Time Series Smoothing

In this assignment, we will explore the reaction of a simple exponential smoothing model to standard input series.

Theoretic part

Let the time series y_t, $t = 1,2,\dots,T$ can be represented as an additive combination of two components: systematic f_t and disturbing u_t :

$$y_t = f_t + u_t,\ t = 1,2,\dots,T.$$

Variable u_t is random and it has zero expected value and the same dispersion for any t value, its sequential values are uncorrelated. Variable f_t is a row level at the moment t, and the law of level change is a trend.

Components f_t and u_t are not observable. The process of model constructing that approximates trend is called time series smoothing.

A number of methods are used to highlight a trend. The simplest of the exponentially smoothing methods is naturally called simple exponential smoothing (SES). We will consider this method, which belongs to the class of adaptive, when the parameters of the model change during the transition from one observed value to another. This method is suitable for forecasting data with no clear trend or seasonal patter.

SES assume that the series has an infinite backstory and using the weighted least squares method find the coefficients of the polynomial P_t from time t selected degree d, that is, minimize

$$S_R = \sum_{i=0}^{\infty} (y_{t-1} - P_{t-1})^2 \beta^i, \qquad 0 < \beta < 1.$$

Polynomial coefficients can be presented as a linear combinations exponential mean corresponding orders. A simple exponential smoothing model defined for a polynomial of degree zero ($P_t = a_t$), has the form:

$$a_t = S_t^{(l)} = (1-\beta)\sum_{i=0}^{\infty} y_{t-1}\beta^i.$$

As examples of the polynomial coefficients, formulas are given for the coefficients of the linear model $y_t = a_t + b_t t + u_t$ (double exponential smoothing):

$$a_t = a_{t-1} + b_{t-1} + (1-\beta^2)e_t,$$

$$b_t = b_{t-1} + (1-\beta^2)e_t.$$

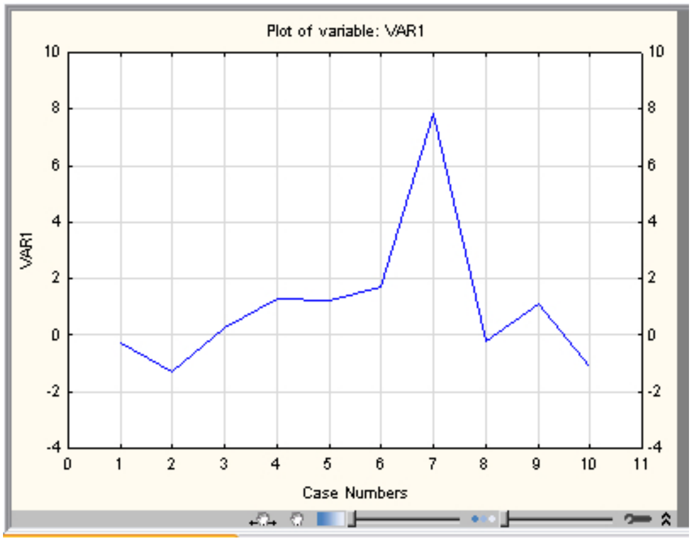
e_t – error of one step forward forecasting, $e_t = y_t - (a_{t-1} + b_{t-1} 1)$.

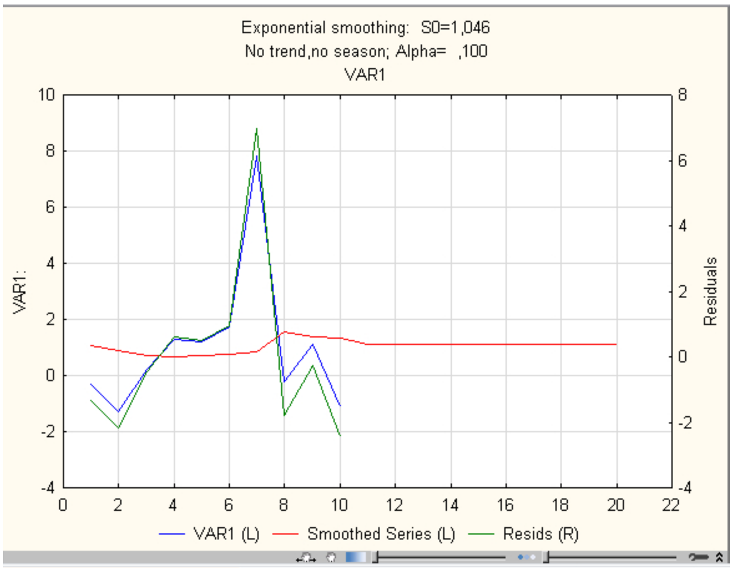
The constant of smoothing $\alpha = 1 - \beta$ is selected empirically, based on the minimum forecast errors for the estimated model.

