

# ECE 459 Project Report

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## Project#4: SSB Modulation and Demodulation

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# 1 Introduction

In the realm of communication systems, Single Sideband (SSB) modulation represents a crucial advancement that addresses some of the key limitations of Amplitude Modulation (AM). While Double-Sideband Suppressed Carrier (DSB-SC) modulation mitigates the inefficiency of AM due to the transmission of the carrier signal, SSB modulation goes a step further by addressing the issue of channel bandwidth. This innovative modulation technique achieves this by suppressing one of the two sidebands generated in DSB-SC modulation, thus significantly improving spectral efficiency.

SSB modulation relies exclusively on either the lower sideband (LSB) or the upper sideband (USB) to convey the message signal across a communication channel. Depending on which sideband is transmitted, we distinguish between lower SSB and upper SSB modulation. This selective transmission of a single sideband is achieved through various methods, such as frequency filtering or phase manipulation.

This project is aimed at implementing the SSB modulation and demodulation, showing its performance of fully utilizing the bandwidth. We will transform two message signal into high frequency domain with same carrier frequency, each utilizing the lower and upper sideband. To make the result more vivid, the amplitude, bandwidth and duration of two signal is different.

## 1.1 Experiment Setup

To gain a practical implementation of SSB modulation and demodulation, we need to set up a basic foundation that simulates the transmission and reception of a message signal using SSB techniques. The following components are required for this experiment:

1. **Time vector:** The Python simulation is based on discrete signal processing, where we set time vector as:

$$fs = 500; t = np.linspace(-100, 100, 200 * fs)$$

2. **Triangle Wave for LSB bandwidth:** A source of a triangle wave, which will occupy the bandwidth associated with the lower sideband.

$$x_m(t) = A_m \cdot \text{tri}\left(\frac{t}{T}\right)$$

Where:

- $A_m$  is the amplitude, we set as 1.
- $T$  is the duration, we set as 0.5.
- $\text{tri}(t)$  is the triangular function, defined as:

$$\text{tri}(t) = \begin{cases} 1 - \frac{|t|}{T}, & \text{if } |t| \leq T \\ 0, & \text{otherwise} \end{cases}$$

3. **Rectangular Wave for USB bandwidth:** A waveform source that occupies the bandwidth associated with the upper sideband. For the sake of comparative analysis and as a reference during the experiment, we set it as double size of triangle Wave.

$$x_m(t) = A_m \cdot \text{rect}\left(\frac{t}{T}\right)$$

Where:

- $A_m$  is the amplitude, we set as 2.
- $T$  is the duration, we set as 1.

- $\text{rect}(t)$  is the rectangular function, defined as:

$$\text{rect}(t) = \begin{cases} 1, & \text{if } |t| \leq \frac{T}{2} \\ 0, & \text{otherwise} \end{cases}$$

4. **Carrier Wave with Sufficient Frequency  $f_c$ :** A carrier signal with a frequency high enough to accommodate the message signal.

$$x_c(t) = A_c \cdot \cos(2\pi f_c t)$$

Where:

- $A_c$  is the amplitude, we set as 1.
- $f_c$  is the frequency, we set as 1000Hz

## 2 Methodology

### 2.1 SSB Modulation

#### 2.1.1 Modulator Parameters

- **carrier frequency:**  $f_c = 1000\text{Hz}$
- **duration:**  $t = 1\text{s}$
- **amplitude:**  $A = 1$
- **sampling rate:**  $f_s = 1000\text{Hz}$ . For the digital representation, it should be at least twice the maximum frequency present in the message signal to satisfy the Nyquist criterion.
- **sideband selection:** Two band-pass filters, one for USB, the other for LSB.
- **filter cutoff frequency:** We set the ideal filter in this experiment.

