CS 5/7350
Quiz #4 Due Mar 8 for Completion Grade

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1. [2.5 pt] Consider two different algorithms that each solve a different problem.

   - Implementation X, I_x, solves Problem P_x and Implementation X is $\Theta(n)$
   - Implementation Y, I_y, solves Problem P_y and Implementation Y is $\Theta(2^n)$
   - Implementation Z, I_z, solves Problem P_z and Implementation Z is $O(n^2)$

   Determine if each of these "**Yes** it is true", "**Maybe** it is true but doesn't have to be", or "**No** it is not true"

   a.__M__ P_x is harder than P_y

   b.__M__ P_y is harder than P_x

   c.__Y__ I_y is harder than I_x

   d.__M__ I_z is harder than I_x

   e.__M__ Problem X is $\Omega(n)$

   f.__N__ Problem X is $\omega(n)$

   g.__Y__ Problem X is $O(n^3)$

   h.__Y__ Problem X is $o(n^2)$

   i.__Y__ Implementation Y is $\Omega(n)$

   j.__N__ Implementation X is $\omega(n)$

   Solution: a. It has an implementation is an upper bound on the problem, but is is not necessarily a tight upper bound.
   b. The same reason as a.
   c. It has a tight x the top bound for both of them, so that makes that really easy.
   d. I_z just an upper bound here, it can even have an upper bound of $log(n)$, it is not a tight bound.
   e. Problem X could have a tight lower bound.
   f. Problem X has an upper bound of n, but this is the implementation is the $\Theta(n)$, not the problem. So $\omega(n)$ is required to be a loose bound. So an upper bound on the Implementation is an upper bound of the problem. It cannot be an asymptotically loose lower bound on the problem.
   g. Problem x has an upper bound of n, but also has an upper bound of n^3.
   h. It has an upper bound at the n, so it's going to have non-tied upper bound of n^2
   i. It has a lower bound of 2^n, but it also has a lower bound of n^3, n^2, n

2. [2 pts] How many edges exist in:

   i A complete graph of $|V|$ vertices

   Solution:

$$C_{|V|}^2 = \frac{(|V|)!}{2![(|V|-2]!} = \frac{(|V|)!}{2(|V|-2)!} = \frac{(|V|)(|V|-1)}{2}$$

   ii A cycle of $|V|$ vertices

Solution:

$$|V|$$

iii A Tree of $|V|$ vertices

Solution:

$$|V - 1|$$

iv A complete bi-partite graph $B_{j,k}$ with j vertices on one part and k vertices on the other part.

Solution:

$$j \times k$$

3. [2 pts] Find an integer for n modulo 14635 that satisfies the following equation. Note that you may use the following: $1/2793$ % 14635 is 2047:

$$(2793n + 91) \ \% \ 14635 = 1374$$

Solution:

$$\begin{aligned}
(2793n + 91) \ \% \ 14635 &= 1374 \\
(2793n) \ \% \ 14635 + 91 \ \% \ 14635 &= 1374 \\
2793n \ \% \ 14635 + 91 &= 1374 \\
2793n \ \% \ 14635 &= 1283
\end{aligned}$$

$$\begin{aligned}
&\because \frac{1}{2793} \ \% \ 14635 = 2047 \\
&\therefore (\frac{1}{2793} \times 2793n) \ \% \ 14635 = (2047 \times 1283) \ \% \ 14635 \\
&\therefore n \ \% \ 14635 = (2626301) \ \% \ 14635 = 6636 \\
&\therefore n = 14635 \times i + 6636, i \ is \ integer
\end{aligned}$$