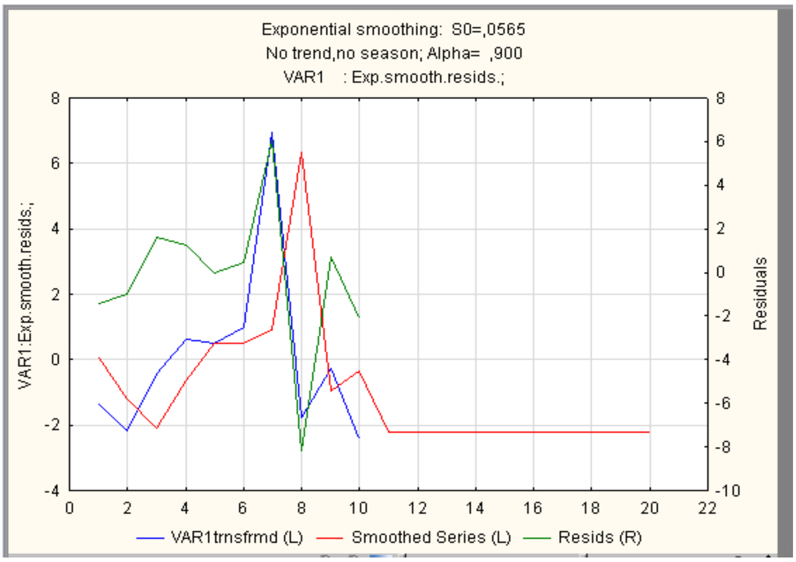
Task 1

Explore reaction of simple exponential smoothing models for standard input series.

