

CS 5/7350 – Test 1
September 29, 2021

Name: Angus

- This exam is **closed book** and **closed notes**.
- Only the approved TI-30Xa calculator
- No cell phones, or other electronics.
- Pencil and/or pen only are permitted.
- Two Scratch Pages are on the back.
- It is 3 **hours** in duration.
- You should have 13 problems. Pay attention to the point value of each problem and dedicate time as appropriate.

On my honor, I have neither given nor received unauthorized aid on this exam.

SIGNED: _____

DATE: _____

CS 5/7350 – Test #1
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Name: _____
[+ 5 pts for CS 5350]

ID: _____

1. [6 pts] Define the following Terms as succinctly as possible:

a) Algorithm _____

b) Big Omega _____

c) Power Set _____

d) Compression _____

e) Entropy _____

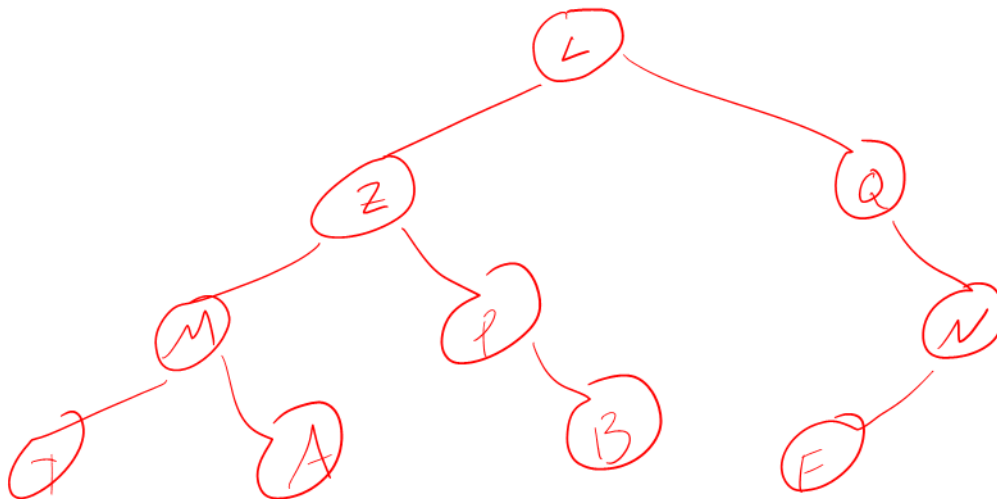
f) Merge Sort _____

2. [7 pts] Argue that the problem, S, of sorting an unsorted array of integers of length greater than 100 elements is at least as hard - and maybe even harder - than the problem, L, of finding the ten largest elements of the same unsorted array of integers.

Since we can use a solver for problem S to solve problem L by sorting the array and then printing the last ten elements, Problem S must be just as hard or possibly harder than Problem L.

3. [8 pts] A tree has the following In-Order and Pre-Order traversals. **Draw the tree and give the Post order traversal**

In Order: T M A Z P B L Q F N
Pre Order: L Z M T A P B Q N F



Post order: T A M B P Z F N Q L

4. [7 pts] Using n_0 equal to 10, show that $f(n) = 8n^2 + 5n + 1$ is $O(n^3)$.

$$0 < 8n^2 + 5n + 1 \leq C_2 n^3 \quad \forall n \geq 10$$

$$0 < \frac{8}{n} + \frac{5}{n^2} + \frac{1}{n^3} \leq C_2 \quad \forall n \geq 10$$

$C_2 = 0.851$ ✓ This is tightest answer

5. [8 pts] You run different programs for various values of “n” and create 4 tables of the runtimes. Give the Asymptotic bounds that each of the tables support?

a.	n	time(ms)	b.	n	time(ms)	c.	n	time(ms)	d.	n	time(ms)
	1000	2120		1000	58913		100	21564		52	20
	2000	4120		2000	60913		200	81564		53	60
	3000	6120		4000	62913		300	181564		54	180
	4000	8120		8000	64913		400	321564		55	540
	5000	10120		16000	66913		500	501564		56	1620
	6000	12120		32000	68913		600	721564		57	4860
	7000	14120		64000	70913		700	981564		58	14580
	8000	16120		128000	72913		800	1281564		59	43740
	9000	18120		256000	74913		900	1621564		60	131220
	10000	20120		512000	76913		1000	2001564		61	393660

$O(n)$ $O(\log n)$ $O(n^2)$ $O(n^3)$

6. [8 pts] Answer the following Questions:

a. Given that $M > 100$ and $7^{31121} \bmod M = 8$; Find $7^{31122} \bmod M =$ 56

b. How much entropy does an entire message with 40A's and 60 B's have?

