ECE 470: Introduction to Robotics Homework 4

Question 1.

You are tasked to design the joint control for cart with a rod as shown below. Using the control partitioning law, and the dynamics formulated in lecture, fill in the required expression (i)-(iv) in the block diagram shown in Figure 1. You may assume insignificant mass for the link *(5 Points)*

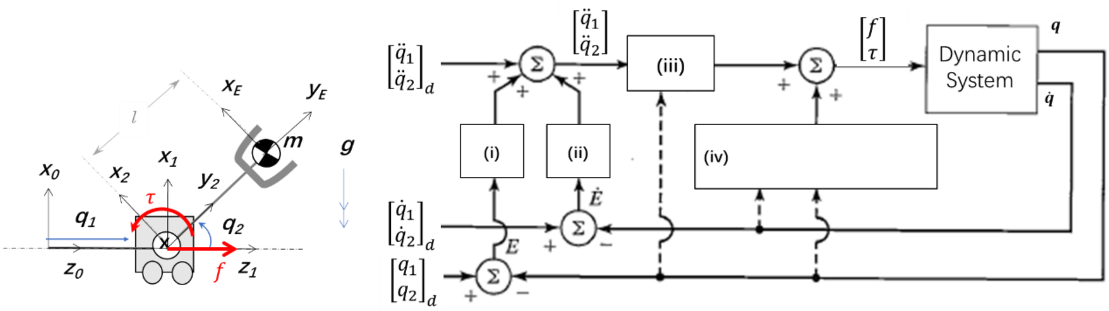


Figure 1.

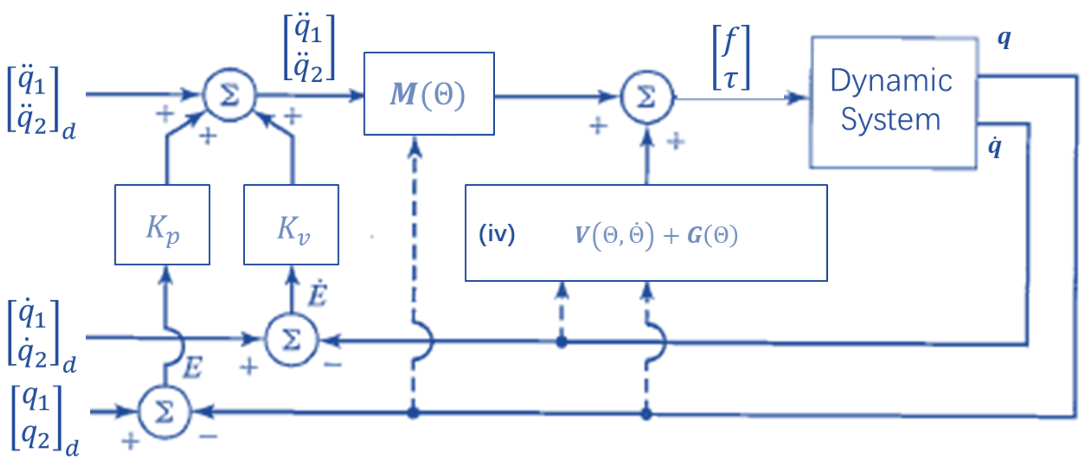
Solution to Question 1

$$\begin{pmatrix} f \\ \tau \end{pmatrix} = \begin{bmatrix} \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot{q}_1} - \dfrac{\partial L}{\partial q_1} \\ \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot{q}_2} - \dfrac{\partial L}{\partial q_2} \end{bmatrix}$$

$$\begin{bmatrix} f \\ \tau \end{bmatrix} = \begin{bmatrix} m\ddot{q}_1 + ml\sin(q_2)\ddot{q}_2 + ml\dot{q}_2^2\cos(q_2) \\ -ml\ddot{q}_1\sin(q_2) - ml^2\ddot{q}_2 + mgl\cos(q_2) \end{bmatrix}$$

$$\boldsymbol{\tau} = \boldsymbol{M}(\mathbf{Q})\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \boldsymbol{V}(\boldsymbol{Q}, \dot{\boldsymbol{Q}}) + \boldsymbol{G}(\boldsymbol{Q})$$

$$\therefore \begin{bmatrix} f \\ \tau \end{bmatrix} = \begin{bmatrix} m & ml\sin(q_2) \\ -ml\sin(q_2) & -ml^2 \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} ml\dot{q}_2^2\cos(q_2) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ mgl\cos(q_2) \end{bmatrix}$$



