ECE 459 Project Report

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Project#6: FM Demodulator

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1 Introduction

In modern communication systems, Frequency Modulation (FM) is a pivotal technique for information transmission. Following Project 5's experiment on FM signal generation, this project deal with FM demodulation. Our goal is to implement a FM receiver, which uses an idealized frequency differentiator and envelope detector for extracting the original message from the modulated carrier wave.

1.1 Problem Statement

> Based on the FM signal generated in Project 5, design and implement and FM receiver (demodulator) using Python. Your demodulator should use an idealized frequency differentiator followed by envelop detector. Compare the reconstructed signal with the original signal (message). Also, compare with the exact theoretical solution obtained in class and comment on your results.

1.2 Objective

- Construct a Python FM demodulator in both theoretical and practical principles
- Evaluate their performance in terms of accuracy and fidelity in recovering the transmitted message.

1.3 Characteristic

The FM receiver will be characterized by two main components, each mathematically defined as follows:

- An **ideal frequency differentiator**, which computes the instantaneous frequency deviation $\Delta f(t)$ from the FM signal $s_{FM}(t)$ as the derivative:

$$\Delta f(t) = \frac{d\phi(t)}{dt} = \frac{1}{2\pi}\frac{d}{dt}\arg\{s_{FM}(t)\}$$

  where $\phi(t)$ is the instantaneous phase of the FM signal.

- An **envelope detector**, which recovers the message signal $m(t)$ from the amplitude of the differentiated FM signal $\Delta f(t)$ using the relation:

$$m(t) = A_{env}\left\{\sqrt{\Delta f(t)^2 + \beta^2}\right\} - \beta$$

  where $A_{env}\{\}$ denotes the envelope operation and β is a constant related to the modulation index.

2 Methodology

2.1 Idealized Frequency Differentiator

2.1.1 Frequency Differentiation in the Frequency Domain

The idealized frequency differentiator is employed to determine the instantaneous frequency of a FM signal. This is achieved by performing operations in the frequency domain and then transforming the result back into the time domain.

- **Fourier Transform**: The FM signal $s(t)$ is initially transformed into the frequency domain using the Fast Fourier Transform (FFT). The resulting frequency spectrum $X(f)$ represents the signal in terms of its frequency components. The FFT is mathematically expressed as:

$$X(f) = \text{FFT}\{s(t)\} \tag{1}$$

This transformation is crucial for facilitating the subsequent frequency domain differentiation.

- **Differentiation**: Differentiation in the frequency domain is achieved by multiplying the spectrum $X(f)$ by $j2\pi f$. Here, j denotes the imaginary unit, making the product involve complex arithmetic. The frequency f scales the spectrum, effectively differentiating the original signal $s(t)$ in the time domain. The differentiation is given by:

$$Y(f) = j2\pi f \cdot X(f) \tag{2}$$

This step is fundamental in highlighting the frequency variations in the FM signal, which are directly related to the modulating message signal.

- **Inverse Fourier Transform**: To convert the differentiated spectrum back to the time domain, the Inverse Fast Fourier Transform (IFFT) is used. The IFFT applied to $Y(f)$ yields the time-domain signal $y(t)$, which is the differentiated version of the original FM signal:

$$y(t) = \text{IFFT}\{Y(f)\} \tag{3}$$

This signal now contains the key information required for envelope detection.

2.1.2 Modulator Parameters

Parameter	**Value**
Carrier frequency, f_c	10 Hz
Duration of rect, T	1 s
Amplitude of carrier, A_c	1
Frequency-sensitivity factor, k_f	10
Sampling rate, F_s	2000 Hz
Amplitude of $m(t)$, A_m	1

Table 1: FM Modulation Parameters

2.2 Envelope Detector

2.2.1 Brief introduction for this method

The envelope detector is a circuit or algorithm used to extract the envelope portion from a modulated signal. Here is a brief overview of the envelope detector method:

1. Rectification: Convert the signal into a single-sided (non-negative) signal.

2. Envelope Extraction: The envelope represents the slow variations or amplitude changes of the signal, ignoring the high-frequency details.

