Appendix

Appendix

- Math basics
  - Modular arithmetic
  - Permutations
  - Probability
  - Linear algebra

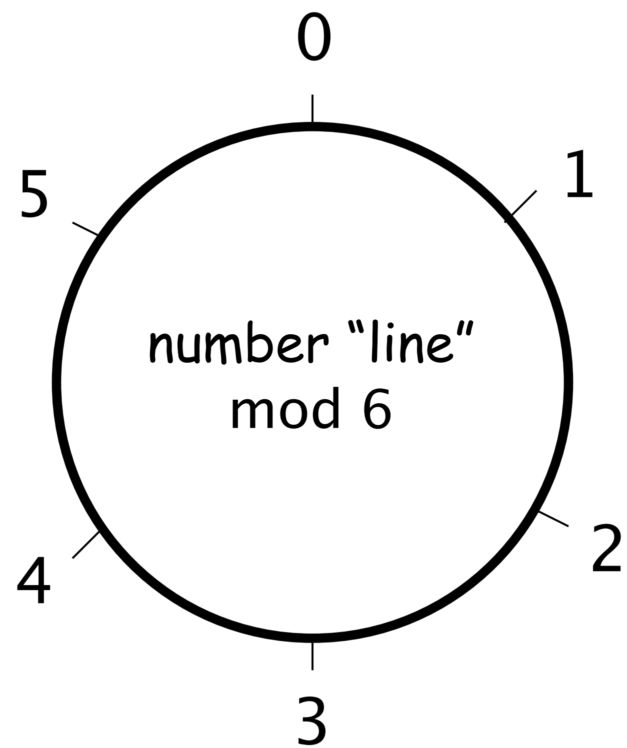
- Networking basics
  - Protocol stack, layers, etc.

Crypto Math Basics

Modular Arithmetic

Clock Arithmetic

- For integers x and n, "x mod n" is the remainder when we compute x ÷ n
  - We can also say "x modulo n"
- Examples
  - 7 mod 6 = 1
  - 33 mod 5 = 3
  - 33 mod 6 = 3
  - 51 mod 17 = 0
  - 17 mod 6 = 5



Modular Addition

- Notation and facts
  - 7 mod 6 = 1
  - 7 = 13 = 1 mod 6
  - ((a mod n) + (b mod n)) mod n = (a + b) mod n
  - ((a mod n)(b mod n)) mod n = ab mod n
- Addition Examples
  - 3 + 5 = 2 mod 6
  - 2 + 4 = 0 mod 6
  - 3 + 3 = 0 mod 6
  - (7 + 12) mod 6 = 19 mod 6 = 1 mod 6
  - (7 + 12) mod 6 = (1 + 0) mod 6 = 1 mod 6

Modular Multiplication

- Multiplication Examples
  - $3 \cdot 4 = 0 \pmod 6$
  - $2 \cdot 4 = 2 \pmod 6$
  - $5 \cdot 5 = 1 \pmod 6$
  - $(7 \cdot 4) \bmod 6 = 28 \bmod 6 = 4 \bmod 6$
  - $(7 \cdot 4) \bmod 6 = (1 \cdot 4) \bmod 6 = 4 \bmod 6$

Modular Inverses

- *Additive inverse* of x mod n, denoted $-x$ mod n, is the number that must be added to x to get 0 mod n
  - $-2 \bmod 6 = 4$, since $2 + 4 = 0 \bmod 6$
- *Multiplicative inverse* of x mod n, denoted x^{-1} mod n, is the number that must be multiplied by x to get 1 mod n
  - $3^{-1} \bmod 7 = 5$, since $3 \cdot 5 = 1 \bmod 7$

Modular Arithmetic Quiz

- Q: What is $-3 \bmod 6$?
- A: 3
- Q: What is $-1 \bmod 6$?
- A: 5
- Q: What is $5^{-1} \bmod 6$?
- A: 5
- Q: What is $2^{-1} \bmod 6$?
- A: No number works!
- Multiplicative inverse might not exist

Relative Primality

- x and y are **relatively prime** if they have no common factor other than 1
- $x^{-1} \bmod y$ exists only when x and y are relatively prime
- If it exists, $x^{-1} \bmod y$ is easy to compute using Euclidean Algorithm
  - We won't do the computation here

Totient Function

- $\varphi(n)$ is "the number of numbers less than n that are relatively prime to n"
  - Here, "numbers" are positive integers
- Examples
  - $\varphi(4) = 2$ since 4 is relatively prime to 3 and 1
  - $\varphi(5) = 4$ since 5 is relatively prime to 1,2,3,4
  - $\varphi(12) = 4$
  - $\varphi(p) = p-1$ if p is prime
  - $\varphi(pq) = (p-1)(q-1)$ if p and q prime

Permutations

Permutation Definition

- Let S be a set
- A permutation of S is an ordered list of the elements of S
  - Each element of S appears exactly once
- Suppose $S = \{0,1,2,\ldots,n-1\}$
  - Then the number of perms is…
  - $n(n-1)(n-2) \cdots (2)(1) = n!$

Permutation Example

- Let $S = \{0,1,2,3\}$
- Then there are 24 perms of S
- For example,
  - $(3,1,2,0)$ is a perm of S
  - $(0,2,3,1)$ is a perm of S, etc.
- Perms are important in cryptography

Probability Basics

Discrete Probability

- We only require some elementary facts
- Suppose that $S=\{0,1,2,\ldots,N-1\}$ is the set of all possible outcomes
- If each outcome is equally likely, then the probability of event $E \subseteq S$ is
  - $P(E) =$ # elements in E / # elements in S

Probability Example

- For example, suppose we flip 2 coins
- Then $S = \{hh, ht, th, tt\}$
  - Suppose $X =$ "at least one tail" $= \{ht, th, tt\}$
  - Then $P(X) = 3/4$
- Often, it's easier to compute
  - $P(X) = 1 - P($complement of $X)$

Complement

- Again, suppose we flip 2 coins
- Let $S = \{hh,ht,th,tt\}$
  - Suppose X = "at least one tail" = $\{ht,th,tt\}$
  - Complement of X is "no tails" = $\{hh\}$
- Then
  - $P(X) = 1 - P(\text{comp. of } X) = 1 - 1/4 = 3/4$
- We make use of this trick often!

