

# CS 5/7330

Transactions : Serializability

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# Concurrency control

- Why concurrency?
  - **increased processor and disk utilization**, leading to better transaction *throughput*: one transaction can be using the CPU while another is reading from or writing to the disk
  - **reduced average response time** for transactions: short transactions need not wait behind long ones.

# Concurrency control

- Why concurrency control?
  - **Shared resources.** Many transaction may want to access the same object/tuple.
  - **Isolation.** One of the key requirement of transactions

# Concurrency control -- schedule

- *Schedules* – sequences that indicate the chronological order in which instructions of concurrent transactions are executed
  - a schedule for a set of transactions must consist of all instructions of those transactions
  - must preserve the order in which the instructions appear in each individual transaction.
- Assumption: at any time, only one operation from one transaction can be executed
- However, DBMS may interleave operations from multiple transactions

# Concurrency control -- schedule

- *Serial schedule*: schedules that does not allow interleaving between transactions (i.e. one transaction finishes before the other begin)
- Equivalent schedules: two schedules are equivalent if they “produce the same results”
  - Still need to define what it means by “produce the same results”

# Concurrency control – schedule (example)

- Let  $T_1$  transfer \$50 from  $A$  to  $B$ , and  $T_2$  transfer 10% of the balance from  $A$  to  $B$ . The following is a serial schedule in which  $T_1$  is followed by  $T_2$ .

$T_1$	$T_2$
read( $A$ ) $A := A - 50$ write( $A$ ) read( $B$ ) $B := B + 50$ write( $B$ )	read( $A$ ) $temp := A * 0.1$ $A := A - temp$ write( $A$ ) read( $B$ ) $B := B + temp$ write( $B$ )

Schedule 1

# Concurrency control – schedule (example)

- Let  $T_1$  and  $T_2$  be the transactions defined previously. The following schedule is not a serial schedule, but it is *equivalent* to Schedule 1.

$T_1$	$T_2$
read( $A$ ) $A := A - 50$ write( $A$ )	
	read( $A$ ) $temp := A * 0.1$ $A := A - temp$ write( $A$ )
read( $B$ ) $B := B + 50$ write( $B$ )	
	read( $B$ ) $B := B + temp$ write( $B$ )

Schedule 3



# Concurrency control – schedule (example)

- The following concurrent schedule does not preserve the value of the sum  $A + B$ .

$T_1$	$T_2$
read( $A$ ) $A := A - 50$	read( $A$ ) $temp := A * 0.1$ $A := A - temp$ write( $A$ ) read( $B$ )
write( $A$ ) read( $B$ ) $B := B + 50$ write( $B$ )	         $B := B + temp$ write( $B$ )

Schedule 4

# Concurrency control – big question

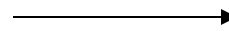
- Why is schedule 3 equivalent to schedule 1, but schedule 4 is not?
  - Conflicts?
  - Order of conflicts?
  - Any general rules we can apply?

## Serializable schedule – example (2)

- Consider the fund transfer operation (last lecture)

1. Find tuple for x's account (database query)
2. Read x's account info into main memory
3. Check if x have at least \$k
4. Subtract \$k from x's account
5. Write x's new balance back to the database (database update)
6. Find tuple for y's account (database query)
7. Read y's account info into main memory
8. Add \$k to y's account
9. Write y's new balance to the database (database update)

*Rewrite  
Using class  
notation*



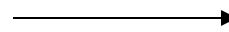
1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 - k$
3.  $\text{Write}(X, A1)$
4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 + k$
6.  $\text{Write}(Y, A2)$

## Serializable schedule – example (2)

- Consider the dividend operation (last lecture)

1. Find tuple for x's account  
(database query)
2. Read x's account info into  
main memory
3. Add 1% to x's account
4. Write x's new balance back  
to the database (database  
update)
5. Find tuple for y's account  
(database query)
6. Read y's account info into  
main memory
7. Add 1% to y's account
8. Write y's new balance back  
to the database (database  
update)

*Rewrite  
Using class  
notation*



1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 * 1.01$
3.  $\text{Write}(X, A1)$
4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 * 1.01$
6.  $\text{Write}(Y, A2)$

## Serializable schedule – example (2)

- Suppose x has \$100, y has \$200
- Consider two operations
  - x transfer \$50 to y
  - Dividend
- For serial schedules
  - If transfer comes before dividend
    - X : 100 -> 50 -> 50.5
    - Y : 200 -> 250 -> 252.5
  - If dividend comes before transfer
    - X : 100 -> 101 -> 51
    - Y : 200 -> 202 -> 252
  - In both case,  $X + Y = 303$

