CS 5/7350 – Test #1
March 10, 2020

Name: Key
[+ 5 pts for CS 5350]

ID:_____________________________

1.5 each

1. [9 pts] Define the following Terms as succinctly as possible:

(i) Algorithm A step by step procedure for solving a problem in a finite amount of time ← This is required.

(ii) Tree A connected, acyclic graph. ← does not have to be rooted

(iii) In-Order Traversal Visiting the vertices of a rooted tree by recursively visiting the left subtree, the current vertex and then the right subtree

(iv) Graph A collection of elements (vertices) and a relation on those elements (edges)

(v) NP-Hard A class of problems that are at least as hard and some are possibly harder than all N-P Problems. ← may or may not also be NP problem itself. ← Solve 1, solve all NP Problems.

(vi) Insertion Sort A method of ordering items by taking each item and placing it where it belongs among the other items already taken.

2 each

2. [8 pts] Solve the following problems:

(i) 2^{55} modulo 11 = $10 = (2^{10} \cdot 2^{10} \cdot 2^{10} \cdot 2^{10} \cdot 2^{10} \cdot 2^5) \% 11 = (1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 10)$.

(ii) Given |M| = 6 & |N| = 5, find | Power Set of (MxN) | $2^{6 \cdot 5} = 2^{30}$.

(iii) Number of edges in a complete graph with 10 vertices = $\frac{10 \cdot 9}{2} = 45$.

(iv) – (¼) modulo 7 = $= -(2) \% 7 = 5 \% 7 = 5$.

2 each

3. [14 pts] For each pair below, circle the choice that has the higher asymptotic upper bound. If they are the same, circle "same".

(i) $1 + 2 + 3 + 4 + \cdots + n$ or $n^2 + 3n + 65$ or same

(ii) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{n}$ or n or same

(iii) 1 * 2 * 3 * 4 * 5 * … * n or 1 * 2 * 3 * … * n * (n+1) or same

(iv) 3 + 3 + … + 3 (n times) or 2 + 2 + … + 2 (n times) or same

(v) 3 * 3 * … * 3 (n times) or 2 * 2 * … * 2 (n times) or same

(vi) 1 + 2 + 3 + 4 + … + n or 1 + 2 + 3 + 4 + … + n + (n+1) same

(vii) n^2 or $(n+1)^2$ or same

4. [5 pts] Argue that the problem, H, of creating a MIN-HEAP from an unsorted array of integers using the HEAPIFY algorithm discussed in class is at least as hard - and maybe even harder - than the problem, M, of finding the minimum element of the same unsorted array of integers.

Given a solution to problem H, I can use that solution to solve problem M by simply reading the first element of the heap created by the solution to H.

5. [9 pts] Determine a Huffman encoding for each symbol in a message that contains:

8 C's, 8 D's, 3 E's, 3 F's, 2 G's, 1 H and 1 K.

See below

How many bits are in the entire message if each symbol is encoded with 3 bits?

2pts

26 letters X3 = 78 bits

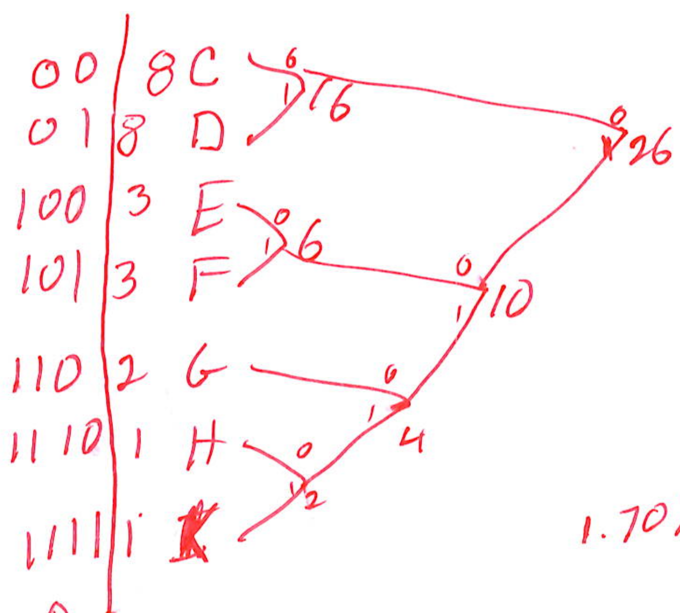
How many bits are in the entire Huffman coded message?