Linear Algebra Basics

Vectors and Dot Product

- Let $\Re$ be the set of real numbers
- Then $v \in \Re^n$ is a vector of n elements
- For example
  - $v = [v_1, v_2, v_3, v_4] = [2, -1, 3.2, 7] \in \Re^4$
- The dot product of $u, v \in \Re^n$ is
  - $u \cdot v = u_1 v_1 + u_2 v_2 + \ldots + u_n v_n$

Matrix

- A matrix is an n x m array
- For example, the matrix A is 2 x 3

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 1 & 7 & 9 \end{bmatrix}$$

- The element in row i column j is a_{ij}
- We can multiply a matrix by a number

$$3A = \begin{bmatrix} 3\cdot 3 & 3\cdot 4 & 3\cdot 2 \\ 3\cdot 1 & 3\cdot 7 & 3\cdot 9 \end{bmatrix} = \begin{bmatrix} 9 & 12 & 6 \\ 3 & 21 & 27 \end{bmatrix}$$

Matrix Addition

- We can add matrices of the same size

$$\begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix} + \begin{bmatrix} -1 & 4 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 7 & 7 \end{bmatrix}.$$

- We can also multiply matrices, but this is not so obvious
- We do **not** simply multiply the elements

Matrix Multiplication

- Suppose A is $m \times n$ and B is $s \times t$
- Then $C=AB$ is only defined if $n=s$, in which case C is $m \times t$
- Why?
- The element c_{ij} is the dot product of row i of A with column j of B

Matrix Multiply Example

- Suppose

$$B = \begin{bmatrix} -1 & 2 \\ 2 & -3 \end{bmatrix} \qquad A = \begin{bmatrix} 3 & 4 & 2 \\ 1 & 7 & 9 \end{bmatrix}$$

- Then

$$BA = C_{2\times 3} = \begin{bmatrix} [-1,2]\cdot\begin{bmatrix} 3 \\ 1 \end{bmatrix} & [-1,2]\cdot\begin{bmatrix} 4 \\ 7 \end{bmatrix} & [-1,2]\cdot\begin{bmatrix} 2 \\ 9 \end{bmatrix} \\ [2,-3]\cdot\begin{bmatrix} 3 \\ 1 \end{bmatrix} & [2,-3]\cdot\begin{bmatrix} 4 \\ 7 \end{bmatrix} & [2,-3]\cdot\begin{bmatrix} 2 \\ 9 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} -1 & 10 & 16 \\ 3 & -13 & -23 \end{bmatrix}$$

- And AB is undefined

Matrix Multiply Useful Fact

- Consider $AU = B$ where A is a matrix and U and B are column vectors
- Let $a_1, a_2, \ldots, a_n$ be columns of A and $u_1, u_2, \ldots, u_n$ the elements of U
- Then $B = u_1 a_1 + u_2 a_2 + \ldots + u_n a_n$

Example:

$$\begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \end{bmatrix} = 2 \begin{bmatrix} 3 \\ 1 \end{bmatrix} + 6 \begin{bmatrix} 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 30 \\ 32 \end{bmatrix}$$

Identity Matrix

- A matrix is square if it has an equal number of rows and columns
- For square matrices, the identity matrix I is the multiplicative identity
  - $AI = IA = A$
- The 3 x 3 identity matrix is

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Block Matricies

- Block matrices are matrices of matrices
- For example

$$M = \begin{bmatrix} I_{n\times n} & C_{n\times 1} \\ A_{m\times n} & B_{m\times 1} \end{bmatrix} \text{ and } V = \begin{bmatrix} U_{n\times \ell} \\ T_{1\times \ell} \end{bmatrix}$$

- We can do arithmetic with block matrices
- Block matrix multiplication works if individual matrix dimensions "match"

Block Matrix Mutliplication

- Block matrices multiplication example
- For matrices

$$M = \begin{bmatrix} I_{n\times n} & C_{n\times 1} \\ A_{m\times n} & B_{m\times 1} \end{bmatrix} \text{ and } V = \begin{bmatrix} U_{n\times \ell} \\ T_{1\times \ell} \end{bmatrix}$$

- We have

$$MV = \begin{bmatrix} X_{n\times \ell} \\ Y_{m\times \ell} \end{bmatrix}$$

- Where X = U+CT and Y = AU+BT

Linear Independence

- Vectors $u,v \in \Re^n$ linearly independent if $au + bv = 0$ implies $a=b=0$
- For example,

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

- Are linearly independent

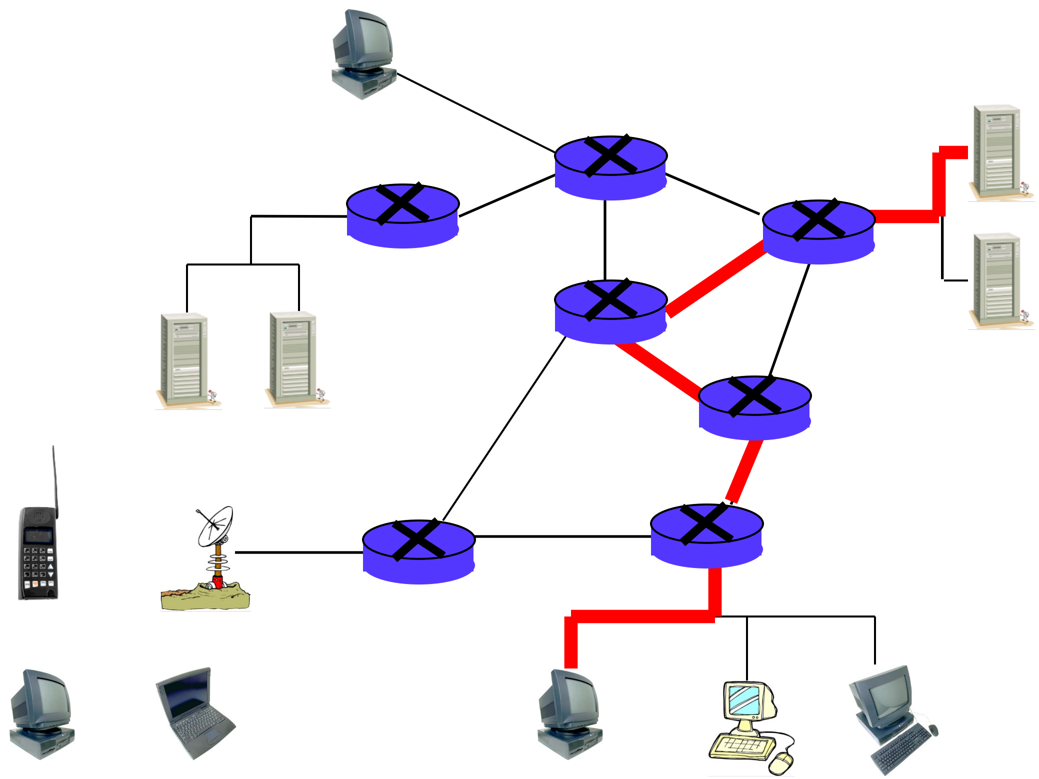
Linear Independence

- Linear independence can be extended to more than 2 vectors
- If vectors are linearly independent, then none of them can be written as a *linear combination* of the others
  - None of the independent vectors is a sum of multiples of the other vectors