3. Filtering: Apply low-pass filtering to the rectified signal to remove high-frequency noise or rapid changes, retaining the slow variations of the signal.

4. Output: The extracted envelope signal is then used to recover the amplitude variations of the original modulated signal, thus achieving demodulation.

In summary, the envelope detector typically involves rectification followed by envelope extraction and filtering to obtain the envelope of the signal.

2.2.2 The formula of the method in practical application

The frequency-modulated wave in this project can be defined as:

$$s(t) = A_c \cdot \cos\left(2\pi f_c t + 2\pi k \int_0^t m(\tau)\, d\tau\right)$$

The derivative equation for the modulated signal is represented as:

$$\frac{ds(t)}{dt} = -A_c \cdot [2\pi f_c + 2\pi k \cdot m(t)] \cdot \sin\left(2\pi f_c t + 2\pi k \int_0^t m(\tau)\, d\tau\right)$$

And for the envelop detector method, we define a signal as:

$$y(t) = A_c \cdot [2\pi f_c + 2\pi k f_m(t)]$$

2.3 Result

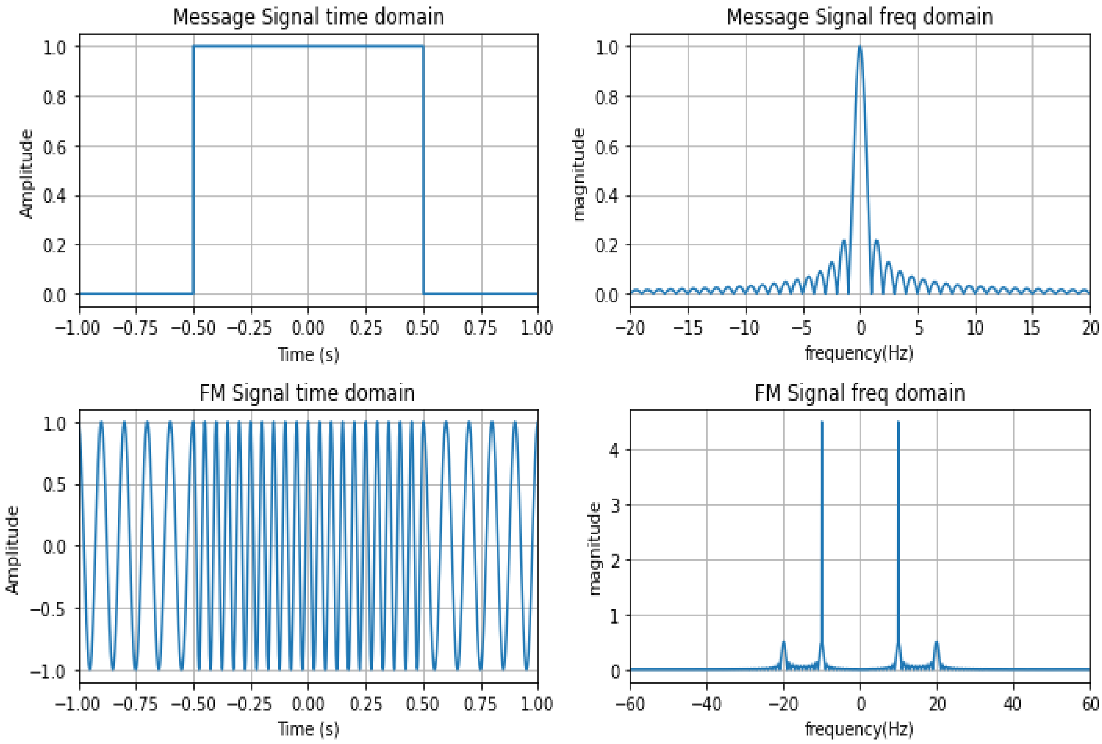

Figure 1: Generation of FM signal

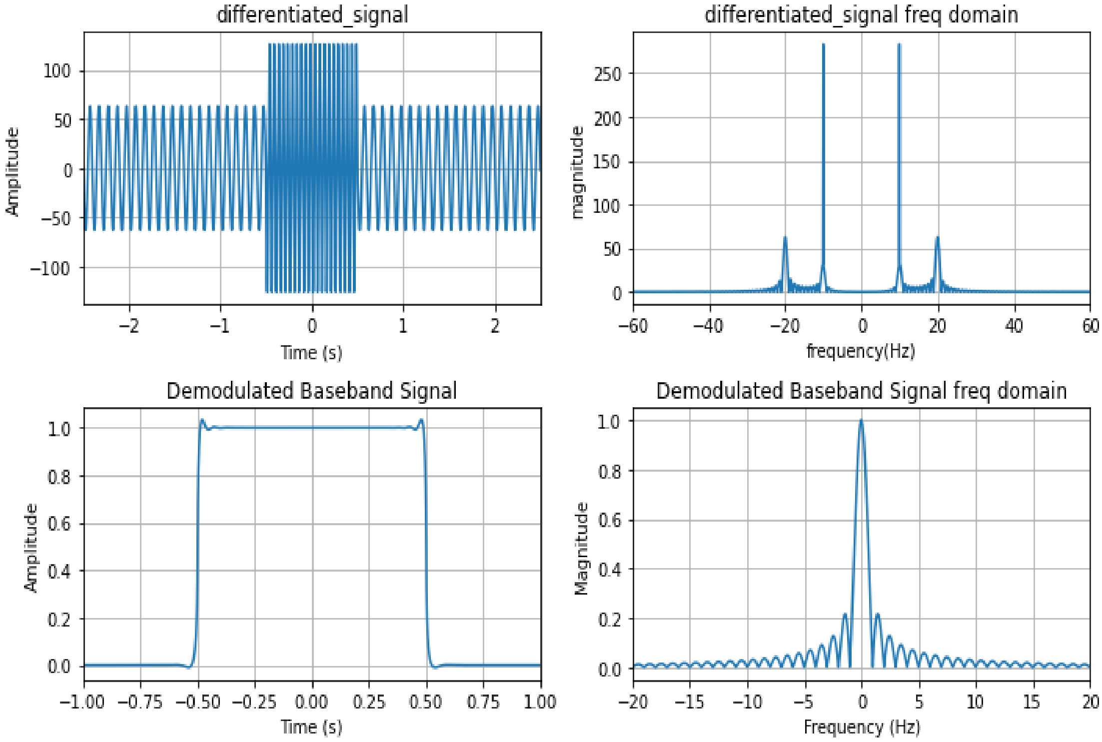

Figure 2: FM ideal Demodulator

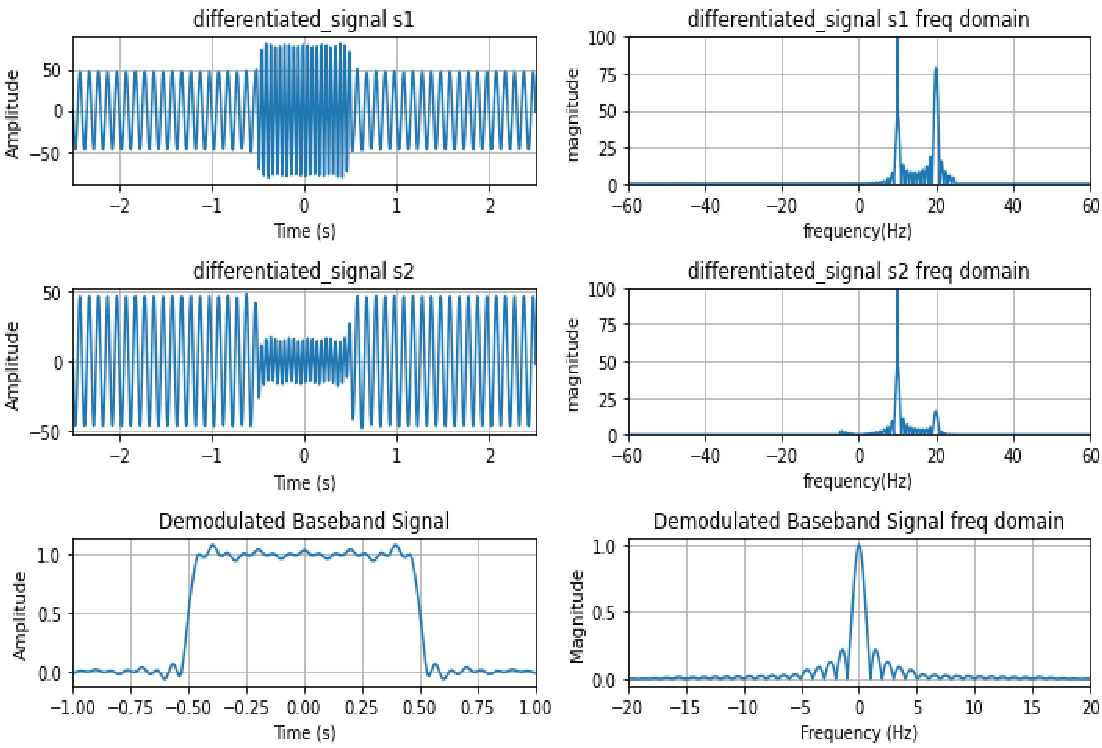

Figure 3: FM practical Demodulator

3 Discussion

This section includes the challenges we met in implementing the FM demodulator and the strategies used to address them, highlighting the complexity of FM demodulation in digital signal processing.

3.1 Result Analysis

3.1.1 Time-Domain Analysis

