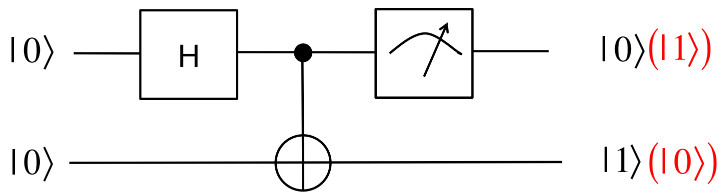
# Information




***Representation, Models, and the Origin of Quantum Mechanics***

# Information

- Encoded in State of a Physical System
  - Voltage level in an IC
  - presence or absence of light
  - color of light
- Computation Changes State in (hopefully) Predictable Way
- Study of Information Linked to Study of Physical Processes of Representation and Computation
- *What about symbols on paper/screen/disk for representation?*
- *What about mathematics for their computation (manipulation)?*

## How do we represent information?

- *What about symbols on paper/screen/disk for representation?*
- *What about mathematics for their computation (manipulation)?*
- “Models” of Information and Computation
- Switching Theory Predominately used for Classical Computing (Shannon/Boole)
- Good Model for Deterministic 2-state Information and Computation Devices

## Current State of Information

- Effects of Quantum Mechanics are “Interfering” with Computation as MOSFET Features Decrease
- “Interference” Since Classical Switching Theory Models becoming Inaccurate
- (some) Nanotechnology Uses Quantum Effects to Model Switching Theory (eg. QCA)
- What about Changing Models to Reflect the Quantum Mechanical Nature of Computation?

***This is the Primary focus of our Class***

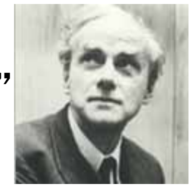
## Information

- Information is Physical
- Computation is a Physical Process
- Point of View is Necessary for Utilization and Advances in Quantum Computing
- Classical Models have “Loose” Connection with Physical Reality
  - Time/Space Complexity of Algorithms
  - Entropy (uncertainty measure) in Information Theory

## Bits and Qubits

- BInary digiT (BIT) Exists in one of two States
- QUantum BIT (QUBIT) Exists in a Continuum of States with 2 Basis States
- Measurement/Observation of a Bit yields its Value Without Affecting its State
- Measuring/Observing a Qubit is Non-deterministic and Alters its State
- QUantum DIgiT (QUDIT) Exists in a continuum of states with more than 2 Basis States

## Qubit Model



- Qubit Notation: $|\psi\rangle$ "ket - phi"
- "Ket: Notation of Paul Dirac
- Represents a Column Vector in 2-Dimensional Complex Vector Space
  - Two Components that are Complex Numbers
- Two Computational Basis States for Qubit

$$|0\rangle \qquad\qquad |1\rangle$$

## Qubit Model

- Qubit exists in Linear Combination of Basis States
- "Ket Notation Represents a Column Vector

$$|\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \alpha\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

## Multiple Bit Systems

- Bits are Modeled as Scalars: $\mathbb{B} = \{0,1\}$
- Multi-bit systems are Formed Using
- the ***Cartesian Product***: $\mathbb{B}^n = \prod \mathbb{B}$
- Example: 3-bit System:

$$f : \mathbb{B}^3 \to \mathbb{B} \qquad \mathbb{B}^3 = \mathbb{B} \times \mathbb{B} \times \mathbb{B}$$

$$\mathbb{B}^3 = \{0,1\} \times \{0,1\} \times \{0,1\}$$

$$\mathbb{B}^3 = \{\{0,0\},\{0,1\},\{1,0\},\{1,1\}\} \times \{0,1\}$$

$$\mathbb{B}^3 = \{\{0,0,0\},\{0,0,1\},\{0,1,0\},\{0,1,1\},$$
$$\{1,0,0\},\{1,0,1\},\{1,1,0\},\{1,1,1\}\}$$

## Multiple Bit Systems (cont.)

- Example: 3-bit System: $f = x + y + z$

$$f : \mathbb{B}^3 \to \mathbb{B}$$

- Mapping of $\mathbb{B}^3$ to $\mathbb{B}$
- One Model is Truth Table

| $x$ | $y$ | $z$ | $f$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

$\mathbb{B}$

$\mathbb{B}^3$

***Multi-bit Systems use Cartesian Product as Model***

## Multiple Qubit Systems

- Qubits are Modeled as 2-D Vectors: $|\psi\rangle \in \mathbb{H}$
- Multi-qubit systems are Formed Using
- the Tensor Product: $\mathbb{H}^n = \bigotimes \mathbb{H}$
- Example: 2-qubit System:

$$\mathbf{A} : \mathbb{H}^2 \rightarrow \mathbb{H}^2 \qquad \mathbb{H}^2 = \mathbb{H} \otimes \mathbb{H}$$

- Let

$$|\psi\rangle, |\phi\rangle \in \mathbb{H} \qquad |\psi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad |\phi\rangle = \begin{bmatrix} \delta \\ \gamma \end{bmatrix}$$

