> *#Лабораторная работа 2(Вариант 10)*
#Мартинович Андрей Александрович
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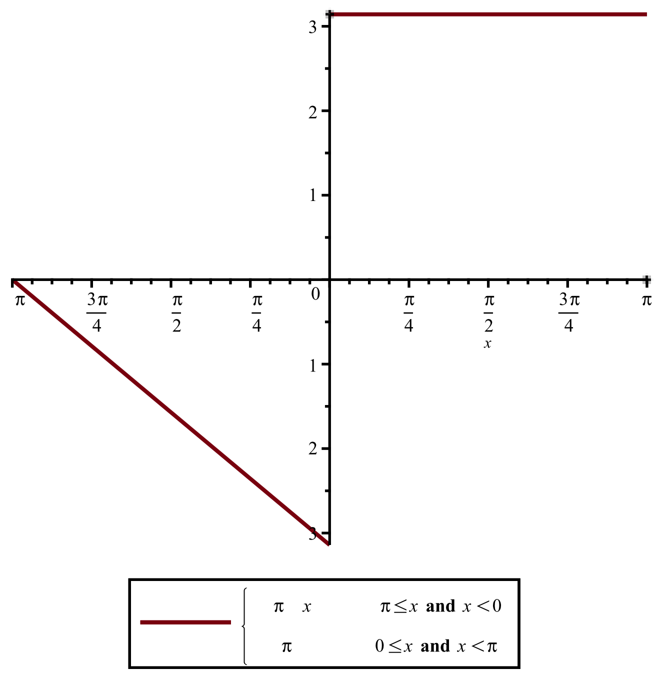
> **#Задание 1. Для 2п-периодической кусочно-непрерывной функции f(x) по ее аналитическому определению на главном периоде**
#получите разложение в тригонометрический ряд Фурье.
#Постройте в одной системе координат на промежутке $[\,-3\,\pi,\ 3\,\pi]$ графики частичных сумм $S1(x)$, $S3(x)$, $S7(x)$ ряда и его суммы $S(x)$.

> $f := x \to piecewise(-\mathrm{Pi} \le x < 0,\ -\mathrm{Pi} - x,\ 0 \le x < \mathrm{Pi},\ Pi)$:
$plot(f(x),\ x = -Pi..Pi,\ discont = true,\ legend = f(x))$



> $a0 := simplify\left(\frac{1}{\mathrm{Pi}} \cdot Int(f(x), x = \ \mathrm{Pi}..\mathrm{Pi})\right) = simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x), x = \ \mathrm{Pi}..\mathrm{Pi})\right)$;

$$a0 := \frac{\displaystyle\int_{\pi}^{\pi} \left(\begin{cases} \pi \quad x & x < 0 \\ \pi & 0 \le x \end{cases}\right) \mathrm{d}x}{\pi} = \frac{\pi}{2} \tag{1}$$

> $an := simplify\left(\frac{1}{\mathrm{Pi}} \cdot Int(f(x) \cdot \cos(n \cdot x), x = \ \mathrm{Pi}..\mathrm{Pi})\right) = simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x) \cdot \cos(n \cdot x), x = \ \mathrm{Pi}..\mathrm{Pi})\right)$ assuming $n :: posint$

$$an := \frac{\displaystyle\int_{\pi}^{\pi} \left(\begin{cases} \pi \quad x & x < 0 \\ \pi & 0 \le x \end{cases}\right) \cos(n\,x)\ \mathrm{d}x}{\pi} = \frac{(-1)^{n} \quad 1}{\pi\,n^{2}} \tag{2}$$

> $bn := simplify\left(\frac{1}{\text{Pi}}\cdot Int(f(x)\cdot\sin(n\cdot x), x=-\text{Pi}..\text{Pi})\right) = simplify\left(\frac{1}{\text{Pi}}\cdot int(f(x)\cdot\sin(n\cdot x), x=-\text{Pi}..\text{Pi})\right)$ assuming $n :: posint$;

$$bn := \frac{\displaystyle\int_{-\pi}^{\pi}\left(\begin{cases}-\pi-x & x<0\\ \pi & 0\leq x\end{cases}\right)\sin(n\,x)\,dx}{\pi} = \frac{-(-1)^n+2}{n} \tag{3}$$

```
> FourierTrigSum :=proc(f, k, a, b)
    local a_0, a_n, b_n, n, l;
    l := (b - a)/2;
    assume(n::posint);
    a_0 := simplify(int(f(x), x = a..b)/l);
    a_n := simplify(int(f(x)·cos(Pi·n/l·x), x = a..b)/l);
    b_n := simplify(int(f(x)·sin(Pi·n/l·x), x = a..b)/l);
    return a_0/2 + sum(a_n·cos(Pi·n/l·x) + b_n·sin(Pi·n/l·x), n = 1..k)
  end proc:
```

```
> S1 := FourierTrigSum(f, 1, -Pi, Pi) ;
  S3 := FourierTrigSum(f, 3, -Pi, Pi) ;
  S7 := FourierTrigSum(f, 7, -Pi, Pi) ;
  S := FourierTrigSum(f, infinity, -Pi, Pi) ;
  S50000 := FourierTrigSum(f, 50000, -Pi, Pi) :
```

$$S1 := \frac{\pi}{4} - \frac{2\cos(x)}{\pi} + 3\sin(x)$$

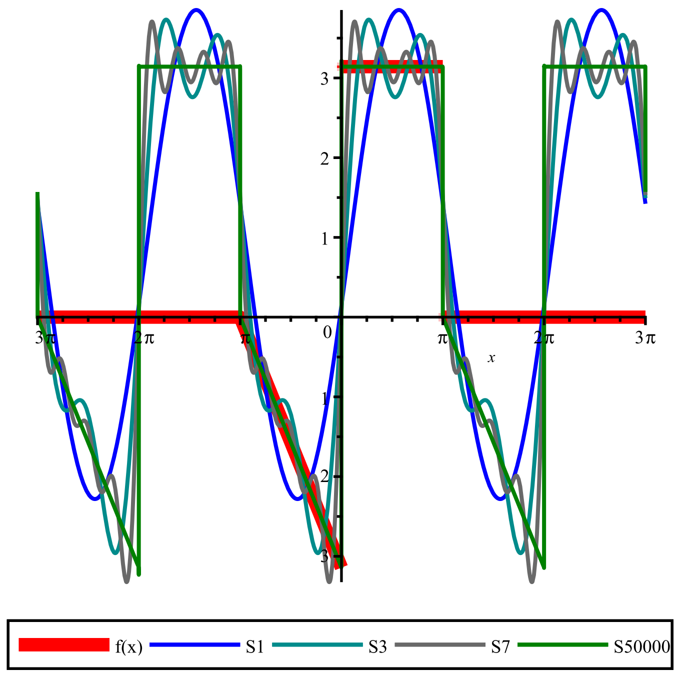
$$S3 := \frac{\pi}{4} - \frac{2\cos(x)}{\pi} + 3\sin(x) + \frac{\sin(2x)}{2} - \frac{2\cos(3x)}{9\pi} + \sin(3x)$$

$$S7 := \frac{\pi}{4} - \frac{2\cos(x)}{\pi} + 3\sin(x) + \frac{\sin(2x)}{2} - \frac{2\cos(3x)}{9\pi} + \sin(3x) + \frac{\sin(4x)}{4} - \frac{2\cos(5x)}{25\pi} + \frac{3\sin(5x)}{5} + \frac{\sin(6x)}{6} - \frac{2\cos(7x)}{49\pi} + \frac{3\sin(7x)}{7}$$

$$S := \frac{\pi}{4} + \sum_{n\sim=1}^{\infty}\left(\frac{((-1)^{n\sim}-1)\cos(n\sim x)}{n\sim^2\,\pi} + \frac{(-(-1)^{n\sim}+2)\sin(n\sim x)}{n\sim}\right) \tag{4}$$

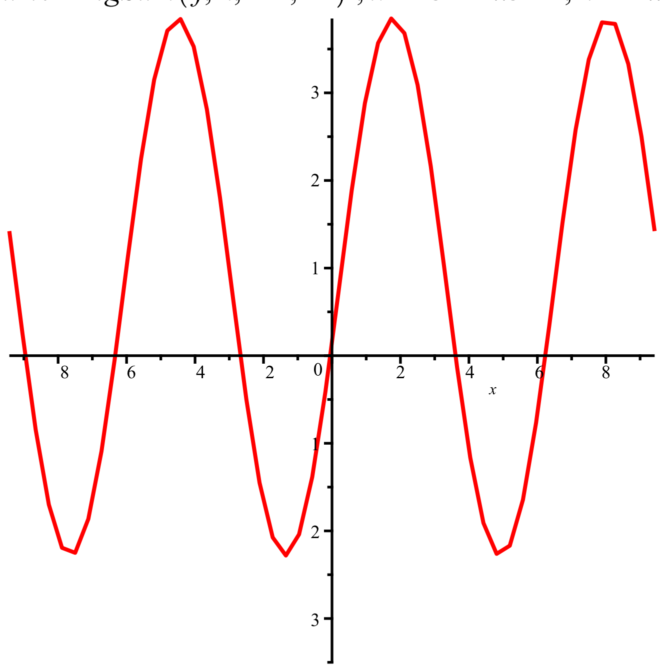
```
> plot([S1, S3, S7, S50000], x = -3·Pi..3·Pi, legend = ["S1", "S3", "S7", "S50000"], color
      = ["Blue", "DarkCyan", "DimGray", "Green"]) :
  plot(f(x), x = -3·Pi..3·Pi, legend = "f(x)", discont = true, color = red, thickness = 5) :
```

plots[*display*](%, %%)



> ***#Анимация***

> *plots*[*animate*](*FourierTrigSum*(*f*, *k*, Pi, Pi) , *x* = 3·Pi ..3·Pi, *k* = 1 ..16, *numpoints* = 50);



> *restart* :

>

> *#Лабораторная работа 2(Вариант 10)*
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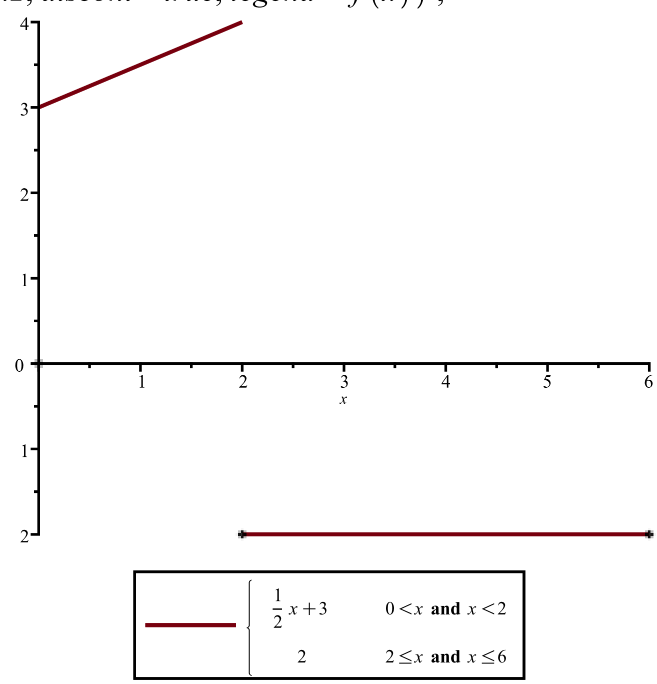
> **#Задание 2. Разложите в ряд Фурье x2 - периодическую функцию y = f(x) , заданную на промежутке (0, x1) формулой**
#y = ax + b, а на [x1, x2] y = c.
#Постройте в одной системе координат на промежутке$[-2\ x2, 2\ x2]$, графики частичных сумм $S1(x)$, $S3(x)$, $S7(x)$ ряда и его суммы $S(x)$

> $x1 := 2;$
$x2 := 6;$
$c := -2;$

$f := x \rightarrow piecewise\left(0 < x < x1, \frac{1}{2} \cdot x + 3, x1 \le x \le x2,\ c\right):$

$$x1 := 2$$
$$x2 := 6$$
$$c := -2 \qquad (1)$$

> $plot(f(x), x = 0 \,..\, x2, discont = true, legend = f(x));$



> $a0 := simplify\left(\frac{2}{x2} \cdot Int(f(x), x = 0 \,..\, x2)\right) = simplify\left(\frac{2}{x2} \cdot int(f(x), x = 0 \,..\, x2)\right);$

$$a0 := \frac{\left(\int_0^6 \left(\begin{cases} \frac{x}{2} + 3 & x < 2 \\ -2 & 2 \le x \end{cases}\right) \mathrm{d}x\right)}{3} = \frac{1}{3} \qquad (2)$$

