CS 5/7350 – Test 1
March 17, 2021

Name: Answers

- This exam is **closed book** and **closed notes**.
- No cell phones, or other electronics except as required for zoom and only used for zoom or other proctoring.
- Pencil and/or pen are permitted.
- Two Scratch Pages are on the back.
- It is **3 hours** in duration plus time for scanning and uploading, etc.
- You should have 15 problems. Pay attention to the point value of each problem and dedicate time as appropriate.

On my honor, I have neither given nor received unauthorized aid on this exam.

SIGNED:_____

DATE:_____

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Name: _____

ID:_____

1. [8 pts] You run different programs for various values of "n" and create 4 tables of the runtimes. Give the Asymptotic bounds that each of the tables support?

a.	n	time(ms)
	10	34290
	20	512980
	30	2540070
	40	7939560
	50	19255450
	60	39751740
	70	73412430
	80	124941520
	90	199763010
	100	304020900

$\Theta(n^4)$

b.	n	time(ms)
	10	17
	11	34
	12	68
	13	136
	14	272
	15	544
	16	1088
	17	2176
	18	4352
	19	8704

$\Theta(2^n)$

c.	n	time(ms)
	1000	3247
	2000	4557
	3000	5562
	4000	6410
	5000	7156
	6000	7831
	7000	8452
	8000	9029
	9000	9572
	10000	10085

$\Theta(\sqrt{n})$

d.	n	time(ms)
	1000	3247
	2000	3447
	4000	3647
	8000	3847
	16000	4047
	32000	4247
	64000	4447
	128000	4647
	256000	4847
	512000	5047

$\Theta(\log n)$

2. [6 pts] Consider two different algorithms that each solve a different problem.
   - Algorithm X solves Problem Px and Algorithm X is $\Theta(n^2)$
   - Algorithm Y solves Problem Py and Algorithm Y is $\Theta(n^3)$

   Determine if each of these "**Yes** it is true", "**Maybe** it is true but doesn't have to be", or "**No** it is not true"

   a. M Problem Px is easier than Problem Py

   b. M Problem Py is easier than Problem Px

   c. Yes Algorithm X is easier than Algorithm Y

   d. Yes Algorithm X is $\Omega(n^2)$

   e. No Algorithm X is $\omega(n^2)$

   f. Yes Algorithm Y is $\Omega(n^2)$

3. [5 pts] Given that M > 100 and 7^{31121} mod M = 9 and A = {1, 2, 3, 4} and B = {A, B, C, D}
   a. 7^{31122} mod M = 63
   b. 7^{62242} mod M = 81
   c. 1/3 mod 11 = 4
   d. |A x B| = 16
   e. 2^{125} % 11 = 10

4. [8 pts] How many bits of entropy are in the following messages?
   a. A message that contains 20 H's and 80 T's?

      72.19 bits

   b. A message that contains 30 H's and 70 T's?

      88.129 bits

   c. A message that contains 0 H's and 100 T's?

      100 bits

   d. A message that contains 50 H's and 50 T's?

5. [5 pts] Define Algorithm

A step by step procedure for solving a problem in a finite amount of time.

6. [8 pts] A 100 character message containing only A, B, C and D's is Huffman encoded. For each of the following codes, determine if they are valid. If they are not, explain why they are not and if they are valid, give a number of A, B, C and Ds that could generate that code:
   a. A = 00; B = 01; C = 10; D = 11

25 each Valid

   b. A = 1; B = 01; C = 00; D = 001

In Valid 001 could Be "CA" or 'D'

   c. A = 1; B = 01; C = 000; D = 001

A: 50 C: 10
B: 30 D: 10
Valid

   d. A = 1; B = 0; C = 10; D= 11

In Valid 10 could be 'AB' or 'C'

7. [6 pts] Argue that the problem, S, of sorting an unsorted array of integers is at least as hard as and possibly harder than the problem, H, of creating a MIN_HEAP of that array.

Since the solution to the problem of sorting an array can be used for creating a Min-Heap (because a sorted array is a min-heap), Problem S must be just as hard or possible harder then problem H

8. [6 pts] Draw all the non-isomorphic trees on 5 vertices.

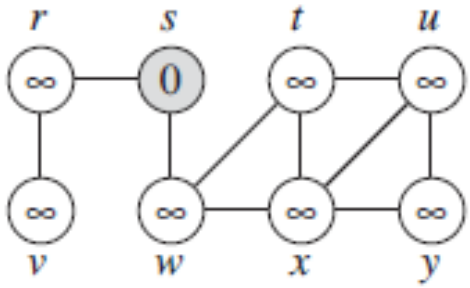
9. [6 pts] We want to play a game called guess an integer "n". I will think of an integer. You can offer guesses and I'll tell you "higher" or "lower. Give an algorithm for guessing n that requires Θ (lg(n)) guesses. (Hint, Θ(lg(n)) could be 2 lg (n) or 3 lg(n) – To answer a common question, there is no upper bound, but the number of guesses is allowed to grow based on the size of the chosen number)

Guess = 1
While (ASK is Guess < answer == yes)
Guess = Guess * 2
Binary search for answer between Guess and Guess / 2

10. [6 pts] You are designing a Diffie-Hellman key exchange. You choose the base of 3. Would 7, 11 or 13 make a better modulus for that base? Why?