## Multiple Qubit Systems

$$|\psi\phi\rangle = |\psi\rangle \otimes |\phi\rangle$$

$$|\psi\phi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \otimes \begin{bmatrix} \delta \\ \gamma \end{bmatrix} = \begin{bmatrix} \alpha \begin{bmatrix} \delta \\ \gamma \end{bmatrix} \\ \beta \begin{bmatrix} \delta \\ \gamma \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \alpha\delta \\ \alpha\gamma \\ \beta\delta \\ \beta\gamma \end{bmatrix}$$

- This is the Tensor Product of two 1-dimensional tensors (also known as vectors)
- We will review more about linear algebra later

## Multiple Qubit Systems

- How is Mapping Performed?

$$\mathbf{A} : \mathbb{H}^2 \rightarrow \mathbb{H}^2$$

- Linear Transformation $\mathbf{A}$ is a Matrix
- Example:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{A} : \mathbb{H}^2 \rightarrow \mathbb{H}^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha\delta \\ \alpha\gamma \\ \beta\delta \\ \beta\gamma \end{bmatrix} = \begin{bmatrix} \alpha\delta \\ \alpha\gamma \\ \beta\gamma \\ \beta\delta \end{bmatrix}$$

## Multiple Qubit Systems

- Tensor Product Means Quantum System of $n$ Particles has Basis State Space of Size $2^n$
- Additionally, Each Qubit Exists in a "Superposition" of those $2^n$ Basis Vectors
- Superposition is the Power of these Systems
  - $n$ particle System Yields $2^n$ Parallelism
- Downside is Measurement Forces Qubits to Some Basis State with Some Probability

## Decoherence

- "Measurement" is Actually Interaction with Outside Observer
- Interaction with the Outside can Occur Inadvertently Forcing Particles to Basis States - ***Decoherence***
- Decoherence is a Major Obstacle to be Overcome for Quantum Computing to Succeed

## Entanglement

- 2 or More Particles May Exist in States of Superposition and Also Related Together
- Entanglement (Verschränkung as Per Schrödinger)
- State of System of Entangled Particles Cannot be Written as a Tensor Product
- Entangled Particles can Exist Even at Far Distance from One Another

## Entanglement

- Measurement of One Entangled Particle Affects the State of the Other!!!
- Einstein called this “Spooky Action at a Distance”
- Will Review Classic Experiments to Learn About this Phenomena
  - Blackbody Radiation Theory (Max Planck)
  - Einstein’s Photoelectric Effect (Einstein)
  - Double Slit Experiment (Thomas Young)

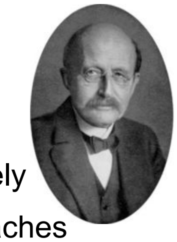
## The Notion of Quanta

*“Planck had put forward a new, previously unimagined thought, the thought of the atomistic structure of energy.”*

- Albert Einstein

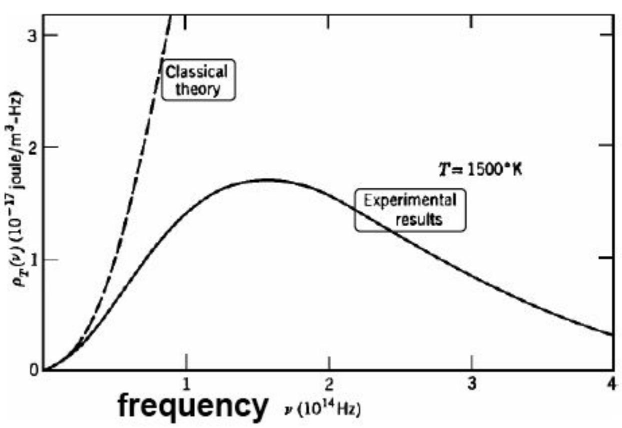
- 1901 Paper by Max Planck on Black Body Radiation
- Classical Physics leads to Conclusion that Radiation from Hot Object is Very Bright at blue or violet end of Spectrum
- eg. A Fireplace Log glowing red ends up Emitting Ultraviolet Rays as well as X-Rays and Gamma Rays - the *Ultraviolet Catastrophe*

# Blackbody Radiation



- Max Planck Formulated a New Law of Blackbody Radiation in 1900
- Blackbody: hypothetical object that completely absorbs all radiant energy falling upon it, reaches some equilibrium temperature, and then reemits that energy as quickly as it absorbs it.
- Classical theory up to time of Planck didn't agree with experimental data and predicted "Ultraviolet Catastrophe"

# Ultraviolet Catastrophe




- Classical Theory Predicts Radiation Energy Increases with Square of Frequency - Not Seen Experimentally - Infinite Total Radiated Energy!!!