## Serializable schedule – example (2)

- But with the following schedule

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 - k$
3.  $\text{Write}(X, A1)$

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 * 1.01$
3.  $\text{Write}(X, A1)$
4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 * 1.01$
6.  $\text{Write}(Y, A2)$

4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 + k$
6.  $\text{Write}(Y, A2)$

$X : 100 \rightarrow 50 \rightarrow 50.5; Y : 200 \rightarrow 202 \rightarrow 252; X+Y = 302.5$

## Serializable schedule – example (2)

- What cause the problem?
  - Contention of resources?
  - Interleaving?
- Is interleaving always bad?

## Serializable schedule – example (2)

- But with the following schedule

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 - k$
3.  $\text{Write}(X, A1)$

4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 + k$
6.  $\text{Write}(Y, A2)$

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 * 1.01$
3.  $\text{Write}(X, A1)$

4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 * 1.01$
6.  $\text{Write}(Y, A2)$

$X : 100 \rightarrow 50 \rightarrow 50.5; Y : 200 \rightarrow 250 \rightarrow 252.5; X+Y = 303$

In this case, interleaving is ok!



## Serializable schedule – example (2)

- Let's change slightly:

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 - k$
3.  $\text{Write}(X, A1)$

4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 + k$
6.  $\text{Write}(Y, A2)$

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 * 1.01$
3.  $\text{Write}(X, A1)$
4.  $A2 \leftarrow \text{Read}(Y)$

5.  $A2 \leftarrow A2 * 1.01$
6.  $\text{Write}(Y, A2)$

$X : 100 \rightarrow 50 \rightarrow 50.5; Y : 200 \rightarrow 250 \rightarrow 202; X+Y = 252.5$

In this case, interleaving is very bad!

## Serializable schedule – example (2)

- Let's change slightly (again):

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 - k$
3.  $\text{Write}(X, A1)$

4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 + k$
6.  $\text{Write}(Y, A2)$

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 * 1.01$

3.  $\text{Write}(X, A1)$
4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 * 1.01$
6.  $\text{Write}(Y, A2)$

$X : 100 \rightarrow 50 \rightarrow 50.5$ ;  $Y : 200 \rightarrow 250 \rightarrow 252.5$ ;  $X+Y = 303$

In this case, interleaving is good again!

## Serializable schedule – example (2)

- What's going on here?
  - Interleaving can be very bad.
  - However, some interleaving does not cause problems.
  - How can we determine what kind of interleaving is “nice”?

## Serializable schedule – example (2)

- Notice in example
  - The value of X (and Y) is changed twice
  - Let's call the values
    - **Old value** (before any change)
    - **Intermediate value** (after one change)
    - **Final value** (after all changes)

## Serializable schedule – example (2)

- But with the following schedule

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 - k$
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1.  $A1 \leftarrow \text{Read}(X)$
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$X : 100 \rightarrow 50 \rightarrow 50.5; Y : 200 \rightarrow 202 \rightarrow 252; X+Y = 302.5$

## Serializable schedule – example (2)

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6.  $\text{Write}(Y, A2)$

4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 + k$
6.  $\text{Write}(Y, A2)$

Notice that the two transaction use inconsistent values as input.

But: how to formalize this notion?

# Serializability

- How to formalize the notion?
  - One can look at final results
  - If the schedule produce the same result as a serial schedule, then it's fine.
  - However, this may be due to luck and/or “commutative” operators

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 - k$
3.  $\text{Write}(X, A1)$

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 + m$
3.  $\text{Write}(X, A1)$
4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 - m$
6.  $\text{Write}(Y, A2)$

4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 + k$
6.  $\text{Write}(Y, A2)$

A better notion is needed

# Conflict serializability

- Suppose two transactions ( $T_1$ ,  $T_2$ ) want to operate on the same data object ( $X$ )
- Four possible scenarios
  - $T_1$  Read( $X$ ),  $T_2$  Read( $X$ )
  - $T_1$  Read( $X$ ),  $T_2$  Write( $X$ )
  - $T_1$  Write( $X$ ),  $T_2$  Read( $X$ )
  - $T_1$  Write( $X$ ),  $T_2$  Write( $X$ )
- How does the order of these operations affect the results of the transactions?



