# STA257

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# recap

# the *functions* so far

1. Probabilitity measure: $P : \mathcal{A} \longrightarrow \mathbb{R}$ and satisfies the three axioms. In general no "picture" possible, because its domain is a collection of events.

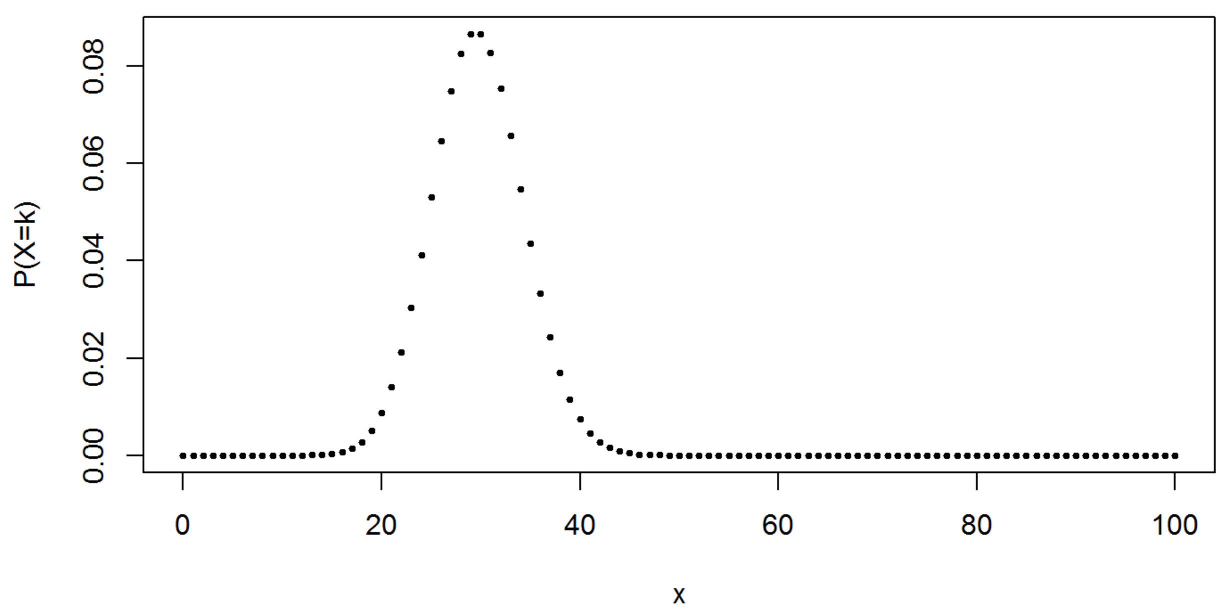
2. **Random variable** $X : S \longrightarrow \mathbb{R}$. In general no "picture" possible, because its domain is a sample space. We care about: *its distribution*.

3. Cumulative distribution function $F$ for the random variable $X$. Defined as $F(x) = P(X \leq x)$. **Completely characterizes a distribution**. A picture is possible, and the picture does give some information of limited use. NEW: Has a few technical properties of interest.

4. For a *discrete* random variable, there is also a probability mass function $p(x) = P(X = x)$. **Completely characterizes a distribution**. A picture is possible, and the picture can be informative. Has a few technical properties of interest.

# Binomial$(n, p)$ distributions

The probability that someone is HIV+ given that their ELISA test comes back positive is $0.2971$. Suppose we have 100 people with a positive ELISA test. How might one visualize the distribution of the number of people who are HIV+?



# …a few more examples

The extreme cases have already been considered.

$$P(X = n)$$

$$P(X = 0)$$

$$1 - P(X = n)$$

$$1 - P(X = 0)$$

Problem solving hints: look for cases where the number of trials is fixed and one is interesting in counting occurences of something.

# the geometric distributions

Consider a Bernoulli($p$) process. Count the number of trials until the first "success".

This is a random variable. Call it $X$.

What is the p.m.f. of $X$?

$$p(k) = P(X = k) = \begin{cases} (1-p)^{k-1} p & x \in \{1, 2, 3, \ldots\} \\ 0 & \text{otherwise} \end{cases}$$

Is this a valid pmf? Yes.

We say $X$ has a geometric distribution with paramter $p$, or $X \sim \text{Geometric}(p)$.

Interesting property: "memoryless"

# the "negative binomial" distributions

Consider a Bernoulli$(p)$ process. Count the number of trials until the $r^{th}$ "success".

This is a random variable. Call it $X$.

What is the p.m.f. of $X$?

$$p(k) = P(X = k) = \begin{cases} \binom{k-1}{r-1} p^k (1-p)^{k-r} & x \in \{r, r+1, r+2, \dots\} \\ 0 & \text{otherwise} \end{cases}$$

Is this a valid pmf? Yes (a little obscure to figure out)

We say $X$ has a negative binomial distribution with paramters $p$ and $r$, or $X \sim \text{NegBin}(p, r)$.