# Planck's Constant

- 19th Century Physics due to Maxwell and Hertz indicate an Oscillating Charge Produces Radiation
  - $\nu$ is Oscillating Frequency
  - $E$ is Energy of Oscillating Charge
- Planck Proposed Discrete Energy Levels of Radiation:

$$E = 0, h\nu, 2h\nu, 3h\nu, 4h\nu, \ldots, nh\nu$$
$$n > 0 \text{ and is an integer}$$

- Theory Successfully Explains why Ultraviolet Catastrophe never Occurs
- When Allotted Energy for an Oscillator is Smaller than a "Package" of Available Energy through Planck's Formula, Radiation Intensity Decreases

# Blackbody Radiation

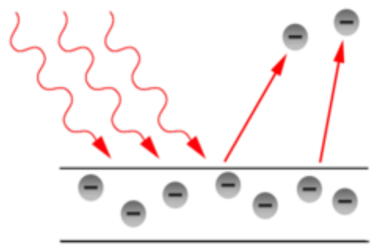
- 1901 Planck Made Assumption that Radiation Energy is Emitted in Packets (quanta)
- Out of "Desperation" to Find Theory that Matched Experimental Data
- Each Packet with Energy: $E = h\nu$
- Where $h$ is a Calculated Scale Factor now Called Planck's Constant

$$h = 6.626 \times 10^{-34}\,\mathrm{J \cdot s} \qquad \hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34}\,\mathrm{J \cdot s}$$

- $\nu$ is the Frequency of the Radiation

***The Notion of Quantum Behavior is Introduced in Physics***

## Photoelectric Effect



- Photoelectrons Emitted from Matter after Absorption of Energy from EM Radiation such as Light
- Demonstrated by Heinrich Rudolf Hertz (Hertz Effect)

## Photoelectric Effect

- Shining a Light on Metal Liberates $e^-$ from Surface
  - Easy for UV Light; harder for Red Light
  - Energy of $e^-$ Depends on Light Frequency
  - Increasing Light Intensity Increases Number of $e^-$
- Einstein Extended Planck's Hypothesis that Light is Quantized (know known as photons) $E_{photon} = h\nu$
- Each photon Interacts with Single $e^-$
  - More Photons=More Liberated $e^-$

## Photoelectric Effect Equations

- Energy of Photon = Energy needed to remove an electron + Kinetic energy of the emitted electron

$$E_{photon} = h\nu \qquad hf = \phi + E_{k_{\max}}$$

- Where
  - $h$ is Planck's Constant
  - $f$ is frequency of incident photon
  - $\phi$ is the Work Function; minimum energy to remove photoelectron
  - $E_{k\max}$ is the maximum kinetic energy of photoelectron

## Photoelectric Effect Equations

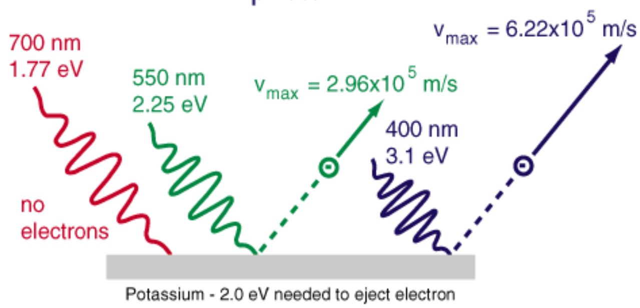
$$hf = \phi + E_{k_{\max}} \qquad \phi = hf_0$$

$$E_{k_{\max}} = \frac{1}{2}mv_m^2 \qquad E = \sqrt{(pc)^2 + (mc^2)^2}$$

- Where
  - $f_0$ is threshold frequency for photoelectric effect
  - $m$ is rest mass of the ejected electron
  - $v_m$ is velocity of the ejected electron
  - if $hf<\phi$, no electron is emitted
  - $p$ is momentum of particle
  - $E$ related to $p$ by Einstein's Special Theory of Relativity

# Photoelectric Effect

$$E_{photon} = h\nu$$




Photoelectric effect

***Theory of Discrete (Quantum) Levels Again used in Physics***

- Einstein Receives Nobel Prize in 1921 for this Discovery

# Einstein's Planck Medal

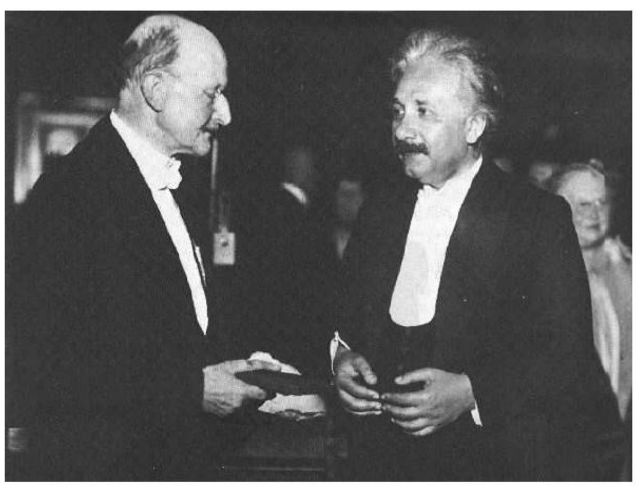


FIGURE 1.2. Planck to Einstein: *I hereby award you the Planck Medal because you expanded my desperate idea of quantum of energy to the even more desperate idea of quantum of light.*

*source: unknown and quote is questionable*

# Thomas Young's Experiment

"We choose to examine a phenomenon (the double-slit experiment) that is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality it contains the *only* mystery."

