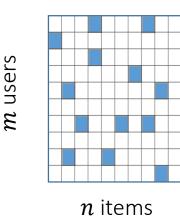
ACA ML 2018 Workshop on Recommender Systems

Part 3

Recap of the 2st part

A general view on matrix factorization

utility matrix A



Incomplete data:

known entriesunknown entries

Task: find utility (or relevance) function f_U such that:

 f_R : Users × Items \rightarrow Relevance score

As optimization problem with some loss function \mathcal{L} :

$$\mathcal{L}(A,R) \to \min$$

Any factorization model consists of:

- Utility function to generate *R*
- ullet Optimization objective defined by ${\cal L}$
- Optimization method (algorithm)

Simplistic view: latent features ↔ genres

$$R = PQ^T$$
Sci-fi
Action
Drama
Comedy

ALS vs SGD vs SVD

ALS SGD

More stable Sensitive to hyper-parameters

Fewer hyper-parameters to tune Requires special treatment of learning rate

Higher complexity, however requires fewer iterations Lower complexity, but slower convergence

Embarrassingly parallel Inherently sequential (parallelization is tricky for RecSys)

Higher communication cost in distributed environment For binary feedback complexity changes: $nnz \rightarrow MN$

Unlike SVD:

More involved model selection (no rank truncation).

No global convergence guarantees!

Asynchronous SGD is non-deterministic.

Allow for custom optimization objectives. $\mathcal{L}(A,R) \to \mathcal{L}(f(A,R))$

For explicit feedback:

Algorithm	Overall complexity	Update complexity	Sensitivity	Optimality
SVD*	$O(nnz_A \cdot r + (M+N)r^2)$	$O(nnz_a \cdot r)$	Stable	Global
ALS	$O(nnz_A \cdot r^2 + (M+N)r^3)$	$O\left(nnz_a\cdot r + r^3\right)$	Stable	Local
CD	$O(nnz_A \cdot r)$	$O(nnz_a \cdot r)$	Stable	Local
SGD	$O(nnz_A \cdot r)$	$O(nnz_a \cdot r)$	Sensitive	Local

^{*} For both standard and randomized implementations [71].

Task

- You have a binary utility matrix (with "true" zeros) resulted from some implicit feedback information.
 - What will be the complexity of SGD?
 - What will be the SGD-based solution if you omit zero values?
 - Is it reasonable to use bias terms?

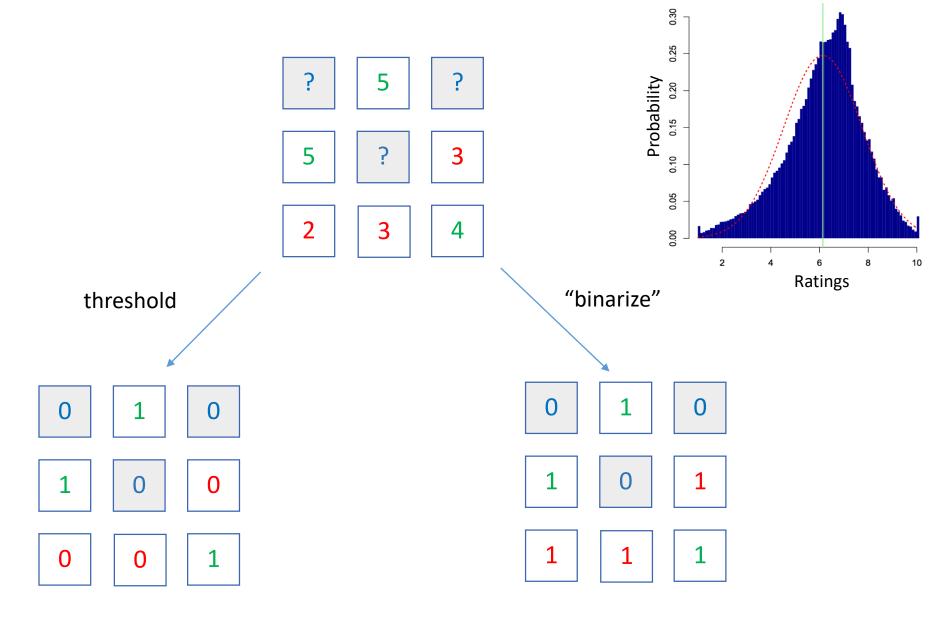
What are your solutions to 2nd home assignment?

Part 3

Today's lecture:

- Advanced matrix factorization techniques
 - Mixed implicit and explicit feedback models (NSVD, SVD++)
 - Bilinear models: SVDFeature and Factorization Machines. Hybrid models.
- What can be optimized
- Tensor factorization

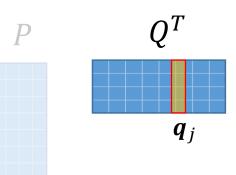
Explicit to implicit



NSVD

Key idea: user is represented as a combination of items

A. Paterek, "Improving regularized singular value decomposition for collaborative filtering", 2007.



$$r_{ij} = \left(\sum_{k \in S^{(i)}} \boldsymbol{q}_k^T\right) \boldsymbol{q}_j$$

omitting bias terms

$$S^{(i)} = \{j: w_{ij} \neq 0\}$$

indices of items, rated by user i ;
forms a "neighborhood" of items

- helpful in the case of extreme sparsity
- reduced storage requirements
- prone to overfitting

What is the corresponding matrix form?

Hint: you are given the matrix of (implicit) interactions S.

$$R = SQQ^T$$

Does it look familiar to you?

SVD:
$$R = A_0 V V^T$$

SVD++

Key idea: user is described by implicit and explicit interactions

Y. Koren, "Factorization meets the neighborhood: a multifaceted collaborative filtering model", 2008.

 $\begin{aligned} \boldsymbol{p}_{\mathrm{i}} \leftarrow \boldsymbol{p}_{\mathrm{i}} + \frac{1}{|S^{(i)}|} \sum_{k \in S^{(i)}} \overline{\boldsymbol{q}}_{k} \\ \text{Explicit part} & \text{Implicit part} \end{aligned}$

$$r_{ij} = \left(\boldsymbol{p}_{i} + \frac{1}{|S^{(i)}|} \sum_{k \in S^{(i)}} \overline{\boldsymbol{q}}_{k}\right)^{T} \boldsymbol{q}_{j}$$

omitting bias terms

$$R = (P + S_{\rm n}\bar{Q})Q^T$$

binary
$$S_{\mathrm{n}} = D^{-1}S$$

$$D = \mathrm{diag}\{\|\boldsymbol{s}_1\|^2, ..., \|\boldsymbol{s}_M\|^2\}$$

$$P + S_{\rm n}^{(1)} \bar{Q}^{(1)} + S_{\rm n}^{(2)} \bar{Q}^{(2)} + \cdots$$

independent latent spaces!