4pts

$2\cdot 8 + 2\cdot 8 + 3\cdot 3 + 3\cdot 3 + 3\cdot 2 + 4\cdot 1 + 4\cdot 1 = 64$ bits

How much entropy (information) is in the entire message?

3pts



$C+D = lg\left(\frac{26}{8}\right) = 1.70$ bits each

$E+F = lg\left(\frac{26}{3}\right) = 3.12$ bits each

$G = lg\left(\frac{26}{2}\right) = 3.70$ bits each

$H+I = lg\left(\frac{26}{1}\right) = 4.70$ bits each

$1.70 \times 16 + 3.12 \times 6 + 3.70 \times 2 + 4.70 \times 2 =$ 62.7 bits

I wanted this as part of the answer, but the question was unclear, so I didn't count off if it was omitted.

6. [6 pts] Two people need to establish a secret key for encrypting communications. They agree to use a Diffie-Hellman key exchange with a modulus of 11 and decide on 2 as the base. Person A chooses a random value of 8 and person B chooses a random value of 9.

   a. What is the value Person A sends to Person B

   3 1.5 point

   b. What is the value Person B sends to Person A

   6 1.5 points

   c. What is the shared secret key between Person A and Person B

   4 3 points

7. [9 pts] The table below gives asymptotic bounds on various cases of 3 algorithms. Add to the table any bounds you can also determine from the bounds given.

Problem			Algorithm			Implementation		
Best	Avg	Worst	Best	Avg	Worst	Best	Avg	Worst
$O(n^2)$	$O(n^2)$	$O(n^2)$	$O(n^2)$	$O(n^2)$	$\Theta(n^2)$			$\Omega(n^2)$
$O(n^2)$	$O(n^2)$		$O(n^2)$	$\Theta(n^2)$	$\Omega(n^2)$		$\Omega(n^2)$	$\Omega(n^2)$
$O(n^2)$			$\Theta(n^2)$	$\Omega(n^2)$	$\Omega(n^2)$	$\Omega(n^2)$	$\Omega(n^2)$	$\Omega(n^2)$

3 pts max each row ; 1 point per box

8. [6 pts] Answer the following questions.:

(i) A program requires 6 seconds to process an input size of 45. If the running time is Θ ($\sqrt{n}$) about how large of an input set could you process in 60s?

4500 seconds

(ii) A program requires 5 days to brute force attack a password of 48 bits. Since the running time is Θ (2^n) about how long would it take for the program to brute force attack a password of 256 bits?

5×2^{208} days

(iii) If a program required 5 days to brute force attack a password 48 bits, how long would it take to attack a password of 256 bits if the running time were $O(n^2)$ instead of exponential?

$\left(\frac{256}{48}\right)^2 \cdot 5 \approx 142$ days

9. [9 pts] Give the tightest asymptotic average case upper and lower bounds you know for the following scenarios:

(i) Deleting the 20th element of an array of size n when order doesn't matter

$\theta(1)$ move last element to place of deletion

(ii) Deleting the 20th element of an array of size n keeping everything else in the same order?

$\theta(n)$

(iii) Finding the k smallest items in an unsorted array of size n

$\theta(n)$ Quick select

(iv) Deleting an element from a heap of size n

$\theta(lg(n))$

(v) The best algorithm finding the middle element (based on value) in a sorted array.