# Conflict serializability

- $T_1$  Read(X),  $T_2$  Read(X)
  - No effect.
- $T_1$  Read(X),  $T_2$  Write(X)
  - Order will determine what value of X  $T_1$  reads
  - Affect the results of  $T_1$
- $T_1$  Write(X),  $T_2$  Read(X)
  - Order will determine what value of X  $T_2$  reads
  - Affect the results of  $T_2$
- $T_1$  Write(X),  $T_2$  Write(X)
  - No effect on  $T_1$  and  $T_2$
  - But affect on the next transaction that read X
- Thus, in 2<sup>nd</sup> to 4<sup>th</sup> case order matters
- We denote that the pair(s) of operations are **in conflict**

# Conflict serializability

- What's conflict have to do with it?
- Consider a schedule of two transactions ( $T_1, T_2$ )
  - Suppose  $T_1$  execute operation  $I_1$ , and than  $T_2$  execute operation  $I_2$
  - If  $I_1$  and  $I_2$  has conflict
    - Then swapping them implies potential problem
  - Else, (we claim) the results are not affected

# Conflict serializability -- example

- Remember this schedule?

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 - k$
3.  $\text{Write}(X, A1)$

4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 + k$
6.  $\text{Write}(Y, A2)$

- i.  $A1 \leftarrow \text{Read}(X)$
- ii.  $A1 \leftarrow A1 * 1.01$
- iii.  $\text{Write}(X, A1)$

- iv.  $A2 \leftarrow \text{Read}(Y)$
- v.  $A2 \leftarrow A2 * 1.01$
- vi.  $\text{Write}(Y, A2)$

$X : 100 \rightarrow 50 \rightarrow 50.5; Y : 200 \rightarrow 250 \rightarrow 252.5; X+Y = 303$

In this case, interleaving is ok!

# Conflict serializability -- example

- Remember this schedule?

1.  $A1 \leftarrow \text{Read}(X)$
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- vi.  $\text{Write}(Y, A2)$

(4) And (iii) are not in conflict, so can swap

# Conflict serializability -- example

- Remember this schedule?

1.  $A1 \leftarrow \text{Read}(X)$
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iii.  $\text{Write}(X, A1)$

iv.  $A2 \leftarrow \text{Read}(Y)$

v.  $A2 \leftarrow A2 * 1.01$

vi.  $\text{Write}(Y, A2)$

(4) And (ii) are not in conflict, so can swap

# Conflict serializability -- example

- Remember this schedule?

1.  $A1 \leftarrow \text{Read}(X)$
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- iii.  $\text{Write}(X, A1)$
- iv.  $A2 \leftarrow \text{Read}(Y)$
- v.  $A2 \leftarrow A2 * 1.01$
- vi.  $\text{Write}(Y, A2)$

(4) And (i) are not in conflict, so can swap

# Conflict serializability -- example

- Remember this schedule?

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 - k$
3.  $\text{Write}(X, A1)$
4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 + k$

6.  $\text{Write}(Y, A2)$

- i.  $A1 \leftarrow \text{Read}(X)$
- ii.  $A1 \leftarrow A1 * 1.01$
- iii.  $\text{Write}(X, A1)$
- iv.  $A2 \leftarrow \text{Read}(Y)$
- v.  $A2 \leftarrow A2 * 1.01$
- vi.  $\text{Write}(Y, A2)$

Similarly (5) And (i) – (iii) are not in conflict, so can swap

# Conflict serializability -- example

- Remember this schedule?

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 - k$
3.  $\text{Write}(X, A1)$
4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 + k$
6.  $\text{Write}(Y, A2)$

- i.  $A1 \leftarrow \text{Read}(X)$
- ii.  $A1 \leftarrow A1 * 1.01$
- iii.  $\text{Write}(X, A1)$
- iv.  $A2 \leftarrow \text{Read}(Y)$
- v.  $A2 \leftarrow A2 * 1.01$
- vi.  $\text{Write}(Y, A2)$

Similarly (6) And (i) – (iii) are not in conflict, so can swap

We obtain a serial schedule by this transformation (swapping process)



# Conflict serializability -- example

- Now, remember this schedule?