Networking Basics

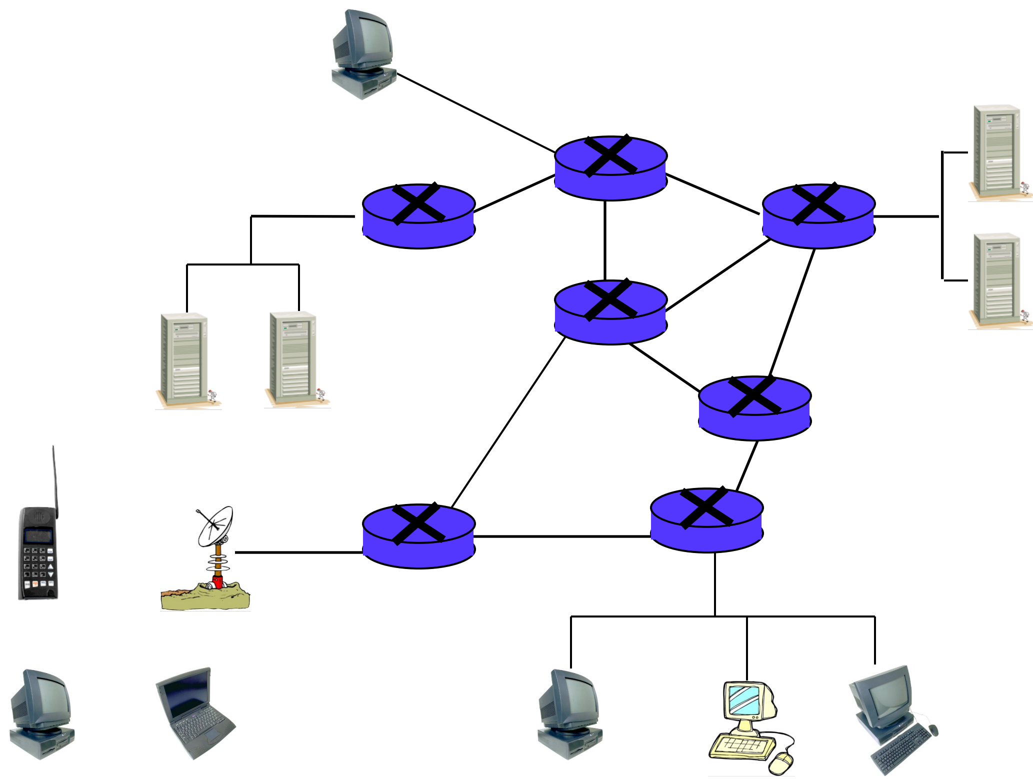
Network

- Includes
  - Computers
  - Servers
  - Routers
  - Wireless devices
  - Etc.
- Purpose is to transmit data



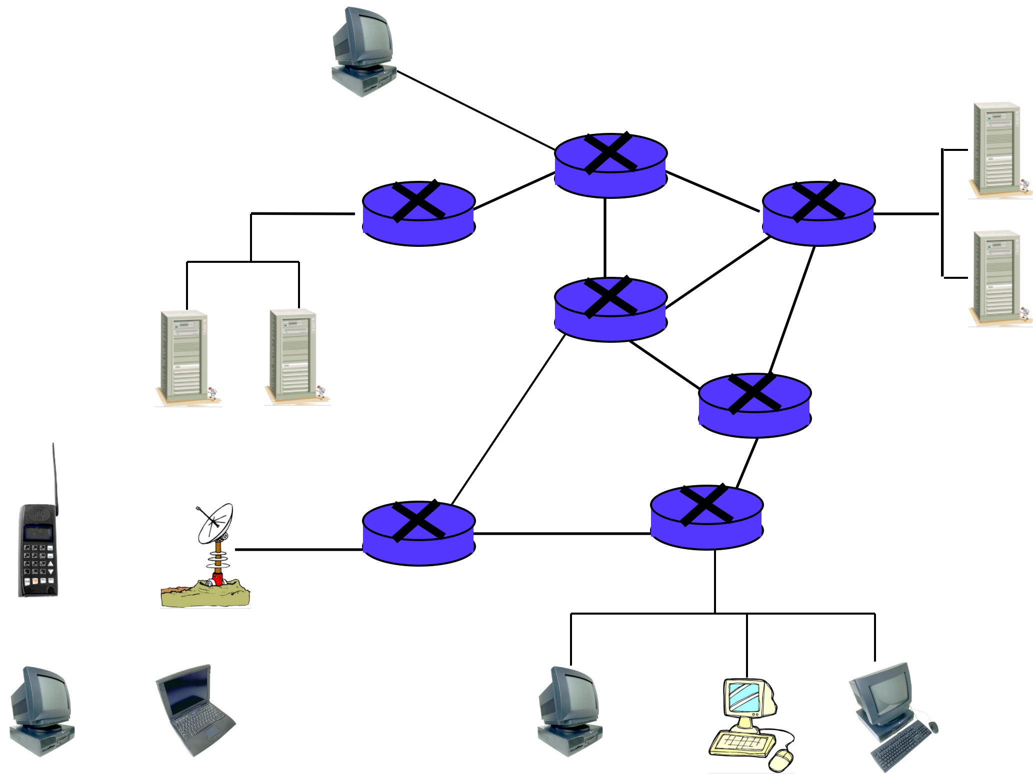
Network Edge

- Network **edge** includes
- Hosts
  - Computers
  - Laptops
  - Servers
  - Cell phones
  - Etc., etc.



Network Core

- Network core consists of
  - Interconnected mesh of routers
- Purpose is to move data from host to host



Packet Switched Network

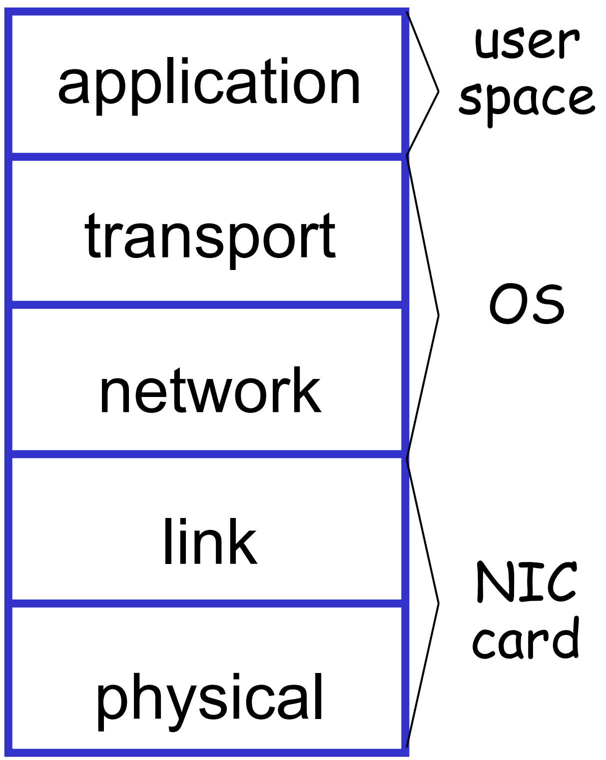
- Telephone network is/was **circuit switched**
  - For each call, a dedicated circuit established
  - Dedicated bandwidth
- Modern data networks are **packet switched**
  - Data is chopped up into discrete packets
  - Packets are transmitted independently
  - No dedicated circuit is established
  - More efficient bandwidth usage
  - But more complex than circuit switched