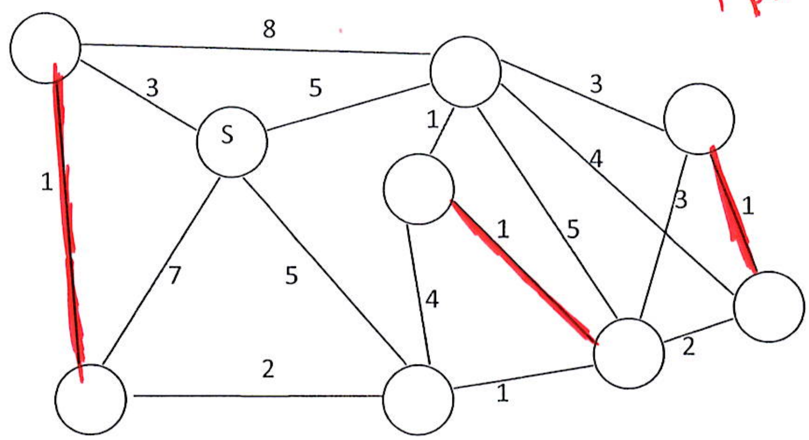
$\theta(1)$ just look it up - middle index

(vi) The best algorithm searching in a sorted linked list to determine if a specific element (based on value) is present.

$\theta(n)$

10. [10 pts] Consider the following graph. For any questions needing a starting vertex, use vertex S as the starting vertex.

1 pts



(i) What is the value of the third edge chosen when finding a minimum spanning tree using Prim's algorithm? 2 1.5 pts

(ii) What is the value of the third edge chosen when finding a minimum spanning tree using Kruskal's algorithm? 1 1.5 pts

(iii) When using Dijkstra's algorithm to find the shortest path from S to all vertices, what is the value of the third edge chosen? 5 1.5 pts

(iv) How many components are in the graph? 1 1.5 pts

(v) What is the minimum number of edges you need to remove so the graph will have an Euler Tour? Mark the edges you would remove. 3 3 pts

11. [6 pts] A particular problem requires 2 seconds to process 200 items and is $\Theta(n^3)$. You want to process 4000 items. You have a choice to either use a computer that is 10 times faster (allowing it to process 200 items in 0.2 seconds) or use the same computer with a different algorithm that still processes 200 items in 2 seconds, but has a growth rate that is $\Theta(n^2)$.

(i) Which is the faster choice for 4000 items?

$\Theta(n^2) = 800$ $\Theta(n^3)$ 10x faster = 1600 use $\Theta(n^2)$ algorithm

3 pts

(ii) For what input sizes is the faster computer better?

$0.2(\frac{n}{200})^3 < 2(\frac{n}{200})^2$ $n < 2000$ items

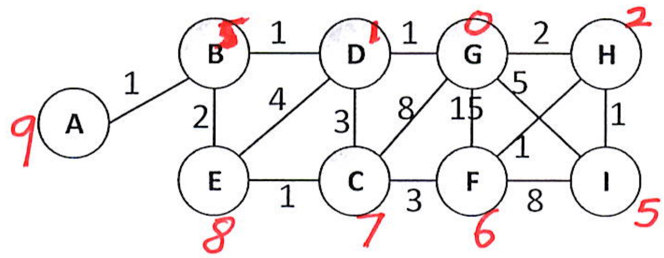
$\frac{n}{200} < 10$

(iii) For what input sizes is the $\Theta(n^2)$ algorithm better?

$0.2(\frac{n}{200})^3 > 2(\frac{n}{200})^2 \Rightarrow \frac{n}{200} > 10 = n > 2000$ items

3 pts

12. [9 pts] You live in city G. You want to know the cost to travel from city G to all other cities (A,B,C,D,E,F,H and I). The edges of the graph below represent the cost to travel the roads between various cities. If an edge doesn't exist, then there is no road between those two cities. In this particular scenario, even though the roads have a different cost, it takes most of a day to travel each road. Therefore, you must spend the night at each intermediate city (vertex) at an additional cost of 3.



(i) How would you modify Dijkstra's Single Source Shortest Path algorithm to find the cost from city G to all other cities in the graph with a vertex costing 3 to pass through it?

when passing through the city, the cost is 3 plus cost to reach the city instead of just the cost to reach the city.

2 pts

(ii) What is the order you reach the cities in your adjusted algorithm.

3 pts G D H $\begin{pmatrix} I & B \\ B & I \end{pmatrix}$ F C E A

(iii) Write the cost to reach each city from City G by its vertex in the graph.

4 pts