> *#Лабораторная работа 2(Вариант 10)*
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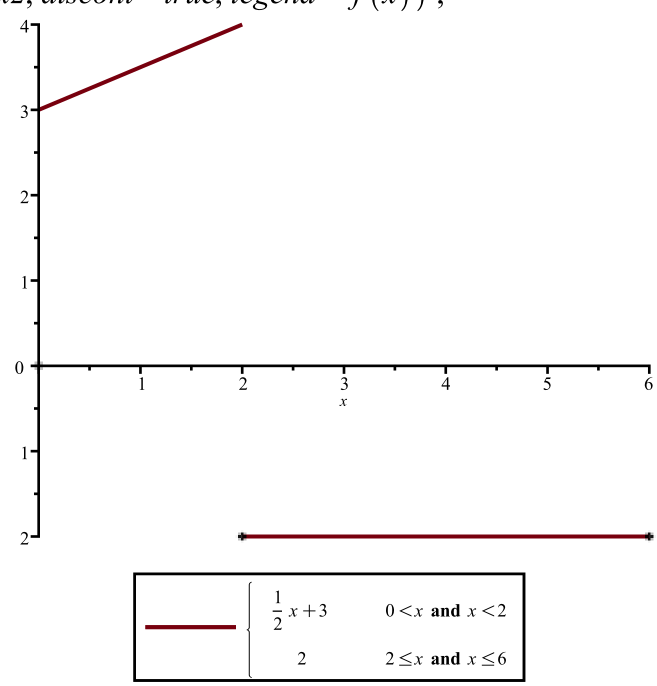
> **#Задание 2. Разложите в ряд Фурье x2 - периодическую функцию y = f(x) , заданную на промежутке (0, x1) формулой**
#y = ax + b, а на [x1, x2] y = c.
#Постройте в одной системе координат на промежутке $[-2\ x2, 2\ x2]$, графики частичных сумм $S1(x)$, $S3(x)$, $S7(x)$ ряда и его суммы $S(x)$

> $x1 := 2;$
$x2 := 6;$
$c := -2;$

$$f := x \rightarrow piecewise\left(0 < x < x1,\ \frac{1}{2}\cdot x + 3,\ x1 \le x \le x2,\ c\right):$$

$$x1 := 2$$
$$x2 := 6$$
$$c := -2 \tag{1}$$

> $plot(f(x), x = 0\,..x2, discont = true, legend = f(x))\ ;$



> $a0 := simplify\left(\frac{2}{x2}\cdot Int(f(x), x = 0\,..x2)\right) = simplify\left(\frac{2}{x2}\cdot int(f(x), x = 0\,..x2)\right)\ ;$

$$a0 := \frac{\left(\int_0^6 \left(\begin{cases} \frac{x}{2} + 3 & x < 2 \\ -2 & 2 \le x \end{cases}\right) dx\right)}{3} = \frac{1}{3} \tag{2}$$

> $an := simplify\left(\frac{2}{x2}\cdot Int\left(f(x)\cdot\cos\left(\frac{2\cdot n\cdot \mathrm{Pi}\cdot x}{x2}\right), x=0..x2\right)\right) = simplify\left(\frac{2}{x2}\cdot int\left(f(x)\cdot\cos\left(\frac{2\cdot n\cdot \mathrm{Pi}\cdot x}{x2}\right), x=0..x2\right)\right)$ assuming n :: $posint$;

$$an := \frac{\left(\int_0^6 \left(\begin{cases}\frac{x}{2}+3 & x<2\\ -2 & 2\le x\end{cases}\right)\cos\left(\frac{n\,\pi\,x}{3}\right)\mathrm{d}x\right)}{3} \tag{3}$$

$$= \frac{3\left(4\,n\,\pi\sin\left(\frac{2\,n\,\pi}{3}\right)+\cos\left(\frac{2\,n\,\pi}{3}\right)-1\right)}{2\,n^2\,\pi^2}$$

