CS 5/7350 – Test 1
March 2, 2022

Name: Answers

- This exam is **closed book** and **closed notes**.
- No calculators, No cell phones or other electronics.
- Pencil and/or pen only are permitted.
- Two Scratch Pages are on the back.
- It is 3 **hours** in duration.
- You should have 14 problems. Pay attention to the point value of each problem and dedicate time as appropriate.

On my honor, I have neither given nor received unauthorized aid on this exam.

SIGNED:_____

DATE:_____

CS 5/7350 – Test #1
March 2, 2022

Name: _____
[+ 5 pts for CS 5350]

ID:_____

1. [9 pts] Define the following Terms as succinctly as possible:

a) Algorithm _____

_____

_____

b) Big Theta _____

_____

_____

c) Set _____

_____

_____

d) Implementation _____

_____

_____

e) Entropy _____

_____

_____

f) Graph _____

_____

_____

2. [8 pts] You have a complete graph with |V| vertices where |V| is >= 2. Each edge in this graph has a capacity of 7. You pick one vertex as the Start Vertex, S, and another vertex as the Sink Vertex, T. Since the is a complete graph, you will get the same answer regardless of which two vertices you pick.

   a) What is the length of the shortest path between Vertex S and Vertex T.

Directly connected, so 1 edge of capacity 7

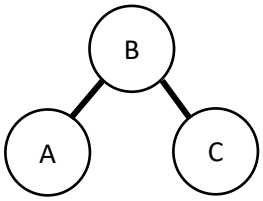
   b) What is the maximum flow (in terms of |V|) between Vertex S and Vertex T.

7 (|V|-1)

3. [6 pts] Argue that the problem, S, of sorting an unsorted array of integers of length greater than 100 elements is at least as hard - and maybe even harder - than the problem, L, of finding the five smallest values of the same array.

Since a solver for problem S can be used to solve Problem L by sorting the array with S and getting the first 5 elements, Problem S must be just has hard or possibly harder than Probalem L

4. [6 pts] The following in-order traversal: A B C and the following pre-order traversal: B A C would form the following tree:



Give a 3 vertex in-order traversal and a 3 vertex pre-order traversal that cannot form a tree. That is, no single tree can have both the in-order traversal and the pre-order traversal.

In Order: A B C
Pre Order: B C A

5. [6 pts] Using n_0 equal to 10, show that $f(n) = 7n^3 + 8n^2 + 5n + 1$ is $\Omega(n^2)$.

$0 < c_1 n^2 \leq 7n^3 + 8n^2 + 5n + 1 \quad \forall n \geq 10$

$0 < c_1 \leq 7n + 8 + \frac{5}{n} + \frac{1}{n^2} \quad \forall n \geq 10$

$c_1 \leq 70.$

Note that 70 is not a tight value, but works for the inequality

6. [8 pts] You run different programs for various values of "n" and create 4 tables of the runtimes. Give the Asymptotic bounds that each of the tables support?

a. n	time(ms)
1000	2120
2000	4120
3000	6120
4000	8120
5000	10120
6000	12120
7000	14120
8000	16120
9000	18120
10000	20120

$\Theta(n)$

b. n	time(ms)
1000	58913
2000	60913
4000	62913
8000	64913
16000	66913
32000	68913
64000	70913
128000	72913
256000	74913
512000	76913

$\Theta(\log n)$

c. n	time(ms)
100	21564
200	81564
300	181564
400	321564
500	501564
600	721564
700	981564
800	1281564
900	1621564
1000	2001564

$\Theta(n^2)$

d. n	time(ms)
52	20
53	60
54	180
55	540
56	1620
57	4860
58	14580
59	43740
60	131220
61	393660

