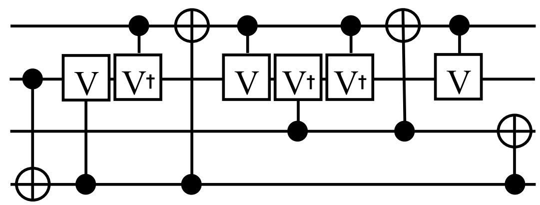
ECE/CS 8381

Quantum Logic and Computing

Instructor: Mitch Thornton



GOAL: Introduction to the Ideas of Reversible and Quantum Logic and Computing

http://lyle.smu.edu/~mitch/class/8381/index.html

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Class Grades

- ***Examination 1*** (25%)- Test of the basic concepts between beginning of class and Mid-term
- ***Examination 2*** (25%)- Test of new concepts discussed in class to date of exam
- ***Project Proposal*** (15%) – Thorough Background on Previous results, proposed approach, (hypothesis, planned experiments)
- ***Homework/Labs*** (10%) –Assigned periodically during the semester
- ***Examination 3*** (25%) – Final Project Report including Prototype/Experimental Results and an Accompanying Presentation

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Desired Student Background

- ***Math*** – linear algebra, discrete mathematics, elementary probability theory
- ***Physics*** – basic Physics courses required for undergraduate in the sciences/engineering, an introduction to quantum mechanical principles is desirable
- ***CS/ECE*** – computer architecture fundamentals, digital design fundamentals, exposure to algorithms and theory is desirable

Anyone with credit in the ECE 5/7383 Intro. to Quantum Informatics Automatically Satisfies the Desired Background

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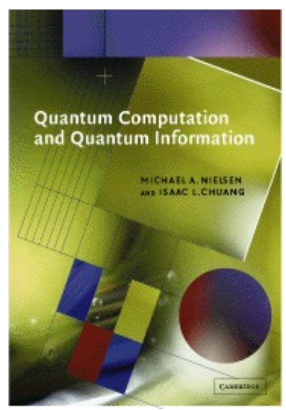
Textbook



This is a comprehensive reference. We will not cover everything in this book.

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Other Material

- Main Textbook contains (*most of*) the general overview of the course material
- Selected Material from:
  - References
  - Historical Readings
  - Archived Papers
  - Other Web Resources
- Will ATTEMPT to place all notes online, BUT,
  - **YOU SHOULD TAKE NOTES ALSO**

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General Topic Outline

- Linear and selected topics from Tensor Algebra (review sessions online)
- Computation and the Laws of Physics (very brief)
- Relevant Concepts in Quantum Mechanics/Electrodynamics
- Hilbert Spaces and the Notation of Dirac
- Quantum States and Measurement
- The Concept of the Qubit

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General Topic Outline (cont)

- The Bloch Sphere and Superposition
- Entanglement
- Reversible Logic and Thermodynamics
- Quantum Logic Gates
- No-Cloning Theorem
- Quantum Algorithms/Circuit Structure
- Survey of Known Algorithms

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Quantum Computing Overview

- New computing paradigm
- Certain algorithms show tremendous speedup
  - Overcomes limitations of Turing model
- Computes Fourier transform in

  $\mathrm{O}(\log n)^2$ rather than $\mathrm{O}(n \log n)$
- Database search in $\mathrm{O}(n^{1/2})$ rather than $\mathrm{O}(n)$ [Grover]
- Factorization in $\mathrm{O}(\log n)$ rather than $\mathrm{O}(n^{1/2})$ [Shor]

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Quantum Characteristics Exploited

- Quantum Superposition (Schrodinger's Cat Paradox)
- EPR paradox - Entanglement
- Teleportation
- Interpretations (*e.g*. the Multi-verse)
- Pure and mixed states
- No cloning theorem

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Quantum Bit (qubit)

- Superposition of basis (1 and 0) states:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

where:

$$|\alpha|^2 + |\beta|^2 = 1 \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- Collapses into a basis state upon observation:

$$\text{Prob}(0) = |\alpha|^2 \qquad \text{Prob}(1) = |\beta|^2$$

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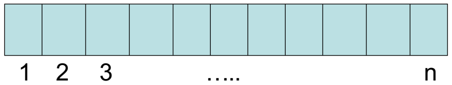
Quantum Register/Computer