97.09

c. How much entropy does an entire message with 50A's and 50 B's have?

100

d. Compute $-(\frac{1}{4}) \bmod 11$ 8

- 30 A's, 14 B's, 10 C's, 9 D's, 7 E's, 4 F's, 2 G's, 1 H and 1 K.

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A hand-drawn diagram illustrating a sequence of points (A through K) on the left, connected by lines to a single point on the right. The points are labeled with numbers and letters. The connections are as follows:

- A (30) connects to point 6.
- B (14) connects to point 6.
- C (10) connects to point 3.
- D (9) connects to point 3.
- E (7) connects to point 15.
- F (4) connects to point 15.
- G (2) connects to point 15.
- H (1) connects to point 24.
- I (222) connects to point 24.
- J (221) connects to point 24.
- K (220) connects to point 24.
- L (21) connects to point 24.
- M (20) connects to point 24.
- N (12) connects to point 24.
- O (11) connects to point 24.
- P (10) connects to point 24.
- Q (6) connects to point 24.

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8. [5 pts] Two people need to establish a secret key for encrypting communications. They agree to use a Diffie-Hellman key exchange with a modulus of 11 and decide on 2 as the base. Person A chooses a random value performs the appropriate computations and sends the value 6 to person B. Person B chooses a random value of 3 and performs the appropriate computations:

a. What is the value Person B sends to Person A

8

b. What is the shared secret key between Person A and Person B

7

9. [10 pts] Consider two different algorithms that each solve a different problem.

- Implementation X solves Problem Px and Implementation X is $\Theta(n)$
- Implementation Y solves Problem Py and Implementation Y is $\Theta(2^n)$

Determine if each of these “**Yes** it is true”, “**Maybe** it is true but doesn’t have to be”, or “**No** it is not true”

a. M Problem Px is harder than Problem Py

b. M Problem Py is harder than Problem Px

c. No Implementation X is harder than Implementation Y

d. M Problem X is $\Omega(n)$

e. No Problem X is $\omega(n)$

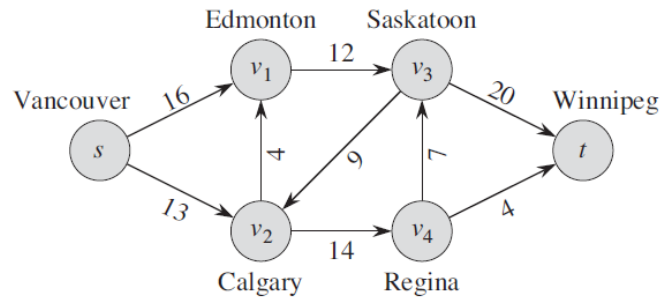
f. Yes Problem X is $O(n)$

g. M Problem X is $o(n)$

h. Yes Implementation X is $\Omega(n)$

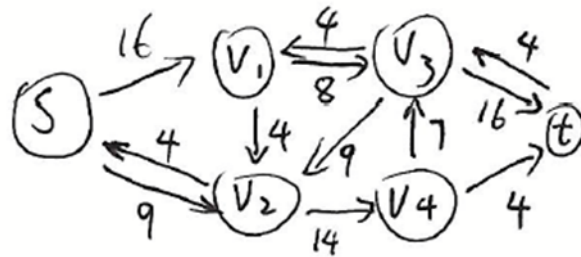
i. No Implementation X is $\omega(n)$

10. [8 pts] The graph below represents containers that are transported between these cities each day. You are determining the maximum flow from vertex S, Vancouver, to vertex T, Winnipeg, using the Ford-Fulkerson algorithm in the graph below.