> $an := simplify\left(\frac{2}{x2}\cdot Int\left(f(x)\cdot\cos\left(\frac{2\cdot n\cdot \mathrm{Pi}\cdot x}{x2}\right), x=0..x2\right)\right) = simplify\left(\frac{2}{x2}\cdot int\left(f(x)\cdot\cos\left(\frac{2\cdot n\cdot \mathrm{Pi}\cdot x}{x2}\right), x=0..x2\right)\right)$ assuming $n :: posint$;

$$an := \frac{\left(\int_0^6 \left(\begin{cases}\frac{x}{2}+3 & x<2\\ -2 & 2\le x\end{cases}\right)\cos\left(\frac{n\,\pi\,x}{3}\right)\mathrm{d}x\right)}{3} \tag{3}$$

$$= \frac{3\left(4\,n\,\pi\sin\left(\frac{2\,n\,\pi}{3}\right)+\cos\left(\frac{2\,n\,\pi}{3}\right)-1\right)}{2\,n^2\,\pi^2}$$

> $bn := simplify\left(\frac{2}{x2}\cdot Int\left(f(x)\cdot\sin\left(\frac{2\cdot n\cdot \mathrm{Pi}\cdot x}{x2}\right), x=0..x2\right)\right) = simplify\left(\frac{2}{x2}\cdot int\left(f(x)\cdot\sin\left(\frac{2\cdot n\cdot \mathrm{Pi}\cdot x}{x2}\right), x=0..x2\right)\right)$ assuming $n :: posint$;

$$bn := \frac{\left(\int_0^6 \left(\begin{cases}\frac{x}{2}+3 & x<2\\ -2 & 2\le x\end{cases}\right)\sin\left(\frac{n\,\pi\,x}{3}\right)\mathrm{d}x\right)}{3} \tag{4}$$

$$= \frac{-12\,n\,\pi\cos\left(\frac{2\,n\,\pi}{3}\right)+10\,n\,\pi+3\sin\left(\frac{2\,n\,\pi}{3}\right)}{2\,n^2\,\pi^2}$$

> $FourierTrigSum :=$ **proc**(f, k, a, b)
local a_0, a_n, b_n, n, l;
$l := \frac{(b-a)}{2}$;
$assume(n::posint)$;
$a\_0 := simplify(int(f(x), x=a..b)/l)$;
$a\_n := simplify\left(\frac{int\left(f(x)\cdot\cos\left(\frac{\mathrm{Pi}\cdot n}{l}\cdot x\right), x=a..b\right)}{l}\right)$;
$b\_n := simplify\left(\frac{int\left(f(x)\cdot\sin\left(\frac{\mathrm{Pi}\cdot n}{l}\cdot x\right), x=a..b\right)}{l}\right)$;
return $\frac{a\_0}{2} + sum\left(a\_n\cdot\cos\left(\frac{\mathrm{Pi}\cdot n}{l}\cdot x\right)+b\_n\cdot\sin\left(\frac{\mathrm{Pi}\cdot n}{l}\cdot x\right), n=1..k\right)$
end proc:

> $S1 := FourierTrigSum(f, 1, 0, x2)$;
$S3 := FourierTrigSum(f, 3, 0, x2)$;
$S7 := FourierTrigSum(f, 7, 0, x2)$;

$S := FourierTrigSum(f, \infty, 0, x2)$;
$S50000 := FourierTrigSum(f, 50000, 0, x2)$:

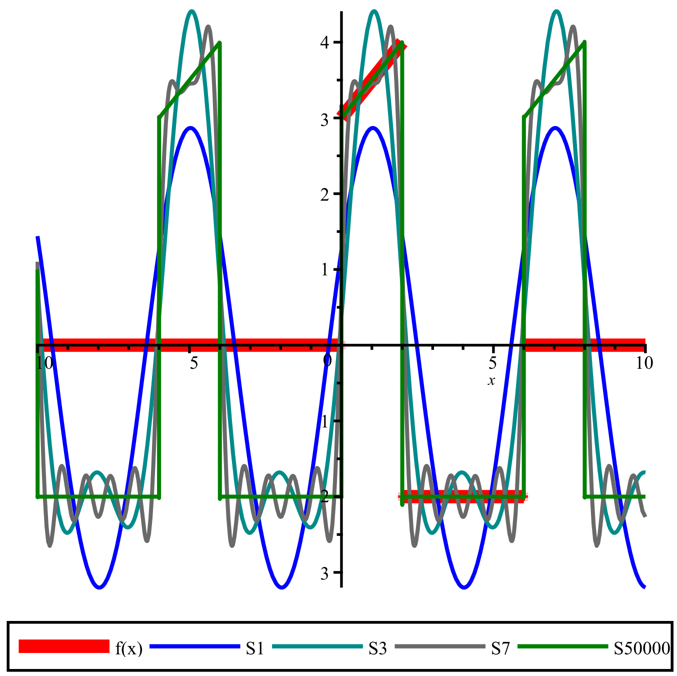
$$S1 := -\frac{1}{6} + \frac{3\left(2\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{\pi x}{3}\right)}{2\pi^2} + \frac{\left(16\pi + \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{\pi x}{3}\right)}{2\pi^2}$$

$$S3 := -\frac{1}{6} + \frac{3\left(2\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{\pi x}{3}\right)}{2\pi^2} + \frac{\left(16\pi + \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{\pi x}{3}\right)}{2\pi^2} + \frac{3\left(-4\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{2\pi x}{3}\right)}{8\pi^2} + \frac{\left(32\pi - \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{2\pi x}{3}\right)}{8\pi^2} - \frac{\sin(\pi x)}{3\pi}$$

$$S7 := -\frac{1}{6} + \frac{3\left(2\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{\pi x}{3}\right)}{2\pi^2} + \frac{\left(16\pi + \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{\pi x}{3}\right)}{2\pi^2} + \frac{3\left(-4\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{2\pi x}{3}\right)}{8\pi^2} + \frac{\left(32\pi - \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{2\pi x}{3}\right)}{8\pi^2} - \frac{\sin(\pi x)}{3\pi} + \frac{3\left(8\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{4\pi x}{3}\right)}{32\pi^2} + \frac{\left(64\pi + \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{4\pi x}{3}\right)}{32\pi^2} + \frac{3\left(-10\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{5\pi x}{3}\right)}{50\pi^2} + \frac{\left(80\pi - \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{5\pi x}{3}\right)}{50\pi^2} - \frac{\sin(2\pi x)}{6\pi} + \frac{3\left(14\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{7\pi x}{3}\right)}{98\pi^2} + \frac{\left(112\pi + \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{7\pi x}{3}\right)}{98\pi^2}$$

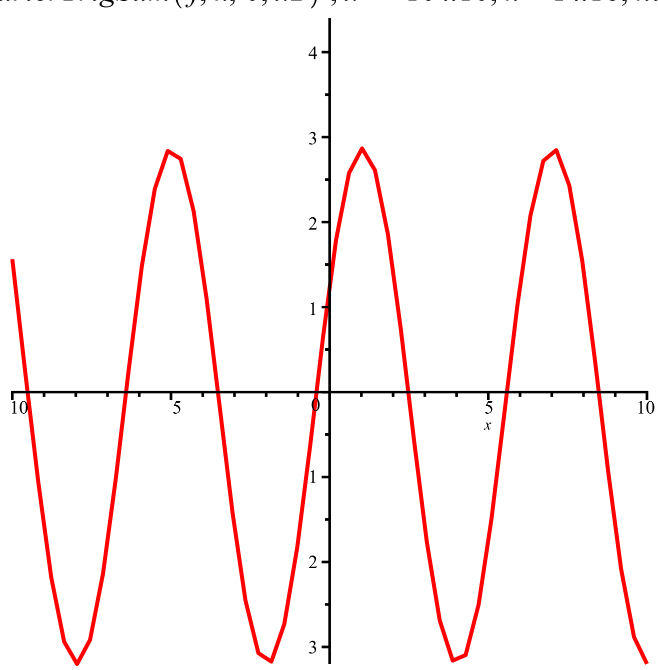
$$S := -\frac{1}{6} + \sum_{n\sim=1}^{\infty}\left(\frac{3\left(4\pi n\sim\sin\left(\frac{2\pi n\sim}{3}\right) + \cos\left(\frac{2\pi n\sim}{3}\right) - 1\right)\cos\left(\frac{\pi n\sim x}{3}\right)}{2\,n\sim^2\pi^2} + \frac{\left(-12\pi n\sim\cos\left(\frac{2\pi n\sim}{3}\right) + 10\pi n\sim + 3\sin\left(\frac{2\pi n\sim}{3}\right)\right)\sin\left(\frac{\pi n\sim x}{3}\right)}{2\,n\sim^2\pi^2}\right) \quad \textbf{(5)}$$

> $plot([S1, S3, S7, S50000], x=-10..10, legend=["S1", "S3", "S7", "S50000"], color=["Blue", "DarkCyan", "DimGray", "Green"])$:
$plot(f(x), x=-10..10, legend="f(x)", discont=true, color=red, thickness=5)$:
$plots[display](\%, \%\%)$



> **#Анимация**

> *plots*[*animate*](*FourierTrigSum*(*f*, *k*, 0, *x2*) , *x* = 10 ..10, *k* = 1 ..16, *numpoints* = 50);



> *restart* :

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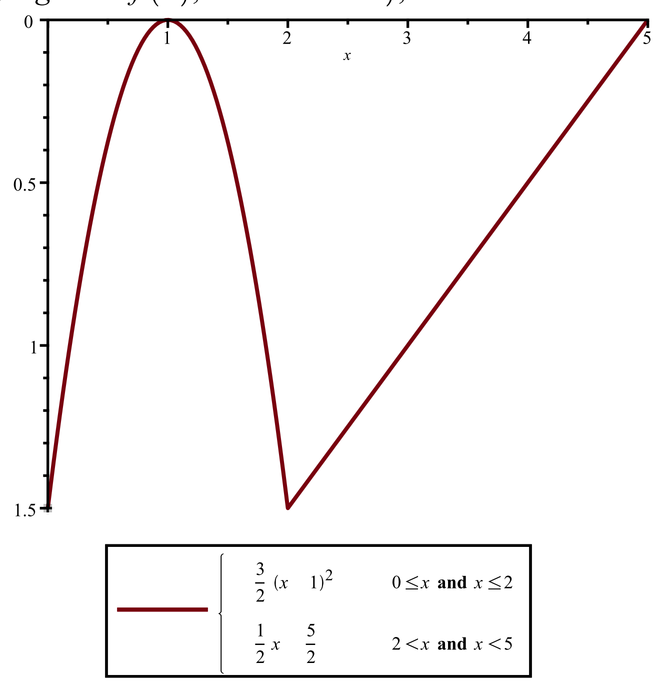
> **#Задание 3. Для графически заданной функции построить три разложения в тригонометрический ряд Фурье.**
#Построить графики сумм рядов на промежутке превышающем длину заданного в три раза.

