Anomalous Hall effect

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The Hamiltonian of the problem is

$$\hat{H} = \frac{\hbar^2 k^2}{2m} + \lambda(k_x \sigma_y - k_y \sigma_x) - \Delta \sigma_z \tag{1}$$

It can be easily diagonalized, the energies are

$$E_k^{(1,2)} = \frac{\hbar^2 k^2}{2m} \pm \sqrt{\lambda^2 k^2 + \Delta^2} \tag{2}$$

It is convenient to introduce $\epsilon_k = \sqrt{\lambda^2 k^2 + \Delta^2}$.

The eigenvectors are, as usual,

$$\begin{aligned} u_\uparrow &= \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} e^{i\phi} \end{pmatrix} \\ u_\downarrow &= \begin{pmatrix} -\sin\frac{\theta}{2} e^{-i\phi} \\ \cos\frac{\theta}{2} \end{pmatrix} \end{aligned} \tag{3}$$

Here

$$\begin{aligned} \cos\theta &= -\frac{\Delta}{\epsilon_k} \\ e^{i\phi} &= \frac{-k_y + i k_x}{k} \end{aligned} \tag{4}$$

The Berry curvature can be evaluated from the gauge–invariant formula. For the upper band,

$$\begin{aligned} \Omega_z = \Omega_{xy} &= -\frac{i\lambda^2}{4\epsilon_k^2} \left[\langle u_\uparrow|\sigma_y|u_\downarrow\rangle\langle u_\downarrow|\sigma_x|u_\uparrow\rangle - \langle u_\uparrow|\sigma_x|u_\downarrow\rangle\langle u_\downarrow|\sigma_y|u_\uparrow\rangle \right] \\ \Omega_x &= \Omega_y = 0 \end{aligned} \tag{5}$$

The matrix elements:

$$\begin{aligned} \langle u_\uparrow|\sigma_x|u_\downarrow\rangle &= \cos\frac{\theta}{2}^2 - e^{-2i\phi}\sin\frac{\theta}{2} \\ \langle u_\downarrow|\sigma_y|u_\uparrow\rangle &= i(\cos\frac{\theta}{2}^2 + e^{2i\phi}\sin\frac{\theta}{2}) \end{aligned} \tag{6}$$

So,

$$\Omega_z^{\text{upper}} = -\frac{\lambda^2}{2\epsilon_k^2}\cos\theta = \frac{\lambda^2\Delta}{2\epsilon_k^3} \tag{7}$$

For lower band, we obtain similarly

$$\Omega_z^{\text{lower}} = -\frac{\lambda^2\Delta}{2\epsilon_k^3} \tag{8}$$

Now let's find the Hall conductivity. Let the electric field be $(E, 0, 0)$. Then the equations of motion for the wave packet read

$$\begin{aligned} \dot{x} &= \frac{1}{\hbar}\frac{\partial E_k}{\partial k_x} - \dot{y}\Omega_z \\ \dot{y} &= \frac{1}{\hbar}\frac{\partial E_k}{\partial k_y} + \dot{x}\Omega_z \\ \hbar\dot{k_x} &= -eE \\ \hbar\dot{k_y} &= 0 \end{aligned} \tag{9}$$