Task solved

- 2.3 - 2 points
- 2.4 - 3 points
- 5.12 - 4 points
- 5.13 - 3 points
- 5.14 - 3 points
- 5.15 - 2 points
- 6.1 - 2 points
- 6.2 - 2 points
- 6.3 - 2 points
- 6.5 - 4 points
- 7.1 - 2 points
- 7.2 - 3 points
- 7.10 - 3 points

35 points in total

Task 2.3

Task statement

What is the most general solution of $\frac{dT}{dt} = k(T - 20)$?

Solution

$$\begin{aligned}
\frac{dT}{dt} &= k(T - 20) \\
dT &= k(T - 20)dt \\
\frac{dT}{T - 20} &= kdt,\ T \neq 20 \\
\ln|T - 20| &= kt + C \\
T &= Ce^{kt} + 20
\end{aligned}$$

While I was solving the equation, I said that $T \neq 20$. This solution is obtained when $C = 0$. Hence this solution is the most general.

Answer

The most general solution is: $T = Ce^{kt} + 20$.

Task 2.4

Task statement

What is the most general solution of $\frac{dT}{dt} = k(T - C)$?

Solution

$$\begin{aligned}
\frac{dT}{dt} &= k(T - C) \\
dT &= k(T - C)dt \\
\frac{dT}{T - C} &= kdt,\ T \neq C \\
\ln|T - C| &= kt + C_1 \\
T &= C_1 e^{kt} + C
\end{aligned}$$

While I was solving the equation, I said that $T \neq C$. This solution is obtained when $C_1 = 0$. Hence this solution is the most general.

Answer

The most general solution is: $T = C_1 e^{kt} + C$.

Task 5.12

Task statement

Let $a \neq b \in \mathbf{R}$ be real numbers and x ranges over $\mathbf{R}$.

a Show that functions e^{ax} and e^{bx} are (linearly) independent.

b Does there exist a second-order linear homogeneous equation that has two partial solutions e^{ax} and e^{bx}?

c Does there exist a second-order linear homogeneous equation with a general solution $C_1e^{ax} + C_2e^{bx}$? $(C_1,\ C_2 \in \mathbf{R})$.

Does there exist a second-order linear homogeneous equation that has Ce^{ax} as the most general solution ($C \in \mathbf{R}$)?

Solution

a Two functions are linearly dependent if $c_1e^{ax} + c_2e^{bx} = 0$ for $c_1,\ c_2 \neq 0$.

$$\begin{aligned} c_1e^{ax} + c_2e^{bx} &= 0 \\ e^{ax} &= -\frac{c_2}{c_1}e^{bx} \\ -\frac{c_2}{c_1} &= e^{(a-b)x} \end{aligned}$$

On a left-hand side we have a constant, on a right-hand side we have a function of argument t. The function can be equal to a constant if it is independent on its argument, it is possible only if $a = b$, but it contradicts to the task statement, therefore the functions are linearly independent.

b Let us write second-order linear homogeneous equation:

$$y'' + py' + qy = 0$$

Then after substuting two partial solutions e^{ax} and e^{bx} we obtain:

$$\begin{cases} a^2e^{ax} + ape^{ax} + qe^{ax} = 0 \\ b^2e^{bx} + bpe^{bx} + qe^{bx} = 0 \end{cases} \implies \begin{cases} a^2 + ap + q = 0 \\ b^2 + bp + q = 0 \end{cases}$$

Applying Vietta's formulas for quadratic polynomials:

$$\begin{cases} p = -(a+b) \\ q = ab \end{cases}$$

There exist a second-order linear homogeneous equation with given partial solutions:

$$y'' - (a+b)y' + (ab)y = 0$$

c Previously I proved that:

(a) functions e^{ax} and e^{bx} are linearly independent

(b) there exist a second-order linear homogeneous equation with partial solutions e^{ax} and e^{bx}.

Hence e^{ax} and e^{bx} form a solution space for the equation $y'' - (a+b)y' + (ab)y = 0$, and a general solution for this equation is $y = C_1 e^{ax} + C_2 e^{bx}$.

Answer

Above I proved that

a The functions e^{ax} and e^{bx} are linearly independent.

b There exist a second-order linear homogeneous equation that has two partial solutions e^{ax} and e^{bx} and it has form $y'' - (a+b)y' + (ab)y = 0$.

c There exist a second-order linear homogeneous equation with a general solution $C_1 e^{ax} + C_2 e^{bx}$? $(C_1,\ C_2 \in \mathbf{R})$.