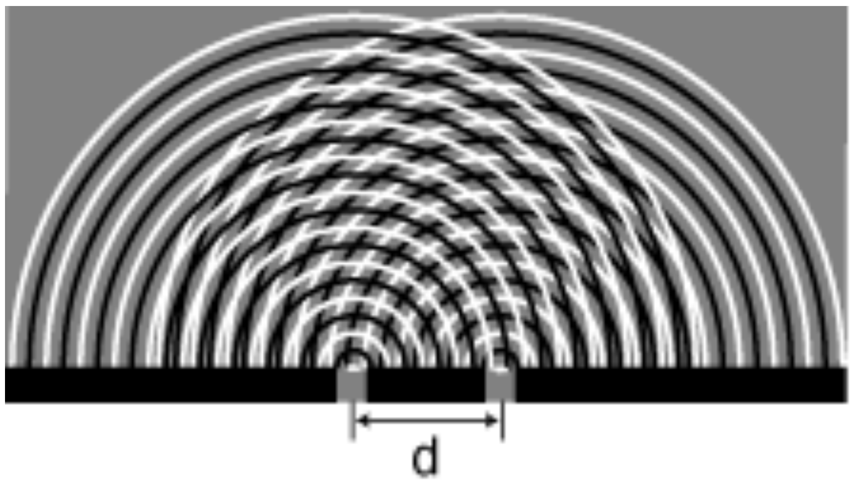
-Richard Feynman

# Wave Theory Effect of Interference

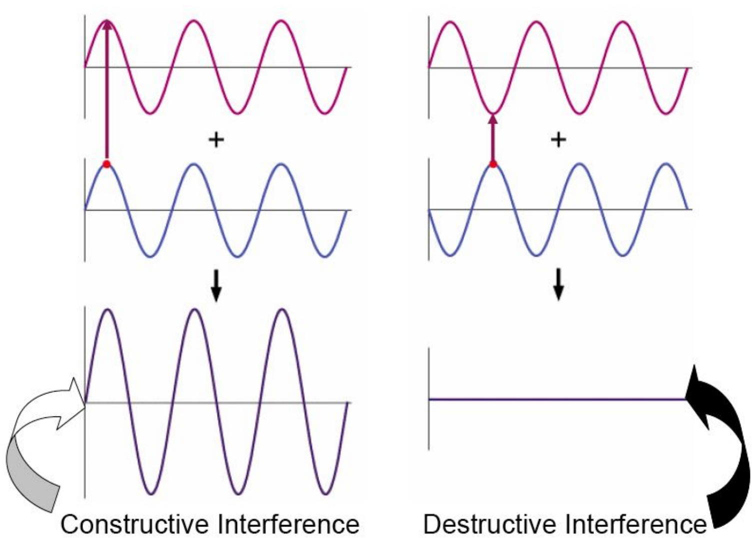
## Demonstrated by Young circa 1800

- Determine if Light is a Wave or Particle
- Known as "Double Slit" Experiment
- Light Source and Barrier with Two Slits
- Screen (second barrier) Behind First
- Interference Pattern Observed similar to what Would Happen if Water Waves used
- Wave-like Phenomena
  - Constructive/Destructive Interference
- Waves Interfere with Each Other while Particles do not
- Light has Wave-like Behavior

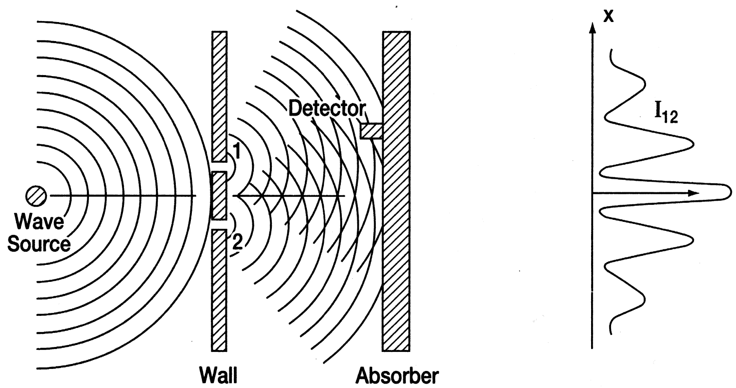
## Wave Interference




## Constructive/Destructive Interference



# Young's Double Slit Experiment

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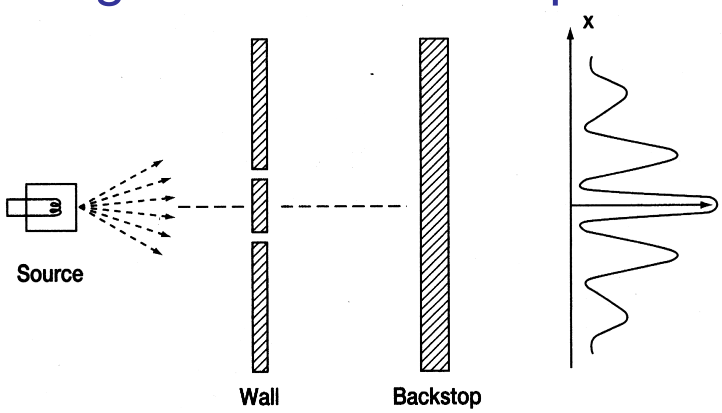
# Young's Double Slit Experiment




