Boston Housing

First Part

Preparation

Import modules for analysis

```
import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
import statsmodels.api as sm
import statsmodels.formula.api as smf
import scipy.stats
```

Create dataframe

```
boston_house = pd.read_csv('BostonHousing.csv')
```

EDA

Check data

```
boston_house
```

```
        crim    zn  indus  chas    nox     rm   age     dis  rad  tax  \
0    0.00632  18.0   2.31     0  0.538  6.575  65.2  4.0900    1  296

1    0.02731   0.0   7.07     0  0.469  6.421  78.9  4.9671    2  242

2    0.02729   0.0   7.07     0  0.469  7.185  61.1  4.9671    2  242

3    0.03237   0.0   2.18     0  0.458  6.998  45.8  6.0622    3  222

4    0.06905   0.0   2.18     0  0.458  7.147  54.2  6.0622    3  222

..       ...   ...    ...   ...    ...    ...   ...     ...  ...  ...

501  0.06263   0.0  11.93     0  0.573  6.593  69.1  2.4786    1  273

502  0.04527   0.0  11.93     0  0.573  6.120  76.7  2.2875    1  273

503  0.06076   0.0  11.93     0  0.573  6.976  91.0  2.1675    1  273

504  0.10959   0.0  11.93     0  0.573  6.794  89.3  2.3889    1  273
```

```
505  0.04741   0.0  11.93     0  0.573  6.030  80.8  2.5050    1  273


     ptratio       b  lstat  medv
0       15.3  396.90   4.98  24.0
1       17.8  396.90   9.14  21.6
2       17.8  392.83   4.03  34.7
3       18.7  394.63   2.94  33.4
4       18.7  396.90   5.33  36.2
..       ...     ...    ...   ...
501     21.0  391.99   9.67  22.4
502     21.0  396.90   9.08  20.6
503     21.0  396.90   5.64  23.9
504     21.0  393.45   6.48  22.0
505     21.0  396.90   7.88  11.9

[506 rows x 14 columns]
```

The description of all the features:

- CRIM: Per capita crime rate by town
- ZN: Proportion of residential land zoned for lots over 25,000 sq. ft
- INDUS: Proportion of non-retail business acres per town
- CHAS: Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)
- NOX: Nitric oxide concentration (parts per 10 million)
- RM: Average number of rooms per dwelling
- AGE: Proportion of owner-occupied units built prior to 1940
- DIS: Weighted distances to five Boston employment centers
- RAD: Index of accessibility to radial highways
- TAX: Full-value property tax rate per 10,000 dollars
- PTRATIO: Pupil-teacher ratio by town
- B: proportion of American African by town
- LSTAT: Percentage of lower status of the population
- MEDV: Median value of owner-occupied homes in $1000s **TARGET**

```
boston_house.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 506 entries, 0 to 505
Data columns (total 14 columns):
 #   Column   Non-Null Count  Dtype
---  ------   --------------  -----
 0   crim     506 non-null    float64
 1   zn       506 non-null    float64
 2   indus    506 non-null    float64
 3   chas     506 non-null    int64
 4   nox      506 non-null    float64
 5   rm       506 non-null    float64
```

```
 6   age      506 non-null    float64
 7   dis      506 non-null    float64
 8   rad      506 non-null    int64
 9   tax      506 non-null    int64
 10  ptratio  506 non-null    float64
 11  b        506 non-null    float64
 12  lstat    506 non-null    float64
 13  medv     506 non-null    float64
dtypes: float64(11), int64(3)
memory usage: 55.5 KB
```

Check NA

```
boston_house.isna().sum()
```

```
crim       0
zn         0
indus      0
chas       0
nox        0
rm         0
age        0
dis        0
rad        0
tax        0
ptratio    0
b          0
lstat      0
medv       0
dtype: int64
```

Lets see on descriptive statistics

```
boston_house.describe()
```

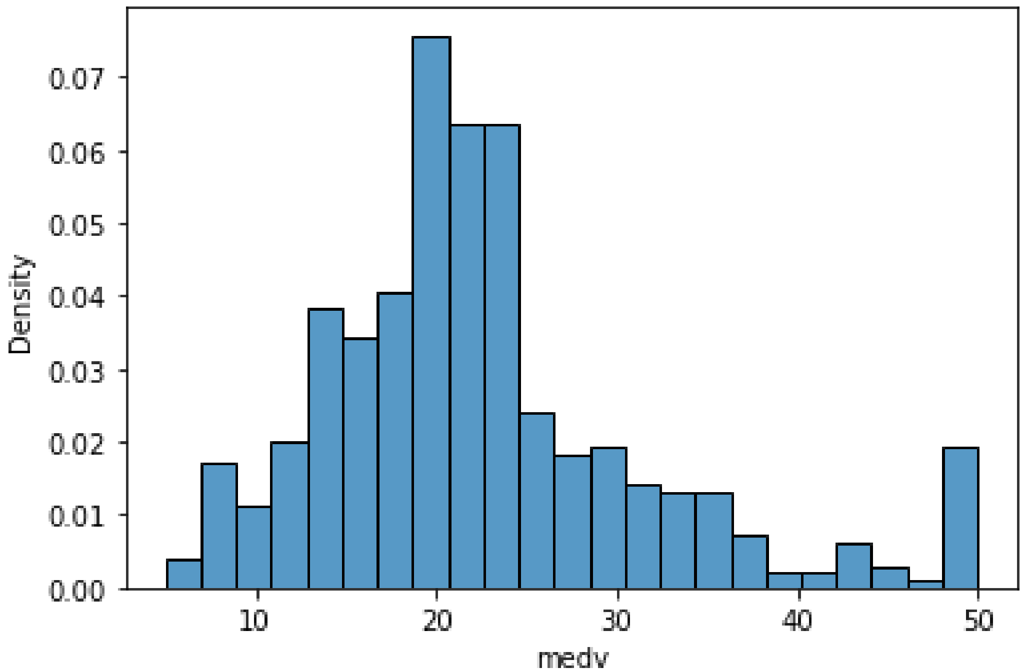
```
             crim          zn       indus        chas         nox
rm  \
count  506.000000  506.000000  506.000000  506.000000  506.000000
506.000000
mean     3.613524   11.363636   11.136779    0.069170    0.554695
6.284634
std      8.601545   23.322453    6.860353    0.253994    0.115878
0.702617
min      0.006320    0.000000    0.460000    0.000000    0.385000
3.561000
25%      0.082045    0.000000    5.190000    0.000000    0.449000
5.885500
50%      0.256510    0.000000    9.690000    0.000000    0.538000
6.208500
75%      3.677083   12.500000   18.100000    0.000000    0.624000
6.623500
```

```
max     88.976200  100.000000   27.740000    1.000000    0.871000
8.780000

              age         dis         rad         tax     ptratio
b  \
count  506.000000  506.000000  506.000000  506.000000  506.000000
506.000000
mean    68.574901    3.795043    9.549407  408.237154   18.455534
356.674032
std     28.148861    2.105710    8.707259  168.537116    2.164946
91.294864
min      2.900000    1.129600    1.000000  187.000000   12.600000
0.320000
25%     45.025000    2.100175    4.000000  279.000000   17.400000
375.377500
50%     77.500000    3.207450    5.000000  330.000000   19.050000
391.440000
75%     94.075000    5.188425   24.000000  666.000000   20.200000
396.225000
max    100.000000   12.126500   24.000000  711.000000   22.000000
396.900000

            lstat        medv
count  506.000000  506.000000
mean    12.653063   22.532806
std      7.141062    9.197104
min      1.730000    5.000000
25%      6.950000   17.025000
50%     11.360000   21.200000
75%     16.955000   25.000000
max     37.970000   50.000000
```

Evaluate the medv distribution. We can see the distribution of target close to normal, except margin measure nearly 50.000$. So we need to use cook distance to select appropriate observation

```
sns.histplot(boston_house.medv, stat='density')
```

```
<AxesSubplot:xlabel='medv', ylabel='Density'>
```



Create target and feature dataframes. And start to analyse

```
boston_house_target  = boston_house.medv
boston_house_feature  = boston_house.drop('medv', axis=1)
boston_house_feature
```

```
        crim    zn  indus  chas    nox     rm   age     dis  rad  tax
\
0    0.00632  18.0   2.31     0  0.538  6.575  65.2  4.0900    1  296

1    0.02731   0.0   7.07     0  0.469  6.421  78.9  4.9671    2  242

2    0.02729   0.0   7.07     0  0.469  7.185  61.1  4.9671    2  242

3    0.03237   0.0   2.18     0  0.458  6.998  45.8  6.0622    3  222

4    0.06905   0.0   2.18     0  0.458  7.147  54.2  6.0622    3  222

..       ...   ...    ...   ...    ...    ...   ...     ...  ...  ...

501  0.06263   0.0  11.93     0  0.573  6.593  69.1  2.4786    1  273

502  0.04527   0.0  11.93     0  0.573  6.120  76.7  2.2875    1  273

503  0.06076   0.0  11.93     0  0.573  6.976  91.0  2.1675    1  273

504  0.10959   0.0  11.93     0  0.573  6.794  89.3  2.3889    1  273
```

```
505  0.04741   0.0  11.93     0  0.573  6.030  80.8  2.5050    1  273


     ptratio       b  lstat
0       15.3  396.90   4.98
1       17.8  396.90   9.14
2       17.8  392.83   4.03
3       18.7  394.63   2.94
4       18.7  396.90   5.33
..       ...     ...    ...
501     21.0  391.99   9.67
502     21.0  396.90   9.08
503     21.0  396.90   5.64
504     21.0  393.45   6.48
505     21.0  396.90   7.88

[506 rows x 13 columns]
```

