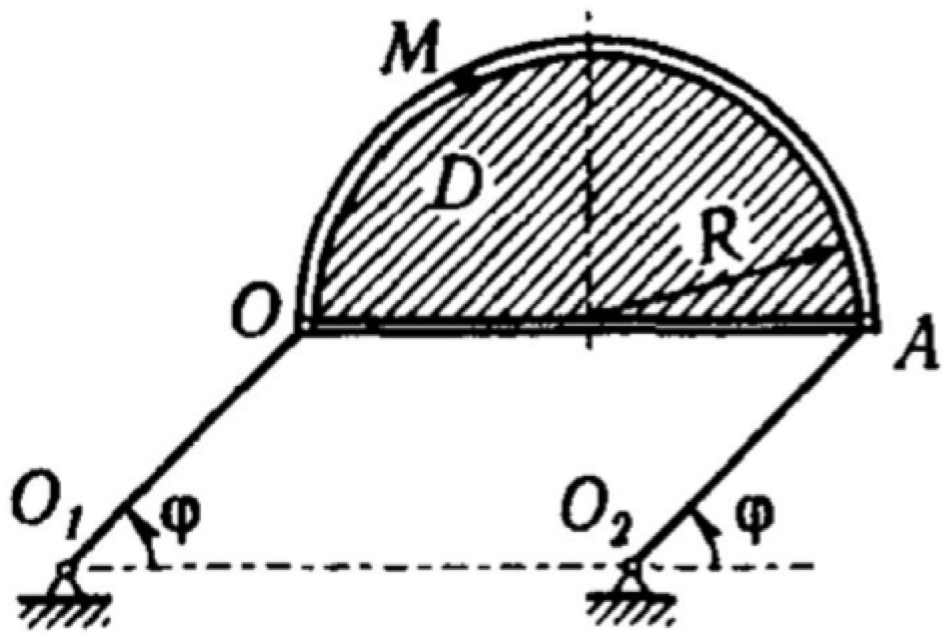
Task 1

Given:



$OM = s_r(t) = 6\pi t^2,\ \phi(t) = \pi \frac{t^3}{6},\ R = 18,\ O_1O = O_2A = 20.$

Simulate the mechanism, find absolute, transport and relative velocities and accelerations for M, find t, when M reaches point A.

Solution:

1. Positions:

   Let D be the center of link OA.

   Point O position: $O_1O \begin{pmatrix} \cos\phi - R \\ \sin\phi \end{pmatrix}$.

   Point A position: $O_1O \begin{pmatrix} \cos\phi + R \\ \sin\phi \end{pmatrix}$.

   Let θ be an angle between DM and OD, hence $\theta(t) = \frac{OM}{R} = \frac{6\pi t^2}{R}$.

   Point M position: $O_1O \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} + R \begin{pmatrix} -\cos\theta \\ \sin\theta \end{pmatrix}$.

2. Velocities:

   Relative velocity of the point M: $v_{rel} = R\dot{\theta} \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}$.

Transport velocity of the point M: $v_{tr} = O_1O\dot{\phi}\begin{pmatrix} -\sin\phi \\ \cos\phi \end{pmatrix}$.

Absolute velocity of the point M: $v_{abs} = v_{rel} + v_{tr} = R\dot{\theta}\begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix} + O_1O\dot{\phi}\begin{pmatrix} -\sin\phi \\ \cos\phi \end{pmatrix}$.

3. Accelerations:

   $\omega_{rel} = \dot{\theta}$, $\varepsilon_{rel} = \ddot{\theta}$.

   Normal relative acceleration of the point M:

$$a^n_{rel} = -\omega^2_{rel}\vec{DM} = -\omega^2_{rel}R\begin{pmatrix} -\cos\theta \\ \sin\theta \end{pmatrix}$$

   Tangential relative acceleration of the point M:

$$a^\tau_{rel} = \vec{\varepsilon_{rel}} \times \vec{DM} = \begin{pmatrix} 0 \\ 0 \\ \varepsilon_{rel} \end{pmatrix} \times \begin{pmatrix} -R\cos\theta \\ R\sin\theta \\ 0 \end{pmatrix}$$

   $\omega_{tr} = \dot{\phi}$, $\varepsilon_{tr} = \ddot{\phi}$.

   Normal transport acceleration of the point M:

$$a^n_{tr} = -\omega^2_{tr}\vec{O_1O} = -\omega^2_{tr}\begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}$$

   Tangential transport acceleration of the point M:

$$a^\tau_{tr} = \vec{\varepsilon_{tr}} \times \vec{O_1O} = \begin{pmatrix} 0 \\ 0 \\ \varepsilon_{tr} \end{pmatrix} \times \begin{pmatrix} O_1O\cos\theta \\ O_1O\sin\theta \\ 0 \end{pmatrix}$$

   OA does translational motion, hence there is no coriolis acceleration.

