ECE 459 Project Report

Fall 2023

Project#2: Computer Experiment of AM and DSB-SC Modulation

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1 Introduction

1.1 Experiment Setup

The first part of this experiment is about the amplitude modulation display and analysis of sinusoidal signals. The influence of different factor that make three conditions: undermodulation, 100 percent modulation and overmodulation, on the amplitude modulation results is discussed.

The experiment focuses on the study of sinusoidal modulation based on the setup of section 3.1 Amplitude Modulation (AM). The parameters are set as follows:

- **Carrier amplitude:** $A_c = 1$.
- **Carrier frequency:** $f_c = 0.4$ Hz.
- **Modulation frequency:** $f_m = 0.05$ Hz.

For the purpose of visualization and analysis, the experiment aims to showcase 10 complete cycles of the modulated wave, which corresponds to a total time span of 200 seconds. To achieve this on a digital system, the modulated wave is sampled at a rate of 10 Hz ($f_s = 10$ Hz). This sampling rate results in a total of 2,000 data points for the entire duration (200 seconds multiplied by the sampling rate of 10 Hz).

The frequency spectrum of the modulated signal occupies a bandwidth that stretches from -5 Hz to 5 Hz. Given the proximity of the carrier frequency to its adjacent side frequencies (a separation of 0.05 Hz, which is the modulation frequency), there is a desire to achieve a fine frequency resolution of 0.005 Hz (f_r). This ensures accurate representation and differentiation between the carrier and the side frequencies.

1.2 Objective

The primary objectives of the project are:

1. **Generation and visualization** of the amplitude modulation (AM) and double sideband-suppressed carrier (DSB-SC).
2. **Computation and analysis** within both time and frequency domain.
3. **Discussion** of the difference between textbook graph and ours.

2 Experiment

2.1 AM

The AM wave $s(t)$ is given by:

$$s(t) = A_c[1 + k_a m(t)]\cos(2\pi f_c t) \tag{1}$$

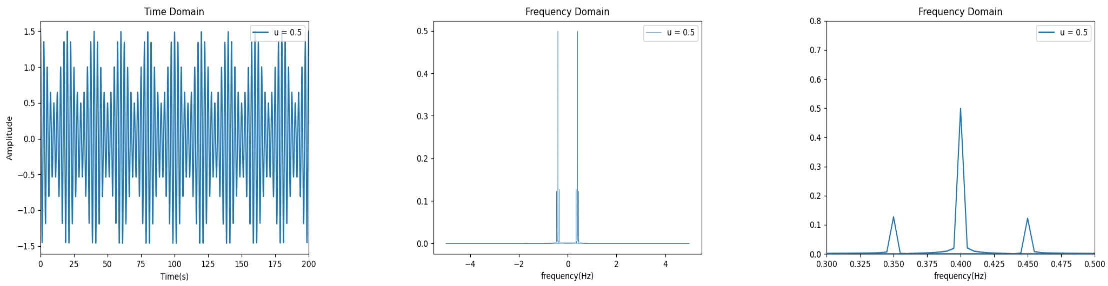
$$= A_c\cos(2\pi f_c t) + \frac{1}{2}A_c\mu\cos[2\pi(f_c - f_m)t] + \frac{1}{2}A_c\mu\cos[2\pi(f_c + f_m)t] \tag{2}$$

where:

- k_a is the amplitude sensitivity.
- μ is the modulation factor.
- $\mu = k_a A_m$.

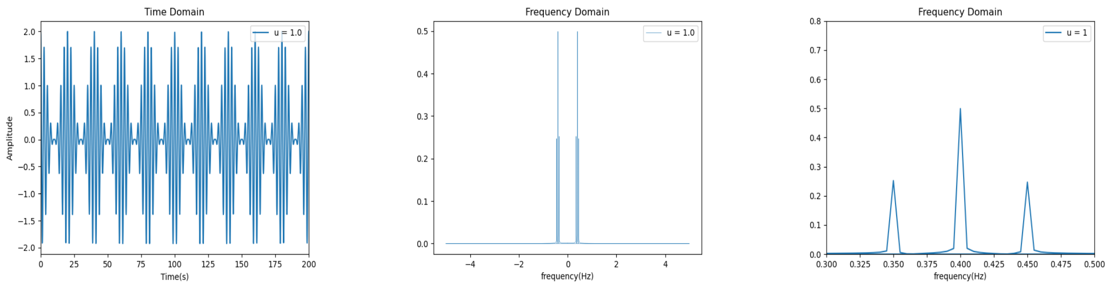
By using the modulation property of the Fourier transform, the spectrum $S(f)$ is:

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] \tag{3}$$
$$+ \frac{A_c \mu}{4}[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)] \tag{4}$$
$$+ \frac{A_c \mu}{4}[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)] \tag{5}$$



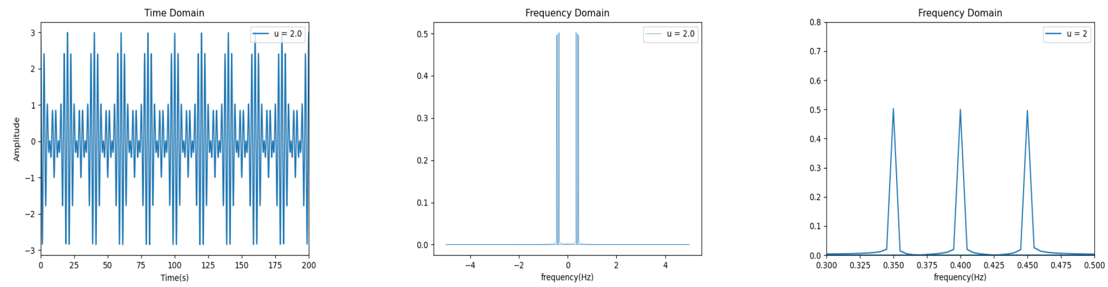
(a) (b) (c)

Figure 1: Amplitude modulation with 50 percent modulation: (a) AM wave, (b) magnitude spectrum of the AM wave, and (c) expanded spectrum around the carrier frequency.



(a) (b) (c)

Figure 2: Amplitude modulation with 100 percent modulation: (a) AM wave, (b) magnitude spectrum of the AM wave, and (c) expanded spectrum around the carrier frequency.



(a) (b) (c)

Figure 3: Amplitude modulation with 200 percent modulation: (a) AM wave, (b) magnitude spectrum of the AM wave, and (c) expanded spectrum around the carrier frequency.

2.2 DSB-SC

2.2.1 Modulation

DSB-SC modulation is defined mathematically by the product of the message signal and the carrier signal.

Let $m(t)$ be the message signal and $c(t) = A_c \cos(2\pi f_c t)$ be the carrier signal. The DSB-SC modulated signal $s(t)$ is given by:

$$s(t) = m(t) \cdot c(t) = A_c m(t) \cos(2\pi f_c t) \tag{6}$$

The spectrum of the modulated signal contains both the upper sideband (USB) and lower sideband (LSB) frequencies, but the carrier frequency component is absent. Therefore, the power of the DSB-SC signal is only in its sidebands.

The frequency components are given by:

$$\text{Lower Sideband (LSB)} : f_c - f_m$$
$$\text{Upper Sideband (USB)} : f_c + f_m$$

The power P_{DSB-SC} of $s(t)$ is given by:

$$P_{DSB-SC} = \frac{1}{T} \int_0^T |s(t)|^2 dt$$

For the DSB-SC signal, this power becomes:

$$P_{DSB-SC} = \frac{1}{T} \int_0^T m^2(t) \cos^2(2\pi f_c t) dt$$

Using the trigonometric identity:

$$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

The power becomes:

$$P_{DSB-SC} = \frac{1}{T} \int_0^T m^2(t) \frac{1 + \cos(4\pi f_c t)}{2} dt$$

Since the double-frequency term averages out to zero over a period, the power of the DSB-SC signal simplifies to:

$$P_{DSB-SC} = \frac{1}{2T} \int_0^T m^2(t) dt$$

This shows that the power of the DSB-SC signal is half the power of the modulating message signal (assuming the carrier has a peak amplitude of 1).

2.2.2 Coherent Detection

For coherent detection, the received signal $r(t)$ is multiplied with a locally generated carrier:

$$r(t) = s(t) \cos(2\pi f_c t) = A_c m(t) \cos^2(2\pi f_c t) \tag{7}$$

Using the trigonometric identity:

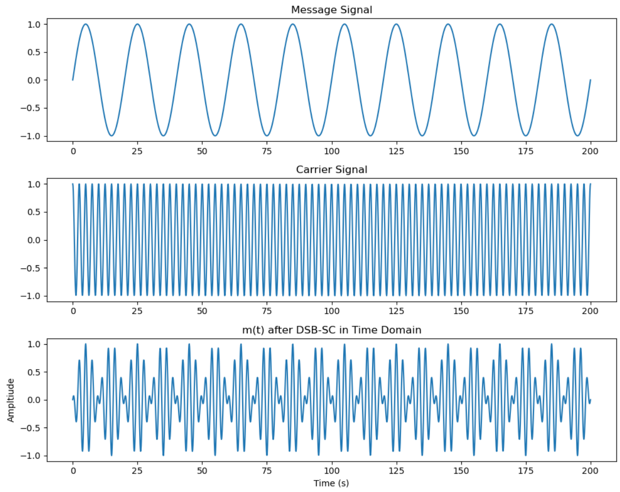
$$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2} \tag{8}$$