Standardization and linear model

Descriptive statistics demonstrates that features obtain different std. It can (and will) influence linear model so we have to standardize this dataset. However, previously look on original data analysis

```
X = sm.add_constant(boston_house_feature)
model = sm.OLS(boston_house_target, X)
results = model.fit()

print(results.summary())
```

```
                            OLS Regression Results

======================================================================
========
Dep. Variable:                   medv   R-squared:
0.741
Model:                            OLS   Adj. R-squared:
0.734
Method:                 Least Squares   F-statistic:
108.1
Date:                Wed, 14 Dec 2022   Prob (F-statistic):
6.72e-135
Time:                        01:38:00   Log-Likelihood:
-1498.8
No. Observations:                 506   AIC:
3026.
Df Residuals:                     492   BIC:
3085.
Df Model:                          13
```

```
Covariance Type:            nonrobust

==============================================================================
========
                 coef    std err          t      P>|t|      [0.025
0.975]
------------------------------------------------------------------------------
--------
const         36.4595      5.103      7.144      0.000      26.432
46.487
crim          -0.1080      0.033     -3.287      0.001      -0.173
-0.043
zn             0.0464      0.014      3.382      0.001       0.019
0.073
indus          0.0206      0.061      0.334      0.738      -0.100
0.141
chas           2.6867      0.862      3.118      0.002       0.994
4.380
nox          -17.7666      3.820     -4.651      0.000     -25.272
-10.262
rm             3.8099      0.418      9.116      0.000       2.989
4.631
age            0.0007      0.013      0.052      0.958      -0.025
0.027
dis           -1.4756      0.199     -7.398      0.000      -1.867
-1.084
rad            0.3060      0.066      4.613      0.000       0.176
0.436
tax           -0.0123      0.004     -3.280      0.001      -0.020
-0.005
ptratio       -0.9527      0.131     -7.283      0.000      -1.210
-0.696
b              0.0093      0.003      3.467      0.001       0.004
0.015
lstat         -0.5248      0.051    -10.347      0.000      -0.624
-0.425
==============================================================================
========
Omnibus:                      178.041   Durbin-Watson:
1.078
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
783.126
Skew:                           1.521   Prob(JB):
8.84e-171
Kurtosis:                       8.281   Cond. No.
1.51e+04
==============================================================================
========

Notes:
```

```
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
[2] The condition number is large, 1.51e+04. This might indicate that
there are
strong multicollinearity or other numerical problems.
```

Adj. R-squared: 0.734 and p-value = 6.72e-135 - very good for raw data but it can not reflect true dependance between target and features. Lets do standardization

```
mean_param = boston_house_feature.mean(axis=0)
stds_param = boston_house_feature.std(axis=0)

scaled_features = (boston_house_feature - mean_param) / stds_param
scaled_features
```

```
         crim        zn     indus      chas       nox        rm
age  \
0   -0.419367  0.284548 -1.286636 -0.272329 -0.144075  0.413263 -
0.119895
1   -0.416927 -0.487240 -0.592794 -0.272329 -0.739530  0.194082
0.366803
2   -0.416929 -0.487240 -0.592794 -0.272329 -0.739530  1.281446 -
0.265549
3   -0.416338 -0.487240 -1.305586 -0.272329 -0.834458  1.015298 -
0.809088
4   -0.412074 -0.487240 -1.305586 -0.272329 -0.834458  1.227362 -
0.510674
..        ...       ...       ...       ...       ...       ...
...
501 -0.412820 -0.487240  0.115624 -0.272329  0.157968  0.438881
0.018654
502 -0.414839 -0.487240  0.115624 -0.272329  0.157968 -0.234316
0.288648
503 -0.413038 -0.487240  0.115624 -0.272329  0.157968  0.983986
0.796661
504 -0.407361 -0.487240  0.115624 -0.272329  0.157968  0.724955
0.736268
505 -0.414590 -0.487240  0.115624 -0.272329  0.157968 -0.362408
0.434302

          dis       rad       tax   ptratio         b     lstat
0    0.140075 -0.981871 -0.665949 -1.457558  0.440616 -1.074499
1    0.556609 -0.867024 -0.986353 -0.302794  0.440616 -0.491953
2    0.556609 -0.867024 -0.986353 -0.302794  0.396035 -1.207532
3    1.076671 -0.752178 -1.105022  0.112920  0.415751 -1.360171
4    1.076671 -0.752178 -1.105022  0.112920  0.440616 -1.025487
..        ...       ...       ...       ...       ...       ...
501 -0.625178 -0.981871 -0.802418  1.175303  0.386834 -0.417734
502 -0.715931 -0.981871 -0.802418  1.175303  0.440616 -0.500355
503 -0.772919 -0.981871 -0.802418  1.175303  0.440616 -0.982076
```

```
504 -0.667776 -0.981871 -0.802418  1.175303  0.402826 -0.864446
505 -0.612640 -0.981871 -0.802418  1.175303  0.440616 -0.668397

[506 rows x 13 columns]
```

Again check prepared data

```
scaled_features.describe()
```

```
               crim            zn         indus          chas
nox  \
count  5.060000e+02  5.060000e+02  5.060000e+02  5.060000e+02
5.060000e+02
mean   8.326673e-17  3.466704e-16 -3.016965e-15  4.677857e-16
3.563575e-15
std    1.000000e+00  1.000000e+00  1.000000e+00  1.000000e+00
1.000000e+00
min   -4.193669e-01 -4.872402e-01 -1.556302e+00 -2.723291e-01 -
1.464433e+00
25%   -4.105633e-01 -4.872402e-01 -8.668328e-01 -2.723291e-01 -
9.121262e-01
50%   -3.902803e-01 -4.872402e-01 -2.108898e-01 -2.723291e-01 -
1.440749e-01
75%    7.389247e-03  4.872402e-02  1.014995e+00 -2.723291e-01
5.980871e-01
max    9.924110e+00  3.800473e+00  2.420170e+00  3.664771e+00
2.729645e+00

                 rm           age           dis           rad
tax  \
count  5.060000e+02  5.060000e+02  5.060000e+02  5.060000e+02
5.060000e+02
mean  -1.149882e-14 -1.158274e-15  7.308603e-16 -1.068535e-15
6.534079e-16
std    1.000000e+00  1.000000e+00  1.000000e+00  1.000000e+00
1.000000e+00
min   -3.876413e+00 -2.333128e+00 -1.265817e+00 -9.818712e-01 -
1.312691e+00
25%   -5.680681e-01 -8.366200e-01 -8.048913e-01 -6.373311e-01 -
7.668172e-01
50%   -1.083583e-01  3.170678e-01 -2.790473e-01 -5.224844e-01 -
4.642132e-01
75%    4.822906e-01  9.059016e-01  6.617161e-01  1.659603e+00
1.529413e+00
max    3.551530e+00  1.116390e+00  3.956602e+00  1.659603e+00
1.796416e+00

            ptratio             b         lstat
count  5.060000e+02  5.060000e+02  5.060000e+02
mean  -1.084420e-14  8.117354e-15 -6.494585e-16
```

```
std    1.000000e+00  1.000000e+00  1.000000e+00
min   -2.704703e+00 -3.903331e+00 -1.529613e+00
25%   -4.875567e-01  2.048688e-01 -7.986296e-01
50%    2.745872e-01  3.808097e-01 -1.810744e-01
75%    8.057784e-01  4.332223e-01  6.024226e-01
max    1.637208e+00  4.406159e-01  3.545262e+00
```

And repeat linear regression by OLS using prepared data

```
X = sm.add_constant(scaled_features)
model_scaled = sm.OLS(boston_house_target, X)
results_scaled = model_scaled.fit()

print(results_scaled.summary())
```

```
                            OLS Regression Results

======================================================================
========
Dep. Variable:                   medv   R-squared:
0.741
Model:                            OLS   Adj. R-squared:
0.734
Method:                 Least Squares   F-statistic:
108.1
Date:                Wed, 14 Dec 2022   Prob (F-statistic):
6.72e-135
Time:                        01:55:41   Log-Likelihood:
-1498.8
No. Observations:                 506   AIC:
3026.
Df Residuals:                     492   BIC:
3085.
Df Model:                          13

Covariance Type:            nonrobust

======================================================================
========
                 coef    std err          t      P>|t|      [0.025
0.975]
----------------------------------------------------------------------
--------
const         22.5328      0.211    106.814      0.000      22.118
22.947
crim          -0.9291      0.283     -3.287      0.001      -1.484
-0.374
zn             1.0826      0.320      3.382      0.001       0.454
1.712
indus          0.1410      0.422      0.334      0.738      -0.688
```

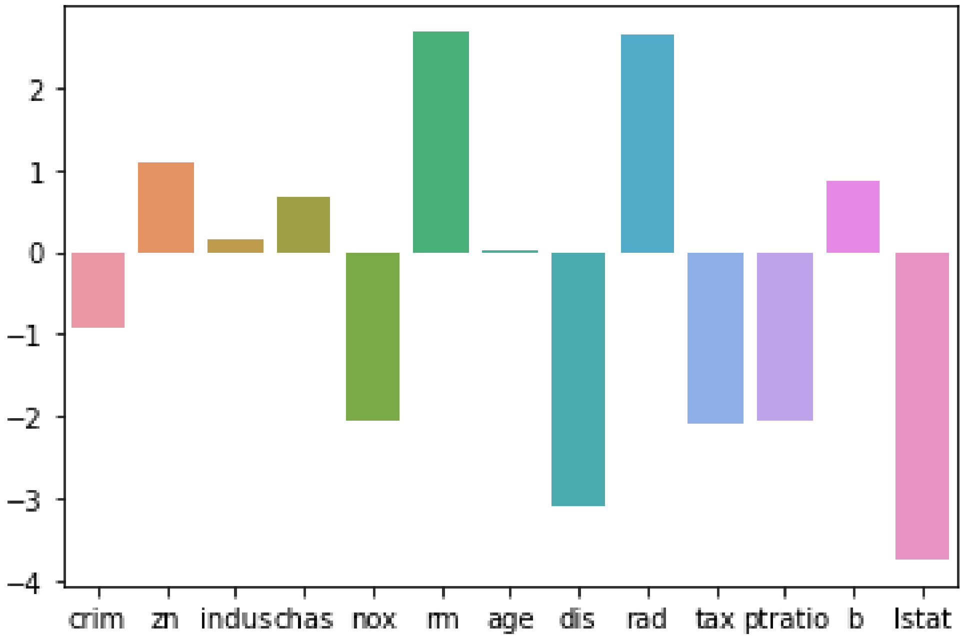
```
0.970
chas            0.6824      0.219      3.118      0.002       0.252
1.112
nox            -2.0588      0.443     -4.651      0.000      -2.928
-1.189
rm              2.6769      0.294      9.116      0.000       2.100
3.254
age             0.0195      0.372      0.052      0.958      -0.711
0.750
dis            -3.1071      0.420     -7.398      0.000      -3.932
-2.282
rad             2.6649      0.578      4.613      0.000       1.530
3.800
tax            -2.0788      0.634     -3.280      0.001      -3.324
-0.834
ptratio        -2.0626      0.283     -7.283      0.000      -2.619
-1.506
b               0.8501      0.245      3.467      0.001       0.368
1.332
lstat          -3.7473      0.362    -10.347      0.000      -4.459
-3.036
======================================================================
========
Omnibus:                     178.041   Durbin-Watson:
1.078
Prob(Omnibus):                 0.000   Jarque-Bera (JB):
783.126
Skew:                          1.521   Prob(JB):
8.84e-171
Kurtosis:                      8.281   Cond. No.
9.82
======================================================================
========

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
```