> $f := x \to piecewise\left(0 \le x \le 2, -\frac{3}{2}\cdot(x-1)^2, 2 < x < 5, \frac{1}{2}\cdot x - \frac{5}{2}\right):$

> $plot(f(x), x = 0..5, legend = f(x), discont = true);$



> $a0 := simplify\left(\frac{2}{5}\cdot int(f(x), x = 0..5)\right)$

$$a0 := \frac{13}{10} \tag{1}$$

> $an := simplify\left(\frac{2}{5}\cdot int\left(f(x)\cdot\cos\left(\frac{2\cdot \mathrm{Pi}\cdot n\cdot x}{5}\right), x = 0..5\right)\right)$ assuming $n :: posint$

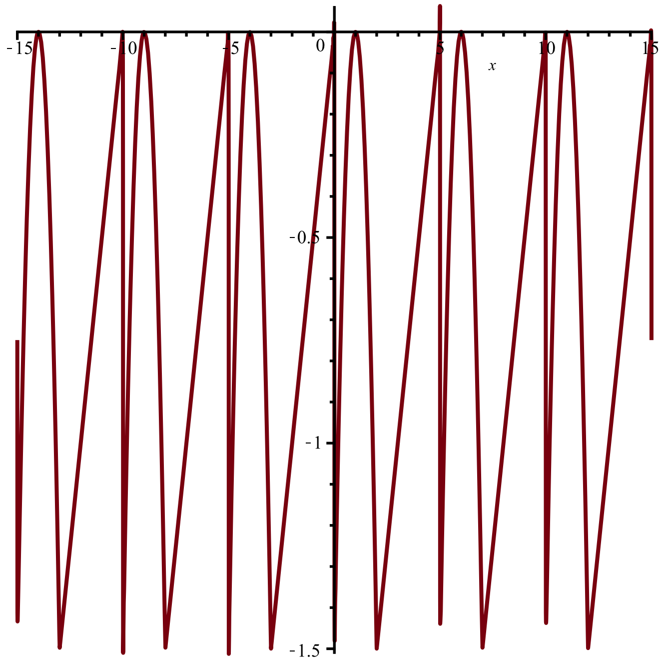
$$an := \frac{5\left(\quad 7\,\pi\,n\cos\left(\frac{4\,\pi\,n}{5}\right) \quad 5\,\pi\,n + 15\sin\left(\frac{4\,\pi\,n}{5}\right)\right)}{4\,\pi^3\,n^3} \tag{2}$$

> $bn := simplify\left(\frac{2}{5}\cdot int\left(f(x)\cdot\sin\left(\frac{2\cdot \mathrm{Pi}\cdot n\cdot x}{5}\right), x = 0..5\right)\right)$ assuming $n :: posint$

$$bn := \frac{6\,\pi^2\,n^2 \quad 35\,\pi\,n\sin\left(\frac{4\,\pi\,n}{5}\right) \quad 75\cos\left(\frac{4\,\pi\,n}{5}\right) + 75}{4\,\pi^3\,n^3} \tag{3}$$

> $S := k \to \frac{a0}{2} + sum\left(an\cdot\cos\left(\frac{2\cdot \mathrm{Pi}\cdot n\cdot x}{5}\right) + bn\cdot\sin\left(\frac{2\cdot \mathrm{Pi}\cdot n\cdot x}{5}\right), n = 1..k\right):$

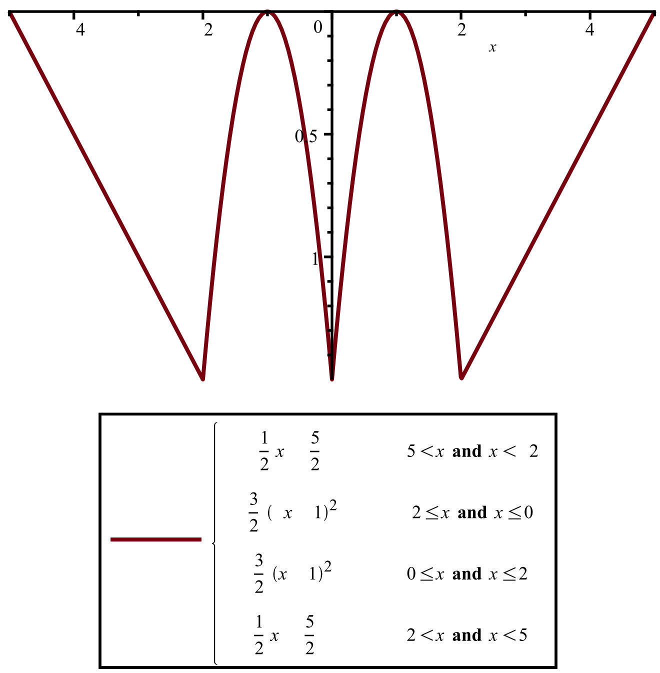
> $plot(S(1000), x=-15..15, discont=true)$



> ***#Определим чётным образом***

> $f\_even := x \rightarrow piecewise\left(-5 < x < -2, -\frac{1}{2}\cdot x - \frac{5}{2}, -2 \le x \le 0, -\frac{3}{2}\cdot(-x-1)^2, 0 \le x \le 2, -\frac{3}{2}\cdot(x-1)^2, 2 < x < 5, \frac{1}{2}\cdot x - \frac{5}{2}\right):$

$plot(f\_even(x), x=-5..5, legend=f\_even(x), discont=true);$



>

> $a0 := simplify\left(\frac{2}{5}\cdot int(f\_even(x), x=0..5)\right);$

(4)

$$a0 := -\frac{13}{10} \tag{4}$$

> $an := simplify\left(\frac{2}{5} \cdot int\left(f\_even(x) \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{5}\right), x = 0..5\right)\right)$ assuming n :: $posint$

$$an := \frac{5\,\pi\,(-1)^n\,n - 35\,\pi\,n\cos\left(\frac{2\,\pi\,n}{5}\right) - 30\,\pi\,n + 150\sin\left(\frac{2\,\pi\,n}{5}\right)}{\pi^3\,n^3} \tag{5}$$

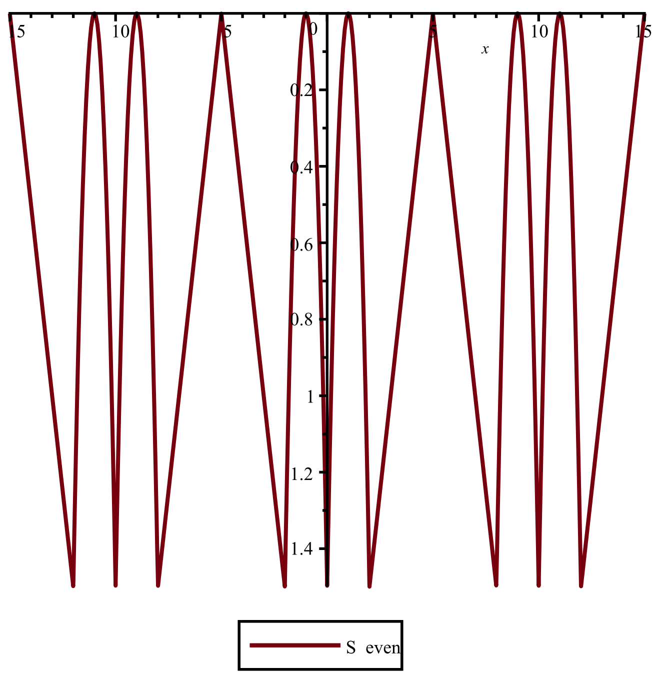
> $bn := simplify\left(\frac{1}{5} \cdot int\left(f\_even(x) \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{5}\right), x = -5..5\right)\right)$ assuming n :: $posint$

$$bn := 0 \tag{6}$$

> $S\_even := k \rightarrow \frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{5}\right) + bn \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{5}\right), n = 1..k\right)$

$$S\_even := k \mapsto \frac{a0}{2} + \sum_{n=1}^{k}\left(an \cdot \cos\left(\frac{\pi \cdot n \cdot x}{5}\right) + bn \cdot \sin\left(\frac{\pi \cdot n \cdot x}{5}\right)\right) \tag{7}$$

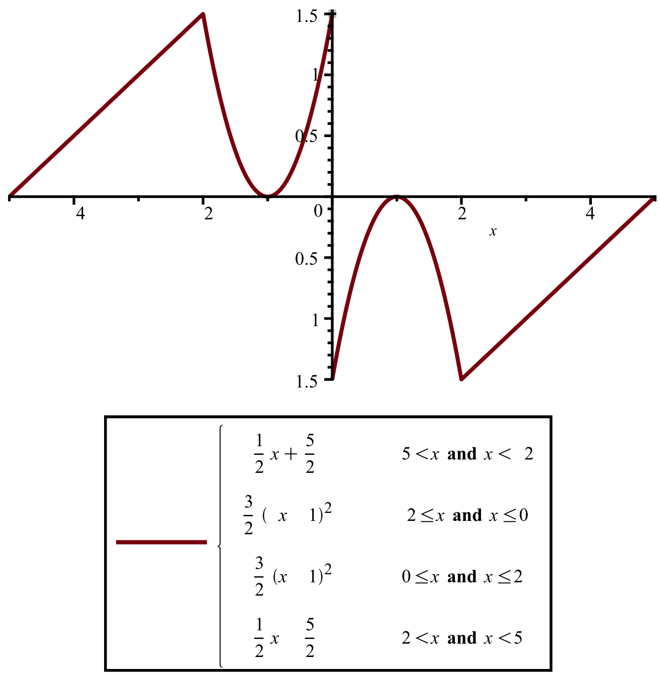
> $plot(S\_even(1000), x = -15..15, discont = true, legend = \text{"S\_even"})$



> **#Определим нечётным образом**

> $f\_odd := x \rightarrow piecewise\left(\ 5 < x < \ 2, \frac{1}{2} \cdot x + \frac{5}{2},\ 2 \le x \le 0, \frac{3}{2} \cdot (\ x \ \ 1)^2, 0 \le x \le 2, \ \frac{3}{2} \cdot (x \ \ 1)^2, 2 < x < 5, \frac{1}{2} \cdot x \ \ \frac{5}{2}\right):$

$plot(f\_odd(x), x = \ 5..5, legend = f\_odd(x), discont = true);$



> $a0 := simplify\left(\frac{1}{5}\cdot int(f\_odd(x), x = \ \ 5..5)\right);$

$$a0 := 0 \tag{8}$$

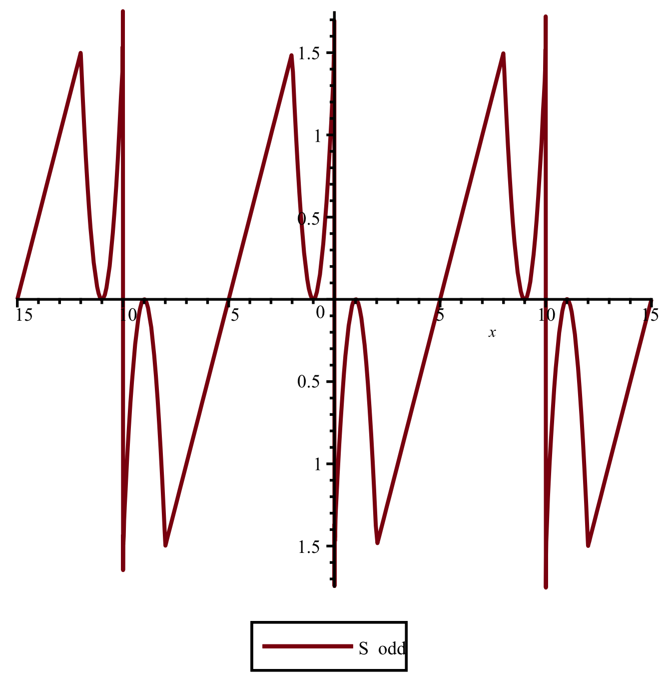
> $an := simplify\left(\frac{1}{5}\cdot int\left(f\_odd(x)\cdot\cos\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right), x = \ \ 5..5\right)\right)$ assuming $n :: posint$

$$an := 0 \tag{9}$$

> $bn := simplify\left(\frac{2}{5}\cdot int\left(f\_odd(x)\ \ \cdot\sin\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right), x = 0..5\right)\right)$ assuming $n :: posint$

$$bn := \frac{3\,\pi^2\,n^2\ \ \ \ 35\,\pi\,n\,\sin\left(\frac{2\,\pi\,n}{5}\right)\ \ \ \ 150\,\cos\left(\frac{2\,\pi\,n}{5}\right) + 150}{\pi^3\,n^3} \tag{10}$$

> $S\_odd := k \rightarrow \frac{a0}{2} + sum\left(an\cdot\cos\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right) + bn\cdot\sin\left(\frac{\mathrm{Pi}\cdot n\cdot x}{5}\right), n = 1..k\right):$
$plot(S\_odd(1000), x = \ \ 15..15, discont = true, legend = \text{"S\_odd"})$



> *restart* :