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 - k$
3.  $\text{Write}(X, A1)$

- i.  $A1 \leftarrow \text{Read}(X)$
- ii.  $A1 \leftarrow A1 * 1.01$
- iii.  $\text{Write}(X, A1)$
- iv.  $A2 \leftarrow \text{Read}(Y)$
- v.  $A2 \leftarrow A2 * 1.01$
- vi.  $\text{Write}(Y, A2)$

4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 + k$
6.  $\text{Write}(Y, A2)$

$X : 100 \rightarrow 50 \rightarrow 50.5; Y : 200 \rightarrow 202 \rightarrow 252; X+Y = 302.5$

# Conflict serializability -- example

- Now, remember this schedule?

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 - k$
3.  $\text{Write}(X, A1)$

- i.  $A1 \leftarrow \text{Read}(X)$
- ii.  $A1 \leftarrow A1 * 1.01$
- iii.  $\text{Write}(X, A1)$
- iv.  $A2 \leftarrow \text{Read}(Y)$
- v.  $A2 \leftarrow A2 * 1.01$
- vi.  $\text{Write}(Y, A2)$

4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 + k$
6.  $\text{Write}(Y, A2)$

(3) And (i) has conflict, so can't swap

(4) And (vi) has conflict, so can't swap

# Conflict serializability -- example

- Can you work through this case?

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 - k$
3.  $\text{Write}(X, A1)$

4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 + k$
6.  $\text{Write}(Y, A2)$

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 * 1.01$
3.  $\text{Write}(X, A1)$
4.  $A2 \leftarrow \text{Read}(Y)$

5.  $A2 \leftarrow A2 * 1.01$
6.  $\text{Write}(Y, A2)$

$X : 100 \rightarrow 50 \rightarrow 50.5; Y : 200 \rightarrow 250 \rightarrow 202; X+Y = 250.5$

In this case, interleaving is very bad!

# Conflict serializability -- example

- From previous slides, it seems
  - A schedule can be transformed to a serial schedule  $\Rightarrow$  good! (achieve isolation)
  - A schedule cannot be transformed to a serial schedule  $\Rightarrow$  bad! (do not achieve isolation)
- Can we generalize?
  - Yes.

# Conflict serializability -- definitions

- Schedule: – sequences that indicate the chronological order in which instructions of concurrent transactions are executed
- Complete schedule: Schedule that contain commit/abort decision for each transaction in the schedule
- Serial schedule: A schedule where there is no interleaving of operations by multiple transactions.
  - Denoted by  $\langle T_1, T_2, \dots, T_n \rangle$

# Conflict serializability -- definitions

- Given a schedule  $S$ , let  $p$  and  $q$  be two operations in  $S$ , we write  $p <_S q$  if  $p$  occurs before  $q$  in  $S$ .
  - We write  $<$  instead of  $<_S$  if the schedule is understood from context.
- We use a subscript to indicate the transaction that issues an operation, e.g.,  $p_i$  is an operation issued by transaction  $T_i$ .

# Conflict serializability -- definitions

- Conflict equivalent transformation: swapping *adjacent* operation on a schedule which does not conflict
- Conflict equivalent: Two schedules  $S$  and  $S'$  are conflict equivalent if  $S$  can be transformed to  $S'$  by successive conflict equivalent transformations

# Conflict serializability -- definitions

- Given a schedule  $S$ , the *committed projection* of  $S$ , denoted by  $C(S)$ , is the schedule obtained from  $S$  by deleting all operations that do not belong to transactions committed in  $S$ .



# Conflict serializability -- definitions

- Conflict serializability: a schedule  $S$  is **conflict serializable** if it is conflict equivalent to a serial schedule
- Example of a schedule that is not conflict serializable:

$T_3$	$T_4$
<b>read</b> ( $Q$ )	
	<b>write</b> ( $Q$ )
<b>write</b> ( $Q$ )	

We are unable to swap instructions in the above schedule to obtain either the serial schedule  $\langle T_3, T_4 \rangle$ , or the serial schedule  $\langle T_4, T_3 \rangle$ .

