CS 5/7350 – Test 1
September 29, 2021

Name: _____

- This exam is **closed book** and **closed notes**.
- Only the approved TI-30Xa calculator
- No cell phones, or other electronics.
- Pencil and/or pen only are permitted.
- Two Scratch Pages are on the back.
- It is 3 **hours** in duration.
- You should have 13 problems. Pay attention to the point value of each problem and dedicate time as appropriate.

On my honor, I have neither given nor received unauthorized aid on this exam.

SIGNED:_____

DATE:_____

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Name: _____
[+ 5 pts for CS 5350]

ID:_____

1. [6 pts] Define the following Terms as succinctly as possible:

a) Algorithm _____

_____

_____

b) Big Omega _____

_____

_____

c) Power Set _____

_____

_____

d) Compression _____

_____

_____

e) Entropy _____

_____

_____

f) Merge Sort _____

_____

_____

2. [7 pts] Argue that the problem, S, of sorting an unsorted array of integers of length greater than 100 elements is at least as hard - and maybe even harder - than the problem, L, of finding the ten largest elements of the same unsorted array of integers.

3. [8 pts] A tree has the following In-Order and Pre-Order traversals. **Draw the tree and give the Post order traversal**

In Order: T M A Z P B L Q F N
Pre Order: L Z M T A P B Q N F

4. [7 pts] Using n_0 equal to 10, show that $f(n) = 8n^2 + 5n + 1$ is $O(n^3)$.

5. [8 pts] You run different programs for various values of "n" and create 4 tables of the runtimes. Give the Asymptotic bounds that each of the tables support?

a.

n	time(ms)
1000	2120
2000	4120
3000	6120
4000	8120
5000	10120
6000	12120
7000	14120
8000	16120
9000	18120
10000	20120

b.

n	time(ms)
1000	58913
2000	60913
4000	62913
8000	64913
16000	66913
32000	68913
64000	70913
128000	72913
256000	74913
512000	76913

c.

n	time(ms)
100	21564
200	81564
300	181564
400	321564
500	501564
600	721564
700	981564
800	1281564
900	1621564
1000	2001564

d.

n	time(ms)
52	20
53	60
54	180
55	540
56	1620
57	4860
58	14580
59	43740
60	131220
61	393660

6. [8 pts] Answer the following Questions:
   a. Given that M > 100 and 7^{31121} mod M = 8; Find 7^{31122} mod M = _____
   b. How much entropy does an entire message with 40A's and 60 B's have?
   c. How much entropy does an entire message with 50A's and 50 B's have?
   d. Compute - (¼) mod 11 _____

7. [8 pts] Consider a method of encoding a string where instead of using regular bits that take on values of 0 or 1, you use “ternary bits” with values 0, 1, or 2. For the following message:

30 A’s, 14 B’s, 10 C's, 9 D's, 7 E's, 4 F's, 2 G's, 1 H and 1 K.

How many of these ternary bits are in the entire message if each symbol is encoded with 2 of these “ternary bits” like this: A = 00; B = 01; C = 02; D = 10; E = 11; F = 12; G = 20; H = 21; K = 22

Draw the tree and create a Huffman encoding of the ternary bits for each symbol:

How many “ternary bits" are in the entire ternary Huffman coded message?

8. [5 pts] Two people need to establish a secret key for encrypting communications. They agree to use a Diffie-Hellman key exchange with a modulus of 11 and decide on 2 as the base. Person A chooses a random value performs the appropriate computations and sends the value 6 to person B. Person B chooses a random value of 3 and performs the appropriate computations:

    a. What is the value Person B sends to Person A

    b. What is the shared secret key between Person A and Person B

9. [10 pts] Consider two different algorithms that each solve a different problem.
    - Implementation X solves Problem Px and Implementation X is $\Theta(n)$
    - Implementation Y solves Problem Py and Implementation Y is $\Theta(2^n)$

    Determine if each of these "**Yes** it is true", "**Maybe** it is true but doesn't have to be", or "**No** it is not true"

    a. _____ Problem Px is harder than Problem Py

    b. _____ Problem Py is harder than Problem Px

    c. _____ Implementation X is harder than Implementation Y

    d. _____ Problem X is Ω (n)

    e. _____ Problem X is ω (n)

