CSE 5/7350 Quiz 1
Background Material
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All Questions are 4 points each:

1. Find a simple formula for $\sum_{i=1}^{k}(2i-1)$

   Solution:

$$\begin{aligned}\sum_{i=1}^{k}(2i-1) &= (2\times 1-1)+(2\times 2-1)+(2\times 3-1)+\cdots+(2\times k-1)\\ &= (2\times 1)+(2\times 2)+(2\times 3)+\cdots+2k-k\\ &= 2(\frac{(1+k)k}{2}-k)\\ &= k(1+k-1)\\ &= k^2\end{aligned}$$

2. Find a simple formula for $\sum_{i=1}^{k}(3+i^2)$

   Solution:

$$\sum_{i=1}^{k}(3+i^2)=\sum_{i=1}^{k}3+\sum_{i=1}^{k}i^2=3k+\sum_{i=1}^{k}i^2$$

$$\begin{aligned}\because \sum_{i=1}^{k}[(i+1)^3-i^3] &= \sum_{i=1}^{k}[(i^3+3i^2+3i+1)-i^3]\\ &= \sum_{i=1}^{k}(3i^2+3i+1)\\ &= 3\sum_{i=1}^{k}i^2+3\sum_{i=1}^{k}i+\sum_{i=1}^{k}1\\ &= 3\sum_{i=1}^{k}i^2+3\frac{(1+k)k}{2}+k\end{aligned}$$

$$\because \sum_{i=1}^{k}[(i+1)^3 - i^3] = [(1+1)^3 - 1^3] + [(2+1)^3 - 2^3] + [(3+1)^3 - 3^3] + \cdots + [(k+1)^3 - k^3]$$
$$= (2^3 - 1^3) + (3^3 - 2^3) + (4^3 - 3^3) + \cdots + [(k+1)^3 - k^3]$$
$$= -1^3 + (k+1)^3$$

$$\therefore -1^3 + (k+1)^3 = 3\sum_{i=1}^{k} i^2 + 3\frac{(1+k)k}{2} + k$$
$$3\sum_{i=1}^{k} i^2 = (k+1)^3 - 1 - 3\frac{(1+k)k}{2} - k$$
$$\sum_{i=1}^{k} i^2 = \frac{2(k+1)^3 - 2(k+1) - 3(k+1)k}{6}$$
$$\sum_{i=1}^{k} i^2 = \frac{(k+1)[2(k+1)^2 - 2 - 3k)]}{6}$$
$$\sum_{i=1}^{k} i^2 = \frac{(k+1)[2(k^2+2k+1) - 2 - 3k]}{6}$$
$$\sum_{i=1}^{k} i^2 = \frac{(k+1)(2k^2+k)}{6}$$
$$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$$

$$\sum_{i=1}^{k}(3 + i^2) = 3k + \frac{k(k+1)(2k+1)}{6}$$

3. Compute the value of $\sum_{i=1}^{\infty}(\frac{1}{2})^{i-1}$

Solution:

$$\sum_{i=1}^{\infty}(\frac{1}{2})^{i-1} = \lim_{n\to\infty}[(\frac{1}{2})^0 + (\frac{1}{2})^1(\frac{1}{2})^3 + \cdots + (\frac{1}{2})^n]$$
$$= \lim_{n\to\infty}[\frac{1(1-(\frac{1}{2})^n)}{1-\frac{1}{2}}]$$
$$= \lim_{n\to\infty}\frac{1-\frac{1}{2^n}}{\frac{1}{2}}$$
$$= \frac{1-0}{\frac{1}{2}}$$
$$= 2$$

4. Compute the value of $\lim_{n\to\infty}(\frac{4n}{3n})$

   Solution:

$$\lim_{n\to\infty}(\frac{4n}{3n}) = \lim_{n\to\infty}\frac{4}{3} = \frac{4}{3}$$

5. Compute the value of $\lim_{n\to\infty}(\frac{n^4}{n^3})$

   Solution:

$$\lim_{n\to\infty}(\frac{n^4}{n^3}) = \lim_{n\to\infty}\frac{\frac{n^4}{n^3}}{\frac{n^3}{n^3}} = \lim_{n\to\infty}\frac{n}{1} = \infty$$

6. Compute the value of $\lim_{n\to\infty}(\frac{2^n}{5^n})$

   Solution:

$$\lim_{n\to\infty}(\frac{2^n}{5^n}) = \lim_{n\to\infty}(\frac{2}{5})^n = 0$$

7. Compute the value of $\lim_{n\to\infty}(\frac{n!}{(n+2)!})$

   Solution:

$$\lim_{n\to\infty}(\frac{n!}{(n+2)!}) = \lim_{n\to\infty}\frac{n!}{n!(n+1)(n+2)} = \lim_{n\to\infty}\frac{1}{(n+1)(n+2)} = 0$$

8. Compute the value of $\lim_{n\to\infty}(\frac{\log_{12} n}{\log_3 n})$

   Solution:

$$\lim_{n\to\infty}(\frac{\log_{12} n}{\log_3 n}) = \lim_{n\to\infty}\frac{\frac{\log_3 n}{\log_3 12}}{\log_3 n} = \lim_{n\to\infty}\frac{1}{\log_3 12} = \frac{1}{\log_3 12}$$

9. Compute $\log_2 487$. Give your answer rounded to 6 decimal places (x.xxxxxx)

   Solution:

$$\log_2 487 \approx 8.9277778$$

Consider a bag of 7 blocks. Each block has a different color. The colors are Red, Orange,Yellow, Green, Blue, Indigo and Violet.

