CS 7350 – Test 3
Online, Fall 2021

Name: _____

- This exam is **closed book** and **closed notes**.
- No cell phones, or other electronics except as required for zoom and only used for zoom or other proctoring.
- Pencil and/or pen and TI - 30Xa calculator only are permitted. No sharing of calculators
- It is **3 hours** in duration plus time for scanning and uploading, etc.
- You should have 14 problems. Pay attention to the point value of each problem and dedicate time as appropriate.

On my honor, I have neither given nor received unauthorized aid on this exam.

SIGNED:_____

DATE:_____

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ID:_____

1. [6 pts] [6 pts] If a smallest last ordering has the largest degree when deleted of 15 and a terminal clique size of 12

   (i) What is the maximum number of colors that might be required by the ordering?

   (ii) What is the minimum number of colors that must be required by the graph?

2. [6 pts] When performing a BB84 key exchange, the person sending generates random 1's and 0's to send. Give two reasons this person should not just send the confidential file instead of random 1's and 0's?

   (i) Reason 1:

   (ii) Reason 2:

3. [10 pts] Consider the following NP completeness questions. Answer them with the best answer of "some" "all" "none" or "unknown"

   (i) Which Problems in NP are also in P? ("some" "all" "none" or "unknown")

   (ii) Which Problems in P are also in NP? ("some" "all" "none" or "unknown")

   (iii) Which Problems in NP-Hard are also in NP? ("some" "all" "none")

   (iv) Which Problems in NP-Complete are in NP-Hard ("some" "all" "none" or "unknown")

   (v) If someone can solve an NP-Complete problem in Polynomial Time, then all NP and all NP-Hard problems can be solved in polynomial time. (true or false)

   (vi) If someone can solve an NP-Complete problem in Polynomial Time, then all NP and all NP-Complete problems can be solved in polynomial time. (true or false)

   (vii) At least 1 NP problem can be solved in polynomial time? (True or False)

   (viii) NP-Complete problems are in P ("true" "false" or "unknown")

   (ix) Which NP-Hard Problems are also NP-Complete? ("some" "all" "none" or "unknown")

   (x) To show a problem is NP-Complete, you must show it is NP and that a solver for another NP-Complete problem can solve it as well. (True or False)

4. [6 pts] Using n_0 equal to 10, show that $f(n) = 8n^2 + 5n + 1$ is $O(n^3)$.

5. [10 pts] Consider two different algorithms that each solve a different problem.
   - Implementation X solves Problem Px and Implementation X is $\Theta(n)$
   - Implementation Y solves Problem Py and Implementation Y is $\Theta(2^n)$

   Determine if each of these "**Yes** it is true", "**Maybe** it is true but doesn't have to be", or "**No** it is not true"

   a. _____ Problem Px is harder than Problem Py

   b. _____ Problem Py is harder than Problem Px

   c. _____ Implementation X is harder than Implementation Y

   d. _____ Problem X is $\Omega(n)$

   e. _____ Problem X is $\omega(n)$

   f. _____ Problem X is $O(n)$

   g. _____ Problem X is $o(n)$

   h. _____ Implementation X is $\Omega(n)$

   i. _____ Implementation X is $\omega(n)$.

   j. _____ Implementation Y is $O(n^n)$

6. [4 pts] How many swaps in the worst case may be required to form a heap using the HEAPIFY algorithm from an array of 16 items?

7. [6 pts] Show the addition table required for addition that adds two numbers from the number system $\beta = 7$ and $D = \{ -3, -2, -1, 0, 1, 2, 3, 4, 5 \}$ giving a number in the same number system. Ensure the addition can be performed in parallel without having to "ripple" a carry. (You do not need to fill in the grey areas

	-3	-2	-1	0	1	2	3	4	5
-3									
-2									
-1									
0									
1									
2									
3									
4									
5									

8. [8 pts] You have 3 dice. Each one is different.
   - Die #1 has sides { 0, 1, 2 } with a
     - 25% chance of rolling a 0, a
     - 35% chance of rolling a 1 and a
     - 40% chance of rolling a 2.
   - Die #2 has sides { 2, 2, 0 } with a
     - 30% chance of rolling the first 2 and a
     - 30% chance of rolling the other 2 and a
     - 40% chance of rolling a 0
   - Die #3 has sides {0, 1} with a
     - 40% chance of rolling a 0 and a
     - 60% chance of rolling a 1

   (i) Fill in the table for the dynamic programming algorithm to solve the problem.
   (ii) What is the probability of rolling a 0?
   (iii) What is the probability of rolling a 1?
   (iv) What is the probability of rolling a 2?
   (v) What is the probability of rolling a 3?
   (vi) What is the probability of rolling a 4?
   (vii) What is the probability of rolling a 5?

9. [8 pts] Create a graph with a starting vertex of "S" (when required) where:
   (i) The weight of the third edge chosen with Prims Minimum Spanning Tree Algorithm is less than the weight of the third edge chosen with Kruskal's Minimum Spanning Tree Algorithm. Mark the third edge chosen by prim's algorithm with a "P" and the third edge chosen by Kruskal's algorithm with a "K".

   (ii) Create a graph and give a Smallest Last Vertex Ordering where the terminal clique is not the largest clique in the graph. Give the smallest last vertex ordering and CIRCLE THE VERTEX YOU REMOVED FIRST IN THE ORDERING

10. [8 pts] Answer the following questions.:
   (i) A program requires 3s to brute force attack an encryption key of 128 bits. If the running time is $\Theta(2^n)$ about how many **<u>years</u>** would it take to brute force attack an encryption key of 512 bits*? (note there are about 32 million seconds in a year)*

   (ii) A program requires 3s to brute force attack an encryption key of 128 bits. If you have access to a quantum computer where the running time is $\Theta(n^2)$ about how many **<u>seconds</u>** would it take to brute force attack an encryption key of 512 bits*?*

11. [6 pts] Use the DGT algorithm discussed in class to determine how to represent the value 281.8 using the number system β=5, D = { -2, -1, 0, 1, 7 }. Show your work. (Hint, 281.8 is 281 and 4/5 which can also be thought of as 282 and -1/5)

12. [6 pts] You have a tree with the following in-order and pre-order traversals. Draw the tree:

IN ORDER: P M W S Z B A Q L R B T
PRE_ORDER: P M L B S W Z Q A T R B

13. [8 pts] You have two strings, A and B.
- String A has a length of 9.
- String B has a length of 7.
- String C has an unknown length.
- The Longest Common Subsequence between String A and C is 5.

(i) What is the minimum length of String C?

(ii) What is the maximum length of String C?

(iii) What is the minimum length of the Levensthein Edit Distance of String A and String B

(iv) What is the maximum length of the Levensthein Edit Distance of String A and String B?

14. [8 pts] You are looking at a message and each symbol in a message that contains:

16 A's, 8 B's, 4 C's, 2 D's, 1 E's and 1 F's.

How much entropy does each “B” have in the message?

How many bits of entropy are in the entire message?

Give a Huffman Coding of the symbols of the message?

How many bits are in the entire Huffman encoded message?

Scratch Paper