Task 5.13

Task statement

Show that the equation $y'' - 2y' - 8y = 0$ has a general solution $y = C_1 e^{4x} + C_2 e^{-2x}$. Is it the most general solution? (Explain why.)

Solution

The equation is a linear homogeneous second order ordinary differential equation.

Let $y = e^{ax}$, then

$$a^2 e^{ax} - 2ae^{ax} - 8e^{ax} = 0$$
$$a^2 - 2a - 8 = 0$$

Solving this equation, we obtain the roots: $a = 4,\ a = -2$.

The partial solutions are:

1. $y_1 = C_1 e^{4x}$
2. $y_2 = C_2 e^{-2x}$

To find a general solution for initial equation, we will combine the partial two:

$$y = C_1 e^{4x} + C_2 e^{-2x}$$

Vectors $\begin{pmatrix} e^{4x} \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ e^{-2x} \end{pmatrix}$ are linearly independent, hence they form a basis of the space of all particular solutions of given differential equation.

Answer

Given solution is a general solution (shown above), and also it is the most general due to linear independency.

Task 5.14

Task statement

Show that the equation $y'' - 6y' - 9y = 0$ has a general solution $y = (C_1 + C_2 x)e^{3x}$. What is the most general solution of this equation?

Solution

The equation is a linear homogeneous second order ordinary differential equation.

Let $y = e^{ax}$, then

$$a^2 e^{ax} - 6ae^{ax} + 9e^{ax} = 0$$
$$a^2 - 6a - 9 = 0$$

Solving this equation, we obtain the root $a = 3$.

We have a mulitple root, hence we should adopt $y_1 = e^{3x}$ as the first partial solution of the equation.

To do it, I will use variable variation to find an independent particular solution $y_2 = zy_1 = ze^{3x}$.

The derivatives of y_2:

$$y_2' = z'e^{3x} + 3ze^{3x}$$
$$y_2'' = z''e^{3x} + 3z'e^{3x} + 3z'e^{3x} + 9ze^{3x} = e^{3x}(z'' + 6z' + 9z)$$

Instantiating $y_2,\ y_2'\ y_2''$ in to the equation we obtain:

$$e^{3x}(z'' + 6z' + 9z) - 6e^{3x}(z' + 3z) + 9ze^{3x} = 0$$
$$z'' + 6z' + 9z - 6z' - 18z + 9z = 0$$
$$z'' = 0$$

Integrating z'' by x we obtain:

$$z' = \int 0dx = C_2$$
$$z = \int C_2 dx = C_2 x + C_1$$

Hence we obtain the solution: $y_2 = (C_1 + C_2 x)e^{3x}$. It contains of two partial solutions: $C_1 e^{3x}$ and $C_2 x e^{3x}$. They are linearly independent, therefore the solution is the most general.

Answer

The solution $y = (C_1 + C_2 x)e^{3x}$ is a general solution (shown above) and it is the most general.

Task 5.15

Task statement

Find a particular solution of the equation $y'' - 4y' + 13y = 2x + 1$.

Solution

The equation is a linear non-homogeneous second order ordinary differential equation.

To find a partial solution, we should find a partial solution in a form of $z = Ax + B$.

$$\begin{aligned} z &= Ax + B \\ z' &= A \\ z'' &= 0 \end{aligned}$$

Instantiating $z,\ z',\ z''$ into the equation, we obtain:

$$-4A + 13Ax + 13B = 2x + 1$$

Since two polynomials are equal iff corresponding coefficients are equal, we have:

$$\begin{cases} 13A = 2 \\ 13B - 4A = 1 \end{cases} \quad \text{or} \begin{cases} A = \frac{2}{13} \\ B = \frac{21}{169} \end{cases}$$

Hence a particular solution for the equation is $y = \frac{2}{13}x + \frac{21}{169}$.

Answer

A particular solution for the equation: $y = \frac{2}{13}x + \frac{21}{169}$.

Task 6.1

Task statement

Is $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = C \begin{pmatrix} e^{-4x} \\ -2e^{-4x} \end{pmatrix} + D \begin{pmatrix} e^{4x} \\ \frac{2}{3}e^{4x} \end{pmatrix}$ on $\mathbf{R}$, where C, D are any real parameters, the most general solution of the system $\begin{cases} y_1' = 2y_1 + 3y_2 \\ y_2' = 4y_1 - 2y_2 \end{cases}$?

