ECE374 Assignment 5

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Group & netid

Chen Si *chensi3*

Jie Wang *jiew5*

Shitian Yang *sy39*

Problem 1

1. Suppose we have a river and on either side are a number of cities numbered from 1 to n (North side: $N[1 \ldots n]$, South side: $S[1 \ldots n]$). The city planner wants to connect certain cities together using bridges and has a list of the desired crossings (x is a $2 \times k$ array where k is the number of planned bridges) . Unfortunately, as we know, bridges cannot cross one-another over water so the city planner must focus on building the most bridges from his plan that do not intersect. Describe an algorithm that finds the maximum number of non-intersecting bridges.

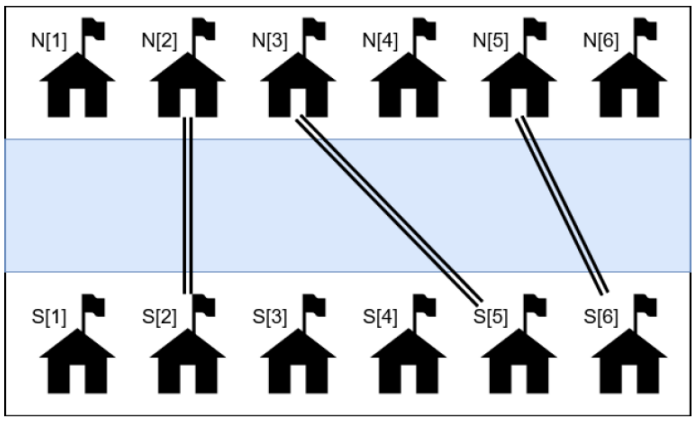


Figure 1. Assuming $n = 6$, $x = \begin{bmatrix} 1 & 5 & 6 & 2 & 3 \\ 4 & 6 & 1 & 2 & 5 \end{bmatrix}$, then the output should be 3 as shown above.

Solution:

Intuition:

(1) We could solve this problem by the following steps:

i. sort x by the first row. e.g. $x \rightarrow \begin{bmatrix} 1 & 2 & 3 & 5 & 6 \\ 4 & 2 & 5 & 6 & 1 \end{bmatrix}$

ii. then, to avoid bridge crossing, we build the bridges from left to right, and each column of a bridge must have the second-row number larger than the last one before building. Therefore, we could find the number of the longest increasing subsequence (LIS) of the second row with

backtracking skills that would be the max number of bridges if we consider in the first-row perspective.

(2) In the LIS algorithm we have the recurrence relation as

$$LIS(i,j) = \begin{cases} 0, & if\ i > n \\ LIS(i+1,j), & if\ i \le n\ and\ A[j] \ge A[i] \\ \max\big(LIS(i+1,j), 1 + LIS(i+1,i)\big), & otherwise \end{cases}$$

(3) Therefore, we could use a n×(n+1) table A to store the intermediate values, with form

		A[1] = 6 1	A[2] = 3 2	A[3]=5 3	A[4]=2 4	A[5]=7 5	A[6]=8 6	A[7]=1 7	inf 8	Represents limiter j
[]	0	0	0	0	0	0	0	0	0	
[6]	1		0	0	0	1	1	0	1	
[6,3]	2			1	0	1	1	0	1	
[6,3,5]	3				0	2	2	0	2	
[6,3,5,2]	4					2	2	0	2	
[6,3,5,2,7]	5						3	0	3	
[6,3,5,2,7,8]	6							0	4	
[6,3,5,2,7,8,1]	7								4	
Represents sub-array	i									

, and the value that we would like to find as result is A[n, n+1].

(4) As LIS algorithm with dynamic programming has run time O(n^2), our algorithm is O(logn + n^2) = O(n^2).

Therefore, we could describe the algorithm as

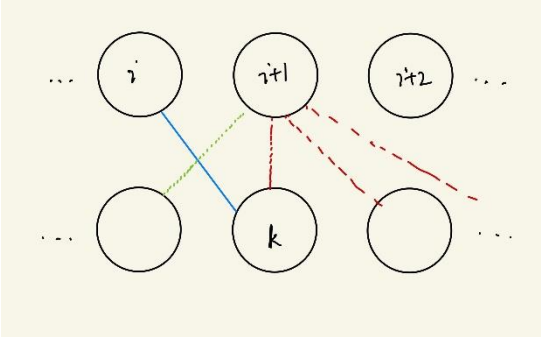
```
BuildBridge(x[1…N,1…N],n):
    sortedMatrix = sort(x, row=1)
    count = LIS(sortedMatrix[2,:])

    return count
```

Some Explanations:

1. Why soring the rows at first?
Answer: Sorting the rows would allow us to determine which next bridge are we discussing on whether we should add it or not in the dynamic programming process, enabling a “sequence” to appear within a set of unordered bridges.

2. Why using LIS?
Answer:



When building bridges, take the left chart as an example. If we previously built a bridge linking the ith node to the kth node (shown in blue line), we then could only build bridges that starts from node i+1 and goes to nodes k and nodes to the right of k (node >= k), as shown with red lines. We can't connect node i+1 with any node less than k below, as shown with green lines. Therefore, we are actually requiring a non-decreasing subsequence in the row below, and the maximum number of bridges is the length of this non-decreasing subsequence.