# Test for serializability

- Consider the following schedules:

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 - k$
3.  $\text{Write}(X, A1)$

- i.  $A1 \leftarrow \text{Read}(X)$
- ii.  $A1 \leftarrow A1 * 1.01$
- iii.  $\text{Write}(X, A1)$
- iv.  $A2 \leftarrow \text{Read}(Y)$
- v.  $A2 \leftarrow A2 * 1.01$
- vi.  $\text{Write}(Y, A2)$

4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 + k$
6.  $\text{Write}(Y, A2)$

Not serializable

1.  $A1 \leftarrow \text{Read}(X)$
2.  $A1 \leftarrow A1 - k$
3.  $\text{Write}(X, A1)$

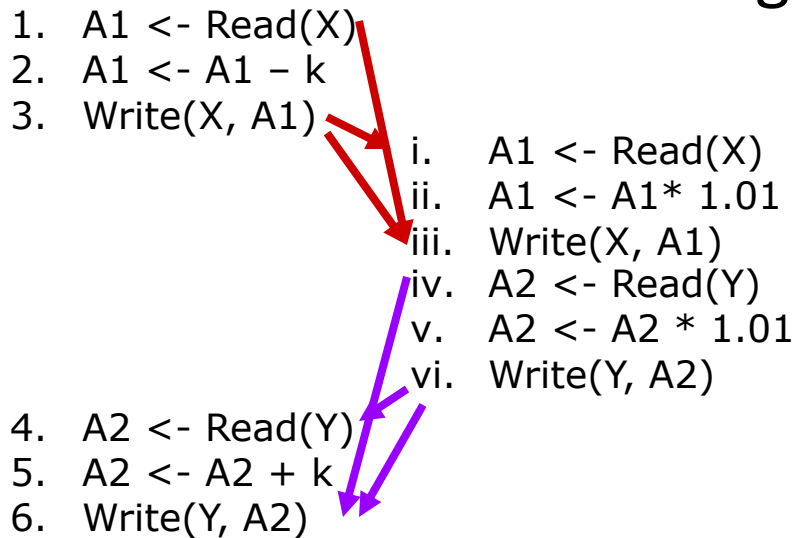
4.  $A2 \leftarrow \text{Read}(Y)$
5.  $A2 \leftarrow A2 + k$
6.  $\text{Write}(Y, A2)$

Serializable

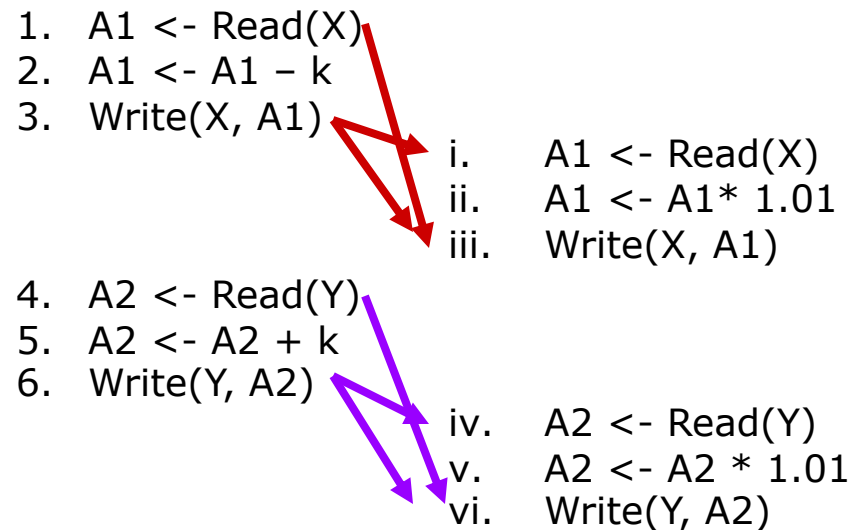
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- v.  $A2 \leftarrow A2 * 1.01$
- vi.  $\text{Write}(Y, A2)$

