> *#Лабораторная работа 2(Вариант 10)*
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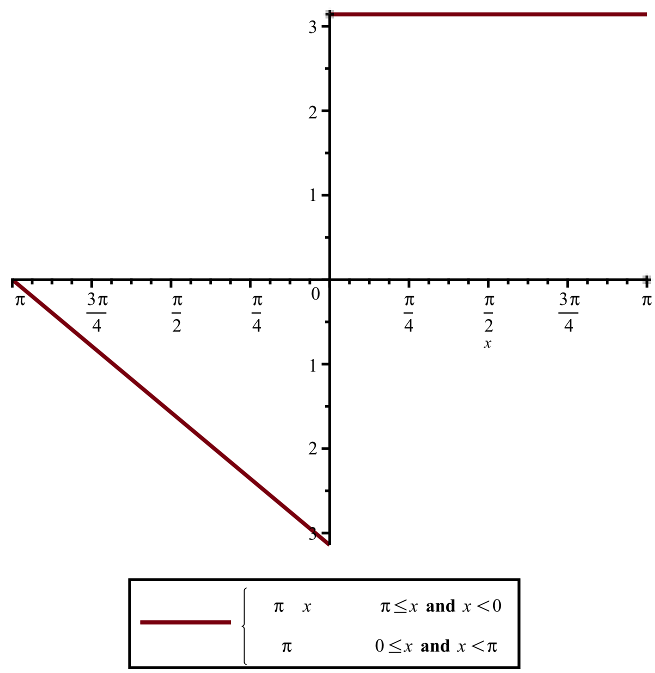
> **#Задание 1. Для 2п-периодической кусочно-непрерывной функции f(x) по ее аналитическому определению на главном периоде**
#получите разложение в тригонометрический ряд Фурье.
#Постройте в одной системе координат на промежутке $[-3\pi, 3\pi]$ графики частичных сумм $S1(x)$, $S3(x)$, $S7(x)$ ряда и его суммы $S(x)$.

> $f := x \to piecewise(-\mathrm{Pi} \le x < 0,\ -\mathrm{Pi} - x,\ 0 \le x < \mathrm{Pi},\ Pi) :$
$plot(f(x), x = -Pi..Pi, discont = true, legend = f(x))$



> $a0 := simplify\left(\frac{1}{\mathrm{Pi}} \cdot Int(f(x), x = \ \mathrm{Pi}..\mathrm{Pi})\right) = simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x), x = \ \mathrm{Pi}..\mathrm{Pi})\right);$

$$a0 := \frac{\int_{\pi}^{\pi} \left(\begin{cases} \pi \quad x & x < 0 \\ \pi & 0 \le x \end{cases}\right) \mathrm{d}x}{\pi} = \frac{\pi}{2} \tag{1}$$

> $an := simplify\left(\frac{1}{\mathrm{Pi}} \cdot Int(f(x) \cdot \cos(n \cdot x), x = \ \mathrm{Pi}..\mathrm{Pi})\right) = simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x) \cdot \cos(n \cdot x), x = \ \mathrm{Pi}..\mathrm{Pi})\right)$ assuming $n :: posint$

$$an := \frac{\int_{\pi}^{\pi} \left(\begin{cases} \pi \quad x & x < 0 \\ \pi & 0 \le x \end{cases}\right) \cos(n\, x)\, \mathrm{d}x}{\pi} = \frac{(-1)^n \quad 1}{\pi\, n^2} \tag{2}$$

```
> bn := simplify(1/Pi · Int(f(x)·sin(n·x), x=-Pi..Pi)) = simplify(1/Pi · int(f(x)·sin(n·x), x=-Pi..Pi)) assuming n :: posint;
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$$bn := \frac{\int_{-\pi}^{\pi} \left(\begin{cases} -\pi - x & x < 0 \\ \pi & 0 \le x \end{cases} \right) \sin(n\,x)\, \mathrm{d}x}{\pi} = \frac{-(-1)^{n} + 2}{n} \tag{3}$$