Network Protocols

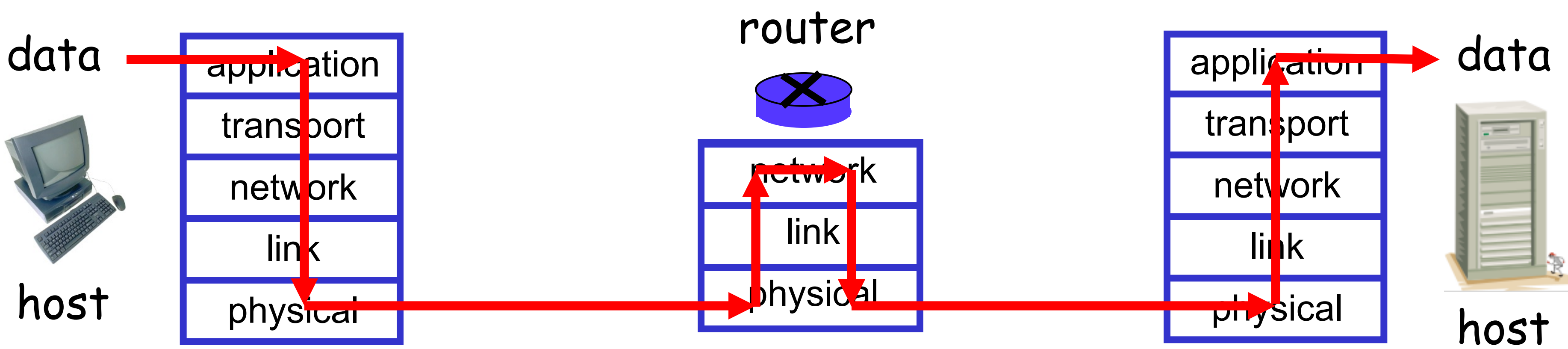
- Study of networking focused on **protocols**
- Networking protocols precisely specify "communication rules"
- Details are given in RFCs
  - RFC is essentially an Internet standard
- Stateless protocols don't remember
- Stateful protocols do remember
- Many security problems related to "state"
  - E.g., DoS is a problem with stateful protocols

Protocol Stack

- Application layer protocols
  - HTTP, FTP, SMTP, etc.
- Transport layer protocols
  - TCP, UDP
- Network layer protocols
  - IP, routing protocols
- Link layer protocols
  - Ethernet, PPP
- Physical layer



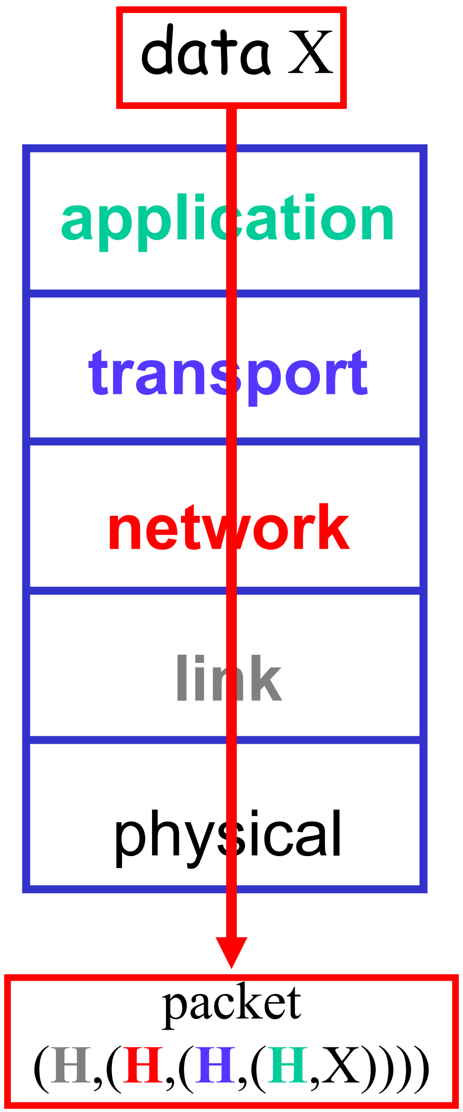
Layering in Action



- At source, data goes "down" the protocol stack
- Each router processes packet "up" to network layer
  - That's where routing info lives
- Router then passes packet down the protocol stack
- Destination processes up to application layer
  - That's where the data lives

Encapsulation

- X = application data at source
- As X goes down protocol stack, each layer adds header information:
  - Application layer: (**H**, X)
  - Transport layer: (**H**, (**H**, X))
  - Network layer: (**H**, (**H**, (**H**, X)))
  - Link layer: (**H**, (**H**, (**H**, (**H**, X))))
- Header has info required by layer
- Note that app data is on the inside



Application Layer

- Applications
  - Web browsing, email, P2P, etc.
  - Running on hosts
  - Hosts want network to be transparent
- Application layer protocols
  - HTTP, SMTP, IMAP, Gnutella, etc.
- Protocol is only one part of an application
  - For example, HTTP only a part of web browsing

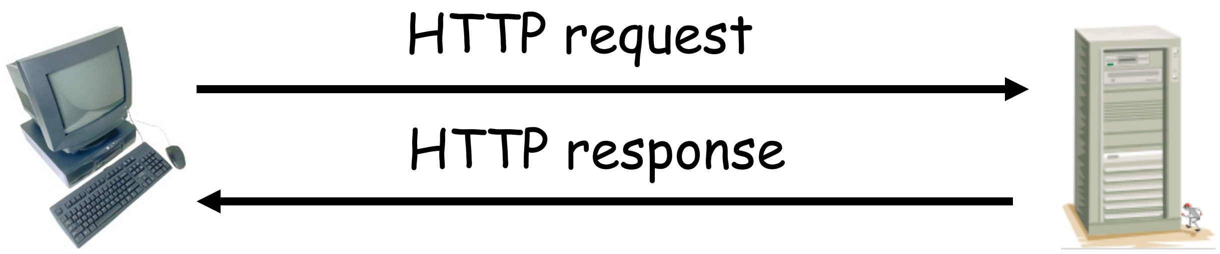
Client-Server Model

- Client
  - "speaks first"
- Server
  - tries to respond to request
- Hosts are clients and/or servers
- Example: Web browsing
  - You are the client (request web page)
  - Web server is the server