Adj. R-squared: 0.734 - its similar to raw data but probably they have difference in the next round. Build graphs of coefficients for every feature

```
sns.barplot(results_scaled.params.index[1:],
results_scaled.params[1:])
```

```
C:\Users\ornfi\anaconda3\lib\site-packages\seaborn\_decorators.py:36:
FutureWarning: Pass the following variables as keyword args: x, y.
From version 0.12, the only valid positional argument will be `data`,
and passing other arguments without an explicit keyword will result in
an error or misinterpretation.
  warnings.warn(
```

```
<AxesSubplot:>
```



Check linear dependance

Predict target values using acquired coefficients

```
prediction = results_scaled.get_prediction(X)
target_predict = prediction.predicted_mean

print(target_predict[:10])

[30.00384338 25.02556238 30.56759672 28.60703649 27.94352423
25.25628446
 23.00180827 19.53598843 11.52363685 18.92026211]
```

Concate predicted and real target values in one dataframe for graphs

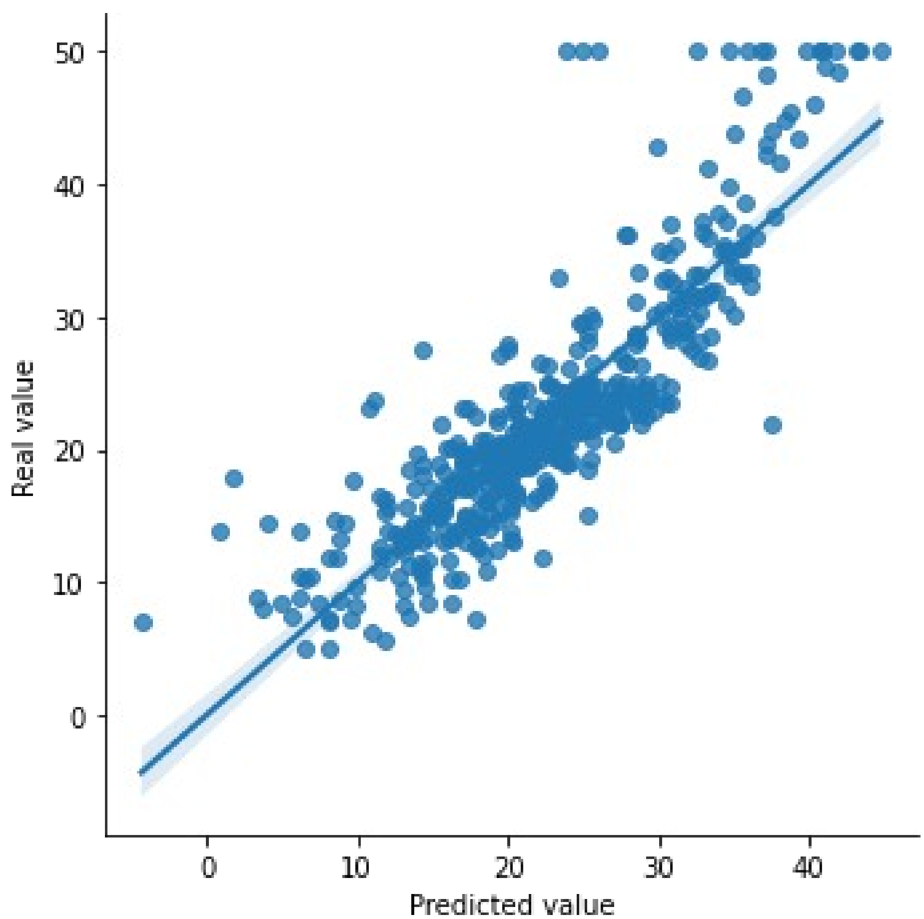
```
predict_target_df = pd.DataFrame(list(zip(target_predict.round(1),
boston_house_target)),
                                columns =['Predicted target', 'Real
target'])
predict_target_df

     Predicted target  Real target
0                30.0         24.0
1                25.0         21.6
2                30.6         34.7
3                28.6         33.4
4                27.9         36.2
..                ...          ...
501              23.5         22.4
502              22.4         20.6
```

```
503                27.6          23.9
504                26.1          22.0
505                22.3          11.9

[506 rows x 2 columns]
```

```
plt.figure(figsize=[12,7], dpi=900)
sns.lmplot(x='Predicted target', y='Real target',
data=predict_target_df, order=1)
plt.xlabel('Predicted value')
plt.ylabel('Real value')
```

```
Text(24.050000000000001, 0.5, 'Real value')
```

```
<Figure size 10800x6300 with 0 Axes>
```



We can see that predicted target well conform with real target. However we take into acount some value in area of 50.000$ dramaticaly differ from expectation. Hence, start to use cook distance for sorting observation decreasing quality of model

Influential observation

```
influence = results_scaled.get_influence()
cooks = influence.cooks_distance
```

I use standard threshold = 0.05

```
(cooks[1] < 0.05).sum()
```

```
0
```

None of the observations prevent us from building qualitative model so in the future we will use the whole dataframe

Independence of observations

Features can interfere and be consistent with each other. Hence, we will check the collinearity of the features using correlation

```
correlation_matrix = boston_house_feature.corr()
correlation_matrix
```

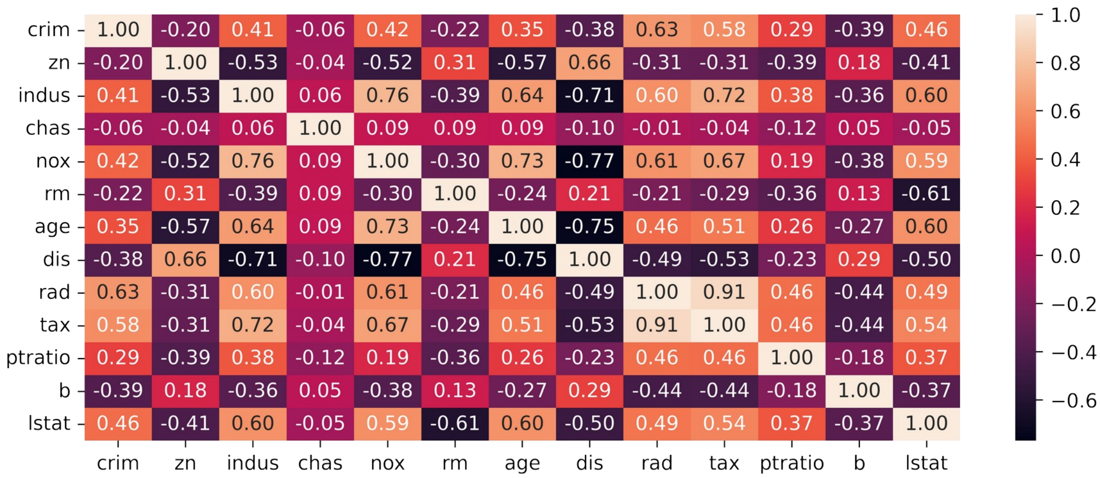
```
             crim        zn     indus      chas       nox        rm
age  \
crim     1.000000 -0.200469  0.406583 -0.055892  0.420972 -0.219247
0.352734
zn      -0.200469  1.000000 -0.533828 -0.042697 -0.516604  0.311991 -
0.569537
indus    0.406583 -0.533828  1.000000  0.062938  0.763651 -0.391676
0.644779
chas    -0.055892 -0.042697  0.062938  1.000000  0.091203  0.091251
0.086518
nox      0.420972 -0.516604  0.763651  0.091203  1.000000 -0.302188
0.731470
rm      -0.219247  0.311991 -0.391676  0.091251 -0.302188  1.000000 -
0.240265
age      0.352734 -0.569537  0.644779  0.086518  0.731470 -0.240265
1.000000
dis     -0.379670  0.664408 -0.708027 -0.099176 -0.769230  0.205246 -
0.747881
rad      0.625505 -0.311948  0.595129 -0.007368  0.611441 -0.209847
0.456022
tax      0.582764 -0.314563  0.720760 -0.035587  0.668023 -0.292048
0.506456
ptratio  0.289946 -0.391679  0.383248 -0.121515  0.188933 -0.355501
0.261515
b       -0.385064  0.175520 -0.356977  0.048788 -0.380051  0.128069 -
0.273534
lstat    0.455621 -0.412995  0.603800 -0.053929  0.590879 -0.613808
0.602339

              dis       rad       tax   ptratio         b     lstat
```

```
crim     -0.379670  0.625505  0.582764  0.289946 -0.385064  0.455621
zn        0.664408 -0.311948 -0.314563 -0.391679  0.175520 -0.412995
indus    -0.708027  0.595129  0.720760  0.383248 -0.356977  0.603800
chas     -0.099176 -0.007368 -0.035587 -0.121515  0.048788 -0.053929
nox      -0.769230  0.611441  0.668023  0.188933 -0.380051  0.590879
rm        0.205246 -0.209847 -0.292048 -0.355501  0.128069 -0.613808
age      -0.747881  0.456022  0.506456  0.261515 -0.273534  0.602339
dis       1.000000 -0.494588 -0.534432 -0.232471  0.291512 -0.496996
rad      -0.494588  1.000000  0.910228  0.464741 -0.444413  0.488676
tax      -0.534432  0.910228  1.000000  0.460853 -0.441808  0.543993
ptratio -0.232471  0.464741  0.460853  1.000000 -0.177383  0.374044
b         0.291512 -0.444413 -0.441808 -0.177383  1.000000 -0.366087
lstat    -0.496996  0.488676  0.543993  0.374044 -0.366087  1.000000
```