# Test for serializability

- Label all the conflicting operations



Not serializable



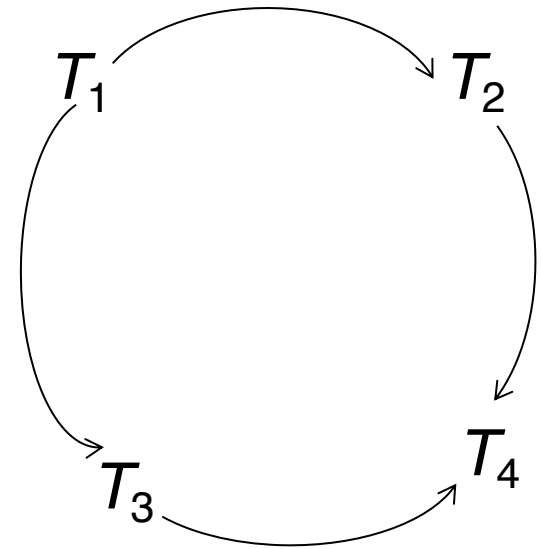
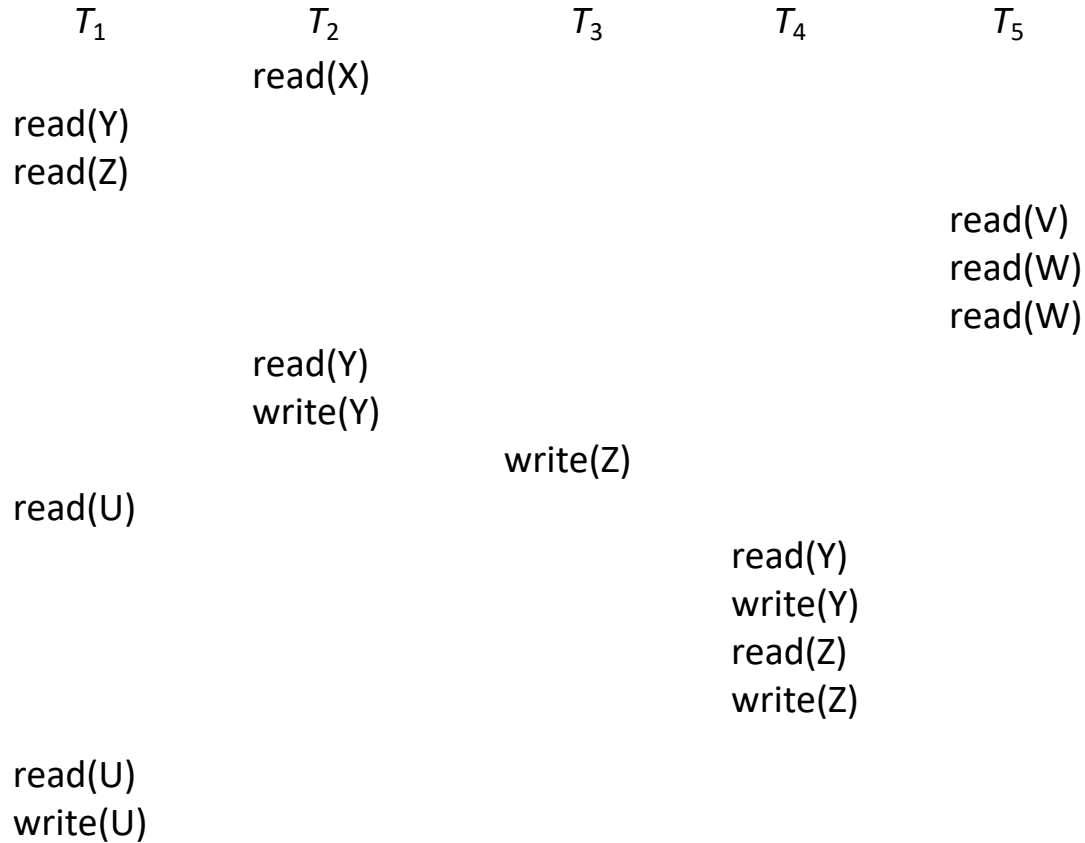
Serializable

What is the difference?

# Test for serializability

- The *serialization graph* ( $SG$ ) (or the precedence graph used in the textbook) for a schedule  $S$ , denoted  $SG(S)$ , is a directed graph  $(V, E)$  such that:
  - $V$  (nodes) = the set of transactions that are *committed* in  $S$ .
  - $E$  (edges) =  $T_i \rightarrow T_j$  ( $i \neq j$ ) if one of  $T_i$ 's operation precedes and conflicts with one of  $T_j$ 's operations in  $S$ .
- Theorem: A schedule is conflict serializable iff its serialization graph is acyclic

# Test of serializability -- example



# Test of serializability

- Proof: (if)
  - $SG(S)$  is acyclic  $\Rightarrow$  there exists a topological order of the committed transactions in  $S$ .
  - W.l.o.g., let  $S' = T_1, T_2, \dots, T_m$  be such an order. We can show that  $C(S) \equiv S'$ .
  - Let  $p_i$  and  $q_j$  ( $i \neq j$ ) be two conflicting operations in  $S$  such that  $T_i$  and  $T_j$  are committed in  $S$ .
  - Clearly,  $p_i$  &  $q_j \in C(S)$ .
  - If  $p_i <_S q_j$ , we have an edge  $T_i \rightarrow T_j$  in  $SG(S)$ .
  - Therefore,  $T_i$  must precede  $T_j$  in  $S'$ . That is, all operations of  $T_i$  appear before all operations of  $T_j$  in  $S'$ .
  - Hence,  $p_i <_{S'} q_j$ .

