Q: What is a State-Space Model (SSM) used for in time series forecasting?

A: SSMs model time series data by capturing both the system's dynamics and observation uncertainty, using hidden latent states that evolve recursively over time.

Q: What are the two main components of a State-Space Model?

A: The state equation (process model) and the observation equation (measurement model).

Q: Define the state equation in a State-Space Model.

A: The state equation describes how the hidden state vector (X_k) evolves based on the previous state, inputs, and a noise term, formalized as (X_{k+1} = f_{ heta_x}(X_k, u_k, \eta_k)).

Q: What role does the observation equation play in a State-Space Model?

A: It maps the hidden state vector to the observable data (Y_k), modeling how latent dynamics are reflected in measurements, formulated as (Y_k = g_{ heta_y}(X_k, u_k, \epsilon_k)).

Q: How do Hidden Markov Models (HMMs) relate to State-Space Models?

A: HMMs are a specialized type of SSM where hidden states follow a Markov process with discrete possible states, often used for systems with distinct regimes.

Q: What is the state transition matrix (P) in an HMM?

A: (P) is a matrix where each element (p_{ij}) represents the probability of transitioning from one state to another, maintaining a row-stochastic property.

Q: Describe the role of emission probabilities in an HMM.

A: Emission probabilities define the likelihood of observing a specific output given a hidden state, organized into an emission matrix.

Q: What is a Linear State-Space Model (LSSM)?

A: An LSSM uses linear transformations to model time-series data, balancing complexity and tractability, with continuous hidden states linked to outputs through matrix operations.

Q: How is the state equation structured in an LSSM?

A: It is expressed as ($X_{k+1} = A_k X_k + B_k u_k + \eta_k$), where (A_k) and (B_k) represent the system dynamics and input effects, respectively.

Q: What is the measurement equation in an LSSM?

A: The measurement equation relates the state vector to observed data through ($Y_k = C_k X_k + D_k u_k + \epsilon_k$), where (C_k) maps latent states to observations.

Q: What is the Markov property in State-Space Models?

A: The Markov property implies that the system’s future evolution depends solely on the current hidden state, not on the full history of past states.

Q: How is an AR(p) model represented as a Linear State-Space Model (LSSM)?

A: An AR(p) model can be represented as an LSSM by defining a state vector with lagged values and using a state transition matrix to evolve the state vector over time.

Q: What is the purpose of similarity transformations in LSSMs?

A: Similarity transformations allow different internal state representations that yield the same input-output behavior, showing that the observable dynamics are invariant under these transformations.

Q: Explain the concept of controllable and observable canonical forms in LSSMs.

A: Controllable and observable canonical forms are specific configurations of system matrices that simplify analysis, with controllable form focused on input control and observable form on reconstructing states from outputs.

Q: What is the primary goal in parameter identification for LSSMs?

A: The goal is to estimate system matrices (A, B, C, D) and initial state (x_0) to best fit observed input-output data, often using gradient descent.

Q: How does Backpropagation Through Time (BPTT) assist in training LSSMs?

A: BPTT calculates gradients over sequential data by unrolling dependencies, enabling the optimization of parameters in LSSMs.

Q: What is the primary challenge when using gradient-based methods on LSSMs or RNNs?

A: The vanishing and exploding gradient problem, often linked to the eigenvalues of the state matrix, which affects stability during training.

Q: Describe the forward pass in the BPTT algorithm for LSSMs.

A: In the forward pass, the model propagates input through the state-space equations to generate predictions at each time step.

Q: What is a Jacobian tensor in the context of BPTT for LSSMs?

A: It represents the gradient of state variables with respect to parameters, essential for calculating parameter updates during backpropagation.

Q: How is the total loss function defined in LSSMs during training?

A: The total loss is the average squared error across time steps, measuring the alignment between predicted and observed outputs over a specified time horizon.