CSE 7350 – Test 2
April 20, 2022

Name: _____

ID_____

- This exam is **closed book** and **closed notes**.
- No cell phones, or other electronics.
- Pencil and/or pen and non-graphing calculator only are permitted. No sharing of calculators.
- It is **3 hours** in duration.
- You should have 13 problems plus one extra credit problem. Pay attention to the point value of each problem and dedicate time as appropriate.

If you are a distance student and would like your graded test emailed back to you, provide your email address

E-MAIL:_____

On my honor, I have neither given nor received unauthorized aid on this exam.

SIGNED:_____

DATE:_____

CSE 5/7350 – Test #2
April 20, 2022

Name: _____
[+5 pts for 5350 students]

ID: _____

[9 pts] Define the following Terms as succinctly as possible:

(i) Algorithm _____

_____

_____

(ii) Dynamic Programming _____

_____

_____

(iii) Φ(N) _____

_____

_____

(iv) Longest Common Subsequence _____

_____

_____

(v) NP-Hard _____

_____

_____

(vi) Fibonacci sequence _____

_____

_____

2. [6 pts] Compute the following {note 91339 is the product of two primes 241 and 379}:

(i) Compute $\Phi(91339) =$ 90720 (240 · 378)

(ii) For which values of |V| does a cycle with V vertices have an Euler Tour all

(iii) Compute 21 $C_2 =$ 210 $\frac{21 \cdot 20}{2}$

3. [8 pts] You have 2 different dice that are not evenly weighted:
   - Dice 1 has sides {1,2,3} and a 10% chance of rolling a 1, a 40% chance of rolling a 2 and a 50% chance of rolling a 3.
   - Dice 2 has sides {2,2,3,3,3,4,4} with a 20% chance for each 2, a 10% chance for each 3 and a 15% chance for each 4.
   - Set up the table for the dynamic programming algorithm and fill in the complete column for Dice 1 and Dice 2.
   - What is the probability of rolling a 5 with these dice? 35%

	D1	D1+2	
1	.1	0	
2	.4	0	
3	.5	.04	
4	0	.19	
5	0	.35	
6	0	.27	
7	0	.15	
8	0		

4. [8 pts] Consider the heapify algorithm for creating a heap from an array of random integers:
   (i) How many swaps (maximum) may be required for an array of 3 integers?

   1

   (ii) How many swaps (maximum) may be required for an array of 7 integers?

   4

   (iii) How many swaps (maximum) may be required for an array of 15 integers?

   11

   (iv) How many swaps (maximum) may be required for an array of 31 integers?

   26

5. [8 pts] Consider the following NP completeness questions. Spr 23 - on Test #3
   (i) Assume you can solve an NP-Complete problem in polynomial time and mark the following as "true" or "false" with this assumption:

   - All P problems can be solved in polynomial time?

     T

   - All NP problems can be solved in polynomial time.

     T

   - All NP-Complete problems can be solved in polynomial time.

     T

   - All NP-Hard Problems can be solved in polynomial time.

     F

   (ii) At least 1 NP problem can be solved in polynomial time? (True or False) T

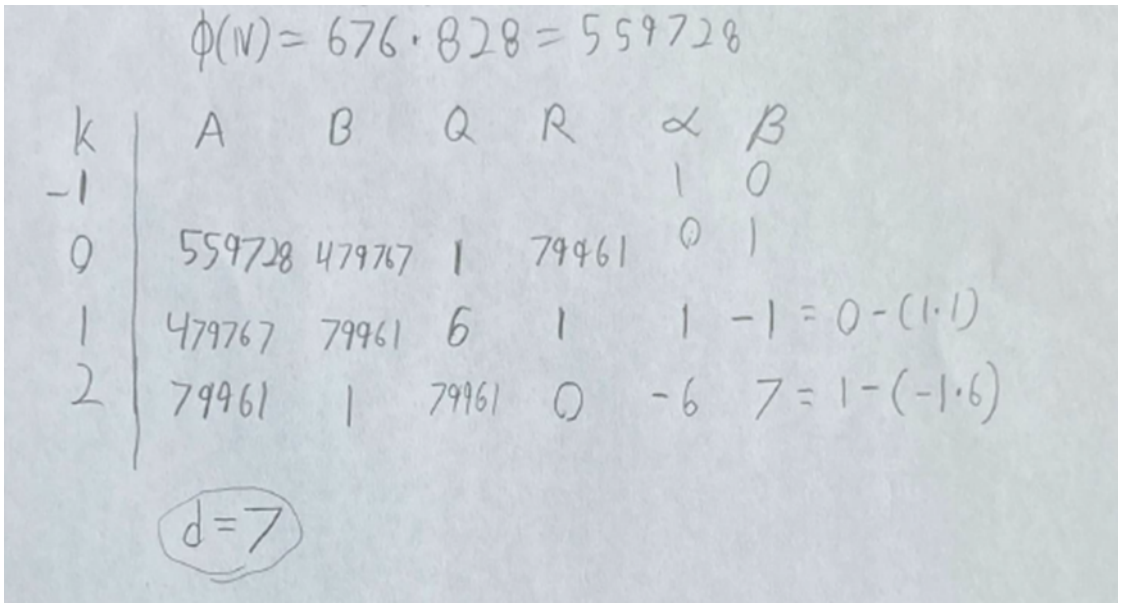
   (iii) NP-Complete problems are in P ("true" "false" or "unknown") unknown