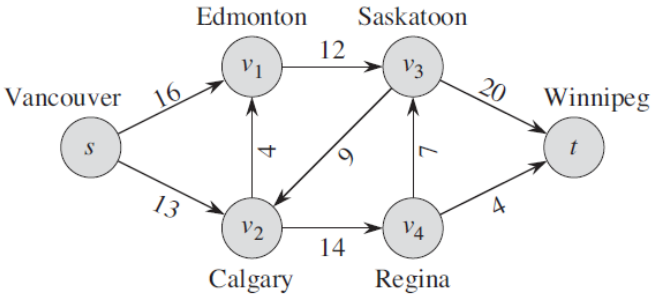
    f. _____ Problem X is O (n)

    g. _____ Problem X is o (n)

    h. _____ Implementation X is Ω (n)

    i. _____ Implementation X is ω (n)

10. [8 pts] The graph below represents containers that are transported between these cities each day. You are determining the maximum flow from vertex S, Vancouver, to vertex T, Winnipeg, using the Ford-Fulkerson algorithm in the graph below.



A Path Search finds the path s -> v2 -> v1 -> v3 -> t.

a. How much flow is in this path?
b. What does the new graph look like after "removing" this flow

A Path Search now finds the path s -> v1 ->v2 -> v4 -> v3 -> t on the new graph.

c. Is it possible to find this path?
d. If so, how much flow is in this path?
e. If so, what does the new graph look like after "removing" this flow?

11. [10 pts] Answer the following questions.:

a) A program requires 6 seconds to process an input size of 80. If the running time is $\Theta(\sqrt{n})$ about how large of an input set could you process in 600s?

b) A program requires 5 **DAYS** to brute force attack a password of 50 bits. Since the running time is $\Theta(2^n)$ about how **MANY YEARS** would it take for the program to brute force attack a password of 512 bits*?*

c) A program requires 5 **DAYS** to brute force attack a password of 50 bits. Since the running time is $\Theta(2^n)$ about how **MANY DAYS** would it take for the program to brute force attack a password of 512 bits if the running time were $\Theta(n^2)$ instead of exponential?

d) A program requires 6 milliseconds to process an input size of 1000. If the running time is $\Theta(n^3)$ about how many seconds would it take to process an input size of 1 trillion items?

e) A program requires 6 milliseconds to process an input size of 1000. If the running time is $\Theta(n)$ about how long many seconds it take to process an input size of 1 trillion items?

12. [8 pts] Answer the following questions:

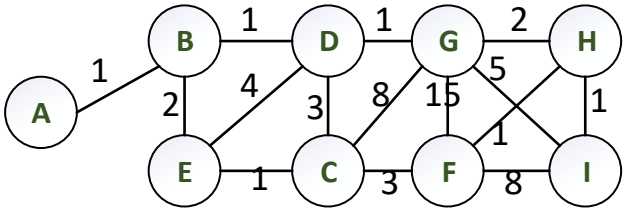
a) What is the weight of a minimum spanning tree for a complete graph with 10 vertices where all edges have a weight of 4?

b) What is the weight of a minimum spanning tree for a cycle with 10 vertices where all edges have a weight of 4?

c) A complete bi-partite graph $B_{j,k}$ is a graph which has J vertices in one partition and k vertices in another partition and all possible edges present between the partitions. For which values of j and k does a cycle exist that spans all the vertices?

d) What is the weight of a minimum spanning tree for a connected bi-partite graph $B_{j,k}$ where all edges have a weight of 4?

13. [7 pts] You live in city G. You want to know the cost to travel from city G to all other cities (A,B,C,D,E,F,H and I). The edges of the graph below represent the cost to travel the roads between various cities. If an edge doesn't exist, then there is no road between those two cities.



a) What is the order in which you explore the cities using Dijkstra's Single Source Shortest Path algorithm to find the cost from city G to all other cities in the graph?

b) Write the cost to reach each city from City G by its vertex in the graph.

Scratch Paper

Scratch Paper