Université Paris Diderot - Paris 7 UFR de Mathématiques
Master première année Janvier 2017
UKMT 42 Francis Comets et Benoît Laslier

Probability and stochastic Processes

We study random sequences with particular dependence structure, mainly martingales and Markov chains. It covers the necessary background to a 2nd year of master with orientation in pure or applied probability and/or mathematical statistics, as well as to "préparation à l'agrégation, option probabilités".

In english. 12 credits.

Organisation:

Course:
Tuesday 12:30–14:30, room 2016
Thursday 16:30-18:30, room 1015

Exercise sessions:
Monday 15:45–17:45, room 1015
Thursday 14:30-16:30, room 1015
Friday 8:00–10:00, room 1015

Midterm exam: March 9, 15:00-18:00 Sophie Germain, 1015

Program:

1- Random variables: law, convergence. Convergence in law, relation to other convergence modes. Characteristic function, Paul Lévy's inversion theorem. Central limit theorem for independent variables (Lyapunov and Lindeberg conditions), multidimensional case. Multivariate Gaussian laws.

2- Conditional expectation. Conditioning by a σ-field. Orthogonal projection in L^2. Extension to L^1. Properties, computations.

3- Martingales. Filtrations, adapted processes. Martingales, super- and sub-martingales. Predictible processes and martingale transform. Doob's decomposition.

4- Martingales and stopping times. Stopping theorems. Applications. Maximal inequalities.

5- Martingale convergence. Upcrossings and downcrossings. Almost sure convergence of sub-martingales bounded in L^1. Convergence theorem in L^2. Random series. Uniform integrability. Uniformly integrable martingales. Doob's inequalities. L^p-convergence. Central limit theorem for martingales.

6- Markov chains with discrete state space. Stochastic matrix, Markov chain. Irreducibility, aperiodicity. Ergodic theorem. Central limit theorem. Coupling and convergence.

7- Complements on Markov chains (if time permits). Entropy, Boltzmann's H theorem. Monté-Carlo Markov Chain (MCMC). Exemples: Metropolis algorithm, simulated annealing. Propp-Wilson algorithm for exact simulation. Random walk on $\mathbb{Z}^d$, discrete Laplacian, Green function.

Bibliography

M. Benaïm, N. El Karoui: Promenade aléatoire. École Polytechnique. 2004
D. Williams: Probability with martingales. Cambridge University Press, 1991

And also:

R. Durrett: Probability: Theory and Examples. Duxbury 1995
G. Grimmett, D. Stirzaker: One thousand exercises in probability. Oxford University Press, 1992
J.-F. Le Gall: "Intégration, Probabilités et Processus Aléatoires"
`https://www.math.u-psud.fr/~jflegall/IPPA2.pdf`
G. Lawler, V. Limic: Random walk: a modern introduction. Cambridge University Press, 2010
J. Norris: Markov chains. Cambridge University Press, 1998

and lecture notes and exercises on Moodle:

`http://moodlesupd.script.univ-paris-diderot.fr/`