

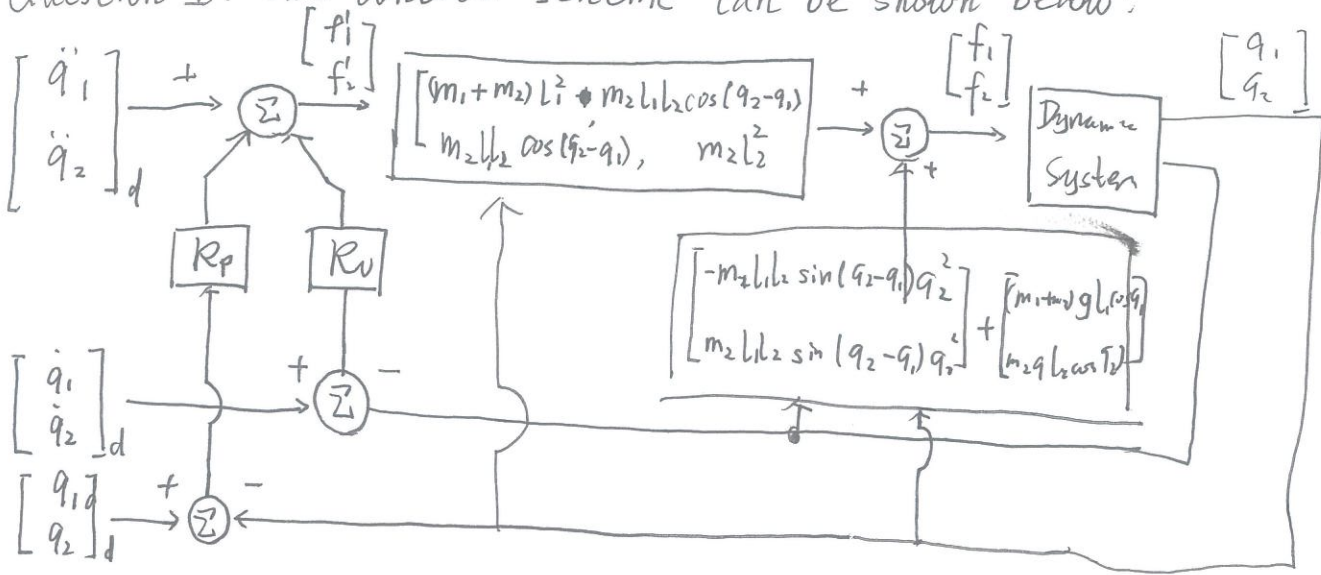
ECE 410 Assignment 4

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Question 1: The control scheme can be shown below:



Because:

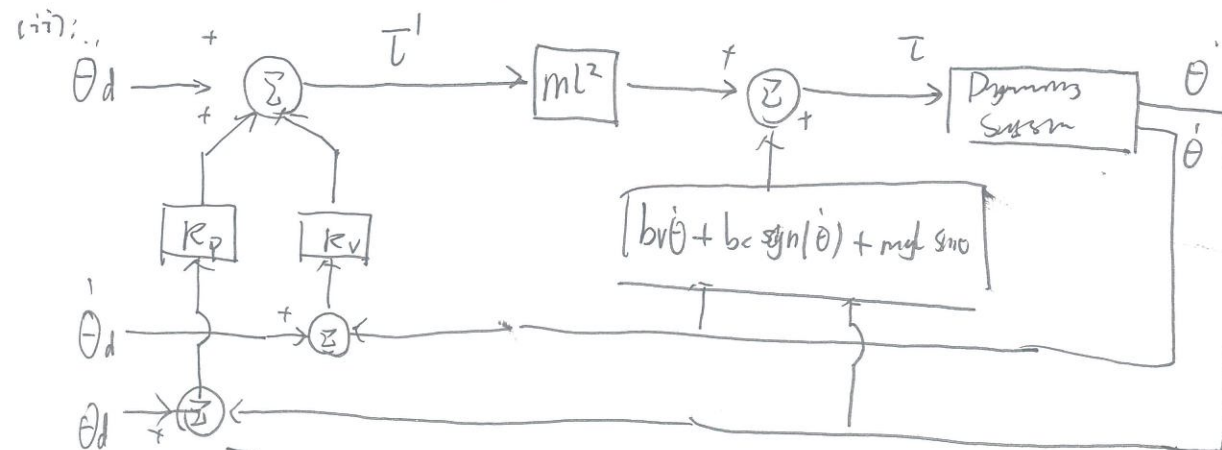
$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} (m_1+m_2)L_1^2 & m_2L_1L_2\cos(q_2-q_1) \\ m_2L_1L_2\cos(q_2-q_1) & m_2L_2^2 \end{bmatrix} \begin{bmatrix} f_1' \\ f_2' \end{bmatrix} + \begin{bmatrix} -m_2L_1L_2\sin(q_2-q_1)\dot{q}_2^2 \\ m_2L_1L_2\sin(q_2-q_1)\dot{q}_1^2 \end{bmatrix}$$

$$\begin{bmatrix} f_1' \\ f_2' \end{bmatrix} = \begin{bmatrix} \ddot{q}_{1d} \\ \ddot{q}_{2d} \end{bmatrix} + K_p \begin{bmatrix} q_{1d} - q_1 \\ q_{2d} - q_2 \end{bmatrix} + K_v \begin{bmatrix} \dot{q}_{1d} - \dot{q}_1 \\ \dot{q}_{2d} - \dot{q}_2 \end{bmatrix} + \begin{bmatrix} (m_1+m_2)gL_1\cos q_1 \\ m_2gL_2\cos q_2 \end{bmatrix}$$

Question 2:

$$\tau - M_v - M_c - mgl\sin\theta = ml^2\ddot{\theta}$$

$$\Rightarrow \tau = ml^2\ddot{\theta} + b_v\dot{\theta} + b_c\operatorname{sgn}(\dot{\theta}) + mgl\sin\theta$$



$$(b): \tau_m = I_m \ddot{\theta}_m + b_m \dot{\theta}_m + \tau (1/\eta)$$

$$= I_m \ddot{\theta}_m + b_m \dot{\theta}_m + \left(\frac{1}{b}\right) (m l^2 \ddot{\theta} + b_v \dot{\theta} + b_c \operatorname{sgn}(\dot{\theta}) + m l g \sin \theta)$$

$$= (I_m \eta + \frac{1}{b} m l^2) \ddot{\theta} + (b_m \eta + b_v \frac{1}{b}) \dot{\theta} + \frac{1}{b} (b_c \operatorname{sgn}(\dot{\theta}) + m l g \sin \theta)$$

$$\text{So } \tau = \eta \tau_m = (\eta^2 I_m + m l^2) \ddot{\theta} + (\eta^2 b_m + b_v) \dot{\theta} + b_c \operatorname{sgn}(\dot{\theta}) + m l g \sin \theta$$

proved

(i): k_p (ii): k_v

$$(iii) \eta^2 I_m + m l^2 \quad (v): \frac{1}{b}$$

$$(iv) (\eta^2 b_m + b_v) \dot{\theta} + b_c \operatorname{sgn}(\dot{\theta}) + m l g \sin \theta$$