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> ***#Лабораторная работа 2 (Вариант 10)***
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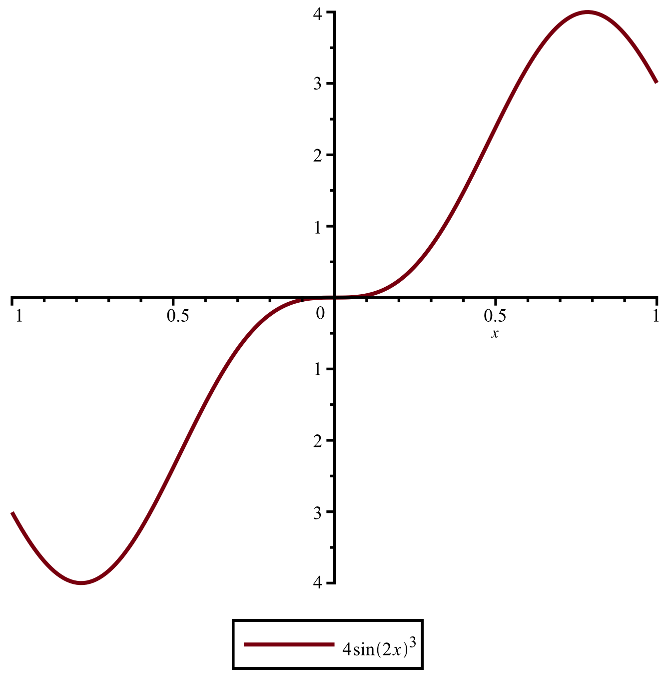
> **#Задание 4. Разложите функцию в ряд Фурье по многочленам Лежандра и Чебышёва на промежутке** $[1, 1]$ **.**

> $f := 4\cdot \sin^3(2\cdot x)$:
$our\_function := plot(f(x), x=-1\,..1, discont=true, legend=f)$



> $with(orthopoly)$

$$[G, H, L, P, T, U] \tag{1}$$

> ***#По многочлену Лежандра***

> **for** n **from** 0 **to** 11 **do** $c[n] := \dfrac{\int_{1}^{1} f\cdot P(n,x)\,\mathrm{d}x}{\int_{1}^{1} P(n,x)^2\mathrm{d}x}$; **end do**

$$c_0 := 0$$

$$c_1 := \quad 2\sin(2)^2\cos(2) \quad 4\cos(2) + \frac{\sin(2)^3}{3} + 2\sin(2)$$

$$c_2 := 0$$

$$c_3 := \frac{49\sin(2)^2\cos(2)}{18} + \frac{266\cos(2)}{9} + \frac{77\sin(2)}{9} + \frac{469\sin(2)^3}{108}$$

$$c_4 := 0$$

$$c_5 := \frac{6215\sin(2)}{24} \quad \frac{6721\cos(2)}{12} + \frac{209\sin(2)^2\cos(2)}{24} + \frac{715\sin(2)^3}{144}$$

$$c_6 := 0$$

$$c_7 := -\frac{8395 \sin(2)^2 \cos(2)}{864} - \frac{123305 \sin(2)^3}{5184} + \frac{2499805 \sin(2)}{216} + \frac{681785 \cos(2)}{27}$$

$$c_8 := 0$$

$$c_9 := \frac{1216361 \sin(2)^2 \cos(2)}{31104} + \frac{24758995 \sin(2)^3}{186624} - \frac{27957486065 \sin(2)}{31104} - \frac{30540881599 \cos(2)}{15552}$$

$$c_{10} := 0$$

$$c_{11} := -\frac{149468881 \sin(2)^2 \cos(2)}{373248} - \frac{3081363659 \sin(2)^3}{2239488} + \frac{19795216570943 \sin(2)}{186624} + \frac{21626467593307 \cos(2)}{93312} \quad \textbf{(2)}$$

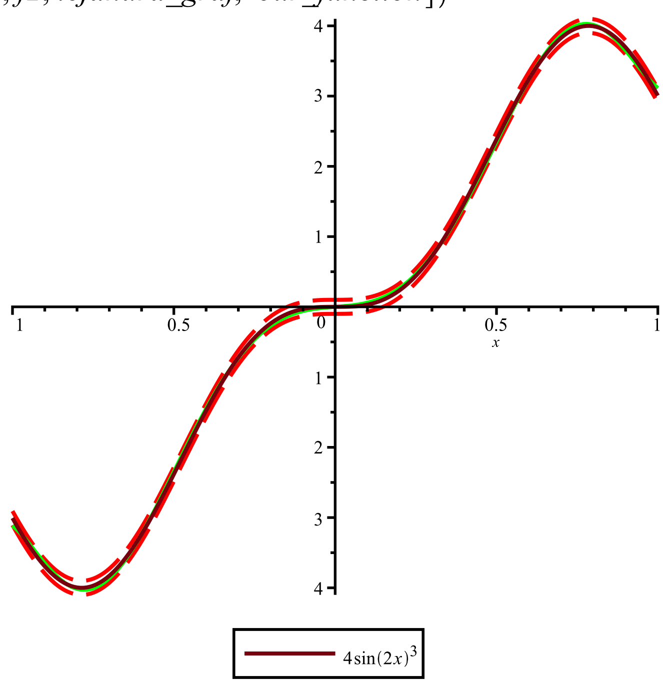
> $lejandra\_graf := plot(add(c[n] \cdot P(n, x), n = 0..7), x = -1..1, color = green)$:

>

> $f1 := plot(f + 0.1, x = -1..1, linestyle = dash, color = red)$:

> $f2 := plot(f - 0.1, x = -1..1, linestyle = dash, color = red)$:

> $plots[display]([f1, f2, lejandra\_graf, our\_function])$

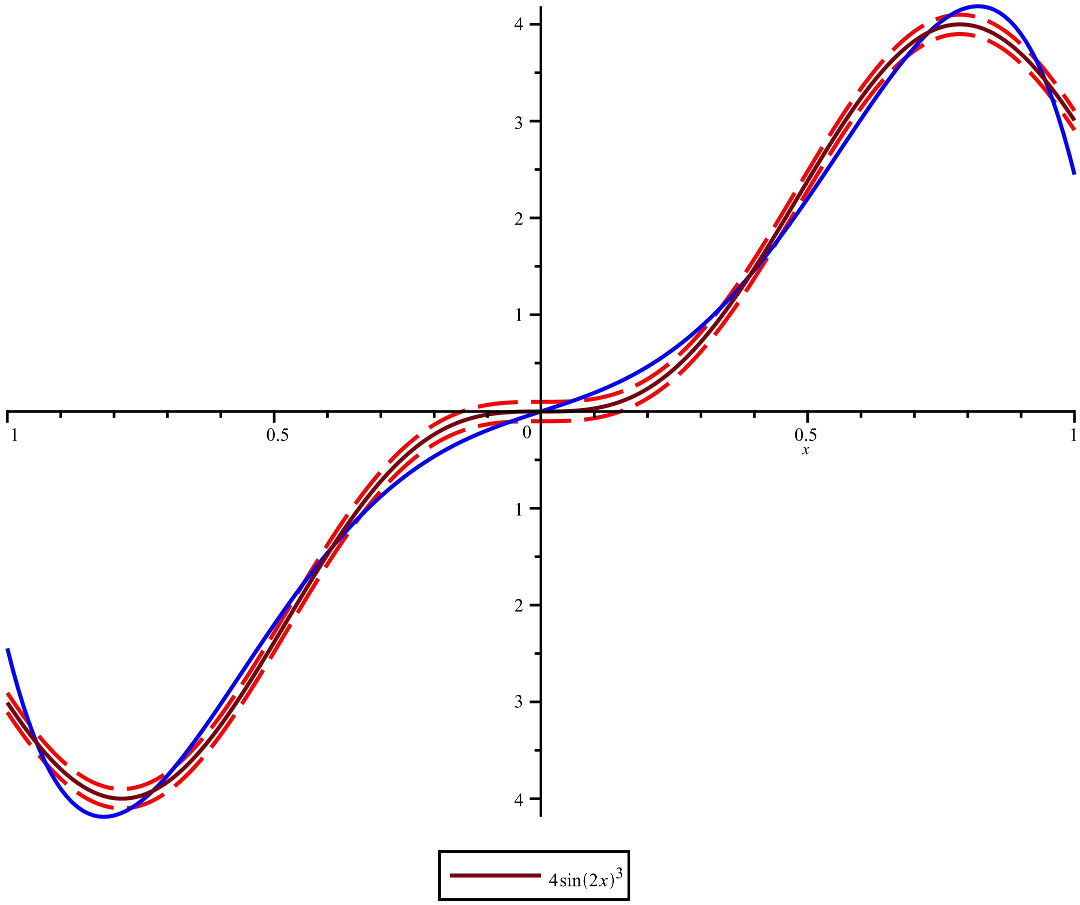


>

> $nmin := plot(add(c[n] \cdot P(n, x), n = 0..6), x = \ 1..1, color = blue)$:

> $plots[display](f1, f2, nmin, our\_function)$

#можем заметить, что когда n = 6 функция отклоняется больше чем на 0, 1 (***Лежандр***)



> **#По многочлену Чебышёва**

> **for** n **from** 0 **to** 11 **do** $c[n] := \dfrac{\displaystyle\int_{1}^{1} \frac{f \cdot T(n,x)}{\mathrm{sqrt}(1 \quad x^2)}\,\mathrm{d}x}{\displaystyle\int_{1}^{1} \frac{T(n,x)^2}{\mathrm{sqrt}(1 \quad x^2)}\,\mathrm{d}x}$; **end do**

$$c_0 := 0$$

$$c_1 := \frac{2 \left(\displaystyle\int_{-1}^{1} \frac{4 \sin(2x)^3 x}{\sqrt{\quad x^2 + 1}}\,\mathrm{d}x \right)}{\pi}$$

$$c_2 := 0$$

$$c_3 := \frac{2 \left(\displaystyle\int_{-1}^{1} \frac{4 \sin(2x)^3 (4x^3 \quad 3x)}{\sqrt{\quad x^2 + 1}}\,\mathrm{d}x \right)}{\pi}$$

$$c_4 := 0$$

$$c_5 := \frac{2\left(\int_{-1}^{1} \frac{4\sin(2x)^3\left(16x^5 - 20x^3 + 5x\right)}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi}$$

$$c_6 := 0$$

$$c_7 := \frac{2\left(\int_{-1}^{1} \frac{4\sin(2x)^3\left(64x^7 - 112x^5 + 56x^3 - 7x\right)}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi}$$

$$c_8 := 0$$

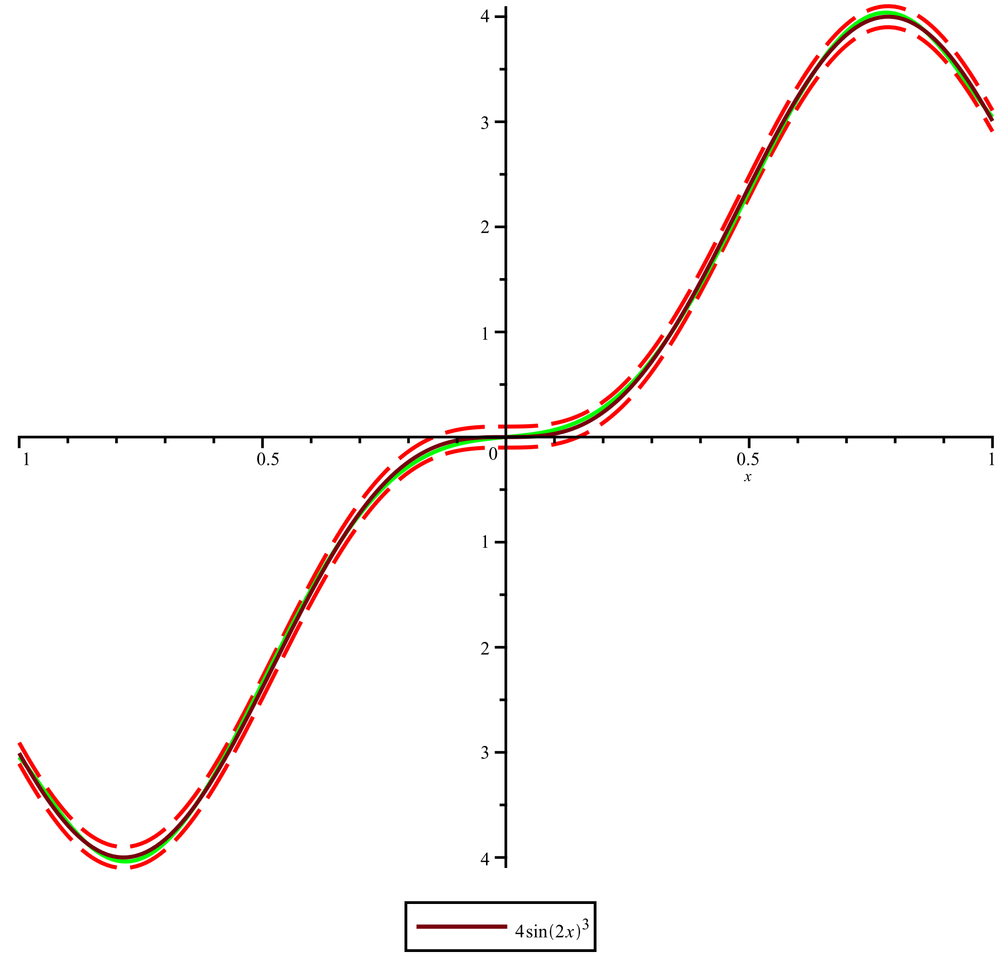
$$c_9 := \frac{2\left(\int_{-1}^{1} \frac{4\sin(2x)^3\left(256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x\right)}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi}$$

$$c_{10} := 0$$

$$c_{11} := \frac{2\left(\int_{-1}^{1} \frac{4\sin(2x)^3\left(1024x^{11} - 2816x^9 + 2816x^7 - 1232x^5 + 220x^3 - 11x\right)}{\sqrt{-x^2+1}}\,\mathrm{d}x\right)}{\pi} \tag{3}$$

