Hands-on Quantum Programming Labs for EECS Students

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Abstract

This report presents a practical approach to teaching quantum computing to Electrical Engineering & Computer Science (EECS) students through dedicated hands-on programming labs. The labs cover a diverse range of topics, encompassing fundamental elements, such as entanglement, quantum gates and circuits, as well as advanced algorithms including Quantum Key Distribution, Deutsch and Deutsch-Jozsa Algorithms, Simon's algorithm, and Grover's algorithm. As educators, we aim to share our teaching insights and resources with fellow instructors in the field. The full lab handouts and program templates are provided for interested instructors. Furthermore, the report elucidates the rationale behind the design of each experiment, enabling a deeper understanding of quantum computing.

Keywords — Quantum Programming, Quantum Computing Education, Hands-On Laboratory, Quantum Parallelism, Qiskit

1 Introduction

Quantum Computing is a rapidly growing field with many potential applications, including cryptography, optimization, pharmaceuticals, materials science, etc. As more and more companies and organizations invest in quantum research and development, the demand for skilled professionals with expertise in quantum computing is likely to increase. That is, having an understanding of quantum computing and practical experiences in quantum programming can be a valuable asset for Electrical Engineering & Computer Science (EECS) students looking to pursue careers in industry or academia. Therefore, in Fall 2022, we decided to offer an introductory-level Quantum Computing course the first time for our EECS senior students at Cleveland State University(CSU).

Recommended by Prof. Kiper at Miami University [8], we chose Bernhardt's book "Quantum Computing for Everyone" [1] as the textbook. This book provides simple explanations of the challenging mathematics of quantum computing and gives elementary examples to illustrate what it means and how it is applied in problem solving. However, like many other quantum computing books, this text also lacks the stuff of practical quantum programming, which would be useful for the EECS students because quantum programming expertise will likely be in high demand as quantum computing continues to advance.

Hands-on programming provides a means to bridge the gap between theoretical knowledge and practical implementation in quantum computing. By engaging in hands-on programming, students can gain a deeper understanding of quantum concepts such as superposition, entanglement, quantum gates and quantum algorithms. For example, it might be easier for students to understand how the Deutsch and the Deutsch-Jozsa algorithms work via the quantum phase kickback circuit implementation than the rigorous proof in math. Furthermore, actually writing code and running it on quantum computers or simulators, students can explore the real-world implications of quantum algorithms and discover novel ways to leverage quantum computing power. As suggested in [11], a course with dedicated programming labs would greatly improve students' learning experience in quantum programming.

Several open-source quantum software development kits, such as Amazon's Braket, Google's Cirq, IBM's Qiskit, Microsoft's Q# and QDK, etc. are available on-line. A good survey and comparison of these frameworks can be found in [4]. For the dedicated hands-on labs, we focused on teaching students a single quantum programming environment throughout the course. We chose IBM's Qiskit because IBM is the only provider which allows unlimited free accesses to actual quantum computers. Furthermore, Cleveland Clinic [2] installed the IBM Quantum System One [5], which supports a 127-qubit Eagle processor, to accelerate biomedical discoveries. Through the collaboration between CSU and the Cleveland Clinic[18], our students will have opportunities to conduct experiments on the cutting-edge quantum computer.

Therefore, our goal of this work was to design programming experiments for students to implement during seven dedicated hands-on labs. Many of the experiments are tied to the textbook and students are asked to modify a program template and then verify their experimental result with the corresponding example described in the textbook. To give students more practices, we selected and revised some experiments from the Qiskit Textbook [13], Qiskit Computing Labs [14], and from the Professors Yanofsky and Mannucci's book [19]. Through this report, we would like to share our teaching experience and materials with other instructors teaching quantum computing. Interested readers can refer to the Appendices for all of the hands-on lab handouts and program templates.

2 Course Content and Schedule

We outline the course content and schedule in Table 1. The class is scheduled weekly, for 3 consecutive hours. and a 75-minute hands-on lab section is carried out immediately after the topic has been discussed in the class. Performing a lab immediately after the topic is taught ensures that the information is still fresh in students' minds. During a hands-on lab, students can observe the results of their actions in real-time. This instant feedback allows them to identify and correct any misconceptions or errors, further reinforcing their learning and understanding of the material.

Table 1: Topics and Schedule

	Topic 1	Topic 2
Week 1	Introduction (Ch. 1)	Overview: Qubit, Superposition,
	, ,	Interference & Entanglement
Week 2	Linear Algebra (Ch. 2)	Linear Algebra (Ch. 2)
Week 3	Spin & Qubits (Ch. 3)	Hands-on Lab 1 (Python & Jupyter)
Week 4	Spin & Qubits (Ch. 3)	Quantum Gates (Ch. 7)
Week 5	Test 1	Entanglement (Ch. 4)
Week 6	Entanglement (Ch. 4)	Hands-on Lab 2 (Entanglement)
Week 7	Bell's inequality (Ch. 5)	Hands-on Lab 3 (Quantum Gates and Circuits)
Week 8	Ekert, QKD (Ch. 5), Hands-on Lab 4 (QKD)	Guest Speaker (Cleveland Clinic)
Week 9	Test 2	Superdense code, Teleportation
		& Error Correction (Ch. 7)
Week 10	Deutsch & Deutsch-Jozsa algorithms (Ch. 8)	Hands-on Lab 5 (Deutsch algorithm)
Week 12	Simon's algorithm (Ch. 8)	Hands-on Lab 6 (Simon's algorithm)
Week 14	Shor's algorithm & Grover's algorithm (Ch. 9)	Hands-on Lab 7 (Grover's algorithm and 3-SAT)
Week 15	Complexity (Ch. 8)	Test 3

3 Hands-on Quantum Programming Labs

The seven dedicated hands-on labs were conducted in our Linux cluster lab. Each workstation has been installed the software packages and libraries, such as Python3, qiskit, jupyter, etc. Note that the Jupyter Notebook [17] is an interactive computing environment and its cell-based approach makes it easy to organize code and text, run code interactively, and create documents that combine executable code, visualizations, and explanations.

In the lab, students need to follow the procedure stated in the handout step by step to implement the programs, record, and explain the results in their lab reports. This section gives a brief description of each lab and its objectives. Some interesting experiments which are important for students to practice are also presented below. Interested instructors can find the full handouts in the Appendices.

3.1 Hands-on Lab 1: Python and Jupyter

Objectives:

• learn the features of Python

practice programming with Jupyter Notebook

In the first warm-up lab, each student will set up his/her working environment (e.g. installing Qiskit tools, creating a dedicated sub-directory for this course, etc.), and get familiar with some Linux commands. Students will also learn how to implement and run Python 3 programs in Jupyter Notebook's code cells, as well as how to load and execute program templates in cells. Furthermore, they will review some features in the Python language, such as using indentation instead of a pair of curly braces in C/C++/Java to indicate a block of code, the dictionary data structure which uses string as the array index, etc. Note that, in this lab, students are asked to copy and run the quantum "Hello World" program which uses a Hadamard gate to set a qubit into a superposition state and then measures it. However, we will not explain how the code works in detail until the next lab. This fosters a sense of curiosity that may lead to more profound and lasting learning outcomes.

3.2 Hands-on Lab 2: Entanglement

Objectives:

- · Qiskit Terra and Qiskit Aer.
- Hadamard gate
- · two-qubit entanglement

In this lab, students will learn how to write simple quantum programs in Qiskit. A Qiskit program typically follows a structured pattern to firstly construct the quantum circuits using the Qiskit Terra module and then execute the circuits either on a simulator (i.e. the Qiskit Aer module) or on a real IBM quantum computer. Instructors should remind students that the Qiskit's qubits order is simply reversed as compared to the order used in the textbook[1] (also in the reference[19]). That is, Qiskit puts q_0 rightmost (e.g. $q_2q_1q_0$) while the textbook puts q_0 leftmost (e.g. $q_0q_1q_2$). Students have to be careful while reading Qiskit documents, Qiskit gate operation matrix, histogram output result, etc.

Students will study in detail the quantum "Hello World" program template given in the previous lab. Next, they will learn that quantum randomness via the H gate is not simply like a classical random coin toss. That is, as shown in Figure 1, students will modify the template by applying two H gates in succession and observe that the result is not exactly the same as tossing the coin twice. They also need to verify the result through the state vector calculation.

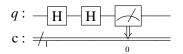


Figure 1: Applying two H gates in succession

The major theme of Lab 2 is entanglement. Students will construct the two-qubit entanglement circuit shown in Figure 2 and observe the entangled results (i.e. either "00" or "11"). In the next experiment, they need to modify the circuit by initializing the qubit q_1 to be 1 like in the Figure 3. Students are asked to explain whether the result is entangled or not in their reports. They can refer to page 59 in the textbook about the simple rule to check the entanglement of two qubits. That is, assume that r, s, t, and u represent the probability amplitudes of two qubits q_0q_1 states: 00, 01, 10, and 11, respectively. These two qubits are entangled if $ru \neq st$. Otherwise, they are not entangled and can be decomposed into the tensor product of two individual qubits.

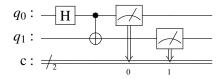


Figure 2: Two-qubit Entanglement: 00 or 11

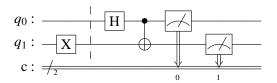


Figure 3: Two-qubit Entanglement: 01 or 10

After finishing Lab 2, students will be given a homework assignment. They are asked to run a two-qubit entanglement program on an actual quantum computer. Each student needs to follow the steps to create a free account on the IBM website, which allows the user to submit and execute quantum programs on IBM quantum computers worldwide[15]. They need to explain in their report why the histogram result is a little different from the result obtained in the Lab2 Experiment using the simulator.

3.3 Hands-on Lab 3: Quantum Gates and Circuits

Objectives:

- entanglement with more than 2 qubits in Qiskit
- constructing circuits with quantum gates such as X, H, CNOT, and Toffoli
- use of quantum gates to emulate classical gates (AND, NAND, and OR)

This lab starts with doing two experiments: one is to entangle three qubits from the same control qubit and the other is to entangle four qubits in a chain manner. Next, students are asked to use quantum gates such as X, H, CNOT, and Toffoli, to construct circuits for emulating the classical AND, NAND, and OR gates. Some students may have learned NP-completeness already. Instructors could tell them that, for NP-complete problems, such as SAT, quantum computers can gives quadratic speedup over the classical computers. Learning how to emulate the classical gates will help them build the oracle circuits used in the Grover's search algorithm for solving the SAT problem. These SAT-related experiments will be conducted in Lab 7 in the future.

Note that there are two different methods to implement the classical OR gate. One is to use two CNOT gates and one Toffoli gate, as shown in Figure 4. If either one of the two inputs is 1, it uses the CNOT to toggle the output result to be 1. If both inputs are 1's, output result will be toggled twice back to 0 and hence we need the Toffoli gate to toggle the output qubit again. The other method is to adopt X and AND gates based on the following equation:

$$q_0 \vee q_1 = \neg(\neg q_0 \wedge \neg q_1)$$

That is, as shown in Figure 5, we invert both inputs and apply the NAND function. Finally, we have to invert the inputs back to their original values.

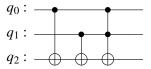


Figure 4: Implementing the OR gate with CNOT and Toffoli gates

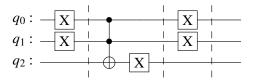


Figure 5: An alternative implementation of the OR gate

There are two slightly more complicated quantum circuits that are useful for students to study. The first circuit is shown in Figure 6. After implementing and executing the program, students will find out, for the four possible inputs q_0q_1 : 00, 01, 10, and 11, the corresponding outputs will be: 00, 10, 01, and 11, respectively. They need to deduce that the circuit swaps the states between q_0 and q_1 .

The second circuit, known as Phase Kickback and depicted on the left of Figure 7, will generate the outputs q_0q_1 as 00, 11, 10, and 01, corresponding to the inputs: 00, 01, 10, and 11, respectively. Namely, the circuit acts like the q_1 is the control while the q_0 becomes the target of a CNOT gate, as shown in the right of Figure 7. Note that the two H gates in q_0 's wire are not successive due to the presence of a CNOT connection in between. The Phase Kickback circuit has been used in many quantum algorithms. In our experience, comprehending the Phase Kickback circuit would be helpful for beginners in elucidating the workings of the Deutsch and Deutsch-Jozsa algorithms.

It is also worth mentioning that, though not included in the Lab3, the circuits in Figure 6 and Figure 7 can be used together. The current IBM quantum computer adopts the heavy-hexagon lattice topology to connect qubits[12]. Hence, the swap gate is needed to facilitate interactions between non-adjacent qubits. Moreover, the control qubits are not adjacent in the lattice and this limitation makes the circuit in Figure 6 impossible if q_0 and q_1 are connected together. If the qubit q_1 in Figure 6 cannot be the control qubit, we can simply use the Phase Kickback to solve the problem[20]. The solution is depicted in the rightmost part in Figure 8. This can be left as a homework exercise for students.

In addition to the swap gate, there is another interesting quantum circuit, called the bridge gate[6], which can let the CNOT gate to operate on non-adjacent qubits. For example, if the control qubit q_0 and the target qubit q_2

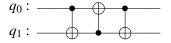


Figure 6: Swap the states in q_0 and q_1

$$q_0:$$
 H H \equiv $q_0:$ $q_1:$ $q_1:$

Figure 7: Phase Kickback

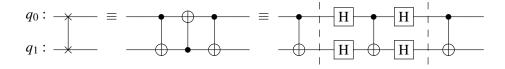


Figure 8: Swap the states in q_0 and q_1 if q_1 cannot be the control qubit

are not adjacent, they can use their common neighbor q_1 as the bridge This involves four direct-connect CNOT gates, as shown in Figure 9. The circuit can be comprehended using the following instructions in sequence:

$$q_2 = q_{1_org} \oplus q_{2_org}$$
 $q_1 = q_{0_org} \oplus q_{1_org}$
 $q_2 = q_1 \oplus q_2 = (q_{0_org} \oplus q_{1_org}) \oplus (q_{1_org} \oplus q_{2_org}) = q_{0_org} \oplus q_{2_org}$
 $q_1 = q_0 \oplus q_1 = q_{0_org} \oplus (q_{0_org} \oplus q_{1_org}) = q_{1_org}$

Note that the value in the intermediate qubit q_1 remains unchanged at the end. We plan to include this example as a new experiment in the future.