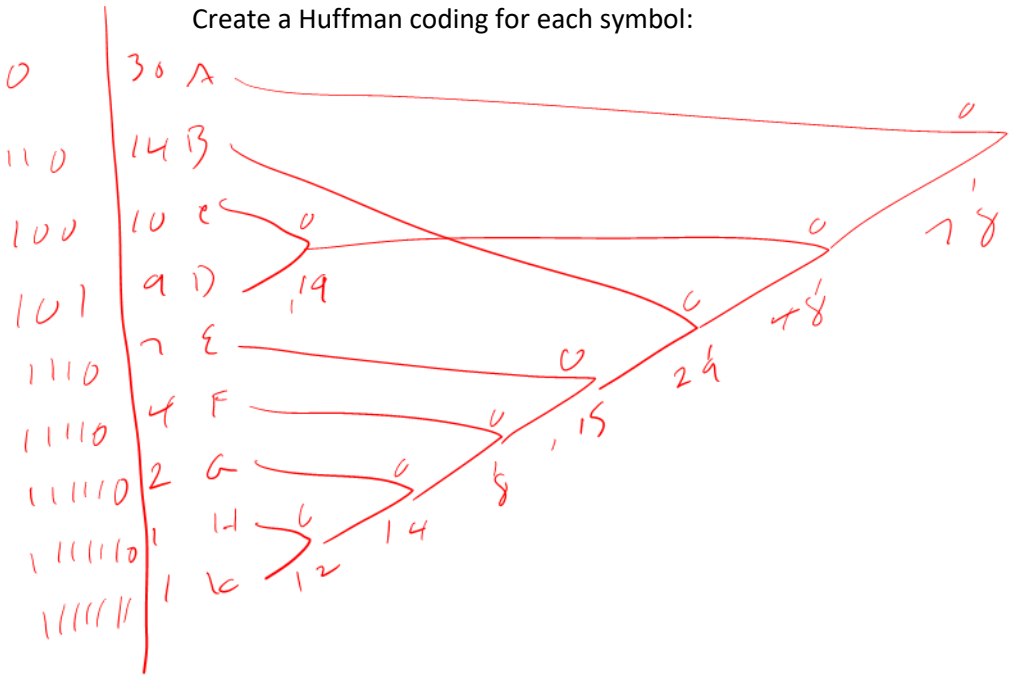
$\Theta(3^n)$

7. [6 pts] Answer the following Questions:
   a. Given that M > 100 and (7^{311} mod M) = 1; Find 7^{313} mod M = 49
   b. Compute 1/3 mod 9 Does Not Exist.
   c. Compute - (½) mod 11 5

8. [8 pts] A message contains the following number of each symbol:

30 A's, 14 B's, 10 C's, 9 D's, 7 E's, 4 F's, 2 G's, 1 H and 1 K.

Create a Huffman coding for each symbol:



How many bits are in the entire Huffman coded message?

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9. [6 pts] Two people need to establish a secret key for encrypting communications. They agree to use a Diffie-Hellman key exchange with a modulus of 11 and decide on 2 as the base. Person A chooses a random value performs the appropriate computations and sends the value 3 to person B. Person B chooses a random value of 4 and performs the appropriate computations:

   a. What is the value Person B sends to Person A

   5

   b. What is the shared secret key between Person A and Person B

   4

10. [9 pts] Consider two different algorithms that each solve a different problem.
    - Implementation X solves Problem Px and Implementation X is Ω (n) and O(n^2)
    - Implementation Y solves Problem Py and Implementation Y is $\Theta(2^n)$

    Determine if each of these "**Yes** it is true", "**Maybe** it is true but doesn't have to be", or "**No** it is not true"

    a. M Problem Px is harder than Problem Py

    b. M Problem Py is harder than Problem Px

    c. No Implementation X is harder than Implementation Y

    d. M Problem X is Ω (n)

    e. M Problem X is Ω (n^2)