Solution

The most general solution given in task 6.5, it is the same as given, but in another form.

Task 6.2

Task statement

Solve system $\begin{cases} y_1' = 6y_1 - y_2 \\ y_2' = y_1 + 4y_2 \end{cases}$

Solution

Let us differentiate the first equation:

$$y_1'' = 6y_1' - y_2'$$

After instantiating the second equation into it, we obtain:

$$y_1'' = 6y_1' - y_1 - 4y_2$$

From the first equation: $y_2 = 6y_1 - y_1'$. Let us instantiate it in the equation above:

$$y_1'' = 10y_1' - 25y_1$$

The equation obtained is a linear homogeneous second order differential equation.

$$\begin{aligned} y_1'' - 10y_1' + 25y_1 &= 0 \\ a^2e^{ax} - 10ae^{ax} + 25e^{ax} &= 0 \\ a^2 - 10a + 25 &= 0 \\ a &= 5 \end{aligned}$$

Hence $y_1 = C_1e^{5x} + C_2xe^{5x}$.
We know that $y_2 = 6y_1 - y_1'$, hence $y_2 = C_1e^{5x} + C_2xe^{5x} - C_2e^{5x}$

Answer

The solution for given system is: $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = C_1 \begin{pmatrix} e^{5x} \\ e^{5x} \end{pmatrix} + C_2 \begin{pmatrix} xe^{5x} \\ -e^{5x} + xe^{5x} \end{pmatrix}$

Task 6.3

Task statement

Solve system $\begin{cases} y_1' = 5y_1 + 2y_2 \\ y_2' = -4y_1 + y_2 \end{cases}$

Solution

Let us differentiate the first equation:

$$y_1'' = 5y_1' + 2y_2'$$

After instantiating the second equation into it, we obtain:

$$y_1'' = 5y_1' - 8y_1 + 2y_2$$

From the first equation: $y_2 = \frac{y_1' - 5y_1}{2}$. Let us instantiate it in the equation above:

$$y_1'' = 6y_1' - 13y_1$$

The equation obtained is a linear homogeneous second order differential equation.

$$\begin{aligned} y_1'' - 6y_1' + 13y_1 &= 0 \\ a^2e^{ax} - 6ae^{ax} + 13e^{ax} &= 0 \\ a^2 - 6a + 13 &= 0 \\ a &= 3 \pm 2i \end{aligned}$$

Hence

$$\begin{aligned} y_1 &= C_1e^{(3+2i)x} + C_2e^{(3-2i)x} \\ &= C_1e^{3x}e^{2ix} + C_2e^{3x}e^{-2ix} \\ &= e^{3x}(C_1\cos 2x + C_1 i\sin 2x + C_2\cos 2x - C_2 i\sin 2x) \\ &= e^{3x}((C_1 + C_2)\cos 2x + i(C_1 - C_2)\sin 2x) \\ &= e^{3x}(D_1\cos 2x + D_2\sin 2x) \end{aligned}$$

We know that $y_2 = \frac{y_1' - 5y_1}{2}$, hence $y_2 = \frac{e^{3x}}{2}(3(D_1\cos 2x + D_2\sin 2x) + 2(D_2\cos 2x - D_1\sin 2x) - 5(D_1\cos 2x + D_2\sin 2x)) = -D_1e^{3x}(\cos 2x + \sin 2x) + D_2(\cos 2x - \sin 2x)$

Answer

The solution for given system is: $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = -D_1e^{3x}\begin{pmatrix} \cos 2x \\ \cos 2x + \sin 2x \end{pmatrix} + D_2e^{3x}\begin{pmatrix} \sin 2x \\ \cos 2x - \sin 2x \end{pmatrix}$

Task 6.5

Task statement

Solve system $y_1' = 2y_1 + 3y_2$ and $y_2' = 4y_1 - 2y_2$ using matrix exponent, eigenvalues and eigenvectors.

