Instructions

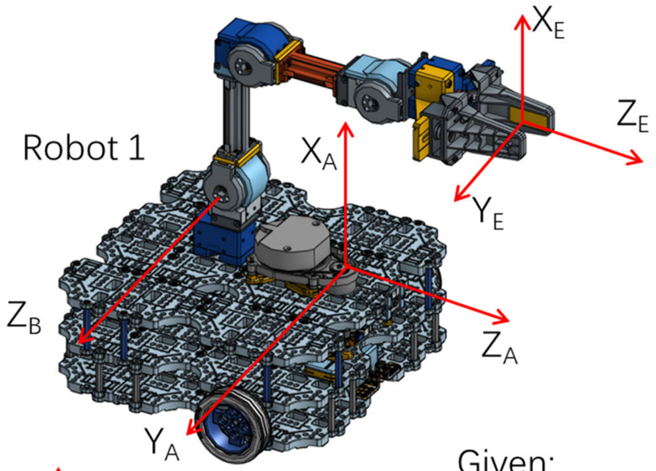
1. Do not start writing until you are instructed to do so.
2. Do not continue to write when you are told to stop.
3. You are not allowed to communicate with one another during the quiz.
4. This is an open-book quiz. Except for a calculator, you are NOT allowed to use other electronic devices.
5. Answer in the answer-sheet and submit both question- and answer-sheet before the end of the quiz.
6. Write your name and student number clearly in the answer sheet.
7. There are two questions (20 points each) with sub-questions

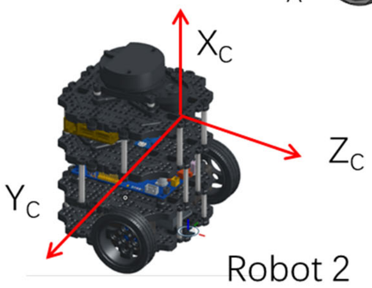
Question 1

a) From what you learned in this course, describe a robot using an example as illustration.

(4 Points)

b) The figure shows two mobile robots, one with a manipulator arm mounted on it. Frame A, B, E are attached rigidly to Robot 1 and Frame C is attached rigidly to Robot 2.





Given:

$$^{B}T_{E}=\begin{bmatrix}0 & 1 & 0 & 2\\ 0 & 0 & 1 & 1\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$^{A}T_{B}=\begin{bmatrix}0 & 1 & 0 & 1\\ 0 & 0 & 1 & 0\\ 1 & 0 & 0 & -1\\ 0 & 0 & 0 & 1\end{bmatrix} \quad ^{A}T_{C}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 3\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Robot 1 and Robot 2 went through some transformations in the following order

1. Robot 2 moves around Robot 1 such that {C} rotates 90° about axis X_A to become {C1}.
2. Robot 2 rotates about itself such that {C1} rotates 90° about axis X_{c1} to become {C2}.
3. Robot 1 moves forward such that {A}, {B} and {E} translate along the vector $^{A}(0, 0, 2)'$ to become {A3}, {B3} and {E3}
4. The arm on Robot 1 moves such that {E3} rotates 90° about axis Z_B to become {E4}

Obtain the expression for

i. $^{A}T_{C1}$ *(3 Points)*

ii. $^{A}T_{C2}$ *(3 Points)*

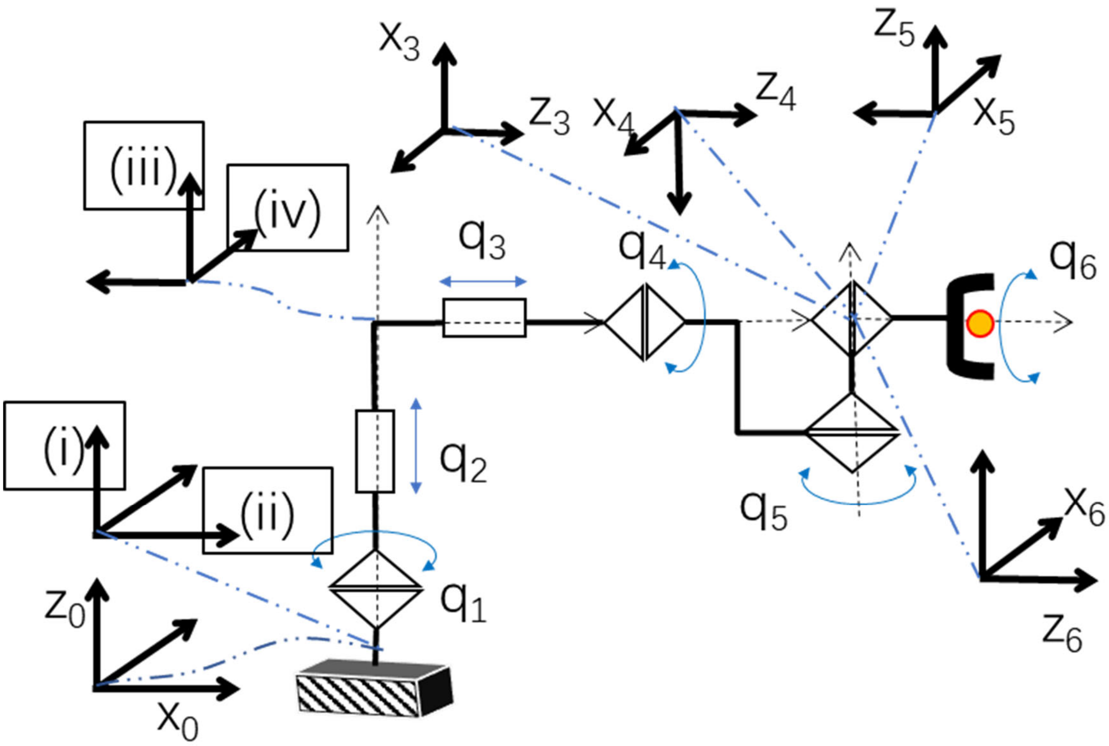
iii. $^{A3}T_{C2}$ *(3 Points)*

iv. $^{E3}T_{C2}$ *(3 Points)*

v. $^{C2}T_{E4}$ *(4 Points)*

Question 2

The following figure shows a 6-joint robot with frames assigned to the links. Joint q_1, q_4, q_5 and q_6 are revolute joints while q_2 and q_3 are translational joints.