***Appears to Confirm Light is a Wave***

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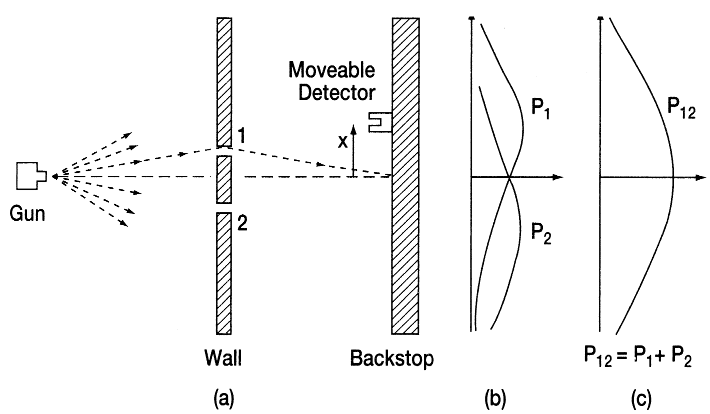
# Interference Waves versus Non-interfering Particles

- Feynman Lectures on Physics (vol. III)
- Consider Two Slit Experiment with Bullets from Gun
- Gun Shoots Bullets at 2-Slit Barrier in Random Directions
- Resulting Accumulation Patterns show Non-interfering Phenomena

# Young's Double Slit Experiment

***Shouldn't this pattern be observed for particles??***

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## Electromagnetic Waves

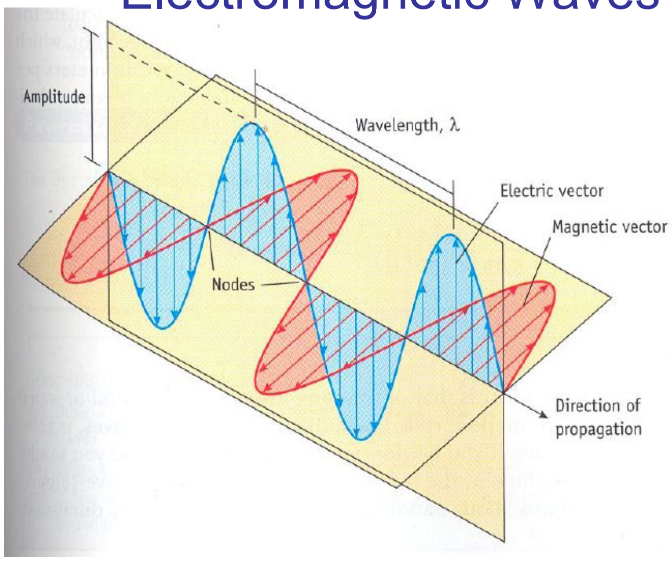



Diagram of Transverse Wave

## Electromagnetic Wavelength

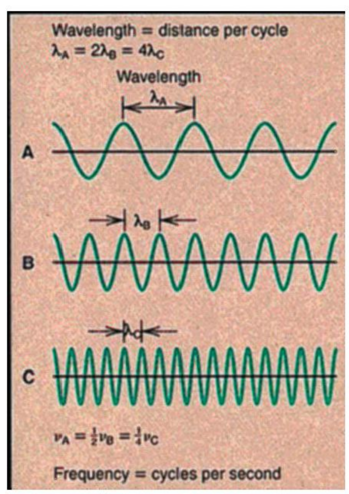
Wavelength and Frequency of Electromagnetic Radiation