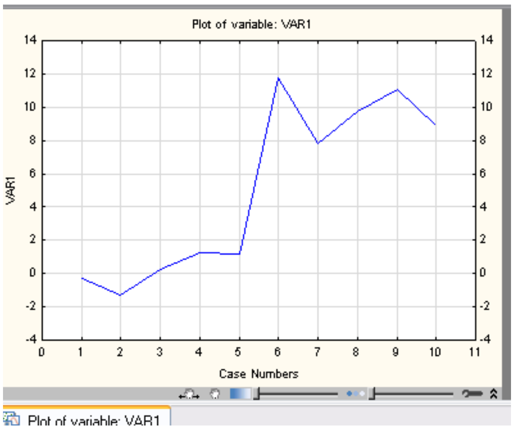
Case 1. Single impulse

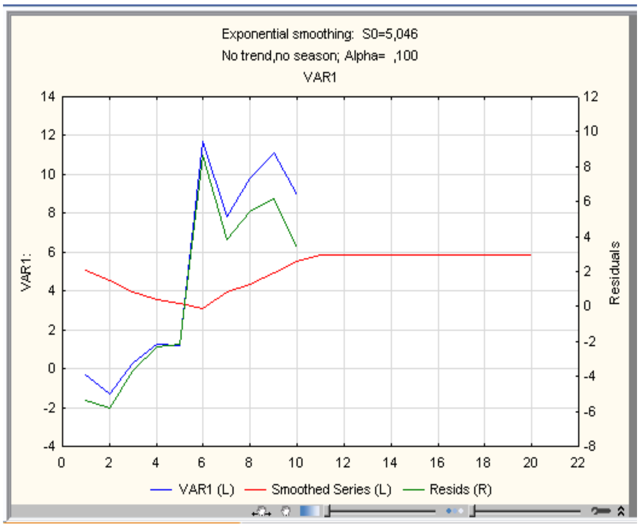


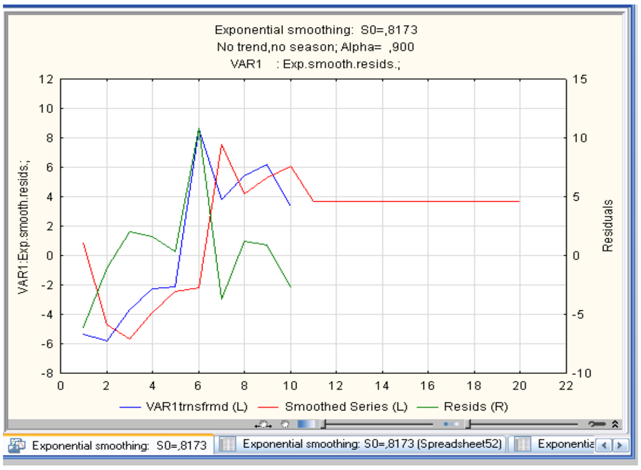




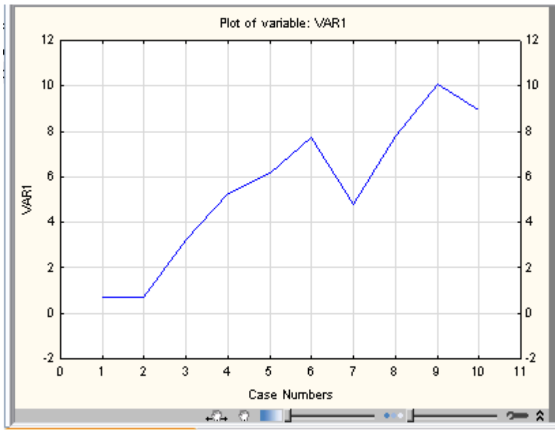
Case 2. Step effect

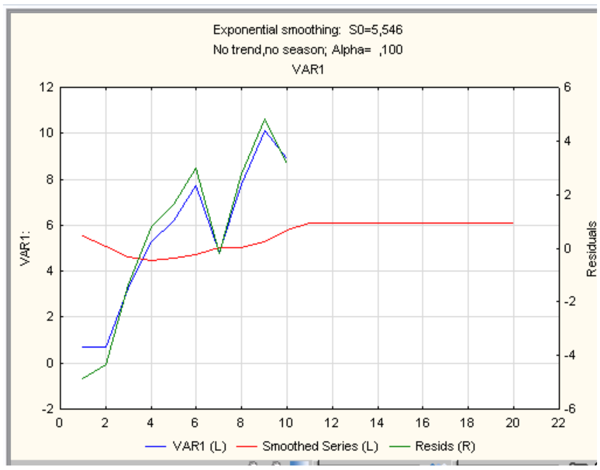


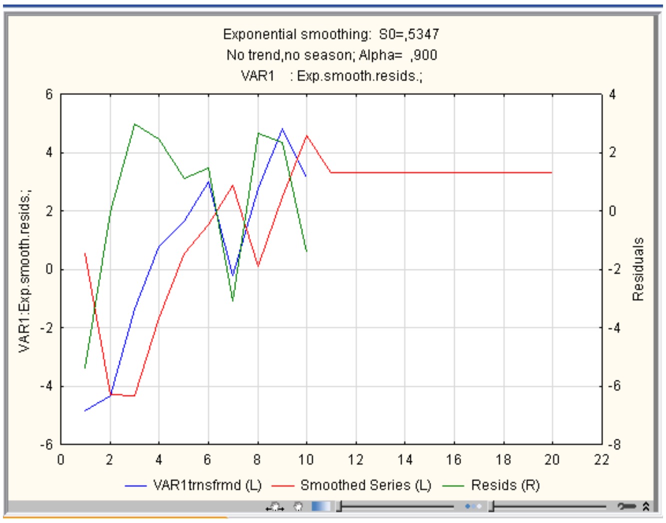




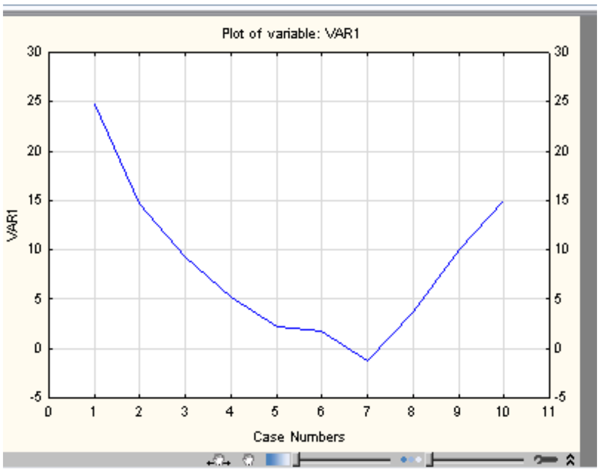
Case 3. Linear function

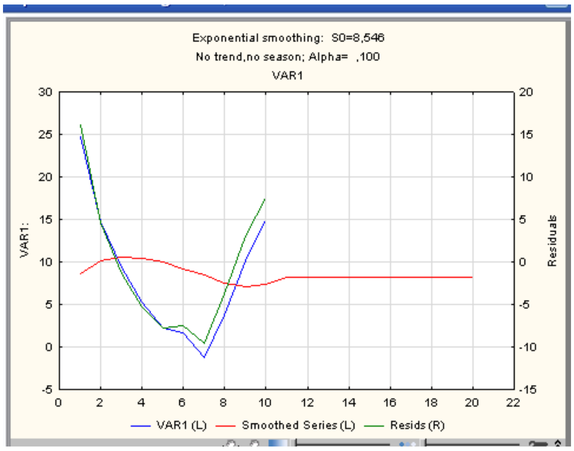


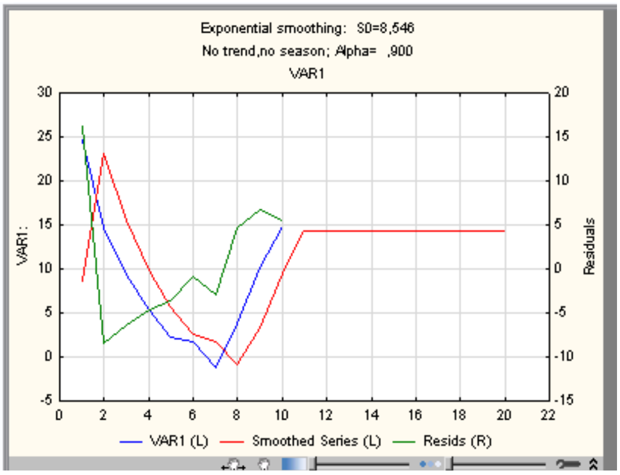




Case 4. Parabolic function







Task 2

For the problem of predicting two steps ahead, choose the appropriate model of exponential smoothing for the real series. Conduct selection by minimum sum of squared deviations.

Case 1. Single impulse

Summary of error	Exponential smoothing: S0=1,046 (Spreadsheet47) No trend,no season; Alpha= ,100 VAR1 Error
Mean error	0,05649173444
Mean absolute error	1,75330071450
Sums of squares	66,01421648117
Mean square	6,60142164812
Mean percentage error	163,28966423475
Mean abs. perc. error	-116,26483182793

Summary of error	Exponential smoothing: S0=,0565 (Spreadsheet47) No trend,no season; Alpha= ,900 VAR1 : Exp.smooth.resids.; Error
Mean error	-0,253446910520
Mean absolute error	2,272692226322
Sums of squares	114,958984165401
Mean square	11,495898416540
Mean percentage error	40,635353926776
Mean abs. perc. error	-96,760990689814

