

Homework 3: Name _____ ID _____

1. [15 pts] Given you have three dice where die D1 has faces {0, 2, 2, 2, 3, 3} die D2 has faces {2,3,3,5} and Die D3 has faces {2,2,2, 1,1,1, 0 0}. Fill in the dynamic programming table for D1 and D2 and D3.

- (i) Indicate how many different ways you can roll the value 7 using all the three dice in the problem.

45

- (ii) Indicate how many different ways you can roll the value 11 using all three dice in the problem.

0

- (iii) What is the Probability of rolling a 7?

45 / 192

| | D1 | With D2 | With D3 |
|----|----|---------|---------|
| 0 | 1 | 1 | 2 |
| 1 | 0 | 0 | 3 |
| 2 | 3 | 1 | 5 |
| 3 | 2 | 2 | 7 |
| 4 | 0 | 3 | 15 |
| 5 | 0 | 9 | 33 |
| 6 | 0 | 4 | 44 |
| 7 | 0 | 3 | 45 |
| 8 | 0 | 2 | 25 |
| 9 | 0 | 0 | 15 |
| 10 | 0 | 0 | 6 |
| 11 | 0 | 0 | 0 |

2. [15 pts] You have 4 items to consider:

- Item 1 weighs 1lb and is worth \$2.
- Item 2 weighs 5 lbs and is worth \$7.
- Item 3 weighs 6 lbs and is worth \$8.
- Item 4 weighs 2 lbs and is worth \$3.

(i) If you can carry 11 lbs and can not divide the items. There is also only one of each item available. Set up the table to show which items you would take.

| | Item 1 | With I2 | With I3 | With I4 |
|----|--------|---------|---------|---------|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 2 | 2 | 2 | 2 |
| 2 | 2 | 2 | 2 | 3 |
| 3 | 2 | 2 | 2 | 5 |
| 4 | 2 | 2 | 2 | 5 |
| 5 | 2 | 7 | 7 | 7 |
| 6 | 2 | 9 | 9 | 9 |
| 7 | 2 | 9 | 9 | 10 |
| 8 | 2 | 9 | 10 | 12 |
| 9 | 2 | 9 | 10 | 12 |
| 10 | 2 | 9 | 10 | 13 |
| 11 | 2 | 9 | 10 | 13 |

Cells in which you take items are highlighted

(ii) Assume you can divide the items. That is 25% of an item weighs 25% as much and is worth 25% of the value. You can take a maximum of 100% of an item. That is there is only one of each item available. Which items would you take and how much of each item would you take?

Item 1 is \$2/lb Item 2 is \$7/5lb = \$1.4/lb Item 3 is \$1.33/lb and Item 4 is \$1.5/lb

Take Item 1, Item 4, Item 2 and half of Item 3 for $\$2 + \$7 + \$3 + \$8/2 = \$16$

3. [10 pts] A table was created and filled in for determining the Longest Increasing Subsequence. The final row of the table is shown below.

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 4 | 5 | 6 | 8 |
|---|---|---|---|---|---|

- (i) What does the value 6 indicate?
 There is a subsequence of length 5 that ends in a 6.
- (ii) What does the value 5 indicate?
 There is a subsequence of length 4 that ends in a 5.
- (iii) How long is the Longest Increasing Subsequence?
 6 values

4. [10 pts] Set up a table and show the Longest Common Subsequence for the following two strings:

A A C C T C G T A C and A C C A T G G T A A C T

(You may do this in Excel and show your results here)

| | | A | A | C | C | T | C | G | T | A | C |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| C | 0 | 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |
| C | 0 | 1 | 1 | 2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| A | 0 | 1 | 2 | 2 | 3 | 3 | 3 | 3 | 3 | 4 | 4 |
| T | 0 | 1 | 2 | 2 | 3 | 4 | 4 | 4 | 4 | 4 | 4 |
| G | 0 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 | 5 | 5 |
| G | 0 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 5 | 5 | 5 |
| T | 0 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 | 6 | 6 |
| A | 0 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 7 |
| A | 0 | 1 | 2 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 7 |
| C | 0 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |
| T | 0 | 1 | 2 | 3 | 3 | 4 | 5 | 5 | 6 | 7 | 8 |

5. [10 pts] Show the table to find the Levensthein Edit Distance of the following two strings: What is the edit distance?

5

ACTGTACGC and ATXTCGCYM

(You may do this in Excel and show your results here)

| | | A | C | T | G | T | A | C | G | C |
|---|---|---|---|---|---|---|---|---|---|---|
| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| A | 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| T | 2 | 1 | 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| X | 3 | 2 | 2 | 2 | 2 | 3 | 4 | 5 | 6 | 7 |
| T | 4 | 3 | 3 | 2 | 3 | 2 | 3 | 4 | 5 | 6 |
| C | 5 | 4 | 3 | 3 | 3 | 3 | 3 | 3 | 4 | 5 |
| G | 6 | 5 | 4 | 4 | 3 | 4 | 4 | 4 | 3 | 4 |
| C | 7 | 6 | 5 | 5 | 4 | 4 | 5 | 4 | 4 | 3 |
| Y | 8 | 7 | 6 | 6 | 5 | 5 | 5 | 5 | 5 | 4 |
| M | 9 | 8 | 7 | 7 | 6 | 6 | 6 | 6 | 6 | 5 |

6. [10 pts] Consider two strings. The length of the first string is unknown and the length of the second string is fifteen. The Levensthein edit distance between the strings is 6.
- What is the maximum possible length of the first string? 21
 - What is the minimum possible length of the first string? 9
 - What is the maximum possible length of the Longest Common Subsequence of the two strings? 15
 - What is the minimum possible length of the Longest Common Subsequence of the two strings? 9

7. [10 pts] Set up the table for the Extended Euclidian Algorithm and use it to find

$1/19$ modulo 3315.

349

| k | A | B | Q | R | alpha | beta |
|----|------|----|-----|---|-------|------|
| -1 | | | | | 1 | 0 |
| 0 | 3315 | 19 | 174 | 9 | 0 | 1 |
| 1 | 19 | 9 | 2 | 1 | 1 | -174 |
| 2 | 9 | 1 | 9 | 0 | -2 | 349 |

8. [20 pts] Bob encoded a message to Alice using RSA. Alice's public key is $e = 82313$ with a modulus of 467807. Using a quantum computer, you were able to factor the modulus into the product of two primes, $677 * 691$.

- a. What is Alice's Private Key (show your table for calculating the Extended Euclidian Algorithm) **17**

| k | A | B | Q | R | alpha | beta |
|----|--------|-------|-------|-------|-------|------|
| -1 | | | | | 1 | 0 |
| 0 | 466440 | 82313 | 5 | 54875 | 0 | 1 |
| 1 | 82313 | 54875 | 1 | 27438 | 1 | -5 |
| 2 | 54875 | 27438 | 1 | 27437 | -1 | 6 |
| 3 | 27438 | 27437 | 1 | 1 | 2 | -11 |
| 4 | 27437 | 1 | 27437 | 0 | -3 | 17 |

- b. What was the un-encrypted message Bob was sending to Alice?

$$2^{17} \% 467807 = 131072$$