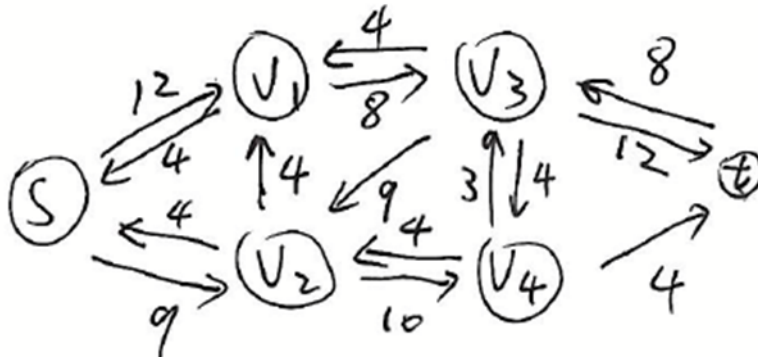
A Path Search finds the path $s \rightarrow v_2 \rightarrow v_1 \rightarrow v_3 \rightarrow t$.

- How much flow is in this path? **4**
- What does the new graph look like after “removing” this flow



A Path Search now finds the path $s \rightarrow v_1 \rightarrow v_2 \rightarrow v_4 \rightarrow v_3 \rightarrow t$ on the new graph.

- Is it possible to find this path? **Yes**
- If so, how much flow is in this path? **4**
- If so, what does the new graph look like after “removing” this flow?



11. [10 pts] Answer the following questions.:

- a) A program requires 6 seconds to process an input size of 80. If the running time is $\Theta(\sqrt{n})$ about how large of an input set could you process in 600s?

$$800,000$$

- b) A program requires 5 **DAYS** to brute force attack a password of 50 bits. Since the running time is $\Theta(2^n)$ about how **MANY YEARS** would it take for the program to brute force attack a password of 512 bits?

$$2^{456}$$

- c) A program requires 5 **DAYS** to brute force attack a password of 50 bits. Since the running time is $\Theta(2^n)$ about how **MANY DAYS** would it take for the program to brute force attack a password of 512 bits if the running time were $\Theta(n^2)$ instead of exponential?

$$5^{24}$$

- d) A program requires 6 milliseconds to process an input size of 1000. If the running time is $\Theta(n^3)$ about how many seconds would it take to process an input size of 1 trillion items?

$$6 \times 10^{24} \text{ sec}$$

- e) A program requires 6 milliseconds to process an input size of 1000. If the running time is $\Theta(n)$ about how long many seconds it take to process an input size of 1 trillion items?

$$6 \times 10^6 \text{ sec}$$

12. [8 pts] Answer the following questions:

- a) What is the weight of a minimum spanning tree for a complete graph with 10 vertices where all edges have a weight of 4?

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- b) What is the weight of a minimum spanning tree for a cycle with 10 vertices where all edges have a weight of 4?

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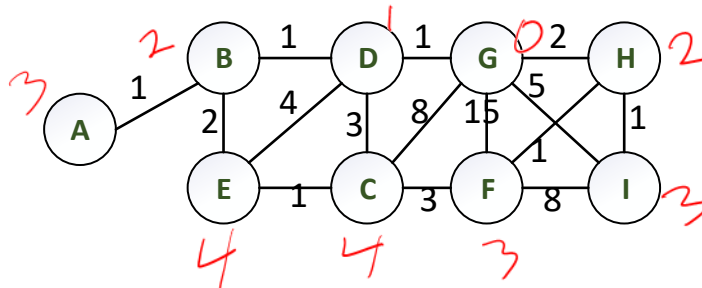
- c) A complete bi-partite graph $B_{j,k}$ is a graph which has j vertices in one partition and k vertices in another partition and all possible edges present between the partitions. For which values of j and k does a cycle exist that spans all the vertices?

$j = k$ $j, k \geq 2$

- d) What is the weight of a minimum spanning tree for a connected bi-partite graph $B_{j,k}$ where all edges have a weight of 4?

$(j + k - 1) \cdot 4$

13. [7 pts] You live in city G. You want to know the cost to travel from city G to all other cities (A,B,C,D,E,F,H and I). The edges of the graph below represent the cost to travel the roads between various cities. If an edge doesn't exist, then there is no road between those two cities.



- a) What is the order in which you explore the cities using Dijkstra's Single Source Shortest Path algorithm to find the cost from city G to all other cities in the graph?

- b) Write the cost to reach each city from City G by its vertex in the graph.

G D H B F ± A C E

Scratch Paper

Scratch Paper