$\lambda\nu = c$

$\lambda$ = wavelength

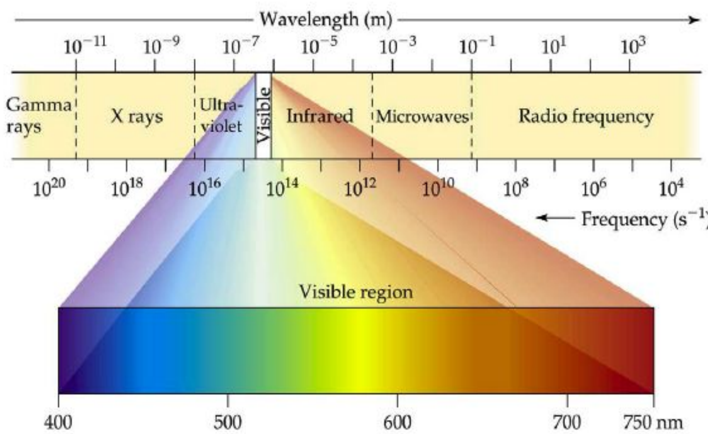
$\nu$ = frequency

$c = 2.997 \times 10^8$ m/s

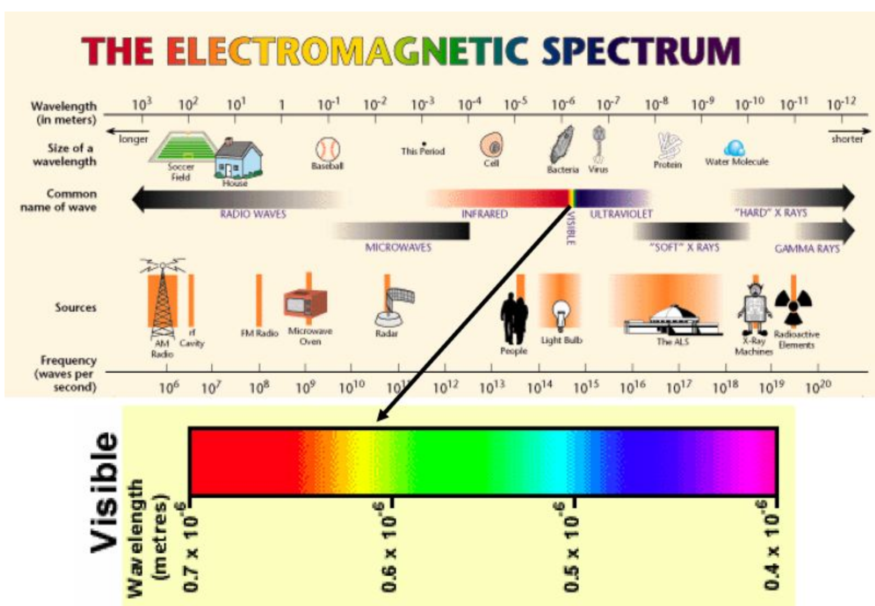



c is free-space velocity

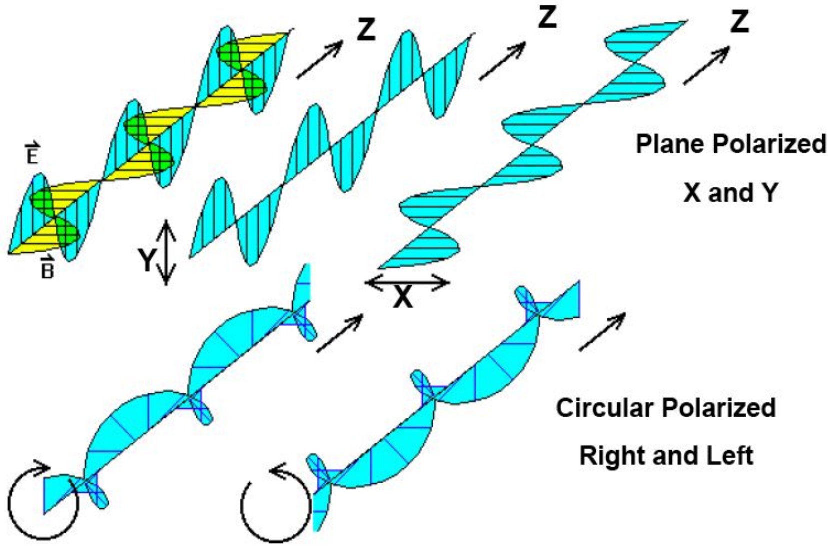
# Wave Nature of Light




Visible Light Small Part of EM Spectrum



# EM Wave Polarization




# Nature of Light

- Young's Experiment Demonstrates Light is a Wave
- Einstein's Photoelectric Effect (1905) Demonstrates Light is a Stream of Particles, photons (as Newton argued)
- Light acts like Waves and Particles
  - Waves: Interference and Diffraction
  - Particles: Interact with Matter: Collisions and Momentum
- Photons have "wave-like" Characteristics such as:
  - Wavelength
  - Polarization

## Light as a Particle

- Einstein's Mathematical Description of Photoelectric effect Described in his 1905 Paper:

  *On a Heuristic Viewpoint Concerning the Production and Transformation of Light*

- Proposed Light Quanta (later known as photons)
- Resulted in his 1921 Nobel Prize
- Motivated by Max Planck's black-body radiation work Described in his 1901 Paper:

  *On the Law of Distribution of Energy in the Normal Spectrum*

- Allowed idea that Light consists of of Discrete Energy Packets or photons

## Wave-Particle Duality

- Young's Experiment Demonstrated Light Behaves in a Wave-like Manner
- Photoelectric effect is Explained based on the Assumption that Light is a Stream of Particles as First Hypothesized by Newton
- These Two Results Illustrate Wave-Particle Duality of the Nature of Light
  - Two Light Rays Interfere with Each Other in a way similar to sound waves from two stereo speakers
  - Light interacts with matter in a way only particles can utilizing mechanical quantities of mass, momentum, and kinetic energy

# de Broglie Hypothesis

- (1924) All Matter has Wave-like Property
- Nobel Prize in 1929 for PhD thesis (a first)

$$\lambda = \frac{h}{p}$$

**EXAMPLE**

Standard 9mm Handgun round: 115 grains, 1115 fps

Standard 9mm Handgun round: 7.45 grams, 340 mps

What is the momentum of the bullet?


# de Broglie Hypothesis



- (1924) All Matter has Wave-like Property
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$$\lambda = \frac{h}{p}$$

**EXAMPLE**

Standard 9mm Handgun round: 115 grains, 1115 fps

Standard 9mm Handgun round: 7.45 grams, 340 mps

What is the momentum of the bullet?


$$p = (0.00745\text{kg})\left(\frac{340\text{m}}{\text{s}}\right) = 2.533\frac{\text{kg}\cdot\text{m}}{\text{s}}$$

