CS 5/7350
Quiz #2 Due Mar 1 for Completion Grade

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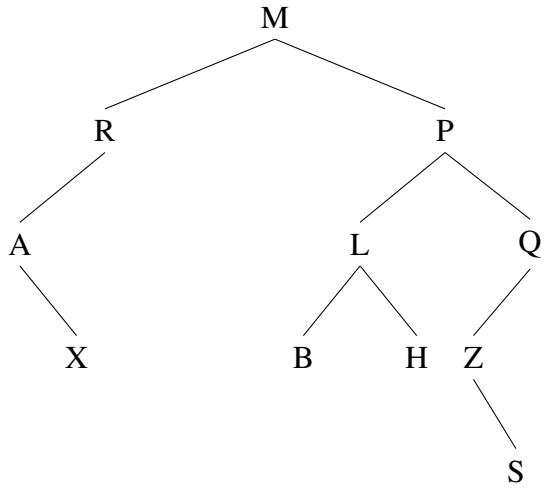
1. [1 pt] Argue that the problem, S, of sorting an unsorted array of integers of length greater than 100 elements is at least as hard - and maybe even harder - than the problem, L, of finding the ten largest elements of the same unsorted array of integers.

   Solution:
   S is at least as hard and maybe even harder than the problem L.
   Because if solve the problem S, and then can use the solution of S to solve the problem L which means the solution of problem L is based on the solution of S.

2. [2 pts] A tree has the following In-Order and Pre-Order traversals. Draw the tree

   In Order: A X R M B L H P Z S Q
   Pre Order: M R A X P L B H Q Z S

   Solution:



3. [1 pts] Answer the following 3 questions:

   (a) How much entropy does an entire message with 40A's and 60 B's have?

   Solution:

$$A : p_A = \frac{40}{100} = \frac{2}{5}, \log_2 \frac{1}{p_A} = \log_2 \frac{1}{\frac{2}{5}} = \log_2 \frac{5}{2}\ bits$$
$$B : p_B = \frac{60}{100} = \frac{3}{5}, \log_2 \frac{1}{p_B} = \log_2 \frac{1}{\frac{3}{5}} = \log_2 \frac{5}{3}\ bits$$
$$Total = 40 \times \log_2(\frac{5}{2}) + 60 \times \log_2(\frac{5}{3}) \approx 97.095\ bits$$

(b) How much entropy does an entire message with 50A's and 50 B's have?

Solution:

$$A : p_A = \frac{50}{100} = \frac{1}{2}, \log_2 \frac{1}{p_A} = \log_2 \frac{1}{\frac{1}{2}} = \log_2 2 = 1\ bits$$
$$B : p_B = \frac{50}{100} = \frac{1}{2}, \log_2 \frac{1}{p_B} = \log_2 \frac{1}{\frac{1}{2}} = \log_2 2 = 1\ bits$$
$$Total = 50 \times 1 + 50 \times 1 = 100\ bits$$

4. [2 pts] You have a complete graph with $|V|$ vertices where $|V|$ is ≥ 2. Each edge in this graph has a capacity of 7. You pick one vertex as the Start Vertex, S, and another vertex as the Sink Vertex, T. Since the is a complete graph, you will get the same answer regardless of which two vertices you pick. Answer the following questions:

(a) What is the length of the shortest path between Vertex S and Vertex T

Solution:
Edge of Vertex S and Vertex T, i.e. $E(S, T)$.
Because it is a complete graph, it must have an edge between vertex V and Vertex T.

(b) What is the maximum flow (in terms of $|V|$) between Vertex S and Vertex T

Solution:

$$(|V| - 1) \times 7$$

(c) What is the weight of the minimum spanning tree of the graph?

Solution:

$$(|V| - 1) \times 7$$