ECE470 Lecture
Trajectory Generation

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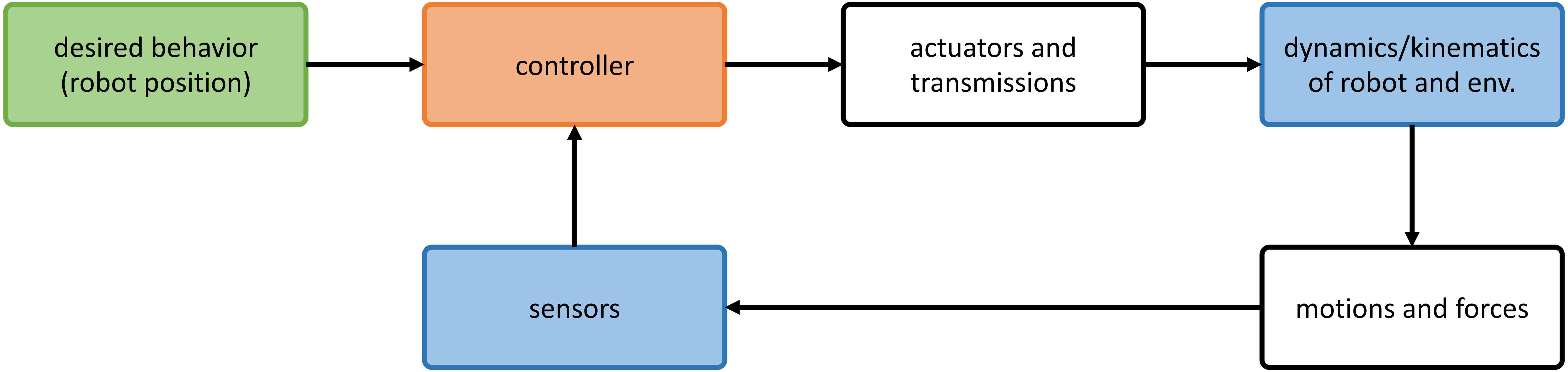
Notes from Modern Robotics Ch 9

Control Paradigm



Trajectories and Paths

- The specification of a robot state as a function of time is called a **trajectory**
- Using forward kinematic maps, we can obtain the position of each link given as joint angles
  - The trajectory of the end-effector is then $T_{sb}(\theta(t))$
- A **path** is a set of points

Normalized Trajectories

- **Path** $\theta(s)$ maps a scalar path parameter $s \in [0,1]$ to a point in the robot's configuration space

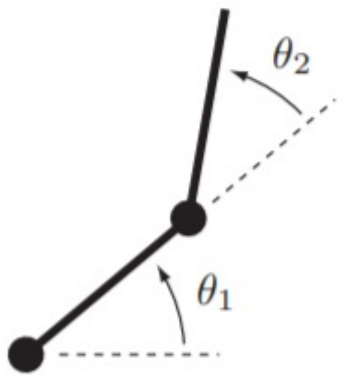
- A **time-scaling** $s(t)$ is a monotonically increasing function:

Straight-Line Paths

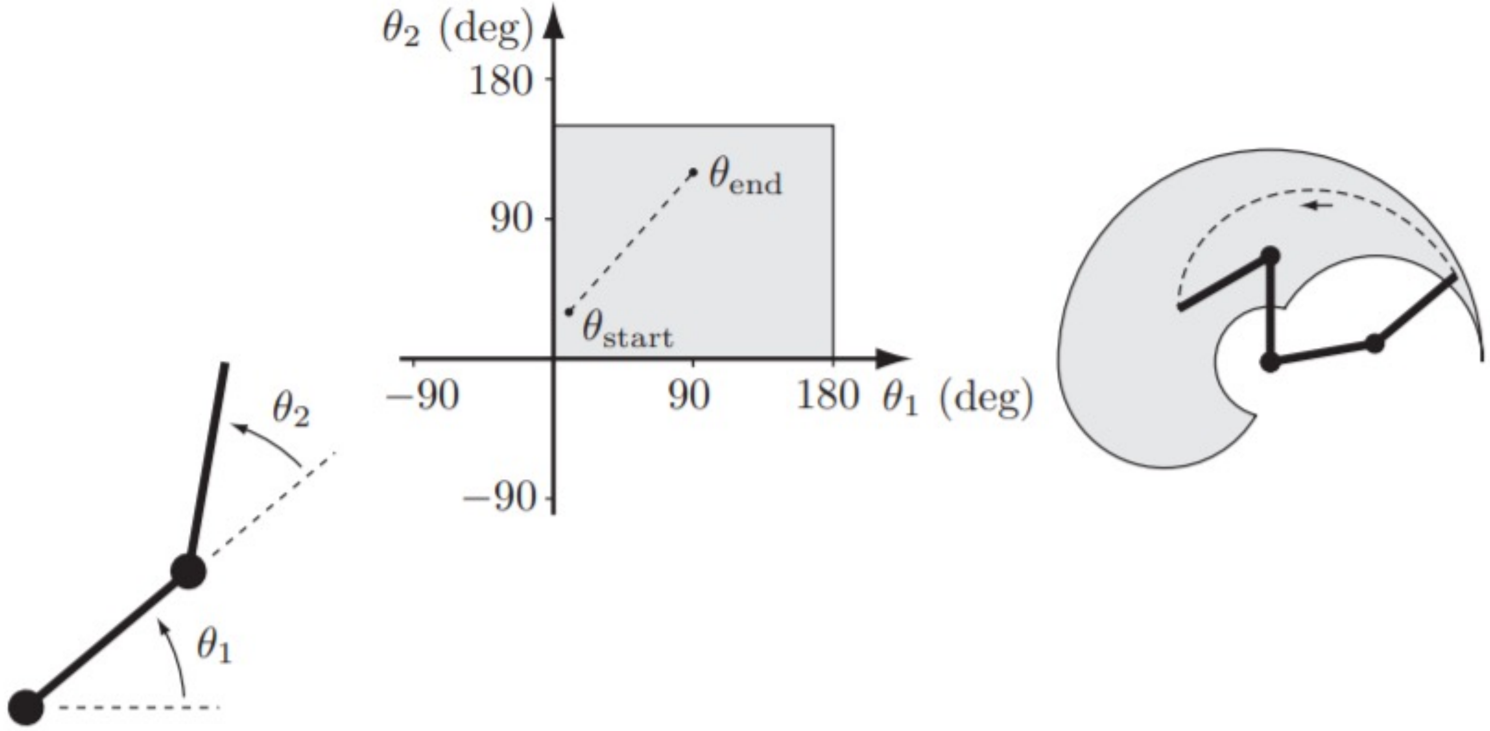
- Given θ_0 and θ_1, find straight-line path:

- Is this in the task or configuration space?
  - Straight lines in joint space do not lead to straight lines in end-effector/task space
- Straight line in task space:

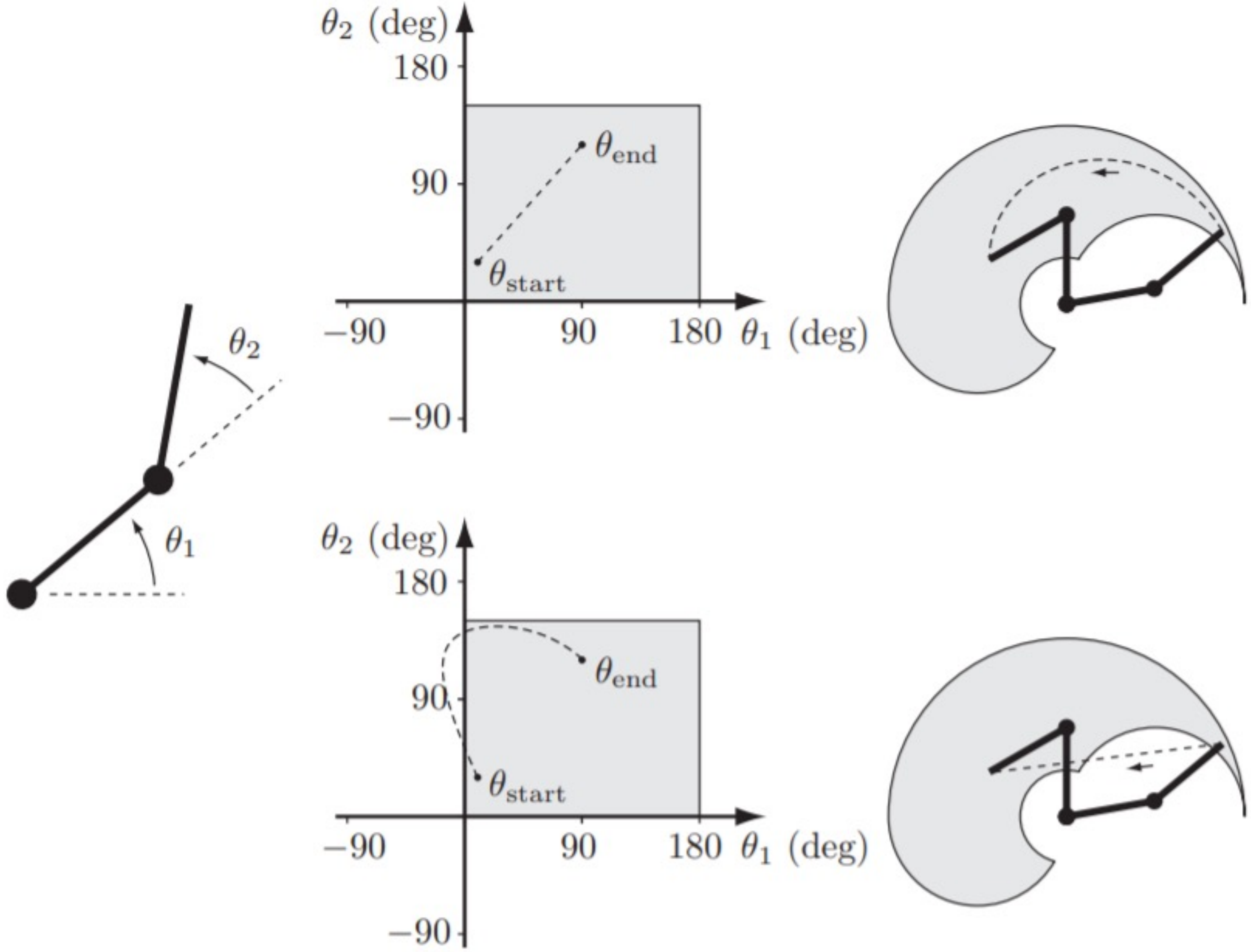
Straight-line Paths



Straight-line Paths



Straight-line Paths



Straight lines in SE(3)

In $\mathbb{R}^2$, straight lines are characterized by a constant velocity

Straight lines in SE(3)

- We can decouple rotation and translation:

- Now pass to **IK solver** to translate into joint space!

Time-scaling of straight-line paths

Time scaling ensures that the motion is smooth and constraints are met

Polynomial Time-Scaling (1)

Polynomial Time-Scaling (2)

Summary

- Defined **paths, time-scaling,** and **trajectories**
- Looked at how to find **straight-line paths** in various spaces
- We choose a **parametrization** $\boldsymbol{s(t)}$, and computed the resulting velocity and acceleration profiles of the trajectory
  - Using a third-order polynomial, we tuned their maximal values to meet requirements with one parameter T
- We can follow the same procedure with different parametrizations for $s(t)$ (e.g., polynomials of order 5, trapezoidal functions, splines, etc.)
  - Having more parameters allows us to meet more constraints. For example, using a fifth order polynomial, we can ensure that $\ddot{\theta}(0) = \ddot{\theta}(T) = 0$, meaning no jerk at beginning and end of the motion
- **Next topics** are on different concepts of / approaches to planning when the path may not be given