Solution

Representation of the system in matrix form:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = e^{\begin{pmatrix} 2 & 3 \\ 4 & -2 \end{pmatrix} x} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

To find matrix exponential, we need to find eigenvalues and eigenvectors:

- Eigenvalues:

$$\begin{aligned} det \begin{pmatrix} 2-\lambda & 3 \\ 4 & -2-\lambda \end{pmatrix} &= 0 \\ (2-\lambda)(-2-\lambda) - 12 &= 0 \\ \lambda^2 - 4 - 12 &= 0 \\ \lambda &= \pm 4 \end{aligned}$$

- Eigenvectors:

  1. $\lambda = 4$:

$$\begin{aligned} \begin{pmatrix} -2 & 3 \\ 4 & -6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= 0 \\ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= \begin{pmatrix} 3 \\ 2 \end{pmatrix} \\ v_1 &= \begin{pmatrix} 3 \\ 2 \end{pmatrix} \end{aligned}$$

  2. $\lambda = -4$:

$$\begin{aligned} \begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= 0 \\ \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} &= \begin{pmatrix} -1 \\ 2 \end{pmatrix} \\ v_2 &= \begin{pmatrix} -1 \\ 2 \end{pmatrix} \end{aligned}$$

- Matrix exponential:

  Let $V = \begin{pmatrix} v_1 & v_2 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix}$

$$
\begin{aligned}
e^{\begin{pmatrix} 2 & 3 \\ 4 & -2 \end{pmatrix} x} &= V e^{\Lambda x} V^{-1} \\
&= \begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix} e^{\begin{pmatrix} 4 & 0 \\ 0 & -4 \end{pmatrix}} \frac{1}{4} \begin{pmatrix} -1 & \frac{3}{2} \\ 1 & \frac{1}{2} \end{pmatrix} \\
&= \frac{1}{4} \begin{pmatrix} e^{-4x} + 3e^{4x} & \frac{3}{2}(e^{4x} - e^{-4x}) \\ 2(e^{4x} - e^{-4x}) & 3e^{-4x} + e^{4x} \end{pmatrix}
\end{aligned}
$$

Hence the solution is $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} e^{-4x} + 3e^{4x} & \frac{3}{2}(e^{4x} - e^{-4x}) \\ 2(e^{4x} - e^{-4x}) & 3e^{-4x} + e^{4x} \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$.
Or:

$$
\begin{aligned}
y_1 &= C_1(e^{-4x} + 3e^{4x}) + C_2(e^{4x} - e^{-4x}) \\
y_2 &= C_1(e^{4x} - e^{-4x}) + C_2(3e^{-4x} + e^{4x})
\end{aligned}
$$

Answer

The solution for the system:

$$
\begin{aligned}
y_1 &= C_1(e^{-4x} + 3e^{4x}) + C_2(e^{4x} - e^{-4x}) = D_1 e^{-4x} + D_2 e^{4x} \\
y_2 &= C_1(e^{4x} - e^{-4x}) + C_2(3e^{-4x} + e^{4x}) = D_3 e^{-4x} + D_4 e^{4x}
\end{aligned}
$$

Task 7.1

Task statement

Show that

1. $L(\sin \omega t) =$ if $s > 0$ then $\frac{\omega}{s^2+\omega^2}$ else undefined
2. $L(\cos \omega t) =$ if $s > 0$ then $\frac{s}{s^2+\omega^2}$ else undefined

Solution

1. $L(\sin \omega t)$

   $$L(\sin \omega t) = \int_0^\infty e^{-st} \sin \omega t dt$$

   First, let us find the indefinite integral:

   $$\begin{aligned}
   \int e^{-st} \sin \omega t dt &= -\frac{e^{-st} \sin \omega t}{s} + \frac{\omega}{s} \int e^{-st} \cos \omega t dt \\
   &= -\frac{e^{-st} \sin \omega t}{s} + \frac{\omega}{s}(-\frac{e^{-st} \cos \omega t}{s} - \frac{\omega}{s} \int e^{-st} \sin \omega t dt) \\
   &= -\frac{e^{-st} \sin \omega t}{s} - \frac{\omega e^{-st} \cos \omega t}{s^2} - \frac{\omega^2}{s^2} \int e^{-st} \sin \omega t dt \\
   s^2 \int e^{-st} \sin \omega t dt &= -s e^{-st} \sin \omega t - \omega e^{-st} - \omega^2 \int e^{-st} \sin \omega t dt \\
   \int e^{-st} \sin \omega t dt &= -\frac{e^{-st}(s \sin \omega t + \omega \cos \omega t)}{s^2 + \omega^2}
   \end{aligned}$$

