This report does not contain all images, more of them can be found in the file trajectory.ipynb

Task 1

Task statement

Calculate and plot position, velocity, and acceleration trajectories of driving your robot model from configuration q_0 to configuration q_f in joint space.

Solution

Firstly, to calculate trajectory, we need to find minimum time to reach desired point:

Blending time can be found as:

$$t_b = \frac{\dot{q}}{\ddot{q}}$$

where $\dot{q}$ is maximum velocity and $\ddot{q}$ is maximum acceleration

Distance travelled during blending:

$$q_b = \frac{\dot{q}^2}{\ddot{q}}$$

Distance travelled during non-acceleration phase:

$$q_{na} = |\Delta q| - q_b$$

where $\Delta q = q_f - q_0$

Time of non-acceleration phase:

$$t_{na} = \frac{q_{na}}{\dot{q}}$$

Then we should check, if t_{na} is negative (or q_{na}), then we have enough time to reach the desired point without non-acceleration phase, hence we should use triangular profile, otherwise we should use trapezoidal trajectory

For triangular trajectory:

For this type of trajectory we apply $t_b = \sqrt{\frac{|\Delta q|}{\ddot{q}}}$

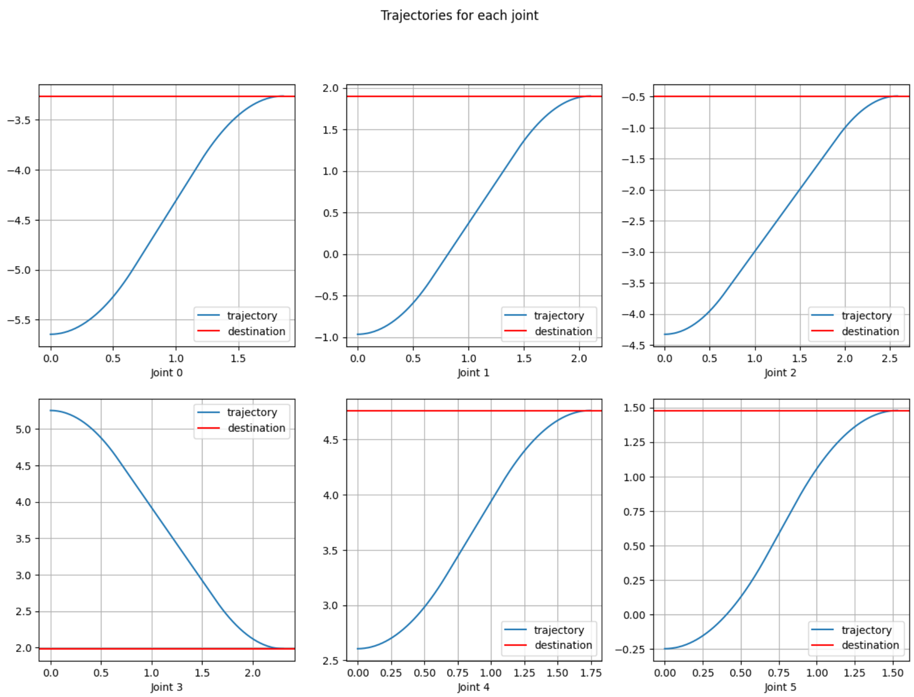
$$q(t) = \begin{cases} q_0 + sign(\Delta q)\frac{\ddot{q}t^2}{2}, \ 0 < t \leq t_b \\ q_0 + sign(\Delta q)(\frac{\ddot{q}t_b^2}{2} + \ddot{q}t_b(t - t_b) - \frac{\ddot{q}(t-t_b)^2}{2}), \ t_b < t \leq t_f \end{cases}$$

where $t_f = 2t_b$

For trapezoidal trajectory:

$$q(t) = \begin{cases} q_0 + sign(\Delta q)\frac{\ddot{q}t^2}{2}, \ 0 < t \leq t_b \\ q_0 + sign(\Delta q)(\frac{\ddot{q}t_b^2}{2} + \dot{q}(t - t_b)), \ t_b < t \leq t_b + t_{na} \\ q_0 + sign(\Delta q)(\frac{\ddot{q}t_b^2}{2} + \dot{q}t_{na} - \frac{\ddot{q}(t - t_b - t_{na})^2}{2}), \ t_b + t_{na} < t \leq t_f \end{cases}$$

where $t_f = 2t_b + t_{na}$



Task 2

Task statement

Synchronize your 6 joints to start and end motion at the same time.

