> *#Лабораторная работа 2(Вариант 10)*
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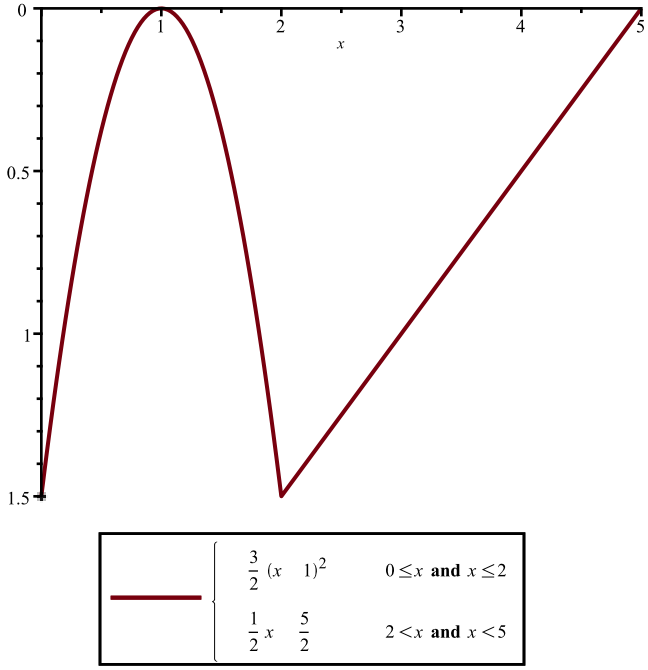
> **#Задание 3. Для графически заданной функции построить три разложения в тригонометрический ряд Фурье.**
#Построить графики сумм рядов на промежутке превышающем длину заданного в три раза.

> $f := x \to piecewise\left(0 \le x \le 2,\ -\frac{3}{2}\cdot(x-1)^2,\ 2 < x < 5,\ \frac{1}{2}\cdot x - \frac{5}{2}\right)$:

> $plot(f(x),\ x = 0..5,\ legend = f(x),\ discont = true)$;



> $a0 := simplify\left(\frac{2}{5}\cdot int(f(x),\ x = 0..5)\right)$

$$a0 := \frac{13}{10} \tag{1}$$

> $an := simplify\left(\frac{2}{5}\cdot int\left(f(x)\cdot\cos\left(\frac{2\cdot \mathrm{Pi}\cdot n\cdot x}{5}\right),\ x = 0..5\right)\right)\text{assuming } n :: posint$

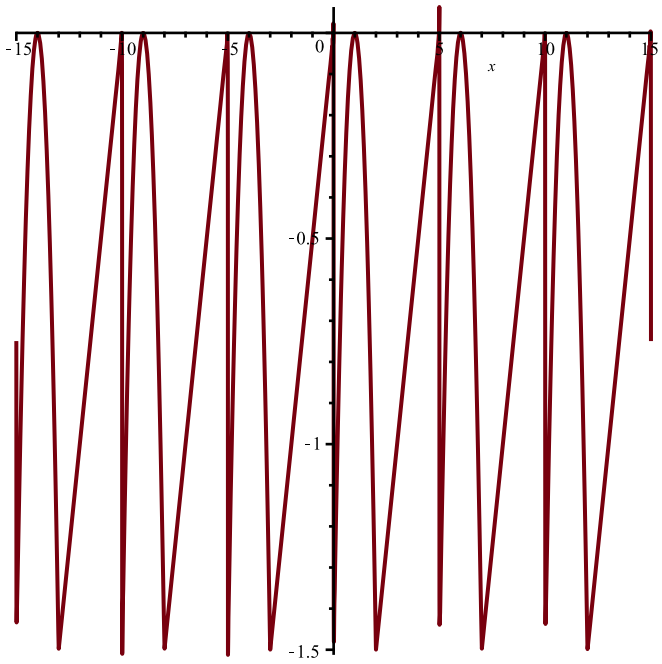
$$an := \frac{5\left(7\,\pi\,n\cos\left(\frac{4\,\pi\,n}{5}\right)\ \ 5\,\pi\,n + 15\sin\left(\frac{4\,\pi\,n}{5}\right)\right)}{4\,\pi^3\,n^3} \tag{2}$$

> $bn := simplify\left(\frac{2}{5}\cdot int\left(f(x)\cdot\sin\left(\frac{2\cdot \mathrm{Pi}\cdot n\cdot x}{5}\right),\ x = 0..5\right)\right)\text{assuming } n :: posint$

$$bn := \frac{6\,\pi^2\,n^2\ \ 35\,\pi\,n\sin\left(\frac{4\,\pi\,n}{5}\right)\ \ 75\cos\left(\frac{4\,\pi\,n}{5}\right) + 75}{4\,\pi^3\,n^3} \tag{3}$$