   Then we will find definite integral (Note that the integral will diverge for $s < 0$ and equal to zero for $s = 0$):

   $$\begin{aligned}
   \int_0^\infty e^{-st} \sin \omega t dt &= -\frac{e^{-st}(s \sin \omega t + \omega \cos \omega t)}{s^2 + \omega^2} \Big|_0^\infty \\
   &= \lim_{l \to \infty} \frac{e^{-sl}(s \sin \omega l + \omega \cos \omega l)}{s^2 + \omega^2} + \frac{e^0(s \sin 0 + \omega \cos 0)}{s^2 + \omega^2} \\
   &= \frac{\omega}{s^2 + \omega^2}
   \end{aligned}$$

   $$L(\cos \omega t) = \int_0^\infty \cos \omega t dt$$

2. $L(\cos \omega t)$

   First, let us find the indefinite integral:

$$
\begin{aligned}
\int \cos\omega t dt &= -\frac{e^{-st}\cos\omega t}{s} - \frac{\omega}{s}\int e^{-st}\sin\omega t dt \\
&= -\frac{e^{-st}\cos\omega t}{s} + \frac{\omega}{s^2}e^{-st}\sin\omega t - \frac{\omega^2}{s^2}\int e^{-st}\cos\omega t dt \\
s^2\int \cos\omega t dt &= se^{-st}\cos\omega t + \omega e^{-st}\sin\omega - \omega^2\int e^{-st}\cos\omega t dt \\
\int \cos\omega t dt &= \frac{e^{-st}(\omega\sin\omega t - s\cos\omega t)}{s^2+\omega^2}
\end{aligned}
$$

Then we will find definite integral (Note that the integral will diverge for $s < 0$ and equal to zero for $s = 0$):

$$
\begin{aligned}
\int_0^\infty \cos\omega t dt &= \frac{e^{-st}(\omega\sin\omega t - s\cos\omega t)}{s^2+\omega^2}\Big|_0^\infty \\
&= \lim_{l\to\infty}\frac{e^{sl}(\omega\sin\omega l - s\cos\omega l)}{s^2+\omega^2} - \frac{e^0\omega\sin 0 - s\cos 0}{s^2+\omega^2} \\
&= \frac{s}{s^2+\omega^2}
\end{aligned}
$$

Answer

Above shown the proofs

Alternative solution

From Euler's identity: $e^{i\omega t} = \cos\omega t + i\sin\omega t$

According to superposition property of Laplace transform: Let $f,\ g:\ [0,\infty] \to \mathbf{R}$ and $\alpha,\ \beta \in \mathbf{R}$; then $L(\alpha f + \beta g) =$ at every point $s \in \mathbf{R}$ where both $L(f)$ and $L(g)$ are defined. (from lecture topic 7, slide 9)

Hence $L(e^{i\omega t} = L(\cos\omega t) + i\sin\omega t)$.

$$
\begin{aligned}
L(e^{i\omega t} &= \int_0^\infty e^{i\omega - s}t)dt \\
&= \frac{1}{i\omega - s}\int_0^\infty e^{i\omega - s}t)d((i\omega - s)t) \\
&= \frac{1}{s - i\omega} \\
&= \frac{s + i\omega}{s^2+\omega^2}
\end{aligned}
$$

Then we will take real part of the transform to obtain transform of cosine and imaginary part to obtain transform of sine:

$$L(\cos \omega t) = Re(L(e^{i\omega t}) = \frac{s}{s^2 + \omega^2}$$
$$L(\sin \omega t) = Im(L(e^{i\omega t}) = \frac{\omega}{s^2 + \omega^2}$$

This approach has two problems:

1. In the slides it is said that $\alpha,\ \beta \in \mathbf{R}$, but here we use it for complex numbers (I don't know why it is like that)

2. We don't know the limits on s

Task 7.2

Task statement

If $n \in \mathbf{N}$ then $L(t^n e^{at}) =$ if $s > a$ then $\frac{n!}{(s-a)^{n+1}}$ else undefined.