More general representation

$$R = (P + S_n \bar{Q})Q^T = [I S_n] \begin{bmatrix} P \\ \bar{Q} \end{bmatrix} \bar{P}$$
 $R = (X\bar{P})Q^T$

X - is a "design" matrix

Matrix *X* encodes implicit information. It can be extended to incorporate any side information.

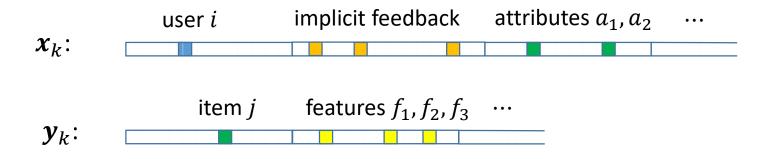


SVDFeature

T. Chen, et al. "Feature-based matrix factorization", 2011

$$R = (XP)(YQ)^T$$

$$X = [X_1 \ X_2 \ ... \ X_m]$$
 $Y = [Y_1 \ Y_2 \ ... \ Y_n]$



Including bias terms:

$$r_{ij} = b_0 + \boldsymbol{t}^T \boldsymbol{x}_i + \boldsymbol{f}^T \boldsymbol{y}_j + \boldsymbol{x}_i^T P Q \boldsymbol{y}_j$$

Optimized with ALS, SGD.

$$b_0 = \sum_{g \in G} \gamma_g \mu_g$$
 is precomputed!

Model parameters: $\Theta = \{t, f, P, Q\}$

Factorization Machines

Idea: polynomial expansion

S. Rendle, "Factorization machines", 2010.

$$f(\mathbf{x}) = b_0 + \mathbf{b}^T \mathbf{x} + \mathbf{x}^T H \mathbf{x} + \cdots$$

Feature vector x										T	arg	et y											
X ⁽¹⁾	1	0	0		1	0	0	0		0.3	0.3	0.3	0		13	0	0	0	0			5	y ⁽¹⁾
X ⁽²⁾	1	0	0		0	1	0	0		0.3	0.3	0.3	0		14	1	0	0	0			3	y ⁽²⁾
X ⁽³⁾	1	0	0		0	0	1	0		0.3	0.3	0.3	0		16	0	1	0	0			1	y ⁽²⁾
X ⁽⁴⁾	0	1	0		0	0	1	0		0	0	0.5	0.5		5	0	0	0	0			4	y ⁽³⁾
X ⁽⁵⁾	0	1	0		0	0	0	1		0	0	0.5	0.5		8	0	0	1	0			5	y ⁽⁴⁾
X ⁽⁶⁾	0	0	1		1	0	0	0		0.5	0	0.5	0		9	0	0	0	0			1	y ⁽⁵⁾
X ⁽⁷⁾	0	0	1		0	0	1	0		0.5	0	0.5	0		12	1	0	0	0			5	y ⁽⁶⁾
	A	B Us	C er		TI		SW Movie	ST		TI Otł	NH ner M	SW lovie	ST s rate	ed	Time	ال	NH ₋ast l	SW Movie		 ed			

Factorization Machines

$$r_{ij} = b_0 + \mathbf{t}^T \mathbf{x}_i + \mathbf{f}^T \mathbf{y}_j + \mathbf{x}_i^T P Q \mathbf{y}_j$$
 $\mathbf{b} = \begin{bmatrix} \mathbf{t} \\ \mathbf{f} \end{bmatrix} \quad \mathbf{z} = \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} \quad \text{for some fixed } i \text{ and } j$

$$r(\mathbf{z}) = b_0 + \mathbf{b}^T \mathbf{z} + \mathbf{z}^T H \mathbf{z} + \cdots$$

characterizes intra- and inter-relations between any encoded entities

Data is sparse \rightarrow impose low-rank structure on H

H is symmetric positive semi-definite

$$H = VV^T$$

V embeds all users, items and their side information

user i

2nd order FM:

$$r(\mathbf{z}) = b_0 + \sum_{k} b_k z_k + \sum_{k} \sum_{k'=k+1} \langle \mathbf{v}_k, \mathbf{v}_{k'} \rangle z_k z_{k'}$$

 $\langle \cdot , \cdot \rangle$ is a scalar product

Model parameters: $\Theta = \{b_0, \mathbf{z}, V\}$

 \mathbf{z}_k :

