Лабораторная работа №3.2

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\> **#Задание1(решить уравнения, построить в одной системе координат несколько интегральных кривых)**

\> ***#1.1***
restart;

\> $$de := x = \cosh\left(\frac{d^2}{dx^2}\,y(x)\right) + \left(\frac{d^2}{dx^2}\,y(x)\right)^2$$

$$de := x = \cosh\left(\frac{d^2}{dx^2}\,y(x)\right) + \left(\frac{d^2}{dx^2}\,y(x)\right)^2 \quad (1)$$

\> $dsolve(de)$

$$y(x) = \iint RootOf\left(-x + \cosh(\_Z) + \_Z^2\right)\,dx\,dx + \_C1\,x + \_C2 \quad (2)$$

\> $x\_t := \cosh(t) + t^2$

$$x\_t := \cosh(t) + t^2 \quad (3)$$

\> $dx := diff(x\_t, t)$

$$dx := \sinh(t) + 2\,t \quad (4)$$

\> $y1 := int(t \cdot dx, t)$

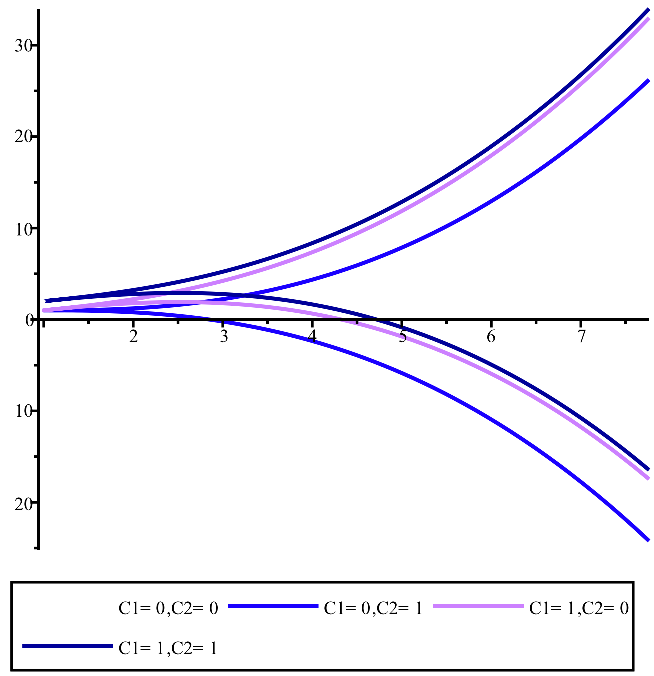
$$y1 := t\cosh(t) - \sinh(t) + \frac{2\,t^3}{3} \quad (5)$$

\> $y\_t := int((y1 + C1)\cdot dx, t) + C2$

$$y\_t := \frac{t\cosh(t)^2}{2} - \frac{3\sinh(t)\cosh(t)}{4} + \frac{t}{4} - 2\,t\cosh(t) + 2\sinh(t) + \frac{2\,t^3\cosh(t)}{3} + \frac{4\,t^5}{15} + C1\cosh(t) + C1\,t^2 + C2 \quad (6)$$

\> $res\_plot := seq(seq(plot([x\_t, y\_t, t=-2..2], colour = COLOUR(RGB, 1.5 - 0.7\cdot(2\cdot C2 + C1), 1 - 0.5\cdot(2\cdot C2 + C1), 3 - 0.8\cdot(2\cdot C2 + C1)), legend = typeset(\text{"C1= "}, C1, \text{",C2= "}, C2)), C2 = 0..1), C1 = 0..1) :$

\> $plots[display](res\_plot);$



> **restart;**
#1.2

> de2 := sin(x)·cos(x) · (y(x) · y''(x)   (y'(x))^2) = 2 · y(x) · y'(x)

$$de2 := \sin(x)\,\cos(x)\left(y(x)\left(\frac{d^2}{dx^2}\,y(x)\right)\quad\left(\frac{d}{dx}\,y(x)\right)^2\right) = 2\,y(x)\left(\frac{d}{dx}\,y(x)\right) \qquad \textbf{(7)}$$

> *y2_finish* := *simplify*(*dsolve*(*de2*)) :

> *y2_finish* := *simplify*(*convert*(*expand*(*simplify*(*convert*(*subs*([_C2 = C2, _C1 = C1], *y2_finish*), tan))), tan)) :