Solution

$$\begin{aligned}
L(t^n e^{at}) &= \int_0^\infty t^n e^{(a-s)t} dt \\
&= \frac{t^n e^{(a-s)t}}{a-s}\Big|_0^\infty - \int_0^\infty n t^{n-1} \frac{e^{(a-s)t}}{a-s} dt \\
&= \frac{n}{s-a} \int_0^\infty t^{n-1} e^{(a-s)t}
\end{aligned}$$

Hence $L(t^n e^{at}) = \frac{n}{s-a} L(e^{at} t^{n-1})$

- For $n = 1$:
  $L(t^1 e^{at}) = \frac{1}{s-a} L(e^{at} t^0) = \frac{1}{(s-a)^2}$ (According to the first shift property)
- For $n = 2$:
  $L(t^2 e^{at}) = \frac{2}{s-a} L(e^{at} t^1) = \frac{2}{(s-a)^3}$
- For $n = 3$:
  $L(t^3 e^{at}) = \frac{3}{s-a} L(e^{at} t^2) = \frac{6}{(s-a)^4}$

Hence $L(t^n e^{at}) = \frac{n!}{(s-a)^{n+1}}$ for $s > a$.

Task 7.10

Task statement

Validate the correspondence $(-e^{2t} + \frac{1}{2}e^{5t} + \frac{5}{2}e^{t}) \leftrightarrow \frac{3+(2s-9)(s-2)-12(s-2)}{(s-1)(s-2)(s-5)}$.

Solution

1. Using Laplace transform:

$$\begin{aligned}
L(-2e^{2t} + \frac{e^{5t}}{2} + \frac{5e^{t}}{2}) &= L(-2e^{2t}) + L(\frac{1}{2}e^{5t}) + L(\frac{5}{2}e^{t}) \\
&= -\frac{1}{s-2} + \frac{1}{2(s-5) + \frac{5}{2(s-1)}} \\
&= \frac{-2(s-1)(s-5) + (s-1)(s-2) + 5(s-2)(s-5)}{2(s-2)(s-1)(s-5)} \\
&= \frac{2s^2 - 13s + 24}{(s-1)(s-2)(s-5)}
\end{aligned}$$

Therefore correspondence is wrong

2. Using Inverse Laplace transform:

$$\begin{aligned}
\frac{3+(2s-9)(s-2)-12(s-2)}{(s-1)(s-2)(s-5)} &= \frac{3}{(s-1)(s-2)(s-5)} + \\
&+ \frac{2s-9}{(s-1)(s-5)} - \frac{12}{(s-1)(s-5)}
\end{aligned}$$

$$\frac{3+(2s-9)(s-2)-12(s-2)}{(s-1)(s-2)(s-5)} =$$
$$= \frac{3}{(s-1)(s-2)(s-5)} + \frac{2s-9}{(s-1)(s-5)} - \frac{12}{(s-1)(s-5)}$$

Using partial fraction decomposition:

(a)

$$\frac{A}{s-1} + \frac{B}{s-2} + \frac{C}{s-5} = \frac{3}{(s-1)(s-2)(s-5)}$$

$$A(s^2 - 7s + 10) + B(s^2 - 6s + 5) + C(s^2 - 3s + 2) = 3$$

$$\begin{cases} A + B + C = 0 \\ -7A - 6B - 3C = 0 \\ 10A + 5B + 2C = 3 \end{cases}$$

Hence $A = \frac{3}{4},\ B = -1,\ C = \frac{1}{4}$.

(b)

$$\frac{A}{s-1} + \frac{B}{s-5} = \frac{2s-9}{(s-1)(s-5)}$$

$$A(s-5) + B(s-1) = 2s-9$$

$$\begin{cases} A+B=2 \\ -5A-B=-9 \end{cases}$$

Hence $A = \frac{7}{4},\ B = \frac{1}{4}$.

(c)

$$\frac{A}{s-1} + \frac{B}{s-5} = \frac{12}{(s-1)(s-5)}$$

$$A(s-5) + B(s-1) = 12$$

$$\begin{cases} A+B=0 \\ -5A-B=12 \end{cases}$$

Hence $A = -3,\ B = 3$.

After decomposition we obtain:

$$F(S) = \frac{3}{4(s-1)} - \frac{1}{s-2} + \frac{1}{4(s-5)} + \frac{7}{4(s-1)} + \frac{1}{4(s-5)} + \frac{3}{s-1} - \frac{3}{s-5}$$

Applying Inverse Laplace transform:

$$f(t) = \frac{3}{4}e^t - e^{2t} + \frac{1}{4}e^{5t} + \frac{7}{4}e^t + \frac{1}{4}e^{5t} + 3e^t - 3e^{5t}$$

$$f(t) = \frac{22}{4}e^t - e^{2t} - \frac{5}{2}e^{5t}$$

As we can see, in both scenarios the correspondence is wrong

Answer

The correspondence is wrong