ECE 470/ ME 445: Introduction to Robotics- Homework 03

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A robot arm is designed as illustrated by the following figure. It can be assumed that the mass distributions of the links are insignificant and can be treated as lumped equivalent masses m_1 and m_2.

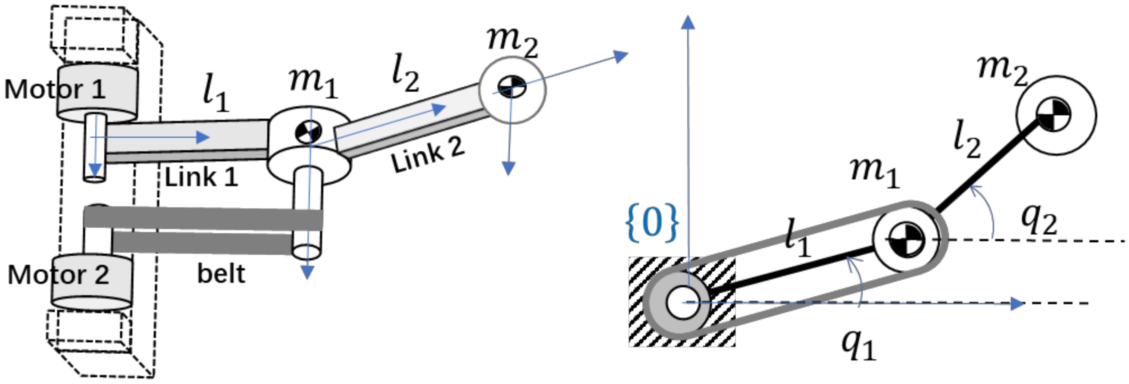


Figure 1

a) Write down the position of masses m_1 and m_2 in terms of q_1 and q_2 referenced from the given frame $\{0\}$. *(2 marks)*

c) Show that the total kinetic energy of the system, K can be written as

$$K = \frac{1}{2}(m_1 l_1^2 + m_2 l_1^2)\dot{q}_1^2 + m_2 l_1 l_2 \cos(q_2 - q_1)\,\dot{q}_1\dot{q}_2 + \frac{1}{2}m_2 l_2^2 \dot{q}_2^2$$ *(4 marks)*

d) Obtain the total potential energy of the system. *(2 marks)*

e) Write down the Lagrangian L. *(2 marks)*

f) Obtain the dynamic equations relating the torque output (τ_1, τ_2) of Motor 1 & 2 with the motion of the masses in $q_{1,2}, \dot{q}_{1,2}, \ddot{q}_{1,2}$ *(5 marks)*

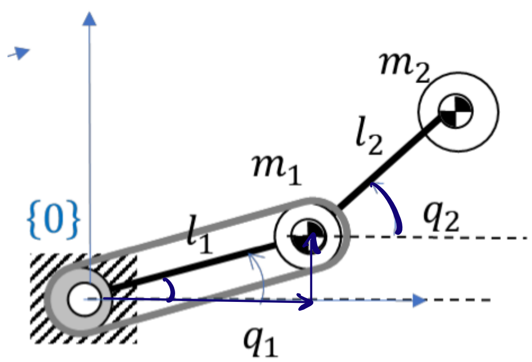
(c): Proof: $K = K_1 + K_2$

$K_1 = \frac{1}{2} I_1 \omega_1^2 = \frac{1}{2}(m_1 l_1^2)\dot{q}_2^2$

$K_2 = \frac{1}{2}(m_2 l_1^2)\dot{q}_1^2 + m_2 l_1 l_1 \cos(q_2 - q_1)\dot{q}_1\dot{q}_2 + \frac{1}{2} m_2 l_2^2 \dot{q}_2^2$

So $K = \ldots$ ☑ Proved

(a):



m_1: $x_1 = l_1 \cos q_1$
$y_1 = l_1 \sin q_1$

$\vec{P}_1 = \begin{bmatrix} l_1 \cos q_1 \\ l_1 \sin q_1 \end{bmatrix}$,

m_2: $x_2 = x_1 + l_2 \cos q_2$
$y_2 = y_1 + l_2 \sin q_2$

$\vec{P} = \begin{bmatrix} l_1 \cos q_1 + l_2 \cos q_2 \\ l_1 \sin q_1 + l_2 \sin q_2 \end{bmatrix}$