Question 2.

a) Consider a single-link manipulator as the cart is fixed stationarily shown Figure 2.

i) Formulate the dynamic equation factoring coulomb and viscous force acting on the joint and loading due to gravity. You may assume that viscous and coulomb frictions result in resisting moment $M_v = b_v\dot{\theta}$ and $M_c = b_c\mathrm{sgn}\,(\dot{\theta})$, respectively. *(2 Points)*

ii) Draw the control block diagram illustrating the use of control law partitioning for the non-linearities in the system. *(6 Points)*

You may assume negligible mass distribution for the link and assumed lumped mass at the distal time as shown.

b) A DC motor is installed to drive the arm with motor torque τ_m input to a gear transmission ratio η. Redraw the control diagram incorporating the dynamics of the actuator. *(4 Points)*

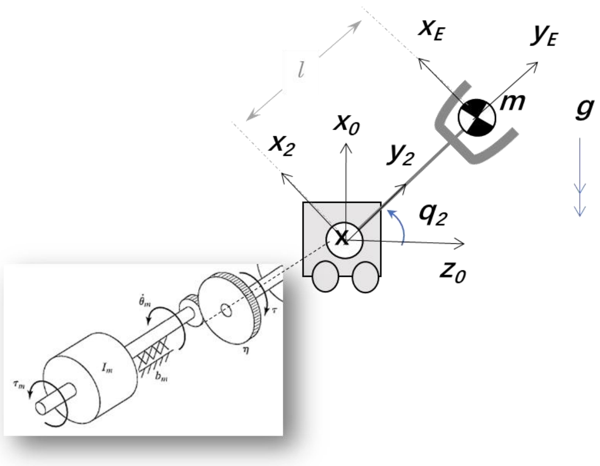


Figure 2.

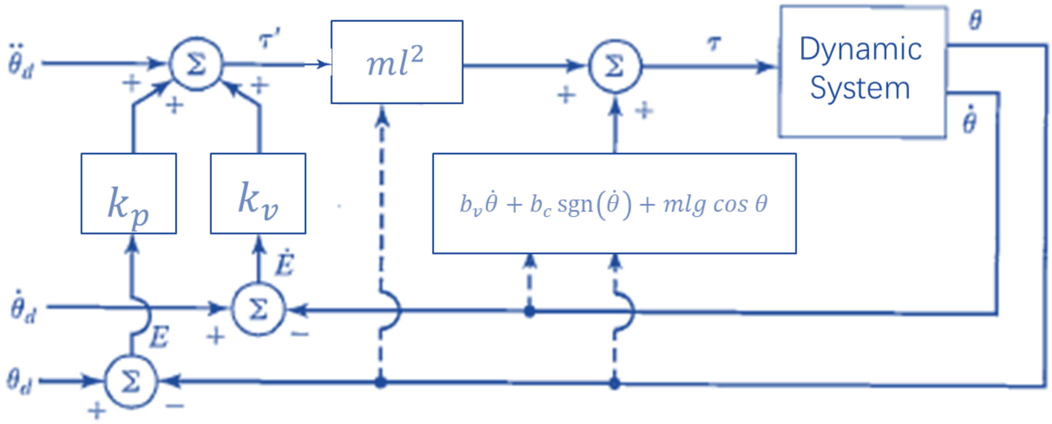
Solution to Question 2:

(a) i)

$$ml^2\ddot{\theta} + b_v\dot{\theta} + b_c\,\mathrm{sgn}(\dot{\theta}) + mlg\cos\theta$$

ii)

$$\tau = ml^2\ddot{\theta} + b_v\dot{\theta} + b_c\,\mathrm{sgn}(\dot{\theta}) + mlg\cos\theta$$



b)