   (iv) Which NP-Hard Problems are also NP-Complete? ("some" "all" "none" or "unknown")

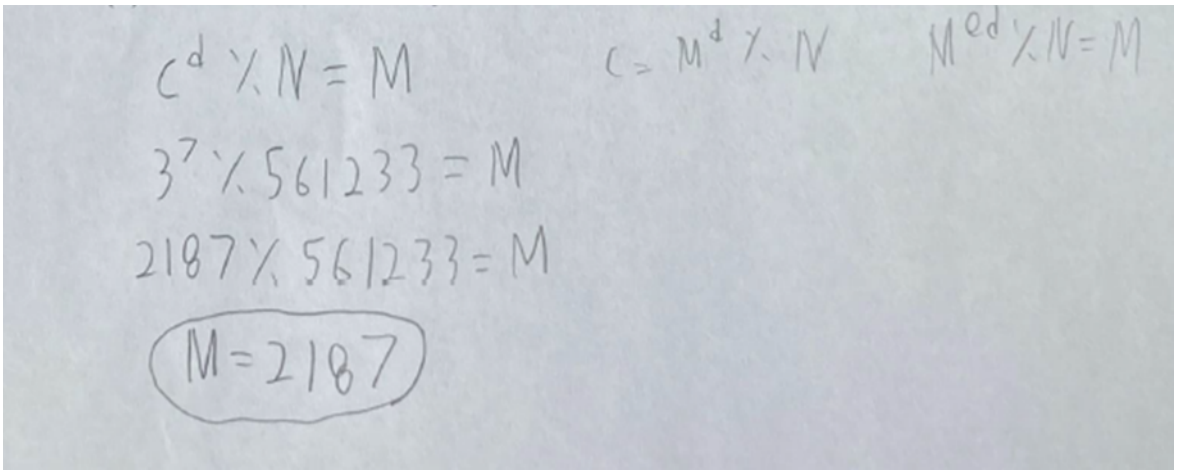
   Some

6. [10 pts] Consider an RSA encryption system that has a public key of 479767 for the value e and 561233 for the value of the modulus N. You also saw a message that had been encrypted by the public key. The value of this encrypted message is 3.

(i) You are able to factor N=561233 into the product of two prime numbers 677 * 829. What is the value of the private key? Show your work including the table for computing the Extended Euclidean Algorithm.



(ii) What was the message before it was encrypted (Give an integer)



7. [8 pts] Set up the table to find the longest increasing sub-sequence of the following sequence: 4, 6, 9, 5, 7, 8, 11, 2, 3, 13

4	4								
6	4	6							
9	4	6	9						
5	4	5	9						
7	4	5	7						
8	4	5	7	8					
11	4	5	7	8	11				
2	2	5	7	8	11				
3	2	3	7	8	11				
13	2	3	7	8	11	13			

4 5 7 8 11 13

8. [8 pts] Consider the following items with the following weights:

(i) Setup the table for the 0-1 knapsack problem and fill it in to determine the value and which items you would take if you can carry a weight of 9.

Item Number	Weight	Value
1	3	21
2	4	30
3	5	32
4	3	22
5	3	24

Weight	I 1	I 1+2	I 1+2+3	I 1+2+3+4	I 1+2+3+4+5		
0	0	0	0	0	0		
1	0	0	0	9	9		
2	0	0	0	0	0		
3	21	21	21	22	24		
4	21	30	30	30	30		
5	21	30	32	32	32		
6	21	30	32	43	46		
7	21	51	51	52	54		
8	21	51	53	54	56		
9	21	51	62	62	67		

(ii) What is the total value and which Items would you take

Items 1+4+5 for $67

9. [9 pts] Consider the Levensthein Edit Distance for two strings A and B.

(i) Write the equation describing what you would put in the table for location T[I,j].