3.4 Hands-on Lab 4: Quantum Key Distribution

Objectives:

- the BB84 protocol (see textbook page 53)
- the Ekert protocol (see textbook page 87)

Quantum Key Distribution (QKD) is a cryptographic technique that utilizes quantum mechanics to enable two communication parties to produce a shared random secret key known only to them. The secret key can be used for encrypting and decrypting messages. There are two QKD protocols discussed in the textbook:

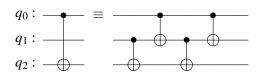


Figure 9: The Bridge gate: The CNOT gate (control: q_0 , target: q_2) by way of q_1

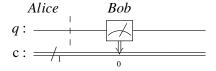


Figure 10: BB84 emulation: Both Alice and Bob choose the Vertical direction

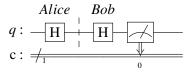


Figure 11: BB84 emulation: Both Alice and Bob choose the Horizontal direction

BB84 and Ekert91. In this lab, students are asked to implement Qiskit programs to emulate the behaviors of these two protocols and verify the output results of the probabilities from their programs with the corresponding descriptions in the textbook.

The BB84 protocol exploits the principles of quantum mechanics, utilizing the properties of quantum states to ensure the security of the key exchange process. As described in the textbook, Alice prepares a sequence of 4n qubits, encodes each qubit randomly in one of two bases (i.e. either vertical or horizontal) and then sends them to Bob. Bob measures each of these qubits using randomly chosen bases. Then, they publicly communicate which bases they used for each qubit and only keep the qubits corresponding to the times when they both use the same basis. The length of these kept qubits is roughly about 2n. These qubits will be identical if Eve is not intercepting them. Therefore, Alice and Bob can compare half of these qubits to detect the presence of any Eve's eavesdropping attempts. If they agree on all of them, they know Eve is not listening in and they can use the other n qubits as the shared key to encrypt/decrypt messages in the future communications.

The first simple experiment is to let students observe that Alice and Bob will obtain the same qubit value if they choose the same basis – either the vertical or horizontal direction, as shown in Figure 10 and Figure 11, respectively. In the next experiment, students are asked to run a template program which emulates the BB84 protocol except detecting any interception. The template initially sets the n to be 4, a small value so students can trace the executions easily. Based on the program output, they need to mark, in their report, the bit positions where Alice and Bob use the same bases and then extract the corresponding bits from "Alice bits" and "Bob results" into two bit sequences. Finally, they need to add the code to select half of the same-basis bits and compare them whether the same or not. If the bits comparison is True, students can increase the variable n to a larger value, say 1000, and run the program again to make sure that their program works for a larger size of bit sequence.

In the third experiment, students are asked to add lines of code to their program implemented in the first experiment to emulate Eve's interception. It can be observed that if Alice, Eve, and Bob choose the same basis, they will all get the same bit and hence Eve's eavesdropping cannot be detected. Figure 12 is an example in which Alice, Eve, and Bob all choose the horizontal basis. Note that after Eve guesses the measurement basis, she will send the qubit to Bob using the same basis. If Eve chooses the wrong basis, as shown in Figure 13, the emulated program will output that Bob gets the right bit with only a 50% chance.

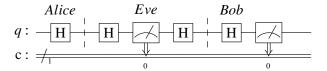


Figure 12: BB84 emulation with Eve's eavesdropping and guessing right

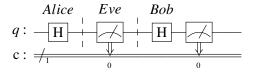


Figure 13: BB84 emulation with Eve's eavesdropping and guessing wrong

In Experiment 4, students need to modify the code implemented in Experiment 2 to emulate Eve's actions. That is, Eve firstly generates a random choice of basis for each bit. Next, she measures each qubit based on the generated random choice of basis. Finally, she encodes the message again based on the generated random choice of basis and then sends the message to Bob. Students are also asked to add some code to find out the percentage of the n sample bits in which Alice and Bob disagree. As described in the textbook, if Alice and Bob disagree about a quarter of the sample bits (i.e. n/4), they know that Eve's intercepting. Students can check their program's output whether it is 1/4 or not.

Experiments 5, 6, and 7 are about the Ekert91 protocol which utilizes entangled qubits as in the Bell's test. There are a few variations of the protocol. Based on the version presented in the textbook, there are 3n pairs of entangled qubits. For each pair, Alice receives one and randomly chooses a direction to measure her qubit, while Bob gets the other and also randomly selects a direction to measure it. There are three directions Alice and Bob can use: 0° , 120° , or 240° . Therefore, they will agree around n bits when both choose the same direction. Because the bases they chose will be revealed over a public insecure line, they can use these same-basis n bits as the shared secure key if Eve is not intercepting. Note that to detect interception, they use the sequence of qubits that come from the times when they chose different bases.

The template program given in Experiment 5 simply tests for one pair of entangled qubits when Alice and Bob use different directions to measure the qubits (see Figure 14). From the program output, students need to calculate the probability both Alice and Bob agree the qubits (i.e. combining the percentages of '00' and '11') and the probability they disagree (i.e. adding the percentages of '10' and '01'). In addition to 0° and 120° different directions, students also need to test the other five cases such as 0° and 240°, 120° and 240°, etc. Students will find out that for any case, the probability Alice and Bob agree is approximate 0.25 and the probability they disagree is around 0.75. The theoretic proof can be found in the section about Bell's inequality in the textbook.

In Experiment 6, students need to study, modify and run another template program which emulate Alice's and Bob's behaviors based on the Ekert protocol. This program outputs the probability that Alice and Bob agree when they use different bases. In the next experiment, they are asked to modify and extend this template program to detect whether Eve is intercepting or not. If Eve is eavesdropping, the probability that Alice and Bob agree when they use different bases will raise from 1/4 as in the Experiment 6 to 3/8. Students can check whether their

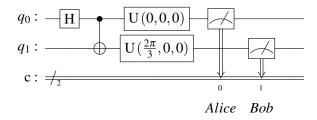


Figure 14: Ekerti91 emulation: Measuring two entangled qubits with different bases

results are consistent with the description in the textbook. Note that the textbook does not derive how to get the probability 3/8 in detail. This can be left as an homework assignment for students.

3.5 Hands-on Lab 5: Deutsch and Deutsch-Jozsa Algorithms

Objectives:

- the Deutsch algorithm
- the Deutsch-Jozsa algorithm

The Deutsch algorithm is the first algorithm to show that a quantum algorithm could be faster than a classic one to solve a specific problem. It specifically addresses the task of determining whether a black-box function $f:\{0,1\}\longrightarrow\{0,1\}$ is constant or balanced. Note that a function is called balanced if it sends half of its inputs to 0 and the other half to 1. There are four such functions. Two are constant: the constant 0 function (f(0)=f(1)=0), or the constant 1 function (f(0)=f(1)=1), while the other two are balanced: the identity function (f(0)=0,f(1)=1), or the inversion function (f(0)=1,f(1)=0). In classical computing, it would typically require two function evaluations to tell whether a black-box function is constant or balanced. By employing quantum principles such as superposition and interference, the quantum algorithm requires only a single query to the function, providing a speedup over the classical method.

In the first four experiments of this lab, students will learned how to construct the circuits for each of the four black-box functions. For a quantum system, every operation must be unitary and thus reversible. For achieving this, each function, say F, needs another input y to map the state $|x,y\rangle$ to the state $|x,y\oplus f(x)\rangle$. Because $|x,(y\oplus f(x))\oplus f(x)\rangle=|x,y\oplus 0\rangle=|x,y\rangle$, the function F is its own inverse. To construct the circuit for the constant 0 function, where f(x)=0, the output of the bottom qubit becomes $y\oplus f(x)=y\oplus 0=y$. Namely, there are only two straight wires in the black box F and there is no connection between the qubit x and the qubit y, as shown in the left part of Figure 15. Hence, the top qubit x goes through two H gates in succession. Based on the experience we learned from the Experiment 2 in the Lab 2, the qubit x will be restored back to its initial value 0 before measurement. Similarly, the circuit for the constant 1 function, where f(x)=1, can be constructed as in the right part of Figure 15. This is because $y\oplus f(x)=y\oplus 1=\overline{y}$. Same as the constant 0 function, there is no connection between the qubit x and the qubit y. The qubit y also goes through two successive H gates and its value will be restored back to its initial value 0. Therefore, both constant functions yield the value 0 in the qubit x.

The left portion of Figure 16 shows how to build the circuit of the balanced identity function, where f(x) = x. That is, the function F adopts a CNOT gate to map the input state $|x,y\rangle$ to the output state $|x,y\oplus f(x)\rangle = x$.

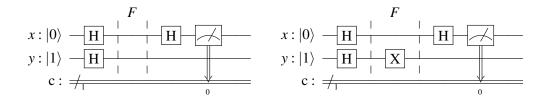


Figure 15: Deutsch — the Constant 0 function (left) and the Constant 1 function (right)

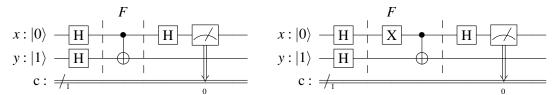


Figure 16: Deutsch — the balanced functions; either Identity (left) or Inversion (right)

 $|x,y\oplus x\rangle$. From the Experiment 7 in the Lab 3 (i.e. the phase kickback circuit), we know that bottom qubit y becomes the control bit of the CNOT gate. Furthermore, another tricky setting is the initial value of the qubit y. The value 1 in y will let the qubit y toggle the value in the qubit x from 0 to 1, which is different than the result of the constant functions. Likewise, the circuit of the balanced inversion function, where $f(x) = \overline{x}$, can be constructed as in the right portion of Figure 16. The function F maps the state from $|x,y\rangle$ to $|x,y\oplus f(x)\rangle = |x,y\oplus \overline{x}\rangle$. To implement it, we need to apply an X gate for the qubit x to invert its value. However, this X gate will not change the superposition state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ in x because $|0\rangle$ becomes $|1\rangle$ and $|1\rangle$ becomes $|0\rangle$. Same as the balanced identity function, the value 1 in y will flip the value in the qubit x from 0 to 1. Hence, if we measure the qubit x and its value is 1, we know that the function is balanced.

In the class lecture, students learned that the Deutsch algorithm doesn't truly demonstrate the power of quantum parallelism because it employs a single query to distinguish between two cases: constant or balanced. The Deutsch-Jozsa algorithm generalizes the Deutsch problem with a larger domain. It determines whether a black-box function $f: \{0,1\}^n \longrightarrow \{0,1\}$ is constant (i.e. it always outputs the same value for all 2^n inputs) or balanced (i.e. half of the inputs go to 0 and the other half go to 1). The Deutsch-Jozsa algorithm shows the power of quantum parallelism by using only one query to the oracle, as compared to the worst case scenario $2^{n-1} + 1$ function evaluations needed in the classical method. In Experiment 5, students are asked to load and run a template program involving a constant 0 function with n = 3, as depicted in Figure 17. Next, they are instructed to add an X gate on the qubit y to make the black box F a constant 1 function. The instructor would remind students that there is no connection between the x qubits and the qubit y for both constant functions. Hence, just like the constant functions in the Deutsch's algorithm, these three qubits $x_0x_1x_2$ goes through two successive H gates and the value in $x_0x_1x_2$ will be restored back to "000".

In Experiment 6 and Experiment 7, students are asked to construct the circuits of the balanced functions $f(x_0, x_1, x_2) = x_0 \oplus x_1 \oplus x_2$ and $f(x_0, x_1, x_2) = x_0 x_1 \oplus x_2$, respectively. As shown in Figure 18 and Figure 19, there is at least one CNOT connection between the x qubits and the qubit y. Since the phase kickback causes the input x qubits changed through the connection(s), the value in $x_0 x_1 x_2$ will not be "000" anymore. That's why we can determine the function is constant or balanced by examining the x qubits.

We believe that these hands-on programming experiments will help students easily understand the Deutsch and the Deutsch-Jozsa algorithms from a different perspective, rather than relying solely on the rigorous mathematical proofs presented in the textbook. The phase kickback circuit, along with the y qubit's initial value set to

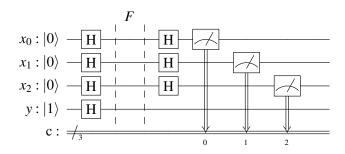


Figure 17: Deutsch-Jozsa – the constant 0 function $f(x_0, x_1, x_2) = 0$

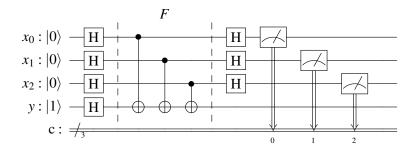


Figure 18: Deutsch-Jozsa – the balanced function $f(x_0, x_1, x_2) = x_0 \oplus x_1 \oplus x_2$

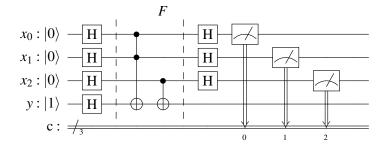


Figure 19: Deutsch-Jozsa – the balanced function $f(x_0, x_1, x_2) = x_0x_1 \oplus x_2$

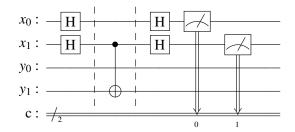


Figure 20: A Simple Example of Simon's Algorithm Circuit with s = 10

1, will flip certain values of the *x*-qubit for the balanced functions. This flipping will not occur for the constant functions due to the lack of any connection. However, while the mathematical derivation might be lengthy, it clearly demonstrates the relationship between the top *x*-qubit(s) and the bottom *y* qubit. As emphasized twice in the textbook, once for the Deutsch algorithm and once for the Deutsch-Jozsa algorithms, the *x*-qubit(s) and the *y* qubit are not entangled because their joint state can be expressed as a tensor product decomposition.

3.6 Hands-on Lab 6: Simon's Algorithm

Objectives:

- the Simon's algorithm
- the Oracle construction method for the Simon's algorithm

The Simon's algorithm solves the problem of finding the hidden string in a black-box function $f: \{0,1\}^n \to \{0,1\}^n$. The black-box 2-to-1 function has the property that there is a secret non-zero binary string s of length n, (i.e. $s \neq 00...0$), s.t. f(x) = f(y) if and only if y = x or $y = x \oplus s$. The Simon's algorithm contains quantum procedures as well as classical post-processing procedures. As described in the textbook, we can run Simon's circuit n+N times, where N does not depend on n, to get n+N equations. These equations contains the n-1 independent vectors, so we can use a classical algorithm (e.g. Gaussian elimination) to recover the secret string s.