Substituting into $r(t)$:

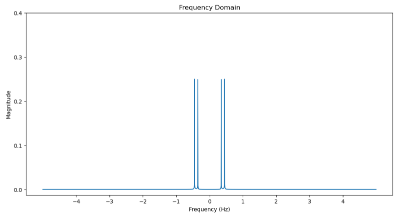
$$r(t) = \frac{A_c m(t)}{2} + \frac{A_c m(t) \cos(4\pi f_c t)}{2} \tag{9}$$

To recover the original message $m(t)$, the signal $r(t)$ is passed through a low-pass filter, which removes the high-frequency term, leaving:

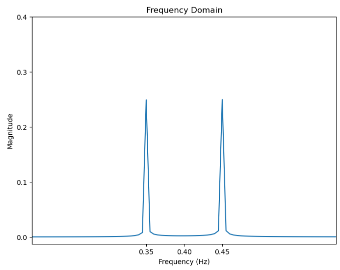
$$m(t) = \frac{2}{A_c} \cdot \text{Output of Low-pass filter} \tag{10}$$



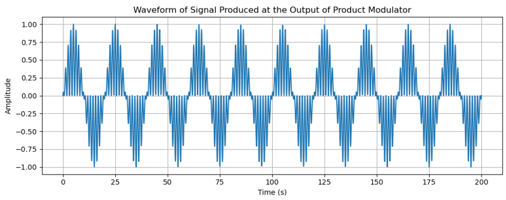
(a) message, carrier and modulated signal



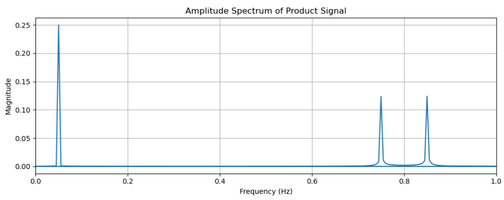
(b) Magnitude Spectrum of the Modulated Wave



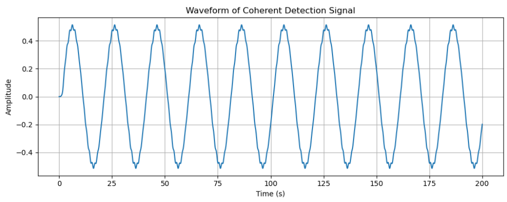
(c) expanded spectrum around the carrier frequency



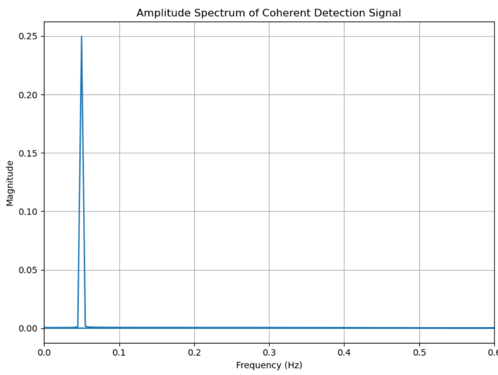
(d) Waveform of Signal Produced at the Output of Product Modulator



(e) Amplitude Spectrum of Product Signal



(f) Waveform of Coherent Detection Signal



(g) Amplitude Spectrum of Coherent Detection Signal

Figure 4: DSB-SC Experiment Results

3 Discussion

3.1 AM

3.1.1 Difference between textbook graph and ours

In the actual operation process due to the picture scale and the code is not rigorous problem, we noticed that the bottom of the frequency diagram is not the same as the textbook is horizontal, but there is a certain fluctuation, and the starting point is slightly different, this is because the FFT in the production process of the instant correspondingly small.

3.1.2 Discussion of the results

In this AM computer experiment, we adjust the modulation factor to observe the effects of undermodulation, 100percent modulation, and overmodulation. In these three cases, the lower band bit is ±0.35Hz, the upper band bit is ±0.45Hz, and the carrier frequency is ±0.4Hz. When μ= 0.5, it is undermodulation, when the maximum amplitude of the message signal or the modulated signal is less than the maximum amplitude of the carrier signal. In undermodulation, the carrier level drops above zero without causing distortion, and the envelope detector can be used for demodulation. When μ= 1, it is the case of 100 percent modulation. At this time, the maximum amplitude of the message signal or the modulated signal is exactly equal to the maximum amplitude of the carrier signal. Nevertheless, full modulation does not cause distortion, but the carrier level drops to zero, and at the same time, the envelope detector is still viable. When $\mu = 2$, it is overmodulated, when the maximum value of the message signal or the modulated signal is greater than the maximum value of the carrier signal, in this process, when the carrier level is below zero, the carrier undergoes a phase reversal of 180°, so the envelope detector does not work in this case. It can be seen that when we do AM, we should avoid over-modulation by limiting the value of μ to avoid data distortion during signal transmission.