> $S := k \to \frac{a0}{2} + sum\left(an\cdot\cos\left(\frac{2\cdot \mathrm{Pi}\cdot n\cdot x}{5}\right) + bn\cdot\sin\left(\frac{2\cdot \mathrm{Pi}\cdot n\cdot x}{5}\right),\ n = 1..k\right)$:

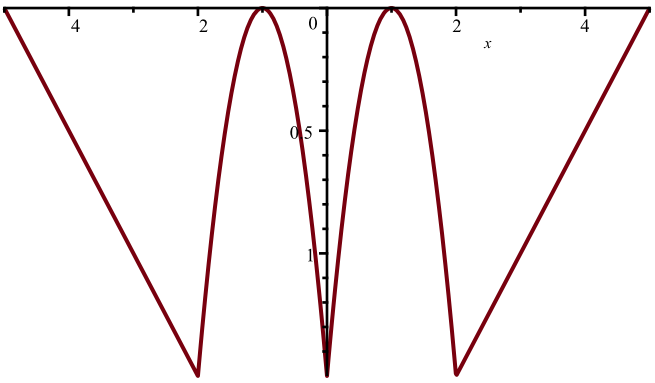
> $plot(S(1000), x = -15..15, discont = true)$



> ***#Определим чётным образом***

> $f\_even := x \to piecewise\left(-5 < x < -2, -\frac{1}{2} \cdot x - \frac{5}{2}, -2 \le x \le 0, -\frac{3}{2} \cdot (-x-1)^2, 0 \le x \le 2, -\frac{3}{2} \cdot (x-1)^2, 2 < x < 5, \frac{1}{2} \cdot x - \frac{5}{2} \right):$

$plot(f\_even(x), x = -5..5, legend = f\_even(x), discont = true);$



$$f\_even(x) = \begin{cases} \frac{1}{2}x - \frac{5}{2} & 5 < x \text{ and } x < 2 \\ \frac{3}{2}(x-1)^2 & 2 \le x \text{ and } x \le 0 \\ \frac{3}{2}(x-1)^2 & 0 \le x \text{ and } x \le 2 \\ \frac{1}{2}x - \frac{5}{2} & 2 < x \text{ and } x < 5 \end{cases}$$

>

> $a0 := simplify\left(\frac{2}{5} \cdot int(f\_even(x), x = 0..5) \right);$

(4)

$$a0 := -\frac{13}{10} \tag{4}$$

> $an := simplify\left(\frac{2}{5}\cdot int\left(f\_even(x)\cdot\cos\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right), x = 0..5\right)\right)$ assuming $n :: posint$

$$an := \frac{5\,\pi\,(-1)^n\,n - 35\,\pi\,n\cos\left(\frac{2\,\pi\,n}{5}\right) - 30\,\pi\,n + 150\sin\left(\frac{2\,\pi\,n}{5}\right)}{\pi^3\,n^3} \tag{5}$$

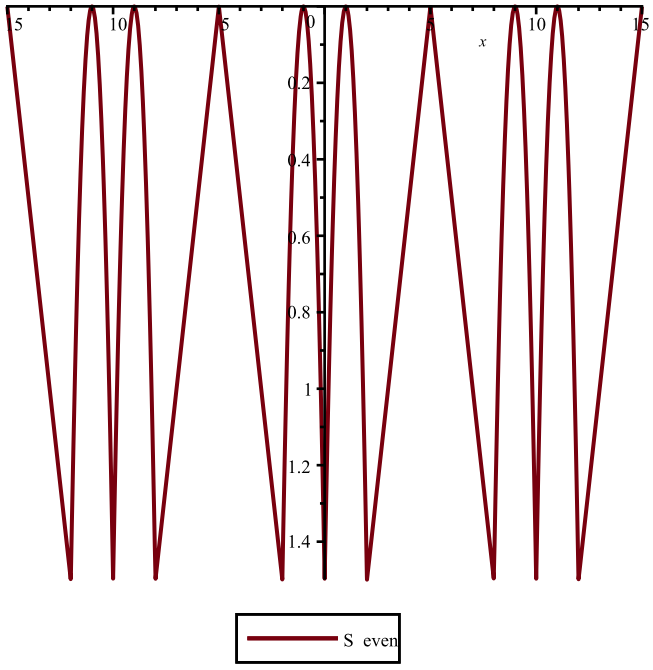
> $bn := simplify\left(\frac{1}{5}\cdot int\left(f\_even(x)\cdot\sin\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right), x = -5..5\right)\right)$ assuming $n :: posint$

$$bn := 0 \tag{6}$$

> $S\_even := k \to \frac{a0}{2} + sum\left(an\cdot\cos\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right) + bn\cdot\sin\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right), n = 1..k\right)$

$$S\_even := k \mapsto \frac{a0}{2} + \sum_{n=1}^{k}\left(an\cdot\cos\left(\frac{\pi\cdot n\cdot x}{5}\right) + bn\cdot\sin\left(\frac{\pi\cdot n\cdot x}{5}\right)\right) \tag{7}$$