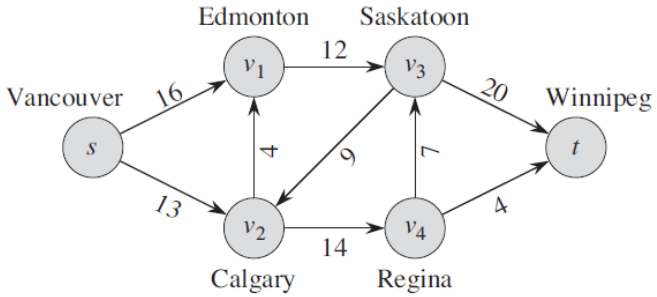
    f. M Problem X is O (n)

    g. Yes Problem X is O (n^2)

    h. Yes Implementation X is Ω (1)

    i. Yes Implementation X is O (2^n)

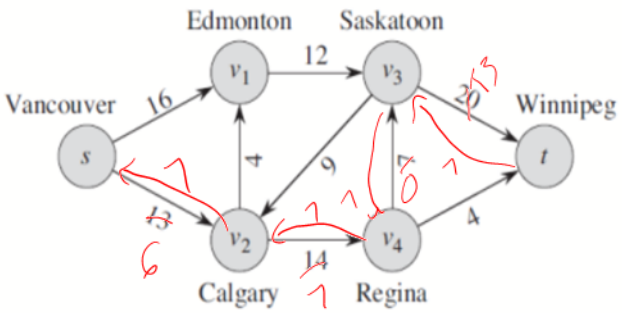
11. [8 pts] The graph below represents containers that are transported between these cities each day. You are determining the maximum flow from vertex S, Vancouver, to vertex T, Winnipeg, using the Ford-Fulkerson algorithm in the graph below.



A Path Search finds the path s -> v2 -> v4 -> v3 -> t.

a. How much flow is in this path? 7

b. What does the new graph look like after "removing" this flow



A Path Search now finds the path s -> v1 ->v3 -> v4 -> t on the new graph.

c. Is it possible to find this path? yes

d. If so, how much flow is in this path?

4

12. [10 pts] Answer the following questions.:

a) A program requires 6 seconds to process an input size of 40. If the running time is $\Theta(\sqrt{n})$ about how large of an input set could you process in 160 seconds?

$40\left(\frac{160}{6}\right)^2 = 28444$ seconds

b) A program requires 2 **DAYS** to brute force attack a password of 64 bits. Since the running time is $\Theta(2^n)$ about how **MANY DAYS** would it take for the program to brute force attack a password of 256 bits?

2^{193}

c) A program requires 2 **DAYS** to brute force attack a password of 64 bits. About how **MANY DAYS** would it take for the program to brute force attack a password of 256 bits if the running time were $\Theta(n^2)$ instead of exponential?

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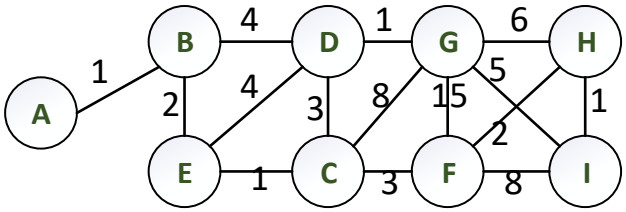
d) A program requires 4 milliseconds to process an input size of 1000. If the running time is $\Theta(n^4)$ about how many seconds would it take to process an input size of 1 million items?

4×10^9 sec

e) A program requires 4 milliseconds to process an input size of 1000. If the running time is $\Theta(n)$ about how long many seconds will it take to process an input size of 1 million items?

4 sec

13. [5 pts] You live in city B. You want to know the cost to travel from city B to all other cities (A,B,C,D,E,F,H and I). The edges of the graph below represent the cost to travel the roads between various cities. If an edge doesn't exist, then there is no road between those two cities.



a) What is the order in which you explore the cities using Dijkstra's Single Source Shortest Path algorithm to find the cost from city B to all other cities in the graph?

B A E C D G F H I

14. [5 pts] Give a smallest last vertex ordering for the graph in Problem #13. Circle in your ordering the first vertex you wrote down for that ordering.

I H F G C D E B (A)

Scratch Paper

Scratch Paper