(d):

$E_P = m_1 g l_1 \sin q_1 + m_2 g l_2 \sin q_2$

(e): $\mathcal{L} = K - P$

$$= \frac{1}{2}(m_2 l_1^2 + m_1 l_2^2)\dot{q}_1^2 + m_2 l_1 l_1 \cos(q_2 - q_1)\dot{q}_1\dot{q}_2 + \frac{1}{2} m_2 l_2^2 \dot{q}_2^2$$

$$- m_1 g l_1 \sin q_1 + m_2 g L_2 \sin q_2$$

(f): *f)* Obtain the dynamic equations relating the torque output (τ_1, τ_2) of Motor 1 & 2 with the motion of the masses in $q_{1,2}, \dot{q}_{1,2}, \ddot{q}_{1,2}$ ***(5 marks)***

Using Euler-Lagrange equation,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \tau_i$$

For $i = 1, 2$, where τ_i is torque provided by motor i

$$\Rightarrow \tau_1 = (m_1 + m_2) L_1^2 \ddot{q}_1 + m_1 L_1 L_2 \cos(q_2 - q_1)\ddot{q}_2$$

$$- m_2 L_1 L_2 \sin(q_2 - q_1)\dot{q}_2 + (m_1 + m_2) g L_1 \cos q_1$$

$$\tau_2 = m_2 L_2^2 \ddot{q}_2 + m_2 L_1 L_2 \cos(q_1 - q_2)\ddot{q}_1$$

$$- m_2 L_1 L_2 \sin(q_1 - q_2)\dot{q}_1^2 + m_2 g l_2 \cos q_2$$

Question 2.

Compare your answer in Question 1 with that of the example discussed in class (Example 5.2 or Section 6.7, Equation (6.58), Reference Textbook J. Craig 3rd Ed.). shown in Figure 2.
State and comment on the differences. ***(5 Points)***

$$\tau_1 = m_2 l_2^2(\ddot\theta_1+\ddot\theta_2) + m_2 l_1 l_2 \cos\theta_2 (2\ddot\theta_1+\ddot\theta_2) + (m_1+m_2) l_1^2 \ddot\theta_1 - m_2 l_1 l_2 \sin\theta_2 \dot\theta_2^2 - 2 m_2 l_1 l_2 \sin\theta_2 \dot\theta_1 \dot\theta_2 + m_2 g l_2 \cos\theta_{12} + (m_1+m_2) g l_1 \cos\theta_1$$

$$\tau_2 = m_2 l_1 l_2 \cos\theta_2 \ddot\theta_1 + m_2 l_1 l_2 \sin\theta_2 \dot\theta_1^2 + m_2 g l_2 \cos\theta_{12} + m_2 l_2^2(\ddot\theta_1+\ddot\theta_2)$$

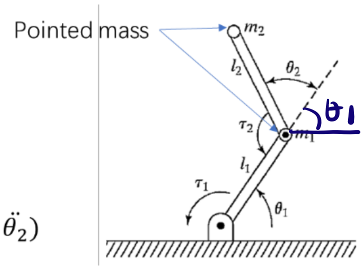
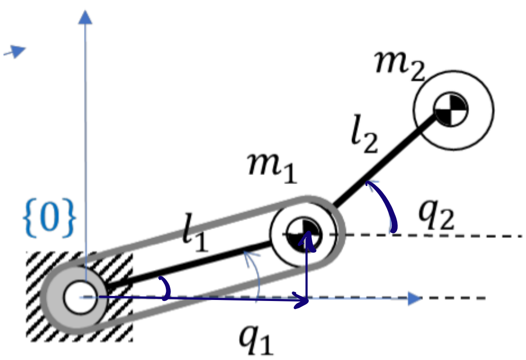


Figure 2

Based on Q(1), here $q_1 = \theta_1$, $q_2 = \theta_1 + \theta_2$. Q2 can be treated as a special case



So the torque differs iff q_2 involves