Its of course nice, but very difficult to distinguish every correlation so build heatmap

```
plt.figure(figsize=[10,4], dpi=700)
sns.heatmap(data=correlation_matrix, annot=True, fmt=".2f")
```

```
<AxesSubplot:>
```



The strong positive correlation between rad and tax suggests that it is better to leave only one value for further analysis. But we'll check it later using VIF

Distribution of prediction

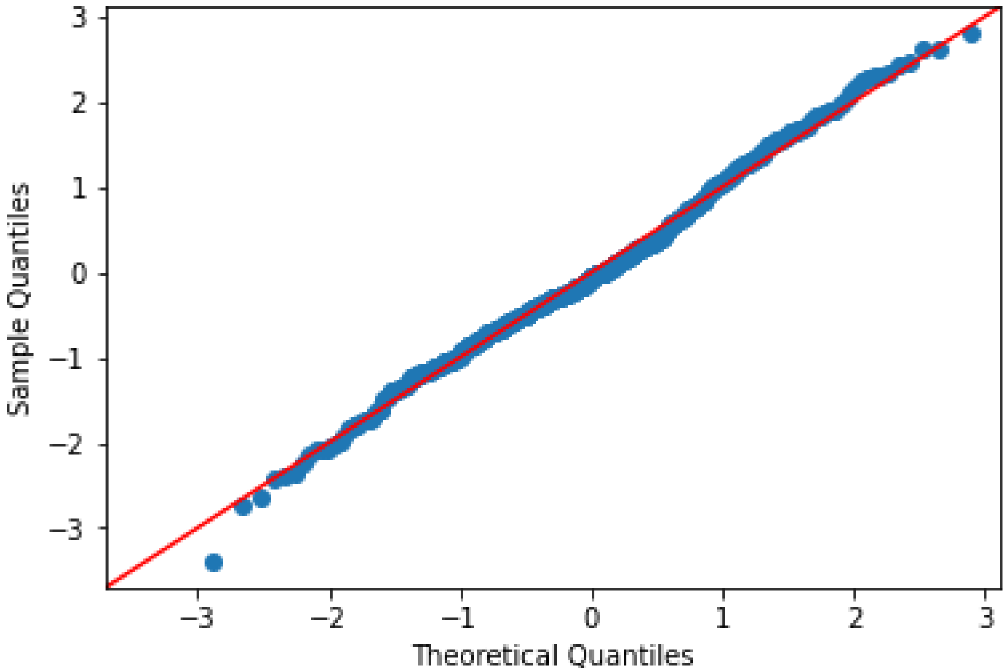
Now we can estimate distribution of predicted value and will be sure that our model predict correctly without outliers

```
scipy.stats.shapiro(target_predict)
```

```
ShapiroResult(statistic=0.9942290186882019,
pvalue=0.05247299373149872)
```

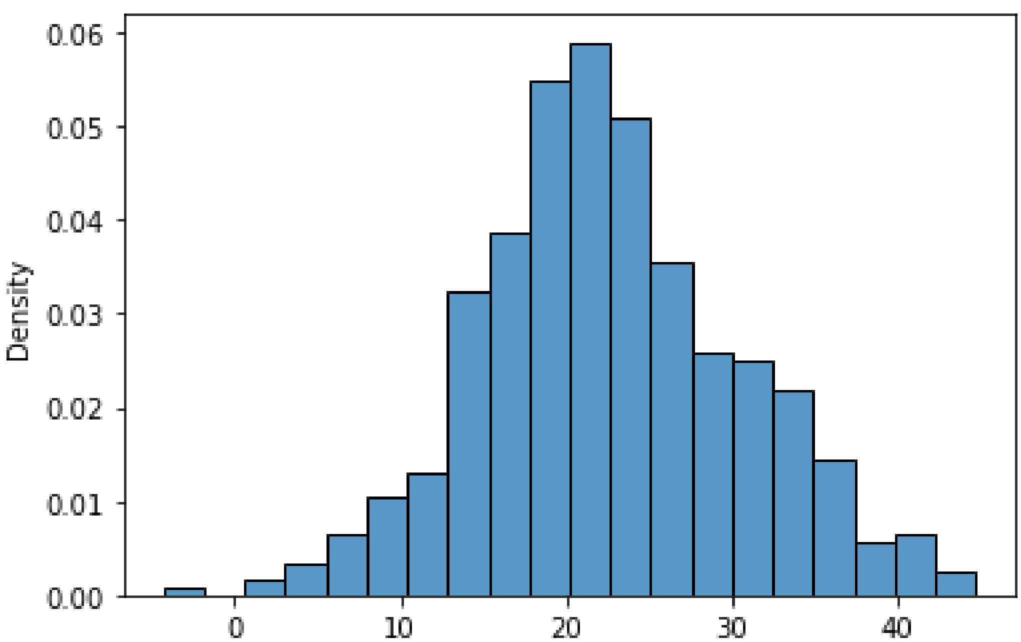
H0 is confirmed - predicted values present normal distribution

```
figure_1 = sm.qqplot(target_predict, line='45', fit=True)
plt.show()
```




```
sns.histplot(target_predict, stat='density')
```

```
<AxesSubplot:ylabel='Density'>
```



QQplot and histplot also confirm good picture of normal distibution

Disribution of error

However it is more interesting to look on distribution of error. Due to this parameter we can say how well the model predicts is not up to predicts or over-predicts.

First of all we will calculate errors

```
errors_full = boston_house_target - target_predict
```

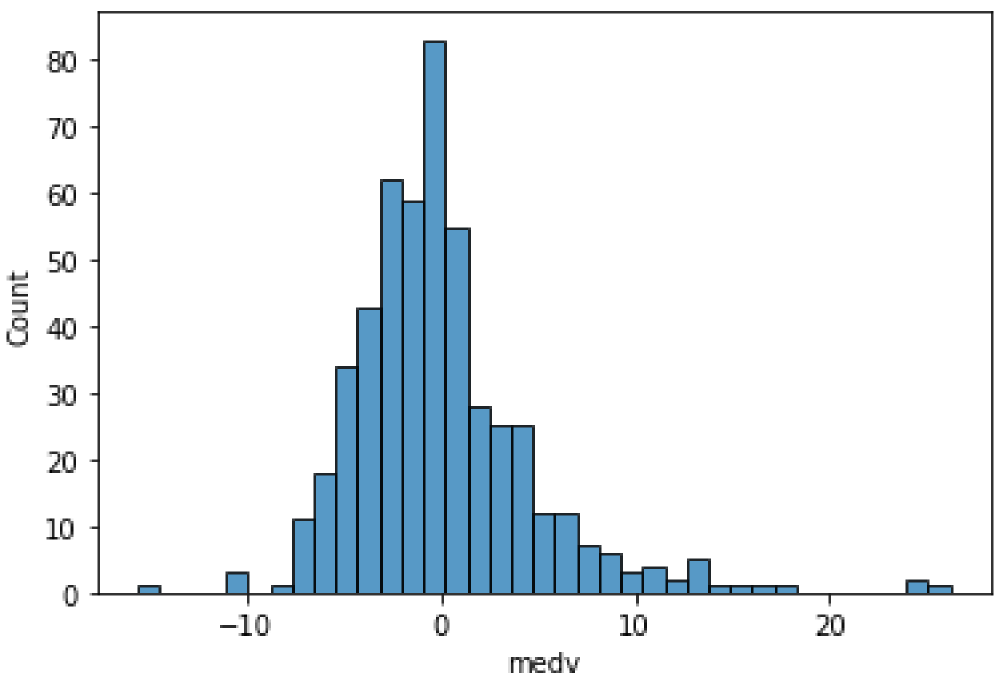
```
scipy.stats.shapiro(errors_full)
```

```
ShapiroResult(statistic=0.9013808369636536,
pvalue=1.4802261404015972e-17)
```