item j features f_1, f_2, f_3

Example of Factorization Machines computation

$$oldsymbol{z} = egin{bmatrix} oldsymbol{x} \\ oldsymbol{y} \\ oldsymbol{f} \end{bmatrix} \qquad V = egin{bmatrix} V_x \\ V_y \\ V_f \end{bmatrix} \qquad V \in \mathbb{R}^{n \times r} \qquad oldsymbol{z} \in \mathbb{R}^n \colon \begin{subarray}{c} & \text{user } i & \text{item } j & \text{features } f_1, f_2, f_3 \\ \hline & V \in \mathbb{R}^{n \times r} & oldsymbol{z} \in \mathbb{R}^n \colon \begin{subarray}{c} & \text{user } i & \text{item } j & \text{features } f_1, f_2, f_3 \\ \hline & V \in \mathbb{R}^{n \times r} & oldsymbol{z} \in \mathbb{R}^n \colon \begin{subarray}{c} & \text{user } i & \text{item } j & \text{features } f_1, f_2, f_3 \\ \hline & V \in \mathbb{R}^{n \times r} & oldsymbol{z} \in \mathbb{R}^n \colon \begin{subarray}{c} & \text{user } i & \text{item } j & \text{features } f_1, f_2, f_3 \\ \hline & V \in \mathbb{R}^n \colon \begin{subarray}{c} & \text{user } i & \text{item } j & \text{features } f_1, f_2, f_3 \\ \hline & V \in \mathbb{R}^n \colon \begin{subarray}{c} & \text{user } i & \text{item } j & \text{features } f_1, f_2, f_3 \\ \hline & V \in \mathbb{R}^n \colon \begin{subarray}{c} & \text{user } i & \text{item } j & \text{features } f_1, f_2, f_3 \\ \hline & V \in \mathbb{R}^n \colon \begin{subarray}{c} & \text{user } i & \text{item } j & \text{features } f_1, f_2, f_3 \\ \hline & V \in \mathbb{R}^n \colon \begin{subarray}{c} & \text{item } j & \text{features } f_1, f_2, f_3 \\ \hline & V \in \mathbb{R}^n \colon \begin{subarray}{c} & \text{item } j & \text{features } f_1, f_2, f_3 \\ \hline & V \in \mathbb{R}^n \colon \begin{subarray}{c} & \text{item } j & \text{features } f_1, f_2, f_3 \\ \hline & V \in \mathbb{R}^n \colon \begin{subarray}{c} & \text{item } j & \text{features } f_1, f_2, f_3 \\ \hline & V \in \mathbb{R}^n \colon \begin{subarray}{c} & \text{item } j & \text{features } f_1, f_2, f_3 \\ \hline & V \in \mathbb{R}^n \colon \begin{subarray}{c} & \text{item } j & \text{features } f_1, f_2, f_3 \\ \hline & V \in \mathbb{R}^n \colon \begin{subarray}{c} & \text{item } j & \text{features } f_1, f_2, f_3 \\ \hline & V \in \mathbb{R}^n \end{split}$$

$$\mathbf{z}^{T}VV^{T}\mathbf{z} = \left(\begin{bmatrix} \mathbf{x}^{T} & \mathbf{y}^{T} & \mathbf{f}^{T} \end{bmatrix} \begin{bmatrix} V_{x} \\ V_{y} \\ V_{f} \end{bmatrix} \right) \left(\begin{bmatrix} V_{x}^{T} & V_{y}^{T} & V_{f}^{T} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{f} \end{bmatrix} \right) = \left(\mathbf{x}^{T}V_{x} + \mathbf{y}^{T}V_{y} + \mathbf{f}^{T}V_{f}\right) \left(\mathbf{x}^{T}V_{x} + \mathbf{y}^{T}V_{y} + \mathbf{f}^{T}V_{f}\right)^{T} = \mathbf{y}^{T}V_{y} + \mathbf{y}^{T$$

"self-interaction" terms

actual 2nd order FM model r(z) (w/o biases)

$$= x^{T} V_{x} V_{x}^{T} x + y^{T} V_{y} V_{y}^{T} y + f^{T} V_{f} V_{f}^{T} f + 2 \left(x^{T} V_{x} V_{y}^{T} y + x^{T} V_{x} V_{f}^{T} f + y^{T} V_{y} V_{f}^{T} f \right)$$
user-item
user-feature
interactions

$$\boldsymbol{v}_{x} = V_{x}^{T} \boldsymbol{x}, \ \boldsymbol{v}_{y} = ... \qquad r(\boldsymbol{z}) = \frac{1}{2}$$

$$v_x = V_x^T x$$
, $v_y = ...$ $r(z) = \frac{1}{2} [(v_x + v_y + v_f)^T (v_x + v_y + v_f) - (||v_x||^2 + ||v_y||^2 + ||v_f||^2)] = 0$

$$=\frac{1}{2}\sum_{l=1}^{r}\left(\left(\sum_{k=1}^{n}v_{kl}z_{k}\right)^{2}-\sum_{k=1}^{n}(v_{kl}z_{k})^{2}\right)$$
 reduces the number of operations

Connection of FM to other models

$$r(\mathbf{z}) = b_0 + \sum_{k} b_k z_k + \sum_{k} \sum_{k'=k+1} \frac{\delta_{kk'} \langle \mathbf{v}_k, \mathbf{v}_{k'} \rangle z_k z_{k'}}{}$$

binary indicator variable, denotes "allowed" interactions

Example:

- restrict interactions to (user, item) pairs only
- forbid intra-relations between entities of the same type
- remove side information

What is the resulting model?

Matrix form of FM

$$\mathbf{z} = \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ f \end{bmatrix}$$
 $V = \begin{bmatrix} V_x \\ V_y \\ V_f \end{bmatrix}$ $V \in \mathbb{R}^{n \times r}$ $\mathbf{z} \in \mathbb{R}^n$: user i item j features f_1, f_2, f_3

$$r(\mathbf{z}) = \frac{1}{2} \mathbf{z}^T \left(\begin{bmatrix} V_x \\ V_y \\ V_f \end{bmatrix} \begin{bmatrix} V_x^T & V_y^T & V_f^T \end{bmatrix} - \begin{bmatrix} V_x & 0 \\ V_y & V_f \end{bmatrix} \begin{bmatrix} V_x & 0 \\ 0 & V_f \end{bmatrix} \begin{bmatrix} V_x & 0 \\ 0 & V_f \end{bmatrix}^T \right) \mathbf{z} = \mathbf{z}^T \left(\begin{bmatrix} V_x \\ V_y \\ V_f \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} \mathbf{z} = \mathbf{z}^T \left(\begin{bmatrix} V_x \\ V_y \\ V_f \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} \mathbf{z} \right) \mathbf{z} = \mathbf{z}^T \left(\begin{bmatrix} V_x \\ V_y \\ V_f \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} \mathbf{z} \right)$$

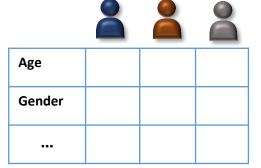
$$= \frac{1}{2} \mathbf{z}^T \begin{bmatrix} 0 & V_{\mathcal{X}} V_{\mathcal{Y}}^T & V_{\mathcal{X}} V_f^T \\ V_{\mathcal{Y}} V_{\mathcal{X}}^T & 0 & V_{\mathcal{Y}} V_f^T \\ V_{\mathcal{Y}} V_{\mathcal{X}}^T & V_{\mathcal{Y}} V_{\mathcal{Y}}^T & 0 \end{bmatrix} \mathbf{z} = \mathbf{z}^T H \mathbf{z}$$