> $cheb\_graf := plot(add(c[n]\cdot T(n, x), n = 1..7), x = -1..1, color = green) :$

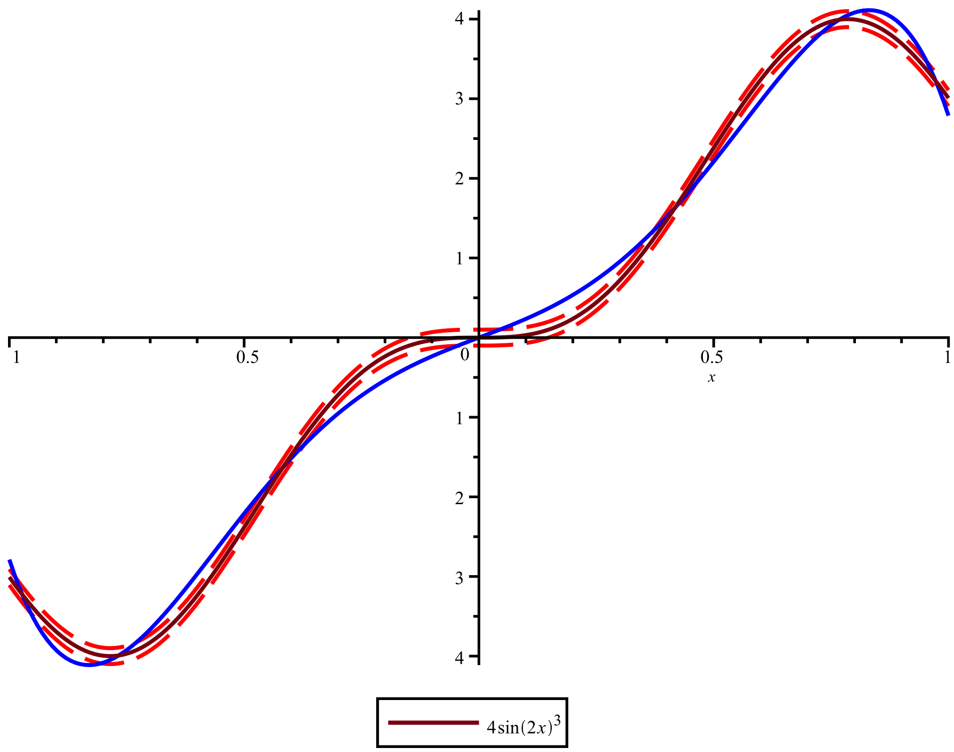
> $plots[display](f1, f2, cheb\_graf, our\_function)$



> $nmin := plot(add(c[n] \cdot T(n, x), n = 1..6), x = \ 1..1, color = blue) :$

> $plots[display](f1, f2, nmin, our\_function)$

#можем заметить, что когда n = 10 функция отклоняется больше чем на 0, 1 (***Чебышев***)



> **#Тригонометрический ряд Фурье**

> $bn := simplify(\ int(\mathrm{f}\cdot\sin(\mathrm{Pi}\cdot m\cdot x), x = \ \ 1\,..1)\,)$ assuming $m :: posint$

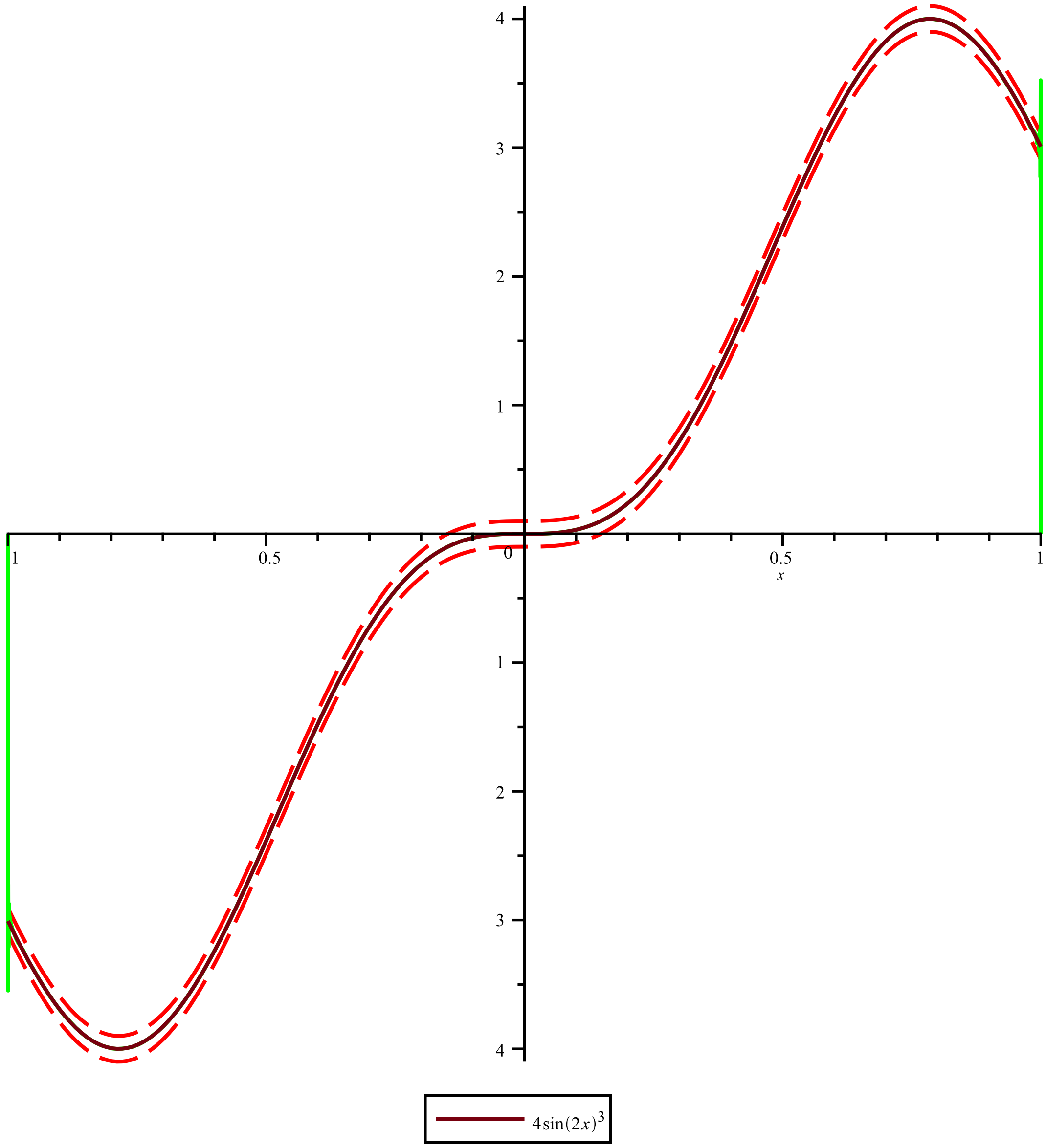
$$bn := \frac{6\,m\,(-1)^{m}\,\pi\left(\pi^{2}\sin(2)\,m^{2}\quad \frac{\pi^{2}\sin(6)\,m^{2}}{3}\quad 36\sin(2)+\frac{4\sin(6)}{3}\right)}{\pi^{4}\,m^{4}\quad 40\,\pi^{2}\,m^{2}+144} \tag{4}$$

> $Sm := \mathrm{k}\rightarrow sum(bn\cdot\sin(\pi\cdot\mathrm{m}\cdot\mathrm{x}), m = 1\,..k)$

$$Sm := k \mapsto \sum_{m=1}^{k} bn\cdot\sin(\pi\cdot m\cdot x) \tag{5}$$

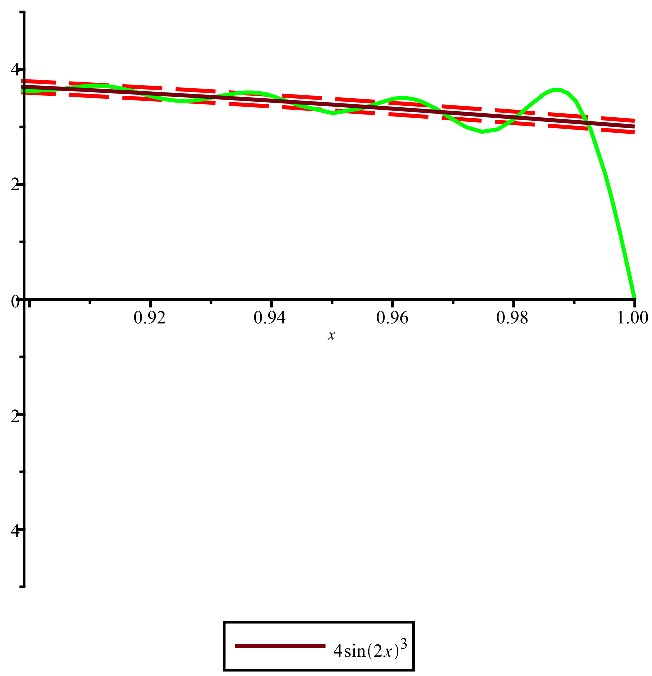
> $fur := plot(Sm(3000), x = \ \ 1\,..1, discont = true, color = green)$:

> $plots[display]([f1, f2, fur, our\_function])$;



> $minFur := plot(Sm(79), x = \ 1 ..1, discont = true, color = green)$:

> $plots[display](f1, f2, minFur, our\_function, view = [0.9 ..1, \ \ 5 ..5])$



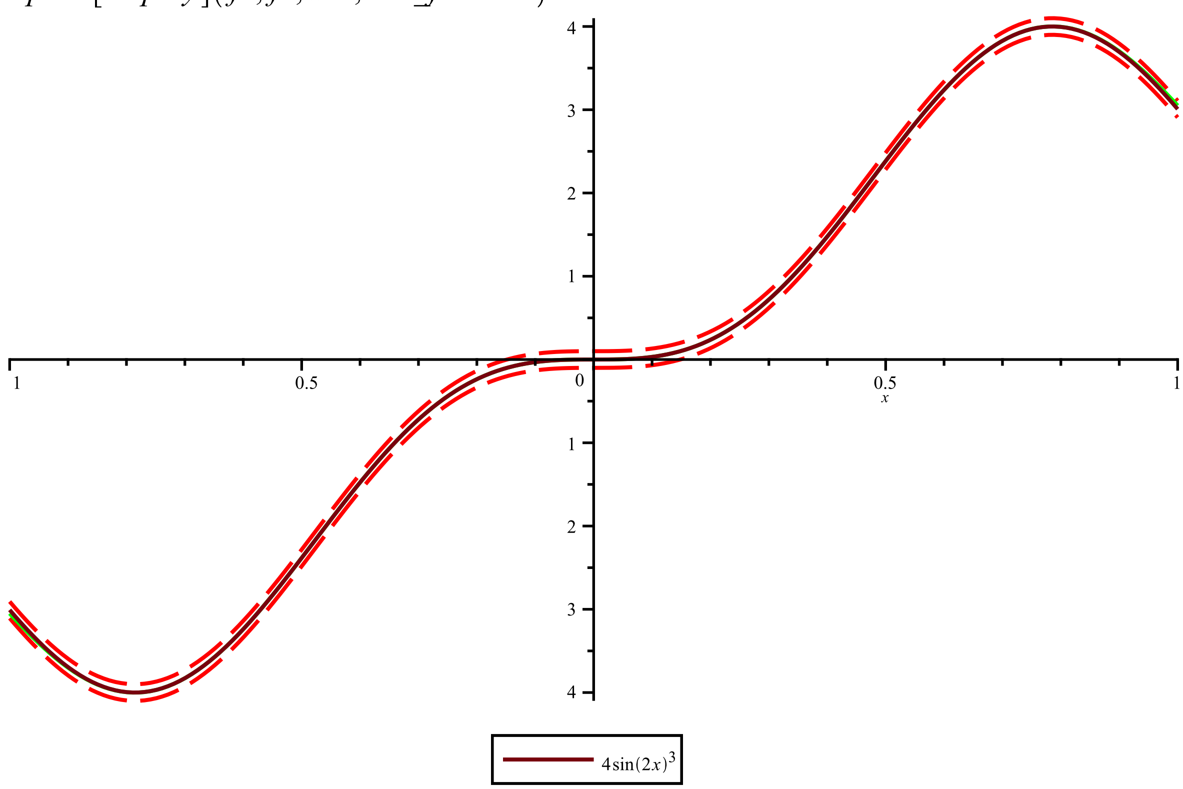
> **#Ряд Тейлора**

> $St := convert(taylor(f, x=0, 16), polynom)$

$$St := 32\,x^{3} \quad 64\,x^{5} + \frac{832}{15}\,x^{7} \quad \frac{5248}{189}\,x^{9} + \frac{42944}{4725}\,x^{11} \quad \frac{9344}{4455}\,x^{13} + \frac{76527488}{212837625}\,x^{15} \qquad \textbf{(6)}$$

