

**CS 5/7350**  
**Quiz #2 Due Feb 22 for Completion Grade**

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CS5350? Yes / No ✓

1. [1.5 pts] Determine a Huffman encoding for each symbol in a message that contains: 11
- 20 As,  
10 20 Bs,  
011 7 Ds,  
010 7 Es,  
0011 3 Fs,  
0010 3 Gs,  
0010 2 Hs,  
0000 2 Ks,

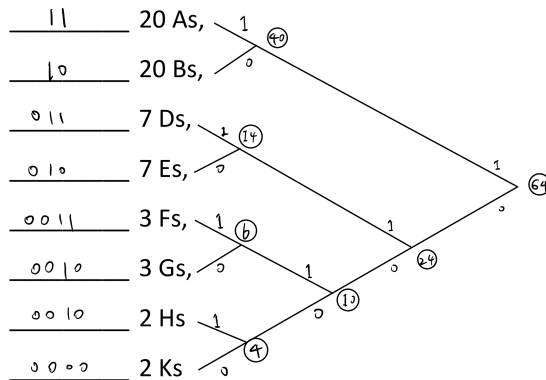
How many bits are in the entire message if each symbol is encoded with 3 bits?

**Solution:**

$$(20 + 20 + 7 + 7 + 3 + 3 + 2 + 2) \times 3 = 64 \times 3 = 192 \text{ bits}$$

How many bits are in the entire Huffman coded message?

**Solution:**



$$(20 + 20) \times 2 + (7 + 7) \times 3 + (3 + 3 + 2 + 2) \times 4 = 80 + 42 + 40 = 162 \text{ bits}$$

2. [2 pts] You run different programs for various value of "n" and create 4 tables of the run-times. Given the Asymptotic bounds that each of the tables support?

| a. | n     | time(ms) | b. | n      | time(ms) | c. | n    | time(ms) | d. | n  | time(ms) |
|----|-------|----------|----|--------|----------|----|------|----------|----|----|----------|
|    | 1000  | 2120     |    | 1000   | 58913    |    | 100  | 21564    |    | 52 | 20       |
|    | 2000  | 4120     |    | 2000   | 60913    |    | 200  | 81564    |    | 53 | 60       |
|    | 3000  | 6120     |    | 4000   | 62913    |    | 300  | 181564   |    | 54 | 180      |
|    | 4000  | 8120     |    | 8000   | 64913    |    | 400  | 321564   |    | 55 | 540      |
|    | 5000  | 10120    |    | 16000  | 66913    |    | 500  | 501564   |    | 56 | 1620     |
|    | 6000  | 12120    |    | 32000  | 68913    |    | 600  | 721564   |    | 57 | 4860     |
|    | 7000  | 14120    |    | 64000  | 70913    |    | 700  | 981564   |    | 58 | 14580    |
|    | 8000  | 16120    |    | 128000 | 72913    |    | 800  | 1281564  |    | 59 | 43740    |
|    | 9000  | 18120    |    | 256000 | 74913    |    | 900  | 1621564  |    | 60 | 131220   |
|    | 10000 | 20120    |    | 512000 | 76913    |    | 1000 | 2001564  |    | 61 | 393660   |

**Solution:**

(a)  $\Theta(n)$ .

$$\frac{10000}{1000} = 10, \frac{f(10000)}{f(1000)} = \frac{20120 \text{ ms}}{2120 \text{ ms}} = 9.49056603773585 \approx 9.5 \approx 10$$

(b)  $\Theta(\log(n))$

$$\frac{f(512000)}{f(12800)} = \frac{76913 \text{ ms}}{72913 \text{ ms}} = 1.0548599015264768 \approx 1.1 \approx \frac{\log_2(512000)}{\log_2(128000)} = \frac{18.96578428}{16.96578428} = 1.1$$

(c)  $\Theta(n^2)$

$$\frac{1000}{100} = 10, \frac{f(1000)}{f(100)} = \frac{2001564 \text{ ms}}{21564 \text{ ms}} = 92.81969949916528 \approx 9.59^2 \approx 10^2$$

$$\frac{900}{300} = 3, \frac{f(900)}{f(300)} = \frac{1621564}{181564} = 8.931087660549448 \approx 9 = 3^2$$

(d)  $\Theta(3^n)$

$$61 - 60 = 1 \frac{f(61)}{f(60)} = \frac{393660}{131220} = 3.0000685871056243 \approx 3^1$$

$$60 - 58 = 2 \frac{f(60)}{f(58)} = \frac{131220}{14580} = 9 = 3^2$$

3. [1 pts] What is  $(-\frac{1}{4}) \text{ modulo } 7$ ?

**Solution:**

| $ab \bmod 7 \backslash a$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---------------------------|---|---|---|---|---|---|---|
| $b$                       | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 0                         | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1                         | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2                         | 0 | 2 | 4 | 6 | 1 | 3 | 5 |
| 3                         | 0 | 3 | 6 | 2 | 5 | 1 | 4 |
| 4                         | 0 | 4 | 1 | 5 | 2 | 6 | 3 |
| 5                         | 0 | 5 | 3 | 1 | 6 | 1 | 2 |
| 6                         | 0 | 6 | 5 | 4 | 3 | 2 | 1 |

$$\therefore (4 \times (-\frac{1}{4}))\%7 = -1\%7 = 6, (4 \times 5)\%7 = 6$$

$$\therefore (-\frac{1}{4})\%7 = 5$$

4. [1 pts] Two people need to establish a secret key for encrypting communications. They agree to use a Diffie-Hellman key exchange with a modulus of 11 and decide on 2 as the base. Person A chooses a random value performs the appropriate computations and sends the value 6 to person B. Person B chooses a random value of 3 and performs the appropriate computations:

(a) What is the value Person B sends to Person A

**Solution:**

$$B \rightarrow A : 2^3 \bmod 11 = 8 \bmod 11 = 8$$

(b) What is the shared secret key between Person A and Person B

**Solution:**

$$(A \text{ sends to Person } B)^3 \bmod 11 = 6^3 \bmod 11 = 216 \bmod 11 = 7$$