Peer-to-Peer Model

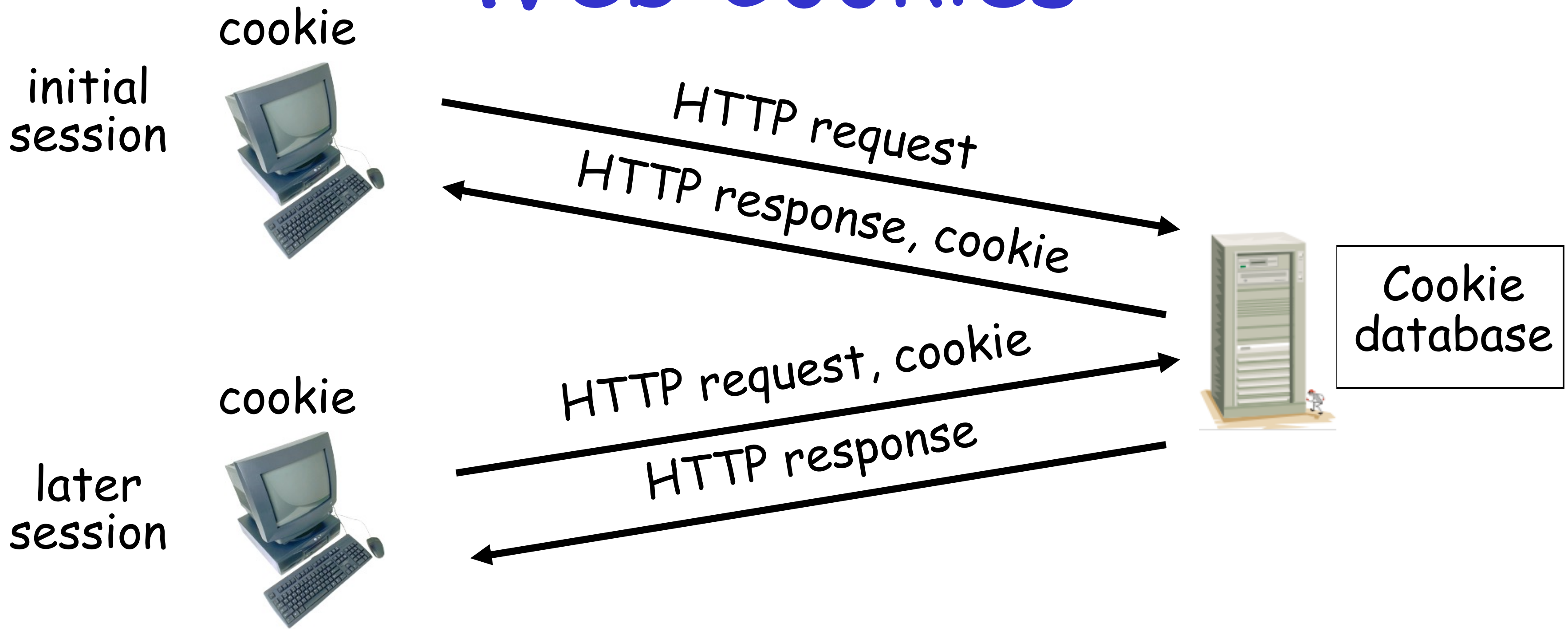
- Hosts act as clients and servers
- For example, when sharing music
  - You are client when requesting a file
  - You are a server when someone downloads a file from you
- In P2P, how does client find server?
  - Many different P2P models for this

HTTP Example



- HTTP --- HyperText Transfer Protocol
- Client (you) requests a web page
- Server responds to your request

Web Cookies



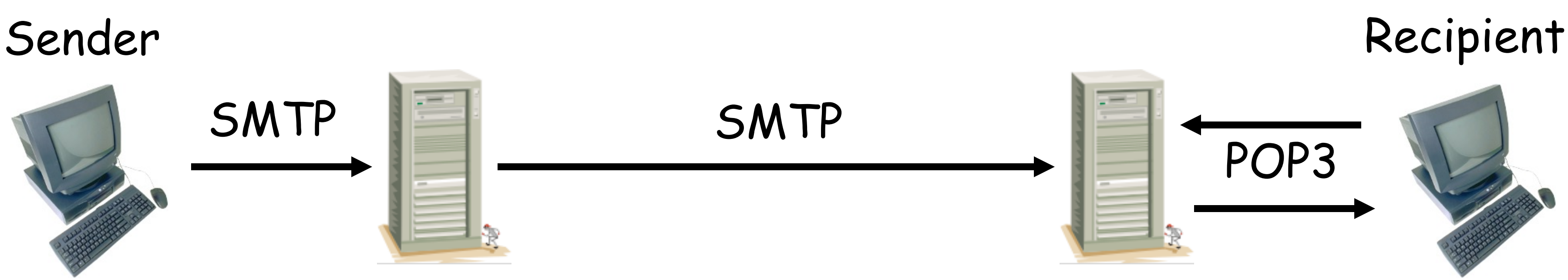
- HTTP is stateless — cookies used to add state
- Initially, cookie sent from server to browser
- Browser manages cookie, sends it to server
- Server looks in cookie database to “remember” you

Web Cookies

- Web cookies used for…
  - Shopping carts
  - Recommendations, etc., etc.
  - A very, very weak form of authentication
- Privacy concerns
  - Web site can learn a lot about you
  - Multiple web sites could learn even more

SMTP

- SMTP used to send email from sender to recipient's mail server
- Then use POP3, IMAP or HTTP (Web mail) to get messages from server
- As with many application protocols, SMTP commands are human readable



Spoofed email with SMTP

User types the red lines:

```
> telnet eniac.cs.sjsu.edu 25
220 eniac.sjsu.edu
HELO ca.gov
250  Hello ca.gov, pleased to meet you
MAIL FROM: <arnold@ca.gov>
250 arnold@ca.gov... Sender ok
RCPT TO: <stamp@cs.sjsu.edu>
250 stamp@cs.sjsu.edu ... Recipient ok
DATA
354 Enter mail, end with "." on a line by itself
It is my pleasure to inform you that you
are terminated
 .
250 Message accepted for delivery
QUIT
221 eniac.sjsu.edu closing connection
```

Application Layer

- DNS --- Domain Name Service
  - Convert human-friendly names such as www.google.com into 32-bit IP address
  - A distributed hierarchical database
- Only 13 “root” DNS server clusters
  - Almost a single point of failure for Internet
  - Attacks on root servers have succeeded
  - But, attacks have not lasted long enough