H0 reject - is not normal disribution so very interesting see histplot of errors

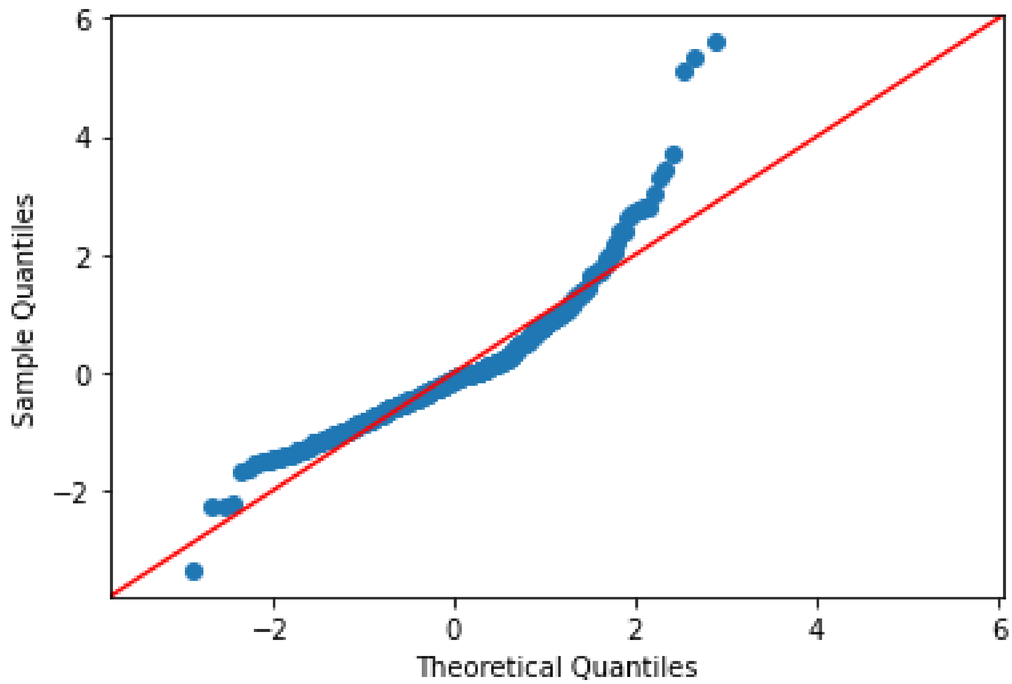
```
sns.histplot(errors_full)
```

```
<AxesSubplot:xlabel='medv', ylabel='Count'>
```



Predictions of our regression model often is not up to predicts

```
figure_errors_ggplot = sm.qqplot(errors_full, line='45', fit=True)
```



QQplot also shows the increasing density of value under zero and some outliersin the highest quartile

Prediction by max value

Find max coefficient for separated prediction

```
max(abs(results_scaled.params[1:]))
```

```
3.7473318548615406
```

```
results_scaled.params
```

```
const      22.532806
crim       -0.929065
zn          1.082639
indus       0.141039
chas        0.682414
nox        -2.058754
rm          2.676877
age         0.019485
dis        -3.107116
rad         2.664852
tax        -2.078837
ptratio    -2.062646
b           0.850109
lstat      -3.747332
dtype: float64
```

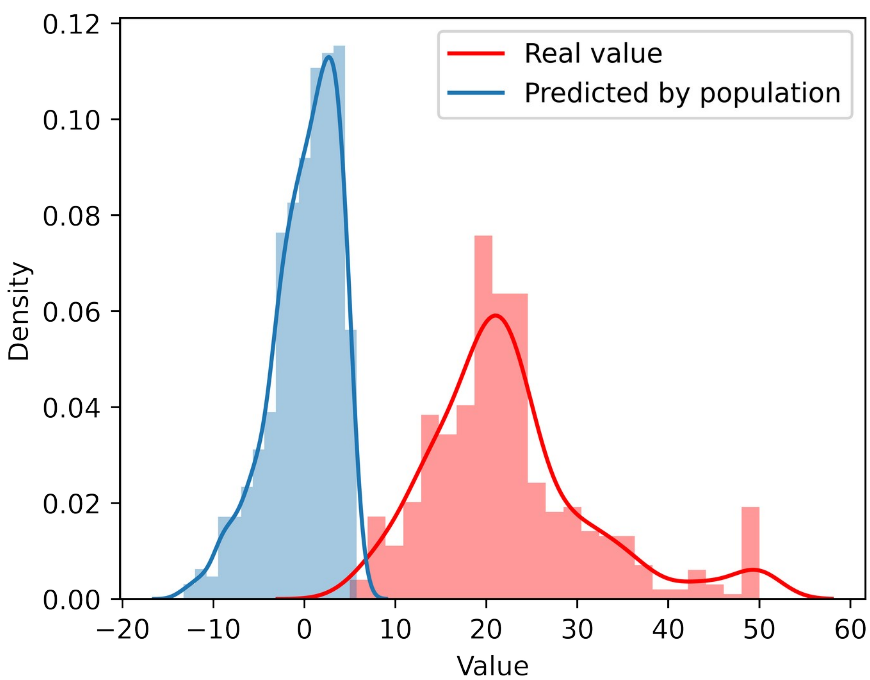
The best abs coefficient consist with lstat feature. So try predict value using only lstat

```
prediction_by_lstat = X.lstat * results_scaled.params[13]
```

```
plt.figure(figsize=[5,4], dpi=700)
sns.distplot(boston_house.medv, color='red')
sns.distplot(prediction_by_lstat)
plt.xlabel('Value')
plt.legend(['Real value', 'Predicted by population'])
```

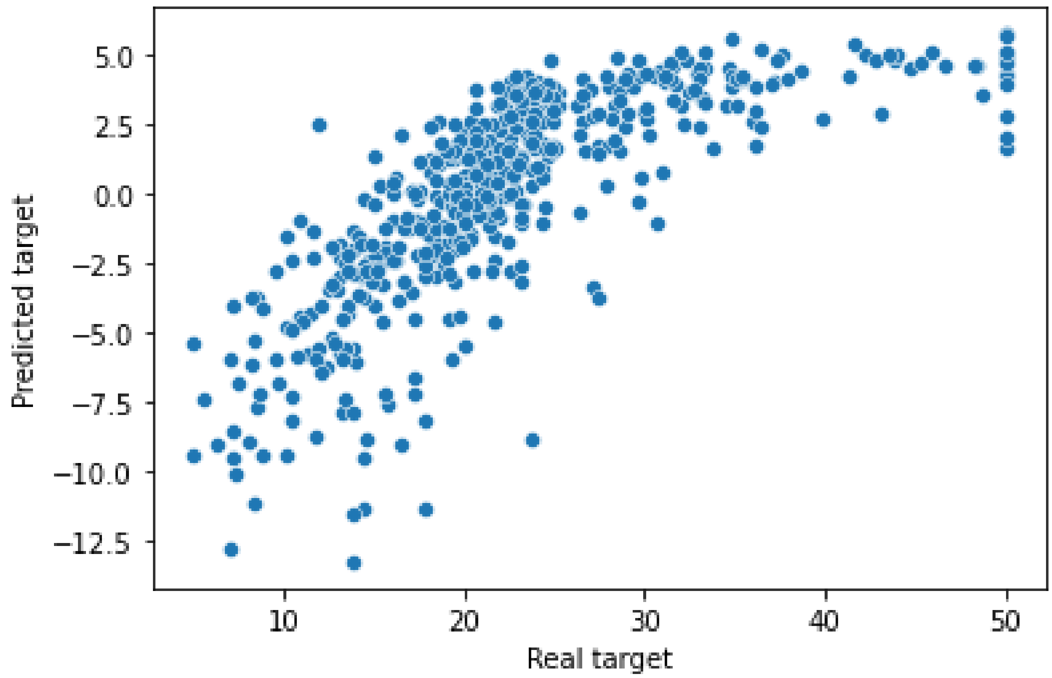
```
C:\Users\ornfi\anaconda3\lib\site-packages\seaborn\
distributions.py:2619: FutureWarning: `distplot` is a deprecated
function and will be removed in a future version. Please adapt your
code to use either `displot` (a figure-level function with similar
flexibility) or `histplot` (an axes-level function for histograms).
  warnings.warn(msg, FutureWarning)
C:\Users\ornfi\anaconda3\lib\site-packages\seaborn\
distributions.py:2619: FutureWarning: `distplot` is a deprecated
function and will be removed in a future version. Please adapt your
code to use either `displot` (a figure-level function with similar
flexibility) or `histplot` (an axes-level function for histograms).
  warnings.warn(msg, FutureWarning)
```

```
<matplotlib.legend.Legend at 0x273b0324a00>
```



The graph demontrates that prediction based only on lstat is under real target value

```
sns.scatterplot(y = prediction_by_lstat, x = boston_house_target);
plt.xlabel("Real target")
plt.ylabel("Predicted target");
```

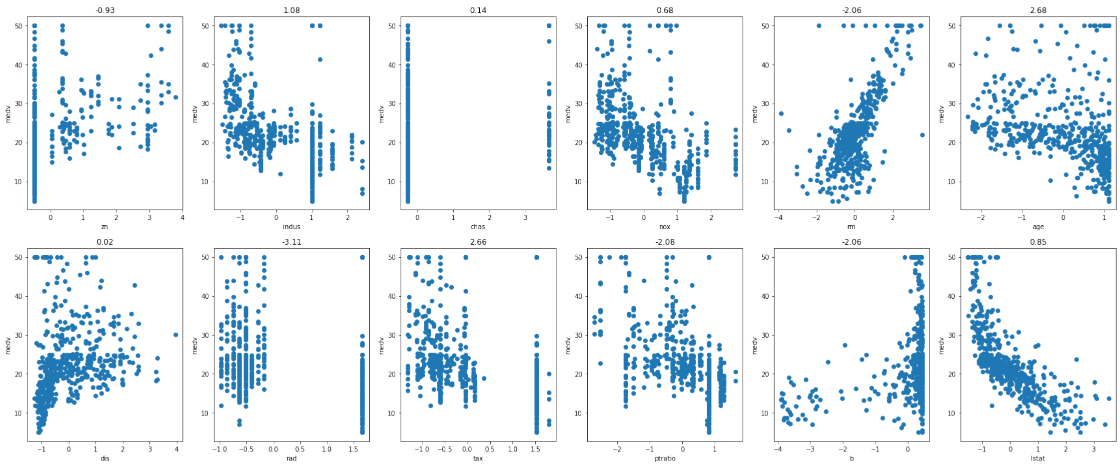


The scatterplot exellently shows that lstat has a linear dependence on the area of values not exceeding 30, after which the prediction does not reflect the real cost of housing

Second Part

Then start to analyse which parameters could help to increase cost of house. To commence with, we can create correlation plot of every standardized features with target

```
name_of_column = list(scaled_features.columns)
column_number = 1

fig, axes = plt.subplots(ncols=6, nrows=2, figsize=(30,12))
for i in range(2):
    for j in range(6):
        ## Store column name in variable
        axes[i, j].scatter(scaled_features.iloc[:,column_number], 
boston_house_target)
        axes[i, j].set_xlabel(name_of_column[column_number])
        axes[i, j].set_ylabel('medv')
        axes[i, 
j].set_title(results_scaled.params[column_number].round(2))
        column_number += 1
```