Case 2. Step effect

Summary of error	Exponential smoothing: S0=5,046 (Spreadsheet52) No trend,no season; Alpha= ,100 VAR1 Error
Mean error	0,817305494837
Mean absolute error	4,667155494934
Sums of squares	253,308239809695
Mean square	25,330823980969
Mean percentage error	63,785708373032
Mean abs. perc. error	-9,479683455832

Summary of error	Exponential smoothing: S0=,8173 (Spreadsheet52) No trend,no season; Alpha= ,900 VAR1 : Exp.smooth.resids.; Error
Mean error	0,315779007222
Mean absolute error	3,042351843711
Sums of squares	185,242445959299
Mean square	18,524244595930
Mean percentage error	-1,948544957375
Mean abs. perc. error	6,631845950160

Case 3. Linear function

Summary of error	Exponential smoothing: S0=5,546 (Spreadsheet57) No trend,no season; Alpha= ,100 VAR1 Error
Mean error	0,53465067579
Mean absolute error	2,69161368691
Sums of squares	97,19993340470
Mean square	9,71999334047
Mean percentage error	-114,20355560106
Mean abs. perc. error	153,89729180326

Summary of error	Exponential smoothing: S0=,5347 (Spreadsheet57) No trend,no season; Alpha= ,900 VAR1 : Exp.smooth.resids.; Error
Mean error	0,30614056172
Mean absolute error	2,28849981544
Sums of squares	71,08381654010
Mean square	7,10838165401
Mean percentage error	185,46853277631
Mean abs. perc. error	-113,26791097330