The time-domain plots of the differentiated signals s_1 and s_2 show the presence of a DC offset, which is indicative of a systematic bias in the demodulation process. This could be the result of non-idealities in the differentiator or the envelope detector circuits. While s_1 maintains a consistent amplitude, s_2 exhibits a lower amplitude, suggesting possible attenuation. This discrepancy might affect the composite signal's accuracy, and therefore, the demodulation quality.

The baseband signal demonstrates a step-like structure, which should correspond to the original digital message. However, the observed fluctuations around these levels are concerning. These deviations could lead to bit errors in a digital communication system and might be attributed to noise, inter-symbol interference, or frequency drift during the modulation or demodulation stages.

3.1.2 Frequency-Domain Analysis

In the frequency domain, the presence of a strong peak at zero frequency in both s_1 and s_2 is consistent with the observed DC offset in the time domain and needs to be addressed to improve signal recovery. Furthermore, the spectral peaks of the demodulated signal should ideally match the spectral content of the original message. The appearance of additional spectral components suggests that the demodulator is introducing unintended frequencies into the signal, possibly due to harmonic distortion or non-linearities within the demodulator.

3.1.3 Potential Improvements

To enhance the demodulator's performance, the following steps should be considered:

- DC Offset Correction: Implementing a high-pass filter could remove the DC offset, which would likely improve the accuracy of the demodulated signal.
- Amplitude Equalization: Investigating the cause of the amplitude difference between s_1 and s_2 and applying appropriate compensation could result in a more balanced demodulation.
- Noise Reduction: Applying more refined filtering techniques could mitigate the impact of noise and thus reduce the fluctuations in the baseband signal.
- Harmonic Distortion Assessment: Further analysis to quantify and reduce harmonic distortion would be beneficial to purify the spectral representation of the demodulated signal.

3.2 Numerical Differentiation and Hilbert Transform

One of the primary challenges in implementing the FM demodulator was the numerical differentiation of the FM signal. The differentiation step, crucial in highlighting frequency variations due to modulation, required precise numerical computation. The use of np.diff in Python only provides a basic approximation, which may not capture subtle changes in the modulated signal, especially in the presence of noise or rapid frequency shifts. To mitigate the potential inaccuracies, the differentiation process was complemented with a Hilbert transform to form an analytic signal. The Hilbert transform, while effective in envelope detection, also introduced complexities due to its nature of producing a complex-valued signal. The extraction of the envelope from this analytic signal required careful handling to ensure that the amplitude variations truly represented the original message signal.

