ECE 470: Introduction to Robotics Homework 7

1) For the image in Figure 1 (a), we can check similarity of a patch with its proximal region using the associated eigenvalues of the SSD expression as discussed in class. Match, with explanation, the appropriate region 1~4 labelled by square boxes with the likely eigenvalue pairs A~D. (4 Points)

2) In trying to detect lines represented by equation $\begin{pmatrix} y \\ 1 \end{pmatrix} = \begin{pmatrix} A & B \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ 1 \end{pmatrix}$ in the cartesian space with coordinates (x, y), we transform the points (x_i,y_i) to a parameter space (A, B).
   a) How will a point (3,4) be transformed to the (A, B) space? (2 Points)
   b) Describe graphically how collinear points P1 to P4 can be identified in Fig. 1 (c)?

      (4 Points)
   c) What is the mathematical limitation in detecting lines in Fig. 1 using (A, B) as parameter space? (2 Points)
   d) Describe a method you learned in class that could deal with this problem using four points to illustrate the procedures.

      (8 Points)

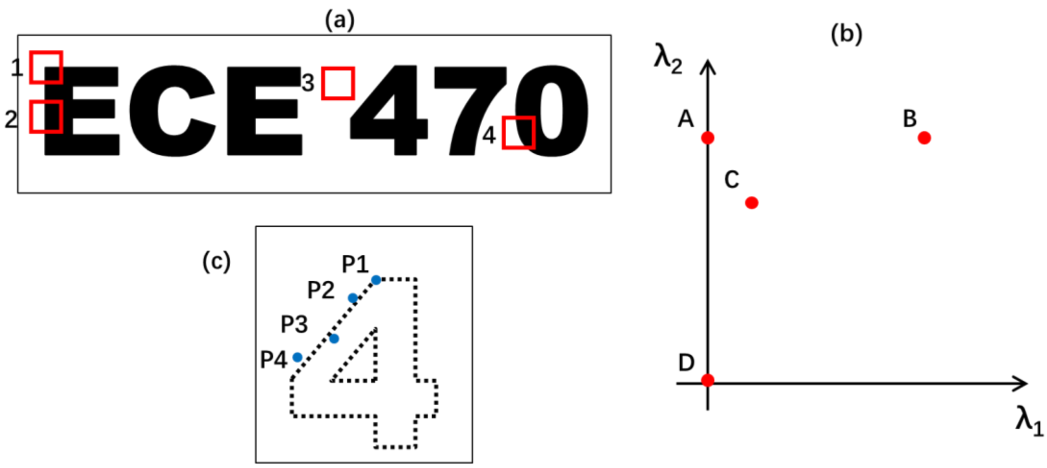

Fig. 1

3) Fig. 2 shows a four-legged robot with a camera {C} mounted on the front and a manipulator robot holding onto a board {B}. An absolute frame {A} is defined by a QR plane fixed at the robot base to define the world coordinate system.

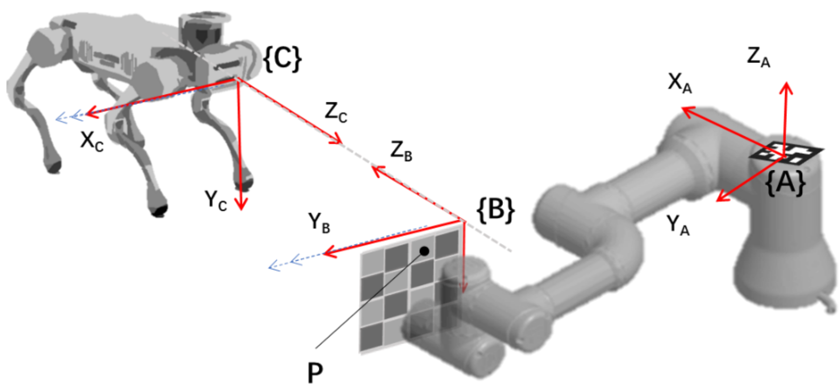

Fig. 2

a) Write down the homogenous transformation matrix ${}^{C}T_{B}$ that represents the pose of {B} in {C}, given that frame {B} and {C} are 10 unit apart, Z_C and Z_B are coaxial and X_C and Y_B are parallel. *(3 Points)*

