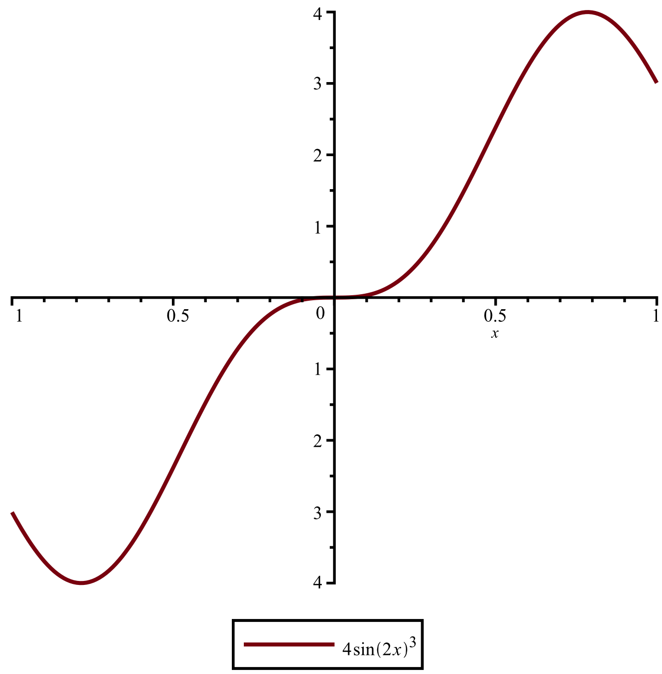
> *#Лабораторная работа 2 (Вариант 10)*
#Мартинович Андрей Александрович

#гр. 353503

> **#Задание 4. Разложите функцию в ряд Фурье по многочленам Лежандра и Чебышёва на промежутке $[1, 1]$.**

> $f := 4 \cdot \sin^3(2 \cdot x)$:
$our\_function := plot(f(x), x = -1 .. 1, discont = true, legend = f)$



> $with(orthopoly)$

$$[G, H, L, P, T, U] \qquad (1)$$

> # *#По многочлену Лежандра*

> **for** n **from** 0 **to** 11 **do** $c[n] := \dfrac{\int_{1}^{1} f \cdot P(n, x)\, \mathrm{d}x}{\int_{1}^{1} P(n, x)^2 \mathrm{d}x}$; **end do**

$$c_0 := 0$$

$$c_1 := \quad 2 \sin(2)^2 \cos(2) \quad 4 \cos(2) + \frac{\sin(2)^3}{3} + 2 \sin(2)$$

$$c_2 := 0$$

$$c_3 := \frac{49 \sin(2)^2 \cos(2)}{18} + \frac{266 \cos(2)}{9} + \frac{77 \sin(2)}{9} + \frac{469 \sin(2)^3}{108}$$

$$c_4 := 0$$

$$c_5 := \frac{6215 \sin(2)}{24} \quad \frac{6721 \cos(2)}{12} + \frac{209 \sin(2)^2 \cos(2)}{24} + \frac{715 \sin(2)^3}{144}$$

$$c_6 := 0$$

$$c_7 := -\frac{8395\sin(2)^2\cos(2)}{864} - \frac{123305\sin(2)^3}{5184} + \frac{2499805\sin(2)}{216} + \frac{681785\cos(2)}{27}$$

$$c_8 := 0$$

$$c_9 := \frac{1216361\sin(2)^2\cos(2)}{31104} + \frac{24758995\sin(2)^3}{186624} - \frac{27957486065\sin(2)}{31104} - \frac{30540881599\cos(2)}{15552}$$

$$c_{10} := 0$$

$$c_{11} := -\frac{149468881\sin(2)^2\cos(2)}{373248} - \frac{3081363659\sin(2)^3}{2239488} + \frac{19795216570943\sin(2)}{186624} + \frac{21626467593307\cos(2)}{93312} \quad \textbf{(2)}$$

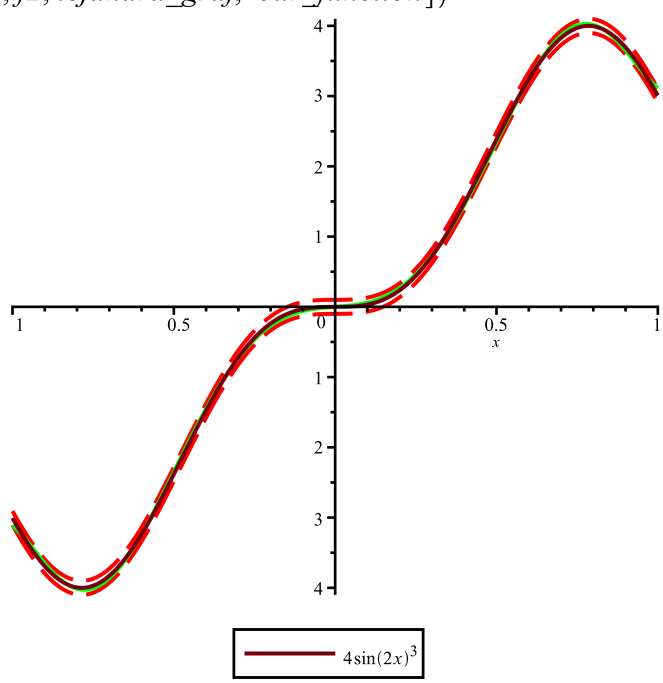
> *lejandra_graf* := *plot*(*add*(*c*[*n*]·*P*(*n*, *x*), *n* = 0 ..7), *x* =-1 ..1, *color* = *green*) :

>

> *f1* := *plot*(*f* + 0.1, *x* =-1 ..1, *linestyle* = *dash*, *color* = *red*) :

> *f2* := *plot*(*f* − 0.1, *x* =-1 ..1, *linestyle* = *dash*, *color* = *red*) :

> *plots*[*display*]([*f1*, *f2*, *lejandra_graf*, *our_function*])