Students begin this lab by running a 2-qubit template program with the hidden string s = 10. Figure 20 depicts the circuit of this program. Students should be reminded that, similar to the Deutsch-Jozsa algorithm, the top qubits x will be measured instead of the bottom qubits. This template outputs the top x qubits either 00 or 01. Note that the template does not call Gaussian elimination to find s and hence students need to manually solve the n-1 linearly independent equations. This will help them know how the whole algorithm works. Because the output string '00' which wouldn't give us any information, students need to use the other output string 01 to calculate its dot product with the secret string s (i.e. s_0s_1) and the result should be 0. That is, they will get

$$0 \times s_0 + 1 \times s_1 = 0.$$

They have to figure out that $s_1 = 0$ and hence s = 10 because not all of the digits in s can be 0. In fact, this example is from the textbook and has been discussed in the class lecture. Students now have the opportunity to practice it once again. In Experiment 2 and Experiment 3, students will modify the oracle circuit in the program based on the given diagrams in the handout. They may need to run their programs a few times to get n - 1 different non-zero strings and then use these strings to derive the bit pattern of the secret string s.

In Experiment 4 and Experiment 5, students will learn a simple method, as presented in [16], for constructing an oracle circuit *F* using a given string *s*. Figure 21 shows the steps to construct such a 2-to-1 mapping function.

- 1. Apply the CNOT gates from qubits of the first register (i.e. *x*) to qubits of the second register (i.e. *y*).
- 2. Because the bits in the string *s* cannot be all 0, find the least index *k* such that $s_k = 1$. Next, apply the CNOT gates from the qubit x_k to any qubit y_i if $s_i = 1$.

Figure 21: A Simon's Oracle Construction Algorithm

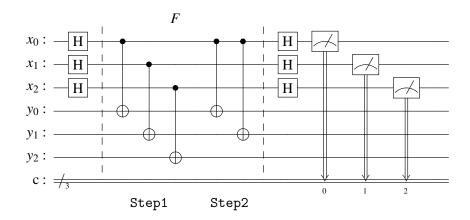


Figure 22: An Example of Constructing Simon's Algorithm Oracle Circuit with s = 110

Note that in the first step, because each qubit in the register y is initialized to be 0, the content of each qubit in the register x, which is the classical information encoded as either a $|0\rangle$ or a $|1\rangle$, will be copied through the CNOT gate to the corresponding qubit in the register y. In the second step, if $x_k == 0$, the register y remains unchanged. Otherwise, the register y will be XORed with s, i.e. $y \leftarrow y \oplus s$. Figure 22 illustrates an example of using these two steps to construct the Simon's Algorithm Oracle Circuit with s = 110. Note that in this example, s = 1 is 0 because s = 1. If the secret string s = 1 is 1 since s = 1 and hence the CNOT control bit is s = 1 in the Step 2.

We need to show that the oracle F constructed using the method in Figure 21 has the property $f(x) = f(x \oplus s)$. Assume that the value in the x register is b when entering F. If b_k is 0, the register y has the value b when leaving F. Otherwise, it has the value $b \oplus s$. Consider another input value d, where $d = b \oplus s$, in the x register when entering F. Note that s_k is 1, so $d_k = b_k \oplus s_k = \overline{b_k}$. There are two cases we need to verify. When $b_k = 0$, $d_k = \overline{b_k} = 1$ and hence the register y will have the value $d \oplus s = (b \oplus s) \oplus s = b$. If $b_k = 1$, then d_k is 0 and the register y will the value d which is $b \oplus s$. Therefore, for both cases, $f(b) = f(d) = f(b \oplus s)$.

There is another way to explain why the measured outputs of the top qubits are limited to certain patterns. Here, we employ the circuit equivalence translation along with the phase kickback property. We use the circuit in Figure 22 as an example. Firstly, we can simplify the circuit by removing the two successive CNOT gates from x_0 to y_0 because CNOT is its own inverse. Next, we add two successive H gates in several places to yield the circuit as shown in Figure 23. Adding two successive H gates does not change the circuit's property. Now, we can use the the phase kickback circuit pattern, as shown in Figure 7, to reverse the control-target positions in each CNOT gate. Note that there are two successive H gates added in the y_1 wire between the CNOT gate controlled from x_1 and the CNOT gate controlled from x_0 . The first H gate added will be used as the ending H

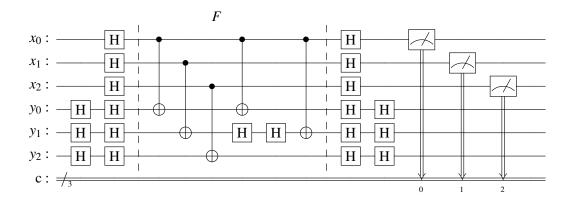


Figure 23: Adding Successive H gates before Circuit Equivalence Translation

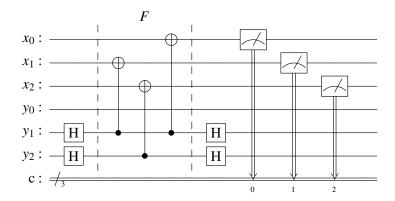


Figure 24: Circuit Equivalence Translation via Phase Kickback in Simon's Algorithm Circuit

gate in the phase kickback pattern, reversing the control position of the CNOT gate from original x_1 to y_1 . The second added H gate will serve as the initial H gate in the phase kickback pattern to reverse the control position of the next CNOT gate from x_0 to y_1 . The translated circuit can be found in Figure 24. It shows that x_0 and x_1 are entangled via y_1 . Hence, the measured output $c_0c_1c_2$ must satisfy the property $c_0 \oplus c_1 = 0$, while c_2 is independent of c_0 and c_1 .

Note that we also need to add additional H gates in the x_k wire if there are 3 or more 1's in s. For example, if s=111, two successive H gates should be inserted in the x_0 wire between the second CNOT gate and the third CNOT gate in Step 2. In general, if the secret string s has the non-zero bits indexed at locations k, l, m, ..., the measured output should satisfy the property $c_k \oplus c_l \oplus c_m \oplus \ldots = 0$. Namely, even number of these bits, c_k , c_l , c_m , ..., will have the value 1 in the measured output.

It's worth noting that the topic of constructing Simon's Algorithm oracle, while intriguing, falls beyond the scope of the textbook's contents. Its intricate details are reserved for advanced students who have already grasped the fundamental principles discussed in the labs. Whether this topic will be addressed depends on the instructor's discretion, allowing flexibility to adapt to the pace and progress of the lab.

3.7 Hands-on Lab 7: Grover's Search Algorithm

Objectives:

- the Grover's algorithm, including the Oracle construction and the Amplitude magnification method
- the Qiskit program for solving the 3-SAT problem via Grover's Search

As described in the textbook, Grover's search algorithm is a quantum computing technique designed to accelerate the search for the specific item(s) in an unstructured database. In the algorithm, an initial quantum state is prepared by applying a Hadamard transform to all possible input states. Subsequently, a specialized quantum oracle is employed to mark the target state by inverting its phase (i.e. flipping the sign of its probability amplitude). The next step involves a process of applying a diffusion operator, which amplifies the probability amplitude of the target state while reducing the amplitudes of other states. The phase inversion oracle and the amplitude amplification procedure may need to be applied repeatedly approximately \sqrt{m} times, where m is the number of items in the database. For example, given an NP-Complete problem 3-SAT with n bits, it can reduce the run-time complexity from $O(2^n)$ to $O(\sqrt{2^n})$, i.e. $O(2^{n/2})$. While quadratic speedup may not be as impressive as exponential speedup, it remains valuable for handling massive data sets.

The first experiment in this lab asks students to load and run a template program which finds the desired element location 11 among 2-bit strings: 00, 01, 10, and 11. In fact, this is a very simple SAT problem that determines the satisfiability of the expression $q_0 \wedge q_1$. Figure 25 shows the circuit which includes the initial Hadamard transforms, the phase inversion oracle, and the amplitude amplification. The phase inversion oracle can be realized using a Controlled-Z gate which flips the phase of the target qubit if the control qubit is in the $|1\rangle$ state. That is, the operation matrix of Controlled-Z is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

It's symmetric and it doesn't matter which qubit is the controlled or target. As described in [7], the Controlled-Z gate is a phase-logic operation which performs phase AND (denoted as pAND). In this experiment, students are asked to use the Operator class defined in the Qiskit quantum_info library in their program to display the oracle operation matrix and the amplitude amplification matrix. They will find out that the operation matrix of amplitude amplification is

$$\begin{bmatrix} -0.5 & 0 & 0 & 0 \\ 0 & -0.5 & 0 & 0 \\ 0 & 0 & -0.5 & 0 \\ 0 & 0 & 0 & -0.5 \end{bmatrix}$$

and check whether it is the same as in the textbook. Students need to look at the output state vector (i.e. probability amplitude) to find the location with the highest probability.

Note that neither the textbook nor the reference book presents how to construct the amplification circuit. The geometric visualization of the Grover's algorithm, as shown in Figure 26, can assist students in understanding how the algorithm works, particularly the construction of amplification. Below is a brief description. Remember that the system is initialized to the uniform superposition over all states

$$|s\rangle = H^{\otimes n} |0^n\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$$

prior to entering the phase operation oracle. The operator of the oracle U_{ω} , which flips the sign of the probability amplitude associated with the target location, will transform the state to $U_{\omega}|s\rangle$ before amplitude magnification.

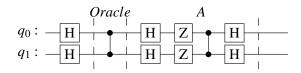


Figure 25: Grover's Algorithm with the Boolean expression $q_0 \wedge q_1$

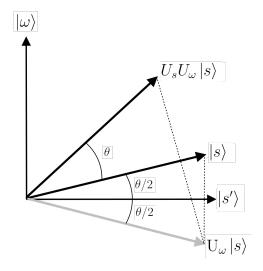


Figure 26: Geometric interpretation of the first iteration of Grover's algorithm (Courtesy of [3])

The amplitude amplification operator, $U_s = 2|s\rangle\langle s| - I$, reflects the state about $|s\rangle$ from $U_{\omega}|s\rangle$ to $U_sU_{\omega}|s\rangle$, which is closer to the target state $|\omega\rangle$. The trick is how to solve the reflection $2|s\rangle\langle s| - I$. Because $|s\rangle = H^{\otimes n}|0^n\rangle$, we can firstly apply the Hadamard gates to transform $|s\rangle$ to $|0^n\rangle$, i.e.,

$$H^{\otimes n}|s\rangle = H^{\otimes n}H^{\otimes n}|0^n\rangle = |0^n\rangle.$$

Now we can do a reflection about the zero state via $2|0^n\rangle\langle 0^n|-I$, which can be implemented by using the Z gates and the Controlled-Z gate. Finally, we apply the Hadamard gates to transform it back. In summary, the amplitude amplification operation is represented as

$$U_s = 2|s\rangle\langle s| - I = H^{\otimes n} (2|0^n\rangle\langle 0^n| - I) H^{\otimes n},$$

which explains the amplitude amplification circuit shown in Figure 25. For advanced students, the instructor may guide them to consult the Qiskit textbook or Nielson's and Chung's book[9] for more detailed explanations.

In Experiment 2, students will work on another example searching for the location indexed at 01. Once more, this can be seen as a SAT problem that determines the satisfiability of the expression $\neg q_0 \land q_1$. Students can modify the previous template program by adding two X-gates, one is before the Controlled-Z and the other is after the Controlled-Z, on the qubit q_0 in the oracle. The X-gate before the Controlled-Z performs the negation of q_0 , while the second X-gate uncomputes it back. Theoretically, this pair of X-gates, one swapping columns and the other swapping rows in the Controlled-Z operation matrix, moves the value -1 in the matrix to the desired element location.

In the next experiment, students will load and run a template program that includes a 3-bit oracle with the desired element location at 111. Because Qiskit does not support the ccz gate, the template program uses a ccx gate instead, but it requires placing a pair of H-gates before and after the X gate in the ccx operation. It's important to note that operation of HXH is equivalent to a Z gate. The program outputs two distinct probability

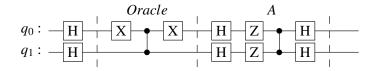


Figure 27: Grover's Search Algorithm with the Boolean expression $\neg q_0 \land q_1$

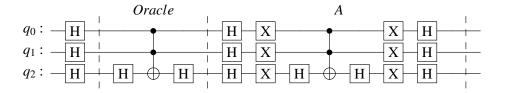


Figure 28: Grover's Algorithm with the Boolean expression $q_0 \wedge q_1 \wedge q_2$

amplitudes. The higher one is located at 111, while the lower one is situated as other locations. Students are asked to compare these two probability amplitudes with the two probability amplitudes mentioned in the textbook. As mentioned earlier, in order to achieve a higher probability of obtaining the correct answer, it may be necessary to apply the oracle and amplitude amplification repeatedly. To repeat the process, we do not recommend that students compose the oracle and amplitude amplification circuits again either manually or via a loop construct in Python. This is because if the transpiler does not aware of the repetition, it may lead the number of gates growing exponentially to $O(2^{n/2})$. Therefore, students are instructed to adapt the repeat() method in Qiskit, which prevents the transpiler from adding the same circuits repeatedly. It will be interesting for students to observe that if the oracle and the amplitude amplification tasks are performed three times, the probability amplitude of the desired location will decrease. In other words, it is overcooked.

We plan to add another experiment with a slightly more complicated Boolean expression, e.g. $(\neg q_0 \lor q_1) \land q_2$, in the future. As shown in Figure 29, after negating q_0 , we use the implementation in Figure 5 to perform the OR operation, which places the result of the clause $(\neg q_0 \land q_1)$ into the ancilla qubit. Next, we use the Controlled-Z gate to phase AND the ancilla qubit with q_2 . Finally, we have to perform uncomputation to clean up temporary effects on the ancilla bit so that it can be re-used. This experiment justifies why students need to learn how to use quantum gates to emulate classical AND and OR gates in Lab 3. An even more complicated example, which contains several clauses in a Boolean formula, can be found in [7].

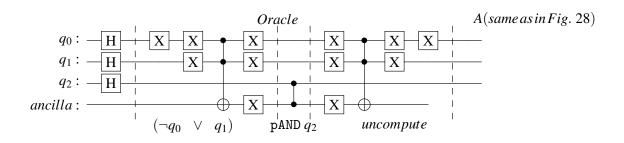


Figure 29: Grover's Algorithm with the Boolean expression $(\neg q_0 \lor q_1) \land q_2$

In the last experiment, students are asked to follow the steps specified in the handout to learn how to solve the 3-SAT problem using the PhaseOracle component and the Grover class provided by Qiskit. They need to study the contents of the web page presented in [10]. The sample program reads a Boolean expression expressed in the DIMACS-CNF format and parses it to construct the corresponding oracle circuit. It then invokes the amplify() method supported in the Grover class in Qiskit to find a solution. Note that utilizing existing library functions can simplify the work of programmers.