10. How many different ways can you pick out 4 blocks from a bag of 7 blocks? The order you pick out the blocks does not matter. (answer with an integer)

    Solution:

$$C_7^4 = \frac{7!}{4!(7-4)!} = \frac{7!}{4!3!} = \frac{7 \times 6 \times 5 \times 4!}{4!3!} = \frac{7 \times 6 \times 5}{6} = 35$$

11. How many different ways can you rearrange the 7 blocks? (answer with an integer)

    Solution:

$$7! = 7 \times 6 \times 4 \times 3 \times 2 \times 1 = 5040$$

12. You reach into the bag and pull out a block. What is the probability that it is Red, Orange or Yellow or Green and not Blue, Indigo or Violet?

    Solution:

$$P = P(Red) + P(Orange) + P(Yellow) + P(Green) = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{4}{7}$$

13. You reach into the bag of 7 blocks and pull out 2 blocks. What is the probability that at least one of the blocks you pulled out is Red, or Orange?

    Solution:

$$P = 1 - P(None\ of\ Red\ or\ Orange) = 1 - \frac{5}{7} \times \frac{4}{6} = 1 - \frac{20}{42} = \frac{42-20}{42} = \frac{22}{42} = \frac{11}{21}$$

Answer the following questions with either:

A. The time depends on size of n and twice as large will likely require about twice the time.
B. The time depends on size of n but twice as large will generally be less than twice the time.
C. Constant amount of time regardless of size of n

14. How long will it take to insert an element at the head of a linked list of size n?

    Solution:

$$C$$

15. How long will it take to remove an element from a doubly linked list of size n if you only have a pointer to the element you wish to remove and are unable to copy the data of the elements?

Solution:

C

16. How long will it take to remove an element from a linked list of size n if you only have a pointer to the element you wish to remove?

    Solution:

    C

17. How long will it take to insert an element at the beginning of an array of size n?

    Solution:

    A

18. How long will it take to delete element k (where k is close to 1/2 n) from an array of size n where order does not matter?

    Solution:

    A

19. How long will it take to delete element k (where k is close to 1/2 n) from an array of size n where order does matter?

    Solution:

    A

20. How long will it take to determine if an element exists in a sorted linked list of size n?

    Solution:

    A

21. How long will it take to determine if an integer exists in a sorted array of n integers?

    Solution:

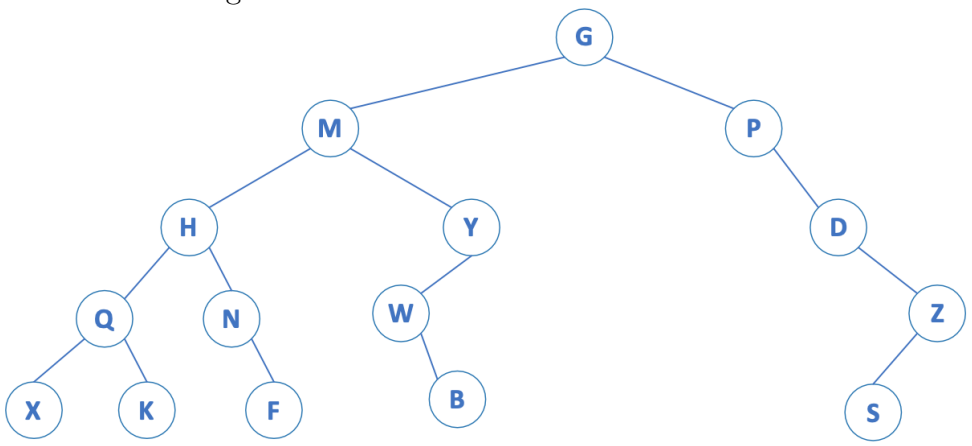
    B

22. How long will it take to correctly insert an element into an AVL tree of size n.

Solution:

$$B$$

Consider the following tree:



23. Give a pre-order traversal of the tree

Solution:

$$G, M, H, Q, X, K, N, F, Y, W, B, P, D, Z, S$$

24. Give a post-order traversal of the tree

Solution:

$$X, K, Q, F, N, H, B, W, Y, M, S, Z, D, P, G$$

25. Give a in-order traversal of the tree

Solution:

$$X, Q, K, H, N, F, M, W, B, Y, G, P, D, S, Z$$