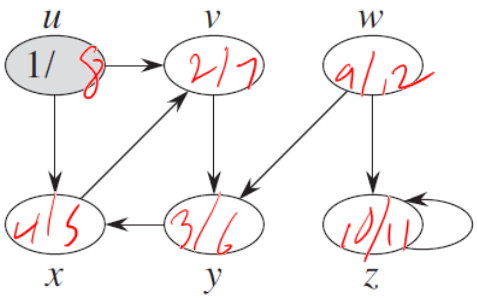
7 is better since it cycles through more values before running to 1.

11. [7 pts] Consider performing a Breath First Search of the following graph starting at S. Give the order the vertices are popped from the queue:

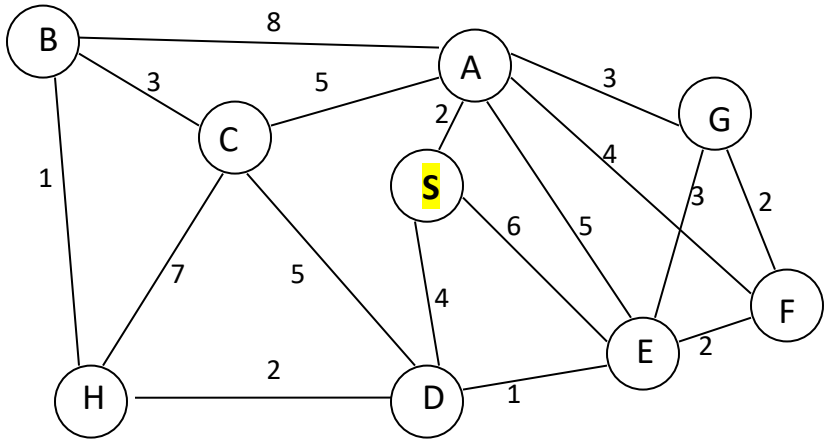


S R W V T X U Y

12. [7 pts] Give the discover and finalize time for each vertex of the following graph when performing a Depth First Search starting at U.



13. [8 pts] Consider the following graph. For any questions needing a starting vertex, use vertex **S** as the starting vertex.



(i) What is the value of the third edge chosen when finding a minimum spanning tree using Prim's algorithm?

2

(ii) What is the value of the third edge chosen when finding a minimum spanning tree using Kruskal's algorithm?

2

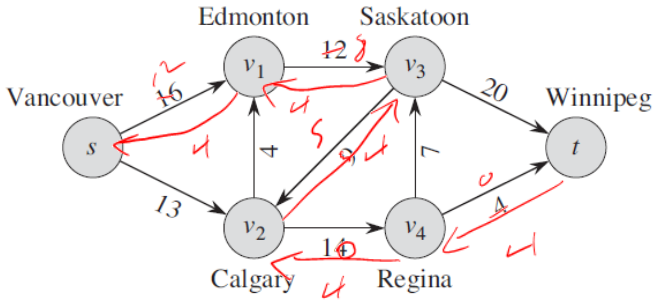
(iii) When using Dijkstra's algorithm to find the shortest path from S to all vertices, what is the order in which the vertices are reached?

S A D E G F H B C

(iv) What is the weight of the minimum spanning tree?

16

14. [8 pts] The graph below represents containers that are transported between these cities each day. You are determining the maximum flow from vertex S, Vancouver, to vertex T, Winnipeg, using the Ford-Fulkerson algorithm in the graph below. A Breadth First Search finds the path s -> v1 -> v3 -> v2 -> v4 -> t. What does the new graph look like after "removing" this flow?



15. [6 pts] A particular algorithm on a computer requires 2 seconds to process 200 items and is $\Theta(n^3)$. You want to process 4000 items. You have a choice to either use a computer that is 10 times faster (allowing it to process 200 items in 0.2 seconds) or use the same computer with a different algorithm that still processes 200 items in 2 seconds, but has a growth rate that is $\Theta(n^2)$.

(i) Which is the faster choice for 4000 items?

Different Alg

(ii) For what input sizes is the faster computer better?

Size < 2000

(iii) For what input sizes is the $\Theta(n^2)$ algorithm better?

Size > 2000

SCRATCH PAPER