# Test of serializability

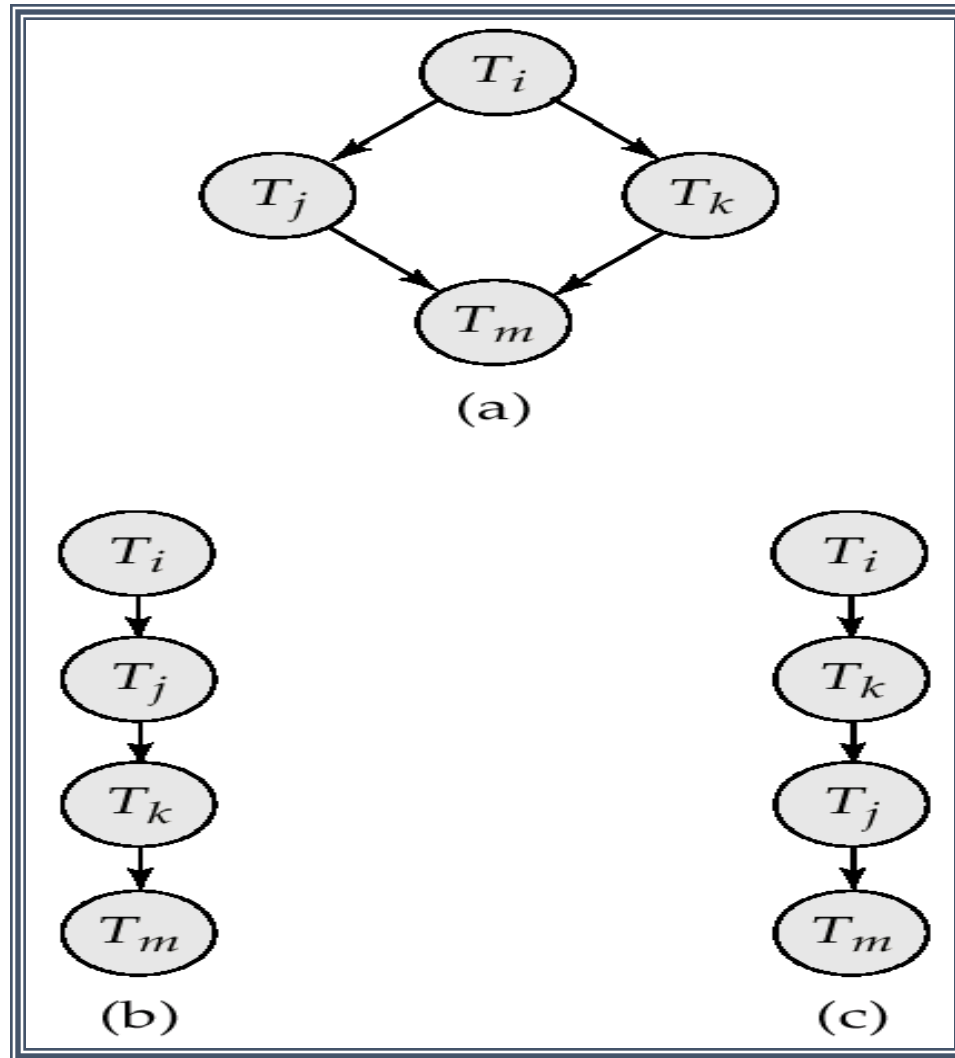
- Proof: (only if)
  - $S$  is serializable  $\Rightarrow C(S) \equiv$  to some serial schedule  $S'$ .
  - If an edge  $T_i \rightarrow T_j$  exists in  $SG(S)$ , then,  $T_i$  must appear before  $T_j$  in  $S'$ .
  - Now, if  $SG(S)$  has a cycle, w.l.o.g., let it be  $T_1 \rightarrow T_2 \rightarrow \dots \rightarrow T_k \rightarrow T_1$ .
  - We have  $T_1$ 's operations must appear before  $T_2$ 's operations, which must appear before  $T_3$ 's operations, ...,  $T_k$ 's operations must appear before  $T_1$ 's operations.
  - A contradiction.

