294, 303
 Version, of Wolfram Language, xiv, 289
 Version control, 303
 VertexLabels, 115, 158
 VertexStyle, 120
 Vertical output (**Column**), 12
 VerticalGauge, 140
 Vertices, of graphs, 115
 Violin, 54
 Visible quotes (**InputForm**), 46
 Vision, color, 28, 127
 Visualization, 137
 annotation of, 228
 for debugging, 292
 of functions, 295
 of lists, 11
 Voice output, 56
 Volcanoes, 97
 Voxels (**Image3D**), 61
 Wait to evaluate, 239
 Wall clock time
 (**AbsoluteTiming**), 288
 Waveforms, for sound, 56
 Weather, data on, 103
 Web
 interactivity on, 36
 Manipulate on, 36
 network of, 115
 plots themed for, 107
 Web address
 in Wolfram Cloud, 218
 length of, 225
 Web APIs, 226
 Web computation (**Delayed**), 218
 Web crawler, 160, 266
 Web forms, 222

- Web search, vs. Wolfram Knowledgebase, 84
- Webcam, picture from, 39
- Websites
 - creating, 217, 303
 - embedding on (**EmbedCode**), 225
- Weeks, and time computations, 101
- Weight, as example of quantity, 88
- White House
 - images from website of, 269
 - location of, 211
- Whitespace**, 256, 259
- Whole number, random, 3
- Wikipedia, importing images from, 266
- WikipediaData**, 148, 206, 213
 - images from, 44
 - text from, 47
- With**, 291
- wolfr.am, 218
- Wolfram|Alpha, vii
 - comparison with, xi
- Wolfram Cloud, 217
 - and **SendMail**, 270
 - storage in, 261
- Wolfram Cloud app, 219, 226
- Wolfram Community, xv
- Wolfram Connector, 269
- Wolfram Data Drop, 261
- Wolfram Demonstrations Project, xv, 37
- Wolfram Knowledgebase, 77, 265
- Wolfram Language
 - home page, xv
 - metadata about (**WolframLanguageData**), 76
 - nature of, xi
 - scope of, 73
 - time to learn, 76
 - uses of, xi
 - version of, xiv
- Wolfram logo
 - 3D printout of, 268
 - net of, 83
- Wolfram Notebooks, xiii, 303
 - as IDE, 288
 - in cloud, 264
- Wolfram Programming Lab, xiii–xv
- Wolfram Research, xv
- Wolfram Workbench, 288
- Wolfram|Alpha
 - as example of large program, 303
 - building a web page like, 223
 - data vs. Wolfram Language, 84
 - interpreting strings like, 214
 - knowledgebase for, 77
- WordBoundary**, 259
- WordCloud**, 47, 213
 - options for, 112
 - weighting in, 280
- WordDefinition**, 214
- WordList**, 47, 124, 137, 166
- WordOrientation**, 112
- Words
 - graph of nearest, 159
 - lengths of common, 137
 - nearest, 124, 127, 159
- Words for integers (**IntegerName**), 48
- Words in text (**TextWords**), 46
- WordTranslation**, 214
- XLS, importing, 266
- XML, 204
- XMLElement**, 259
- XYZColor**, 28
- Yellow**, 25
- Yellow box, for entities, 77
- Yesterday**, 102
- Zeta** (Riemann zeta function), 134

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Stephen Wolfram is the creator of the Wolfram Language, as well as Wolfram|Alpha and Mathematica. He has been a pioneer in the science and technology of computation for nearly four decades. He started using computers while doing physics research in his early teens, and made many important discoveries that led eventually to his groundbreaking 2002 book *A New Kind of Science*. Since founding it in 1987, Wolfram has been CEO of Wolfram Research, one of the world's most respected software companies, known for its long history of technical innovation.



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All code in this book is available for interactive use in the Wolfram Open Cloud at wolfr.am/eiwl-c, as well as in web and desktop versions of Wolfram Programming Lab. Additional material is available at wolfr.am/eiwl.