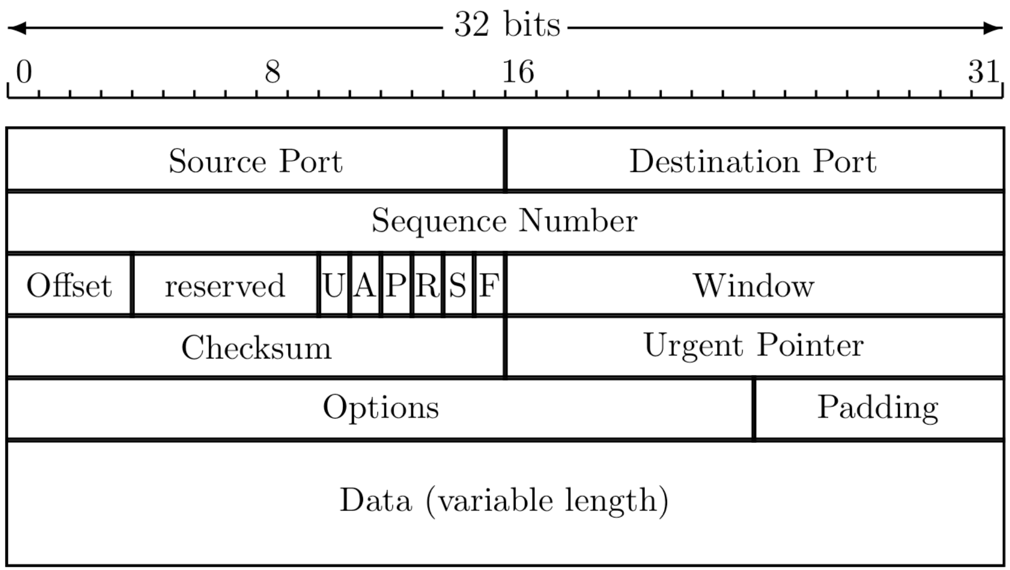
Transport Layer

- The network layer offers unreliable, "best effort" delivery of packets
- Any improved service must be provided by the hosts
- Transport layer: two protocols of interest
  - TCP — more service, more overhead
  - UDP — less service, less overhead
- TCP and UDP runs on hosts, not routers

TCP

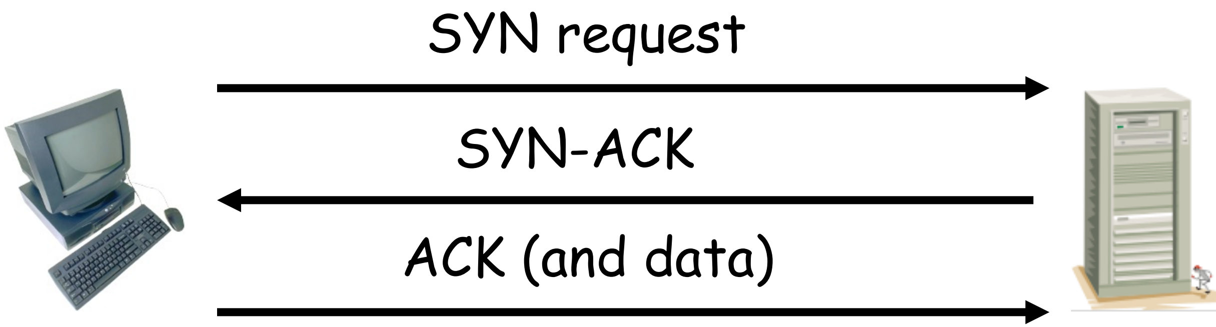
- TCP assures that packets…
  - Arrive at destination
  - Are processed in order
  - Are not sent too fast for receiver: **flow control**
- TCP also provides…
  - Network-wide **congestion control**
- TCP is **connection-oriented**
  - TCP contacts server before sending data
  - Orderly setup and take down of "connection"
  - No true connection, only a logical connection

TCP Header



- Source and destination port
- Sequence number
- Flags (ACK, SYN, RST, etc.)
- Usually 20 bytes (if no options)

TCP Three-Way Handshake



- **SYN**: synchronization requested
- **SYN-ACK**: acknowledge SYN request
- **ACK**: acknowledge msg 2 and send data
- Then TCP “connection” established
    - Connection terminated by FIN or RST

Denial of Service Attack

- The TCP 3-way handshake makes denial of service (DoS) attacks possible
- Whenever SYN packet is received, server must remember "half-open" connection
  - Remembering consumes resources
  - Too many half-open connections and server's resources will be exhausted, and then…
  - …server can't respond to legitimate connections

UDP

- UDP is minimalist, "no frills" service
  - No assurance that packets arrive
  - No assurance packets are in order, etc., etc.
- Why does UDP exist?
  - More efficient (smaller header)
  - No flow control to slow down sender
  - No congestion control to slow down sender
- Packets sent too fast, they will be dropped
  - Either at intermediate router or at destination
  - But in some apps this is OK (audio/video)

Network Layer

- Core of network/Internet
  - Interconnected mesh of routers
- Purpose of network layer
  - Route packets through this mesh
- Network layer protocol is known as IP
  - Follows a **best effort** approach
- IP runs in every host and every router
- Routers also run routing protocols
  - Used to determine the path to send packets
  - Routing protocols: RIP, OSPF, BGP, …

IP Addresses

- IP address is 32 bits
- Every host has an IP address
- Not enough IP addresses!
  - Lots of tricks used to extend address space
- IP addresses given in dotted decimal notation
  - For example: 195.72.180.27
  - Each number is between 0 and 255
- Usually, host's IP address can change

Socket

- Each host has a 32 bit IP address
- But many processes on one host
  - You can browse web, send email at same time
- How to distinguish processes on a host?
- Each process has a 16 bit **port number**
  - Port numbers < 1024 are "well-known" ports (HTTP is port 80, POP3 is port 110, etc.)
  - Port numbers above 1024 are dynamic (as needed)
- IP address and port number define a **socket**
  - Socket uniquely identifies process, Internet-wide

Network Address Translation

- Network Address Translation (**NAT**)
- Used to extend IP address space
- Use one IP address, different port numbers, for multiple hosts
  - "Translates" outside packet (based on port number) to IP for inside host