 $V_x \rightarrow P$, $V_y \rightarrow Q$ – conventional matrix factorization

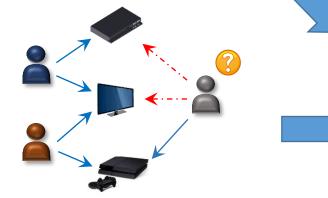
$$H = \frac{1}{2} \begin{bmatrix} 0 & V_{x}V_{y}^{T} & V_{x}V_{f}^{T} \\ V_{y}V_{x}^{T} & 0 & V_{y}V_{f}^{T} \\ V_{f}V_{x}^{T} & V_{f}V_{y}^{T} & 0 \end{bmatrix}$$

Hybrid approach

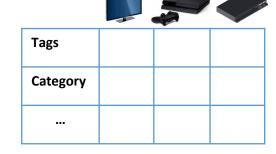
user side information



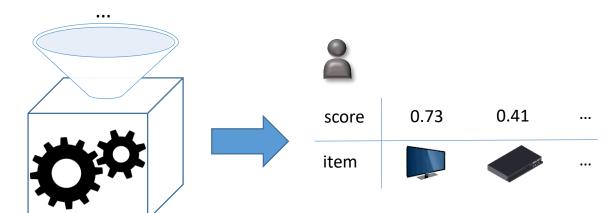
consumption history



item side information



Content-based
Collaborative Filtering
Knowledge-based



hybrid recommendation engine

Taxonomy

Ensemble design

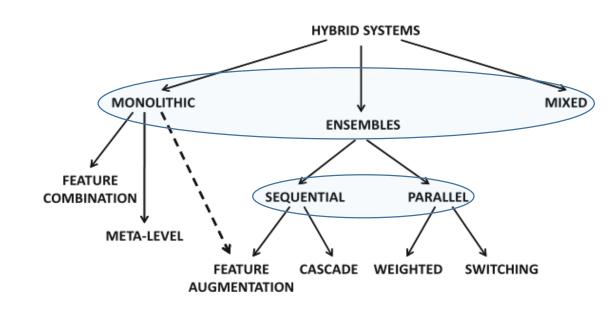
combines several "black-box" algorithms to produce a single more robust output

Monolithic design

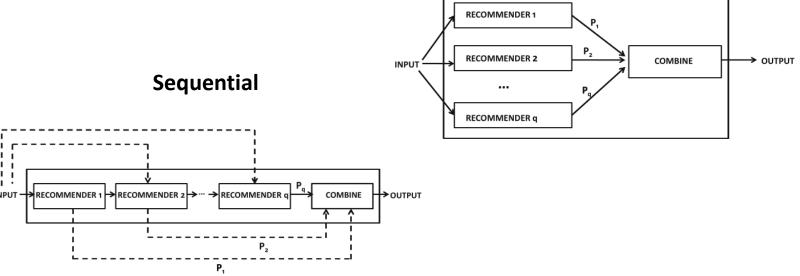
obtained by integration of heterogeneous data sources and customization of models

Mixed systems

present outputs of several recommender models simultaneously



Parallel



Taxonomy

Ensembles

Weighted

combines outputs of several models with varying degree of importance

Switching

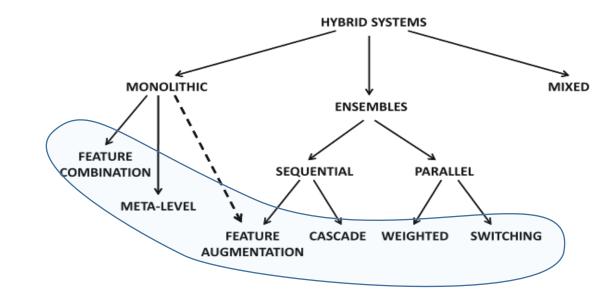
situational choice of the most suitable recommendation model

Cascade

every consequent model tries to improve upon the results of the previous models

Feature Augmentation

one model generates new features or scores and feeds them into another model (stacking)



Monolithic

- Feature Combination
 - several data sources are fed into a single model
- Meta-level

several models are glued together

no "off-the-shelf" solution

Task

Give an example of a feature combination approach

Meta-level system example

Rating matrix

	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}	I_{11}	I_{12}
U_1	5	3	5	4	1	1	ı	3	-	5	-	-
U_2	3	-	-	-	4	5	1	-	5	-	-	1
U_3	1	-	5	4	5	-	5	-	-	3	5	-

(a)

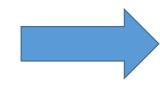
Test user

	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}	I_{11}	I_{12}
U_4	5	-	1	-	-	4	-	ı	3	ı	ı	5

(b)

Item features

	f_1	f_2	f_3	f_4
I_1	1	1	0	0
I_2	1	0	0	0
I_3	1	0	1	1
I_4	1	0	0	1
I_5	0	1	1	0
I_6	0	1	0	0
I_7	0	0	1	1
I_8	0	0	0	1
I_9	0	1	1	0
I_{10}	0	0	0	1
I_{11}	0	0	1	1
I_{12}	0	1	0	0



User profiles

	f_1	f_2	f_3	f_4
U_1	4	1	1	4
U_2	1	4	2	0
U_3	2	1	4	5

	f_1	f_2	f_3	f_4
U_4	1	4	1	0



Model: SVD over profiles

Prediction: folding-in + neighborhood

(c)

Some weighted hybrids

Randomness Injection
 Many matrix factorization methods are inherently randomized due to initialization

Single data, several models → single model, several data samples

- Bagging
 - Row-wise bootstrapping
 - Entry-wise bagging
- Subsampling
 - Row-wise subsampling
 - Entry-wise subsampling

Not storage-efficient



Read more:

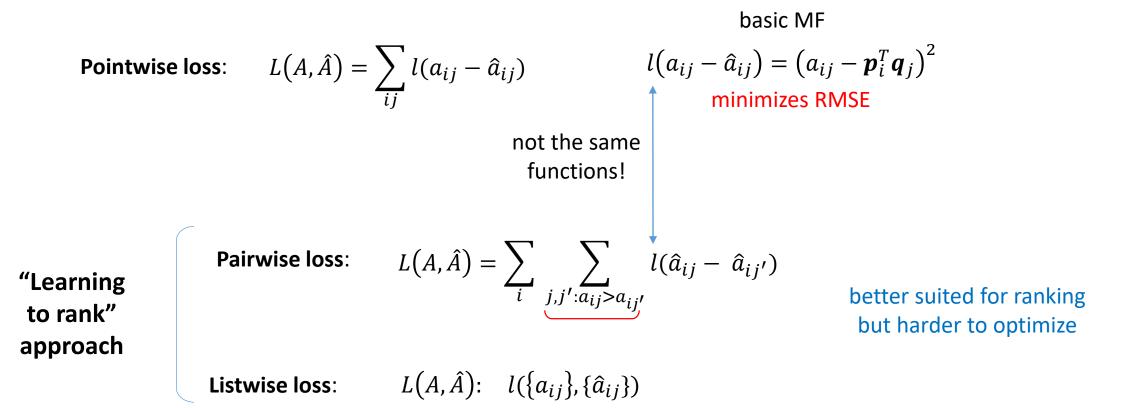
Charu C. Aggarwal. "Recommender Systems. The Textbook", 2016.