Solution

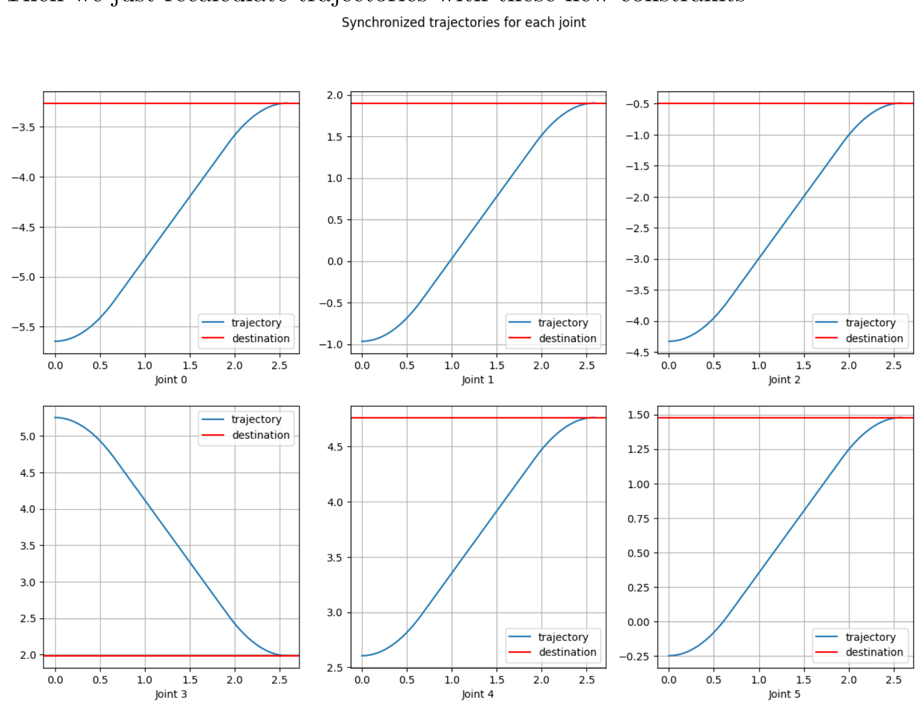
To synchronize the joints, we should find maximum t_b and t_{na} (for triangular ones I set $t_{na} = 0$), then $t_{b_{max}}, t_{na_{max}}$ are the maximum time spend on blend phase and non-acceleration phase.

Then we define new constraints on velocity and acceleration for each joint:

$$\dot{q}_i = \frac{\Delta q_i}{t_{b_{max}} + t_{na_{max}}}$$

$$\ddot{q}_i = \frac{\dot{q}_i}{t_{b_{max}}}$$

Then we just recalculate trajectories with these new constraints



Task 3

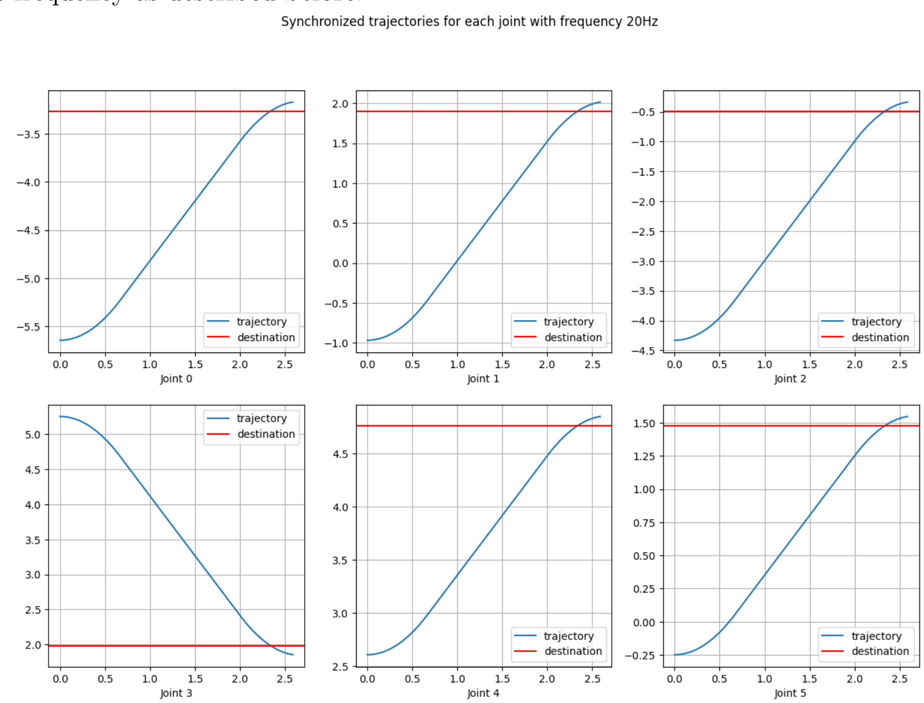
Task statement

Consider you have controller frequency of 20 Hz

Solution

Let $\phi = 20\ Hz$ is the frequency.

Now we should redefine $t_b = \frac{\lceil t_b \phi \rceil}{\phi}$ and $t_b = \frac{\lceil t_{na} \phi \rceil}{\phi}$, now they are satisfy the frequency of the controller. Then we can find the trajectory taking to account the frequency as described before.