Case 4. Parabolic function

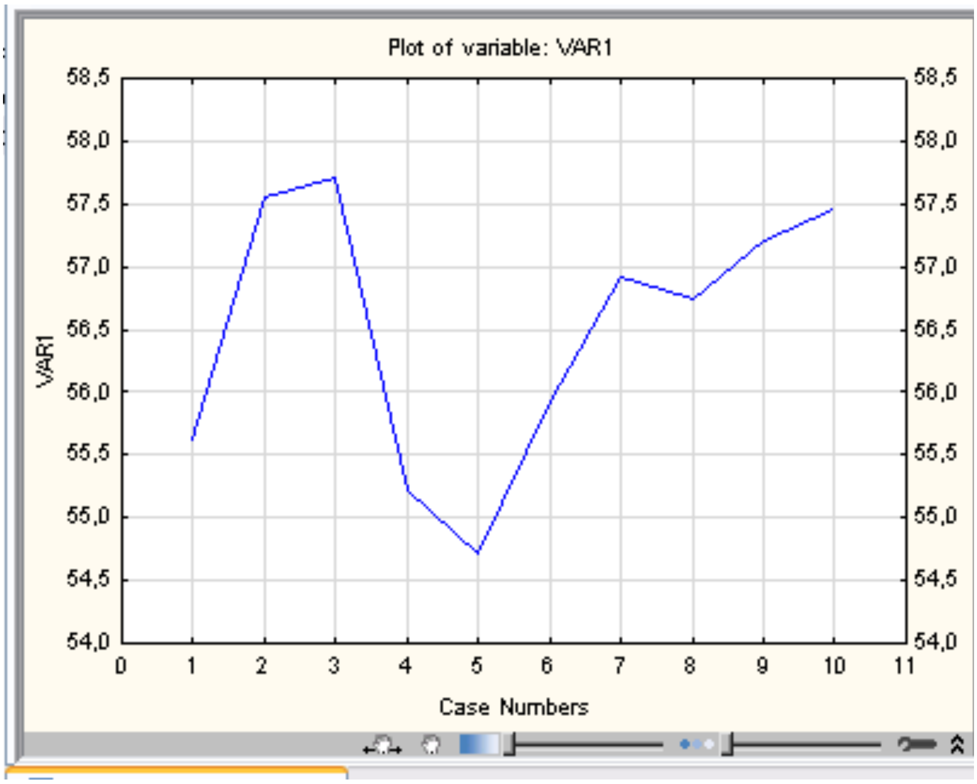
Summary of error	Exponential smoothing: S0=8,546 (Spreadsheet62) No trend,no season; Alpha= ,100 VAR1 Error
Mean error	-0,391128596313
Mean absolute error	6,633550522227
Sums of squares	598,135218467054
Mean square	59,813521846705
Mean percentage error	-0,567180676211
Mean abs. perc. error	35,834935089672

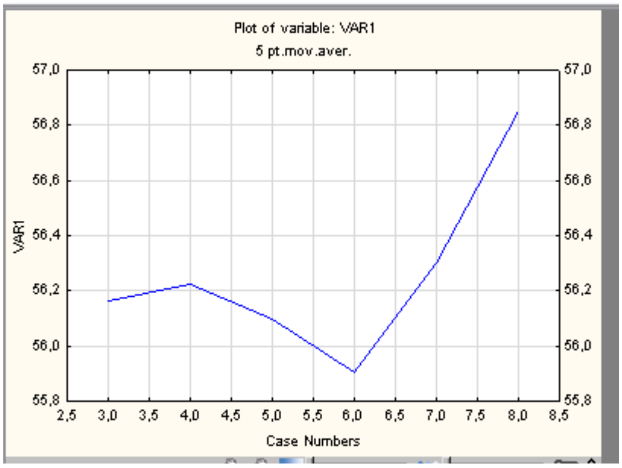
Summary of error	Exponential smoothing: S0=-,391 (Spreadsheet62) No trend,no season; Alpha= ,900 VAR1 : Exp.smooth.resids.; Error
Mean error	0,819480420281
Mean absolute error	6,133517938721
Sums of squares	567,517270450136
Mean square	56,751727045014
Mean percentage error	69,656762197584
Mean abs. perc. error	-17,282414971859

Conclusion: Model of exponential smoothing with input as a single pulse (66,01) and *alpha* =0,1 has the minimum sum of squares

Task 4

In the smoothing mode, we examined a series with dollar values in various Russia's banks (2015y)





Plot of variable: VAR1
5 pt.mov.aver.
VAR1
Case Numbers
57,0
56,8
56,6
56,4
56,2
56,0
55,8
2,5
3,0
3,5
4,0
4,5
5,0
5,5
6,0
6,5
7,0
7,5
8,0
8,5