CS 5/7350 – Test 1
March 2, 2022

Name: _____

- This exam is **closed book** and **closed notes**.

- No calculators, No cell phones or other electronics.

- Pencil and/or pen only are permitted.

- Two Scratch Pages are on the back.

- It is 3 **hours** in duration.

- You should have 14 problems. Pay attention to the point value of each problem and dedicate time as appropriate.

On my honor, I have neither given nor received unauthorized aid on this exam.

SIGNED:_____

DATE:_____

Name: _____

[+ 5 pts for CS 5350]

ID:_____

1. [9 pts] Define the following Terms as succinctly as possible:

   a)   Algorithm _____

   _____

   _____

   b)   Big Theta _____

   _____

   _____

   c)   Set _____

   _____

   _____

   d)   Implementation _____

   _____

   _____

   e)   Entropy _____

   _____

   _____

   f)   Graph _____

   _____

   _____

2. [8 pts] You have a complete graph with |V| vertices where |V| is >= 2. Each edge in this graph has a capacity of 7. You pick one vertex as the Start Vertex, S, and another vertex as the Sink Vertex, T. Since the is a complete graph, you will get the same answer regardless of which two vertices you pick.

    a) What is the length of the shortest path between Vertex S and Vertex T.

    b) What is the maximum flow (in terms of |V|) between Vertex S and Vertex T.

3. [6 pts] Argue that the problem, S, of sorting an unsorted array of integers of length greater than 100 elements is at least as hard - and maybe even harder - than the problem, L, of finding the five smallest values of the same array.

4. [6 pts] The following in-order traversal: A B C and the following pre-order traversal: B A C would form the following tree:
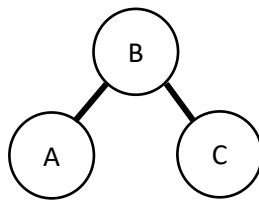
Give a 3 vertex in-order traversal and a 3 vertex pre-order traversal that cannot form a tree. That is, no single tree can have both the in-order traversal and the pre-order traversal.

5. [6 pts] Using n_0 equal to 10, show that $f(n) = 7n^3 + 8n^2 + 5n + 1$ is $\Omega(n^2)$.

6. [8 pts] You run different programs for various values of "n" and create 4 tables of the runtimes. Give the Asymptotic bounds that each of the tables support?

a.	n	time(ms)	b.	n	time(ms)	c.	n	time(ms)	d.	n	time(ms)
	1000	2120		1000	58913		100	21564		52	20
	2000	4120		2000	60913		200	81564		53	60
	3000	6120		4000	62913		300	181564		54	180
	4000	8120		8000	64913		400	321564		55	540
	5000	10120		16000	66913		500	501564		56	1620
	6000	12120		32000	68913		600	721564		57	4860
	7000	14120		64000	70913		700	981564		58	14580
	8000	16120		128000	72913		800	1281564		59	43740
	9000	18120		256000	74913		900	1621564		60	131220
	10000	20120		512000	76913		1000	2001564		61	393660

7. [6 pts] Answer the following Questions:
   a. Given that M > 100 and (7^{311} mod M) = 1; Find 7^{313} mod M = _____

   b. Compute 1/3 mod 9 _____ .

   c. Compute - (½) mod 11 _____

8. [8 pts] A message contains the following number of each symbol:

30 A's, 14 B's, 10 C's, 9 D's, 7 E's, 4 F's, 2 G's, 1 H and 1 K.

Create a Huffman coding for each symbol:

How many bits are in the entire Huffman coded message?