R. Burke. Hybrid recommender systems: Survey and experiments. User Modeling and User-adapted Interaction, 12(4), pp. 331–370, 2002.

Other factorization methods

- NMF
- MMF
- PMF
- CAMF
- SLIM
- FISM
- CliMF
- OrdRec
- CobaFi
- ...

Remark on optimization objectives



Bayesian personalized ranking (BPR)

Key idea − 2 step: 1. get predicted ratings (from any model)

2. try to improve ranking with a posteriori modification (max likelihood)

$$p(\Theta|>_u) \propto p(>_u|\Theta) p(\Theta)$$

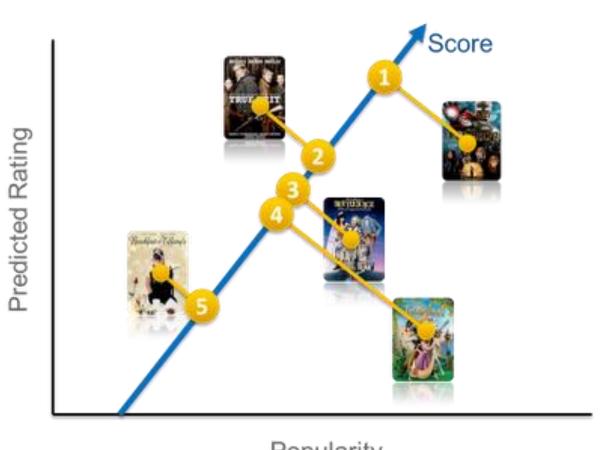
$$p(i >_u j | \Theta) := \sigma(\hat{x}_{uij}(\Theta))$$

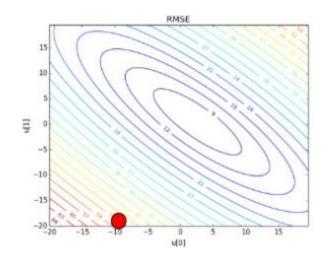
$$\hat{x}_{uij} := \hat{x}_{ui} - \hat{x}_{uj}$$
 x_{ui}, x_{uj} are from some Collaborative Filtering model

pair-wise objective:
$$\sum_{(u,i,j)\in D_S} \ln \sigma(\hat{x}_{uij}) - \lambda_{\Theta} ||\Theta||^2$$

Optimization with stochastic gradient descent.

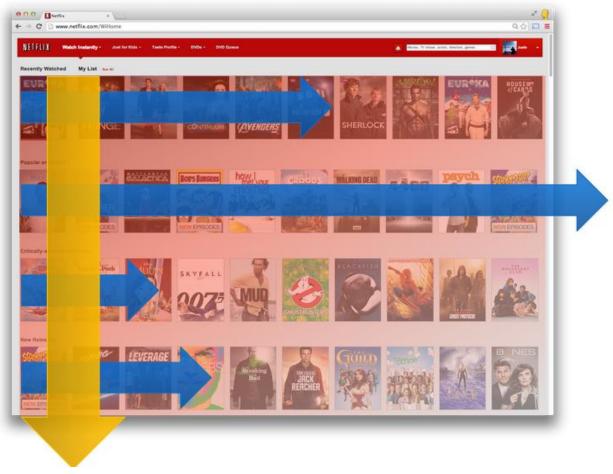
Learning to rank – simple approach





Popularity

User Experience



Reading: How important is the user interface for a recommender system?

https://www.quora.com/How-important-is-the-user-interface-for-a-recommender-system/answer/Abhinav-Sharma?srid=cgo&share=e3430560

Image source: http://technocalifornia.blogspot.ru/2013/07/recommendations-as-personalized.html#!/2014/12/ten-lessons-learned-from-building-real.html

Lessons learned from building real-life recommender systems

http://www.slideshare.net/xamat/recsys-2016-tutorial-lessons-learned-from-building-reallife-recommender-systems

Tensor factorization

Guess standard algorithm behavior

What is likely to be recommended in this case?

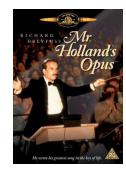


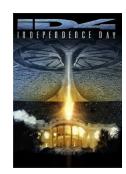












Explanation

predicted scores (folding-in):

$$r \approx VV^T p$$

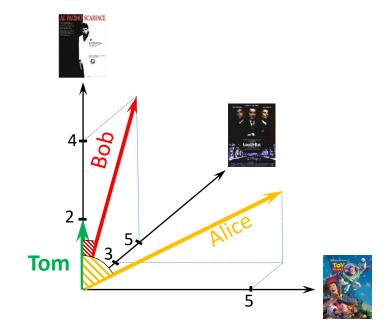
top-*n* recommendations task:

$$toprec(\boldsymbol{p},n) \coloneqq arg \max^{n} \boldsymbol{r}$$

$$\arg\max VV^T(0,...,0,\frac{\mathbf{2}}{\mathbf{2}},0,...,0)^T \equiv \arg\max VV^T(0,...,0,\frac{\mathbf{5}}{\mathbf{5}},0,...,0)^T$$

Rating value doesn't change ranking of the items!

Same for naïve similarity-based model:



	AL PACINO SCARFACE		Condition
Alice	2	5	3
Bob	4		5
Carol	2	5	
Tom	2	???	???
		Predi	ction
		2.6	3.1

Explicit feedback









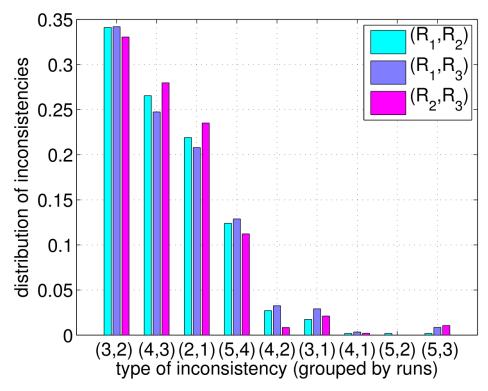
From neoclassical economics: utility is an **ordinal concept**.