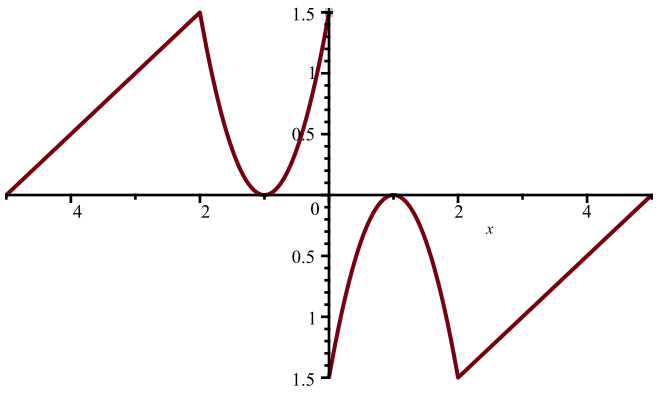
> $plot(S\_even(1000), x = -15..15, discont = true, legend = \text{"S\_even"})$



> ***#Определим нечётным образом***

> $f\_odd := x \to piecewise\left(-5 < x < -2, \frac{1}{2}\cdot x + \frac{5}{2}, -2 \le x \le 0, \frac{3}{2}\cdot(x-1)^2, 0 \le x \le 2, \frac{3}{2}\cdot(x-1)^2, 2 < x < 5, \frac{1}{2}\cdot x - \frac{5}{2}\right):$

$plot(f\_odd(x), x = -5..5, legend = f\_odd(x), discont = true);$



$$\begin{cases} \frac{1}{2}x+\frac{5}{2} & 5<x \text{ and } x<2 \\ \frac{3}{2}(x\ \ 1)^2 & 2\le x \text{ and } x\le 0 \\ \frac{3}{2}(x\ \ 1)^2 & 0\le x \text{ and } x\le 2 \\ \frac{1}{2}x\ \ \frac{5}{2} & 2<x \text{ and } x<5 \end{cases}$$

> $a0 := simplify\left(\frac{1}{5}\cdot int(f\_odd(x), x = \ 5..5)\right);$

$$a0 := 0 \tag{8}$$

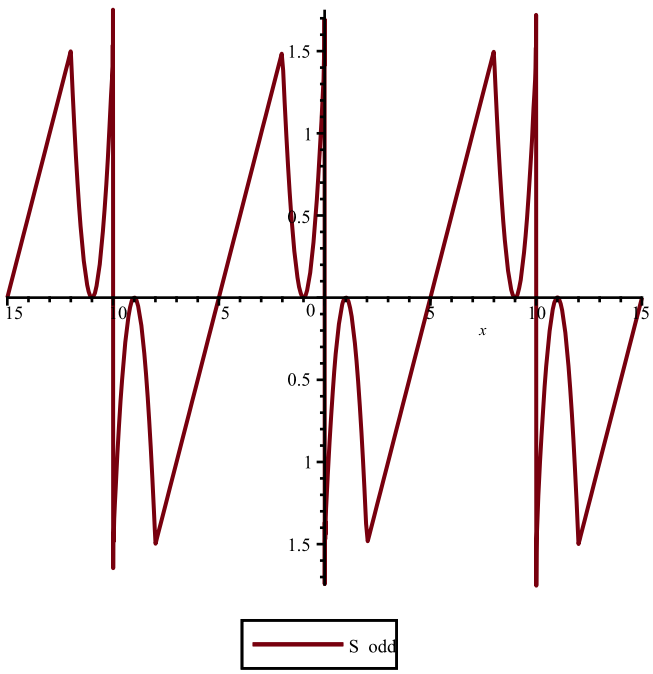
> $an := simplify\left(\frac{1}{5}\cdot int\left(f\_odd(x)\cdot\cos\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right), x = \ 5..5\right)\right)$ assuming $n :: posint$

$$an := 0 \tag{9}$$

> $bn := simplify\left(\frac{2}{5}\cdot int\left(f\_odd(x)\ \cdot\sin\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right), x = 0..5\right)\right)$ assuming $n :: posint$

$$bn := \frac{3\,\pi^2 n^2 \quad 35\,\pi n\sin\left(\frac{2\,\pi n}{5}\right) \quad 150\cos\left(\frac{2\,\pi n}{5}\right)+150}{\pi^3 n^3} \tag{10}$$

> $S\_odd := k\to\frac{a0}{2}+sum\left(an\cdot\cos\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right)+bn\cdot\sin\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right), n=1..k\right):$
$plot(S\_odd(1000), x = \ 15..15, discont = true, legend = \text{"S\_odd"})$



> *restart* :

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