9. [6 pts] Two people need to establish a secret key for encrypting communications. They agree to use a Diffie-Hellman key exchange with a modulus of 11 and decide on 2 as the base. Person A chooses a random value performs the appropriate computations and sends the value 3 to person B. Person B chooses a random value of 4 and performs the appropriate computations:

    a. What is the value Person B sends to Person A

   b. What is the shared secret key between Person A and Person B

10. [9 pts] Consider two different algorithms that each solve a different problem.
- Implementation X solves Problem Px and Implementation X is $\Omega(n)$ and $O(n^2)$
- Implementation Y solves Problem Py and Implementation Y is $\Theta(2^n)$

Determine if each of these "**Yes** it is true", "**Maybe** it is true but doesn't have to be", or "**No** it is not true"

a. _____ Problem Px is harder than Problem Py

b. _____ Problem Py is harder than Problem Px

c. _____ Implementation X is harder than Implementation Y

d. _____ Problem X is $\Omega(n)$

e. _____ Problem X is $\Omega(n^2)$

f. _____ Problem X is $O(n)$

g. _____ Problem X is $O(n^2)$

h. _____ Implementation X is $\Omega(1)$

i. _____ Implementation X is $O(2^n)$

11. [8 pts] The graph below represents containers that are transported between these cities each day. You are determining the maximum flow from vertex S, Vancouver, to vertex T, Winnipeg, using the Ford-Fulkerson algorithm in the graph below.
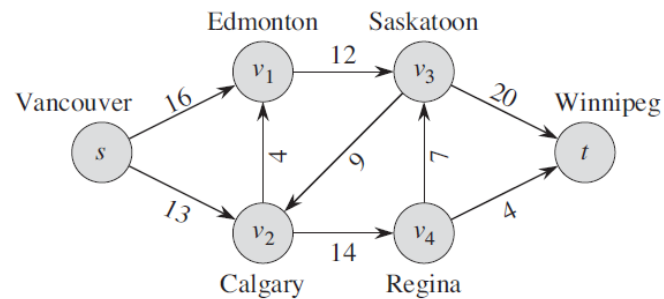
A Path Search finds the path s -> v2 -> v4 -> v3 -> t.

a. How much flow is in this path?

b. What does the new graph look like after "removing" this flow

A Path Search now finds the path s -> v1 ->v3 -> v4 -> t on the new graph.

c. Is it possible to find this path?

d. If so, how much flow is in this path?

12. [10 pts] Answer the following questions.:

   a)   A program requires 6 seconds to process an input size of 40. If the running time is $\Theta(\sqrt{n})$ about how large of an input set could you process in 160 seconds?

   b)   A program requires 2 **DAYS** to brute force attack a password of 64 bits. Since the running time is $\Theta(2^n)$ about how **MANY DAYS** would it take for the program to brute force attack a password of 256 bits?

   c)   A program requires 2 **DAYS** to brute force attack a password of 64 bits. About how **MANY DAYS** would it take for the program to brute force attack a password of 256 bits if the running time were $\Theta(n^2)$ instead of exponential?

   d)   A program requires 4 milliseconds to process an input size of 1000. If the running time is $\Theta(n^4)$ about how many seconds would it take to process an input size of 1 million items?

   e)   A program requires 4 milliseconds to process an input size of 1000. If the running time is $\Theta(n)$ about how long many seconds will it take to process an input size of 1 million items?

13. [5 pts] You live in city B. You want to know the cost to travel from city B to all other cities (A,B,C,D,E,F,H and I). The edges of the graph below represent the cost to travel the roads between various cities. If an edge doesn't exist, then there is no road between those two cities.
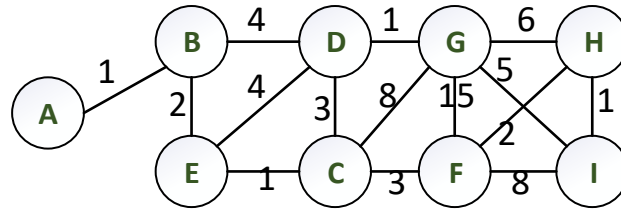
    a)   What is the order in which you explore the cities using Dijkstra's Single Source Shortest Path algorithm to find the cost from city B to all other cities in the graph?

14. [5 pts] Give a smallest last vertex ordering for the graph in Problem #13. Circle in your ordering the first vertex you wrote down for that ordering.

Scratch Paper

Scratch Paper