b) Write down the 3x4 extrinsic matrix of the camera representing the orientation of the world frame {A} with respect to the camera frame {C}, given

$$^{A}T_{B} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 50 \\ -1 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ *(4 Points)*

c) A point referenced from the board frame {B} is ${}^{B}(15, 30, 0)^{T}$ and is observed to have image coordinates (400, 200). Given that *fx=fy* and *ic=jc* and assuming skew coefficient a=0, solve for the intrinsic camera matrix

$$K = \begin{bmatrix} fx & a & ic \\ 0 & fy & jc \\ 0 & 0 & 1 \end{bmatrix}$$ *(6 Points)*

d) If the four-legged robot moves such that the camera translation from {C} to {C'} is ${}^{C}P_{C'}$=(0, 0, -20), obtain the new pixel coordinates of point P. *(3 Points)*

e) If the four-legged robot is moving towards the board with its motion controlled by feedbacking images until it reaches close enough to the end-effector, is this an end-point closed-loop or end-point open-loop configuration? Explain your answer. (2 Points)

f) If the four-legged robot is stationary, it uses spatial information recovered from its camera to guide the robot arm in reaching for a target in the world, is this configuration an end-point closed-loop or end-point open-loop visual servo? Explain your answer. (2 Points)

Solution

1） 1→B: Corner with both large eigenvalues; 2→A: An edge with a single large eigenvalue (note that the axis is NOT necessarily related to a particular horizontal or vertical orientation); 3→D: low change in intensity denoted by small eigenvalues; 4→C has a strong deviation more prominently in one direction.

2)

a) It is represented as a line in (A, B) space. The line has equation B = -Ax+ y with -x as the gradient and y as the vertical intercept. For a point (3, 4), It is a line with gradient=-3 and vertical interception=4 in the (x, y) space.

b) The intersection points between the 4 lines in the parameter space is the coordinates (A, B) that represents the line in image space.

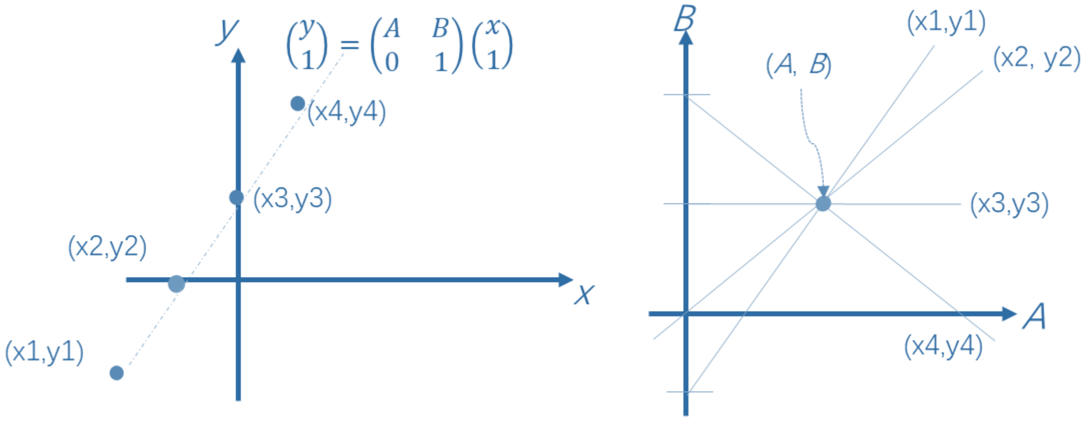

Image Space

Parameter Space

c) Vertical lines associate with character “E” and “4” are not possible when using (A, B) as the parameter-space as the gradient B will need to be infinite.

d) See Lecture Notes on Hough Transform for line detection and list the steps.

