ECE374 Assignment 2

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T3: Proof of closure

3. Let B and C be languages over $\Sigma = \{0, 1\}$. Define:

$$B \xrightarrow{0} C = \{w \in C | \text{ for some } x \in B \text{ strings } w \text{ and } x \text{ contain an equal number of } 0\text{'s}\} \quad (1)$$

Show that the class of regular languages is closed under the $\xrightarrow{0}$ operation.

The class of regular languages is indeed closed under the "$\xrightarrow{0}$ " operation.

- Given a DFA **D1** that recognizes B and a DFA **D2** that recognizes C, we can prove B can be adopted into a NFA **N1** , to include all possible string in **D2**.

Here's how it works:

1. Based on the question, the important thing is number of 0, so **all 1-transition function can be treated as** $\epsilon - reach$, modifying D1 into NFA N1.
2. Then for all string in B1, we have 1 to be 1*. For example,

$$101001 \rightarrow 1^*01^*001^* \quad (1)$$

3. Therefore, the 0's in N1 can be treated as subsequence, and the language C, D2 can be connected to N1. There must exists some x in **N1** can also be recognized as string in C.

According to Piazza#103:

> Existence of a DFA or an NFA for a language L proves that L is regular.
>
> For P3 you need to prove that the class of regular languages is closed under the operation.
>
> The term class here can be understood as a set. People often use terms like class, family, collection, etc to avoid confusion - You wouldn't want to say things like set of sets of sets.

All in all, since B and C both regular, D1 and D2 can be connected using $\epsilon - reach$, transferred as N1 and N2, all string w in N2 can be expressed by some string x in N1.

By following this construction, it can be shown that the language recognized by the new NFA is exactly B $\xrightarrow{0}$ C. Vice versa, C $\xrightarrow{0}$ B also established, this shows that the class of regular languages is closed under the"$\xrightarrow{0}$ "operation.