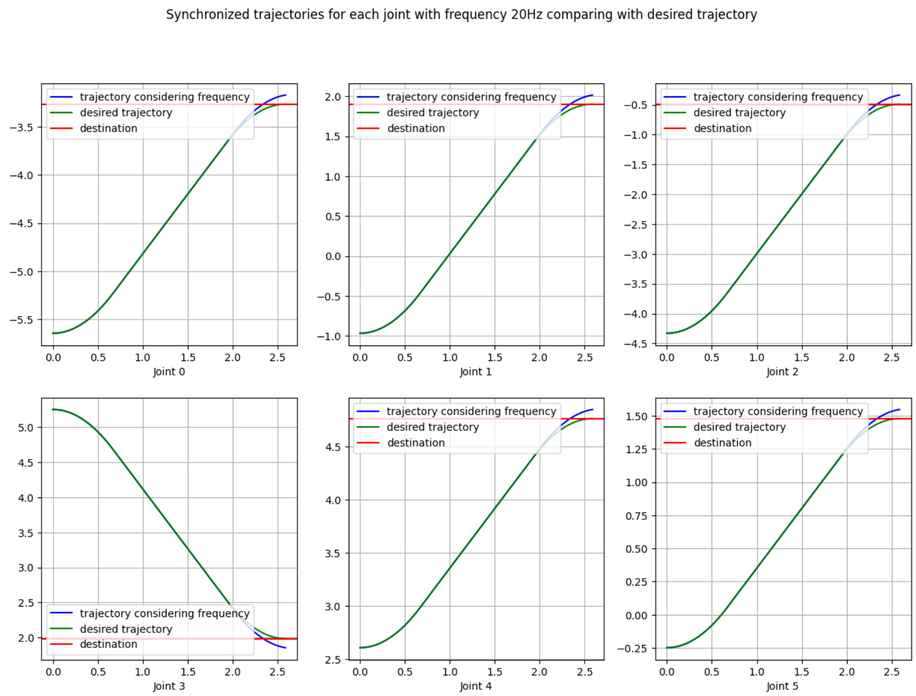
Task 4

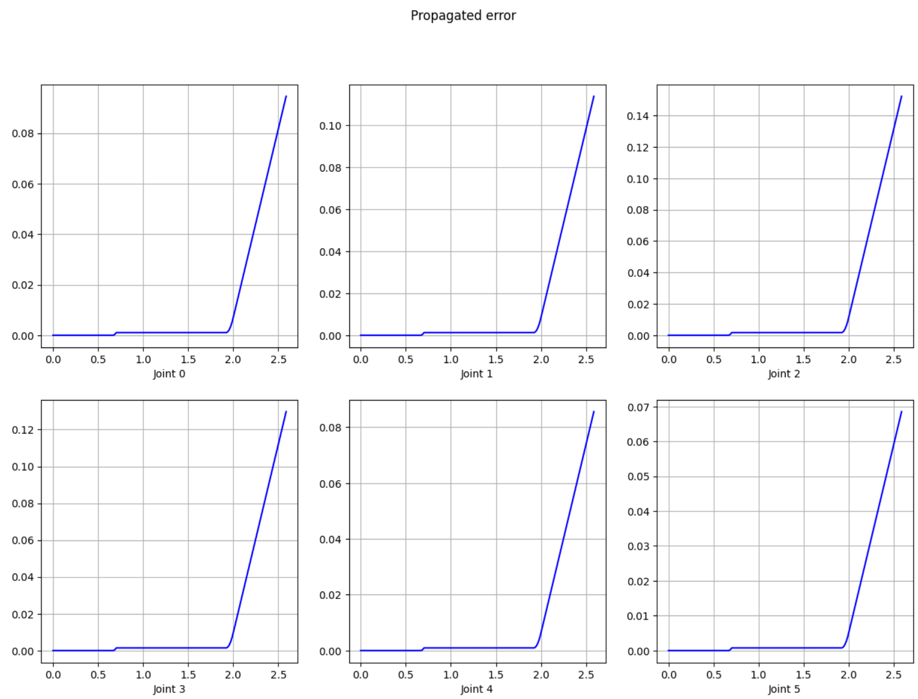
Task statement

Calculate propagated error in end-effector position.

Solution

We can introduce a new variable $e = q_{actual} - q_{desired}$.




Propagated error
Joint 0
Joint 1
Joint 2
Joint 3
Joint 4
Joint 5

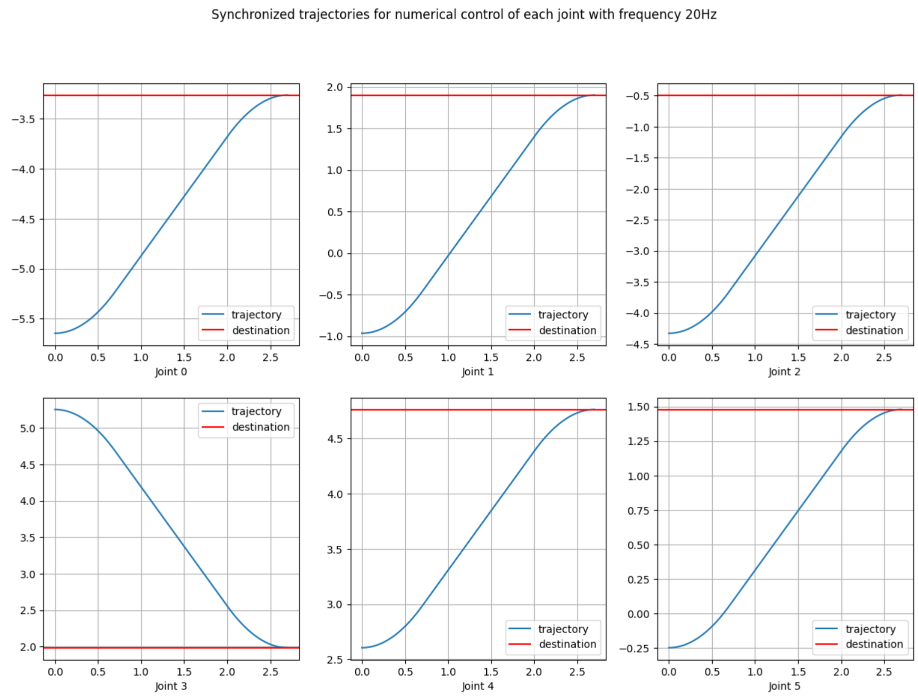
Task 5

Task statement

Redefine synchronized trajectories for numerical control.