4 Conclusion

A collection of engaging laboratory experiments has been developed to serve as supplementary resources for the textbook. Many of the experiments are closely linked with the content of the textbook. These experiments capture students' interest while also offering a concrete platform for validating their comprehension through the comparison of experimental outcomes with textbook examples. The feedback collected from the students has, overall, been very positive. The students indicated that these practical labs help them not only improve their hands-on skills in quantum programming, but also understand the rationale behind them. We believe that this report is valuable for utilization as an instructor's manual for Quantum programming education.

Acknowledgment

This work was supported in part by the fund from JobsOhio, a private non-profit corporation.

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CIS 492/593 Hands-On Lab 1 Python and Jupyter Fall 2022

In the first hands-on lab, you will:

- learn the features of Python
- practice programming with Jupyter Notebook

Task I: python3

Login to a Linux workstation. Type CTRL-ALT-T to open a terminal. On the left sidebar, right click the Terminal icon and choose "Add to Favorites". Perform the following steps:

• Inside the terminal window, type the following command to install qiskit tools:

```
pip3 install qiskit qiskit-aer
```

• In the terminal window, use the commands below to create a sub-directory QC (i.e. a folder) and change your current working directory to it:

```
mkdir QC cd QC
```

• Use the following command to copy a sample python/qiskit program to the current directory:

```
cp ~cis492s/pub/Hgate.py .
```

(Note: Don't forget the dot (.) at the end. In Linux, the dot means the current directory.)

• To run the sample program program, type:

```
python3 Hgate.py
```

Task II: jupyter notebook

Do the following experiments:

Experiment 1

• Use CTRL-ALT-T (or middle click the terminal icon on the left side bar) to open a second terminal. In the second terminal window, type

```
cd QC
```

to change the current directory. Then, use

```
jupyter notebook
```

to launch the Jupyter Notebook App. You will find that the jupyter notebook is inside a web browser.

- Inside the browser, click "New" and choose "Python 3 (ipykernel)". You will see an entry box beginning with "In []:" which we call it a cell.
- In the cell, type the following lines of python code:

```
a = 5
b = 7
print(a,b)
```

(Note: Do NOT type any spaces or tabs at the beginning of each line above). Next, press SHIFT-Enter (or click "Run") to execute the code in the cell. Record the output result.

Experiment 2

• After you run the cell, you will have a new empty cell. Type

```
a,b = b,a
print(a,b)
```

to exchange two variable values without using a third temporary variable. Run the code and record the result.

Experiment 3

• Python uses indentation (i.e. the spaces at the beginning of a line) to indicate a block of code. In the third cell, type

```
if a > b :
    c = a + b
    d = a - b
    print(c,d)
else:
    x = a * b
    y = a / b
    print(x,y)
```

In C/C++ or Java, a block of statements should be included in a pair of curly braces. Run the code and record the result.

Experiment 4

• Click the first cell and change variable a's value to 9. Click the menu "Cell" and choose "Run All". Record the result changed in each cell.

Experiment 5

• Python "for" loop can iterate over a range of numbers, lists, or strings. In a new empty cell, type

Run the code and record the result of the nested loops.

Experiment 6

• Python's dictionary (also know as an associative array) is a data structure which consists of a collection key-value pairs. The feature is that the array index (i.e. the key) can be a string. Type

```
counts = {} # initialize counts as a dictionary
counts['00'] = 477
counts['11'] = 522
counts['00'] += 1
print(counts)
```

Run the code and record the result.

Experiment 7

• In a new empty cell, type

```
%load Hgate.py
```

and click "Run" to load the sample program into the cell. Click to select the cell and then click "Run" to execute the sample program. Record the result of total counts.

Turnin

Click "File" and choose "Save as". Type "lab1" in the entry box and click "Save" to save your work today into the file lab1.ipynb. To turn in your file, use CTRL-ALT-T to open a terminal and type

```
ssh grail
and type your password to login to the server grail. Then, type
cd QC
turnin -c cis492s -p lab1 lab1.ipynb
to electronically submit your file lab1.ipynb.
```

To shutdown the jupyter notebook, hit CTRL-C in the terminal window which you launch the jupyter notebook. Type y and press Enter.

To log out from Ubuntu desktop session, go to the top right corner, click to select "Power Off / Log Out", and choose "Log Out", or simply use the keyboard shortcut CTRL + ALT + DEL and click "Log Out".

Hand in your hard-copy lab report before your leave.

```
import numpy as np
from qiskit import QuantumCircuit, transpile
{\tt from\ qiskit.providers.aer\ import\ QasmSimulator}
from qiskit.visualization import plot_histogram
## for display
from IPython.display import display
# Create a Quantum Circuit
qc = QuantumCircuit(1,1)
# Add a H gate on qubit 0
qc.h(0)
\# Map the quantum measurement from qubit 0 to the classical bit 0
qc.measure(0,0)
# Draw the circuit
display(qc.draw())
# Use Aer's qasm_simulator
simulator = QasmSimulator()
# compile the circuit down to low-level QASM instructions
# supported by the backend (not needed for simple circuits)
compiled_circuit = transpile(qc, simulator)
# Execute the circuit on the qasm simulator
job = simulator.run(compiled_circuit, shots=1000)
# Grab results from the job
result = job.result()
# Returns counts
counts = result.get_counts(compiled_circuit)
print("\nTotal counts:",counts)
```

CIS 492/593 Hands-On Lab 2 Entanglement Fall 2022

In this hands-on lab, you will learn:

- Qiskit Terra and Qiskit Aer. The former is a module which handles quantum circuit construction, circuit analysis, and transformation, while the latter is a high performance simulator for quantum circuits.
- Hadamard gate
- two-qubit entanglement

Login to a workstation and open a terminal (either middle click the terminal icon on the left side bar or press CTRL-ALT-T). In the terminal window, type

cd QC

jupyter notebook

Inside the browser, click "New" and choose "Python 3 (ipykernel)". For each experiment, you need to put the experiment number as a comment (e.g. # Experiment 1) in the top of the code.

Experiment 1: Qiskit Programming

• In the cell, type

%load Hgate.py

and click "Run" to load the sample Qiskit program into the cell and then put the experiment number as a comment.

Below is a brief description of the program. The code imports several modules at beginning. Then, it invokes the constructor QuantumCircuit(1,1) to create a quantum circuit qc which contains 1 qubit and 1 classical bit. Next, we apply an H gate (i.e. qc.h(0) to the qubit 0. Note that the qubit is allocated using the array format and the first qubit is indexed with 0. Same for the classical bit. Finally, we use qc.measure(0,0) to measure the qubit 0 and put the result to the classical bit 0.

After building the circuit, the program calls the function transpile() to compile the abstract circuit qc to low-level instructions and then uses the Aer simulator to execute the instructions 1000 times.

- When calling simulator.run(), change "shots=1000" to "shots=10000" and also add plot_histogram(counts)
 - at the end of the program. Click "Run" to execute the code in the cell. Record the numbers for '0' and '1'.
- Click the cell and click "Run" again. Do you get EXACTLY the same numbers for '0' and '1' as in the previous run? Explain your result. Moreover, what is the data structure of the variable counts?

Experiment 2: Apply two H gates in succession

If we consider the H gate to be analogous to a coin toss, two H gates in succession should be tossing the coin twice. We would still expect close to a 50/50 distribution of zeros and ones for the second flipping. However, quantum randomness is not simply like a classical random coin toss. Let's do the following experiment.

- Click the cell which contains the code in Experiment 1. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Apply the H gates twice in a row, like the diagram below:

$$q: -H-H$$
 $c: \frac{1}{\sqrt{1}}$

That is, you can add another qc.h(0) immediately after qc.h(0) in the code.

• Click "Run" to see and record the result. To verify, calculate by hand the state vector after applying the first H gate in your hard-copy lab report. Note that the qubit is initialized to be 0. *Hint:*

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \times \begin{bmatrix} \\ \end{bmatrix} =$$

Then, calculate the state vector after applying the second H gate.

Experiment 3: Apply two H gates in succession with initial value 1

- Click the cell above (i.e. two H gates in succession). Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Instead of using the default value 0 for the qubit, we would like to initialize the qubit to 1 before applying the two qubits. To achieve this, you can add an X gate (i.e. the NOT gate) by using qc.x(0) before the first calling of qc.h(0), like below:

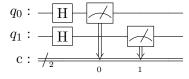
$$q: -X - H - H$$

$$c: \frac{1}{\sqrt{1}}$$

• Click "Run" to see and record the result. Again, calculate by hand the state vector after applying the first H gate in your report. Then, calculate the state vector after applying the second H gate.

Experiment 4: Apply an H gate for each of two parallel qubits

- Click the cell which contains the code done in Experiment 1. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Modify the code in the cell to build the circuit below:



Hint: The circuit needs 2 qubits and 2 classical bits when calling QuantumCircuit(). You also need to add a H gate on qubit 1. Lastly, use qc.measure([0,1],[0,1]) to map the qubits 0 and 1 to the classical bits 0 and 1, respectively.

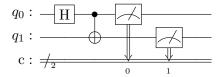
• Click "Run" to get and record the result. Explain.

• What is the initial state vector?

Calculate the state vector after applying the parallel H gates. Hint:

Experiment 5: Entangle two qubits

- Click the cell which contains the code done in Experiment 4. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Modify the code in the cell to build the circuit below:



Hint: Replace the H gate on qubit 1 with a CNOT gate using qubit 0 as the control qubit and qubit 1 as the target qubit (i.e. qc.cx(0,1)).

- Click "Run" to get and record the result. Explain.
- To save the code in this cell into a file entangle2.py, put the following %%writefile entangle2.py

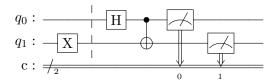
as the FIRST line of the code. Click "Run".

• To check the file entangle2.py has been saved correctly, open a terminal and type

Experiment 6: Are they entangled?

- Click the cell which contains the code done in Experiment 5. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below". Remove the first line %%writefile entangle2.py
- Add the following two lines of code

immediately after calling the function QuantumCircuit(), just like the diagram below:



The purpose of using the X gate on qubit 1 is to invert its initial value from 0 to 1. The barrier() (i.e. a dashed line in the circuit diagram) is a directive which tells the circuit transpiler not to optimize the code crossing the barrier separation line. Here we just use it for more readability.

- Click "Run" to see and record the result.
- Calculate the state vector (i.e. the tensor product of the two qubits) before applying the CNOT gate.

$$\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \otimes \begin{bmatrix} \\ \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

Calculate the state vector after applying the CNOT gate. Hint: similar calculation can be found on Textbook Page 67.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} \\ \\ \end{bmatrix} =$$

Are these two qubits entangled? Explain. Hint: See Textbook Page 59

Turnin

Click "File" and choose "Save as". Type "lab2" in the entry box and click "Save" to save your work today into the file lab2.ipynb. To turn in your file, use CTRL-ALT-T to open a terminal and type

ssh grail

and type your password to login to the server grail. Then, type

cd QC

turnin -c cis492s -p lab2 lab2.ipynb to electronically submit your file lab2.ipynb.

If you finish early, you can start doing the "IBM QC Homework" posted on the CSU Blackboard to run Qiskit programs on a real IBM quantum computer.

Shutdown the jupyter notebook and logout the workstation.

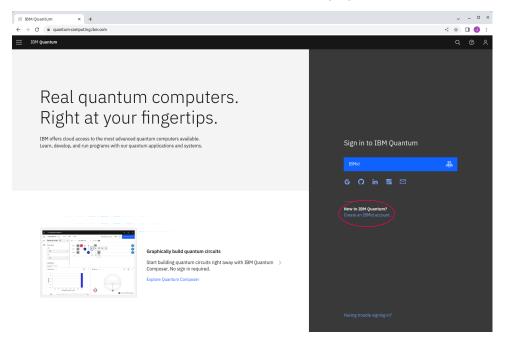
Hand in your lab report before your leave.

CIS 492/593 Hands-On Exercise Fall 2022 Running Qiskit Programs on an IBM Quantum Computer

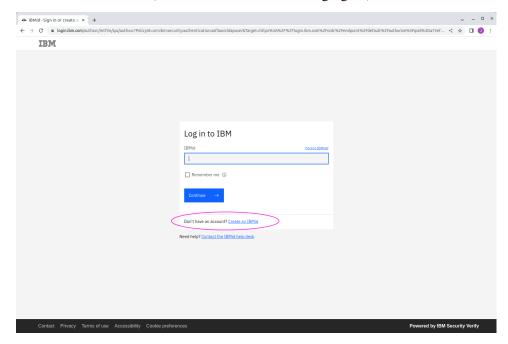
1. Create an IBMid account

• Open a browser and type the following URL into browser: https://quantum-computing.ibm.com/

• Click "Create an IBMid account" (circled in red in the following figure).



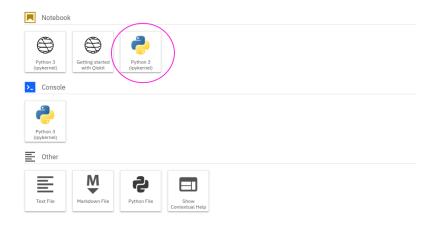
• Again, click "Create an IBMid" (circled in red in the following figure).



• Fill in your information. Click "Next" and then type "CSU" as the Company name. Check your email and then type the verification code.

Running a Qiskit Program on a real Quantum Computer

- After you sign in, you will be asked again to fill in your name, institution, and select an item from the "What is your familiarity with quantum?" list. Click "Continue".
- Click "Launch Lab".
- Click "Python 3 (ipykernel)" (circled in red in the following figure).



• In the empty cell, copy and paste the following code (i.e. the file IBMQ1.py on Blackboard)

```
from qiskit import *

IBMQ.load_account()
provider = IBMQ.get_provider('ibm-q')
print(provider.backends())
```

- Click "Run" and choose "Run Selected Cells". Be patient. It may take a few seconds to get a list of the available backends, e.g. 'ibmq_lima', 'ibmq_quito', etc.
- Assume that we want to submit the job to the backed 'ibmq_lima'. Copy and paste the following code which entangles two qubits (i.e. the file IBMQ2.py on Blackboard) into the next empty cell.