Explicit feedback

Traditional recommender models treat ratings as **cardinal numbers**.



Ratings scale consist of **unequal intervals***:



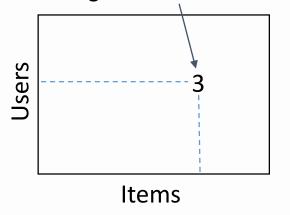
^{*}X. Amatriain, J. M. Pujol, and N. Oliver "I like it... I like it not: Evaluating User Ratings Noise in Recommender Systems", UMAP '09 Proceedings of the 17th International Conference on User Modeling, Adaptation, and Personalization

Restating the problem

Standard model

 $User \times Item \rightarrow Rating$

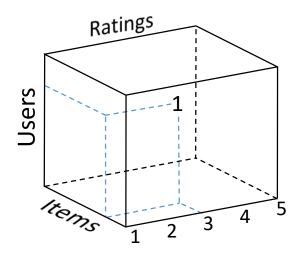
ratings are **cardinal** values



Technique: Matrix factorization

Collaborative Full Feedback model CoFFee (proposed approach)

 $User \times Item \times Rating \rightarrow Relevance Score$



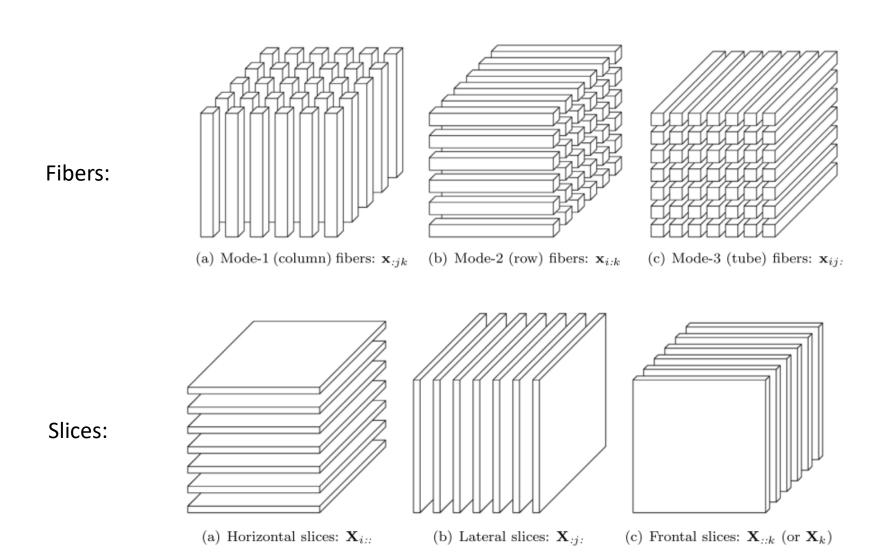
Technique: Tensor Factorization

based on Tucker Decomposition*

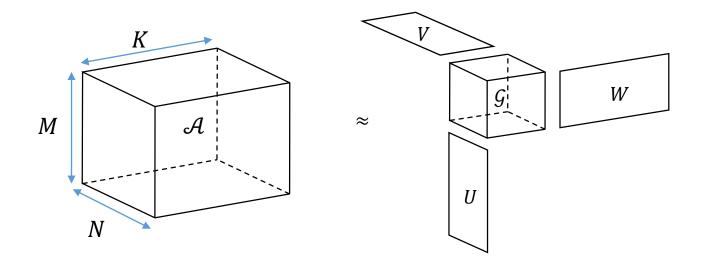
$$\mathcal{A} \approx \mathcal{G} \times_1 U \times_2 V \times_3 W$$

^{*} T. G. Kolda and B. W. Bader, "Tensor Decompositions and Applications", 2009

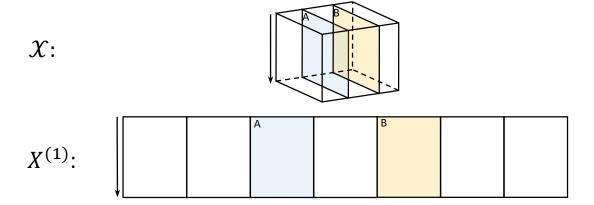
General tensor concepts



Tucker decomposition



Tensor matricization (unfolding):



$$\mathcal{A} \approx \mathcal{G} \times_1 U \times_2 V \times_3 W$$
$$\mathcal{G} \in \mathbb{R}^{r_1 \times r_2 \times r_3}$$

$$U \in \mathbb{R}^{M \times r_1}, V \in \mathbb{R}^{N \times r_2}, W \in \mathbb{R}^{K \times r_3}$$

$$a_{ijk} \approx \sum_{\alpha,\beta,\gamma}^{r_1,r_2,r_3} g_{\alpha\beta\gamma} u_{i\alpha} v_{j\beta} w_{k\gamma}$$

General ALS procedure (HOOI)

Initialize V, W randomly; require orthonormal columns Repeat until convergence:

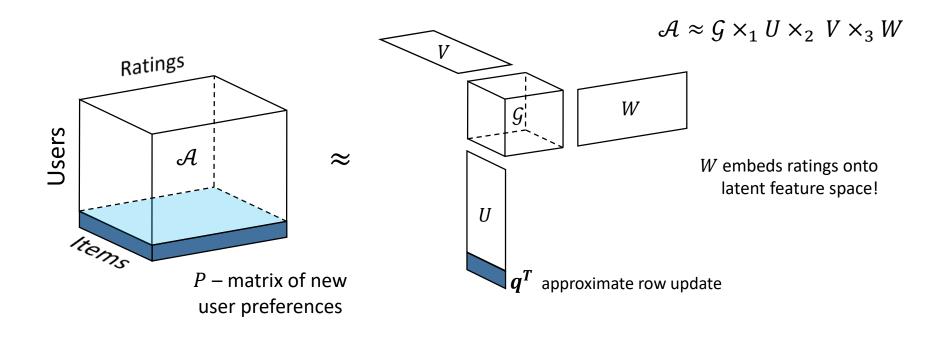
 $U \leftarrow r_1$ left principal vectors of $(\mathcal{A} \times_2 V \times_3 W)^{(1)}$