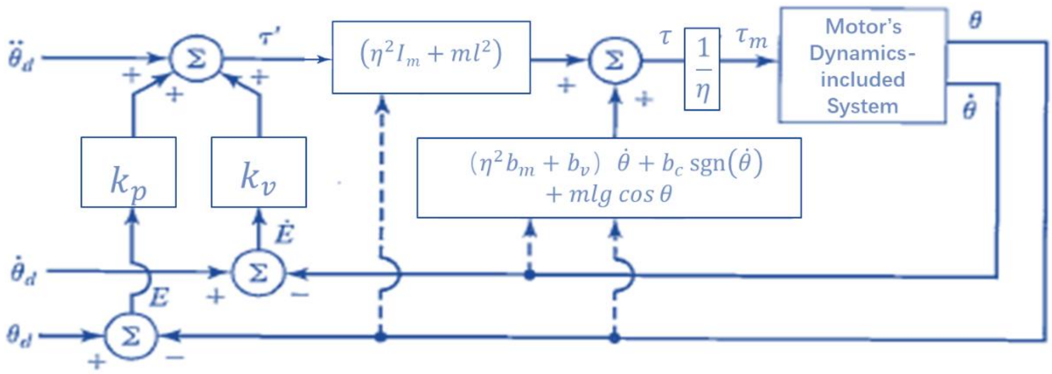
$$\tau - \left(b_v\dot{\theta} + b_c\,\mathrm{sgn}(\dot{\theta}) + mlg\cos\theta\right) - |r_o \times F_m| = ml^2\ddot{\theta}$$

$$\tau = \left(ml^2\ddot{\theta} + b_v\dot{\theta} + b_c\,\mathrm{sgn}(\dot{\theta}) + mlg\cos\theta\right) + |r_o \times F_m|$$

$$\boxed{|r_o \times F_m| = r_o\frac{\tau_m}{r_i} = \eta\tau_m = \eta\left(I_m\ddot{\theta}_m + b_m\dot{\theta}_m\right) = \eta^2\,(I_m\ddot{\theta} + b_m\dot{\theta})}$$

$$\tau = \left(ml^2\ddot{\theta} + b_v\dot{\theta} + b_c\,\mathrm{sgn}(\dot{\theta}) + mlg\cos\theta\right) + \eta^2\,(I_m\ddot{\theta} + b_m\dot{\theta})$$

$$\boxed{\tau = (\eta^2 I_m + ml^2)\ddot{\theta} + (\eta^2 b_m + b_v)\,\dot{\theta} + b_c\,\mathrm{sgn}(\dot{\theta}) + mlg\cos\theta}$$



Question 3.

Explain how you would design a hybrid force-displacement control to clean a glass surface as shown. *(3 Points)*

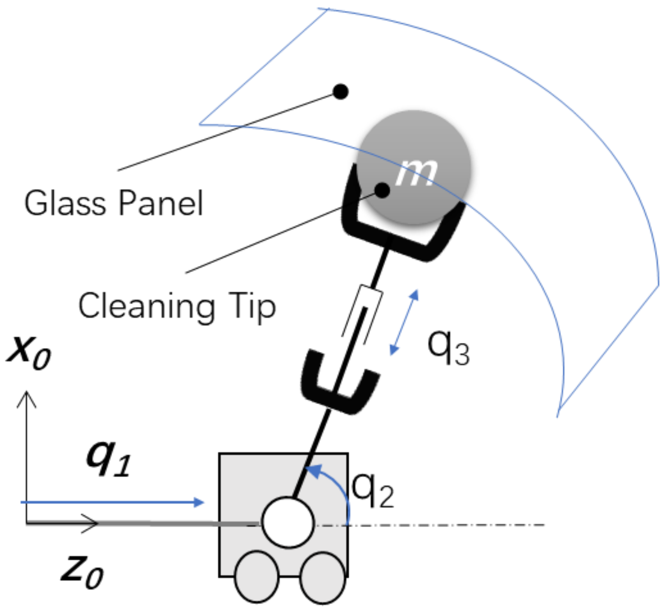


Figure 3

Solution to Question 3:

Considering a planar motion for the cleaning task where the cleaning tip is constrained to the surface of the glass panel with a normal contact force, we can then set the task-space coordinates as $(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\theta})$, where $(\boldsymbol{x}, \boldsymbol{z})$ is the point of contact and $\boldsymbol{\theta}$ is the direction of the normal force vector.

Using hybrid position-force control, we could decouple the joints (q_1, q_2, q_3) to correspond to the 3 independent variables and perform independent joint control where we implement force control with joint 3 to maintain $|\vec{n}|$ and position control with joint 1 and 2 to achieve contact point location at (x, z)

task-space coordinates: $(\boldsymbol{x}, \boldsymbol{z}, \boldsymbol{\theta})$