To generate an SSB signal, the following steps are typically employed:

- **Modulation:** The message signal  $m(t)$  is first used to modulate a carrier wave  $A_c \cos(2\pi f_c t)$ , resulting in a standard amplitude-modulated (AM) signal with upper and lower sidebands.
- **Sideband Separation:** The AM signal is then passed through a filter, which can be a Hilbert transform filter, a phasing filter, or a mechanical filter, to separate the desired sideband from the undesired one.
- **Carrier Suppression:** The carrier frequency component is usually suppressed or reduced to conserve power and reduce bandwidth.

#### 2.1.2 Mathematical Representation

The mathematical representation of an SSB signal can be expressed by:

$$s_{\text{SSB}}(t) = m(t) \cdot \cos(2\pi f_c t) \pm \hat{m}(t) \cdot \sin(2\pi f_c t) \quad (1)$$

where  $\hat{m}(t)$  is the Hilbert transform of the message signal  $m(t)$ , and the  $\pm$  sign corresponds to choosing either the USB or LSB.

#### 2.1.3 Band-pass Filtering Design

The design of the band-pass filter is critical in defining the quality of the SSB signal. The filter must have sharp cutoffs to ensure that only one sideband is passed while the other is attenuated, along with the carrier suppression.

## 2.2 SSB Demodulation

### 2.2.1 Demodulator Parameters

- **carrier frequency:**  $f_c = 1000\text{Hz}$
- **duration:**  $t = 1\text{s}$
- **amplitude:**  $A = 1$
- **sampling rate:**  $f_s = 1000\text{Hz}$

### 2.2.2 Coherent Detection

The received SSB signal  $s(t)$  is mixed with a coherent carrier wave  $c(t) = \cos(2\pi f_c t)$ , where  $f_c$  is the carrier frequency. The mixing operation can be represented as:

$$s_{\text{mixed}}(t) = s(t) \cdot c(t) \quad (2)$$

This mixing results in the sum and difference of the frequencies in  $s(t)$  and  $c(t)$ . For frequencies in  $s(t)$  around  $f_c$ , this results in frequency components around the baseband (0 Hz) and around  $2f_c$ .

### 2.2.3 Low-Pass Filtering Design

A low-pass filter with a cutoff frequency  $f_{\text{cutoff}}$  is applied to  $s_{\text{mixed}}(t)$ . The output  $y(t)$  is the convolution of  $s_{\text{mixed}}(t)$  with the filter's impulse response  $h(t)$ :

$$y(t) = \int_{-\infty}^{\infty} s_{\text{mixed}}(\tau) \cdot h(t - \tau) d\tau \quad (3)$$

The low-pass filter effectively removes the components around  $2f_c$ , retaining only the baseband frequencies which constitute the original information signals.