 $V \leftarrow r_2$ left principal vectors of $(\mathcal{A} \times_1 U \times_3 W)^{(2)}$

 $W \leftarrow r_3$ left principal vectors of $(\mathcal{A} \times_1 U \times_2 V)^{(3)}$

$$\mathcal{G} \leftarrow \mathcal{A} \times_1 U \times_2 V \times_3 W$$

Recommendations in real-time



"Shades of ratings"

Higher order *folding-in*:

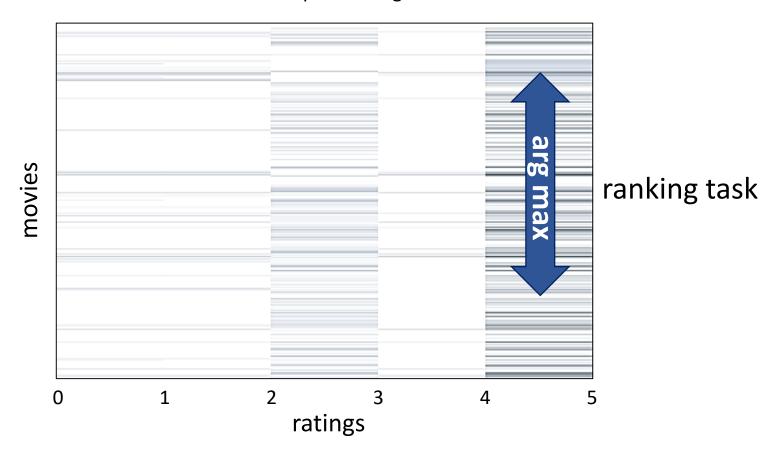
$$R = VV^T PWW^T$$
 items relevance matrix

Compare to SVD: $r = VV^T p$

"Shades" of ratings

$$R = VV^T PWW^T$$

More dense colors correspond to higher relevance score.





Granular view of user preferences, concerning **all possible ratings**.



Model is **equally sensitive** to any kind of feedback.

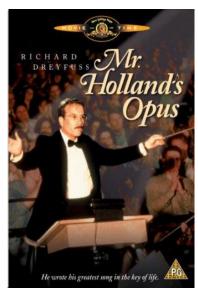
Warm-start with CoFFee

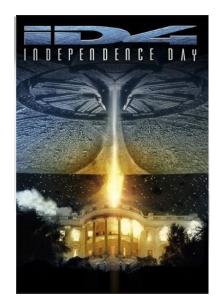












	Scarface ★★☆☆☆	LOTR: The Two Towers ★★☆☆☆	Star Wars: Episode VII - The Force Awakens ★★★★
CoFFee	Toy Story	Net, The	Dark Knight, The
	Mr. Holland's Opus	$\operatorname{Cliffhanger}$	Batman Begins
	Independence Day	Batman Forever	Star Wars: Episode IV - A New Hope
SVD	Reservoir Dogs	LOTR: The Fellowship of the Ring	Dark Knight, The
	$\operatorname{Goodfellas}$	Shrek	${\bf Inception}$
	Godfather: Part II, The	LOTR: The Return of the King	Iron Man

nDCG calculation

$$DCG = \sum_{i} \frac{2^{rel_i} - 1}{\log_2(i+1)}$$
 rel_i - true rating of a recommended item at position i

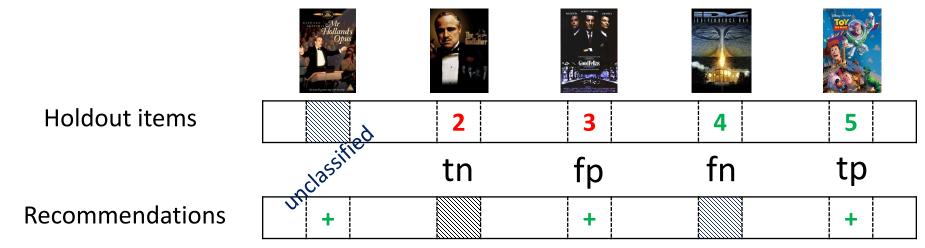
$$nDCG = \frac{DCG}{iDCG}$$

iDCG = DCG with ideal ranking of items

Doesn't take into account the type of feedback: positive or negative.

Need to redefine metrics.

Using classical definition



"presumption of innocence"

Relevance based

$$Precision = \frac{tp}{tp + fp} \qquad Recall = \frac{tp}{tp + fn}$$

$$Recall = \frac{tp}{tp + fn}$$

Ranking based

$$DCG = \sum_{p} \frac{2^{r_p} - 1}{\log_2(p+1)}$$

 $p: \{r_p \ge \text{positivity threshold}\}$ r_p - value of positive feedback

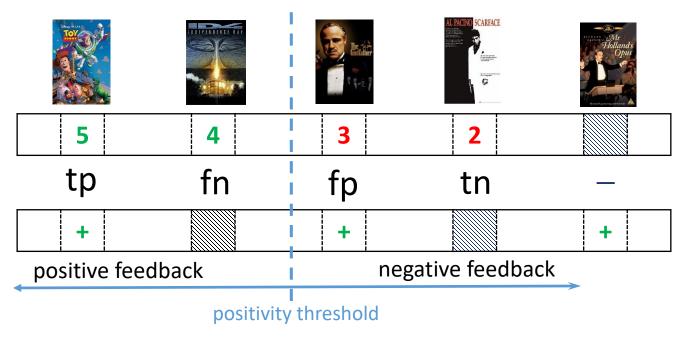
New ranking metric

Methodology:

- split feedback into positive and negative
- treat negative part differently

Known preferences (rel_i)