> $StF := plot(St, x = \;\; 1..1, color=green):$

> $plots[display](f1, f2, StF, our\_function)$



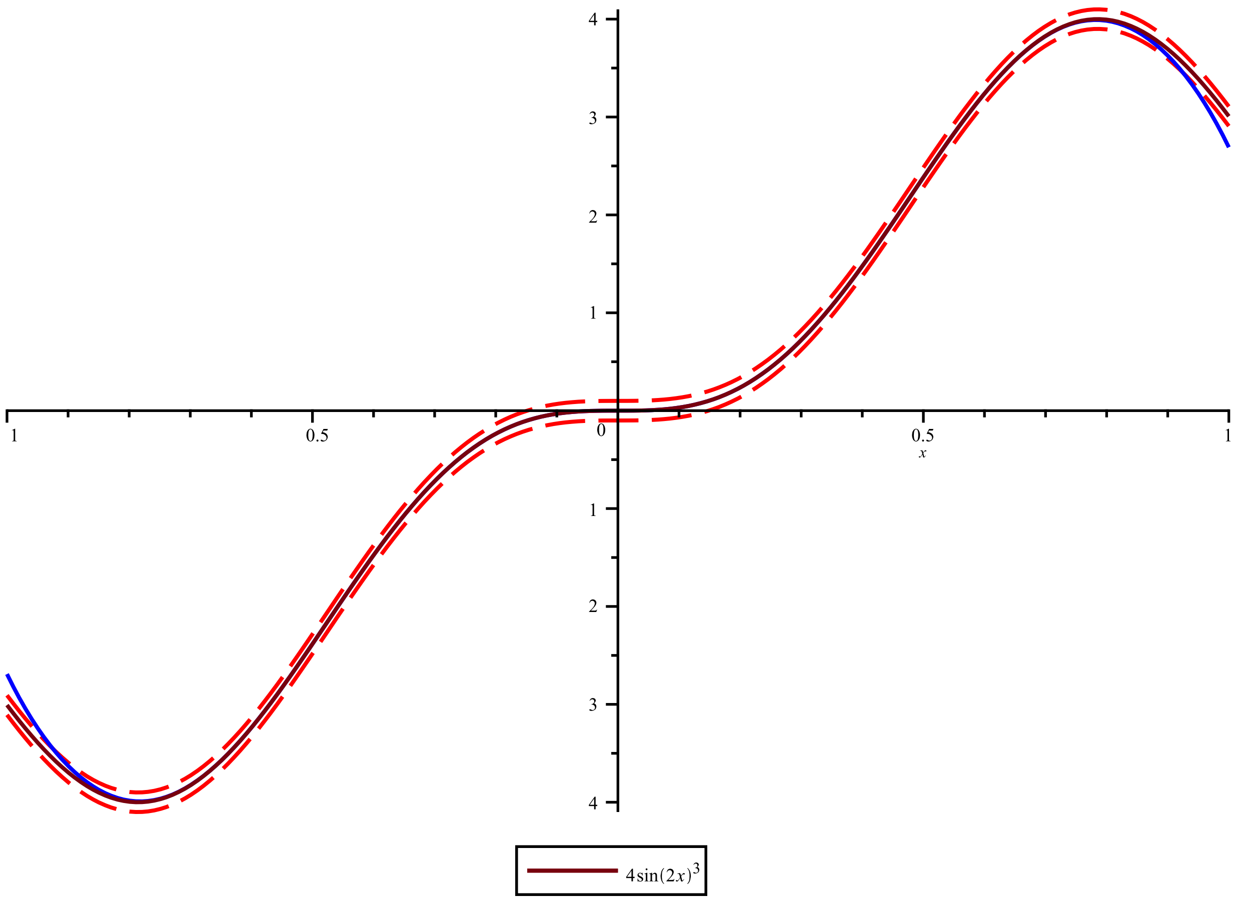
> $St := convert(taylor(f, x=0, 15), polynom)$

(7)

$$St := 32\,x^3 - 64\,x^5 + \frac{832}{15}\,x^7 - \frac{5248}{189}\,x^9 + \frac{42944}{4725}\,x^{11} - \frac{9344}{4455}\,x^{13} \tag{7}$$

> $StF := plot(St, x = -1\,..1, color = blue)$:

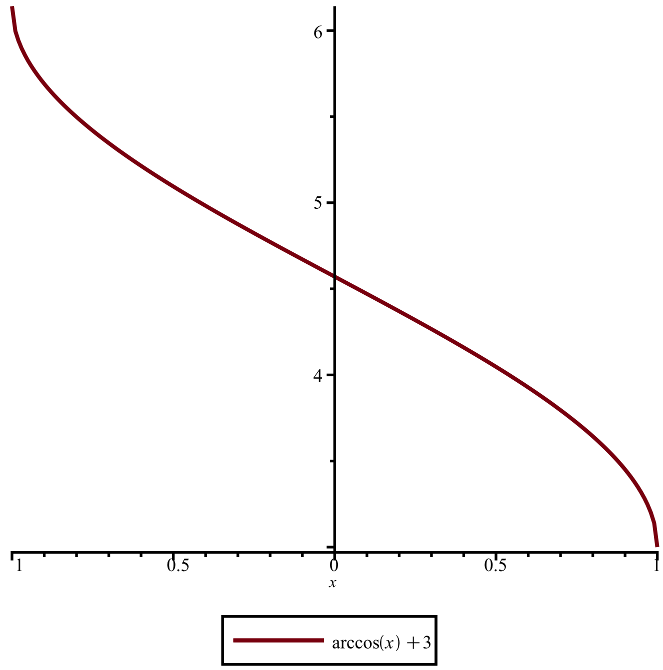
> $plots[display](f1, f2, StF, our\_function)$# *при x =0..15* ***(Тейлор)***



> *restart*

> $f := \arccos(x) + 3$:

> $our\_function1 := plot(f, x = 1\,..1, legend = f)$



> **#По многочлену Лежандра**

> *with*(*orthopoly*)

$$[G, H, L, P, T, U] \tag{8}$$

> **for** n **from** 0 **to** 11 **do** $c[n] := \dfrac{\displaystyle\int_{1}^{1} f \cdot P(n, x)\, \mathrm{d}x}{\displaystyle\int_{1}^{1} P(n, x)^2 \mathrm{d}x}$; **end do**

$$c_0 := 3 + \frac{\pi}{2}$$

$$c_1 := \frac{3\,\pi}{8}$$

$$c_2 := 0$$

$$c_3 := \frac{7\,\pi}{128}$$

$$c_4 := 0$$

$$c_5 := \frac{11\,\pi}{512}$$

$$c_6 := 0$$

$$c_7 := \frac{375\,\pi}{32768}$$

$$c_8 := 0$$

$$c_9 := -\frac{931\,\pi}{131072}$$

$$c_{10} := 0$$

$$c_{11} := -\frac{10143\,\pi}{2097152} \tag{9}$$

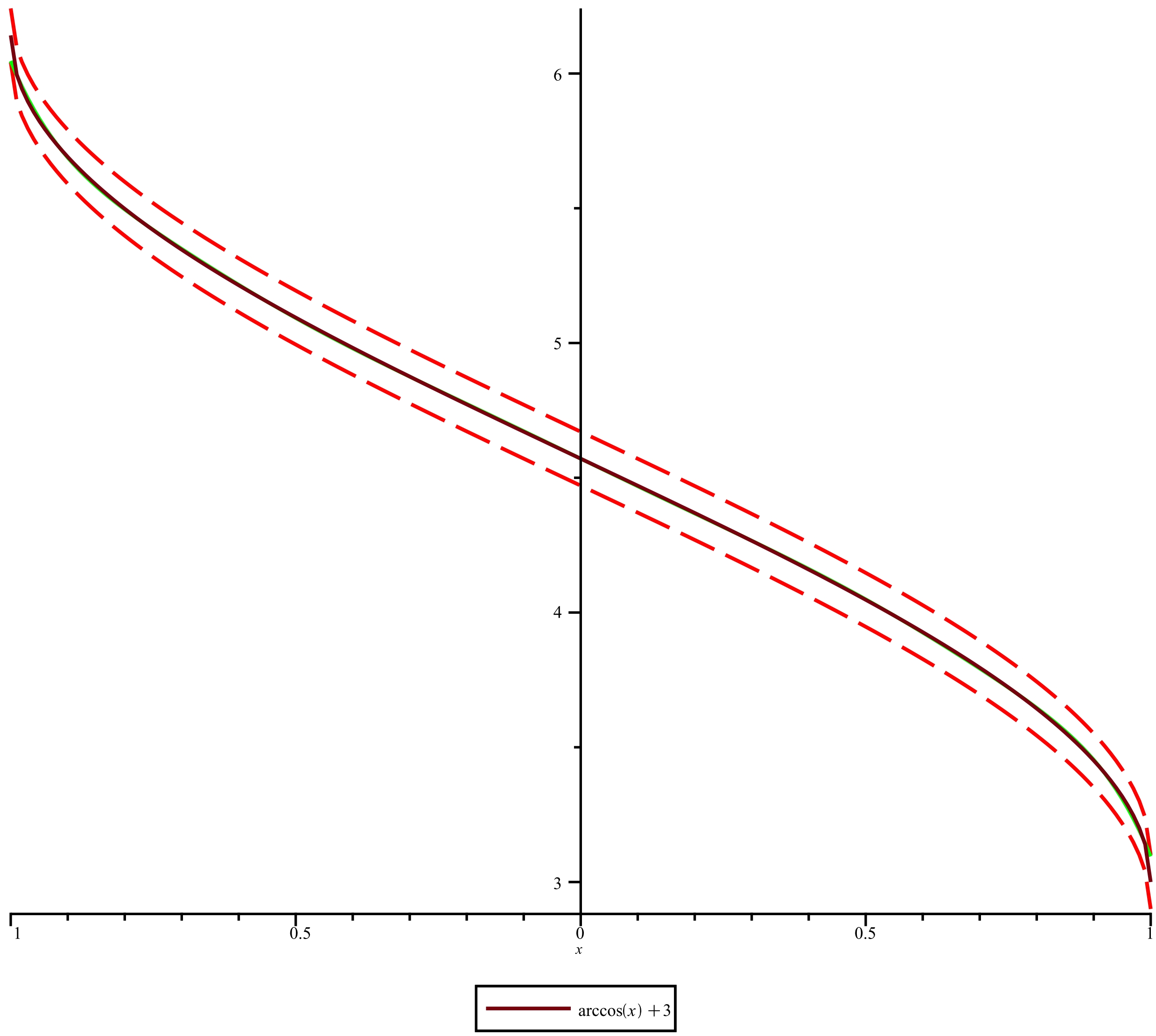
> *lejandra_graf1* := *plot*(*add*(*c*[*n*]·*P*(*n*, *x*), *n* = 0 ..9), *x* = -1 ..1, *color* = *green*) :

>

> *f1* := *plot*(*f* + 0.1, *x* = -1 ..1, *linestyle* = *dash*, *color* = *red*) :

> *f2* := *plot*(*f* − 0.1, *x* = -1 ..1, *linestyle* = *dash*, *color* = *red*) :

> *plots*[*display*]([*f1*, *f2*, *lejandra_graf1*, *our_function1*])