```
> FourierTrigSum := proc(f, k, a, b)
    local a_0, a_n, b_n, n, l;
    l := (b - a)/2;
    assume(n::posint);
    a_0 := simplify(int(f(x), x = a..b)/l);
    a_n := simplify(int(f(x)·cos(Pi·n/l·x), x = a..b)/l);
    b_n := simplify(int(f(x)·sin(Pi·n/l·x), x = a..b)/l);
    return a_0/2 + sum(a_n·cos(Pi·n/l·x) + b_n·sin(Pi·n/l·x), n = 1..k)
  end proc:
> S1 := FourierTrigSum(f, 1, -Pi, Pi);
  S3 := FourierTrigSum(f, 3, -Pi, Pi);
  S7 := FourierTrigSum(f, 7, -Pi, Pi);
  S := FourierTrigSum(f, infinity, -Pi, Pi);
  S50000 := FourierTrigSum(f, 50000, -Pi, Pi):
```

$$S1 := \frac{\pi}{4} - \frac{2\cos(x)}{\pi} + 3\sin(x)$$

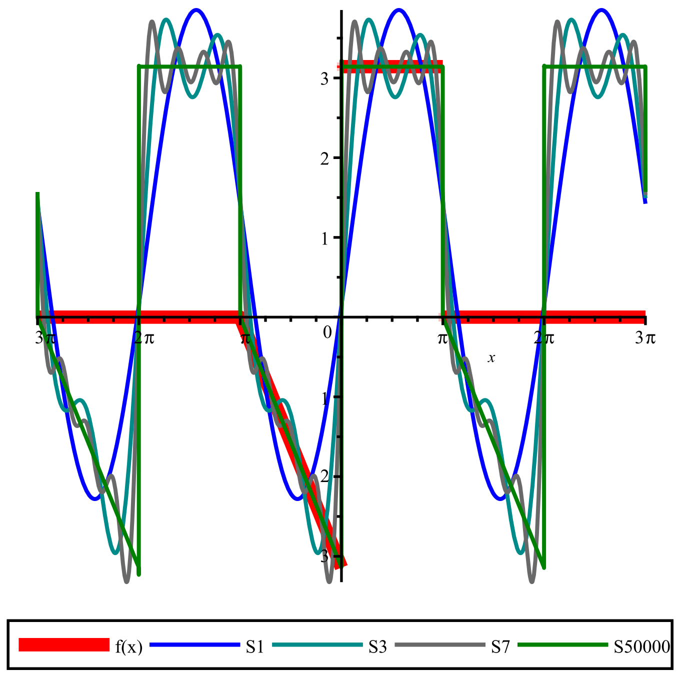
$$S3 := \frac{\pi}{4} - \frac{2\cos(x)}{\pi} + 3\sin(x) + \frac{\sin(2\,x)}{2} - \frac{2\cos(3\,x)}{9\,\pi} + \sin(3\,x)$$

$$S7 := \frac{\pi}{4} - \frac{2\cos(x)}{\pi} + 3\sin(x) + \frac{\sin(2\,x)}{2} - \frac{2\cos(3\,x)}{9\,\pi} + \sin(3\,x) + \frac{\sin(4\,x)}{4} - \frac{2\cos(5\,x)}{25\,\pi} + \frac{3\sin(5\,x)}{5} + \frac{\sin(6\,x)}{6} - \frac{2\cos(7\,x)}{49\,\pi} + \frac{3\sin(7\,x)}{7}$$

$$S := \frac{\pi}{4} + \sum_{n\sim=1}^{\infty} \left(\frac{\left((-1)^{n\sim} - 1\right)\cos(n\sim x)}{n\sim^{2}\,\pi} + \frac{\left(-(-1)^{n\sim} + 2\right)\sin(n\sim x)}{n\sim} \right) \tag{4}$$

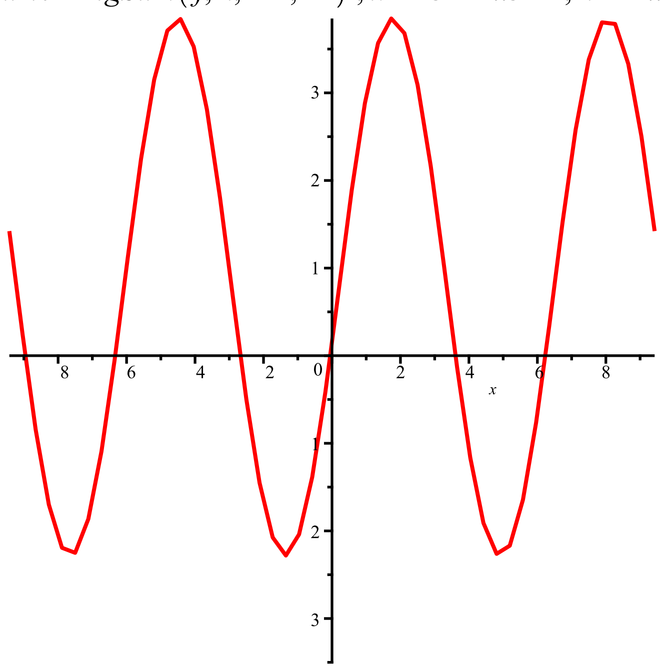
```
> plot([S1, S3, S7, S50000], x=-3·Pi..3·Pi, legend=["S1", "S3", "S7", "S50000"], color=["Blue", "DarkCyan", "DimGray", "Green"]):
  plot(f(x), x=-3·Pi..3·Pi, legend="f(x)", discont=true, color=red, thickness=5):
```

plots[display](%, %%)



> ***#Анимация***

> *plots[animate](FourierTrigSum(f, k,   Pi, Pi) , x =   3·Pi ..3·Pi, k = 1 ..16, numpoints = 50);*



> *restart :*

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