CS 5/7350 – Test 1
October 5, 2022

Name: Answers

- This exam is **closed book** and **closed notes**.
- Only the approved TI-30Xa calculator
- No cell phones, or other electronics.
- Pencil and/or pen only are permitted.
- Two Scratch Pages are on the back.
- It is 3 **hours** in duration.
- You should have 15 problems. Pay attention to the point value of each problem and dedicate time as appropriate.

On my honor, I have neither given nor received unauthorized aid on this exam.

SIGNED:_____

DATE:_____

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Name: _____
[+ 5 pts for CS 5350]

ID:_____

1. [6 pts] Argue that the problem, S, of sorting an unsorted array of integers of length greater than 100 elements is at least as hard - and maybe even harder – within a constant factor of O(1) than the problem, L, of finding the three largest elements of the same unsorted array of integers.

Since a solver for S can be used to solve L by sorting the array and then printing the last 3 elements, problem S must be just as hard or possible harder than Problem L.

2. [6 pts] What is an algorithm?

A step-by-step procedure for solving a problem in a finite amount of time.

3. [5 pts] Draw two different trees where both trees have a pre order traversal of ABC and a post order traversal of CBA.

4. [5 pts] Using n_0 equal to 10, show that $f(n) = 8n^2 + 5n + 1$ is $\Omega(n)$.

$0 \leq c_1 n \leq 8n^2 + 5n + 1 \quad \forall n \geq 10$

$0 \leq c_1 \leq 8n + 5 + \frac{1}{n} \quad \forall n \geq 0$

c_1 can be 8

Note 8 is not the tightest value, but it is always less than 8n + 5 +1/n

5. [8 pts] You run different programs for various values of "n" and create 4 tables of the runtimes. Give the Asymptotic bounds that each of the tables support?

a.	n	time(ms)	b.	n	time(ms)	c.	n	time(ms)	d.	n	time(ms)
	10	1348		10	5698		100	52		4	1
	20	8348		20	5698		200	73		5	5
	30	27348		30	5698		300	89		6	30
	40	64348		40	5698		400	102		7	210
	50	125348		50	5698		500	114		8	1680
	60	216348		60	5698		600	124		9	15120
	70	343348		70	5698		700	134		10	151200
	80	512348		80	5698		800	143		11	1663200
	90	729348		90	5698		900	152		12	19958400
	100	1000348		100	5698		1000	160		13	259459200

$\theta(n^3)$ $\theta(1)$ $\theta(\sqrt{n})$ $\theta(n!)$

6. [5 **pts**] Two people need to establish a secret key for encrypting communications. They agree to use a Diffie-Hellman key exchange with a modulus of 11 and decide on 2 as the base. Person A chooses a random value performs the appropriate computations and sends the value 5 to person B. Person B chooses a random value of 3 and performs the appropriate computations:

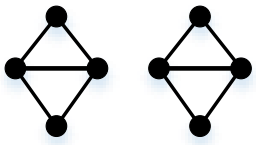
   a. What is the value Person B sends to Person A

   8

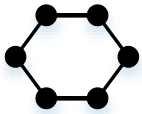
   b. What is the shared secret key between Person A and Person B

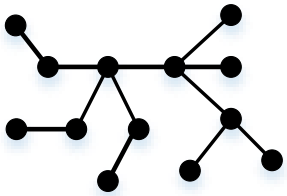
   4

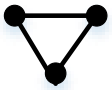
7. [7 pts] For the following graphs, indicate whether they are a tree, a complete graph, a cycle and/or are bipartite or none of the above.

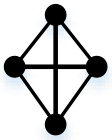
(i)  None

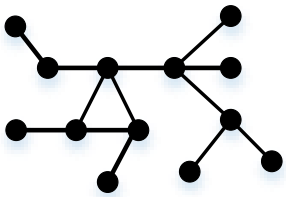
(ii)  Tree, Complete, Bipartite

(iii)  Cycle, Bipartite

(iv)  Tree, Bipartite

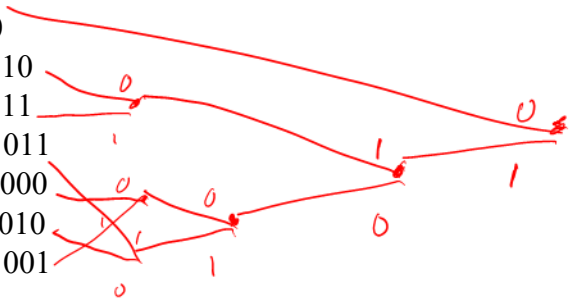
(v)  Cycle, Complete

(vi)  Complete

(vii)  None

8. [9 pts] You had a message that was 11 characters long. It was created from the letters A, B, C, D, E, F and G. You Huffman compressed the message. As a result of your compression, each letter had the following bit patterns:

- A - 0
- B - 110
- C - 111
- D - 1011
- E - 1000
- F - 1010
- G - 1001



Draw the Tree that produced this compression.

How many of each letter were present in the original message?