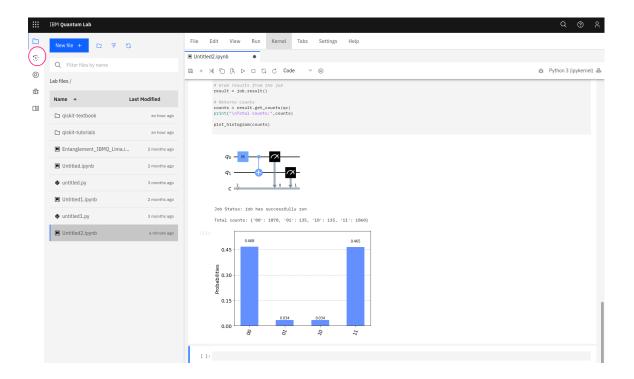
```
import numpy as np
from qiskit import *
from qiskit.visualization import plot_histogram
## for display
from IPython.display import display
# Create a Quantum Circuit
```

```
qc = QuantumCircuit(2,2)
# Add a H gate on qubit 0
qc.h(0)
# apply a CNOT gate on qubit 0 and qubit 1
qc.cx(0,1)
\# Map the quantum measurement from qubit 0 to the classical bit 0
qc.measure([0,1],[0,1])
#qc.measure_all()
# Draw the circuit
display(qc.draw())
provider = IBMQ.get_provider('ibm-q')
# assume that the backend ibmq_lima is in the previous cell's list result
qcomp = provider.get_backend('ibmq_lima')
# qcomp = provider.get_backend('ibmq_belem')
# qcomp = provider.get_backend('ibmq_quito')
# Execute the circuit on the qasm simulator
job = execute(qc, backend = qcomp)
from qiskit.tools.monitor import job_monitor
job_monitor(job)
# Grab results from the job
result = job.result()
# Returns counts
counts = result.get_counts(qc)
print("\nTotal counts:",counts)
plot_histogram(counts)
```

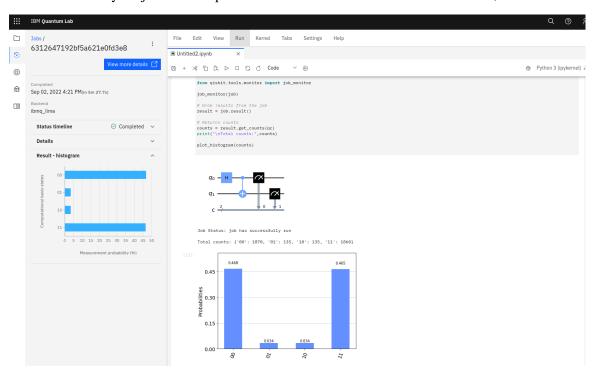
- Click "Run" and choose "Run Selected Cells". You will see something like

 Job Status: job is queued (35)

 at the end. It may take 5–20 or more minutes to execute your job, depending how many jobs already in the queue. Take a cup of coffee or drink while waiting for the result.
- Finally, the Job Status becomes "job has successfully run". Click the "Jobs" icon (circled in red on the top left corner of the following figure to get the job information.



• You can see when your job was completed and on which backend in the left window, like below:



- Take a screenshot of the whole browser window (i.e. similar to the above figure which contains the job completion and backend information) and put it in your report file. Explain in your report why the histogram is a little different than the one done in the Hands-on Lab2 Experiment 5 using simulator.
- Upload your report file to Blackboard. Due: Oct. 18, 11:59PM

```
from qiskit import *

IBMQ.load_account()
provider = IBMQ.get_provider('ibm-q')
print(provider.backends())
```

```
import numpy as np
from qiskit import *
from qiskit.visualization import plot_histogram
## for display
from IPython.display import display
# Create a Quantum Circuit
qc = QuantumCircuit(2,2)
# Add a H gate on qubit 0
qc.h(0)
\# apply a CNOT gate on qubit 0 and qubit 1
qc.cx(0,1)
\# Map the quantum measurement from qubit 0 to the classical bit 0
qc.measure([0,1],[0,1])
#qc.measure_all()
# Draw the circuit
display(qc.draw())
provider = IBMQ.get_provider('ibm-q')
# assume that the backend ibmq_lima is in the previous cell's list result
gcomp = provider.get_backend('ibmg_lima')
# qcomp = provider.get_backend('ibmq_lima')
# qcomp = provider.get_backend('ibmq_belem')
# qcomp = provider.get_backend('ibmq_quito')
# qcomp = provider.get_backend('ibmq_manila')
# qcomp = provider.get_backend('ibmq_nairobi')
# qcomp = provider.get_backend('ibmq_oslo')
\# Execute the circuit on the qasm simulator
job = execute(qc, backend = qcomp)
from qiskit.tools.monitor import job_monitor
job_monitor(job)
# Grab results from the job
result = job.result()
# Returns counts
counts = result.get_counts(qc)
print("\nTotal counts:",counts)
plot_histogram(counts)
```

CIS 492/593 Hands-On Lab 3 Quantum Gates and Circuits Fall 2022

The aim of this hands-on lab is to familiarize yourself with:

- entanglement with more than 2 qubits in Qiskit
- constructing circuits with quantum gates such as X, H, CNOT, and Toffoli
- use of quantum gates to emulate classical gates (AND, NAND, and OR)

Login to a workstation and open a terminal (either middle click the terminal icon on the left side bar or press CTRL-ALT-T). In the terminal window, type

cd QC

jupyter notebook

Inside the browser, click "New" and choose "Python 3 (ipykernel)". For each experiment, you need to put the experiment number as a comment (e.g. # Experiment 1) in the top of the code.

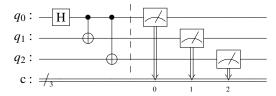
Experiment 1: 3-qubit Entanglement

• In the cell, type

%load entangle2.py

and click "Run" to load the Qiskit program which you did in Lab 2 into the cell.

• Modify the code in the cell to build the circuit below which entangles 3 qubits:

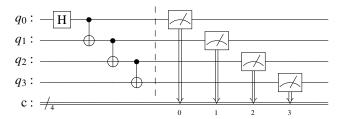


Note that you may call qc.barrier() before the measurements. This will generate a synchronization point (i.e. a dashed line in the circuit diagram), though it will not affect the result.

• Click "Run" to get, record, and explain the result in your report.

Experiment 2: 4-qubit Entanglement

- Click the cell which contains the code done in Experiment 1. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- In this experiment, we use an alternative way to entangle 4 qubits. Modify the code in the cell to build the circuit below:

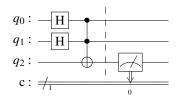


• Click "Run" to get, record, and explain the result in your report.

Experiment 3: Toffoli gate

In the class, we study the the Toffoli gate (also called CCNOT gate or "controlled-controlled-not" gate) which is a universal reversible logic gate. That is, if the first two control bits are both set to 1, it inverts the third bit, otherwise all bits remain the same.

• Copy and paste the code cell done in Experiment 1. Modify the code in the cell to build the circuit below:



Note that we only need one classical bit and it has the value measured from q_2 . In Qiskit, the interface of the Toffoli gate is

where c0 and c1 are the control qubits and t is the target qubit.

• Click "Run" to get and record the result. Explain the result.

Experiment 4: Implementing AND and NAND gates

• Type the following in a new empty cell:

%load ~cis492s/pub/ANDgate.py

Click "Run" to load the code template into the cell.

- The AND function is incomplete. Add one line of code to complete it. Describe your circuit design in the report.
- Click "Run" to execute the code and verify your result. Is your AND quantum circuit reversible?
- Copy and paste the code cell into a new cell. To implement the NAND gate function, simply add a NOT gate (i.e. the quantum X gate) to flip the result of qubit 2 at the end. Also, replace every occurrence of "AND" with "NAND" in the code, including comments.
- Click "Run" to execute the code and verify your result

Experiment 5: Implementation of the OR gate

• Type the following in a new empty cell:

%load ~cis492s/pub/ORgate.py

Click "Run" to load the code template into the cell.

• The OR function is incomplete. Add three lines of code to complete it. Describe your circuit design in the report.

Hint: Use two CNOT gates and one Toffoli gate to flip the target qubit in succession. An alternative way is to invert both inputs, use a NAND gate to get the output, and then invert inputs back.

Click "Run" to execute the code and verify your result.

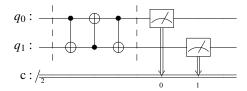
Experiment 6: Application of CNOT gates

• Type the following in a new empty cell:

%load ~cis492s/pub/UNKNOWNfunc.py

Click "Run" to load the code template into the cell.

• The function UNKNOWN is incomplete. Add the code between two barrier calls based on the circuit below:



• Click "Run" to execute the code. Figure out what the the unknown function is.

Hint: The CNOT function is like an XOR gate. Hence, the circuit is like executing the following instructions in sequence:

 $q_1 = q_0 \oplus q_1$

 $q_0 = q_0 \oplus q_1$

 $q_1 = q_0 \oplus q_1$

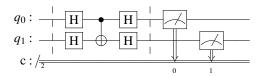
Experiment 7: Application of one CNOT and four H gates

• Type the following in a new empty cell:

%load ~cis492s/pub/UNKNOWNfunc.py

Click "Run" to load the code template into the cell.

• Add the code between two barrier calls in the UNKNOWN function based on the circuit below:



• Click "Run" to execute the code. Figure out what the the unknown function is.

Hint: How does the qubit 1 affect the qubit 0?

Click "File" and choose "Save as". Type "lab3" in the entry box and click "Save" to save your work today into the file lab3.ipynb. To turn in your file, use CTRL-ALT-T to open a terminal and type

ssh grail

and type your password to login to the server grail. Then, type

cd QC

turnin -c cis492s -p lab3 lab3.ipynb

to electronically submit your file lab3.ipynb.

Shutdown the jupyter notebook and logout the workstation.

```
from qiskit import *
from qiskit.visualization import plot_histogram
import numpy as np
## for display
from IPython.display import display
def AND(inp1,inp2):
    """An AND gate.
    Parameters:
        inpt1 (str): Input 1, encoded in qubit 0.
        inpt2 (str): Input 2, encoded in qubit 1.
    Returns:
        QuantumCircuit: Output AND circuit.
        str: Output value measured from qubit 2.
    qc = QuantumCircuit(3, 1)
    qc.reset(range(2))
    if inp1=='1':
        qc.x(0)
    if inp2=='1':
        qc.x(1)
    qc.barrier()
    # this is where your program for quantum AND gate goes
    qc.barrier()
    qc.measure(2, 0) # output from qubit 2 is measured
    \# We'll run the program on a simulator
    backend = Aer.get_backend('aer_simulator')
    # Since the output will be deterministic, we can use just a single shot to get it
    job = backend.run(qc, shots=1, memory=True)
    output = job.result().get_memory()[0]
    return qc, output
## Test the function
for inpl in ['0', '1']:
    for inp2 in ['0', '1']:
        qc, output = AND(inp1, inp2)
        print('AND with inputs',inp1,inp2,'gives output',output)
        display(qc.draw())
print('\n')
```

```
from qiskit import *
from qiskit.visualization import plot_histogram
import numpy as np
## for display
from IPython.display import display
def OR(inp1,inp2):
    """An OR gate.
    Parameters:
        inpt1 (str): Input 1, encoded in qubit 0.
        inpt2 (str): Input 2, encoded in qubit 1.
    Returns:
        QuantumCircuit: Output XOR circuit.
        str: Output value measured from qubit 2.
    qc = QuantumCircuit(3, 1)
    qc.reset(range(3))
    if inp1=='1':
        qc.x(0)
    if inp2=='1':
        qc.x(1)
    qc.barrier()
    # this is where your program for quantum OR gate goes
    qc.barrier()
    qc.measure(2, 0) # output from qubit 2 is measured
    \ensuremath{\text{\#}} We'll run the program on a simulator
    backend = Aer.get_backend('aer_simulator')
    # Since the output will be deterministic, we can use just a single shot to get it
    job = backend.run(qc, shots=1, memory=True)
    output = job.result().get_memory()[0]
    return qc, output## Test the function
for inp1 in ['0', '1']:
    for inp2 in ['0', '1']:
        qc, output = OR(inp1, inp2)
        print('OR with inputs',inp1,inp2,'gives output',output)
        display(qc.draw())
        print('\n')
```

```
from qiskit import *
from qiskit.visualization import plot_histogram
import numpy as np
## for display
from IPython.display import display
def UNKNOWN(inp1,inp2):
    """An unknown function
    Parameters:
        inpt1 (str): Input 1, encoded in qubit 0.
        inpt2 (str): Input 2, encoded in qubit 1.
    Returns:
        QuantumCircuit: Output the circuit.
        str: Output values measured from qubit 0 and qubit 1.
    qc = QuantumCircuit(2, 2)
    qc.reset(range(2))
    if inp1=='1':
        qc.x(0)
    if inp2=='1':
        qc.x(1)
    # barrier between input state and gate operation
    qc.barrier()
    # add the code based on the circuit diagram in the handout
    # barrier between gate operation and measurement
    qc.barrier()
    qc.measure([0,1],[0,1]) \# output from qubit 0 and qubit 1 are measured
    #We'll run the program on a simulator
    backend = Aer.get_backend('aer_simulator')
    #Since the output will be deterministic, we can use just a single shot to get it
    job = backend.run(qc, shots=1, memory=True)
    out = job.result().get_memory()[0]
    return qc, out
## Test the function
for inp1 in ['0', '1']:
    for inp2 in ['0', '1']:
        qc, out = UNKNOWN(inp1, inp2)
        # the output string bit positions are reversed
        print('UNKNOWN func with inputs',inp1,inp2,'gives outputs',out[1],out[0])
        display(qc.draw())
        print('\n')
```

CIS 492/593 Hands-On Lab 4 Quantum Key Distribution Fall 2022

In this hands-on lab, you will study and practice two Quantum Key Distribution methods:

- the BB84 protocol (see textbook page 53)
- the Ekert protocol (see textbook page 87)

Login to a workstation and open a terminal (either middle click the terminal icon on the left side bar or press CTRL-ALT-T). In the terminal window, type

cd QC

jupyter notebook

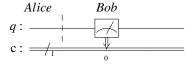
Inside the browser, click "New" and choose "Python 3 (ipykernel)". For each experiment, you need to put the experiment number as a comment (e.g. # Experiment 1) in the top of the code.

Experiment 1: Alice's Encoding and Bob's Measurement using the same basis

• In the cell, type

%load ~cis492s/pub/BB84Test.py

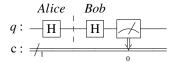
and click "Run" to load a simple program. In this program, Alice prepares and sends a qubit with value 0 in the vertical direction to Bob. Bob measures the qubit in the same direction. The procedure is like the diagram below.



Note that the barrier used here is only an indication of work separation between Alice and Bob. Click "Run" to get and record the result.