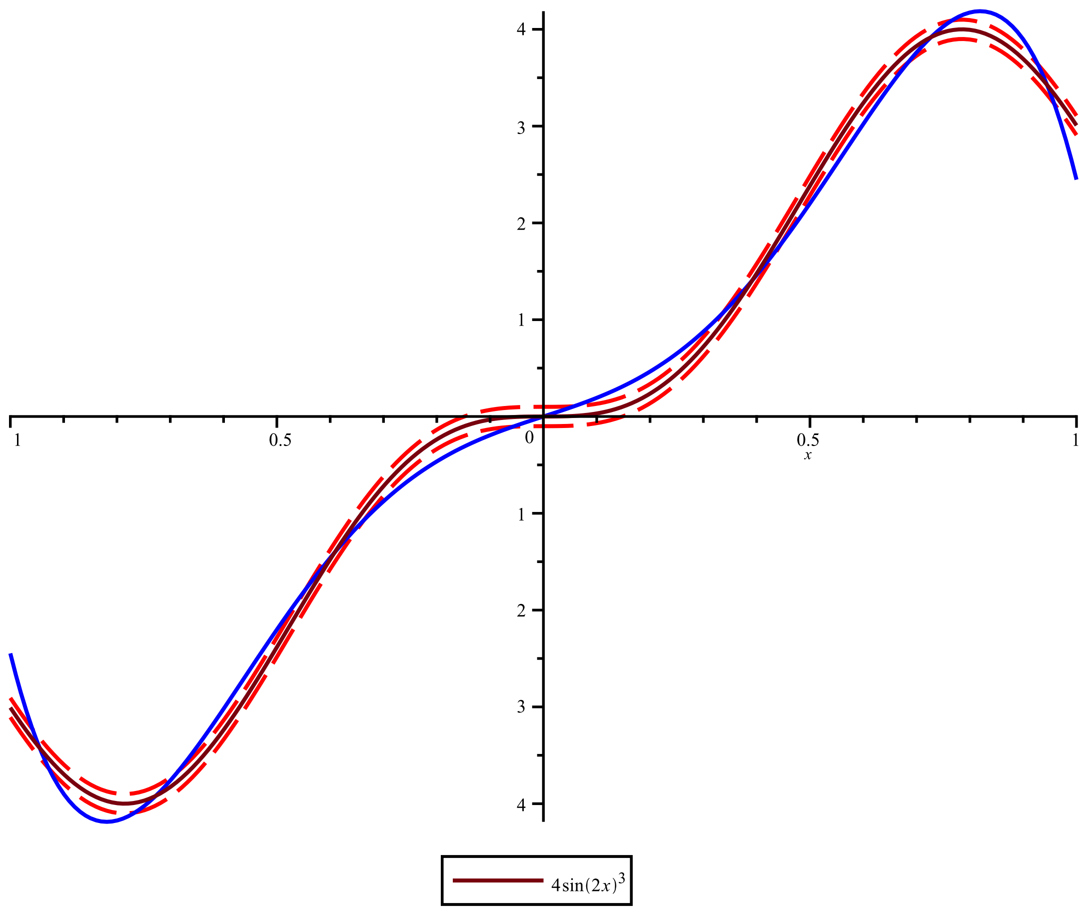


>

> *nmin* := *plot*(*add*(*c*[*n*]·*P*(*n*, *x*), *n* = 0 ..6), *x* = 1 ..1, *color* = *blue*) :

> *plots*[*display*](*f1*, *f2*, *nmin*, *our_function*)

#можем заметить**,** что когда n = 6 функция отклоняется больше чем на 0, 1 (***Лежандр***)



> **#По многочлену Чебышёва**

> **for** n **from** 0 **to** 11 **do** $c[n] := \dfrac{\displaystyle\int_{1}^{1} \frac{f \cdot T(n,x)}{\mathrm{sqrt}(1 \quad x^2)}\,\mathrm{d}x}{\displaystyle\int_{1}^{1} \frac{T(n,x)^2}{\mathrm{sqrt}(1 \quad x^2)}\,\mathrm{d}x}$; **end do**

$$c_0 := 0$$

$$c_1 := \frac{2\left(\displaystyle\int_{-1}^{1} \frac{4\sin(2x)^3 x}{\sqrt{\quad x^2+1}}\,\mathrm{d}x\right)}{\pi}$$

$$c_2 := 0$$

$$c_3 := \frac{2\left(\displaystyle\int_{-1}^{1} \frac{4\sin(2x)^3 (4x^3 \quad 3x)}{\sqrt{\quad x^2+1}}\,\mathrm{d}x\right)}{\pi}$$

$$c_4 := 0$$

$$c_5 := \frac{2\left(\int_{-1}^{1} \frac{4\sin(2x)^3\left(16x^5-20x^3+5x\right)}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi}$$

$$c_6 := 0$$

$$c_7 := \frac{2\left(\int_{-1}^{1} \frac{4\sin(2x)^3\left(64x^7-112x^5+56x^3-7x\right)}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi}$$

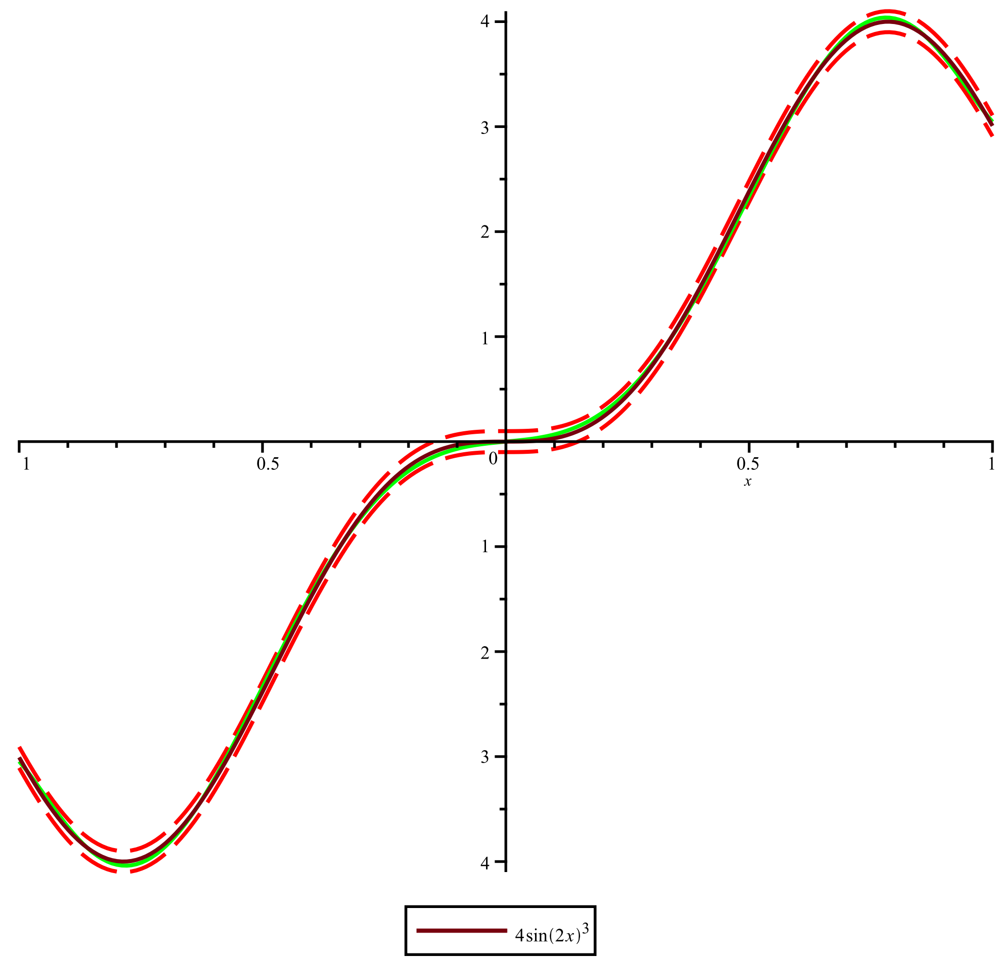
$$c_8 := 0$$

$$c_9 := \frac{2\left(\int_{-1}^{1} \frac{4\sin(2x)^3\left(256x^9-576x^7+432x^5-120x^3+9x\right)}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi}$$

$$c_{10} := 0$$

$$c_{11} := \frac{2\left(\int_{-1}^{1} \frac{4\sin(2x)^3\left(1024x^{11}-2816x^9+2816x^7-1232x^5+220x^3-11x\right)}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi} \quad \textbf{(3)}$$

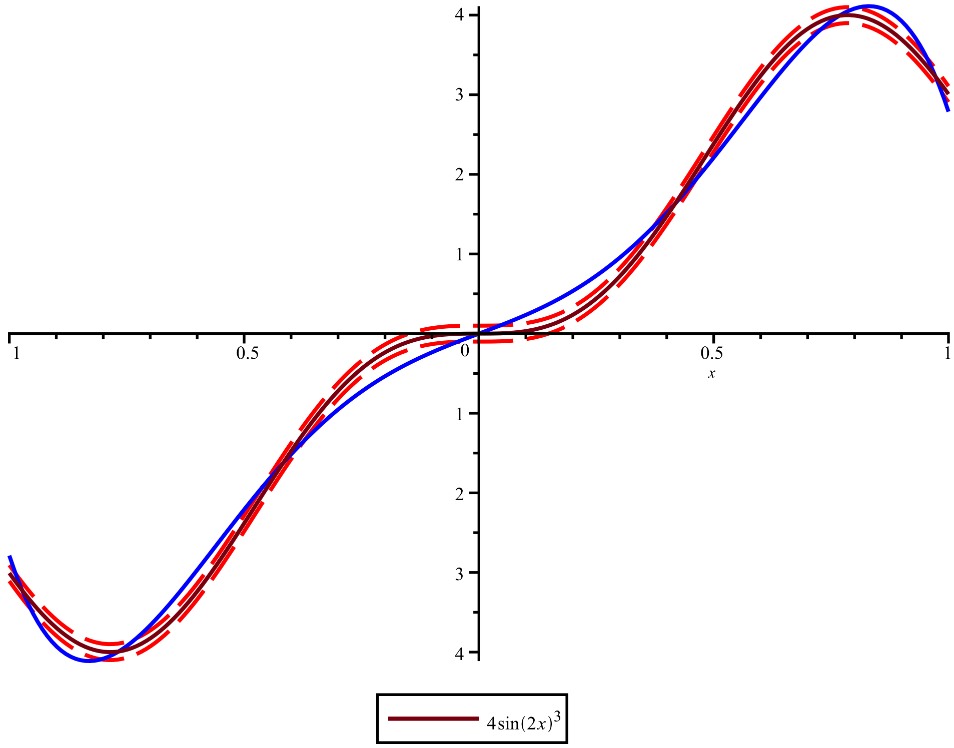
```
> cheb_graf := plot(add(c[n]·T(n, x), n = 1 ..7), x =-1 ..1, color = green) :
> plots[display](f1, f2, cheb_graf, our_function)
```