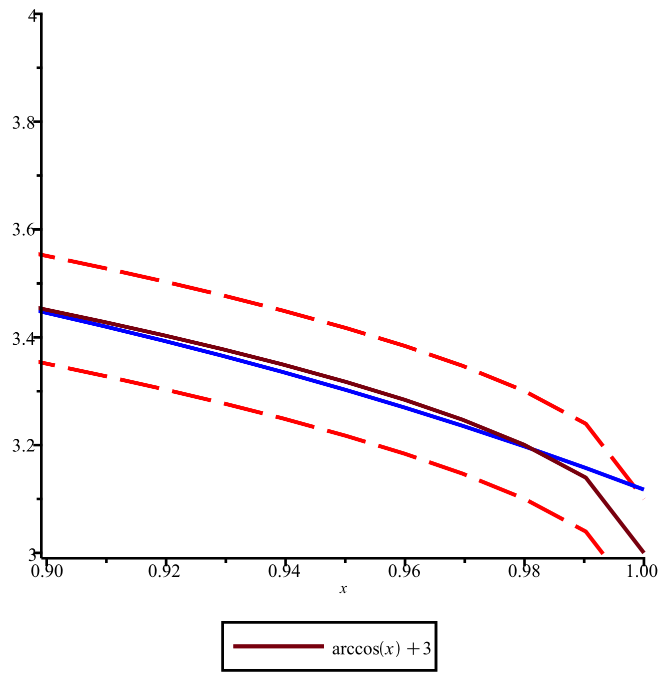


>

> *nmin1* := *plot*(*add*(*c*[*n*]·*P*(*n*, *x*), *n* = 0 ..8), *x* = 1 ..1, *color* = *blue*) :

> *plots*[*display*](*f1*, *f2*, *nmin1*, *our_function1*, *view* = [0.9 ..1, 3 ..4])

#можем заметить, что когда n = 8 функция отклоняется больше чем на 0,
1 (***Лежандр***)



> ***#По многочлену Чебышёва***

> **for** n **from** 0 **to** 10 **do** $c[n] := \dfrac{\displaystyle\int_{1}^{1} \frac{f \cdot T(n, x)}{\mathrm{sqrt}(1 \quad x^2)}\,\mathrm{d}x}{\displaystyle\int_{1}^{1} \frac{T(n, x)^2}{\mathrm{sqrt}(1 \quad x^2)}\,\mathrm{d}x}$; **end do**

$$c_0 := \frac{\frac{1}{2}\pi^2 + 3\pi}{\pi}$$

$$c_1 := \frac{4}{\pi}$$

$$c_2 := 0$$

$$c_3 := \frac{4}{9\pi}$$

$$c_4 := 0$$

$$c_5 := \frac{4}{25\pi}$$

$$c_6 := 0$$

$$c_7 := \frac{4}{49\pi}$$

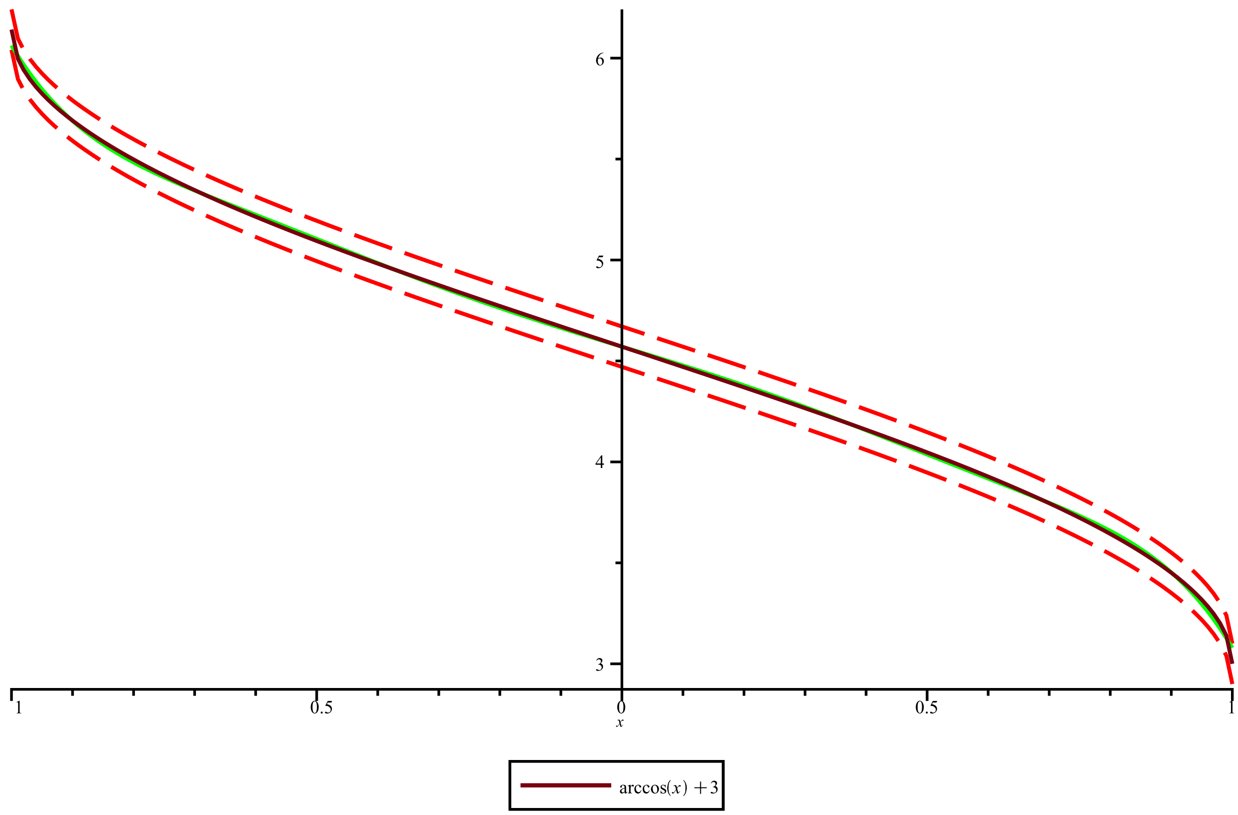
$$c_8 := 0$$

$$c_9 := -\frac{4}{81\,\pi}$$

$$c_{10} := 0 \tag{10}$$

> $cheb\_graf1 := plot(add(c[n]\cdot T(n, x), n = 0..7), x = -1..1, color = green) :$

> $plots[display](f1, f2, cheb\_graf1, our\_function1)$

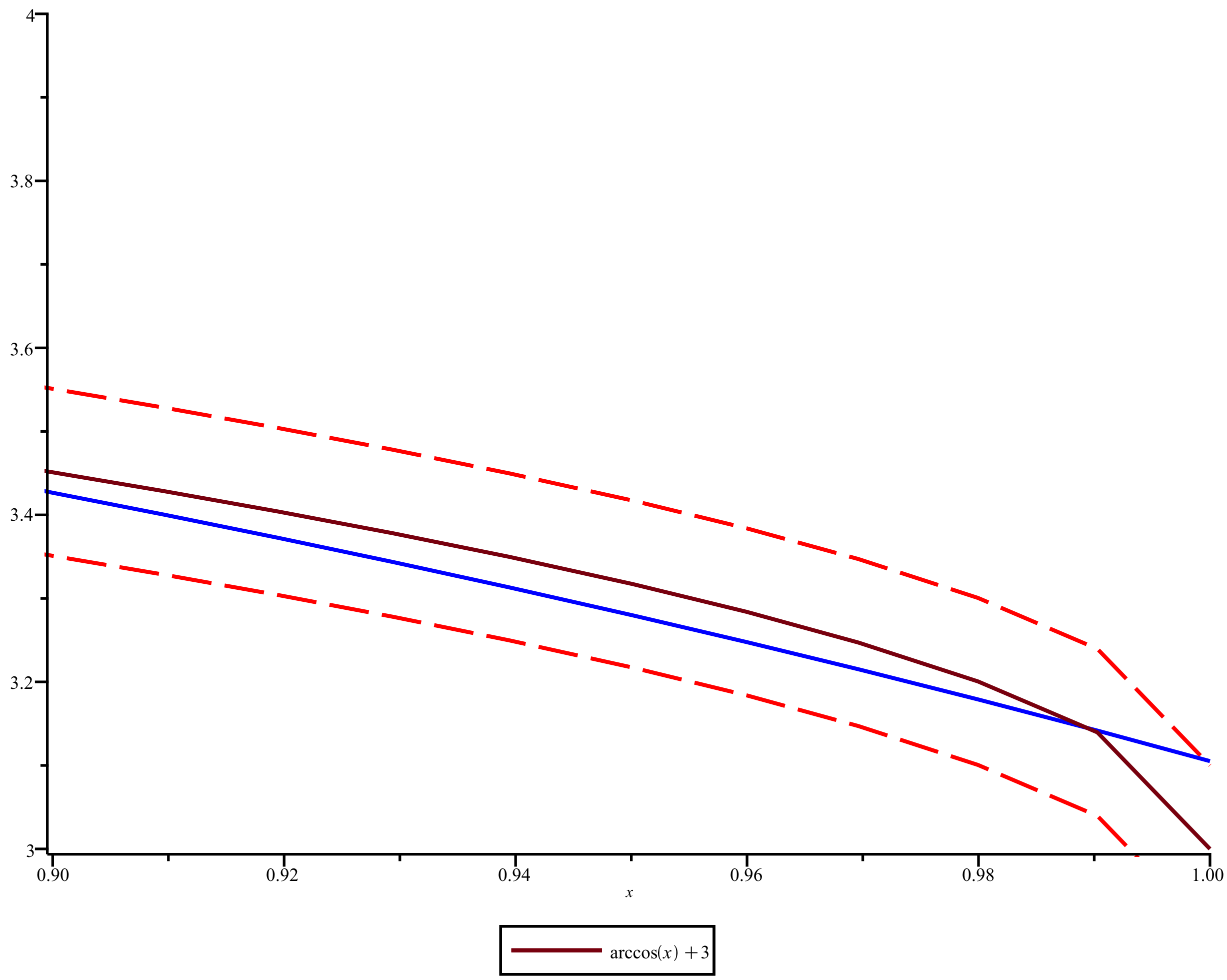


>

> $nmin1 := plot(add(c[n]\cdot T(n, x), n = 0..6), x = \ 1..1, color = blue) :$

> $plots[display](f1, f2, nmin1, our\_function1, view = [0.9..1, 3..4])$

#можем заметить, что когда n = 6 функция отклоняется больше чем на 0,
1 (***Чебышев***)



> # *Тригонометрический ряд Фурье*

> $a0 := simplify(int(f, x = \ 1..1))$

$$a0 := 6 + \pi \tag{11}$$

> $an := simplify(int(\mathrm{f}\cdot\cos(\mathrm{Pi}\cdot nn\cdot x), x = \ 1..1))$ assuming $nn :: posint$

$$an := 0 \tag{12}$$

> $bn := simplify(int(\mathrm{f}\cdot\sin(\mathrm{Pi}\cdot nn\cdot x), x = \ 1..1))$ assuming $nn :: posint$

$$bn := \int_{-1}^{1} (\arccos(x) + 3)\sin(\pi\, nn\, x)\, \mathrm{d}x \tag{13}$$