3)

a)

$$^{C}T_B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) $$^{C}T_A = {}^{C}T_B\,{}^{B}T_A = {}^{C}T_B\,{}^{A}T_B{}^{-1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 50 \\ -1 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} =$$

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 10 \\ 0 & 1 & 0 & -50 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -50 \\ 0 & 0 & -1 & 10 \\ -1 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{C}[\mathrm{R}| \quad \mathrm{t}] = \begin{bmatrix} 0 & 1 & 0 & -50 \\ 0 & 0 & -1 & 10 \\ -1 & 0 & 0 & 10 \end{bmatrix}$$

c) Using the camera model: $s\,[u\ v\ 1]^T = K\ {}^c[R \mid t]_W\,{}^W P$

$$s\begin{bmatrix}u\\v\\1\end{bmatrix}=\begin{bmatrix}fx & a & ic\\0 & fy & jc\\0 & 0 & 1\end{bmatrix}\ {}^C[\mathrm{R}|\quad \mathrm{t}]_B\ {}^B\widetilde{P}$$

Since $fx = fy$ and $ic = jc$, let $f = fx = fy$ and $c = ic = jc$. Also substitute $a = 0$.

$$s\begin{bmatrix}400\\200\\1\end{bmatrix}=\begin{bmatrix}f & 0 & c\\0 & f & c\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & -1 & 10\end{bmatrix}\begin{pmatrix}15\\30\\0\\1\end{pmatrix}$$

$$s\begin{bmatrix}400\\200\\1\end{bmatrix}=\begin{bmatrix}f & 0 & c\\0 & f & c\\0 & 0 & 1\end{bmatrix}\begin{pmatrix}30\\15\\10\end{pmatrix}$$

From 3rd row of equation, s=10

$$4000 = 30f + 10c$$
$$2000 = 15f + 10c$$
$$f = \frac{400}{3};\ c = 0$$
$$\therefore K = \begin{bmatrix}133.3 & 0 & 0\\0 & 133.3 & 0\\0 & 0 & 1\end{bmatrix}$$

d) New Extrinsic matrix: ${}^C[\mathrm{R}|\quad \mathrm{t}]_B = \begin{bmatrix}0 & 1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & -1 & 30\end{bmatrix}$ ** $\begin{bmatrix}0 & 1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & -1 & -10\end{bmatrix}$ *is ok*

due to typo error in original question paper

$$s\begin{bmatrix}u\\v\\1\end{bmatrix}=\begin{bmatrix}fx & a & ic\\0 & fy & jc\\0 & 0 & 1\end{bmatrix}\ {}^C[\mathrm{R}|\quad \mathrm{t}]_B\ {}^B\widetilde{P}$$

$$\begin{bmatrix}su\\sv\\s\end{bmatrix}=\begin{bmatrix}133.3 & 0 & 0\\0 & 133.3 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & -1 & 30\end{bmatrix}\begin{pmatrix}15\\30\\0\\1\end{pmatrix}$$

$$\begin{bmatrix}su\\sv\\s\end{bmatrix}=\begin{bmatrix}133.3 & 0 & 0\\0 & 133.3 & 0\\0 & 0 & 1\end{bmatrix}\begin{pmatrix}30\\15\\30\end{pmatrix}$$

From 3rd row of equation s=30,

$$30u = 4000 \Longrightarrow u = \frac{400}{3}$$
$$30v = 2000 \Longrightarrow v = \frac{200}{3}$$

The new pixel coordinates $(u, v) = (133.3, 66.67)$

e) End-point closed-loop as the camera is moving with the robot in control (four-legged robot)

f) End-point open-loop as the camera is independent of the pose of the robot in control (manipulator robot)