> $bn := simplify\left(\frac{2}{x2}\cdot Int\left(f(x)\cdot\sin\left(\frac{2\cdot n\cdot \mathrm{Pi}\cdot x}{x2}\right), x=0..x2\right)\right) = simplify\left(\frac{2}{x2}\cdot int\left(f(x)\cdot\sin\left(\frac{2\cdot n\cdot \mathrm{Pi}\cdot x}{x2}\right), x=0..x2\right)\right)$ assuming n :: $posint$;

$$bn := \frac{\left(\int_0^6 \left(\begin{cases}\frac{x}{2}+3 & x<2\\ -2 & 2\le x\end{cases}\right)\sin\left(\frac{n\,\pi\,x}{3}\right)\mathrm{d}x\right)}{3} \tag{4}$$

$$= \frac{-12\,n\,\pi\cos\left(\frac{2\,n\,\pi}{3}\right)+10\,n\,\pi+3\sin\left(\frac{2\,n\,\pi}{3}\right)}{2\,n^2\,\pi^2}$$

> $FourierTrigSum :=$ **proc**(f, k, a, b)

local a_0, a_n, b_n, n, l;

$l := \frac{(b-a)}{2}$;

$assume(n::posint)$;

$a\_0 := simplify(int(f(x), x=a..b)/l)$;

$a\_n := simplify\left(\frac{int\left(f(x)\cdot\cos\left(\frac{\mathrm{Pi}\cdot n}{l}\cdot x\right), x=a..b\right)}{l}\right)$;

$b\_n := simplify\left(\frac{int\left(f(x)\cdot\sin\left(\frac{\mathrm{Pi}\cdot n}{l}\cdot x\right), x=a..b\right)}{l}\right)$;

return $\frac{a\_0}{2} + sum\left(a\_n\cdot\cos\left(\frac{\mathrm{Pi}\cdot n}{l}\cdot x\right)+b\_n\cdot\sin\left(\frac{\mathrm{Pi}\cdot n}{l}\cdot x\right), n=1..k\right)$

end proc:

> $S1 := FourierTrigSum(f, 1, 0, x2)$;
$S3 := FourierTrigSum(f, 3, 0, x2)$;
$S7 := FourierTrigSum(f, 7, 0, x2)$;

$S := FourierTrigSum(f, \infty, 0, x2)$;
$S50000 := FourierTrigSum(f, 50000, 0, x2)$:

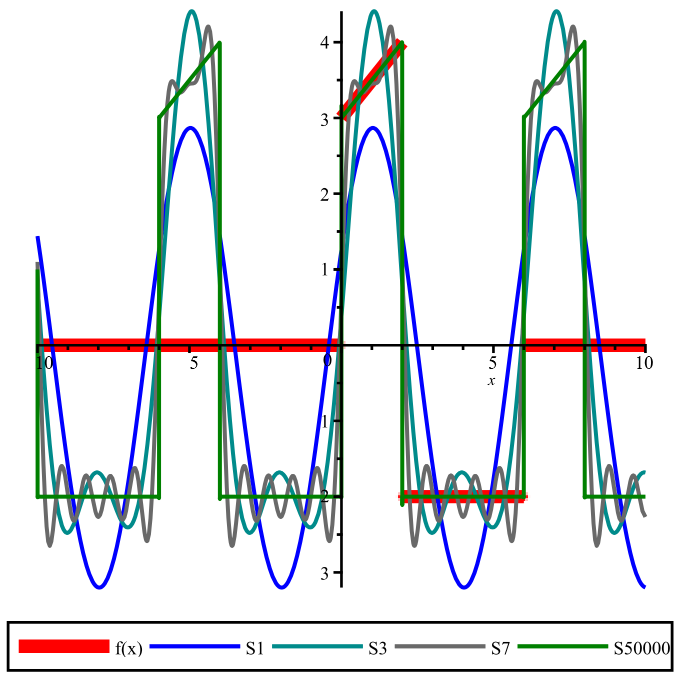
$$S1 := -\frac{1}{6} + \frac{3\left(2\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{\pi x}{3}\right)}{2\pi^2} + \frac{\left(16\pi + \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{\pi x}{3}\right)}{2\pi^2}$$