# de Broglie Hypothesis

- (1924) All Matter has Wave-like Property
- Nobel Prize in 1929 for PhD thesis (a first)

$$\lambda = \frac{h}{p}$$

**EXAMPLE**

Standard 9mm Handgun round: 115 grains, 1115 f/s

Standard 9mm Handgun round: 7.45 grams, 340 m/s

$$\lambda = \left(\frac{1\,\mathrm{s}}{2.533\,\mathrm{kg}\cdot\mathrm{m}}\right)\left(\frac{6.626\times10^{-34}\ \mathrm{m}^2\mathrm{kg}}{1\,\mathrm{s}}\right) = 2.616\times10^{-34}\ \mathrm{m}$$

# Modern Version of Young's Experiment

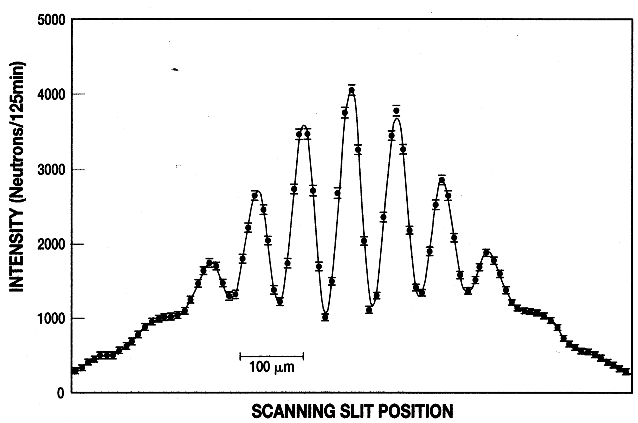
- Young's Experiment Repeated (in 20$^{th}$ century) with very Weak Light source - emits a single photon at a time
- Very Unlikely that Several Photons found in the Apparatus at Same Time
- **SAME** Interference Pattern Appeared in Absorbing Barrier
- What was each individual photon interfering with?
- Answer appears to be with itself
- Appears each photon went through both slits (not one) and Interfered with itself!

## Other Versions of Young's Experiment

- Experiment Carried out with Different Sources:
  - electrons (1950s)
  - neutrons (1970s)
  - atoms (1980s)
- Same Interference Pattern Resulted
- Empirically Demonstrates Wave-Particle Duality (aka the de Broglie Principle)

## Zeilinger et al. Experiment with Neutrons

- Double Slit Experiment with Neutrons in 1991
- Neutrons traveling at 2 km/s

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## Particles act like Waves

- Experiments Demonstrate Particles act like Waves
- Even One Particle at a Time
- What are these Single Particles Interfering with?
- Appears if Individual Particles travel through *BOTH* Slits
- Particles Interfere with *THEMSELVES*

## Qubit System State

- New state of a System Composed from Two or More States
- New State Shares Properties of Each of Combined States
- Assume A and B are Two Different Properties of a Particle (such as being at two different places at the same time)
- Superposition of States is A+B and has something in common with state A and state B
- Particle has non-zero probability of being in each of A and B but not Elsewhere - *IF* the Particle Position is Observed

## Superposition - Double Slit Exp.

- Particle $\psi$ in State A when Passes through Slit A
- Particle $\psi$ in State B when Passes through Slit B
- Superposition of states is Combination of States A and B
- Two Paths are Combined and thus there are two nonzero probabilities
- 50% chance passes through Slit A and 50% chance through Slit B if Observed/Measured
- If not Observed, Particle Passes through both Slits A and B
- Particle Must Pass through Both in order to Interfere with Itself
- This Superposition of States is part of the "mystery" of Quantum Mechanics that Feynman Refers to

## Single Qubit Example

- Let A Slit Represent Basis $|0\rangle$ and B Slit Represent Basis State $|1\rangle$
- If Equally Probable that Particle Observed through Slit A or Slit B, then:

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

- What is the value of $\alpha$ and $\beta$?

# Single Qubit Example

- Let A Slit Represent Basis |0〉 and B Slit Represent Basis State |1〉
- If Equally Probable that Particle Observed through Slit A or Slit B, then:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

- What is the value of $\alpha$ and $\beta$?

$$\mathrm{P}[|\psi\rangle = |0\rangle] = 0.5 \text{ and } \mathrm{P}[|\psi\rangle = |1\rangle] = 0.5$$

$$|\alpha|^2 + |\beta|^2 = 1 \qquad |\alpha|^2 = 0.5 \qquad |\beta|^2 = 0.5$$

$$|\alpha| = \frac{1}{\sqrt{2}} \quad \text{and} \quad |\beta| = \frac{1}{\sqrt{2}}$$

# Superposition of 2 Particles

- Application of the Superposition Principle to a Composite System consisting of two (or more) Subsystems
- In our Example, a Subsystem is a single particle
- Particle $\psi$ Can be in State |0〉 or State B (example property could be location of particle)
- |0〉 and |1〉 are contradictory states (example two different locations)
- Particle $\Phi$ Can be in State |0〉 or State |1〉
- State |00〉 is a "Product State" Meaning Particle $\psi$ is in State |0〉 and Particle $\Phi$ is in State |0〉
- Similarly there are Product States |01〉, |10〉, |01〉
- By Superposition, the two-particle System can be in State |01〉+|01〉+|10〉+|11〉

## Superposition of 2 Particles

- The Product State:

$$|\psi\phi\rangle = |\psi\rangle \otimes |\phi\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \otimes \begin{bmatrix} \delta \\ \gamma \end{bmatrix} = \begin{bmatrix} \alpha\delta \\ \alpha\gamma \\ \beta\delta \\ \beta\gamma \end{bmatrix}$$

- Superposition of Two-particle System:

$$|\psi\phi\rangle = w|00\rangle + x|01\rangle + y|10\rangle + z|11\rangle$$

- Assuming Equally Likely to Pass through Slit A and Slit B, What are the probabilities of observing one of these four states?

