Current–current correlator and Landau susceptibility

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It is possible to calculate the diamagnetic susceptibility of electron gas using the linear responce of electric current.

The perturbation caused by magnetic field:

$$V = -A_\alpha j_\alpha(-k)e^{-i\omega t} \tag{1}$$

The bare current operator:

$$j(k) = \frac{e}{m}\int \frac{d^3k'}{(2\pi)^3}\Psi^\dagger_{k'-\frac{k}{2}}\Psi_{k'+\frac{k}{2}}k' \tag{2}$$

The response of current to the perturbation:

$$\frac{\langle J_\alpha\rangle}{V} = -\frac{ne^2}{mc}A_\alpha e^{-i\omega t} + \frac{ie^{-i\omega t}}{cV}\int_0^\infty d\tau e^{i\omega\tau}\langle[j_\alpha(k,\tau)j_\beta(-k,0)]\rangle A_\beta \tag{3}$$

We need to calculate the current–current Green's function:

$$\begin{aligned} iG^F_{\alpha\beta} &\equiv \frac{1}{V}\int_{-\infty}^\infty d\tau e^{i\omega\tau}\langle \mathrm{T} j_\alpha(k,\tau)j_\beta(-k,0)\rangle = \\ &= \frac{e^2}{m^2}\int\frac{d\omega'}{2\pi}\int\frac{d^3k'}{(2\pi)^3}\frac{k'_\alpha k'_\beta}{(\omega+\omega'-\xi_{k'+\frac{k}{2}}+i0\,\mathrm{sign}\,\xi_{k'+\frac{k}{2}})(\omega'-\xi_{k'-\frac{k}{2}}+i0\,\mathrm{sign}\,\xi_{k'-\frac{k}{2}})} \end{aligned} \tag{4}$$

After integration by ω we get

$$iG^F_{\alpha\beta} = \frac{-ie^2}{m^2}\left[\int_{(1)}\frac{d^3k'}{(2\pi)^3}\frac{k'_\alpha k'_\beta}{\xi_{k'+\frac{k}{2}}-\xi_{k'-\frac{k}{2}}-\omega-i0} + \int_{(2)}\frac{d^3k'}{(2\pi)^3}\frac{k'_\alpha k'_\beta}{\omega+\xi_{k'-\frac{k}{2}}-\xi_{k'+\frac{k}{2}}-i0}\right] \tag{5}$$

Subscripts (1) and (2) denote the areas $\xi_{k'+\frac{k}{2}} > 0 > \xi_{k'-\frac{k}{2}}$ and $\xi_{k'+\frac{k}{2}} < 0 < \xi_{k'-\frac{k}{2}}$ respectively. As we are interested in diamagnetic susceptibility, we need to consider the limit $\omega \to 0$. In such case, the integrals may be simplified so that k expansion becomes easy.

Let us take k in the form $(0,0,k)$. The interesting term is the response of J_x to A_x, which after some calculus reads

$$\frac{J_x}{V} = A_x \frac{ne^2}{mc}\left(-1+\frac{3}{2}\int_0^1 dx\,(1-x^2)\sqrt{1-\frac{k^2}{k_f^2}(1-x^2)}\left(1-\frac{k^2}{4k_f^2}+\frac{k^2x^2}{2k_f^2}\right)\right) \tag{6}$$

By k^2 expansion, it's easy to recover Landau susceptibility.