Base case: $T[I,0] = I$; $T[0,j] = j$

If $A_I = B_j$: $T[I,j] = T[I-1,j-1]$

If $A_I \neq B_j$: $T[I,j] = \min \begin{cases} T[I-1,j]+1 \\ T[I,j-1]+1 \\ T[I-1,j-1]+1 \end{cases}$

(ii) How would you modify this equation for a different version of the Levensthein Edit Distance where substitution is not allowed?

Base case: $T[I,0] = I$; $T[0,j] = j$

If $A_I = B_j$: $T[I,j] = T[I-1,j-1]$

If $A_I \neq B_j$: $\min \begin{cases} T[I-1,j]+1 \\ T[I,j-1]+1 \end{cases}$

(iii) Fill in the following table for finding the regular, unmodified "Levensthein Edit Distance" for two strings, M and N

M = L B B Y C N = L Z B C Y Y

	-	L	B	B	Y	C			
-	0	1	2	3	4	5			
L	1	0	1	2	3	4			
Z	2	1	1	2	3	4			
B	3	2	1	1	2	3			
C	4	3	2	2	2	2			
Y	5	4	3	3	2	3			
Y	6	5	4	4	3	3			

10. [6 pts] You have two strings; String A and String B.
    - The Levensthein Edit Distance between the strings is 9
    - The Longest Common Subsequence between the two strings is 5.
    - The length of String A is < the length of String B

    (i) What is the minimum length of String A?

    5

    (ii) If String A has a length of 15, what is the minimum length of string B?

    (iii) If String A has a length of 15, what is the maximum length of string B?

Dual problem

string A can not have len >15 with LCS = 5 & LED = 9

11. [6 pts] You know that problem C is NP-Complete and you want to use that to prove that problem A is NP-Complete. What two things must you show to do this?

Test #3 Spr 2023.

It is not

12. [6 pts] Give an argument that sorting an array of integers is just as hard and possibly harder than creating a Heap of that array of integers

by sorting the integers (indexed) it creates a heap. since a solver for sorting solves the heapify problem sorting must be just as hard & harder than heapify

13. [8 pts] Consider the following LCS problem:

(i) Fill in the following table for finding the longest common subsequence for two strings, M and N

M = L B B Y C N = L Z B C Y Y

		L	B	B	Y	C			
	0	0	0	0	0	0			
L	0	1	1	1	1	1			
Z	0	1	1	1	1	1			
B	0	1	2	2	2	2			
C	0	1	2	2	2	3			
Y	0	1	2	2	3	3			
Y	0	1	2	2	3	3			

The Shortest Common Supersequence is the shortest sequence that contains both the string M as a subsequence and the string N as a subsequence. The following would be examples:

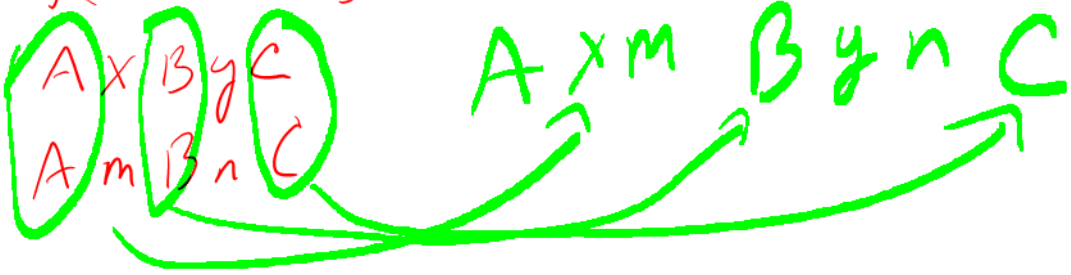
Example 1: L B Z B Y C Y Y Example 2: L Z B B Y C Y Y

(ii) Given the length of string M, |M| the length of string N, |N| and the length of the longest common subsequence, |LCS|, write an equation for the length of the shortest common supersequence?

$|M| + |N| - |LCS|$

(iii) How can you use your solution for the longest common subsequence to determine the shortest common supersequence?

Use the lcs as "anchor points" and fill in between anchor points. Eg



[Extra Credit] Consider the following problems for Extra Credit:

(i) [2 pts] How many swaps may be required (maximum) to heapify an array of size $2^n - 1$ integers? (You may write a summation for this)

$$n\left(\frac{0}{2} + \frac{1}{4} + \frac{2}{8} + \frac{3}{16} + \frac{4}{32} + \cdots\right)$$

(ii) [3 pts] Setup the table for the extended Euclidian algorithm and compute

- 1/21 modulo 98

k	A	B	Q	R	α	β
					1	0
-1						
0	98	21	4	14	0	1
1	21	14	1	7	1	-4
2	14	7	2	0	-1	5

Does Not exist since GCD (98, 21) = 7 and not 1

Scratch Paper