• Click the cell done in the previous step. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".

Assume that Alice encodes the qubit in the horizontal direction instead of the vertical direction. That is, she applies the H-gate on the qubit. Also assume that Bob measures the qubit in the horizontal direction. Modify the code just like the figure below:



Click "Run" to get and record the result. What is your conclusion about the result if Alice and Bob use the same basis?

Experiment 2: BB84 emulation (without Eve's eavesdropping)

• In a new and empty cell, type

%load ~cis492s/pub/BB84.py

and click "Run" to load the Qiskit program which emulates the BB84 protocol into the cell. Study the code.

- Click "Run" to get the outputs. Record the outputs (i.e. "Alice bits", "Alice bases". "Bob bases", and "Bob results") in your report. Mark the bit positions where Alice and Bob use the same bases and then extract the corresponding bits from "Alice bits" and "Bob results" into two bit sequences.
- Alice and Bob will select half of the same-basis bits and compare them over an unencrypted line. For simplicity, we just choose the first half of the sequence as the sample bits. Add the code at the end of the cell:

```
sample_size = len(alice_bits_samebasis) // 2
alice_sample = alice_bits_samebasis[0:sample_size]
bob_sample = bob_bits_samebasis[0:sample_size]

print("sample size:", sample_size)
print("alice_sample = ", alice_sample)
print(" bob_sample = " , bob_sample)

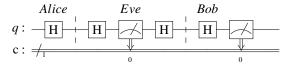
print(bob_sample == alice_sample)
```

Click "Run" to execute the code. What is the size of the sampling bits? Note that if the result is True, Alice and Bob can use the other half of the bit sequence as the key.

• Modify the code to change the value of the variable n from 4 to 1000. Click "Run" to execute the code to make sure that the program works for a larger size of bit sequence.

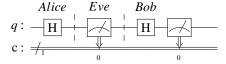
Experiment 3: Alice's Encoding and Bob's Measurement using the same basis (with Eve's eavesdropping)

- Click the cell done in Experiment 1 which contains the code based on the horizontal direction. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Assume that Eve intercepts the qubit on the way from Alice to Bob and she luckily uses the same basis (i.e. the horizontal direction) to measure the qubit. She then prepares and sends Bob a qubit based on the horizontal basis. To emulate Eve's interception, add four lines of code (including a barrier call at the end) before Bob's action, like the diagram below.



Click "Run" to get the result. Is there any difference as compared with the result in Experiment 1 (the horizontal direction)? Can Eve's eavesdropping be detected if Alice, Eve, and Bob use the same basis?

Now assume that Eve guesses the wrong basis to measure the qubit. Click the cell done above.
click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
Modify the code so Eve will use the vertical direction to measure and encode the qubit before sending to Bob, like the figure below:



Click "Run" to get and explain the result.

Experiment 4: BB84 emulation (with Eve's eavesdropping)

- Click the cell which contains the code done in Experiment 2. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- In this experiment, you are asked to add some code between Step 2 and Step 3 to emulate the actions of Eve. That is, she firstly generates a random choice of basis for each bit. Next, she measures each qubit based on the generated random choice of basis. Finally, she encodes the message again based on the generated random choice of basis and then sends the message to Bob. Click "Run" to execute your code and get the result.
- In addition to getting the result False at the end of the previous step, we are also interested in getting the percentage of the sample bits in which Alice and Bob disagree. Add a few lines of code at the end to get the percentage and verify it with the textbook (page 55, lines 23–26).
 - *Hint:* Use a counter and a loop to compare each Alice's sample bit with Bob's sample bit. Increment the counter if they disagree. Output the ratio of the counter over the size of the sample bits.

Experiment 5: Ekert – Measuring two entangled bits with different bases

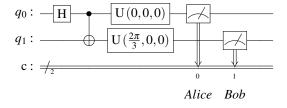
Based on the textbook description about the Ekert91 protocol, Alice and Bob receive a stream of qubits. For each entangled pair of qubits, Alice receives one and randomly chooses a direction to measure her qubit, while Bob gets the other and also randomly selects a direction to measure it. There are three directions Alice and Bob can use: 0° , 120° , or 240° . If they choose the same direction, they will get the same result (i.e. both get 0 or both get 1). However, if they use different directions, they may not get the same result. In this experiment, we are interested in getting the probability that they will AGREE the result, and the probability that they will DISAGREE the result, under the condition that thy will use different directions to measure.

• Note that these three directions 0° , 120° , and 240° can be implemented by a Qiskit U-gate via passing the first parameter with a value 0, 2*pi/3, 4*pi/3, respectively. The fourth parameter is the index of the qubit where the U-gate will be applied.

In a new cell, type

%load ~cis492s/pub/EkertTest.py

and click "Run" to load a simple program. In this program, Alice always measures her qubit in the direction 0° , while Bob always uses the direction 120° to measure. Below is the circuit diagram:



- Study the program. Click "Run" to execute the code. Calculate the probability they AGREE (i.e. combining the percentages of '00' and '11') and the probability they DISAGREE (i.e. adding the percentages of '10' and '01').
- Repeat previous step with Alice using 0° and Bob using 240°. Calculate the two probabilities. Repeat previous step with Alice using 120° and Bob using 240°. Calculate the two probabilities. Repeat previous step with Alice using 120° and Bob using 0°. Calculate the two probabilities. Repeat previous step with Alice using 240° and Bob using 0°. Calculate the two probabilities. Repeat previous step with Alice using 240° and Bob using 120°. Calculate the two probabilities.

Experiment 6: Ekert emulation (without Eve's eavesdropping)

- In a new cell, type

 %load ~cis492s/pub/Ekert.py
 and click "Run" to load the program which emulates the Ekert protocol.
- Study the program. Add a line of code at the end of the program to print the probability that Alice and Bob AGREE the result when they use different bases.
- Click "Run" to execute the code. Verify your result with the equation on Textbook page 81, line 5.

Experiment 7: Ekert emulation (with Eve's eavesdropping)

- Click the cell which contains the code done in Experiment 6. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- To emulate Eve's eavesdropping, add one line of code

 message[q].measure(1,1)

 before Alice measures her entangled qubit in the function measure_message().
- Click "Run" to execute the code. Compare the probability that Alice and Bob AGREE when they use different bases with the corresponding probability in Experiment 6. Verify your result with the description on Textbook page 87, lines 14–16.
- Is the probability that Alice and Bob AGREE the result when they use the same basis still 1 as in the Experiment 6?

Click "File" and choose "Save as". Type "lab4" in the entry box and click "Save" to save your work today into the file lab4.ipynb.

```
To turn in your file, use CTRL-ALT-T to open a terminal and type ssh grail
and type your password to login to the server grail. Then, type
cd QC
turnin -c cis492s -p lab4 lab4.ipynb
to electronically submit your file lab4.ipynb.
```

Shutdown the jupyter notebook and logout the workstation.

```
import numpy as np
from qiskit import QuantumCircuit, Aer
from qiskit.visualization import plot_histogram
## for display
from IPython.display import display
# Create a Quantum Circuit
qc = QuantumCircuit(1,1)
# Alice encodes the qubit
qc.barrier()
# Bob receives the qubit and measures it
qc.measure(0,0)
# Draw and simulate circuit
display(qc.draw())
aersim = Aer.get_backend('aer_simulator')
job = aersim.run(qc,shots=1000)
counts = job.result().get_counts()
print("Total counts:",counts)
plot_histogram(counts)
```

```
## original code from https://qiskit.org/textbook/ch-algorithms/quantum-key-distribution.html
## modified based on Bernhardt's book page 53
from numpy.random import randint
import numpy as np
from qiskit import QuantumCircuit, transpile, Aer
{\tt from\ qiskit.providers.aer\ import\ QasmSimulator}
from qiskit.visualization import plot_histogram
## for display
from IPython.display import display
np.random.seed(seed=23)
n = 4
four\_times\_n = 4 * n
def encode_message(bits, bases):
    message = []
    for i in range(four_times_n):
        qc = QuantumCircuit(1,1)
        if bases[i] == 'V': # Prepare qubit in V direction (i.e. Z-basis)
            if bits[i] == 0:
               pass
            else:
                qc.x(0)
        else: # Prepare qubit in H direction (i.e. X-basis)
            if bits[i] == 0:
                qc.h(0)
            else:
                qc.x(0)
                qc.h(0)
        qc.barrier()
        message.append(qc)
    return message
def measure_message(message, bases):
   backend = Aer.get_backend('aer_simulator')
    measurements = []
    for q in range(four_times_n):
        if bases[q] == 'V': # measuring in V direction
            message[q].measure(0,0)
        else:
                             # measuring in H direction
            message[q].h(0)
            message[q].measure(0,0)
        aer_sim = Aer.get_backend('aer_simulator')
        job = aer_sim.run(message[q], shots=1, memory=True)
        measured_bit = int(job.result().get_memory()[0])
        measurements.append(measured_bit)
    return measurements
def remove_garbage(a_bases, b_bases, bits):
    same\_basis\_bits = []
    for q in range(four_times_n):
        if a_bases[q] == b_bases[q]:
            \# select the bit If both used the same basis
            same_basis_bits.append(bits[q])
    return same_basis_bits
## Step 1
# Alice generates bits
alice_bits = list(randint(2, size=four_times_n))
print("Alice bits: \n", alice_bits)
## Step 2
```

```
# Create an array to tell us which qubits
# are encoded in which bases
#alice_bases = list( map(lambda bit: 'V' if bit == 0 else 'H', randint(2, size=four_times_n))
dir = ['V','H']
alice_bases = list( map(lambda bit: dir[bit], randint(2, size=four_times_n)) )
print("Alice bases: \n", alice_bases)
message = encode_message(alice_bits, alice_bases)
## Step 3
# Decide which basis to measure in:
bob_bases = list( map(lambda bit: dir[bit], randint(2, size=four_times_n)) )
print("Bob bases: \n", bob_bases)
bob_results = measure_message(message, bob_bases)
print("Bob results: \n", bob_results)
## Step 4
alice_bits_samebasis = remove_garbage(alice_bases, bob_bases, alice_bits)
bob_bits_samebasis = remove_garbage(alice_bases, bob_bases, bob_results)
# print("Bob's same-basis bits :\n", bob_bits_samebasis)
## Step 5: The size of the bits which Alice and Bob use the same basis is roughly 2 \star n.
           Textbook uses half of the bits to compare over the unencrypted line.
```

```
import numpy as np
from qiskit import QuantumCircuit, Aer
from qiskit.visualization import plot_histogram
from math import pi
## for display
from IPython.display import display
# Create a Quantum Circuit
qc = QuantumCircuit(2,2)
# entangle two qubits
qc.h(0)
qc.cx(0,1)
# Alice measures qubit 0
qc.u(0,0,0,0)
qc.measure([0],[0])
# Bob measures qubit 1
qc.u(2*pi/3,0,0,1)
qc.measure([1],[1])
# Draw and simulate circuit
display(qc.draw())
aersim = Aer.get_backend('aer_simulator')
job = aersim.run(qc,shots=1000)
counts = job.result().get_counts()
print("Total counts:", counts)
plot_histogram(counts)
```

```
from numpy.random import randint
import numpy as np
from qiskit import QuantumCircuit, transpile, Aer
from qiskit.providers.aer import QasmSimulator
from qiskit.visualization import plot_histogram
from math import pi
## for display
from IPython.display import display
np.random.seed(seed=23)
n = 1000
three\_times\_n = 3 * n
def gen_entangled_pairs():
    message = []
    for i in range(three_times_n):
        qc = QuantumCircuit(2,2)
        qc.h(0)
        qc.cx(0,1)
        qc.barrier()
        message.append(qc)
    return message
def measure_message(message, a_bases, b_bases):
    backend = Aer.get_backend('aer_simulator')
    a_measurements = []
    b_measurements = []
    for q in range(three_times_n):
        # Alice measures one of the entangled qubits (i.e. qubit 0)
        if a_bases[q] == 0:
                                       # measuring in 0 degree
            message[q].measure(0,0)
        elif a\_bases[q] == 1:
                                       # measuring in 120 degree
            message[q].u(2*pi/3,0,0,0)
            message[q].measure(0,0)
        else: ## i.e. a_bases[q] = 2,
                                        measuring in 240 degree
            message[q].u(4*pi/3,0,0,0)
            message[q].measure(0,0)
        # Bob measures the other entangled qubit (i.e. qubit 1)
        if b_bases[q] == 0:
                                       # measuring in 0 degree
            message[q].measure(1,1)
        elif b_bases[q] == 1:
                                       # measuring in 120 degree
            message[q].u(2*pi/3,0,0,1)
            message[q].measure(1,1)
        else: ## i.e. b_bases[q] = 2,
                                       measuring in 240 degree
            message[q].u(4*pi/3,0,0,1)
            message[q].measure(1,1)
        aersim = Aer.get_backend('aer_simulator')
        job = aersim.run(message[q], shots=1, memory=True)
        result = job.result()
        # get_memory[0] returns a string with bob's bit first
        a_measured_bit = result.get_memory()[0][1]
        a_measurements.append(a_measured_bit)
        b_measured_bit = result.get_memory()[0][0]
        b_measurements.append(b_measured_bit)
    return a_measurements, b_measurements
alice_bases = randint(3, size=three_times_n)
bob_bases = randint(3, size=three_times_n)
```

```
m = gen_entangled_pairs()
a_bits, b_bits = measure_message(m, alice_bases, bob_bases)
cnt\_samebasis = 0
cnt\_diffbases = 0
cnt\_agree\_samebasis = 0
cnt\_agree\_diffbases = 0
for i in range(three_times_n):
    if alice_bases[i] == bob_bases[i]:
       cnt\_samebasis += 1
       if a_bits[i] == b_bits[i] :
          cnt\_agree\_samebasis += 1
    else: ## different bases
       cnt_diffbases +=1
       if a_bits[i] == b_bits[i] :
          cnt\_agree\_diffbases += 1
# the probability that they use same basis
print("Probability that they use same basis:" , "{:0.3f}".format(cnt_samebasis / three_times_n
# the probability that they AGREE when they use same basis
print("Probability that they AGREE when they use same basis:", "{:0.3f}".format(cnt_agree_same
basis / cnt_samebasis))
# the probability that they use different bases
print("Probability that they use different bases:", "{:0.3f}".format(cnt_diffbases / three_tim
es_n))
# the probability that they AGREE when they use different bases
```

CIS 492/593 Lab 5 Deutsch and Deutsch-Jozsa Algorithms Fall 2022

The purpose of this hands-on lab is to familiarize yourself with:

- the Deutsch algorithm
- the Deutsch-Jozsa algorithm

Login to a workstation and open a terminal (either middle click the terminal icon on the left side bar or press CTRL-ALT-T). In the terminal window, type

cd QC

jupyter notebook

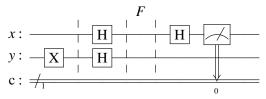
Inside the browser, click "New" and choose "Python 3 (ipykernel)". For each experiment, you need to put the experiment number as a comment (e.g. # Experiment 1) in the top of the code.