The D-H parameters are tabulated as follows.

	Link Twist α_{i-1}	Link Length a_{i-1}	Joint Angle θ_i	Link offset d_i
${}^0_1T$	0	0	$q_1=0$	0
${}^1_2T$	0	0	90°	$q_2=d_2$
${}^2_3T$	90°	0	(v)______________?	(vi)______________?
${}^3_4T$	(vii)______________?	(viii)______________?	(ix)______________?	0
${}^4_5T$	(x)______________?	(xi)______________?	$q_5=180^\circ$	(xii)______________?
${}^5_6T$	90°_	0	$q_6=0$	0

a) Fill in the missing details from (i)-(xii) *(12 Points)*

b) For ${}^0_3P = (\frac{1}{\sqrt{2}}, 1, \frac{1}{\sqrt{2}})^T$, determine a possible set of (q_1, q_2, q_3). *(3 Points)*

c) Describe the reachable workspace for Frame 6 *(2 Points)*

d) Explain what happens when $q_5=180^\circ$ (i.e. current configuration) *(3 Points)*

Solution

Question 1

a) Robots are machines/agents designed by human to carry out tasks while interacting with the environment. An example is a robotic manipulator arm for sorting objects. It is designed to recognize and classify objects in the scene while manipulating the objects as specified.

b)

i.

$$^{A}T_{C1} = {}^{A}T_{A1}\,{}^{A1}T_{C1} = Rotate_X(90)\,{}^{A}T_C$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 90 & -\sin 90 & 0 \\ 0 & \sin 90 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ii.

$$^{A}T_{C2} = {}^{A}T_{C1}\,{}^{C1}T_{C2} = {}^{A}T_{C1}Rotate_X(90)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 90 & -\sin 90 & 0 \\ 0 & \sin 90 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii.

$$^{A3}T_{C2} = {}^{A3}T_{A}\,{}^{A}T_{C2} = Translate_Z(-2)\,{}^{A}T_{C2}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iv.

$$^{E3}T_{C2} = {}^{E3}T_{A3}\,{}^{A3}T_{C2} = {}^{E}T_{A}\,{}^{A3}T_{C2} = \left({}^{A}T_{B}\,{}^{B}T_{E}\right)^{-1}\,{}^{A3}T_{C2}$$

$$= \left(\begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)^{-1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

v.

$$^{C2}T_{E4} = {}^{C2}T_{A3}\,{}^{A3}T_{B3}\,{}^{B3}T_{B4}\,{}^{B4}T_{E4} = \left({}^{A3}T_{C2}\right)^{-1}\,{}^{A}T_{B}Rotate_Z(90)\,{}^{B}T_{E}$$

$$= \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)^{-1} \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 3 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Solution

Question 2

a)

(i) Z_1 (ii) X_1 (iii) Z_2 (iv) X_2 (v) 90° (vi) $q_3=d_3$ (vii) 0 (viii) 0 (ix) $q_4=90^\circ$ (x) 90° (xi) 0 (xii) 0

b)

Forward kinematics:

$$ {}^0_3P = \begin{pmatrix} q_3 \cos q_1 \\ q_3 \sin q_1 \\ q_2 \end{pmatrix} $$

Inverse kinematics:

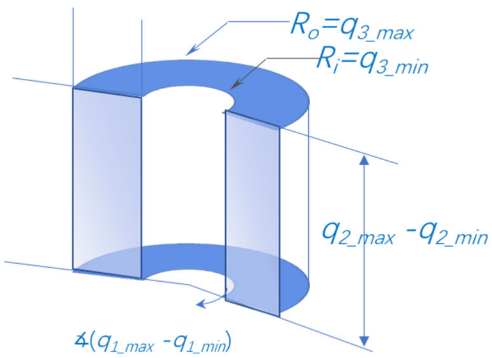
$$ q_2 = \frac{1}{\sqrt{2}} $$

$$ q_3 = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + 1^2} = \sqrt{\frac{3}{2}} $$

$$ q_1 = \mathrm{atan2}(1, \frac{1}{\sqrt{2}}) $$

c)

It is a partial cylinder with height $q_{2\_max} - q_{2\_min}$, angle of sweep $q_{1\_max} - q_{1\_min}$, and radius of sweep $q_{3\_max} - q_{3\_min}$ as shown in the following figure.



d)

This configuration is associated with singularity as the axes of q_4 and q_6 align losing a degree of freedom. Mathematically, the Jacobian matrix $J(\dot{q}_1 \quad \dots\dots \quad \dot{q}_6)$ mapping $(\dot{q}_1 \quad \dots\dots \quad \dot{q}_6)^{\mathrm{T}}$ to $(\dot{\boldsymbol{x}} \quad \dot{\boldsymbol{\theta}})$ is not full rank.