NAT-less Example

source 11.0.0.1:1025
destination 12.0.0.1:80

source 12.0.0.1:80
destination 11.0.0.1:1025

Web server

IP: 12.0.0.1
Port: 80

Alice

IP: 11.0.0.1
Port: 1025

NAT Example

Web server

IP: 12.0.0.1

src 11.0.0.1:4000
dest 12.0.0.1:80

src 12.0.0.1:80
dest 11.0.0.1:4000

Firewall

IP: 11.0.0.1

NAT Table	
4000	10.0.0.1:1025

src 10.0.0.1:1025
dest 12.0.0.1:80

src 12.0.0.1:80
dest 10.0.0.1:1025

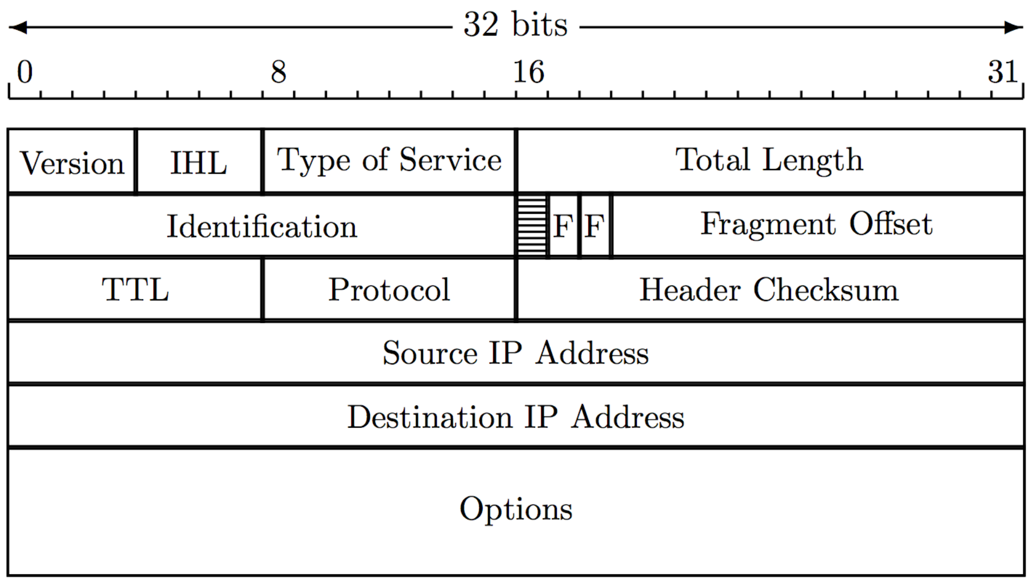
Alice

IP: 10.0.0.1

NAT: The Last Word

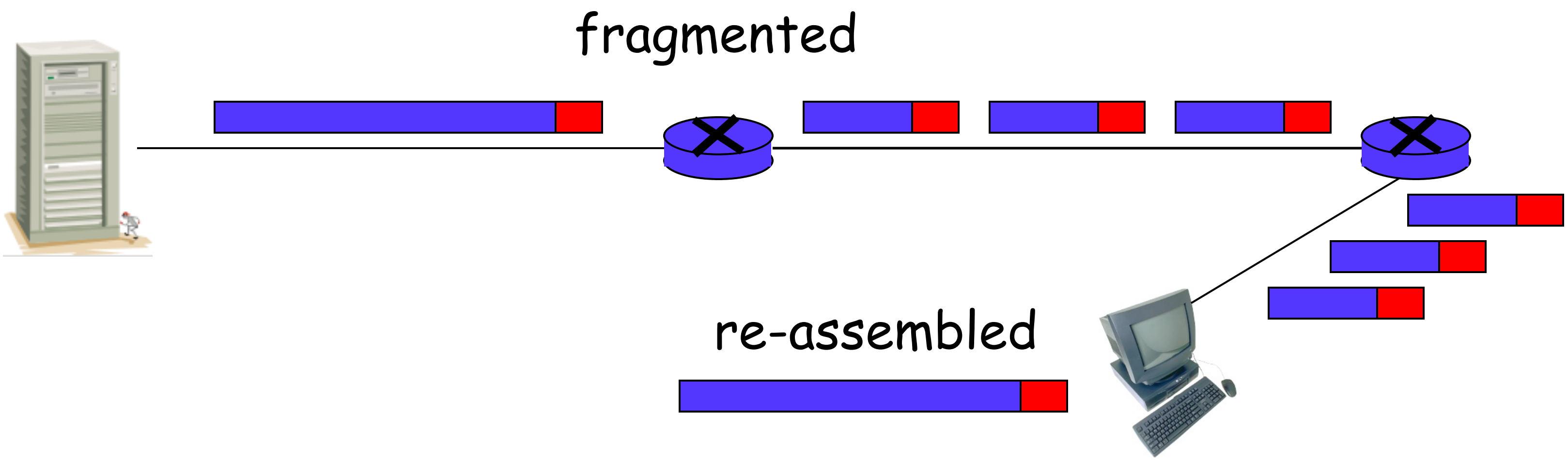
- Advantage(s)?
  - Extends IP address space
  - One (or a few) IP address(es) can be shared by many users
- Disadvantage(s)?
  - Makes end-to-end security difficult
  - Might make IPSec less effective (IPSec discussed in Chapter 10)

IP Header



- IP header used by routers
  - Note source and destination IP addresses
- Time to live (TTL) limits number of "hops"
  - So packets can't circulate forever
- Fragmentation information (see next slide)

IP Fragmentation



- Each link limits maximum size of packets
- If packet is too big, router fragments it
- Re-assembly occurs at destination

IP Fragmentation

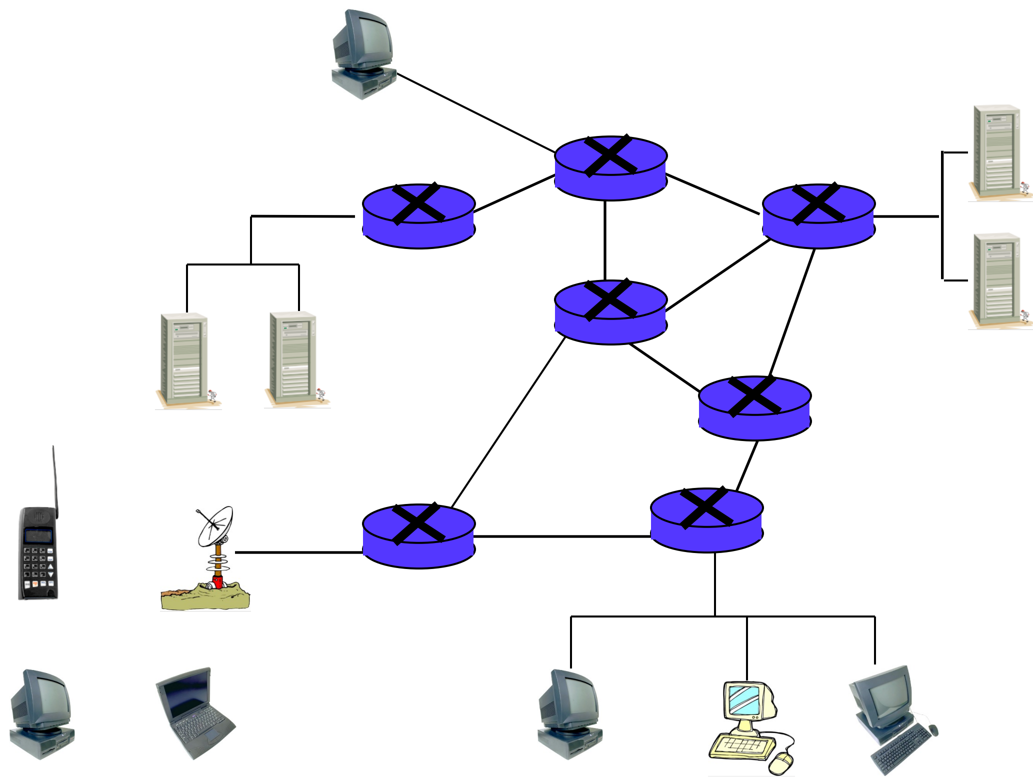
- One packet becomes multiple packets
- Packets reassembled at **destination**
  - Prevents multiple fragmentation/re-assemble
- Fragmentation is a security issue…
  - Fragments may obscure real purpose of packet
  - Fragments can overlap when re-assembled
  - Must re-assemble packet to fully understand it
  - Lots of work for firewalls, for example

IPv6

- Current version of IP is IPv4
- IPv6 is a "new-and-improved" version
- IPv6 is "bigger and better" than IPv4
  - *Bigger* addresses: 128 bits
  - *Better* security: IPSec
- How to migrate from IPv4 to IPv6?
  - Unfortunately, nobody has a good answer...
- So IPv6 has not taken hold (yet?)