- Each cell contains a qubit
- Number of BASIS states is 2^n
- Gate Model of Computation (Deutsch'85):
  1. Initialize Qubits to known States (usually 0 basis state)
  2. Apply a sequence of Operations (called "gates")
  3. Read/Observe the final State of the Register

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Qubit Basis States

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Some Single Qubit Quantum Gates

Hadamard Pauli-X Pauli-Y Pauli-Z

$$\mathbf{H} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \mathbf{Y} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \quad \mathbf{Z} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

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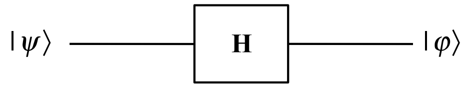
Example Computation

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha|0\rangle + \beta|1\rangle$$



$$|\varphi\rangle = \mathbf{H}|\psi\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \alpha+\beta \\ \alpha-\beta \end{bmatrix}$$

$$\mathbf{H}|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{|0\rangle+|1\rangle}{\sqrt{2}} \qquad \mathbf{H}|1\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{|0\rangle-|1\rangle}{\sqrt{2}}$$

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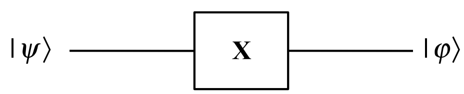
Another Example Computation

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \alpha|0\rangle + \beta|1\rangle$$



$$|\varphi\rangle = \mathbf{X}|\psi\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix} = \beta|0\rangle + \alpha|1\rangle$$

$$\mathbf{X}|0\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle \qquad \mathbf{X}|1\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

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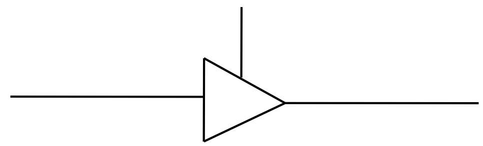
Controlled Gates

- Allows State of one qubit to Control Transformation of Another
- Analogous to a “Control or Enable” Input on a Classical Electronic Logic Gate



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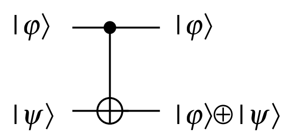
Controlled-X Gate



$$\mathbf{C_X} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Also, a “controlled-NOT” operator

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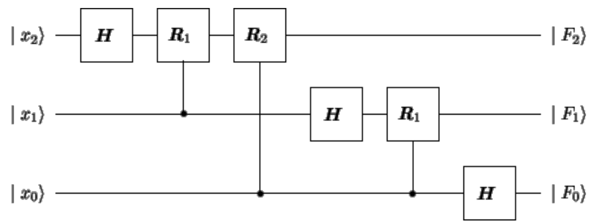
Quantum Fourier Transform Circuit



6 gates suffice to compute an 8 component Discrete Fourier transform that would require 24 operations in FFT and 64 in straight DFT

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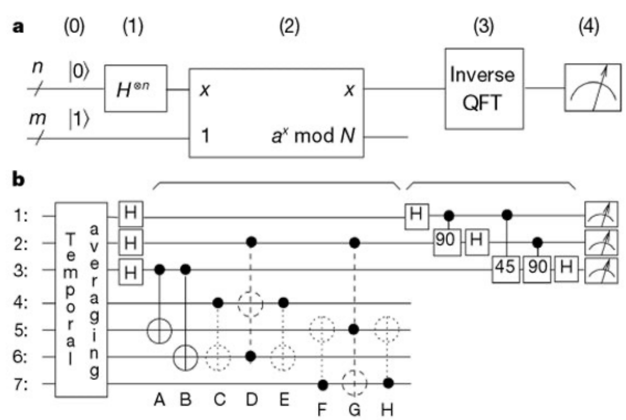
Shor's Algorithm



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Problems with Quantum Computing

- Decoherence
- Error-correction
- Realizability of gates
- Initialization of the register
- Quantum Memory

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