A: 4 B = 1 C = 2 D = 1 E = 1 F = 1 G = 1 others may work, too

How much entropy was present in the original message?

28.054 bits

9. [6 pts] A particular algorithm on a computer requires 2 seconds to process 400 items and is $\Theta(n^3)$. You want to process 8000 items. You have a choice to either use a computer that is 10 times faster (allowing it to process 400 items in 0.2 seconds) or use the same computer with a different algorithm that still processes 400 items in 2 seconds, but has a growth rate that is $\Theta(n^2)$.

a) Which is the faster choice for 8000 items?

Use $\Theta(n^2)$ Alg

b) For what input sizes is the faster computer better?

< 4000

c) For what input sizes is the $\Theta(n^2)$ algorithm better?

> 4000

10. [9 pts] Consider two different algorithms that each solve a different problem.
    - Algorithm X solves Problem Px and Algorithm X is $O(n^3)$ and $\Omega(n)$
    - Algorithm Y solves Problem Py and Algorithm Y is $O(n^4)$ and $\Omega(n^2)$
    - Algorithm Z solves Problem Pz and Algorithm Z is $\Theta(n^5)$

    From the information above, determine if each of these "**Yes** it is true", "**Maybe** it is true but doesn't have to be", or "**No** it is not true"

    a. M Problem Py is harder than Problem Pz

    b. M Problem Pz is harder than Problem Px

    c. M Algorithm Y is harder than Algorithm X

    d. M Problem Y is $\Omega(n^2)$

    e. M Problem X is $\omega(\log(n))$

    f. Yes Problem Y is $O(n^4)$

    g. Yes Problem X is $o(n^4)$

    h. Yes Algorithm Z is $\Omega(n)$

    i. Yes Algorithm Z is $\omega(n)$

11. [10 pts] Setup the table as shown in class and determine 1/10 modulo 7657.

N/A

$\frac{1}{10} \bmod 7657 = 5360$

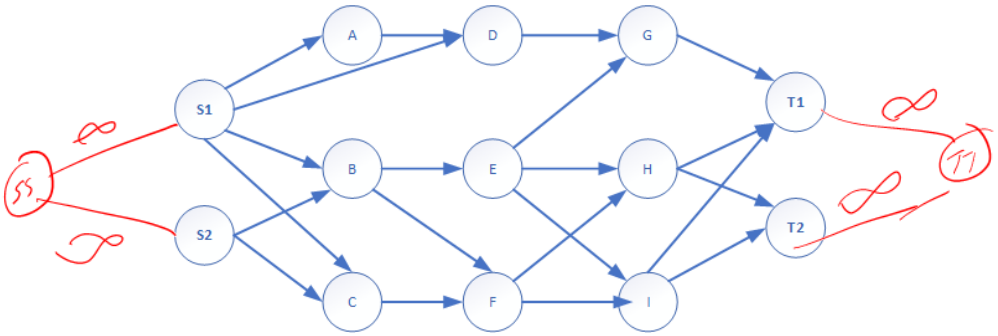
12. [6 pts] Solve the following for the value of A. Ensure your answer is an integer between 1 and 3058 inclusive.

Use $\frac{1}{5} \bmod 3059 = 612$

1822 = ((5 * A) + 89) modulo 3059

A = 2182

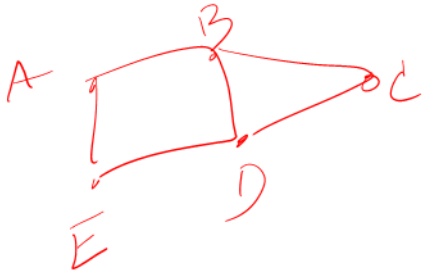
13. [6 pts] You have the following graph:



Assume each of the edges have numbers associated with them that represent the flow of goods. You can think of S1 and S2 as being warehouses that need to be emptied and you can think of T1 and T2 as being storage facilities that can store the items from the S1 and S2 warehouses which are being emptied. Items leaving the S1 and S2 warehouses can be split between the T1 and T2 storage facilities. That is, you do not care which storage facility T1 or T2 each item arrives at. You just want to empty them from S1 and S2 as fast as possible.

How would you modify the graph above to allow you to use the Ford-Fulkerson maximum flow algorithm we learned in class that works for a single source, S, and a single sink, T, to solve this problem?

14. [6 pts] Draw a graph and give a smallest last vertex ordering where the terminal clique is not the largest complete subgraph. Circle the first vertex you write down in your smallest last ordering.



A E D B C

15. [6 pts] Answer the following questions:

a) What is the weight of a minimum spanning tree for a connected, acyclic graph with |V| vertices where all edges have a weight of 4?

$4(|V|-1)$

b) What is the weight of a minimum spanning tree for a cycle with 10 vertices where all edges have a weight of 4?

$4(|V|-1) = 4 \cdot 9 = 36$

c) A complete bi-partite graph $B_{j,k}$ is a graph which has J vertices in one partition and k vertices in another partition and all possible edges present between the partitions. For which values of j and which values of k are the degrees of the vertices even?

when j + k are both even

Scratch Paper

Scratch Paper