## Superposition of 2 Particles

- Assuming Equally Likely to Pass through Slit A and Slit B, What are the probabilities of observing one of these four states?

$$|w|^2 + |x|^2 + |y|^2 + |z|^2 = 1$$

$$|w|^2 + |x|^2 + |y|^2 + |z|^2 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

$$|w| = |x| = |y| = |z| = \frac{1}{2}$$

$$|\psi\phi\rangle = \frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$$

# Entanglement

- Product State $|00\rangle$ or $|11\rangle$ ascribes Definite Properties to Particle 1 and Particle 2
- Superimposed State Does NOT Ascribe such a Definite Property
- Superimposed State Only Says there are Possibilities Concerning Particles 1 and 2
- Superimposed State Implies that the Properties of Particles 1 and 2 are Correlated
  - If Particle 1 is Measured to be in State $|0\rangle$ , then Particle 2 must be in State $|0\rangle$
  - If Particle 1 is Measured to be in State $|1\rangle$ , then Particle 2 must be in State $|1\rangle$
- When Particles 1 and 2 are Entangled, No Way to Characterize one By Itself without Referring to the Other as Well

*What about states $|01\rangle$ and $|10\rangle$ ?*

# An Entangled State

- What if we could somehow "change" the Superimposed Product State to Be:

$$|\psi\phi\rangle = Q|00\rangle + 0|01\rangle + 0|10\rangle + R|11\rangle$$

$$|\psi\phi\rangle = Q|00\rangle + R|11\rangle$$

$$|\psi\phi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

- This is an ENTANGLED state
- Very Interesting Since Observation of One Particle Immediately Implies the State of the Other
- If Particle $|\psi\rangle$ is Observed to be $|0\rangle$, then Particle $|\phi\rangle$ Must also be $|0\rangle$ !!!!!!
- Even if Particles are Very Far Apart
- Einstein Called this "Spooky Action at a Distance"
- Referred to as an EPR Pair

# An Entangled State

- How is it Possible to Ensure Product State is of Form:

$$|\psi\phi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

- Must Transform Product
- Use a Quantum Program or Quantum Circuit
- Quantum Transforms Represented Mathematically by a Linear Transformation (Vector-Matrix Product)
- Consider the Transformation, $T$

$$T = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

# Product State Initialization

- Assume we can "Initialize" the Qubit States:

$$|\psi\rangle = |0\rangle \quad \text{and} \quad |\phi\rangle = |0\rangle$$

$$|\psi\phi\rangle = |\psi\rangle \otimes |\phi\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|\psi\phi\rangle = |\psi\rangle \otimes |\phi\rangle = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

# An Entangled State

- After Initialization, apply the Transform, $T$:

$$T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

$$T\,|\psi\phi\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

# An Entangled State

- After Initialization, apply the Transform, $T$:

$$T\,|\psi\phi\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$T\,|\psi\phi\rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \left( \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right)$$

# An Entangled State

$$T\,|\psi\phi\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\0\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right)$$

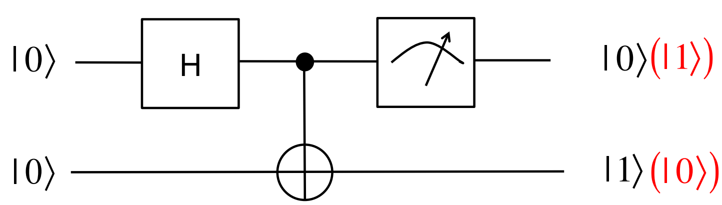
$$T\,|\psi\phi\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix} = \frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\0\end{bmatrix}\otimes\begin{bmatrix}1\\0\end{bmatrix} + \begin{bmatrix}0\\1\end{bmatrix}\otimes\begin{bmatrix}0\\1\end{bmatrix}\right)$$

$$T\,|\psi\phi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle\otimes|0\rangle + |1\rangle\otimes|1\rangle\right) = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

# One of the Bell States

$$T\,|\psi\phi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle\otimes|0\rangle + |1\rangle\otimes|1\rangle\right) = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

- Need a Quantum Program (or Circuit) to Perform $T$ Transform
- Called a "Bell State Generator"
- One Such Circuit:



# The Bell States

$$|\Phi^+\rangle = \frac{(|00\rangle + |11\rangle)}{\sqrt{2}} \qquad |\Psi^+\rangle = \frac{(|01\rangle + |10\rangle)}{\sqrt{2}}$$

$$|\Phi^-\rangle = \frac{(|00\rangle - |11\rangle)}{\sqrt{2}} \qquad |\Psi^-\rangle = \frac{(|01\rangle - |10\rangle)}{\sqrt{2}}$$

These States are the Basis of the Phenomena Such as *Teleportation* and *Quantum Channels*

Anyone who is not shocked about quantum theory has not understood it

Niels Bohr