Recommendations



Standard metric

$$DCG = \sum_{i \in I^{+}} \frac{2^{rel_i} - 1}{\log_2(i+1)}$$

Discounted Cumulative Gain

$$I^+ = \{i: rel_i \geq r_p\}$$
 r_p - positivity threshold

measures potential user's enjoyment

New metric

$$DCL = \sum_{i \in I^{-}} \frac{2^{r_p - rel_i} - 1}{-\log_2(i+1)}$$

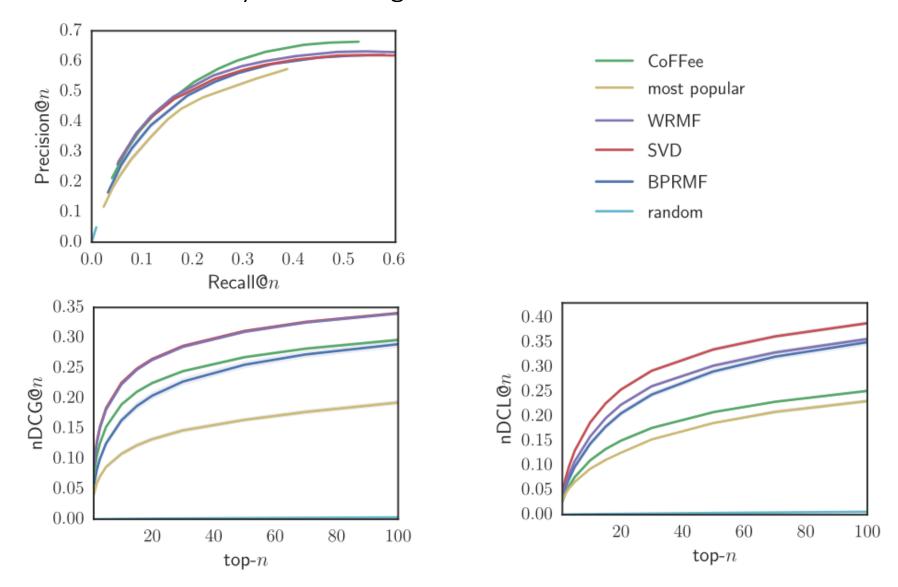
Discounted Cumulative Loss

$$I^{-} = \{i: rel_i < r_p\}$$

measures potential user's disappointment

45

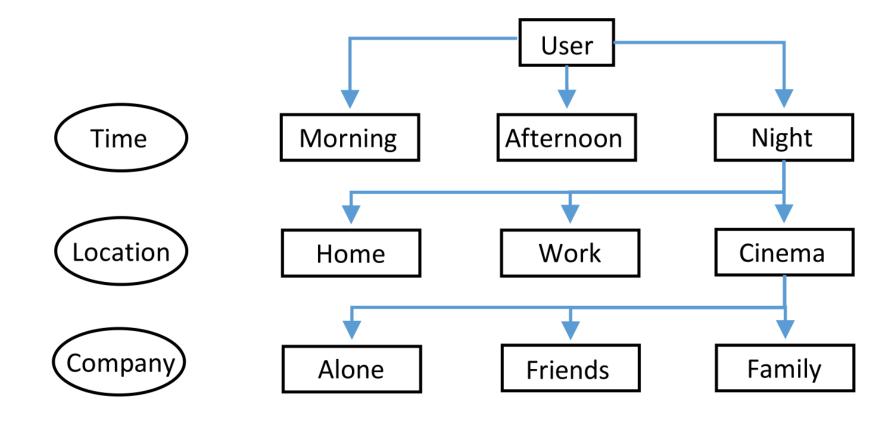
Case study – which algorithm is better?



Read more: "Fifty shades of ratings: How to Benefit from Negative Feedback in Top-n Recommendations Tasks", Evgeny Frolov and Ivan Oseledets.

Proceedings of the 10th ACM Conference on Recommender Systems, 2016, pp. 91-98.

Tensors are also good for context-aware RecSys



Also: folksonomies, cross-domain RS, temporal models

Read more about tensors in recsys: "*Tensor methods and recommender systems*", Evgeny Frolov and Ivan Oseledets.

WIRES Data Mining Knowledge Discovery 2017, vol. 7, issue 3.

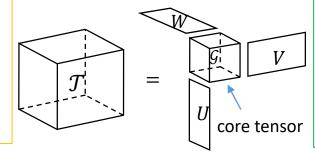
Tensor decompositions – higher order generalization of SVD



$$r_{ij} = \sum_{\alpha=1}^{R} \sigma_{\alpha} u_{i\alpha} v_{j\alpha}$$

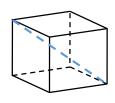
$$t_{ijk} = \sum_{\alpha=1}^{r} \lambda_{\alpha} u_{i\alpha} v_{j\alpha} w_{k\alpha}$$

$$\mathcal{T} = [\![\boldsymbol{\lambda}; U, V, W]\!]$$



$$t_{ijk} = \sum_{\alpha,\beta,\gamma=1}^{r_1,r_2,r_3} g_{\alpha\beta\gamma} u_{i\alpha} v_{j\beta} w_{k\gamma}$$

$$\mathcal{T} = \mathcal{G} \times_1 U \times_2 V \times_3 W$$



diagonal core with λ_{α} on diagonal

Rank can exceed $\max(M, N, K)$.

No generalization of Eckart-Young theorem.

Manifold of rank-r tensors is **not closed**.

Tensor decompositions have much broader set of properties.

dense core of size $r_1 \times r_2 \times r_3$ with $g_{\alpha\beta\gamma}$ values

 r_1, r_2, r_3 - multilinear rank

Tensor product \times_i is calculated with help of tensor matricization.

Two efficient algorithms: HOSVD, HOOI.

Key properties of tensor decompositions

Properties	TD	СР	
Storage requirements	$dnr + r^{d}$ curse of dimensionality	dnr	
Uniqueness	No	Yes (under mild conditions)	
Stability	stable	ill-posed*	

^{*}V. De Silva and L.-H. Lim. Tensor rank and the ill-posedness of the best low-rank approximation problem.

In recommender systems*:

Method	Year	Type	Algorithm
TagTR	2008	TD	HOSVD
Multiverse	2010	TD	SGD
PITF	2010	СР	SGD
TFMAP	2012	СР	SGD
GFF	2015	СР	ALS

^{*}More examples in "Tensor Methods and Recommender Systems", E.Frolov, I.Oseledets http://arxiv.org/abs/1603.06038

Your home task

- Download Movielens-10M dataset: http://files.grouplens.org/datasets/movielens/ml-10m.zip
 - test data will be provided separately
 - it's not a part of the dataset
- Try your favorite models on it

Try comparing several models/configs via cross-validation

This is the dataset that will be used during the team competition!