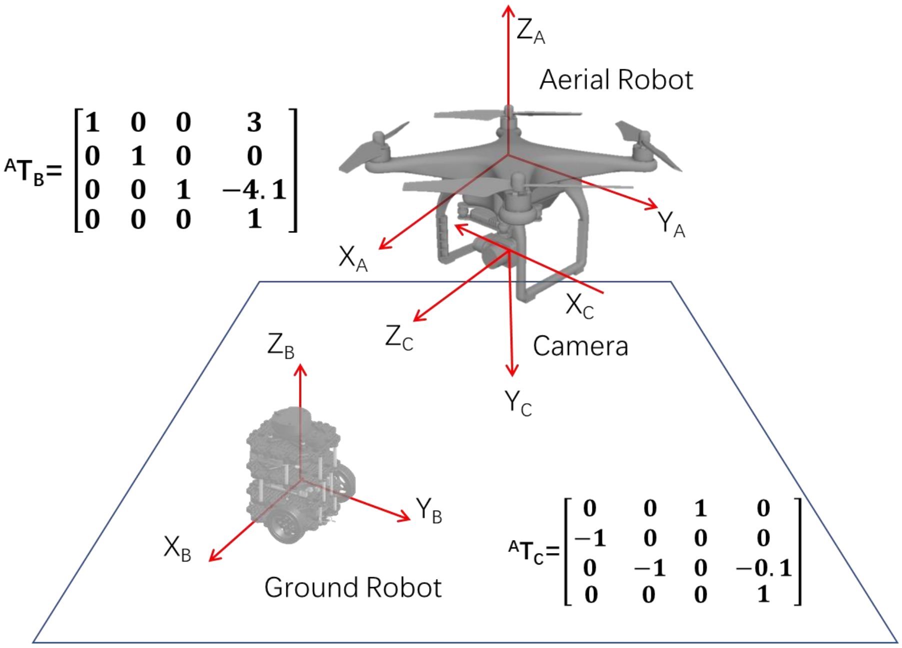
Question 1 (15 Points)

The figure shows a ground mobile robot {B} and an aerial robot {A} with a camera {C} mounted at its bottom.



a) How many independent variables are needed to completely define the pose of the aerial robot in 3D space? Explain. *(3 Points)*

b) Represent the camera's pose with respect to the ground robot's frame in homogenous transformation matrix $^{B}T_{C}$. *(3 Points)*

c) The aerial robot rotates -90° about axis Z_B, transforming frame {A} and {C} to {A1} and {C1}, Represent the new camera's pose with respect to the ground robot's frame in homogenous transformation matrix $^{B}T_{C1}$. *(3 Points)*

d) The camera rotates -45° about axis X_{c1}, transforming frame {C1} to {C2}, Represent the new camera's pose with respect to the ground robot's frame in homogenous transformation matrix $^{B}T_{C2}$. *(3 Points)*

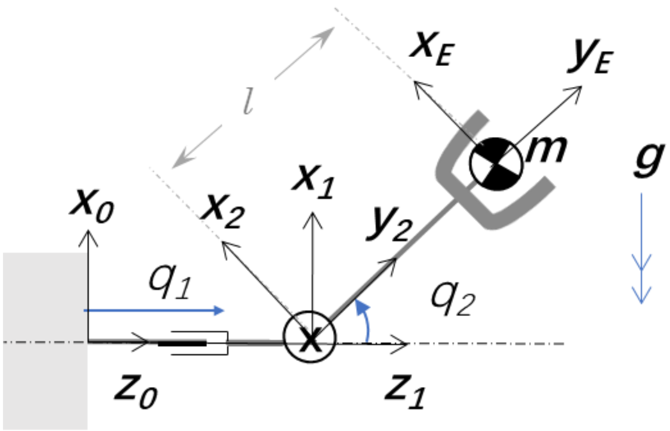
e) The ground robot moves from frame {B} to {B2} via transformation $^{B}T_{B2}$. Using the camera to retrieve the new pose of the ground robot in the camera's frame {C2}, we obtained results:

$$^{C2}T_{B2}=\begin{pmatrix}-1 & 0 & 0 & 0\\ 0 & {}^{1}/_{\sqrt{2}} & -{}^{1}/_{\sqrt{2}} & 0\\ 0 & -{}^{1}/_{\sqrt{2}} & -{}^{1}/_{\sqrt{2}} & {}^{3}/_{\sqrt{2}}\\ 0 & 0 & 0 & 1\end{pmatrix}$$

Using the results, represent the ground robot's movement in homogenous transformation matrix $^{B}T_{B2}$. *(3 Points)*

Question 2 (15 Points)

The figure below shows a robotic manipulator with a prismatic joint and rotational joint with joint variables q_1 and q_2 respectively.



a) Fill in the D-H table based on the frame assignment in the figure. *(2 Points)*

	Link Twist α_{i-1}	Link Length a_{i-1}	Joint Angle θ_i	Link offset d_i
${}^0T_1$	0	0	0	q_1
${}^1T_2$	**(i)_______?**	**(ii)_______?**	**(iii)_____?**	**(iv)_____?**

b) The robot is manipulating a mass, *m* with the end effector {E} under the influence of gravity. Ignore the mass of the linkage structure and treat the motion as a pointed mass at the end effector. State any other assumption you are making while answering the following.