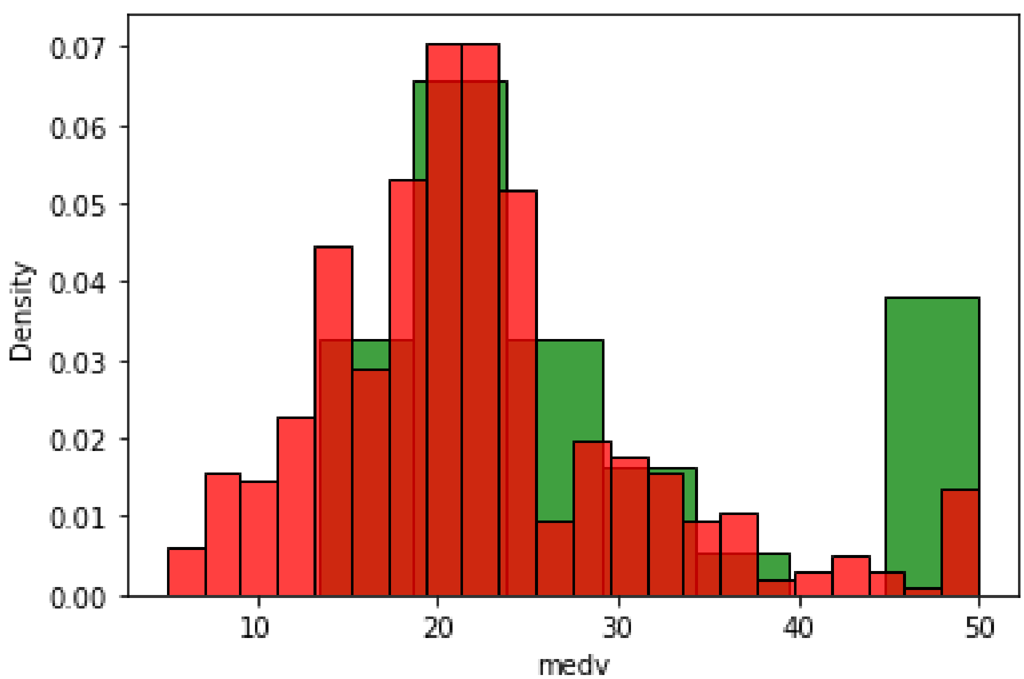


This plot give opportunity to assume what features is better for linear model and what features we have to analyse additionally.

For example, rad and chas are categorical variables so it is worth considering how they affect the value of target.

Divide all value by chas

```
target_no_river = boston_house_target[boston_house["chas"] == 0]
target_river = boston_house_target[boston_house["chas"] == 1]

sns.histplot(target_river, stat="density", color="green")
sns.histplot(target_no_river, stat="density", color="red");
```



The histogram shows that separated targets have abnormal disribution and different outliers. So we have to use nonparametric criteria to understand how chas impacts target

```
scipy.stats.mannwhitneyu(target_river, target_no_river)
```

```
MannwhitneyuResult(statistic=10879.5, pvalue=0.0015816705011294974)
```

The difference between groups is significant

```
target_river.mean(), target_no_river.mean()
```

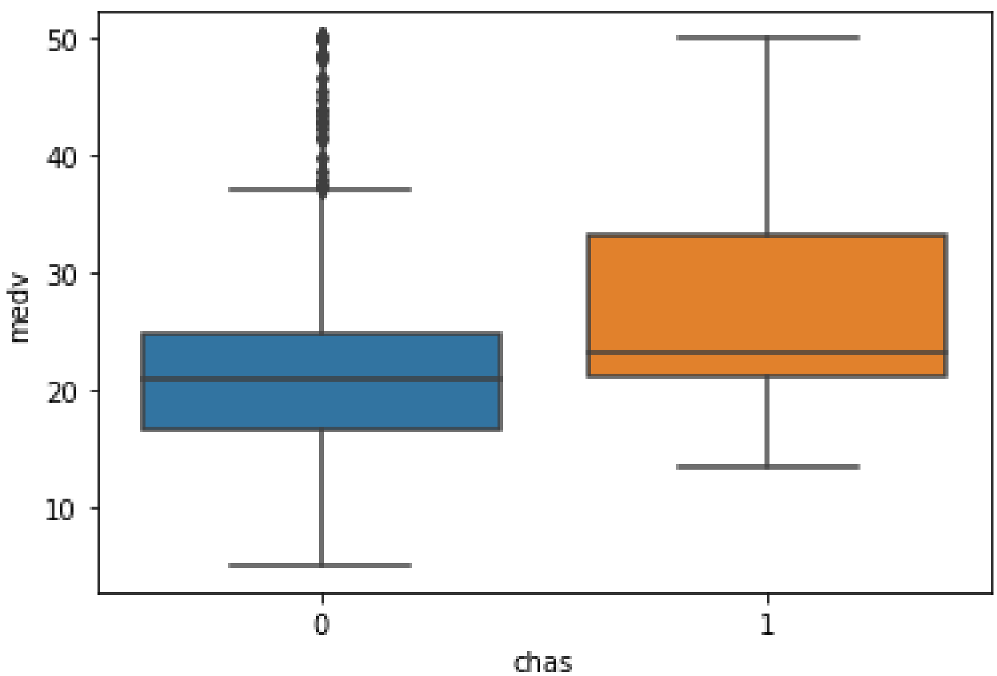
```
(28.44, 22.093842887473482)
```

```
target_river.median(), target_no_river.median()
```

```
(23.3, 20.9)
```

```
sns.boxplot(data=boston_house, x="chas", y="medv")
```

```
<AxesSubplot:xlabel='chas', ylabel='medv'>
```



Housing closed to river obtains higher price

To look at rad feature

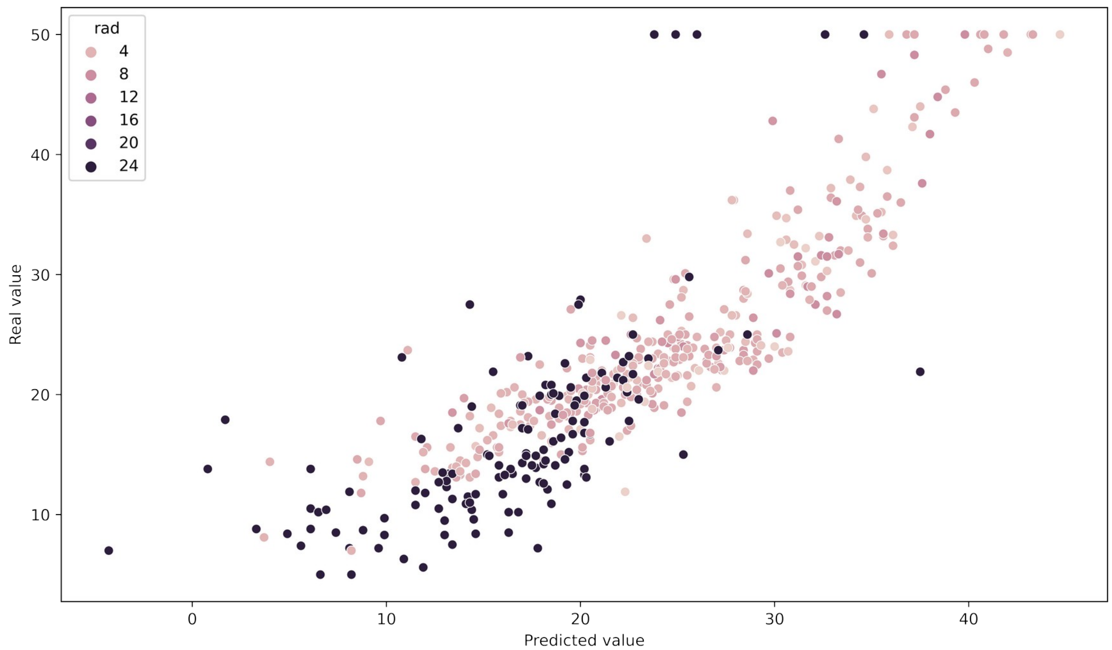
```
boston_house.rad.unique()
```

```
array([ 1,  2,  3,  5,  4,  8,  6,  7, 24], dtype=int64)
```

```
predict_target_rad = pd.DataFrame(list(zip(target_predict.round(1),
boston_house_target, boston_house.rad )),
                                columns =['Predicted target', 'Real
```

```
target', 'rad'])
predict_target_ch
plt.figure(figsize=[12,7], dpi=900)
sns.scatterplot(x='Predicted target', y='Real target',
data=predict_target_rad, hue='rad')
plt.xlabel('Predicted value')
plt.ylabel('Real value')
```

```
Text(0, 0.5, 'Real value')
```