Link Layer

- Link layer sends packet from one node to next
- Links can be different
  - Wired
  - Wireless
  - Ethernet
  - Point-to-point…



Link Layer

- On host, implemented in adapter: Network Interface Card (NIC)
  - Ethernet card, wireless 802.11 card, etc.
  - NIC is "semi-autonomous" device
- NIC is (mostly) out of host's control
  - Implements both link and physical layers

Ethernet

- Ethernet is a **multiple access** protocol
- Many hosts access a shared media
  - On a local area network, or LAN
- With multiple access, packets can "collide"
  - Data is corrupted and packets must be resent
- How to efficiently deal with collisions in distributed environment?
  - Many possibilities, but ethernet is most popular
- We won't discuss details here...

Link Layer Addressing

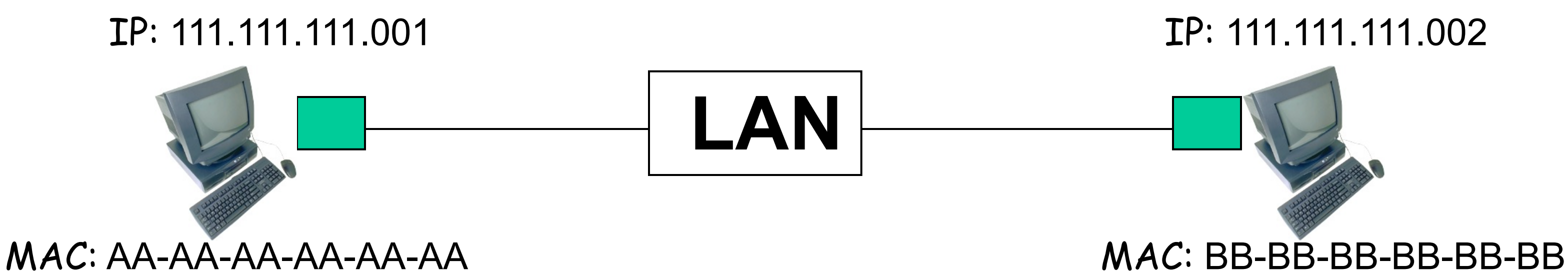
- IP addresses live at network layer
- Link layer also requires addresses (why?)
  - MAC address (LAN address, physical address)
- MAC address
  - 48 bits, globally unique
  - Used to forward packets over one link
- Analogy…
  - IP address is like your home address
  - MAC address is like a social security number

ARP

- Address Resolution Protocol (ARP)
- Used by link layer — given IP address, find corresponding MAC address
- Each host has ARP table, or **ARP cache**
  - Generated automatically
  - Entries expire after some time (about 20 min)
  - ARP used to find ARP table entries

ARP

- ARP is ***stateless***
- ARP sends **request** and receives ARP **reply**
- Replies used to fill ARP cache



111.111.111.002	BB-BB-BB-BB-BB-BB

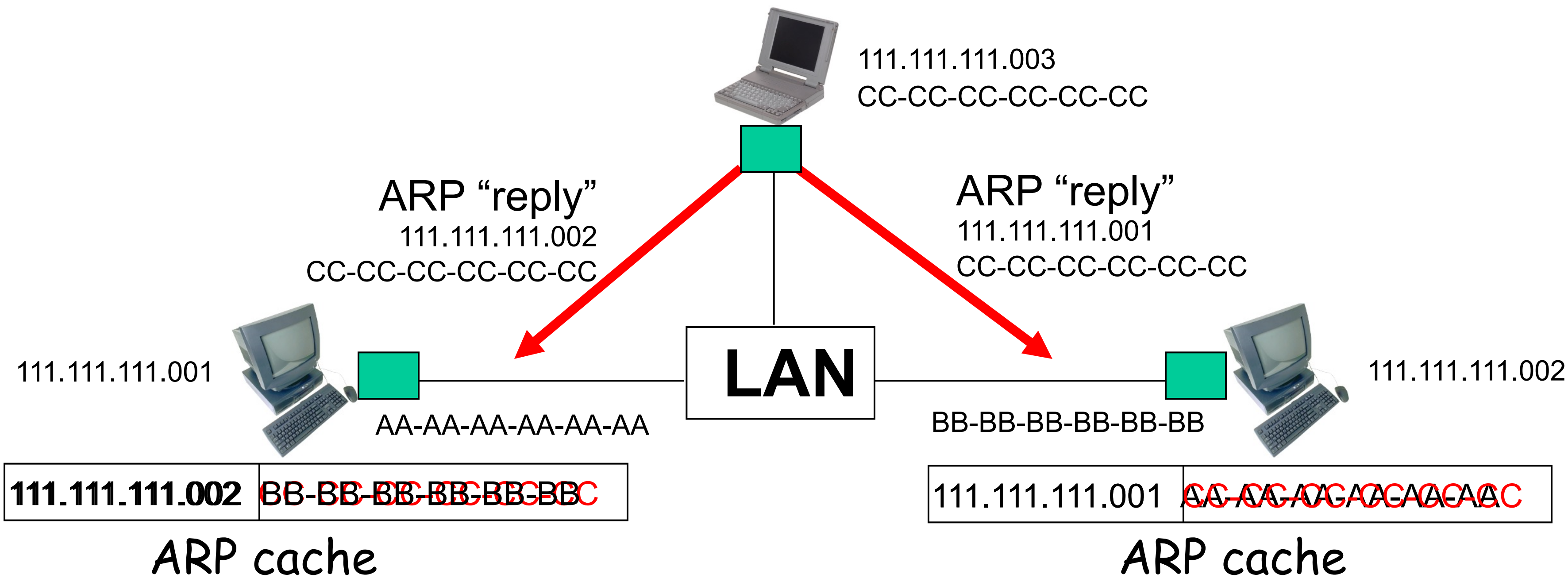
ARP cache

111.111.111.001	AA-AA-AA-AA-AA-AA

ARP cache

ARP Cache Poisoning

- ARP is stateless, so...
- Accepts "reply", even if no request sent



- Host CC-CC-CC-CC-CC-CC is man-in-the-middle