(i) Express the position of the mass, m in frame {0}, ${}^0P_E$ in terms of q_1 and q_2. *(3 Points)*

(ii) Write down the potential energy of the system in terms of q_1 and/or q_2. *(1 Points)*

(iii) Show that the kinetic energy of the system can be expressed as

$$K = \frac{m}{2}\dot{q}_1{}^2 + \frac{m}{2}l^2\dot{q}_2{}^2 - ml\dot{q}_1\dot{q}_2 \sin q_2$$

(3 Points)

(iv) Write down an expression for the Lagrangian, L *(1 Points)*

(v) Obtain the dynamic equations relating the joint force and torque $(f, \tau)^T$ with the joint motion. *(5 Points)*

Solution:

Question 1

a) 6 independent variables: 3 for position (xyz translation), 3 for orientation (roll, pitch, yaw)

b) Knowing that ${}^{B}R_C$ and ${}^{A}R_C$ are the same and hence ${}^{B}P_C = {}^{A}P_C - {}^{B}P_C$, directly write down

$$ {}^{B}T_C = \begin{bmatrix} 0 & 0 & 1 & -3 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Alternatively,

$${}^{B}T_C = {}^{B}T_A\,{}^{A}T_C$$

$$= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -4.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4.1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & -3 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) ${}^{B}T_{C1} = {}^{B}T_{B1}\,{}^{B1}T_{C1} = \text{Rotate}_Z(-90^\circ)\,{}^{B}T_C$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & -3 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

d) ${}^{B}T_{C2} = {}^{B}T_{C1}\,{}^{C1}T_{C2} = {}^{B}T_{C1}\,\text{Rotate}_x(-45^\circ)$

$$= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & -1 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 3 \\ 0 & -1/\sqrt{2} & -1/\sqrt{2} & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

e) ${}^{B}T_{B2} = {}^{B}T_{C2}\,{}^{C2}T_{B2}$

$$= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 3 \\ 0 & -1/\sqrt{2} & -1/\sqrt{2} & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & -1/\sqrt{2} & -1/\sqrt{2} & 3/\sqrt{2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 2.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution:

Question 2

${}^{1}T_{2}$	(i) 90	(ii) 0	(iii) $-q_2$	(iv) 0

2

b) i) $${}^{0}p_{E} = \begin{bmatrix} l\sin q_2 \\ 0 \\ l\cos q_2 + q_1 \end{bmatrix}$$ 3

ii) Potential Energy, $U = mgl\sin q_2$ 1

iii) from (i) $${}^{0}v_E = \begin{bmatrix} l\cos q_2 \dot{q}_2 \\ 0 \\ -l\sin q_2 \dot{q}_2 + \dot{q}_1 \end{bmatrix}$$ 1 (or $v^2 = \dots$)

K.E: $K = \frac{1}{2}m\underbrace{[l^2\cos^2 q_2 \dot{q}_2^2 + (\dot{q}_1 - l\sin q_2 \dot{q}_2)^2]}_{v^2}$

$= \frac{1}{2}m[l^2\dot{q}_2^2 + \dot{q}_1^2 - 2\dot{q}_1\dot{q}_2 l\sin q_2]$ 2

$= \frac{m\dot{q}_1^2}{2} + \frac{m}{2}l^2\dot{q}_2^2 - ml\dot{q}_1\dot{q}_2\sin q_2$ (shown)

iv) $L = K - U \stackrel{1}{=} \frac{m}{2}q_1^2 + \frac{m}{2}l^2 q_2^2 - mlq_1q_2\sin q_2 - mgl\sin q_2$

v) $$\frac{\partial L}{\partial q} = \begin{bmatrix} 0 \\ -ml\dot{q}_1\dot{q}_2\cos q_2 - mlg\cos q_2 \end{bmatrix}$$ 1

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = \begin{bmatrix} m\ddot{q}_1 - ml\sin q_2 \ddot{q}_2 - m_2 \dot{q}_2 l\cos q_2 \\ ml^2\ddot{q}_2 - ml\sin q_2 \ddot{q}_1 - ml\cos q_2 \dot{q}_1 \dot{q}_2 \end{bmatrix}$$ 2

$$\begin{bmatrix} f \\ \tau \end{bmatrix} = \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = \begin{bmatrix} m\ddot{q}_1 - ml\sin q_2 \ddot{q}_2 - m\dot{q}_2^2 l\cos q_2 \\ ml^2\ddot{q}_2 - m_2 l\sin q_2 \ddot{q}_1 - m_2 lg\cos q_2 \end{bmatrix}$$ 2