The scatterplot demonstrates that all options except 24 index spread equal so the distance from radial highways reduces cost but at very large differences of indexes.

```
model_updated = sm.OLS(boston_house_target, X.drop(columns=["chas",
"rad"]))
results_updated = model_updated.fit()

print(results_updated.summary())
```

```
                            OLS Regression Results                            

==============================================================================
========
Dep. Variable:                   medv   R-squared:                       
0.722
Model:                            OLS   Adj. R-squared:                  
0.716
Method:                 Least Squares   F-statistic:                     
116.9
Date:                Thu, 15 Dec 2022   Prob (F-statistic):          
8.71e-130
```

```
Time:                        00:18:31   Log-Likelihood:
-1515.9
No. Observations:                 506   AIC:
3056.
Df Residuals:                     494   BIC:
3107.
Df Model:                          11

Covariance Type:            nonrobust

==============================================================================
========
                 coef    std err          t      P>|t|      [0.025
0.975]
------------------------------------------------------------------------------
--------
const         22.5328      0.218    103.465      0.000      22.105
22.961
crim          -0.6030      0.281     -2.144      0.032      -1.155
-0.051
zn             0.9303      0.329      2.831      0.005       0.285
1.576
indus         -0.2880      0.417     -0.691      0.490      -1.108
0.531
nox           -1.6894      0.452     -3.740      0.000      -2.577
-0.802
rm             2.9427      0.299      9.843      0.000       2.355
3.530
age           -0.0526      0.382     -0.137      0.891      -0.804
0.699
dis           -3.1647      0.433     -7.301      0.000      -4.016
-2.313
tax            0.1405      0.402      0.349      0.727      -0.650
0.931
ptratio       -1.8918      0.286     -6.607      0.000      -2.454
-1.329
b              0.8073      0.252      3.200      0.001       0.312
1.303
lstat         -3.7614      0.373    -10.083      0.000      -4.494
-3.028
==============================================================================
========
Omnibus:                      204.480   Durbin-Watson:
0.981
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
1061.249
Skew:                           1.714   Prob(JB):
3.57e-231
Kurtosis:                       9.212   Cond. No.
5.84
```

```
==============================================================================
========

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
```

If we drop categorical variables, it will lead to decreasing of model quality - *Adj. R-squared: 0.716*. So firstly should to estimate VIF

VIF

Function for VIF

```python
from statsmodels.stats.outliers_influence import
variance_inflation_factor

def calculate_vif(X):
    vif_data = pd.DataFrame()
    vif_data["feature"] = X.columns
    vif_data["VIF"] = [variance_inflation_factor(X.values, i)
                        for i in range(len(X.columns))]

    return vif_data

calculate_vif(X)
```

```
    feature       VIF
0     const  1.000000
1      crim  1.792192
2        zn  2.298758
3     indus  3.991596
4      chas  1.073995
5       nox  4.393720
6        rm  1.933744
7       age  3.100826
8       dis  3.955945
9       rad  7.484496
10      tax  9.008554
11  ptratio  1.799084
12        b  1.348521
13    lstat  2.941491
```

Rad really has a huge colinearity but, like attempt, try to drop chas and rad simultaniously

```python
calculate_vif(X.drop(columns=["chas", "rad"]))
```

```
    feature       VIF
0     const  1.000000
1      crim  1.663648
2        zn  2.272992
3     indus  3.660714
```

```
4       nox  4.294324
5        rm  1.880883
6       age  3.077311
7       dis  3.953729
8       tax  3.403205
9   ptratio  1.725085
10        b  1.338875
11    lstat  2.928554
```

It seems like a strange idea so will act subsequently.

Initially delete rad and compare new model and VIF.

```
model_updated_1 = sm.OLS(boston_house_target, X.drop(columns=["rad"]))
results_updated_1 = model_updated_1.fit()

print(results_updated_1.summary())
```

```
                            OLS Regression Results

======================================================================
========
Dep. Variable:                   medv   R-squared:
0.729
Model:                            OLS   Adj. R-squared:
0.723
Method:                 Least Squares   F-statistic:
110.8
Date:                Thu, 15 Dec 2022   Prob (F-statistic):
1.92e-131
Time:                        03:13:52   Log-Likelihood:
-1509.5
No. Observations:                 506   AIC:
3045.
Df Residuals:                     493   BIC:
3100.
Df Model:                          12

Covariance Type:            nonrobust

======================================================================
========
                 coef    std err          t      P>|t|      [0.025
0.975]
----------------------------------------------------------------------
--------
const         22.5328      0.215    104.682      0.000      22.110
22.956
crim          -0.5810      0.278     -2.090      0.037      -1.127
-0.035
zn             0.9264      0.325      2.852      0.005       0.288
```

```
1.565
indus         -0.4007      0.413     -0.969      0.333      -1.213
0.412
chas           0.7910      0.222      3.563      0.000       0.355
1.227
nox           -1.7685      0.447     -3.956      0.000      -2.647
-0.890
rm             2.8910      0.296      9.772      0.000       2.310
3.472
age           -0.1105      0.378     -0.292      0.770      -0.854
0.633
dis           -3.1378      0.428     -7.323      0.000      -3.980
-2.296
tax            0.2249      0.398      0.565      0.573      -0.557
1.007
ptratio       -1.8158      0.284     -6.398      0.000      -2.373
-1.258
b              0.7683      0.250      3.079      0.002       0.278
1.259
lstat         -3.6878      0.369     -9.986      0.000      -4.413
-2.962
======================================================================
========
Omnibus:                      189.296   Durbin-Watson:
1.059
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
907.772
Skew:                           1.596   Prob(JB):
7.58e-198
Kurtosis:                       8.733   Cond. No.
5.84
======================================================================
========

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
```

```
calculate_vif(X.drop(columns=["rad"]))
```

```
    feature       VIF
0     const  1.000000
1      crim  1.664471
2        zn  2.273018
3     indus  3.682265
4      chas  1.061561
5       nox  4.304929
6        rm  1.885425
7       age  3.083009
8       dis  3.954951
```

```
9        tax  3.415289
10   ptratio  1.734873
11         b  1.341459
12     lstat  2.937752
```

It reduced the quality of the model (Adj. R-squared: 0.723) and imrove VIF. Since our goal to reveal the most important feature (not prediction) we can allow to sacrifice the quality of the model for discovering hidden dependencies

```
calculate_vif(X.drop(columns=["rad", "dis"]))
```

```
    feature       VIF
0     const  1.000000
1      crim  1.638205
2        zn  1.897074
3     indus  3.511704
4      chas  1.061233
5       nox  3.949993
6        rm  1.849614
7       age  2.821975
8       tax  3.414851
9   ptratio  1.720096
10        b  1.340875
11    lstat  2.933530
```

```
calculate_vif(X.drop(columns=["nox", "rad", "dis"]))
```

```
    feature       VIF
0     const  1.000000
1      crim  1.637882
2        zn  1.826807
3     indus  3.155735
4      chas  1.057774
5        rm  1.847368
6       age  2.430656
7       tax  3.073277
8   ptratio  1.504126
9         b  1.331457
10    lstat  2.920516
```

etc... I will skip how I did each stage, but in the end it turned out the following

```
calculate_vif(X.drop(columns=["tax", "indus", "nox", "rad", "zn",
"lstat", "dis"]))
```

```
   feature       VIF
0    const  1.000000
1     crim  1.333301
2     chas  1.041442
3       rm  1.190387
4      age  1.254365
```

```
5  ptratio  1.247573
6        b  1.209671
```

```
model_updated_3 = sm.OLS(boston_house_target, X.drop(columns=["tax",
"indus", "nox", "rad", "zn", "lstat", "dis"]))
results_updated_3 = model_updated_3.fit()

print(results_updated_3.summary())
```

```
                            OLS Regression Results

======================================================================
========
Dep. Variable:                   medv   R-squared:
0.637
Model:                            OLS   Adj. R-squared:
0.633
Method:                 Least Squares   F-statistic:
145.9
Date:                Thu, 15 Dec 2022   Prob (F-statistic):
2.18e-106
Time:                        01:02:46   Log-Likelihood:
-1583.9
No. Observations:                 506   AIC:
3182.
Df Residuals:                     499   BIC:
3211.
Df Model:                           6

Covariance Type:            nonrobust

======================================================================
========
                 coef    std err          t      P>|t|      [0.025
0.975]
----------------------------------------------------------------------
--------
const         22.5328      0.248     90.913      0.000      22.046
23.020
crim          -0.9323      0.286     -3.254      0.001      -1.495
-0.369
chas           0.8968      0.253      3.542      0.000       0.399
1.394
rm             4.9544      0.271     18.303      0.000       4.423
5.486
age           -1.1305      0.278     -4.069      0.000      -1.676
-0.585
ptratio       -1.9914      0.277     -7.186      0.000      -2.536
-1.447
b              1.3671      0.273      5.010      0.000       0.831
```

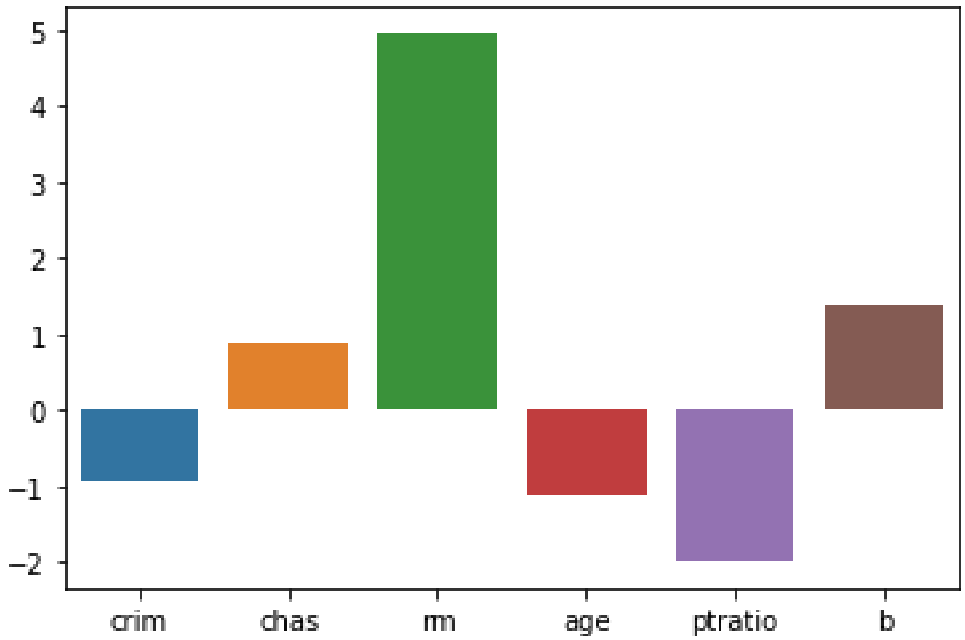
```
1.903
======================================================================
========
Omnibus:                      283.471   Durbin-Watson:
0.863
Prob(Omnibus):                  0.000   Jarque-Bera (JB):
3000.941
Skew:                           2.236   Prob(JB):
0.00
Kurtosis:                      14.060   Cond. No.
1.90
======================================================================
========

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is
correctly specified.
```

It's decrease the model quality (Adj. R-squared: 0.633) but help to idetify the most crucial featuries.

