

ECE374 Assignment 2

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T2: Proof of regularity

2. Let

$$\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

Consider each row to be a binary number and let

$$C = \{w \in \Sigma^* \mid \text{the bottom row of } w \text{ is three times the top row.}\}$$

For example

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \in C, \text{ but } \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \notin C.$$

Show that C is regular. (Hint, it is easier to look at the matrices in reverse order).

According to Lecture 5, the regularity of C is equal to it exists DFA. And this language can be recognized by a DFA as follows:

Inductive Definition:

1. Start with two states:

- one col for the bottom row being 0 times the top row $c_0 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- for the bottom row being 3 times the top row. $c_1 = \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

2. In the NFA, we use an extra bit to record whether the transition create add-on the current states. The

$$DFA = (Q, \Sigma, \sigma, S, A), \text{ where}$$

$$Q = \{q_0, q_1, q_2, q_3, q_5\} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} \times \{0, 1\} + \text{wrong state}$$

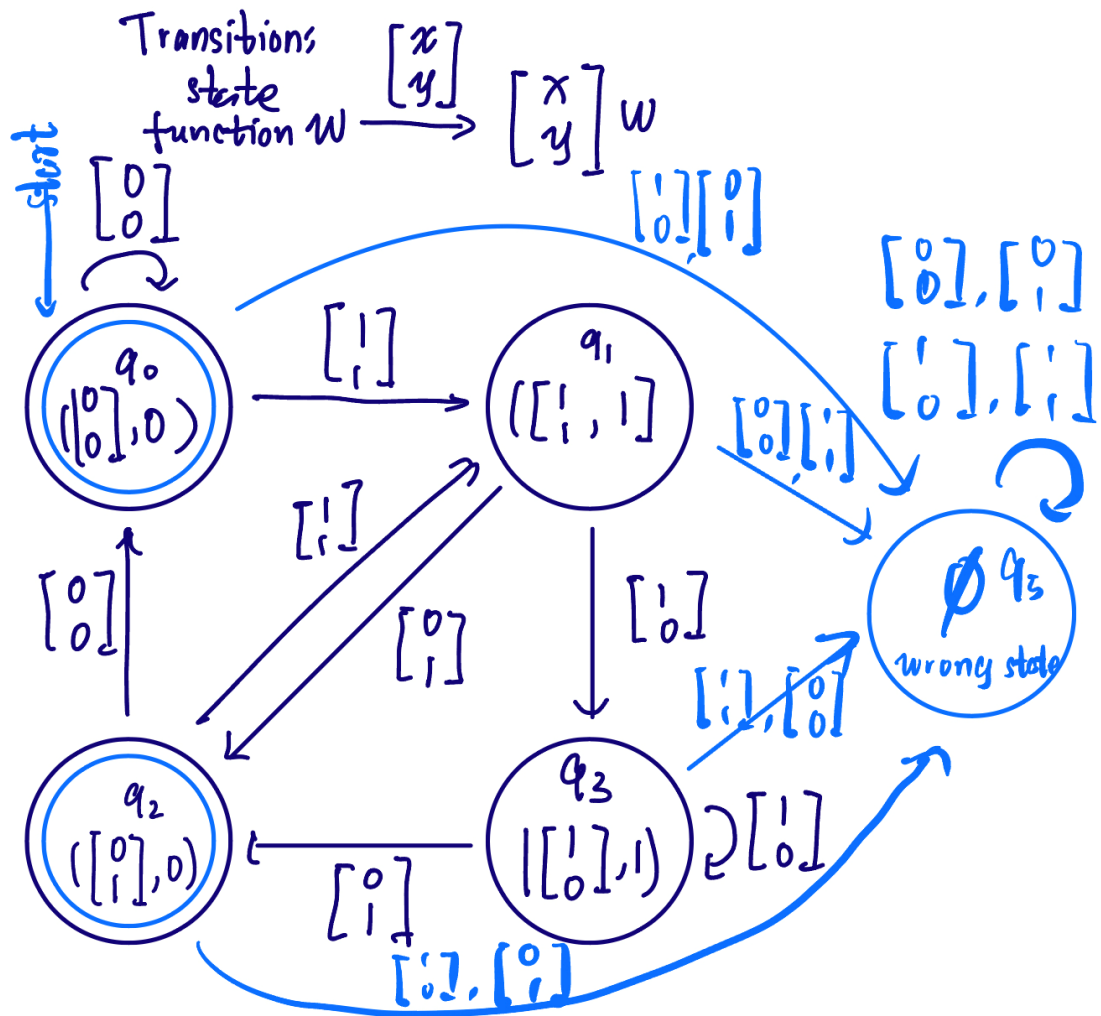
$$\Sigma = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} \quad (1)$$

$$S = q_0 = \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, 0 \right)$$

$$A = \{q_0, q_2\}$$

σ is shown on the picture[1], all the else are ϵ — reach to wrong state.

For each binary digit in the top row, the state changes according to the transition function shown below:



Picture[1] : DFA of the Language C

For instance, the following transition carry on 1 , and further operation can cancel the 1 to 0.

$$\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} \xrightarrow{\begin{bmatrix} 1 \\ 1 \end{bmatrix}} \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} \quad (2)$$

In the DFA, we look at the matrices in reverse order, starting with initial state c_0 , we take a single matrix from \sum , add in front of the "matrix string", judge if there exist add-on.

Therefore, all the cases can be included in our DFA, in other word, there exists regular language C to have bottom row is 3-times upper row. Thus, C is regular and can be recognized by a DFA.