### 3 Result

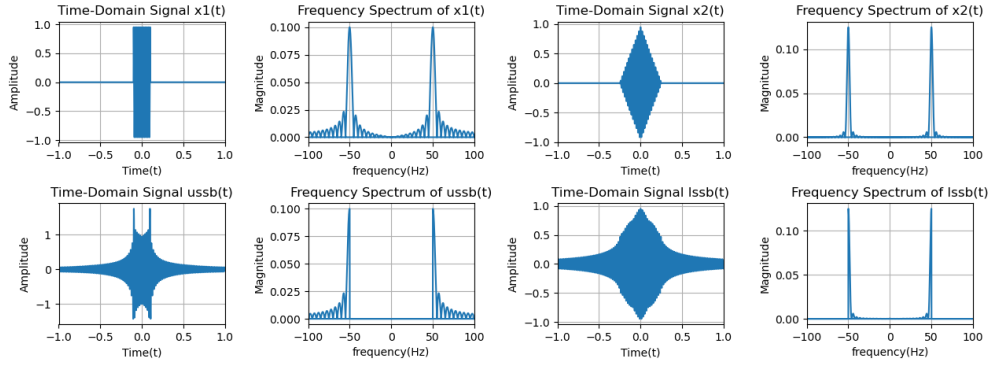


Figure 1: SSB Modulation

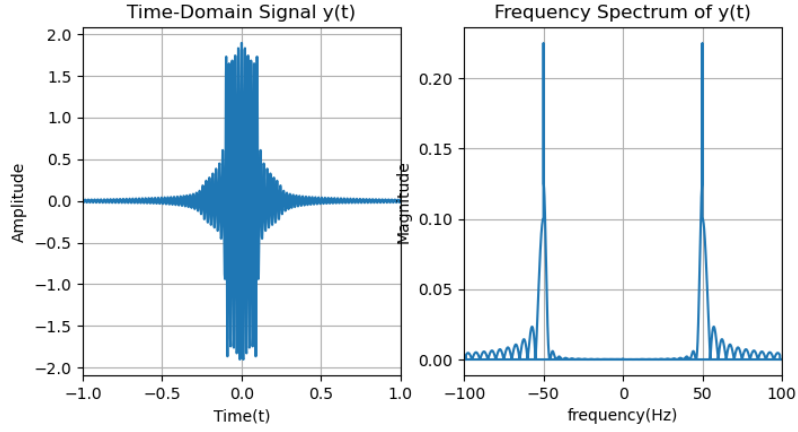


Figure 2: Combined SSB in frequency domain and time domain

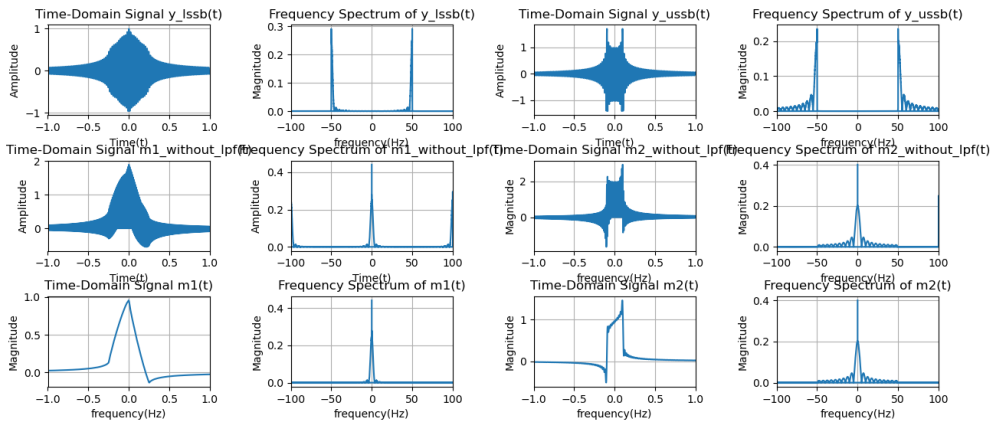


Figure 3: SSB Demodulation

## 4 Discussion

### 4.1 Bandwidth Computation

To compute the bandwidth of message signal, it is hard to use Carson's rule when the frequency of message signal and carrier signal is relative close. Therefore, we choose to directly compute the 99.9% of message energy .

### 4.2 Choice of carrier frequency

In coherent detection, the demodulation process requires a carrier wave that is perfectly aligned in frequency and phase with the carrier used during modulation. Any mismatch in frequency or phase can lead to incomplete or incorrect demodulation. To address this, a phase-locked loop (PLL) can be implemented to ensure that the carrier used in demodulation is synchronized with the received signal. A PLL can track and adjust the phase and frequency of the local oscillator to match the carrier signal of the received SSB signal.

### 4.3 Design of low-pass filter

We have tried using *scipy.signal.lfilter* to demodulate the SSB signal. However, we find it very hard to obtain the original rectangular signal, the distortion is very severe.