> $nmin := plot(add(c[n]\cdot T(n, x), n = 1 ..6), x = \ 1 ..1, color = blue) :$

> $plots[display](f1, f2, nmin, our\_function)$

#можем заметить**,** что когда n = 10 функция отклоняется больше чем на 0,
1 (***Чебышев***)



> # #Тригонометрический ряд Фурье

> $bn := simplify(\ int(\mathrm{f}\cdot\sin(\mathrm{Pi}\cdot m\cdot x), x = \ 1..1))$ assuming $m :: posint$

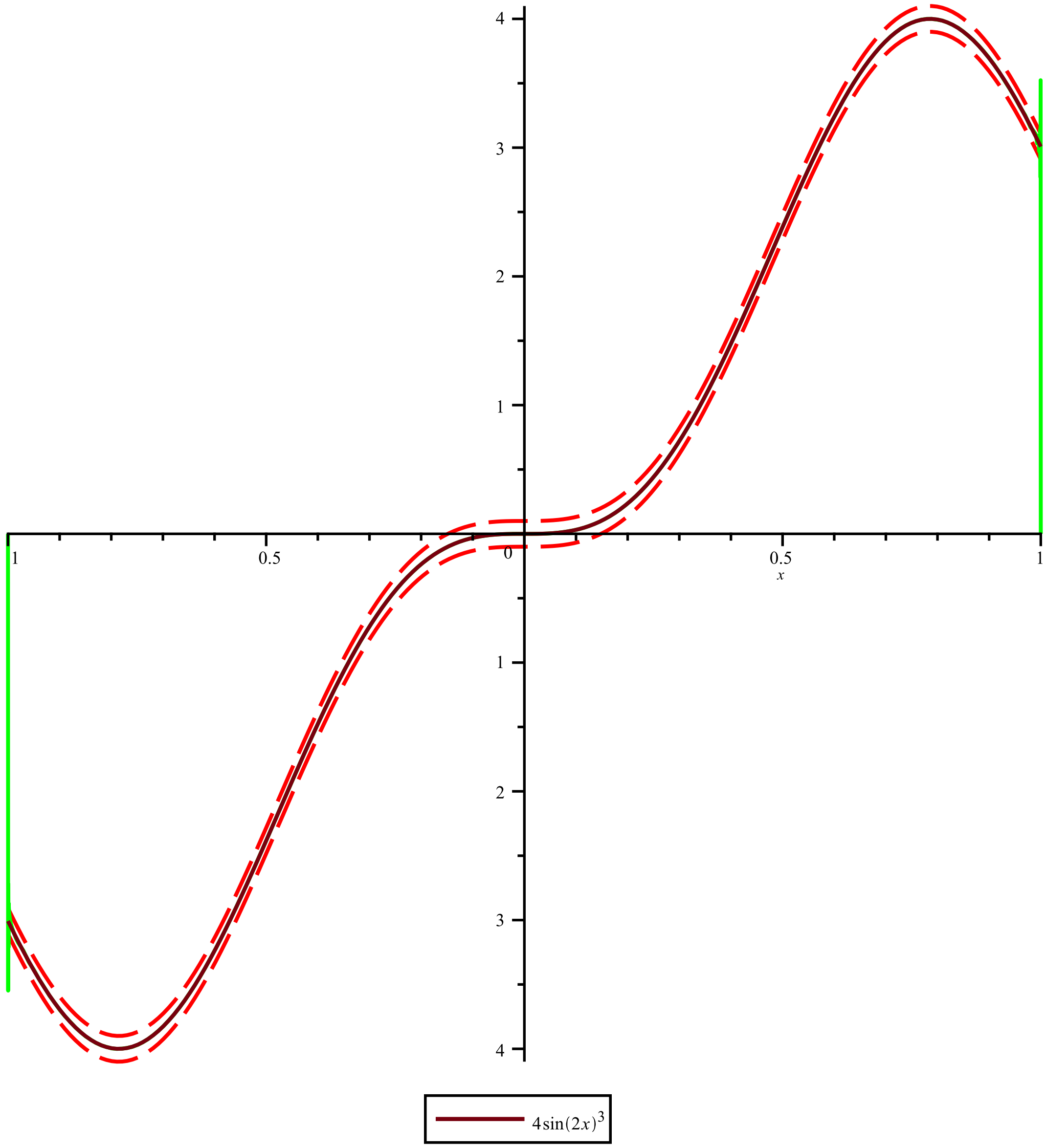
$$bn := \frac{6\, m\, (-1)^m\, \pi \left(\pi^2 \sin(2)\, m^2 \quad \frac{\pi^2 \sin(6)\, m^2}{3} \quad 36 \sin(2) + \frac{4 \sin(6)}{3}\right)}{\pi^4\, m^4 \quad 40\, \pi^2\, m^2 + 144} \tag{4}$$

> $Sm := \mathrm{k} \rightarrow sum(bn\cdot\sin(\pi\cdot\mathrm{m}\cdot\mathrm{x}), m = 1..k)$

$$Sm := k \mapsto \sum_{m=1}^{k} bn\cdot\sin(\pi\cdot m\cdot x) \tag{5}$$

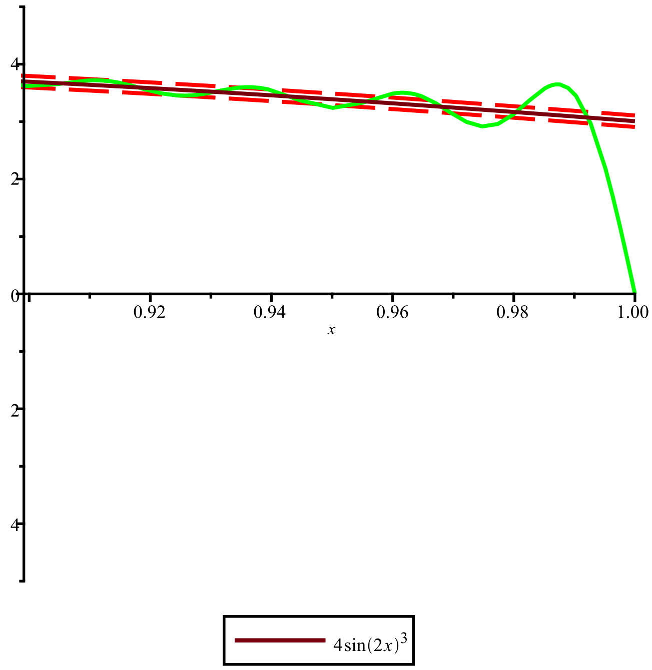
> $fur := plot(Sm(3000), x = \ 1..1, discont = true, color = green)$:

> $plots[display]([f1, f2, fur, our\_function]);$



> $minFur := plot(Sm(79), x = 1 ..1, discont = true, color = green) :$

> $plots[display](f1, f2, minFur, our\_function, view = [0.9 ..1, 5 ..5])$



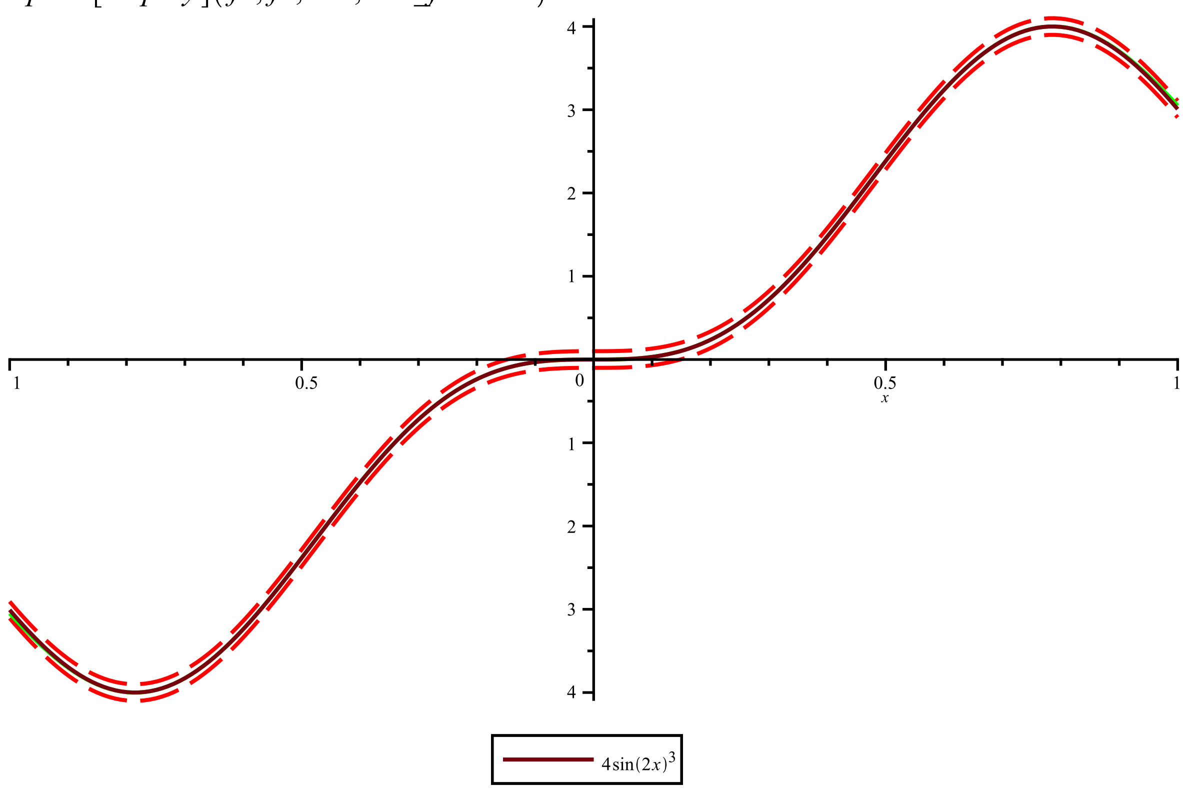
> # #Ряд Тейлора

> *St* := *convert*(*taylor*(*f*, *x* = 0, 16), *polynom*)

$$St := 32\,x^3 \quad 64\,x^5 + \frac{832}{15}\,x^7 \quad \frac{5248}{189}\,x^9 + \frac{42944}{4725}\,x^{11} \quad \frac{9344}{4455}\,x^{13} + \frac{76527488}{212837625}\,x^{15} \tag{6}$$

> *StF* := *plot*(*St*, *x* = 1 ..1, *color* = *green*) :

> *plots*[*display*](*f1*, *f2*, *StF*, *our_function*)



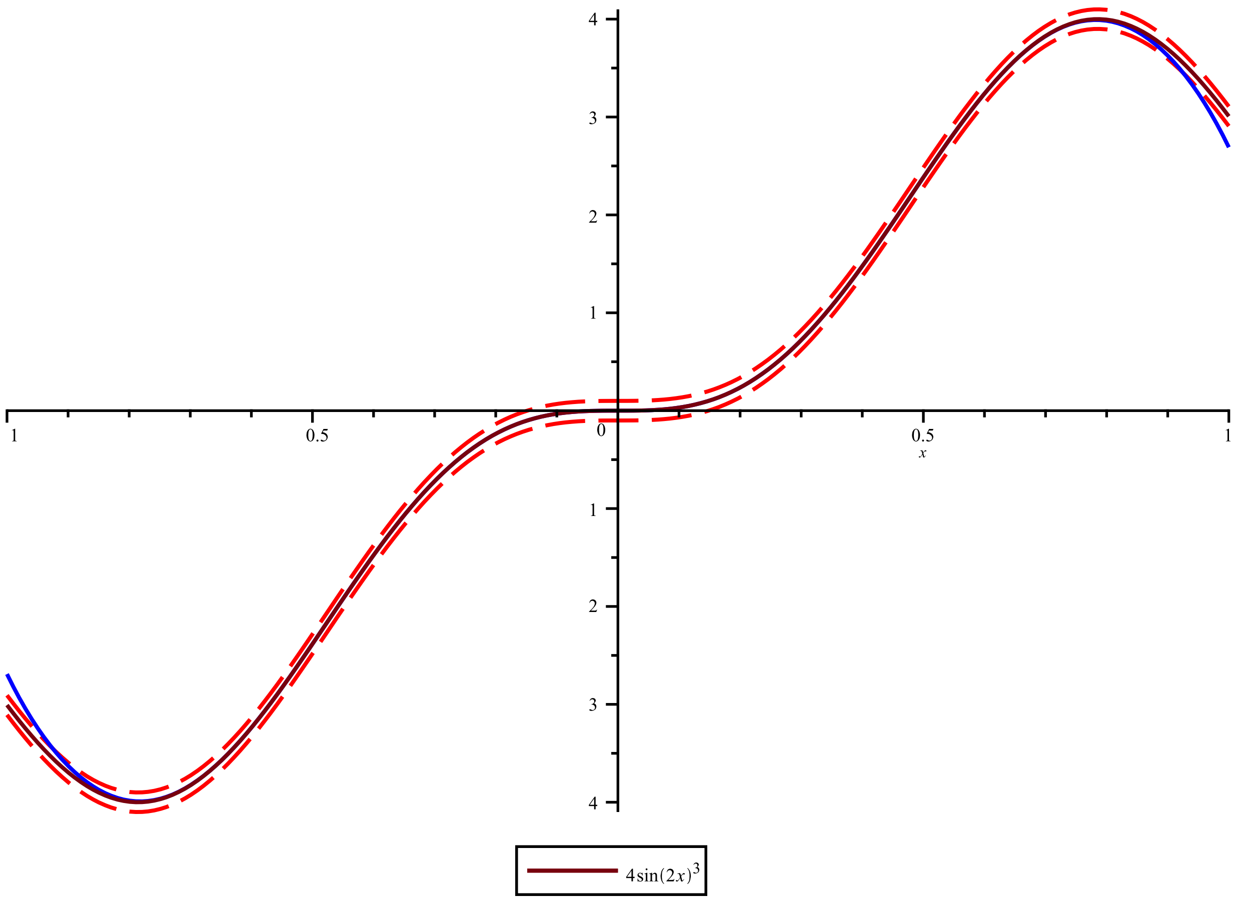
> *St* := *convert*(*taylor*(*f*, *x* = 0, 15), *polynom*)

(7)

$$St := 32\,x^3 - 64\,x^5 + \frac{832}{15}\,x^7 - \frac{5248}{189}\,x^9 + \frac{42944}{4725}\,x^{11} - \frac{9344}{4455}\,x^{13} \tag{7}$$

> $StF := plot(St, x=-1\,..1, color=blue)$:

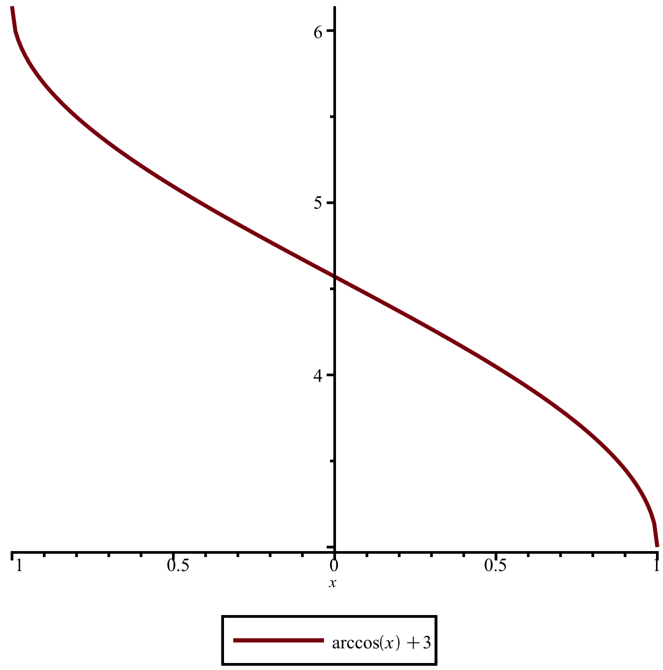
> $plots[display](f1, f2, StF, our\_function)$# при x =0..15 (**Тейлор**)



> *restart*

> $f := \arccos(x) + 3$:

> $our\_function1 := plot(f, x=\ 1\,..1, legend=f)$



> **#По многочлену Лежандра**

> *with*(*orthopoly*)

$$[G, H, L, P, T, U] \qquad (8)$$

> **for** n **from** 0 **to** 11 **do** $c[n] := \dfrac{\int_{1}^{1} f \cdot P(n, x)\, \mathrm{d}x}{\int_{1}^{1} P(n, x)^2 \mathrm{d}x}$; **end do**

$$c_0 := 3 + \frac{\pi}{2}$$

$$c_1 := \frac{3\,\pi}{8}$$

$$c_2 := 0$$

$$c_3 := \frac{7\,\pi}{128}$$

$$c_4 := 0$$

$$c_5 := \frac{11\,\pi}{512}$$

$$c_6 := 0$$

$$c_7 := \frac{375\,\pi}{32768}$$

$$c_8 := 0$$

$$c_9 := -\frac{931\,\pi}{131072}$$

$$c_{10} := 0$$

$$c_{11} := -\frac{10143\,\pi}{2097152} \tag{9}$$

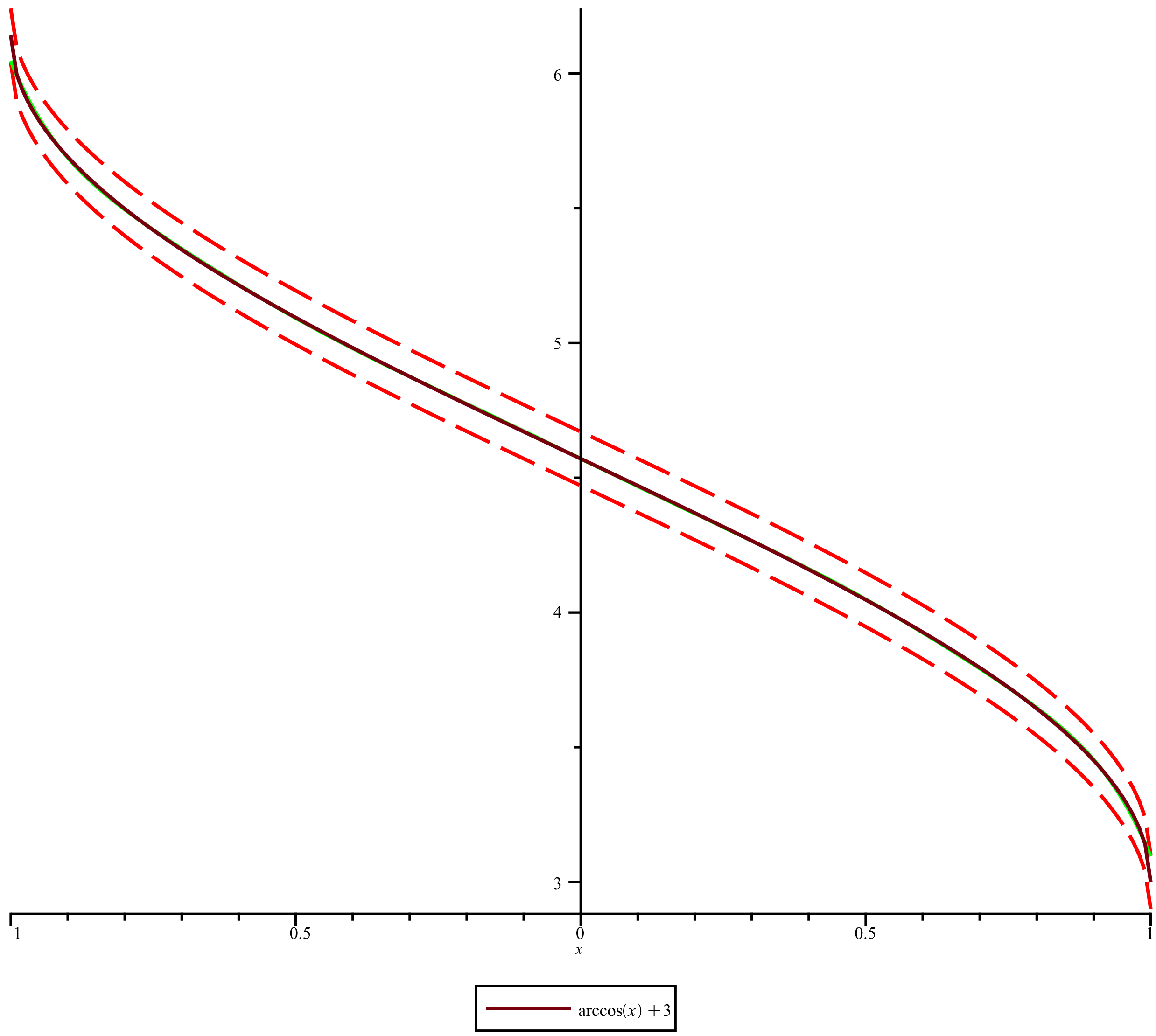
> $lejandra\_graf1 := plot(add(c[n]\cdot P(n, x), n = 0\,..9), x = -1\,..1, color = green)$:

>

> $f1 := plot(f + 0.1, x = -1\,..1, linestyle = dash, color = red)$:

> $f2 := plot(f - 0.1, x = -1\,..1, linestyle = dash, color = red)$:

> $plots[display]([f1, f2, lejandra\_graf1, our\_function1])$

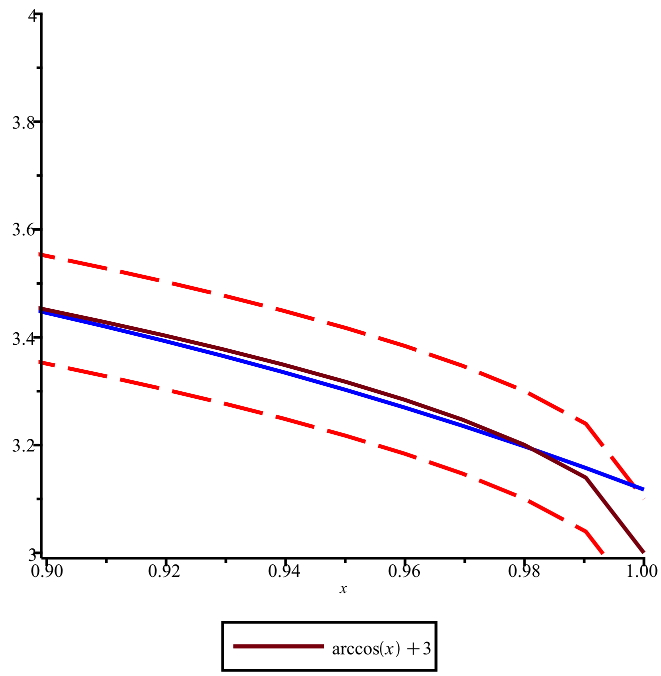


>

> $nmin1 := plot(add(c[n]\cdot P(n, x), n = 0\,..8), x = \ 1\,..1, color = blue)$:

> $plots[display](f1, f2, nmin1, our\_function1, view = [0.9\,..1, 3\,..4])$

#можем заметить, что когда n = 8 функция отклоняется больше чем на 0, 1 (***Лежандр***)



> ### *#По многочлену Чебышёва*

> **for** n **from** 0 **to** 10 **do** $c[n] := \dfrac{\displaystyle\int_{1}^{1} \frac{f \cdot T(n,x)}{\mathrm{sqrt}(1 \quad x^2)}\,\mathrm{d}x}{\displaystyle\int_{1}^{1} \frac{T(n,x)^2}{\mathrm{sqrt}(1 \quad x^2)}\,\mathrm{d}x}$; **end do**

$$c_0 := \frac{\frac{1}{2}\pi^2 + 3\pi}{\pi}$$

$$c_1 := \frac{4}{\pi}$$

$$c_2 := 0$$

$$c_3 := \frac{4}{9\pi}$$

$$c_4 := 0$$

$$c_5 := \frac{4}{25\pi}$$

$$c_6 := 0$$

$$c_7 := \frac{4}{49\pi}$$

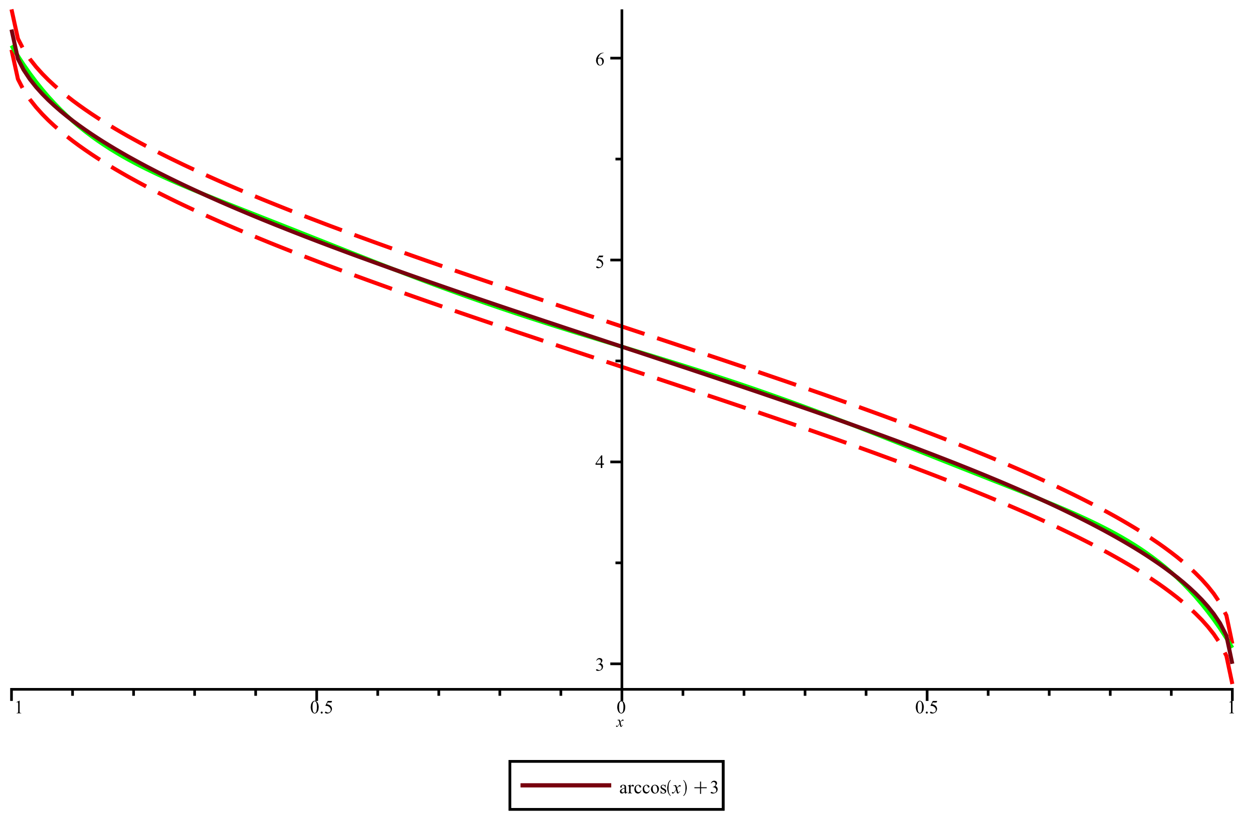
$$c_8 := 0$$

$$c_9 := -\frac{4}{81\,\pi}$$

$$c_{10} := 0 \tag{10}$$

> $cheb\_graf1 := plot(add(c[n] \cdot T(n, x), n = 0 ..7), x = -1 ..1, color = green)$:

> $plots[display](f1, f2, cheb\_graf1, our\_function1)$

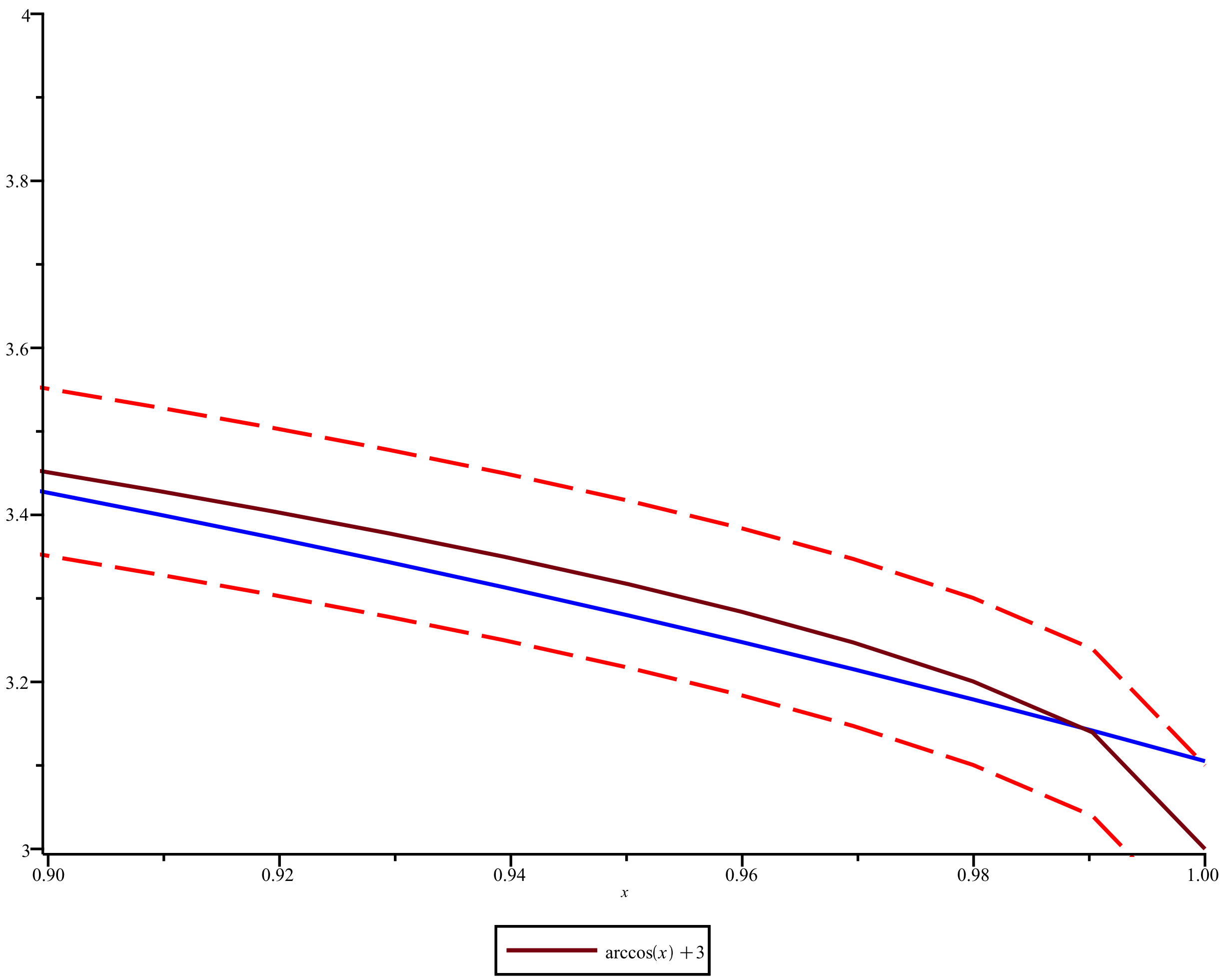


>

> $nmin1 := plot(add(c[n] \cdot T(n, x), n = 0 ..6), x = \; 1 ..1, color = blue)$:

> $plots[display](f1, f2, nmin1, our\_function1, view = [0.9 ..1, 3 ..4])$

#можем заметить, что когда n = 6 функция отклоняется больше чем на 0,
1 (***Чебышев***)



> # #Тригонометрический ряд Фурье

> $a0 := simplify(int(f, x = \ 1..1))$

$$a0 := 6 + \pi \tag{11}$$

> $an := simplify(int(\mathrm{f} \cdot \cos(\mathrm{Pi} \cdot nn \cdot x), x = \ 1..1))$ assuming $nn :: posint$

$$an := 0 \tag{12}$$

> $bn := simplify(int(\mathrm{f} \cdot \sin(\mathrm{Pi} \cdot nn \cdot x), x = \ 1..1))$ assuming $nn :: posint$

$$bn := \int_{-1}^{1} (\arccos(x) + 3) \sin(\pi\, nn\, x)\, \mathrm{d}x \tag{13}$$

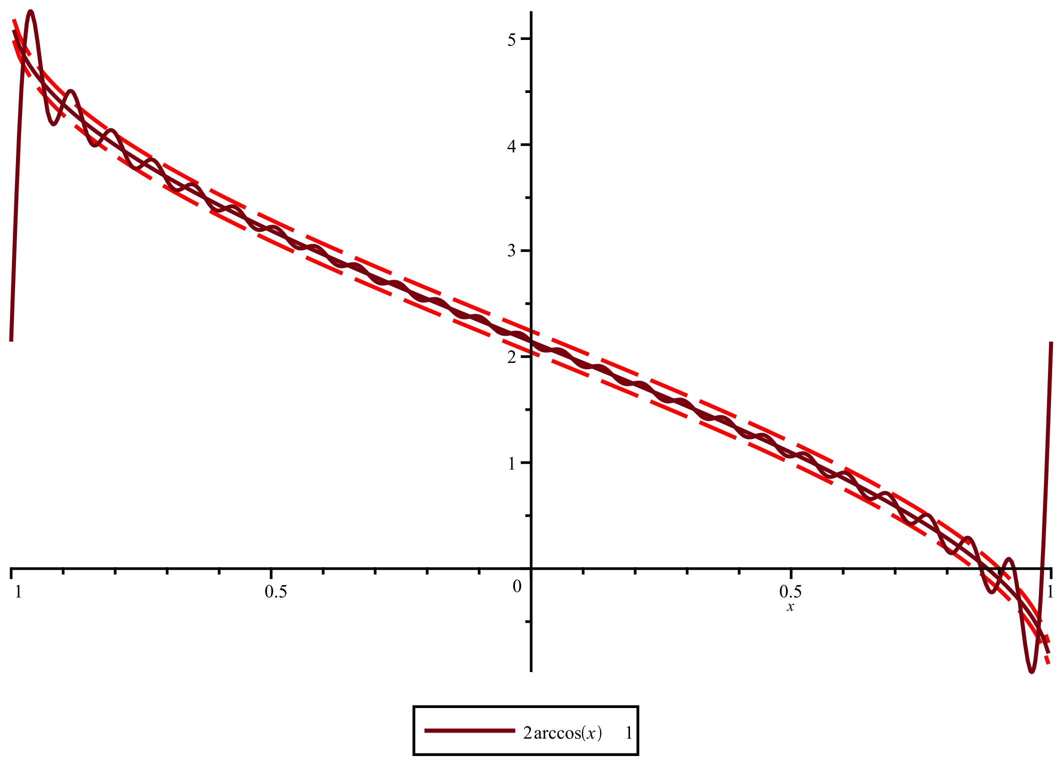
> $Sm := \mathrm{k} \rightarrow \frac{a0}{2} + sum(bn \cdot \sin(\pi \cdot \mathrm{nn} \cdot \mathrm{x}), nn = 1..k)$

$$Sm := k \mapsto \frac{a0}{2} + \left(\sum_{nn=1}^{k} bn \cdot \sin(\pi \cdot nn \cdot x) \right) \tag{14}$$

> $fur := plot(\mathrm{Sm}(25), x = \ 1..1, discont = true)$:

> $plots[display](f1, f2, fur, our\_function1)$

 # тут можно взять промежуток поменьше, например от -0.75 до 0.75 и показать что он будет внутри f +0.1 и f - 0.1()



> #Ряд Тейлора

> $St := convert(taylor(f, x = 0, 64), polynom)$

$$St := 3 + \frac{1}{2}\pi \quad x \quad \frac{1}{6}x^3 \quad \frac{3}{40}x^5 \quad \frac{5}{112}x^7 \quad \frac{63}{2816}x^{11} \quad \frac{34461632205}{11269994184704}x^{41}$$ **(15)**

$$\frac{116680311}{30064771072}x^{35} \quad \frac{2268783825}{635655159808}x^{37} \quad \frac{1472719325}{446676598784}x^{39} \quad \frac{35}{1152}x^{9}$$

$$\frac{100180065}{23622320128}x^{33} \quad \frac{231}{13312}x^{13} \quad \frac{143}{10240}x^{15} \quad \frac{6435}{557056}x^{17} \quad \frac{12155}{1245184}x^{19}$$

$$\frac{46189}{5505024}x^{21} \quad \frac{88179}{12058624}x^{23} \quad \frac{676039}{104857600}x^{25} \quad \frac{1300075}{226492416}x^{27}$$

$$\frac{5014575}{973078528}x^{29} \quad \frac{9694845}{2080374784}x^{31} \quad \frac{67282234305}{23639499997184}x^{43} \quad \frac{17534158031}{6597069766656}x^{45}$$

$$\frac{514589420475}{206708186021888}x^{47} \quad \frac{8061900920775}{3448068464705536}x^{49} \quad \frac{5267108601573}{2392537302040576}x^{51}$$