$$S3 := -\frac{1}{6} + \frac{3\left(2\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{\pi x}{3}\right)}{2\pi^2} + \frac{\left(16\pi + \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{\pi x}{3}\right)}{2\pi^2} + \frac{3\left(-4\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{2\pi x}{3}\right)}{8\pi^2} + \frac{\left(32\pi - \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{2\pi x}{3}\right)}{8\pi^2} - \frac{\sin(\pi x)}{3\pi}$$

$$S7 := -\frac{1}{6} + \frac{3\left(2\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{\pi x}{3}\right)}{2\pi^2} + \frac{\left(16\pi + \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{\pi x}{3}\right)}{2\pi^2} + \frac{3\left(-4\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{2\pi x}{3}\right)}{8\pi^2} + \frac{\left(32\pi - \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{2\pi x}{3}\right)}{8\pi^2} - \frac{\sin(\pi x)}{3\pi} + \frac{3\left(8\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{4\pi x}{3}\right)}{32\pi^2} + \frac{\left(64\pi + \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{4\pi x}{3}\right)}{32\pi^2} + \frac{3\left(-10\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{5\pi x}{3}\right)}{50\pi^2} + \frac{\left(80\pi - \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{5\pi x}{3}\right)}{50\pi^2} - \frac{\sin(2\pi x)}{6\pi} + \frac{3\left(14\pi\sqrt{3} - \frac{3}{2}\right)\cos\left(\frac{7\pi x}{3}\right)}{98\pi^2} + \frac{\left(112\pi + \frac{3\sqrt{3}}{2}\right)\sin\left(\frac{7\pi x}{3}\right)}{98\pi^2}$$

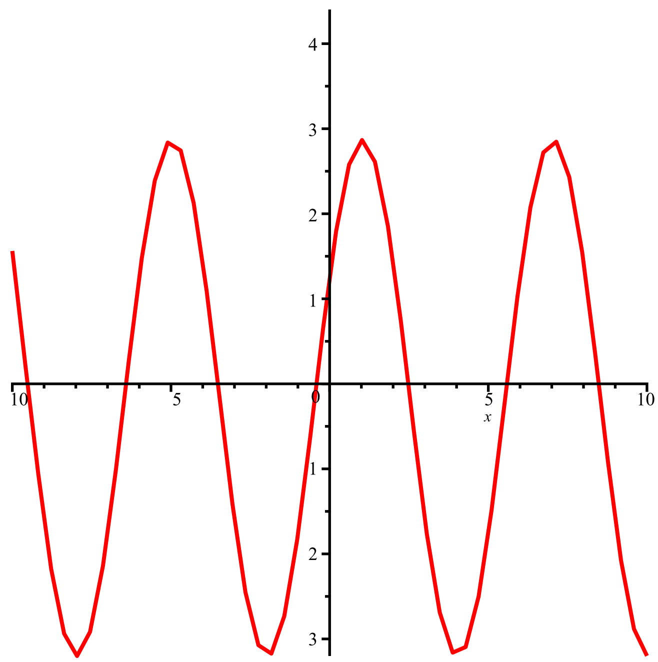
$$S := -\frac{1}{6} + \sum_{n\sim=1}^{\infty}\left(\frac{3\left(4\pi n\sim\sin\left(\frac{2\pi n\sim}{3}\right) + \cos\left(\frac{2\pi n\sim}{3}\right) - 1\right)\cos\left(\frac{\pi n\sim x}{3}\right)}{2 n\sim^2 \pi^2} + \frac{\left(-12\pi n\sim\cos\left(\frac{2\pi n\sim}{3}\right) + 10\pi n\sim + 3\sin\left(\frac{2\pi n\sim}{3}\right)\right)\sin\left(\frac{\pi n\sim x}{3}\right)}{2 n\sim^2 \pi^2}\right) \tag{5}$$

> $plot([S1, S3, S7, S50000], x = -10..10, legend = [$"S1", "S3", "S7", "S50000"$], color = [$"Blue", "DarkCyan", "DimGray", "Green"$])$:
$plot(f(x), x = -10..10, legend = $"f(x)"$, discont = true, color = red, thickness = 5)$:
$plots[display](\%, \%\%)$



> **#Анимация**

> $plots[animate](FourierTrigSum(f, k, 0, x2)\ , x = \ \ 10\,..10, k = 1\,..16, numpoints = 50);$



> $restart:$

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