ECE374 Assignment 3

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Group & netid

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Problem 4

4. An all-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if **every** possible state that M could be in after reading input x is a state from F. Note, this is in contrast to an ordinary NFA that accepts a string if some state among these possible states is a an accept state. Prove that all-NFAs recognize the class of regular languages.

Solution:

To prove that all-NFAs recognize the class of regular languages, we turn this statement into two statements:

1. ***all-NFA can accept all regular languages.***
2. ***if language belongs to all-NFA, then it is regular.***

I. regular DFA -> all-NFA

These can be proved easily, because for a DFA D_1, we have the following definition:

DFA $D_1 = (\Sigma, Q, \delta, s, A)$

in which,

Q is the set of states,

Σ is the set of input symbols in this language,

δ is the set of all transitions,

s is the starting state,

A is the set of accepting states.

Since every regular language has a DFA, which can be transformed into NFA, and by the definition of all-NFA, this NFA must be an all-NFA,

Then, we could determine that $L(\mathrm{D}_1) = L(all - DFA)$ is regular, that is, <u>given a regular language that is represented with a DFA, we could prove that it's all-NFA</u> i.e. all-NFA accepts regular languages.

II. $\boldsymbol{all\text{-}NFA \rightarrow DFA, is\ regular}$

Reversely thinking, we could also transform an arbitrary all-NFA language into the form of a DFA with the following method:

$$M = (Q, \Sigma, \delta, q_0, F)$$

Given an arbitrary regular language L in the form of M

We have

Σ is the set of input symbols in this languae,

Q is the set of states,

q_0 is the starting state,

F is the set of all states transferring to acceptable state,

δ is the set of all transitions, where:
$\delta(q_i, a) = q_f, q_i, q_f \in Q, a \in \Sigma$, which indicates that M takes in a symbol a at initial state q_i and transitions to the next final state q_f.
We could construct a DFA $= (\Sigma', Q', \delta', s', A')$ based on M that:
$Q' = P(Q) \cup \emptyset$ is the set of states in the all-NFA, P(Q) is a power set of Q, Ø is the empty set (the rejecting state),
$\Sigma' = \Sigma$ is the set of input symbols,
$s' = \{s\}$ is the starting state,
$A' = S$ where all $s \in A$ for all $s \in S$
δ'=
For $s \in S, s, q \in Q, a \in \Sigma$
(1) $\delta'(q_i, a) = \emptyset$ for some $s\ in\ Q$**, because the original all-NFA may not fit in every transition. So , we deal with those ϵ-reach paths by simply rejecting them into Ø**
(2) $\delta'(q_i, a) = q |\ q \in Q, q \in E(\delta(s, a)) for\ any\ s \in S$

Therefore, we could determine that $L(M) = L(\text{all\_NFA})$ is regular, that is, <u>given an arbitrary all-NFA, we can turn it into DFA and prove its regularity.</u>

In a nutshell, we could prove that ***all-NFA*** recognize the class of regular languages.