Name: ECE 470/ ME 445 Spring 2023 ZJUI: Midterm

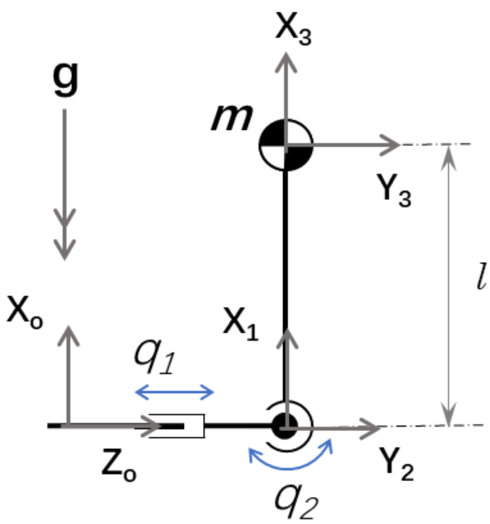
Student ID:

Instructions

1. Do NOT turn over the page NOR start writing until you are instructed to do so.
2. Do NOT continue to write when you are told to stop.
3. You are NOT allowed to communicate with one another during the quiz.
4. This is an open-book quiz. Except for a calculator, you are NOT allowed to use other electronic devices.
5. Write your name and student number clearly in the answer sheet.

Question 1 (Show your working clearly for marks to be awarded)

The figure below shows a robotic manipulator with a translational joint and rotational joint with joint variables q_1 and q_2 respectively.



a) Concept of degree-of-freedom:

(i) How many degree-of-freedom does the manipulator have? (ii) How many independent variables would be required to describe the pose of Frame {3} in Frame {0}? Explain your answer. *(3 Points)*

b) Using D-H convention, taking clockwise rotation of q_2 as positive, sketch the axis of (i) Z_1 and (ii) X_2 on the figure

.Fill in the D-H table based on the frame assignment.

	α_{i-1}	a_{i-1}	θ_i	d_i
${}^0T_1$	0	0	(iii) _____	(iv) _____
${}^1T_2$	(v) _____	0	(vi)_____	0
${}^2T_3$	0	(vii)_____	0	0

(7 Points)

viii) Obtain the homogenous transformation matrix based on the following conditions: Given $l = \sqrt{2}$. If the last digit of your student ID is odd, set joint variables as (1m, 45 deg). Else set joint variables as (2m, -45 deg). *(4 Points)*

c) Product of Exponential: *(8 Points)*

i) Find the M matrix.

ii) Find the Screw axis for each joint

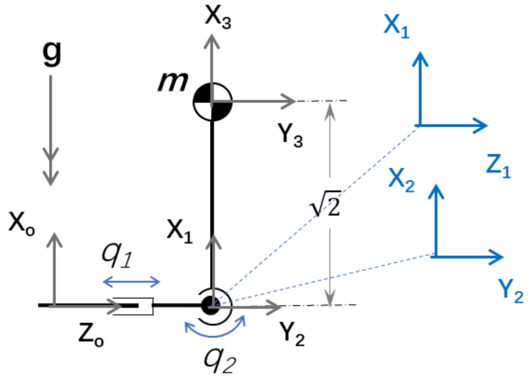
Solution Q1:

a) (3 points)

(i) 2 dof

(ii) Minimally 2 INDEPENDENT variables; (q1, q2) or (x, θ)

b) (7 points)



(iii) 0; (iv) q1; (v) 90°; (vi) q2=0; (vii) /

(viii) (4 points)

$$^{0}T_1=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&q_1\\0&0&0&1\end{bmatrix}$$

$$^{1}T_2=\begin{bmatrix}c_2&-s_2&0&0\\0&0&-1&0\\s_2&c_2&0&0\\0&0&0&1\end{bmatrix}$$

$$^{2}T_3=\begin{bmatrix}1&0&0&l\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$^{0}T_3={}^{0}T_1^{1}T_2^{2}T_3=\begin{bmatrix}c_2&-s_2&0&\phi l c_2\\0&0&-1&0\\s_2&c_2&0&q_1+ls_2\\0&0&0&1\end{bmatrix}$$

Given $l=\sqrt{2}$

For $(q_1,q_2)=(2m,-45^{\circ})$;

$$^{0}T_3=\begin{bmatrix}1/\sqrt{2}&1/\sqrt{2}&0&1\\0&0&-1&0\\-1/\sqrt{2}&1/\sqrt{2}&0&1\\0&0&0&1\end{bmatrix}$$

For $(q_1,q_2)=(1m,45^{\circ})$;

$$^{0}T_3=\begin{bmatrix}1/\sqrt{2}&-1/\sqrt{2}&0&1\\0&0&-1&0\\1/\sqrt{2}&1/\sqrt{2}&0&2\\0&0&0&1\end{bmatrix}$$

c) (8 points)

(i) At$(q_1,q_2)=(0,0)$, $M=\begin{bmatrix}1&0&0&l\\0&0&-1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}$