Question 3 (20 Points)

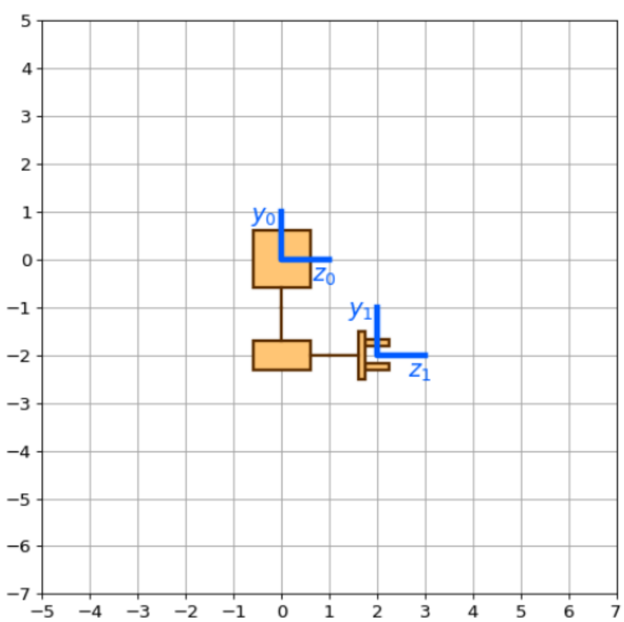
For the following robot schematics, compute the forward kinematics using the Product of Exponentials approach. Each of the following figures shows a robot with n joints in its zero position. You must find the missing pieces so we may write the pose of the end-effector (frame 1) as:

$$T_{01} = e^{[\mathcal{S}_1]\theta_1} \cdots e^{[\mathcal{S}_n]\theta_n} M$$

where $\theta_1, ..., \theta_n$ are the joint variables.

Recall that the base of the robot (the fixed frame $\{0\}$) is visualized by the large square. The end-effector or tool of the robot is denoted by the two-finger gripper at frame $\{1\}$. The order of the joints is determined by the following the mechanisms from the base to end-effector. A *revolute joint* is visualized as a cylinder (i.e., a rectangle from the side view, a circle from the top view). If working with a top view of a revolute joint, you may assume that the axis of rotation is pointing out of the page. A *prismatic joint* is visualized as a small square followed by a small rectangle that is in the direction of actuation. A topview of a prismatic joint is visualized as two nested squares.

a) Find the configuration for a simple robot. *(4 Points)*



(i) Find M and the screw axes for each joint.

(ii) Write out the matrix exponential that gives the forward kinematics of this simple robot.

(iii) If $\theta_1 = 0.85$, what is the pose of the end effector? Given $\cos\theta_1 = 0.66$ and $\sin\theta_1 = 0.75$.

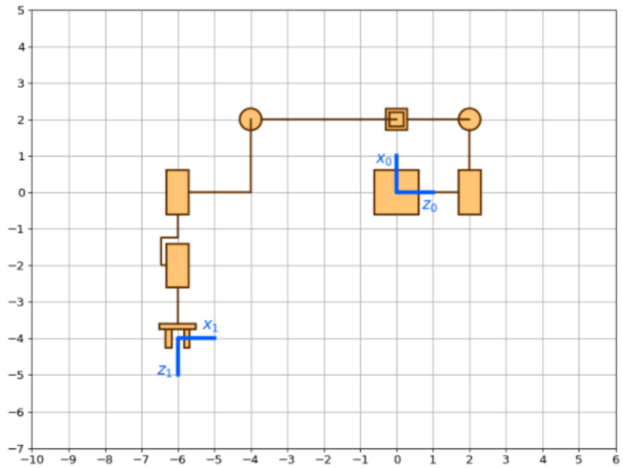
Solution:

(i) M = [1 0 0 0; 0 1 0 -2; 0 0 1 -2; 0 0 0 1] S = [0 0 1 -2 0 0]

(ii) plug into matrix exponential equation as seen in slides

(iii) $T_01 = [0.659983145885 - 0.7512804051400; 0.751280405140.6599831458850 - 2; 0012; 0001]$

b) A more complex robot



i) Find M. *(4 Points)*

ii) Find the screw axes for each joint. *(12 Points)*

Solution:

i) M = [0 0 -1 -4; 0 1 0 0; 1 0 0 -6; 0 0 0 1]

ii) S = [1 0 0 0 -1 -1; 0 1 0 1 0 0; 0 0 0 0 0 0; 0 -2 0 4 0 0; 2 0 1 0 6 6; 0 2 0 2 0 0]