CS 5/7350 – Test 1
March 8, 2023

Name: _____

- This exam is **closed book** and **closed notes**.
- Only the approved TI-30Xa calculator
- No cell phones, or other electronics.
- Pencil and/or pen only are permitted.
- Two Scratch Pages are on the back.
- It is **3 hours** in duration.
- You should have 15 problems. Pay attention to the point value of each problem and dedicate time as appropriate.

On my honor, I have neither given nor received unauthorized aid on this exam.

SIGNED:_____

DATE:_____

CS 7350 – Test #1
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Name: _____

ID: _____

[+5 pts CS-5350]

1. [5 pts] Circle the asymptotically larger function OR circle both if they are the same.

   a. f(n) = 2n and g(n) = 8n
   b. f(n) = 2^n and g(n) = 3^n
   c. f(n) = n! and g(n) = n^n
   d. f(n) = n^2 and g(n) = n^3
   e. f(n) = lg (n^2) and g(n) = lg (n^3)
   f. f(n) = n! and g(n) = (n+1)!
   g. f(n) = $\log_{10}$ n and g(n) = $\log_2$ n
   h. f(n) = lg(2^n) and g(n) = n
   i. f(n) = lg(n!) and g(n) = n lg(n)
   j. f(n) = lg(2^n) and g(n) = lg(3^n)

2. [6 pts] Argue that the problem, S, of sorting an unsorted array of integers is at least as hard - and maybe even harder - than the problem, M, of finding the minimum element of the same unsorted array of integers.

The problem, S, of sorting an unsorted array of integers is at least as hard or possibly harder than the problem M of finding the median because we can use a solver for "S" to solve "M" by sorting the array with the "S" solver and then accessing the middle element in O(1) time

3. [5 pts] Using n_0 equal to 100, find the tightest C_1 and C_2 to show that f(n) = $7n^2 + 4n + 7$ is $\Theta(n^2)$.

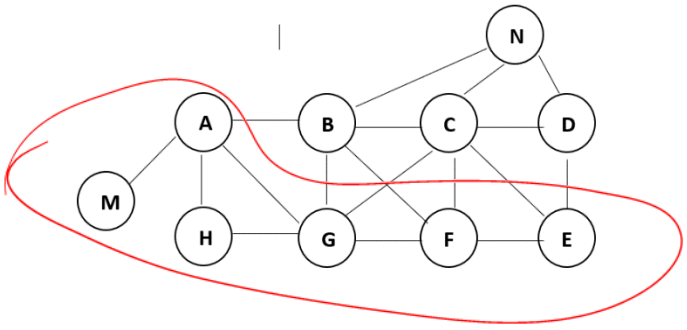
$$0 < C_1 n^2 \le 7n^2 + 4n + 7 \le C_2 n^2$$

$$0 < C_1 \le 7 + \frac{4}{n} + \frac{7}{n^2} \le C_2 \quad \forall n \ge 100$$

$C_1 \le 7$ as n approaches ∞

$C_2 = 7.0407$ with $n = 100$

4. [8 pts] Consider the following graph:



a) Give a Smallest Last Vertex Ordering for the graph where the terminal clique is the largest complete subgraph. Circle the vertex you removed first in your ordering.

G B C F E D N A H M

Largest complete

b) Give a Smallest Last Vertex Ordering for the graph where the terminal clique is not the largest complete subgraph. Circle the vertex you removed first in your ordering.

N D C B F E G A H M

c) A smallest last ordering for a different graph has a terminal clique of size 15 and a largest degree when deleted of 17.

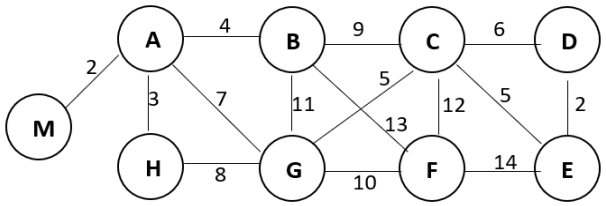
i. As an upper bound, how many colors might be needed for coloring the graph ?

18

ii. As a lower bound, how many colors must be required for coloring the graph?

15

5. [6 pts]Consider the following graph: For any algorithms below requiring a starting vertex, **use vertex H**



a) What is the value of the third edge chosen when computing the minimum spanning tree with Kruskal's Algorithm

3

b) What is the value of the third edge chosen when computing the minimum spanning tree with Prim's Algorithm

4

c) What is the value of the minimum spanning tree.

38

d) You want to find the shortest path from vertex H to all other vertices. What is the order you reach the other vertices using Dijkstra's Single Source Shortest Path algorithm?

H A M B G C F E D

6. [6 pts] Describe how you could write an algorithm which uses Dijkstra's Single Source Shortest Path algorithm as a building block to find the shortest path between all pairs of vertices in the graph above.

Run Dijkstra's algorithm starting with each vertex in the graph and remembering the answers for each pair.

If Dijkstra's Single Source Shortest Path algorithm had an asymptotically bounded running time of Θ(f(n)), what is the running time of your algorithm?

Dijkstra's algorithm is run "V" times, so if it is O(f(n)) time then the overall approach would be O(V * f(n))

7. [15 pts] Consider three different implementations that each solve a different problem.
   - Implementation X solves Problem Px and Implementation X is $\Theta(n)$
   - Implementation Y solves Problem Py and Implementation Y is $\Theta(2^n)$
   - Implementation Z solves Problem Pz and Implementation Z is $O(n^2)$
   -

   Determine if each of these "**Yes** it is true", "**Maybe** it is true but doesn't have to be", or "**No** it is not true"

   a. M Problem Py is harder than Problem Px
   b. Yes Implementation Y is harder than Implementation X
   c. M Problem X is Ω (n)
   d. N Problem X is ω (n)
   e. Yes Problem Z is O (n^3)
   f. Yes Problem Z is O (n^2)
   g. M Problem Y is O (n)
   h. M Problem X is o (n)
   i. Yes Implementation X is Ω (n)
   j. N Implementation X is ω (n)
   k. Yes Implementation X is O (n^4)
   l. M Implementation Z is O (n)
   m. Yes Implementation Z is O (n^3)
   n. M Implementation Z is Ω (n)
   o. N Implementation Y is O (n)

8. [6 pts] Answer the following questions:

a) What is the maximum flow between two vertices for a complete graph with |V| vertices where all edges have a weight of w?