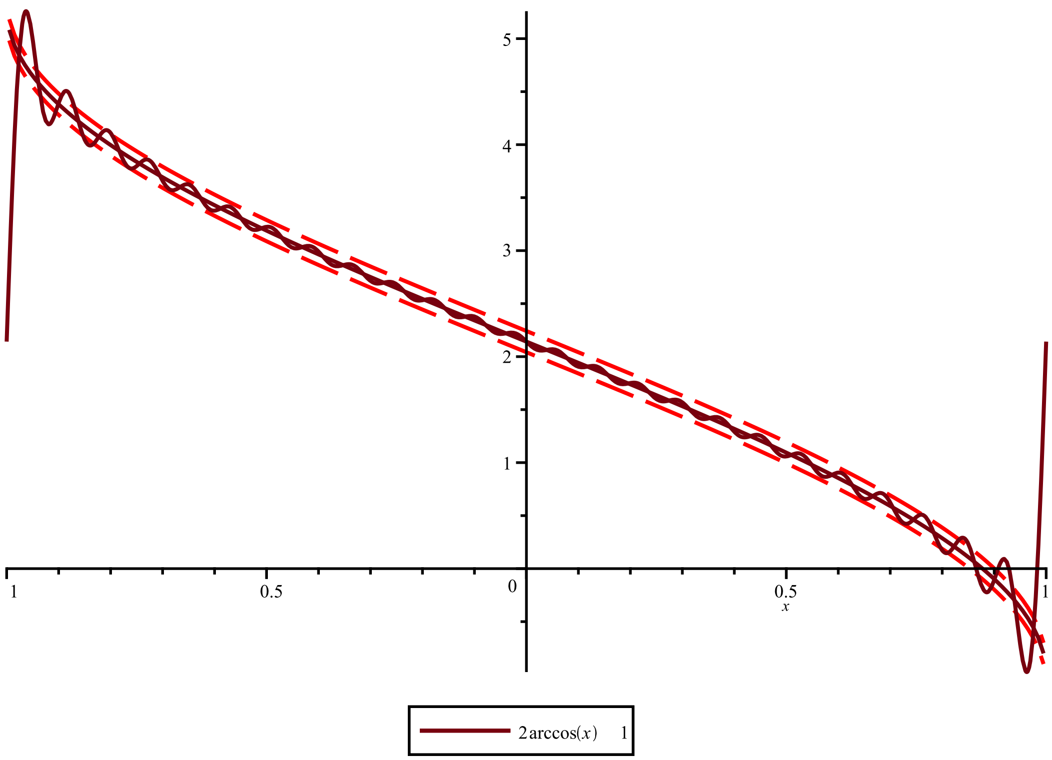
> $Sm := \mathrm{k} \rightarrow \frac{a0}{2} + sum(bn\cdot\sin(\pi\cdot \mathrm{nn}\cdot \mathrm{x}), nn = 1..k)$

$$Sm := k \mapsto \frac{a0}{2} + \left(\sum_{nn=1}^{k} bn\cdot\sin(\pi\cdot nn\cdot x)\right) \tag{14}$$

> $fur := plot(\mathrm{Sm}(25), x = \ 1..1, discont = true)$:

> $plots[display](f1, f2, fur, our\_function1)$

 # тут можно взять промежуток поменьше, например от -0.75 до 0.75 и показать что он будет внутри f +0.1 и f - 0.1()



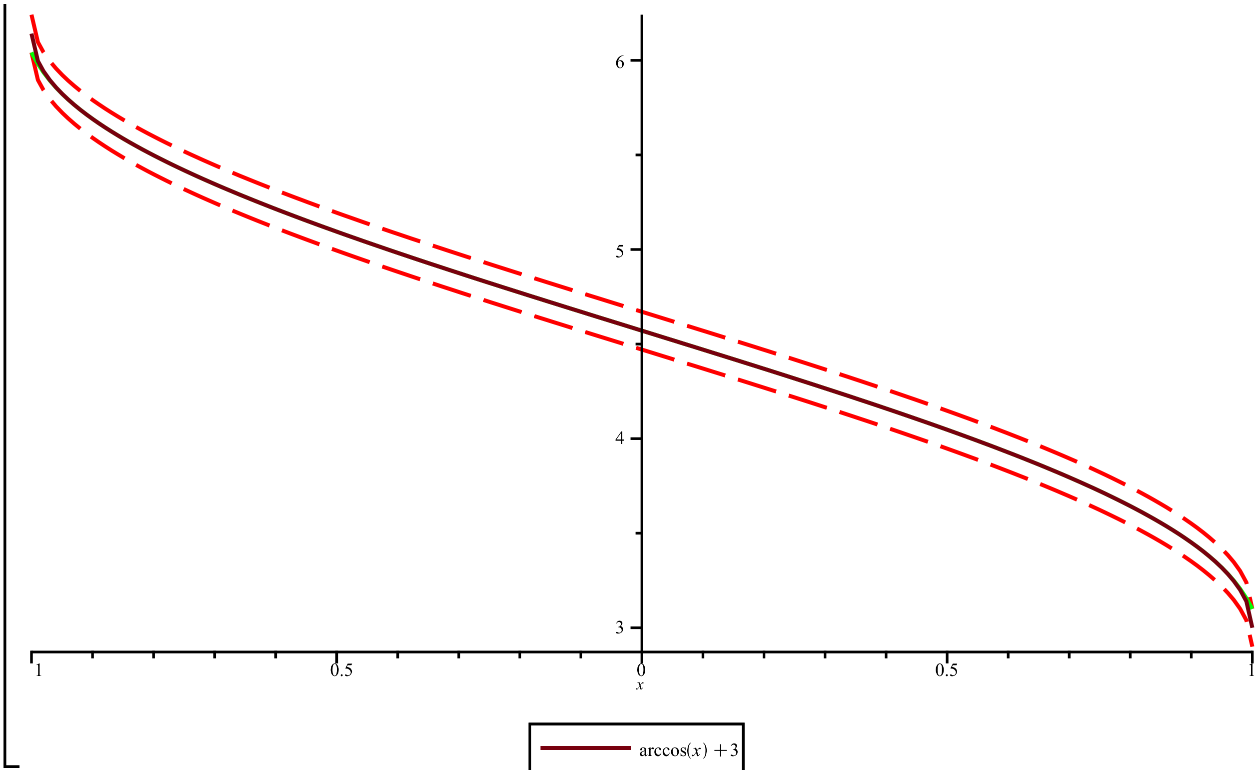
> **#Ряд Тейлора**

> $St := convert(taylor(f, x = 0, 64), polynom)$

$$St := 3 + \frac{1}{2}\pi \quad x \quad \frac{1}{6}x^{3} \quad \frac{3}{40}x^{5} \quad \frac{5}{112}x^{7} \quad \frac{63}{2816}x^{11} \quad \frac{34461632205}{11269994184704}x^{41} \quad \frac{116680311}{30064771072}x^{35} \quad \frac{2268783825}{635655159808}x^{37} \quad \frac{1472719325}{446676598784}x^{39} \quad \frac{35}{1152}x^{9} \quad \frac{100180065}{23622320128}x^{33} \quad \frac{231}{13312}x^{13} \quad \frac{143}{10240}x^{15} \quad \frac{6435}{557056}x^{17} \quad \frac{12155}{1245184}x^{19} \quad \frac{46189}{5505024}x^{21} \quad \frac{88179}{12058624}x^{23} \quad \frac{676039}{104857600}x^{25} \quad \frac{1300075}{226492416}x^{27} \quad \frac{5014575}{973078528}x^{29} \quad \frac{9694845}{2080374784}x^{31} \quad \frac{67282234305}{23639499997184}x^{43} \quad \frac{17534158031}{6597069766656}x^{45} \quad \frac{514589420475}{206708186021888}x^{47} \quad \frac{8061900920775}{3448068464705536}x^{49} \quad \frac{5267108601573}{2392537302040576}x^{51} \quad \frac{61989816618513}{29836347531329536}x^{53} \quad \frac{121683714103007}{61924494876344320}x^{55} \quad \frac{956086325095055}{513410357520236544}x^{57} \quad \frac{1879204156221315}{1062849512059437056}x^{59} \quad \frac{7391536347803839}{4395513236313604096}x^{61} \quad \frac{2077805148460987}{1297036692682702848}x^{63} \tag{15}$$

> $StF := plot(St, x = \ 1..1, color = green):$

> $plots[display](f1, f2, StF, our\_function1)$

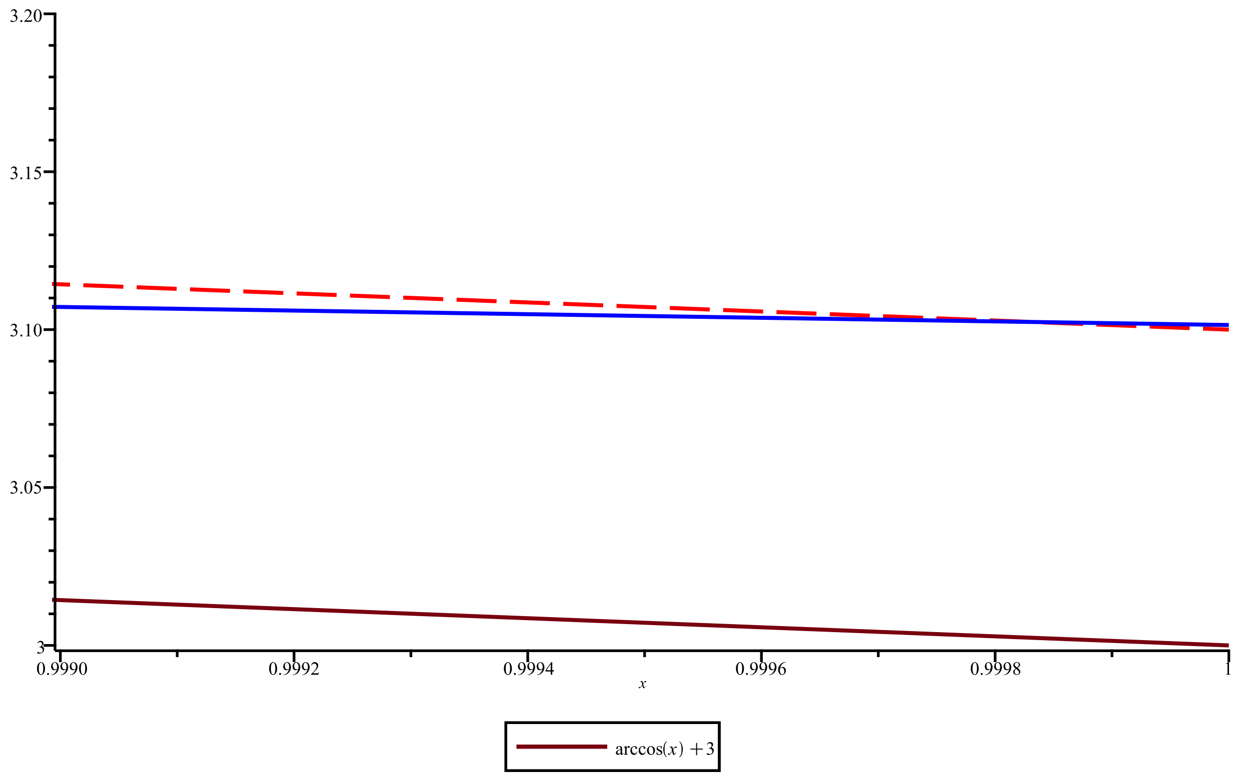


> *minT* := *convert*(*taylor*(*f*, *x* = 0, 63), *polynom*)

$$minT := 3 + \frac{1}{2}\pi \quad x \quad \frac{1}{6}x^{3} \quad \frac{3}{40}x^{5} \quad \frac{5}{112}x^{7} \quad \frac{63}{2816}x^{11} \quad \frac{34461632205}{11269994184704}x^{41} \quad \frac{116680311}{30064771072}x^{35} \quad \frac{2268783825}{635655159808}x^{37} \quad \frac{1472719325}{446676598784}x^{39} \quad \frac{35}{1152}x^{9} \quad \frac{100180065}{23622320128}x^{33} \quad \frac{231}{13312}x^{13} \quad \frac{143}{10240}x^{15} \quad \frac{6435}{557056}x^{17} \quad \frac{12155}{1245184}x^{19} \quad \frac{46189}{5505024}x^{21} \quad \frac{88179}{12058624}x^{23} \quad \frac{676039}{104857600}x^{25} \quad \frac{1300075}{226492416}x^{27} \quad \frac{5014575}{973078528}x^{29} \quad \frac{9694845}{2080374784}x^{31} \quad \frac{67282234305}{23639499997184}x^{43} \quad \frac{17534158031}{6597069766656}x^{45} \quad \frac{514589420475}{206708186021888}x^{47} \quad \frac{8061900920775}{3448068464705536}x^{49} \quad \frac{5267108601573}{2392537302040576}x^{51} \quad \frac{61989816618513}{29836347531329536}x^{53} \quad \frac{121683714103007}{61924494876344320}x^{55} \quad \frac{956086325095055}{513410357520236544}x^{57} \quad \frac{1879204156221315}{1062849512059437056}x^{59} \quad \frac{7391536347803839}{4395513236313604096}x^{61} \tag{16}$$

> *minStF* := *plot*(*minT*, *x* = 1 ..1, *color* = *blue*) :

> *plots*[*display*](*f1*, *f2*, *minStF*, *our_function1*, *view* = [0.999 ..1, 3 ..3.2])



3.20
3.15
3.10
3.05
3
0.9990
0.9992
0.9994
0.9996
0.9998
1
x
arccos(x) + 3

>