3.3 Discrete Fourier Transform Limitations

Another challenge arose from using the Fast Fourier Transform (FFT) for frequency domain analysis. The FFT, while powerful, is constrained by the resolution and windowing effects, which can obscure finer details in the frequency spectrum, particularly for signals with closely spaced frequency components. The application of window functions and careful selection of the sampling rate were critical in addressing these limitations. Ensuring that the sampling rate adhered to the Nyquist criterion was paramount to avoid aliasing effects, while the use of window functions helped in reducing spectral leakage.

3.4 Alignment of Sampling Rates

In the digital representation of the FM signal, aligning the sampling rates during modulation and demodulation posed a significant challenge. Discrepancies in sampling rates could lead to misalignment in the time domain and inaccuracies in frequency extraction. A consistent sampling rate was maintained throughout the modulation and demodulation processes. Careful consideration was given to the choice of sampling rate, ensuring it was sufficiently high to capture the nuances of the modulated signal while remaining computationally manageable.

3.5 Signal Reconstruction and Amplitude Scaling

Reconstructing the original message signal from the demodulated envelope presented challenges, particularly in amplitude scaling. The amplitude of the demodulated signal often did not match the original message signal, mainly due to the approximations in numerical differentiation and Hilbert transform. A post-processing amplitude scaling step was introduced. By comparing the peak amplitudes of the original and demodulated signals, a scaling factor was computed and applied to the demodulated signal. This step significantly improved the alignment of the amplitudes, enhancing the accuracy of the recovered message signal.

4 Conclusion

4.1 Summary

Based on project 5, we successfully realized FM demodulation in both theoretical and practical ways. Through the implementation of a FM receiver, we successfully demonstrated the practical application of theoretical concepts, including the use of an idealized frequency differentiator and an envelope detector. The comparison between the demodulated and original signals afforded us valuable insights into the efficacy of our approach and highlighted the nuances of real-world signal processing.

4.2 Application

The conclusions drawn from this project hold significance for several domains, potentially enhancing and influencing future advancements in these areas:

1. **Radio broadcasting** FM radio broadcasting is the most common application of FM demodulation. FM radio signals are less susceptible to noise and interference than AM radio signals, providing higher-quality audio.

2. **Mobile communications** FM demodulation is used in some mobile communications systems, such as the Global System for Mobile Communications (GSM). Because it allows for efficient use of the available bandwidth, which is crucial for accommodating multiple users in a shared frequency spectrum, as is the case in GSM networks.

3. **Envelop detector** Envelope detectors are simple and inexpensive to build, and they can be used with a variety of different types of modulated signals. However, they are not as selective as other types of demodulators, and they can be susceptible to noise and interference.

5 Appendix

5.1 References

[1] https://en.wikipedia.org/wiki/Envelope_detector

[2] https://en.wikipedia.org/wiki/Demodulation

[3] https://wiki.analog.com/university/courses/electronics/electronics_lab_fm_detectors

[4]https://wirelesspi.com/frequency-modulation-fm-and-demodulation-using-dsp-techniques/

5.2 Python Realization

The experiment is recorded in the attached Jupyter notebook in an interactive way.

5.3 Used Functions Documentation

- **numpy.arange():** Generates evenly spaced numbers over a specified interval.
- **numpy.where():** Returns elements based on conditional expression.
- **numpy.fft.fft():** Computes the one-dimensional n-point discrete Fourier Transform.
- **numpy.fft.fftfreq():** Returns the discrete Fourier Transform sample frequencies.
- **numpy.cumsum():** calculates the cumulative sum of elements in an array.
- **scipy.signal.hibert():** applies the Hilbert transform to a real-valued signal, returning its analytic signal.
- **numpy.fft.fftshift()**: this helps to keep the alignment between time vector and magintude vector.

5.4 Task Separation

Task 1: FM: By Junhao Zhu

1. All the task that is related to demodulation of FM signal

Task 2: Envelope Detector: By Ruiqi Zhao

1. All the task that is related to the envelope detector

Task 3: General Analysis: By Suhao Wang, Jie Wang

1. Comprehensive analysis of experimental data, engage in critical discussion regarding the outcomes, and perform iterative debugging to refine system performance.
2. Craft the introductory and concluding sections of the project documentation, elucidating the scope, objectives, and synthesizing the findings, as well as delineating potential applications of the research.