Solution

Here we should repeat procedure described in Task 2 but for time from Task 3



Task 6

Task statement

Drive your robot model between 2 consequent points. (Solve polynomial)

Solution

Here I used quintic polynomial in the following form:

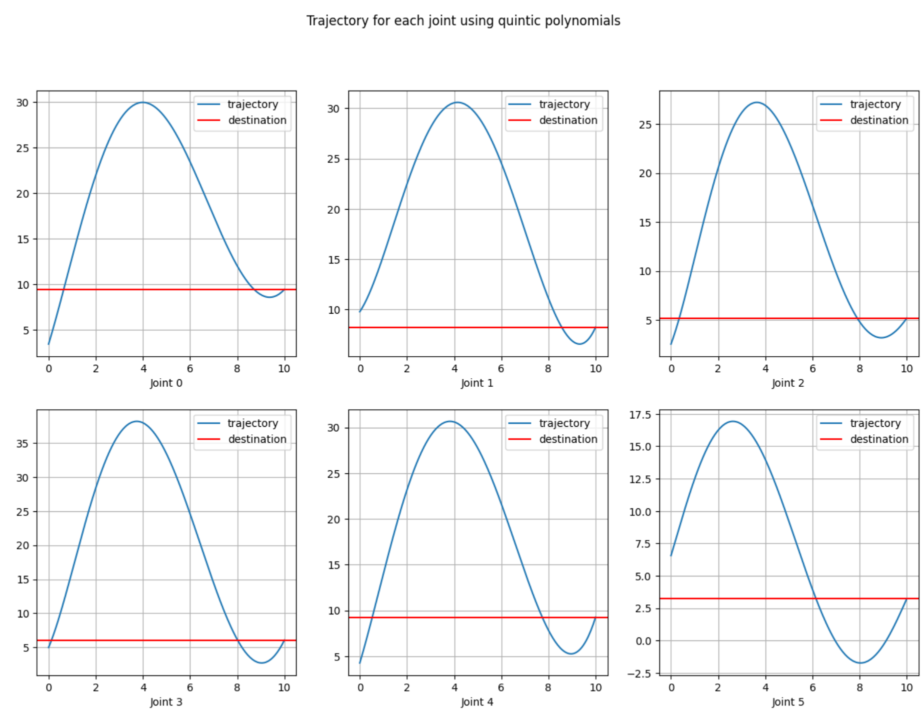
$$q(t) = a_5 t^5 + a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t_1 + a_0$$

To find coefficients of the polynomial, I solved the following linear system of equations:

$$\begin{pmatrix} t_0^5 & t_0^4 & t_0^3 & t_0^2 & t_0 & 1 \\ t_f^5 & t_f^4 & t_f^3 & t_f^2 & t_f & 1 \\ 5t_0^4 & 4t_0^3 & 3t_0^2 & 2t_0 & 1 & 0 \\ 5t_f^4 & 4t_f^3 & 3t_f^2 & 2t_f & 1 & 0 \\ 20t_0^3 & 12t_0^2 & 6t_0 & 2 & 0 & 0 \\ 20t_f^3 & 12t_f^2 & 6t_f & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{pmatrix} = \begin{pmatrix} q_0 \\ q_f \\ \dot{q}_0 \\ \dot{q}_f \\ \ddot{q}_0 \\ \ddot{q}_f \end{pmatrix}$$

Then we can calculate positions for each moment of time:

$$q(t) = \begin{pmatrix} t_0^5 & t_0^4 & t_0^3 & t_0^2 & t_0 & 1 \end{pmatrix} \begin{pmatrix} a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{pmatrix}$$


Trajectory for each joint using quintic polynomials
trajectory
destination
Joint 0
Joint 1
Joint 2
Joint 3
Joint 4
Joint 5