(ii) $S_1=\begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}$; $S_2=\begin{bmatrix}0\\-1\\0\\\begin{pmatrix}0\\-1\\0\end{pmatrix}\times\begin{pmatrix}0\\0\\q_1\end{pmatrix}\end{bmatrix}=\begin{pmatrix}0\\-1\\0\\q_1\\0\\0\end{pmatrix}$

Question 2 (Show your working clearly for marks to be awarded)

The robot in **Fig. 1** is manipulating a mass, *m* with the end effector {E} under the influence of gravity. Ignore the mass of the linkage structure and treat the motion as a pointed mass at the end effector. State any other assumption you are making while answering the following. Letting m and l be the first and last two digits of your student ID, respectively (example: if ID= 1234567890, then $m = 12$ and $l = 90$),

(i) Write down the potential energy of the system in terms of the joint motion. *(2 Points)*

(ii) Evaluate A, B, C such that the system kinetic energy is expressed as

$$K = A\dot{q}_1{}^2 + B\dot{q}_2{}^2 + C\dot{q}_1\dot{q}_2 \cos q_2$$

(6 Points)

(iii) Obtain an expression of the Lagrangian, *L* in terms of the joint motion. *(2 Points)*

(iv) Obtain the force and torque at the joints in terms of the joint motion. *(8 Points)*

Solution Q2.(i)------2 Points

$$u = mg\Delta = mgl\,cos(q_2)$$

Q2.(ii) ------6 Points

$$k = \frac{1}{2}mv^2, \qquad v^2 = |\dot{P}_3|^2 = \left|\begin{pmatrix} \dot{q}_2 l\,sin(q_2) \\ \dot{q}_1 + q_2 l\,cos(q_2) \end{pmatrix}\right|^2$$

$$\therefore K = \frac{1}{2}m(\dot{q}_2^2 l^2 \sin^2(q_2) + (\dot{q}_1 + q_2\, l \cos q_2)^2)$$

$$= \frac{1}{2}m(\dot{q}_2^2 l^2 \sin^2(q_2) + \dot{q}_1^2 + \dot{q}_2^2 l^2 \cos^2(q_2) + 2\dot{q}_1\dot{q}_2 l \cos(q_2))$$

$$= \frac{m}{2}\dot{q}_1^2 + \frac{m}{2}\dot{q}_2^2 l^2 \sin^2(q_2) + \frac{m}{2}\dot{q}_2^2 l^2 \cos^2(q_2) + m\dot{q}_1\dot{q}_2 l \cos(q_2)$$

$$\therefore K = \frac{m}{2}\dot{q}_1 + \frac{ml^2}{2}\dot{q}_2^2 + ml\dot{q}_1\dot{q}_2\cos(q_2)$$

$$\mathrm{A} = \frac{m}{2},\ B = \frac{ml^2}{2},\ C = ml$$

Q2.(iii) ------2 Points

$$L = k - u = \frac{m}{2}\dot{q}_1^2 + \frac{ml^2}{2}\dot{q}_2^2 + ml\dot{q}_1\dot{q}_2\cos(q_2) - mgl\cos(q_2)$$

Q2.(iv) ------8 Points

$$\begin{pmatrix} f \\ \tau \end{pmatrix} = \begin{bmatrix} \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_1} - \frac{\partial L}{\partial q_1} \\ \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_2} - \frac{\partial L}{\partial q_2} \end{bmatrix}$$

$$\begin{cases} \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_1} = \frac{d}{dt}(m\dot{q}_1 + ml\dot{q}_2\cos(q_2)) = m\ddot{q}_1 + ml\cos(q_2)\ddot{q}_2 - ml\dot{q}_2^2\sin(q_2) \\ \frac{\partial L}{\partial q_1} = 0 \end{cases}$$

$$\therefore f = m\ddot{q}_1 + ml\cos(q_2)\ddot{q}_2 - ml\dot{q}_2^2\sin(q_2)$$

$$\begin{cases} \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_2} = \frac{d}{dt}(ml^2\dot{q}_2 + ml\dot{q}_1\cos(q_2)) = ml^2\ddot{q}_2 + ml\ddot{q}_1\cos(q_2) - ml_2\dot{q}_1\dot{q}_2\sin(q_2) \\ \frac{\partial L}{\partial q_2} = -ml\dot{q}_1\dot{q}_2\sin(q_2) + mgl\sin(q_2) \end{cases}$$

$$\therefore \tau = ml^2\ddot{q}_2 + ml\ddot{q}_1\cos(q_2) - ml_2\dot{q}_1\dot{q}_2\sin(q_2) + ml_2\dot{q}_1\dot{q}_2\sin(q_2) - mgl\sin(q_2)$$

$$= ml\ddot{q}_1\cos(q_2) + ml^2\ddot{q}_2 - mgl\sin(q_2)$$

$$\therefore \begin{bmatrix} f \\ \tau \end{bmatrix} = \begin{bmatrix} m\ddot{q}_1 + ml\cos(q_2)\ddot{q}_2 - ml\dot{q}_2^2\sin(q_2) \\ ml\ddot{q}_1\cos(q_2) + ml^2\ddot{q}_2 - mgl\sin(q_2) \end{bmatrix}$$