Then create barplot of every coefficient without constant

```
sns.barplot(results_updated_3.params.index[1:],
results_updated_3.params[1:])
```

```
C:\Users\ornfi\anaconda3\lib\site-packages\seaborn\_decorators.py:36:
FutureWarning: Pass the following variables as keyword args: x, y.
From version 0.12, the only valid positional argument will be `data`,
and passing other arguments without an explicit keyword will result in
an error or misinterpretation.
  warnings.warn(

<AxesSubplot:>
```



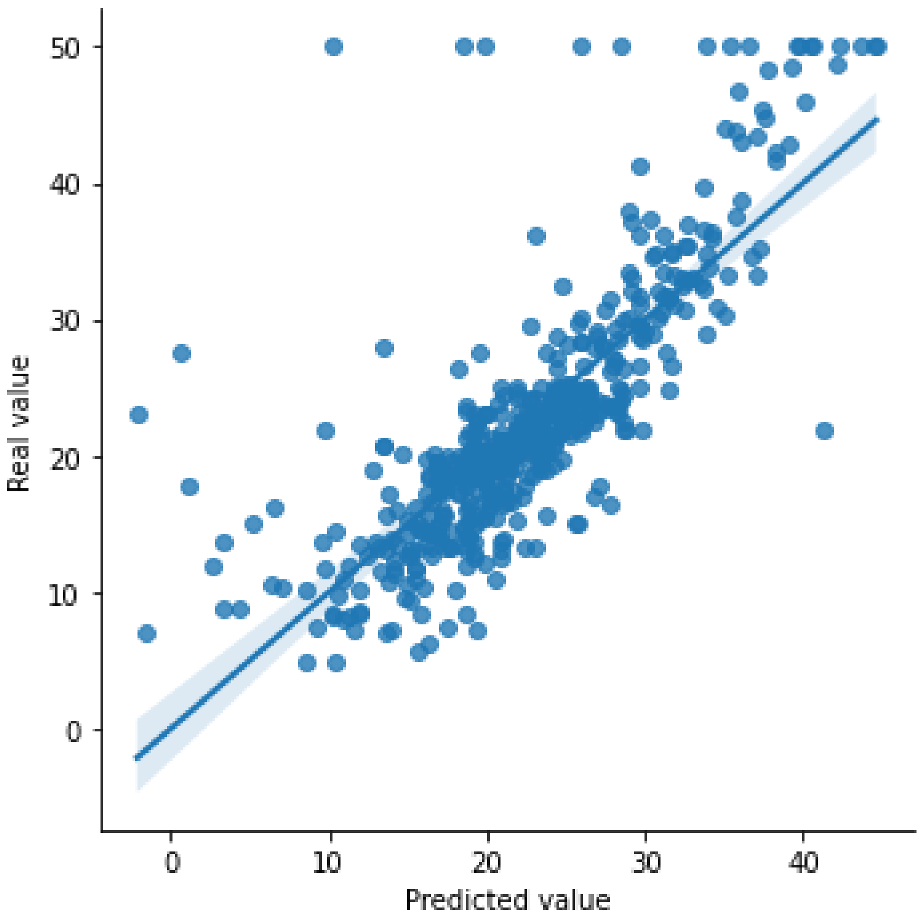
Try to compare the prediction of target by current model and error distribution

```
prediction_ult =
results_updated_3.get_prediction(X.drop(columns=["tax", "indus",
"nox", "rad", "zn", "lstat", "dis"]))
target_predict_ult = prediction_ult.predicted_mean

predict_target_ult =
pd.DataFrame(list(zip(target_predict_ult.round(1),
boston_house_target)),
                                columns =['Predicted target', 'Real
target'])
predict_target_ult

     Predicted target  Real target
0                28.4         24.0
1                24.4         21.6
2                30.5         34.7
3                29.0         33.4
4                29.7         36.2
..                ...          ...
501              23.0         22.4
502              19.4         20.6
503              24.9         23.9
504              23.6         22.0
505              18.7         11.9

[506 rows x 2 columns]

plt.figure(figsize=[12,7], dpi=900)
sns.lmplot(x='Predicted target', y='Real target',
```

```
data=predict_target_ult, order=1)
plt.xlabel('Predicted value')
plt.ylabel('Real value')
```

```
Text(24.05000000000001, 0.5, 'Real value')
```

```
<Figure size 10800x6300 with 0 Axes>
```



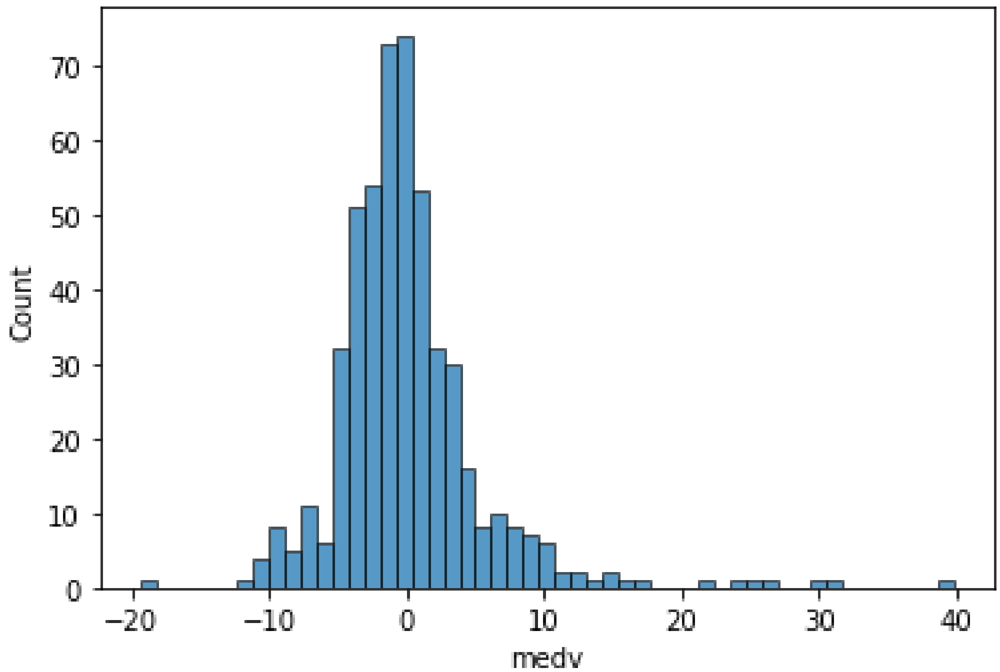

```
errors_upd = boston_house_target - target_predict_ult
```

```
scipy.stats.shapiro(errors_upd)  #H0 reject - isn't normal disribution
```

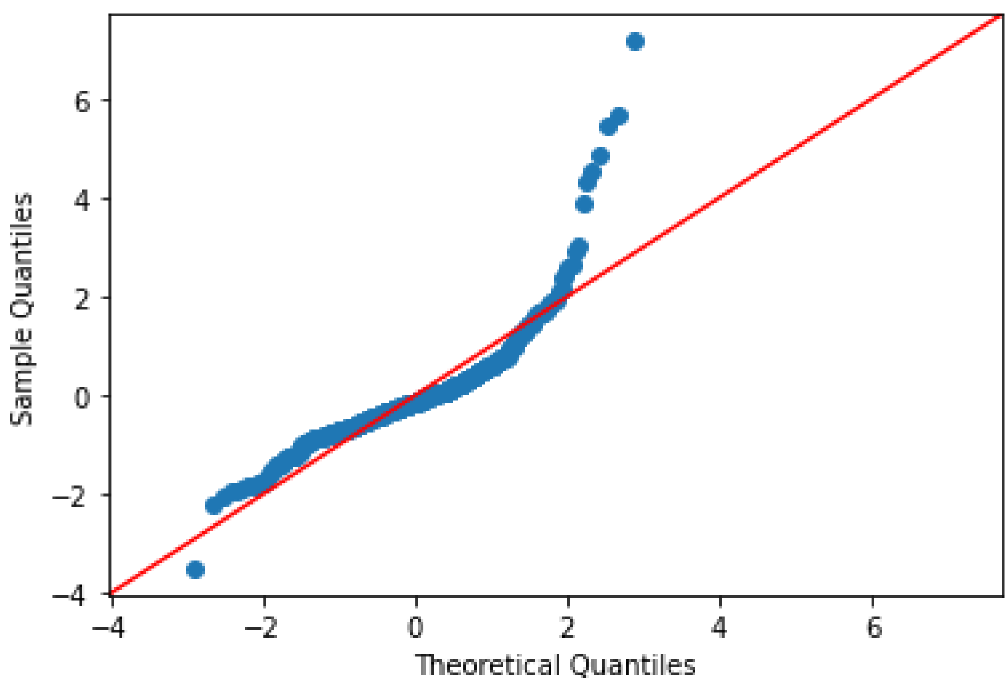
```
ShapiroResult(statistic=0.8326656818389893,
pvalue=1.2416456078012506e-22)
```

```
sns.histplot(errors_upd)
```

```
<AxesSubplot:xlabel='medv', ylabel='Count'>
```




```
result = sm.qqplot(errors_upd, line='45', fit=True)
```



As result the prediction became worse and error distribution also reflects this observation.

However, we can identify the important for prices features:

- the house price will become higher if tract bounds river
- the lower the crime rate per capita - the more expensive the house will be

- average number of rooms per dwelling: the more rooms - the higher price
- proportion of owner-occupied units built prior to 1940 has a negative correlation with price
- increasing of pupil-teacher ratio leads to decreasing of price
- the more people of African-American descent, the higher the price