ECE374 Assignment 3

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T5: Length Limit

5. Prove this language is not regular by providing a fooling set. Be sure to include the fooling set you construct is i) infinite and ii) a valid fooling set.

$$L_{P5} = \{w|w \text{ such that } |w| = \lceil k\sqrt{k}\rceil, \text{ for some natural number} k\}$$

Hint: since this one is more difficult, we'll even give you a fooling set that works: try $F = \{0^{m^6}|m \geq 1\}$. We'll also provide a bound that can help: the difference between consecutive strings in the language, $\lceil(k+1)^{1.5}\rceil - \lceil k^{1.5}\rceil$, is bounded above and below as follows

$$1.5\sqrt{k} - 1 \leq \lceil(k+1)^{1.5}\rceil - \lceil k^{1.5}\rceil \leq 1.5\sqrt{k} + 3$$

All that's left is you need to carefully prove that F is a fooling set for L.

Solution:

We have fooling set $F = \{0^{m^6}|m \geq 1\}$

Then for string w1 and w2 , let their length

$$\begin{gathered} |w1| = \ulcorner k^{1.5}\urcorner, k \in \{0,1,2,3,\dots\} \\ |w2| = \ulcorner(k+1)^{1.5}\urcorner, k \in \{0,1,2,3,\dots\} \\ According\ to\ hint, \\ 1.5\sqrt{k} - 1 \leq |w_2| - |w_1| = difference \leq 1.5\sqrt{k} + 3 \end{gathered} \tag{1}$$

To prove the Fooling set is valid, we Let

$$\begin{gathered} x = 0^{m^6}, y = 0^{n^6}, 1 \leq m < n \\ |0^{m^6}| = m^6, let\ k = m^4, m^6 = k\sqrt{k} \end{gathered} \tag{2}$$

Then, take **z to be a smallest string such that:**

$xz \in L, |xz| - |x| = |z| \leq 1.5\sqrt{m^4} + 3$ ***(First)***

To prove the F is valid, we assume $yz \in L, |yz| - |y| = |z| \geq 1.5\sqrt{n^4} + 1$ ***(Second)***

Then, let us use scaling method to prove ***(Second)*** contradicts with ***(First)*** :

$$
\begin{aligned}
\because m < n, \therefore m \leq n+1 \\
\rightarrow |z| \leq 1.5\sqrt{m^4}+3 \leq 1.5(n+1)^2+3 \\
= 1.5n^2 - 3n + 4.5 = 1.5n^2 - 1 + (5.5 - 3n) \leq 1.5n^2 - 1
\end{aligned} \tag{3}
$$

Therefore,$\rightarrow |z| < 1.5n^2 - 1$, which contradicts with ***(Second)***
Then we can say the assumption is wrong,

- there are infinite different x and y,

- Since the size of fooling set F is infinite. However, we need finite states to describe the DFA,
- So the language is not regular.

Q.E.D