Experiment 1: Deutsch – the Constant 0 function

• In the cell, type

%load ~cis492s/pub/DeutschConstO.py

and click "Run" to load a program in which the black box function F is a constant 0 function (i.e. f(0) = 0, and f(1) = 0). Instead of using the default name 'q' for the qubits, we call the function QuantumRegister() twice to label the top qubit as 'x' and the output qubit as 'y', respectively. The corresponding circuit diagram is like below:

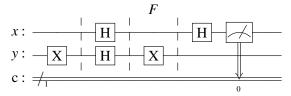


Note that because f(x) = 0, the output of the bottom qubit becomes $y \oplus f(x) = y \oplus 0 = y$. Hence, there are only two straight wires in the black box F and there is no connection between the qubit x and the qubit y.

• Click "Run" to execute the code. What is the output of the top qubit x? Refer the textbook page 149 about how to determine the black box whether it is a constant or a balanced function. Explain the result.

Experiment 2: Deutsch – the Constant 1 function

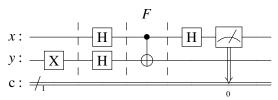
- Click the cell which contains the code done in Experiment 1. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Assume that the black box F is a constant 1 function (i.e. f(0) = 1, and f(1) = 1). Hence, $y \oplus f(x) = y \oplus 1 = \overline{y}$. That is, you need to modify the code (i.e. putting a NOT gate on the qubit y and changing the corresponding comment), like below:



• Click "Run" to execute the code. What is the output of the top qubit x? Explain the result.

Experiment 3: Deutsch – the Identity function (balanced)

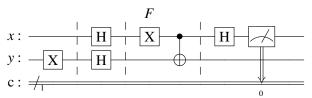
- Click the cell which contains the code done in Experiment 1. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Assume that the black box F is the identity function (i.e. f(0) = 0, and f(1) = 1). To realize $y \oplus f(x)$, you need to modify the code (i.e. adding a CNOT gate with the qubit x as the control while the qubit y as the target, and changing the corresponding comment), just like below:



• Click "Run" to execute the code. What is the output of the top qubit x? Explain the result.

Experiment 4: Deutsch – the Inversion function (balanced)

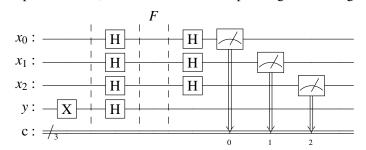
- Click the cell which contains the code done in Experiment 3. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Assume that the black box F is the inversion function (i.e. f(0) = 1, and f(1) = 0). You need to modify the code (i.e. adding a NOT gate to the qubit x, and changing the corresponding comment), like the figure below:



• Click "Run" to execute the code. What is the output of the top qubit x? Explain the result.

Experiment 5: Deutsch-Jozsa – the Constant function

Using a new and empty cell, type
 %load ~cis492s/pub/DJ-Const0.py
 and click "Run" to load a program in which the black box function F is a constant 0 function with
 three parameters x₀, x₁, and x₂. The corresponding circuit diagram is like below:



- Click "Run" to execute the code. What is the output of the top 3-qubit x? Do you see any output string other than '000' during the 1000 times of executions? (Note that in fact we only need one execution to tell the black box is a constant or a balanced function).
- Click the cell which contains the code done above. Click the "Edit" button and select "Copy Cells".

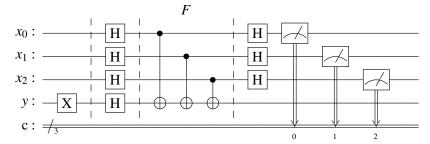
Click "Edit" again and choose "Paste Cells Below".

- Modify the code to make the black box F a constant 1 function. Hint: see Experiment 2.
- Click "Run" to execute the code. What is the output of the top 3-qubit x? Do you see any output string other than '000' during the 1000 times of executions?

Experiment 6: Deutsch-Jozsa – the balanced function $f(x_0, x_1, x_2) = x_0 \oplus x_1 \oplus x_2$

- Construct the truth table for $f(x_0, x_1, x_2) = x_0 \oplus x_1 \oplus x_2$ to verify it is a balanced function.
- Using a new and empty cell, type

 %load ~cis492s/pub/DJ-Const0.py
 and click "Run" to load a template program.
- Modify the code to implement $x_0 \oplus x_1 \oplus x_2$, like the circuit below:



• Click "Run" to execute the code. What is the output of the top 3-qubit x? Do you see '000' appeared during the 1000 times of executions? (Note that in fact we only need one execution to tell the black box is a constant or a balanced function).

Experiment 7: Deutsch-Jozsa – the balanced function $f(x_0, x_1, x_2) = x_0x_1 \oplus x_2$

- Construct the truth table for $f(x_0, x_1, x_2) = x_0x_1 \oplus x_2$ to verify it is a balanced function.
- Modify the code to implement the balanced function $f(x_0, x_1, x_2) = x_0x_1 \oplus x_2$.
- Click "Run" to execute the code. What is the output of the top 3-qubit x? Do you see '000' appeared during the 1000 times of executions? (Note that in fact we only need one execution to tell the black box is a constant or a balanced function).

Click "File" and choose "Save as". Type "lab5" in the entry box and click "Save" to save your work today into the file lab5.ipynb.

```
Open a terminal and type
ssh grail
to login to the server grail. Then, type
cd QC
turnin -c cis492s -p lab5 lab5.ipynb
to electronically turn in your file lab5.ipynb.
```

Shutdown the jupyter notebook and logout the workstation.

Questions:

Below are some questions which will be included in the Homework 3 (Due: Dec. 6). You may start working on these questions now.

- If the black box is a constant function as in the Experiments 1, 2, and 5,
 - a) Is there any wire connection between the qubit(s) x and qubit y?
 - b) What is the initial value for the *x* qubit(s)?
 - c) Explain why the measured value for each *x* qubit(s) is 0. (*Hint*: How do the gates applied on each *x* qubit(s) work? Review the Experiment 2 in Lab2)
- If the black box is a balanced function as in the Experiments 3, 4, 6 and 7,
 - d) Is there any connection via the CNOT gate between the qubit(s) x and qubit y?
 - e) If the answer above is Yes, what happen if we put an H-gate for the corresponding *x* qubit and the qubit *y*? (*Hint*: Review the Experiment 7 in Lab3)
 - f) What is the initial value for the y qubit before applying the H-gate?
 - g) Explain why the measured value for each x qubit(s) is not 0 (as in the Experiments 3 and 4) or not '000' (as in the Experiments 6 and 7).

```
# initialization
import numpy as np
# importing Qiskit
from qiskit import QuantumCircuit, QuantumRegister, ClassicalRegister, transpile, Aer
## for display
from IPython.display import display
# import basic plot tools
from qiskit.visualization import plot_histogram
x = QuantumRegister(1,'x')

y = QuantumRegister(1,'y')
c = ClassicalRegister(1, 'c')
qc = QuantumCircuit(x, y, c)
# set qubit y to 1
qc.x(y)
qc.barrier()
# Apply H-gates for x and y
qc.h(x)
qc.h(y)
qc.barrier()
# f(x) is a constant 0 function; i.e. f(x) is always 0
\# hence, y xor f(x) is y xor 0 which yields y
qc.barrier()
\# Apply the H-gate for the top qubit x
qc.h(x)
# Measure the top qubit, i.e. q0
qc.measure(x,c)
# Display circuit
display(qc.draw())
# use local simulator
aer_sim = Aer.get_backend('aer_simulator')
results = aer_sim.run(qc, shots=1).result()
answer = results.get_counts()
print('the top qubit x: ', answer)
plot_histogram(answer)
```

```
# initialization
import numpy as np
# importing Qiskit
from qiskit import QuantumCircuit, QuantumRegister, ClassicalRegister, transpile, Aer
## for display
from IPython.display import display
# import basic plot tools
from qiskit.visualization import plot_histogram
n = 3
x = QuantumRegister(n,'x')
y = QuantumRegister(1,'y')
c = ClassicalRegister(n, 'c')
qc = QuantumCircuit(x,y,c)
# set qubit y to 1
qc.x(y)
qc.barrier()
\mbox{\#} Apply H-gates for the top n-qubit x and y
qc.h(y)
qc.barrier()
\# f(x) is a constant 0 function; i.e. f(x) is always 0
\# hence, y xor f(x) is y xor 0 which yields y
qc.barrier()
\# Apply the H-gate for the top n-qubit x
qc.h(x)
# Measure the top n qubits
qc.measure(x,c)
# Display circuit
display(qc.draw())
# use local simulator
aer_sim = Aer.get_backend('aer_simulator')
results = aer_sim.run(qc, shots=1000).result()
answer = results.get_counts()
print('the top qubit: ', answer)
plot_histogram(answer)
```

CIS 492/593 Lab 6 Simon's Algorithm Fall 2022

In this hands-on lab, you will study and practice:

- the Simon's algorithm
- the Oracle construction method for the Simon's algorithm

Login to a workstation and open a terminal (either middle click the terminal icon on the left side bar or press CTRL-ALT-T). In the terminal window, type

cd QC

jupyter notebook

Inside the browser, click "New" and choose "Python 3 (ipykernel)". For each experiment, you need to put the experiment number as a comment (e.g. # Experiment 1) in the top of the code.

Experiment 1: Simon's Algorithm with a 2-bit secret string

• In the empty cell, type

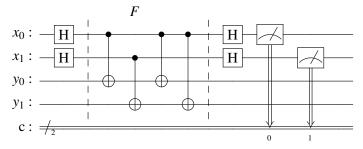
%load ~cis492s/pub/Simon.py

and click "Run" to load a program which will give us a 2-bit string whose dot product with the secret string *s* is 0.

- Study the code. Note that the Oracle() function builds and returns the circuit for the oracle function *F*. We also use the method compose() in Qiskit to combine two quantum circuits together. That is, we add the oracle circuit Fc to the circuit qc.
- Click the cell containing the code and then click "Run" to execute it. Record the output string of the top 2-qubit x.
- Repeat above step to query the oracle until you get two different strings. How many times have you queried the oracle?
- Ignore the output string '00' which wouldn't give us any information. Use the other output string to calculate its dot product with the secret string s (i.e. s_0s_1) and the result should be 0. Hence, you can get one equation regarding s_0 and/or s_1 . Based on the equation, figure out the bit pattern of the secret string s.

Experiment 2: Simon's Algorithm with another 2-bit secret string

- Click the cell which contains the code done in Experiment 1. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Modify the code in the Oracle() function to construct the circuit of *F* (i.e. between two barrier lines), like below:

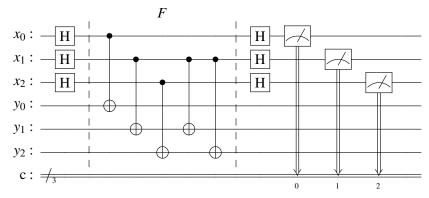


- Click "Run" to execute the code. Record the output string of the top 2-qubit x.
- Repeat above step to query the oracle until you get two different strings. How many times have you queried the oracle?

• Use the output string which is not '00. We know that this string's dot product with the secret string s (i.e. s_0s_1 should be 0. Hence, you can obtain one equation related to s_0 and/or s_1 . Based on the equation, derive the bit pattern of the secret string s. Note that the secret string cannot be '00'.

Experiment 3: Simon's Algorithm with a 3-bit secret string

- Click the cell which contains the code done in Experiment 1. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Change the line of the code 'n = 2' to be 'n = 3'.
- Modify the code in the Oracle() function to construct the circuit of *F* (i.e. between two barrier lines), like below:



- Click "Run" to execute the code. Record the output string of the top 3-qubit x.
- Repeat above step to query the oracle until you get three different strings. How many times have you queried the oracle?
- Use the output strings except the string '000'. We know that each of these strings has a dot product of 0 with the secret string s (i.e. $s_0s_1s_2$). Hence, you can obtain two (or three) equations related to s_0 , s_1 , and/or s_2 . Based on the equations, figure out the bit pattern of the secret string s. Note that the secret string cannot be '000'.

Experiment 4: The Oracle Construction method for Simon's Algorithm

In this experiment, you will learn how to construct the Oracle circuit for a given string *s*. There are two steps in the construction method:

Step I: Apply the CNOT gates from qubits of the first register (i.e. x) to qubits of the second register (i.e. y). So the content of each qubit in the first register, which is the classical information encoded as either a $|0\rangle$ or a $|0\rangle$, will be copied to the corresponding qubit in the second register. In fact, this can be simply implemented by using one line of code:

 $fc.cx(x,y) \\ instead of using a loop : \\ for i in range n: \\ fc.cx(x[i],y[i]) \\$

Step II: Because the bits in the string s cannot be all 0, find the least index k such that $s_k = 1$. Next, apply the CNOT gates from the qubit x_k to any qubit y_i if $s_i = 1$.

For example, if s = 110, the least index k is 0 because $s_0 = 1$. Therefore, we apply two CNOT gates from s_0 to s_0 and from s_0 to s_1 .

In fact, the Oracle circuits in Experiment 2 and Experiment 3 are also based on the two-step procedure.

- Click the cell which contains the code done in Experiment 3. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Assume s is '110'. Replace the line

```
n = 3 with s = '110' n = len(s)
```

- Modify the code in the Oracle() function to construct the circuit of *F* for *s* = 110 using the two-step procedure described above.
- Increase the parameter shots from 1 to 1000 when calling aer_sim.run() because we want to collect all possible strings whose dot product with s is 0.
- To verify that those output strings which have a dot product of 0 with s, add the code below at the end of the program.

```
# Calculate the dot product of the results
def bdotz(s, a):
    accum = 0
    for i in range(len(s)):
        accum += int(s[i]) * int(a[i])
    return (accum % 2)

for a in answer:
    print( '{}.{} = {} (mod 2)'.format(s, a, bdotz(s,a)) )
```

• Click "Run" to execute the code and record the result. Use the first two output strings which are not '000' to figure out the string s.