$(|V|-1)w$

b) What is the maximum flow between two vertices for a tree with |V| vertices where all edges have a weight of w?

w

c) A complete bi-partite graph $B_{j,k}$ is a graph which has J vertices in one partition and k vertices in another partition and all possible edges present between the partitions. What is the maximum flow between the two partitions for a complete bi-partite graph $B_{j,k}$ where all edges have a weight of 3?

$J \cdot k \cdot 3$

d) What is the weight of a minimum spanning tree for a connected bi-partite graph $B_{j,k}$ where all edges have a weight of 3?

$(J+k-1)\cdot 3$

9. [4 pts] Two people need to establish a secret key for encrypting communications. They agree to use a Diffie-Hellman key exchange with a modulus of 11 and decide on 2 as the base. Person A chooses a random value performs the appropriate computations and sends the value 4 to person B. Person B chooses a random value of 5 and performs the appropriate computations:

a. What is the value Person B sends to Person A

10

b. What is the shared secret key between Person A and Person B

?

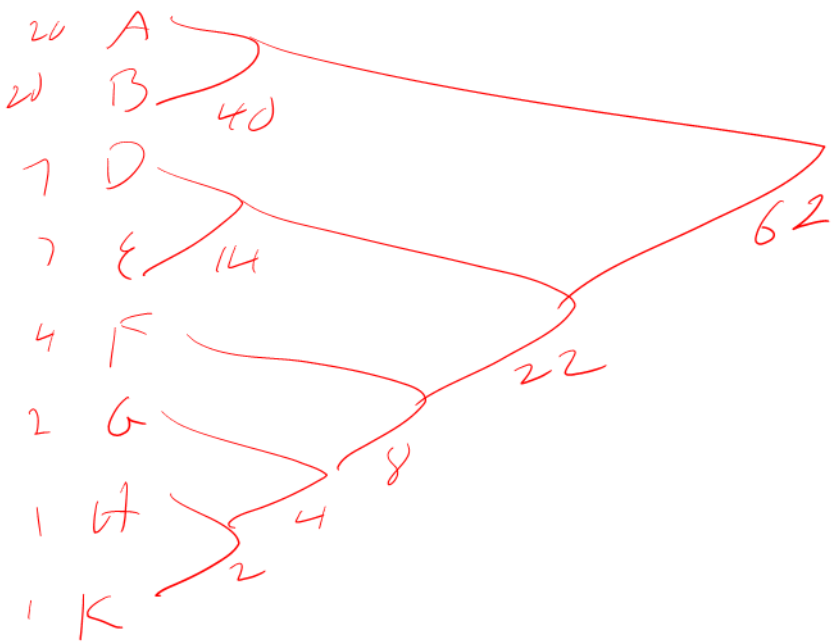
10. [8 pts] Consider a Huffman encoding of the following string.

20 A's, 20 B's, 7 D's, 7 E's, 4 F's, 2 G's, 1 H and 1 K.

How many bits are in the entire message if each symbol is encoded with 3 bits?

186

Create a Huffman encoding of the bits for each symbol:



How many bits are in the entire Huffman coded message?

152

How much entropy is in the entire message?

146.98

11. [6 pts] Answer the following Questions:

a. Compute $\Phi(31*29)$ 840

b. Compute $11^{\Phi(35879)} \% 35879$ 1

c. Compute $11^{\Phi(35879)+1} \% 35879$ 11

12. [7 pts] Answer the following Questions:

a. Given that M > 100 and 3^{31} mod M = 4; Find 3^{32} mod M = 12

b. Given that M > 100 and 3^{32} mod M = 4; Find 3^{64} mod M = 16

c. How much entropy does an entire message with 50A's and 50 B's have?

d. How much entropy does an entire message with 100A's and 0 B's have? 100

e. How much entropy does an entire message with 20 A's, 10 B's, 5 C's and 5 D's have? 0

f. Compute - 7 mod 11 4

g. Compute (½) mod 7 4

h. Compute - (½) mod 13 6

13. [6 pts] What is an algorithm?

A step by step procedure for solving a problem in a finite amount of time.

14. [6 pts] Answer the following questions.:

a) A program requires 9 days to brute force attack a password of 64 bits. Since the running time is $\Theta(2^n)$ about how days would it take for the program to brute force attack a password of 128 bits?

$9 \cdot 2^{64}$

b) A program requires 9 days to brute force attack a password of 64 bits. About how days would it take for the program to brute force attack a password of 128 bits if the running were $O(n^2)$ instead of exponential?

36

15. [6 pts] A particular algorithm on a computer requires 3 seconds to process 50 items and is $\Theta(n^2)$. You want to process 4000 items. You have a choice to either use a computer that is 10 times faster (allowing it to process 50 items in 0.3 seconds) or use the same computer with a different algorithm that still processes 50 items in 3 seconds, but has a growth rate that is $\Theta(n)$.

a) Which is the faster choice for 4000 items?

$\Theta(n)$

b) For what input sizes is the faster computer better?

input < 500

c) For what input sizes is the $\Theta(n)$ algorithm better?

input > 500