ECE374 Assignment 2

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Group & netid

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T2: Proof of regularity

2. Let

$$\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

Consider each row to be a binary number and let

$$C = \{w \in \Sigma^* \mid \text{the bottom row of } w \text{ is three times the top row.}\}$$

For example

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} \in C, \text{ but } \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \notin C.$$

Show that C is regular. (Hint, it is easier to to look at the matrices in reverse order).

According to Lecture 5, the regularity of C is equal to it exists DFA. And this language can be recognized by a DFA as follows:

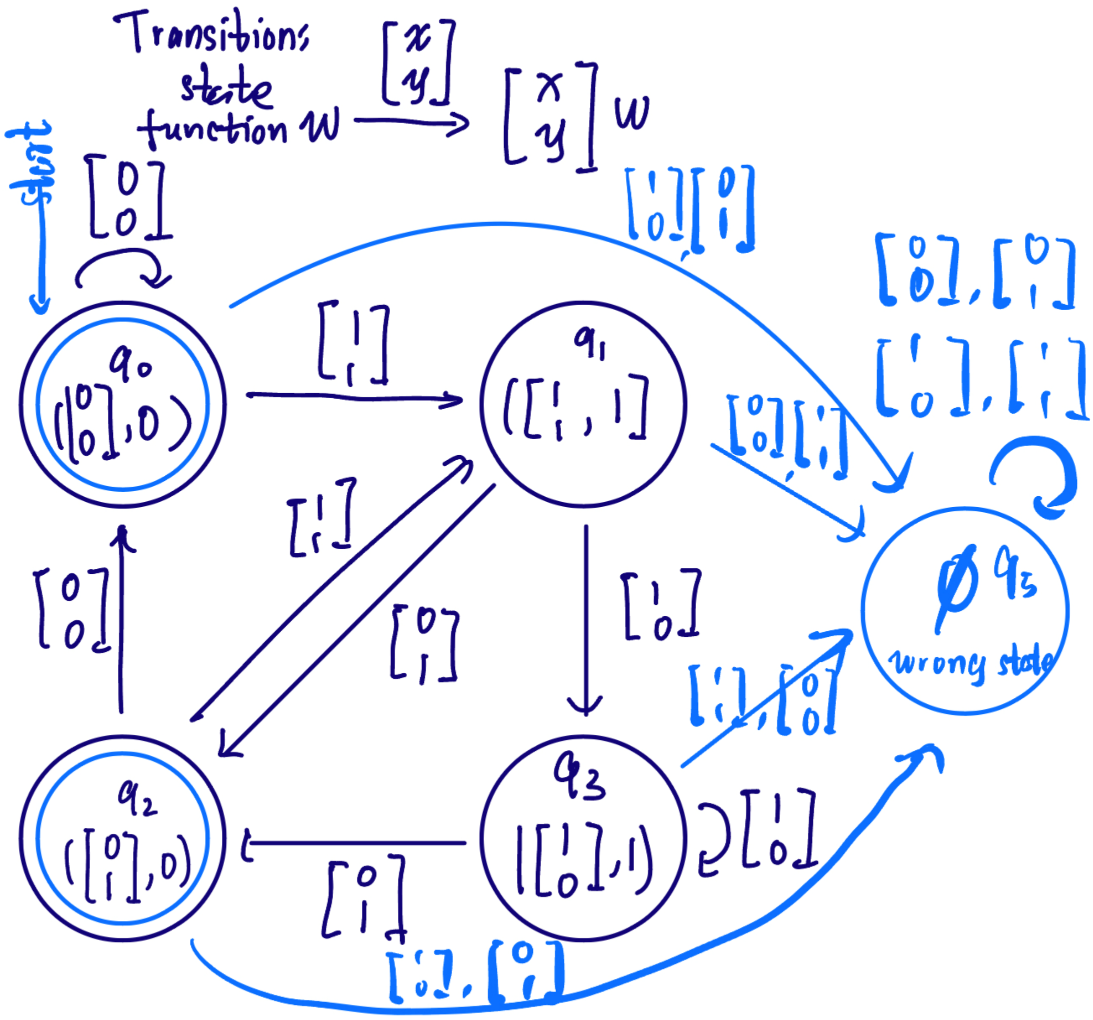
Inductive Definition:

1. Start with two states:
   - one col for the bottom row being 0 times the top row $c_0 = \{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\}$
   - for the bottom row being 3 times the top row. $c_1 = \{\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\}$
2. In the NFA, we use an extra bit to record whether the transition create add-on the current states. The

$$\begin{gathered} DFA = (Q, \sum, \sigma, S, A), where \\ Q = \{q0, q1, q2, q3, q5\} \\ \sum = \{\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}\} \\ S = q0 = (\begin{bmatrix} 0 \\ 0 \end{bmatrix}, 0) \\ A = \{q0, q2\} \end{gathered} \tag{1}$$

$\sigma\ is\ shown\ on\ the\ picture[1]$, all the else are $\epsilon - reach$ to wrong state.

For each binary digit in the top row, the state changes according to the transition function shown below:



Picture[1] : DFA of the Language C

For instance, the following transition carry on 1 , and further operation can cancel the 1 to 0.

$$\{\begin{bmatrix}0\\0\end{bmatrix}\} \xrightarrow{\begin{bmatrix}1\\1\end{bmatrix}} \{\begin{bmatrix}1\\1\end{bmatrix} \begin{bmatrix}0\\0\end{bmatrix}\} \tag{2}$$

In the DFA, we look at the matrices in reverse order, starting with initial state c0, we take a single matrix from $\sum$, add in front of the "matrix string", judge if there exist add-on.

Therefore, all the cases can be included in our DFA, in other word, there exists regular language C to have bottom row is 3-times upper row. Thus, C is regular and can be recognized by a DFA.