Experiment 5: The Oracle Construction method for Simon's Algorithm - II

- Click the cell which contains the code done in Experiment 4. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Change the string s from '110' to '01011'.
- Modify the code in the Oracle() function to construct the circuit of F for s=01011 using the two-step procedure.
- Click "Run" to execute the code and record the result. Use the four linear independent output strings which are not '000' to figure out the string s.

Click "File" and choose "Save as". Type "lab6" in the entry box and click "Save" to save your work today into the file lab6.ipynb.

```
To turn in your file, use CTRL-ALT-T to open a terminal and type
```

```
ssh grail and type your password to login to the server grail. Then, type
```

```
turnin -c cis492s -p lab6 lab6.ipynb to electronically turn in your file lab6.ipynb.
```

Shutdown the jupyter notebook and logout the workstation.

```
# initialization
import numpy as np
# importing Qiskit
from qiskit import IBMQ, Aer
from qiskit.providers.ibmq import least_busy
from qiskit import QuantumCircuit, QuantumRegister, ClassicalRegister, transpile
## for display
from IPython.display import display
# import basic plot tools
from qiskit.visualization import plot_histogram
n = 2
def Oracle():
    x = QuantumRegister(n)
    y = QuantumRegister(n)
    c = ClassicalRegister(n)
    fc = QuantumCircuit(x,y,c)
    fc.cx(x[1],y[1])
    return fc
xq = QuantumRegister(n,'x')
yq = QuantumRegister(n,'y')
c = ClassicalRegister(n)
qc = QuantumCircuit(xq,yq,c)
# Apply H-gates for the top n-qubit xq
qc.h(xq)
qc.barrier()
# get the Oracle circuit and compose it
Fc = Oracle()
qc.compose(Fc,inplace=True)
qc.barrier()
# Apply H-gates for the top n-qubit xq
qc.h(xq)
# Measure
qc.measure(xq,c)
# Display circuit
display(qc.draw())
# use local simulator
aer_sim = Aer.get_backend('aer_simulator')
results = aer_sim.run(qc, shots=1).result()
counts = results.get_counts()
# Because c0 is the rightmost bit in the Qiskit output string,
# we have to reverse the string in the count dictionary
answer = {}
for str in counts:
    answer[str[::-1]] = counts[str]
print(answer)
plot_histogram(answer)
```

CIS 492/593 Lab 7 Grover's Search Algorithm Fall 2022

In this hands-on lab, you will learn and practice:

- the Grover's algorithm, including the Oracle construction and the Amplitude magnification method
- the Qiskit program for solving the 3-SAT problem via Grover's Search

Login to a workstation and open a terminal (either middle click the terminal icon on the left side bar or press CTRL-ALT-T). In the terminal window, type

cd QC

jupyter notebook

Inside the browser, click "New" and choose "Python 3 (ipykernel)". For each experiment, you need to put the experiment number as a comment (e.g. # Experiment 1) in the top of the code.

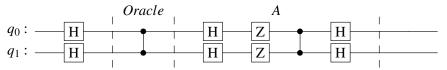
Experiment 1: Grover's Algorithm with a 2-bit oracle

• In the empty cell, type

%load ~cis492s/pub/Grover2.py

and click "Run" to load the program which will find the desired element location 11 among four binary strings: 00, 01, 10, and 11.

• Study the code. Note that the Oracle() function builds and returns the circuit which flips the sign of the probability amplitude of the location we are trying to find. The function A() performs the work of amplitude amplification by flipping a sequence of numbers about their mean. Below is the circuit generated from the program:



Recall that

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad Controlled - Z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

• We are interested in the operation matrices for the oracle and for the amplitude amplification circuits. Instead of calculating the matrices by hand, you can use the Operator class defined in the Qiskit quantum_info library. Add the following line of the code

display(rdm(quantum_info.Operator(oc).data,n))

to the Oracle() function before it returns. Also, Add the following line of the code

display(rdm(quantum_info.Operator(ac).data,n))

to the A() function before it returns. Because the Qiskit Operator uses a reverse bit sequence (i.e. q_1q_0), we implement and call the function rdm() to reverse the bit sequence to be consistent with the order (i.e. q_0q_1) used in the Bernhardt's textbook.

• Click the cell containing the code and then click "Run" to execute it. Record the matrix for the oracle. You may ignore the imaginary part (i.e. the term associated with the letter 'j'). Show which element location has the value -1. Note that -1 will flip the probability amplitude of the element we are trying to find.

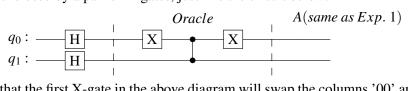
Record the matrix for the amplitude amplification. Also ignore the imaginary part. Is the matrix the same as the the matrix A in the textbook (page 179, line 18)?

Record and explain the result of the state vector (i.e. probability amplitude) (with the transpose operator).

What is the probability for each location?

Experiment 2: Grover's Algorithm with another 2-bit oracle

- Click the cell which contains the code done in Experiment 1. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Instead of finding '11' as in the Experiment 1, you are asked to search the element location '01'. To achieve this, you only need to add two X-gates, one is before the Controlled-Z and the other is after the the Controlled-Z, on the qubits that should be zero. In this experiment, it is the qubit q_0 which has to be enclosed by a pair of X-gates, just like the circuit below:



Note that the first X-gate in the above diagram will swap the columns '00' and '10', and also swap the columns '01' and '11', in the diagonal matrix defined by the Controlled-Z, while the second X-gate will exchange the rows '00' and '10', and also exchange the rows '01' and '11'. These swappings will move the value -1 to the desired element location.

Modify the code in the Oracle() function based on the diagram above.

• Click "Run" to execute the code. Record the matrix for the oracle. Again, ignore the imaginary part. Indicate the location which contains the value -1.

Record and explain the result of the state vector (i.e. probability amplitude) (with the transpose operator).

What is the probability for each location?

Experiment 3: Grover's Algorithm with a 3-bit Oracle

• In the empty cell, type

%load ~cis492s/pub/Grover3.py

and click "Run" to load a program which will find the desired element location 111 among eight binary strings: 000, 001, 010, 011. 100, 101, 110, and 111.

Qiskit does not support the ccz gate. As shown in the Oracle() function, we use a ccx gate and put a pair of H-gates before and after the X gate in ccx to emulate the ccz operation. Note that HXH = Z.

- Click "Run" to execute the code. Record the state vector result (with the transpose operator). Ignore the imaginary part. You should get two different probability amplitudes. Compare these two probability amplitudes with the two probability amplitudes mentioned in the textbook (page 180, line 4). Before you compare, the two probability amplitudes in the textbook should be divided by $\sqrt{8}$ due to $H^{\otimes 3}$.
- To see whether we can get a higher probability of the correct answer, we may flip the sign of the probability amplitude associated with the desired location and then perform the amplitude amplification again. That is, we need to execute the function Oracle() and A() again. This can be implemented by invoking the repeat(iter) method in Qiskit where iter is the number of repetition times. Modify the code

```
# put the OA after the H-gates
qc.compose(OA,inplace=True)
```

to be

```
# put the repeated OA after the H-gates
qc.compose( OA.repeat(2) , inplace=True)
```

- Click "Run" to execute the code.
 - Record the result of the state vector (i.e. probability amplitude) (with the transpose operator).
 - What is the probability for each location? Does the probability of the desired location become higher?
- Modify the code again to repeatedly execute the function Oracle() and A() three times. Record the result of the state vector (i.e. probability amplitude) (with the transpose operator). Does the probability of the desired location become higher? If not, that means we 'overcook' the numbers. Change the repetition times back to 2.

Experiment 4: Grover's Algorithm with another 3-bit Oracle

- Click the cell which contains the code done in Experiment 3. Click the "Edit" button and select "Copy Cells". Click "Edit" again and choose "Paste Cells Below".
- Using the method you learned in Experiment 2, modify the code in the Oracle() function to make the desired location be '001'.
- Click "Run" to execute the code. Record and explain the result of the state vector (i.e. probability amplitude) (with the transpose operator).

Experiment 5: Solving 3-SAT problems using Grover's Algorithm

Note that Grover's algorithm is not practical for most database searches. However, it is useful for solving the NP-complete problems such as 3-SAT, etc. and can give a quadratic speedup over classical algorithms. Follow the steps below to learn how to use Qiskit to solve the 3-SAT problem with Grover's algorithm.

- Open the browser in a new window to browse:
 https://github.com/Qiskit/qiskit-tutorials/blob/master/tutorials/algorithms/07_grover_examples.ipynb
- Study the contents in the above web page.
- Copy the code from the web page to the cell in your jupyter and click "Run" to execute the code.
- · Record the results.

Click "File" and choose "Save as". Type "lab7" in the entry box and click "Save" to save your work today into the file lab7.ipynb.

```
To turn in your file, use CTRL-ALT-T to open a terminal and type ssh grail
and type your password to login to the server grail. Then, type
cd QC
turnin -c cis492s -p lab7 lab7.ipynb
to electronically submit your file lab7.ipynb.
```

Shutdown the jupyter notebook and logout the workstation.

```
#initialization
import numpy as np
# importing Qiskit
from qiskit import QuantumCircuit, transpile, quantum_info
from qiskit.providers.aer import QasmSimulator
{\tt from\ qiskit\ import\ ClassicalRegister,\ QuantumRegister}
from IPython.display import display
# import basic plot tools
from qiskit.visualization import plot_histogram
# reverse the bit sequence of the nb-bit integer t
def rbits(t,nb):
    m = nb-1
    k = 0
    while t > 0:
        k = (t \& 1) << m
        t >>= 1
       m -= 1
    return k
# exchange the elements in the vector v based on the reverse bits of indices
def rv(v,nb) :
    newv = np.copy(v)
    for i in range(1<<nb):
        ni = rbits(i, nb)
        newv[i] = v[ni]
    return newv
# exchange the diagonal elements in the matrix m based on the reverse bits of indices
def rdm(m, nb) :
    newm = np.copy(m)
    for i in range(1<<nb):
        ni = rbits(i,nb)
        newm[i][i] = m[ni][ni]
    return newm
def Oracle(): ## select 11
    oc = QuantumCircuit(n)
    # cz: control q0, target q1
    oc.cz(0, 1)
    return oc
\operatorname{def} A(): ## amplitude amplification via fliiping around the mean
    ac = QuantumCircuit(2)
    ac.h(range(2))
    ac.z(range(2))
    ac.cz(0,1)
    ac.h(range(2))
    return ac
qc = QuantumCircuit(n)
qc.h(range(n))
```

```
qc.barrier()
# query the Oracle: phase inversion
qc.compose(Oracle(),inplace=True)
qc.barrier()
# amplitude magnification
qc.compose(A(),inplace=True)
qc.barrier()
# Draw the circuit
display(qc.draw())
# Do NOT call measure() because we want to see the probability amplitudes in the statevector
qc.save_statevector()
# Use Aer's qasm_simulator
simulator = QasmSimulator()
# compile the circuit down to low-level QASM instructions
compiled_circuit = transpile(qc, simulator)
# Execute the circuit on the qasm simulator
job = simulator.run(compiled_circuit, shots=1000)
# Grab results from the job
result = job.result()
out_state = result.get_statevector()
print(rv(out_state.data,n))
counts = result.get_counts()
# we have to reverse the string in the dictionary counts
answer = {}
for str in counts:
    answer[str[::-1]] = counts[str]
print(answer)
plot_histogram(answer)
```

```
#initialization
import numpy as np
# importing Qiskit
from qiskit import QuantumCircuit, transpile, quantum_info
from qiskit.providers.aer import QasmSimulator
{\tt from\ qiskit\ import\ ClassicalRegister,\ QuantumRegister}
from IPython.display import display
# import basic plot tools
from qiskit.visualization import plot_histogram
# reverse the bit sequence of the nb-bit integer t
def rbits(t,nb):
    m = nb-1
    k = 0
    while t > 0:
       k = (t \& 1) << m
        t >>= 1
        m = 1
    return k
# exchange the elements in the vector v based on the reverse bits of indices
def rv(v, nb):
    newv = np.copy(v)
    for i in range(1<<nb):
        ni = rbits(i, nb)
        newv[i] = v[ni]
    return newv
# exchange the diagonal elements in the matrix m based on the reverse bits of indices
def rdm(m, nb):
    newm = np.copy(m)
    for i in range(1<<nb):</pre>
        ni = rbits(i, nb)
        newm[i][i] = m[ni][ni]
    return newm
n = 3
def Oracle(): ## select q0 q1 q2: 111
    oc = QuantumCircuit(n)
    # use ccx and H-gates to achieve ccz
    oc.h(2)
    oc.ccx(0,1,2)
    oc.h(2)
    return oc
def A(nb): ## amplitude amplification via fliiping around the mean
    ac = QuantumCircuit(nb)
    ac.h(range(nb))
    ac.x(range(nb))
    # Do multi-controlled-Z gate
    ac.h(nb-1)
    ac.mcx(list(range(nb-1)), nb-1) # multi-controlled
    ac.h(nb-1)
    ac.x(range(nb))
```

```
ac.h(range(nb))
    return ac
qc = QuantumCircuit(n)
qc.h(range(n))
# construct the Oracle_Amplication circuits
OA = QuantumCircuit(n)
# query the Oracle
OA.compose(Oracle(), inplace=True)
# amplitude magnification
OA.compose(A(n),inplace=True)
# put the OA after the H-gates
qc.compose(OA,inplace=True)
# Draw the circuit
display(qc.draw())
# Do NOT call measure() because we want to see the probability amplitudes in the statevector
qc.save_statevector()
# Use Aer's qasm_simulator
simulator = QasmSimulator()
# compile the circuit down to low-level QASM instructions
compiled_circuit = transpile(qc, simulator)
# Execute the circuit on the gasm simulator
job = simulator.run(compiled_circuit, shots=1000)
# Grab results from the job
result = job.result()
out_state = result.get_statevector()
print(rv(out_state.data,n))
counts = result.get_counts()
\ensuremath{\sharp} we have to reverse the string in the dictionary counts
answer = {}
for str in counts:
    answer[str[::-1]] = counts[str]
print(answer)
plot_histogram(answer)
```

import qiskit.tools.jupyter
%qiskit_version_table
%qiskit_copyright

Version Information Qiskit Software Version qiskit-terra 0.24.1 0.12.1 qiskit-aer qiskit-ignis 0.7.1 qiskit-ibmq-provider 0.19.1 qiskit 0.36.2 System information Python version 3.8.10 Python compiler GCC 9.4.0 default, May 26 2023 14:05:08 Python build OS Linux CPUs Memory (Gb) 15.396270751953125 Thu Aug 03 16:47:06 2023 EDT

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