3.2 DSB-SC

3.2.1 Difference between textbook graph and ours

1. For Waveform of Coherent Detection Signal, we observed that the initial position of demodulated output is slightly different than the original message signal $m(t)$ and the graph in the textbook.

    - Prof explains this is caused by the small transient response when applying FFT to the product signal.
    - After consideration, we think such a tiny difference is hard to be realized using Python and it's unnecessary, as it is not the expecting feature of low-pass filter.

2. The peak of our graph has many spikes, we consider some reasons behind it:

    - **Filtering Artifacts**: The abrupt peaks might be a result of the filtering process. For instance, the initial transient response of the filter might not have been completely settled, leading to these spikes.
    - **Sampling Issues**: The resolution of the sampling is just only 10Hz, which isn't fine enough. A higher sampling rate could potentially alleviate this.
    - **FFT Spectral Leakage**: As the Fast Fourier Transform (FFT) and its inverse was used, spectral leakage could cause such sharp transitions in the time domain.
    - **Numerical Instabilities**: The numerical limitation of digital signals can generate certain unexpected artifacts.

4 Conclusion

4.1 Summary

This project successfully displays 10 cycles of modulated waves of both amplitude modulation and DSB-SC modulation. The figures we get clearly showcase the relationship between side frequencies and carrier frequencies, which demonstrates the amplitude modulation theory. The demodulation methods of envelope detection and coherent detection also give the different ideas about dealing with different modulated waves.

4.2 Application

The conclusion of this project can be applied to various fields:

- **AM Radio Broadcasting:** In traditional AM radio, the carrier wave is modulated by the audio signal to produce a DSB-SC signal. The carrier is suppressed, and only the upper and lower sidebands containing the audio information are transmitted. DSB-SC allows for the efficient use of bandwidth in radio broadcasting.
- **Single-Sideband (SSB) Modulation:** DSB-SC modulation can be used to generate a single sideband (either upper or lower) by selectively filtering one sideband while discarding the other. SSB modulation is used in applications where bandwidth efficiency is crucial, such as long-distance voice communication in radio and telephony.
- **Amplitude Shift Keying:** In digital communication, DSB-SC can be used in amplitude shift keying (ASK) modulation. In ASK, the presence or absence of a carrier wave represents binary data. The carrier is suppressed when there is no signal, and it is present when transmitting data.
- **Carrier Recovery:** DSB-SC modulation can be used in carrier recovery circuits, where the original carrier is reconstituted from the sidebands. This is particularly useful in demodulation processes.

5 Appendix

5.1 References

[1] Wikipedia. Butterworth Filter. `https://en.wikipedia.org/wiki/Butterworth_filter`.

[2] Jie Wang. GitHub Repository. ECE311 Signal Processing Lab - Lab 6. `https://github.com/Everloom-129/ECE311_Signal_Processing_Lab/blob/main/Lab6/Lab6.ipynb`.

5.2 Code of AM and DSB-SC

We will submit attached file for this part.

5.3 Used Functions Documentation

- **numpy.linspace():** Generates evenly spaced numbers over a specified interval.
- **numpy.where():** Returns elements based on conditional expression.
- **numpy.fft.fft():** Computes the one-dimensional n-point discrete Fourier Transform.
- **numpy.fft.fftfreq():** Returns the discrete Fourier Transform sample frequencies.

5.4 Task Separation

Task 1: AM (P106-P109): By Zhao Ruiqi

1. Precisely replicate 9 images (3*3) and insert them into the methodology section. The drill problem is not required (as mentioned in the group).
2. Write relevant portions for the introduction (interpret the topic, explain the terms), methodology, discussion, conclusion, and appendix, following the format of Project 1's content.

Task 2: DSB-SC (P118-P120): By Jie Wang

1. Accurately reproduce 7 images (3+4). In the task 2 section, mention coherent detection, even though there is no code in the init section.
2. Follow the same instructions as above, with particular attention to coherent detection as explained on page P116.

Task 3: Review and Coordination: By Junhao Zhu

1. Include the objective in the introduction.
2. Mention the application in the conclusion.
3. Integrate both parts, check for inconsistencies, and produce the final product.