We think it might be because of the Filter Edges: If the band-pass filter has very sharp cutoff edges or a non-linear phase response, it can introduce ringing artifacts due to the Gibbs phenomenon. Although we can use a filter with smoother transition bands and a linear phase response to minimize distortion, we decide to use ideal HPF as following:

The low-pass filter's design and implementation are crucial in the demodulation process. An inappropriate cutoff frequency or filter order can lead to either insufficient filtering of high-frequency components or excessive attenuation of the signal's components. Careful design of the filter parameters is necessary. A Butterworth filter is often used for its flat response in the passband. The cutoff frequency should be chosen just above the highest frequency component of the information signal. Additionally, using digital filtering techniques like *filtfilt* in Python can help in minimizing phase distortions.

## 5 Conclusion

### 5.1 Summary

In this project, we successfully implemented a Single Sideband (SSB) modulator and demodulator in Python to process two distinct time-domain signals: a rectangular pulse and a triangular pulse. Utilizing the principles of SSB communication, the rectangular pulse was modulated onto the Upper Side Band (USB) and the triangular pulse onto the Lower Side Band (LSB). The modulation involved the generation of signals, their transformation using the Hilbert transform, and subsequent combination after modulation. For demodulation, coherent detection techniques were employed to accurately recover the original signals. The effectiveness of the system was validated by generating and analyzing both time-domain and frequency-domain plots, which confirmed the successful transmission and recovery of the original signals, thus demonstrating a practical application of SSB techniques in digital signal processing.

### 5.2 Application

The conclusion of this project can be applied to various fields:

1. **Long-Distance Radio Communication:** SSB modulation remains widely used in global shortwave radio communications, including international broadcasting and shortwave radio stations. It excels in combating various noise and interference in radio transmission, particularly suitable for long-distance and international communications. Additionally, it serves as a vital choice when addressing spectrum congestion and limited frequency resources.

2. **Amateur Radio:** Amateur radio enthusiasts extensively utilize SSB modulation worldwide for voice and data communication. In modern contexts, it serves as a means of communication within the amateur radio community and is used for emergency communication and participation in various amateur radio contests.
3. **Military Communications:** Military organizations continue to rely on SSB modulation for secure and reliable long-range communication. It offers high security and interference resistance, commonly used for internal military communication needs, joint military operations, and strategic communication.
4. **Aerospace Communication:** In the aviation and aerospace industries, SSB modulation supports air traffic control, inter-aircraft communication, and signal transmission in space exploration missions. It's extensively employed for voice communication and data transmission between aircraft and ground control centers.
5. **Emergency Communication:** During natural disasters and emergencies, SSB modulation serves as a valuable alternative communication method. It provides a reliable means of communication when traditional communication infrastructure is damaged or fails, supporting rescue and emergency response efforts.

These five modern applications highlight the significance and widespread use of SSB modulation across diverse domains. They demonstrate the efficiency and reliability of SSB modulation in specific scenarios, offering crucial support for modern communication and emergency response needs.

## 6 Appendix

### 6.1 References

- [ 1 ] Simon Haykin & Michel Mother. an-introduction-to-analog-and-digital-communications. 2nd-edition
- [ 2 ] Wikipedia. SSB. [https://en.wikipedia.org/wiki/Single-sideband\\_modulation](https://en.wikipedia.org/wiki/Single-sideband_modulation)

### 6.2 Python Realization

The experiment is recorded in the attached Jupyter notebook in an interactive way.

### 6.3 Used Functions Documentation

- **numpy.linspace():** Generates evenly spaced numbers over a specified interval.
- **numpy.where():** Returns elements based on conditional expression.
- **numpy.fft.fft():** Computes the one-dimensional n-point discrete Fourier Transform.
- **numpy.fft.fftfreq():** Returns the discrete Fourier Transform sample frequencies.
- **numpy.fft.ifft():** compute the inverse discrete Fourier Transform.
- **numpy.logical\_and():** used as the filter in frequency domain.

### 6.4 Task Separation

**Task 1: Modulation:** By Jie Wang

1. All the task that is related to modulation

**Task 2: Demodulation:** By Junhao Zhu

1. All the task that is related to demodulation