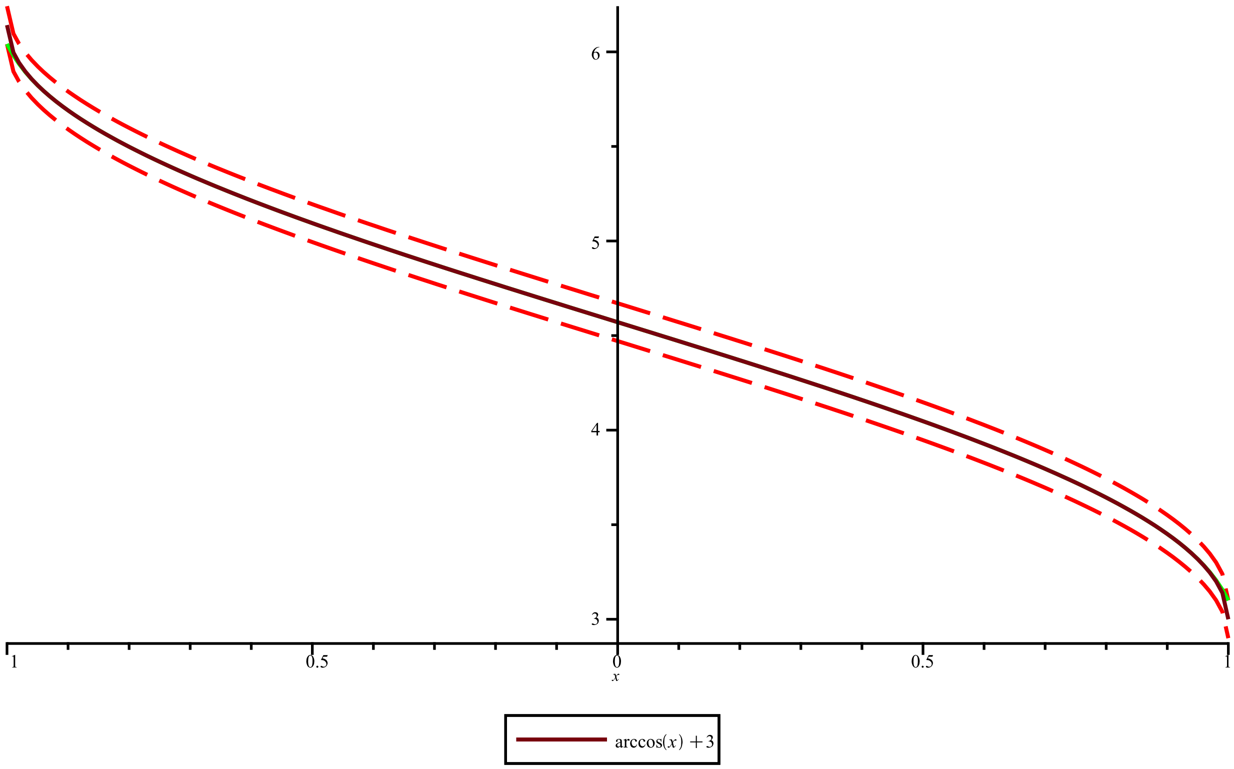
$$\frac{61989816618513}{29836347531329536}x^{53} \quad \frac{121683714103007}{61924494876344320}x^{55} \quad \frac{956086325095055}{513410357520236544}x^{57}$$

$$\frac{1879204156221315}{1062849512059437056}x^{59} \quad \frac{7391536347803839}{43955132363136040\!96}x^{61}$$

$$\frac{2077805148460987}{12970366926827028\!48}x^{63}$$

> $StF := plot(St, x = \; 1..1, color = green):$

> $plots[display](f1, f2, StF, our\_function1)$

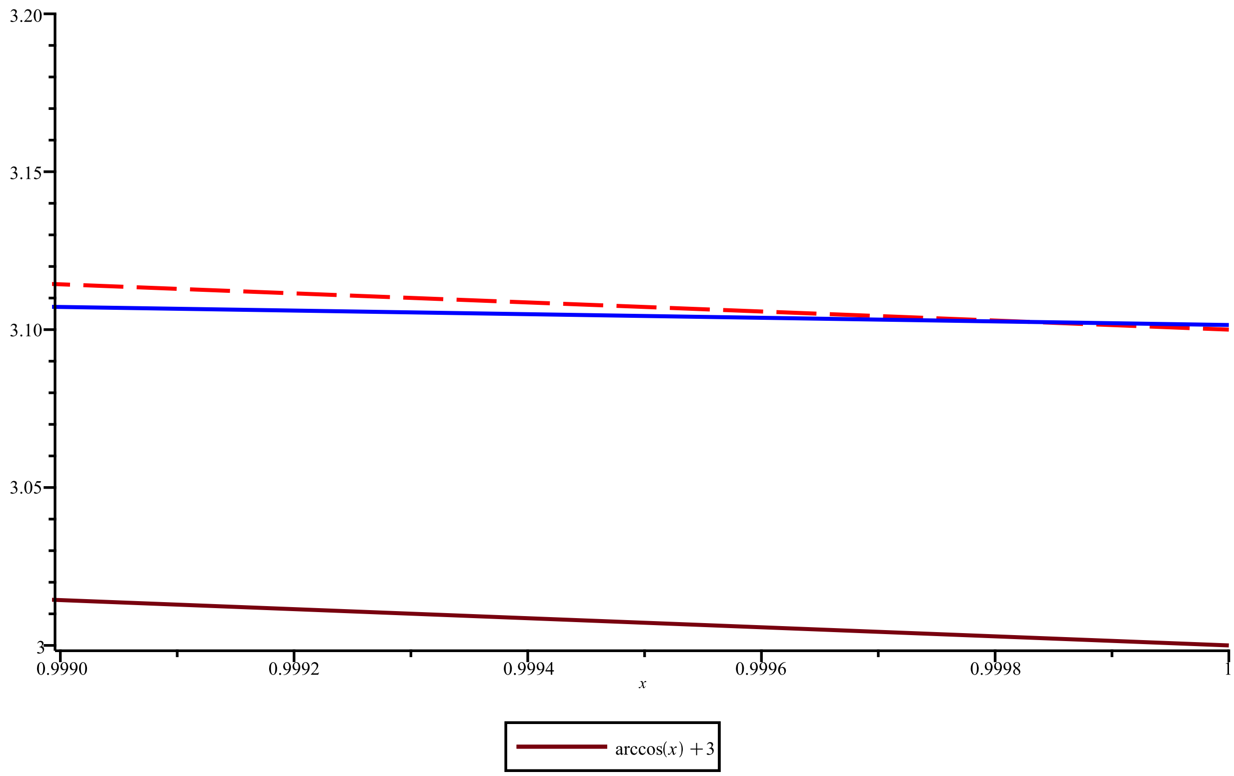


> *minT* := *convert*(*taylor*(*f*, *x* = 0, 63), *polynom*)

$$minT := 3 + \frac{1}{2}\pi \quad x \quad \frac{1}{6}x^{3} \quad \frac{3}{40}x^{5} \quad \frac{5}{112}x^{7} \quad \frac{63}{2816}x^{11} \quad \frac{34461632205}{11269994184704}x^{41} \quad \frac{116680311}{30064771072}x^{35} \quad \frac{2268783825}{635655159808}x^{37} \quad \frac{1472719325}{446676598784}x^{39} \quad \frac{35}{1152}x^{9} \quad \frac{100180065}{23622320128}x^{33} \quad \frac{231}{13312}x^{13} \quad \frac{143}{10240}x^{15} \quad \frac{6435}{557056}x^{17} \quad \frac{12155}{1245184}x^{19} \quad \frac{46189}{5505024}x^{21} \quad \frac{88179}{12058624}x^{23} \quad \frac{676039}{104857600}x^{25} \quad \frac{1300075}{226492416}x^{27} \quad \frac{5014575}{973078528}x^{29} \quad \frac{9694845}{2080374784}x^{31} \quad \frac{67282234305}{23639499997184}x^{43} \quad \frac{17534158031}{6597069766656}x^{45} \quad \frac{514589420475}{206708186021888}x^{47} \quad \frac{8061900920775}{3448068464705536}x^{49} \quad \frac{5267108601573}{2392537302040576}x^{51} \quad \frac{61989816618513}{29836347531329536}x^{53} \quad \frac{121683714103007}{61924494876344320}x^{55} \quad \frac{956086325095055}{513410357520236544}x^{57} \quad \frac{1879204156221315}{1062849512059437056}x^{59} \quad \frac{7391536347803839}{4395513236313604096}x^{61} \tag{16}$$

> *minStF* := *plot*(*minT*, *x* = 1 ..1, *color* = *blue*) :

> *plots*[*display*](*f1*, *f2*, *minStF*, *our_function1*, *view* = [0.999 ..1, 3 ..3.2])


3.20
3.15
3.10
3.05
3
0.9990
0.9992
0.9994
0.9996
0.9998
1
x
arccos(x) + 3

>