4. Find t when M reaches point A.

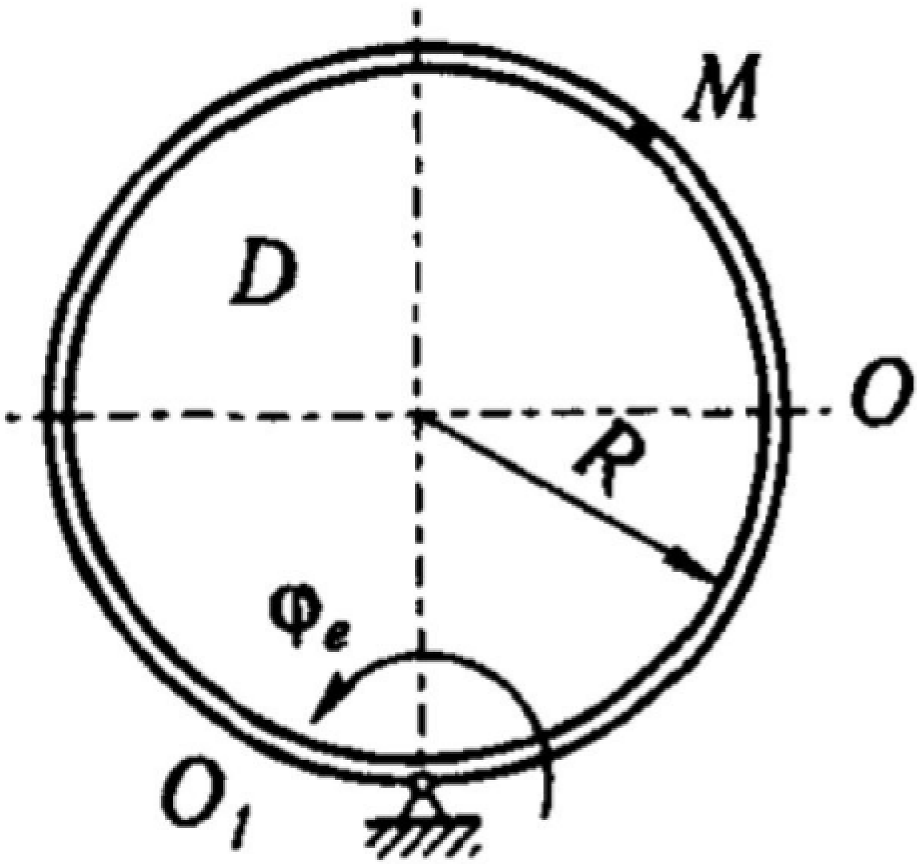
   If point M reached point A, then it travelled distance πR.

$$s_r(t) = \pi R \longrightarrow 6\pi t^2 = \pi R \longrightarrow t = \sqrt{\frac{R}{6}}$$

   For $R = 18$ $t = \sqrt{3}$.

Task 2

Given:



$OM = s_r(t) = 75\pi(0.1t + 0.3t^2),\ \phi(t) = 2t - 0.3t^2,\ R = 30.$

Simulate the mechanism, find absolute, transport and relative velocities and accelerations for M, find t, when M reaches point O second time.

Solution:

1. Positions:

   Let O_2 be the center of the circle.

   Point O_2 position: $R\begin{pmatrix}-\sin\phi \\ \cos\phi\end{pmatrix}$.

   Point O position: $\sqrt{2}R\begin{pmatrix}\cos\phi + \frac{\pi}{4} \\ \sin\phi + \frac{\pi}{4}\end{pmatrix}$.

   Point M position: $R\begin{pmatrix}-\sin\phi \\ \cos\phi\end{pmatrix} + R\begin{pmatrix}\cos(\theta+\phi) \\ \sin(\theta+\phi)\end{pmatrix}$

2. Velocities:

   Relative velocity of the point M: $v_{rel} = R(\dot{\theta} + \dot{\phi}) \begin{pmatrix} -\sin(\theta + \phi) \\ \cos(\theta + \phi) \end{pmatrix}$.

   Transport velocity of the point M: $v_{tr} = -|\vec{O_1M}|\dot{\phi} \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}$

   Absolute velocity of the point M:

$$v_{abs} = v_{rel} + v_{tr} = R(\dot{\theta} + \dot{\phi}) \begin{pmatrix} -\sin(\theta + \phi) \\ \cos(\theta + \phi) \end{pmatrix} - |\vec{O_1M}|\dot{\phi} \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix}$$

3. Accelerations:

   Relative acceleration of the point M:

$$a_{rel} = R(\ddot{\theta} + \ddot{\phi}) \begin{pmatrix} -\sin(\theta + \phi) \\ \cos(\theta + \phi) \end{pmatrix} - R(\dot{\theta} + \dot{\phi})^2 \begin{pmatrix} \cos(\theta + \phi) \\ \sin(\theta + \phi) \end{pmatrix}$$

   Transport acceleration of the point M:

$$a_{tr} = \vec{\varepsilon_{tr}} \times \vec{O_1M} + \vec{\omega_{tr}} \times (\vec{\omega_{tr}} \times \vec{O_1M})$$

$$a_{tr} = \begin{pmatrix} 0 \\ 0 \\ \ddot{\phi} \end{pmatrix} \times \vec{O_1M} + \begin{pmatrix} 0 \\ 0 \\ \dot{\phi} \end{pmatrix} \times (\begin{pmatrix} 0 \\ 0 \\ \dot{\phi} \end{pmatrix} \times \vec{O_1M})$$

   Coriolis acceleration of the point M:

$$a_{cor} = 2 \cdot \vec{\omega_t} r \times \vec{v_{rel}} = 2 \cdot \begin{pmatrix} 0 \\ 0 \\ \dot{\phi} \end{pmatrix} \times (R(\dot{\theta} + \dot{\phi}) \begin{pmatrix} -\sin(\theta + \phi) \\ \cos(\theta + \phi) \end{pmatrix})$$

4. Find t, when M reaches point O second time:

   To reach point O second time, point M should travel distance $2\pi R$:

$$75\pi(0.1t + 0.3t^2) = 2\pi R$$

   After solving this equation, we obtain: $t = \frac{-1+\sqrt{1+\frac{16}{5}R}}{6}$, or $t \approx 1.47481$