# Test of serializability

- Cycle-detection algorithms exist which take order  $n^2$  time, where  $n$  is the number of vertices in the graph.
  - (Better algorithms take order  $n + e$  where  $e$  is the number of edges.)
- If precedence graph is acyclic, the serializability order can be obtained by a *topological sorting* of the graph. This is a linear order consistent with the partial order of the graph.



# Example of topological sort



# Recoverability

- Abort transactions caused another problem:
- Suppose  $T_9$  commits.
- What happen when  $T_8$  has to abort after read(B)?
- $T_9$  will in fact be reading an inconsistant value
- Can be problematic (e.g. if  $T_9$  print the value of A)

$T_8$	$T_9$
read(A) write(A)  read(B)	   read(A)

# Recoverability

- We say that  $T_i$  reads  $x$  from  $T_j$  in a schedule  $S$  if
  - $W_j[x] <_S R_i[x]$ , ( $T_i$  read an  $x$  value written by  $T_j$ )
  - $A_j$  not  $<_S R_i[x]$ , ( $T_j$  has not aborted when  $T_i$  read  $x$ )
  - If  $\exists W_k[x]$  s.t.  $W_j[x] <_S W_k[x] <_S R_i[x]$ , then  $A_k <_S R_i[x]$ . (Any transaction that update  $x$  between  $T_j$  write  $x$  and  $T_i$  read  $x$  is aborted before the read – that means the value of  $x$  read by  $T_i$  is actually written by  $T_j$ )
- We say that  $T_i$  reads from  $T_j$  in  $S$  if  $T_i$  reads some data item from  $T_j$  in  $S$ .

# Recoverability

- A schedule  $S$  is called *recoverable* if, whenever  $T_i$  reads from  $T_j$  ( $i \neq j$ ) in  $S$  and  $C_i \in S$  then  $C_j < C_i$ 
  - I.e., each transaction commits after the commitment of all transactions (other than itself) from which it read from
- This implies all aborted transaction will not make values that are read by committed transaction obsolete/inconsistent.

# Recoverability

- Even for recoverable schedule, trouble may still bestows.
- Suppose none of the transactions committed yet.
- Suppose T10 aborts after read(A)
- T11 needs to be aborted (as it read values from T10)
- T12 needs to be aborted too
- Cascade aborts

$T_{10}$	$T_{11}$	$T_{12}$
read(A) read(B) write(A)	read(A) write(A)	read(A)

# Recoverability

- We say that a schedule  $S$  *avoids cascading abort* if, when  $T_i$  reads  $x$  from  $T_j$  ( $i \neq j$ ),  $C_j < R_i[x]$ .
  - I.e., a transaction may read only those values that are written by committed transaction or by itself.
- A schedule  $S$  is *strict* if whenever  $W_j[x] < O_i[x]$  ( $i \neq j$ ), either  $A_j < O_i[x]$  or  $C_j < O_i[x]$  where  $O_i[x]$  is  $R_i[x]$  or  $W_i[x]$ .
  - I.e., no data item may be read or written until the transaction that previously wrote into it terminates, either by aborting or by committing.

# Serializability – In practice

- Scheduling is usually not up to the DBMS
  - Operating systems do it, so why spend time reinventing the wheel?
- Detection of serializability is limited in usage
  - Cycle detection (while not NP complete) is not very cheap
  - Testing for serializability after execution is not really helpful
- Goal – to develop concurrency control protocols that will assure serializability.
  - not examine the precedence graph as it is being created;
  - instead a protocol will impose a discipline that avoids nonserializable schedules.
  - However, test for serializability will help one to understand and proof the correctness of such protocols

# Serializability – In practice

- Data manipulation language must include a construct for specifying the set of actions that comprise a transaction.
- In SQL, a transaction begins implicitly.
- A transaction in SQL ends by:
  - **Commit work** commits current transaction and begins a new one.
  - **Rollback work** causes current transaction to abort.
- Levels of consistency specified by SQL-92:
  - **Serializable** — default
  - **Repeatable read**
  - **Read committed**
  - **Read uncommitted**



# Serializability – In practice

- **Serializable** — default
- **Repeatable read** — only committed records to be read, repeated reads of same record must return same value. However, a transaction may not be serializable – it may find some records inserted by a transaction but not find others.
- **Read committed** — only committed records can be read, but successive reads of record may return different (but committed) values.
- **Read uncommitted** — even uncommitted records may be read.

(We will revisit these terms later)