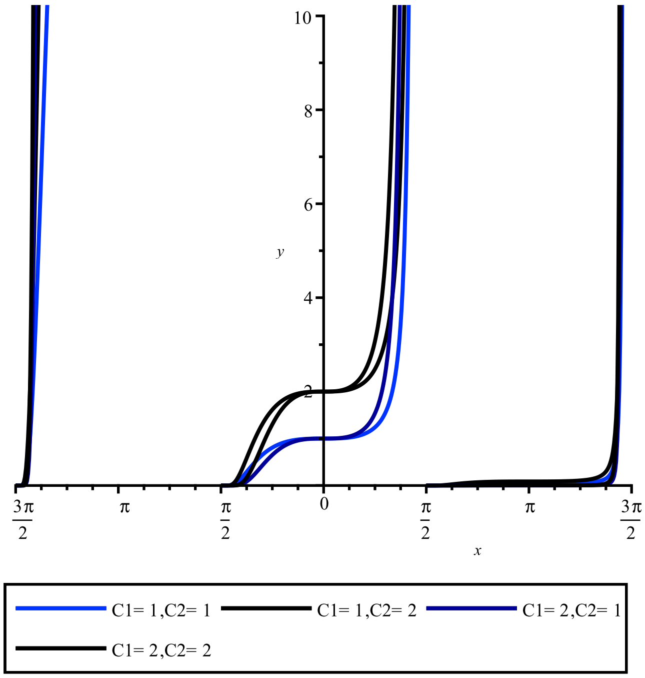
> *y2_finish*;

$$y(x) = C2\,e^{(\tan(x)\quad x)\,C1} \qquad \textbf{(8)}$$

>

> *res_plot2* := *seq*(*seq*(*plot*(*rhs*(*y2_finish*), *x* = $\frac{3}{2}$·Pi .. $\frac{3}{2}$·Pi, *y* = 0 ..10, *colour* = *COLOUR*(*RGB*, 1   0.6·(2·*C2* + *C1*), 2   0.6·(2 ·*C2* + *C1*), 3   0.6·(2·*C2* + *C1*)), *legend* = *typeset*("C1= ", *C1*, ",C2= ", *C2*), *discont* = *true*), *C2* = 1 ..2), *C1* = 1 ..2) :

> *plots*[*display*](*res_plot2*);



> **restart;**

> **#1.3**

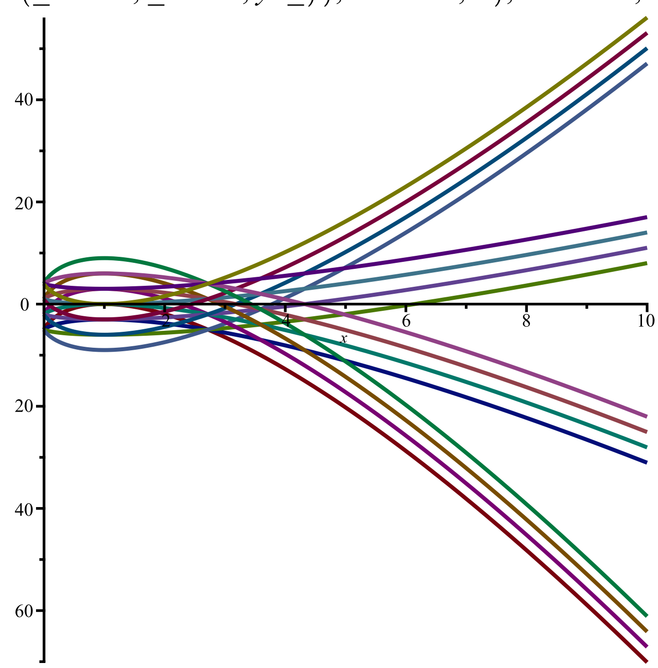
> de3 := y″ · x · ln(x)   y′ = 0

$$de3 := \left(\frac{d^2}{dx^2}\, y(x)\right) x \ln(x) \quad \frac{d}{dx}\, y(x) = 0 \qquad \textbf{(9)}$$

> *y3_ := dsolve(de3)*

$$y3\_ := y(x) = \_C1 + (x \ln(x) \quad x)\ \_C2 \qquad \textbf{(10)}$$

> *plot([seq(seq(rhs(subs(_C1 = v, _C2 = t, y3_)), t = 5..5, 3), v = 5..5, 3)])*



> **restart;**

> ***#1.4***

> de4 := $y'' + \frac{3 \cdot y'}{x} - \frac{3 \cdot y}{(x)^2} = 16 \cdot (x)^3 \cdot (\mathrm{e})^{(x)^4}$

$$de4 := \frac{d^2}{dx^2}\, y(x) + \frac{3\left(\frac{d}{dx}\, y(x)\right)}{x} - \frac{3\, y(x)}{x^2} = 16\, x^3\, (\mathrm{e})^{x^4} \quad \mathbf{(11)}$$

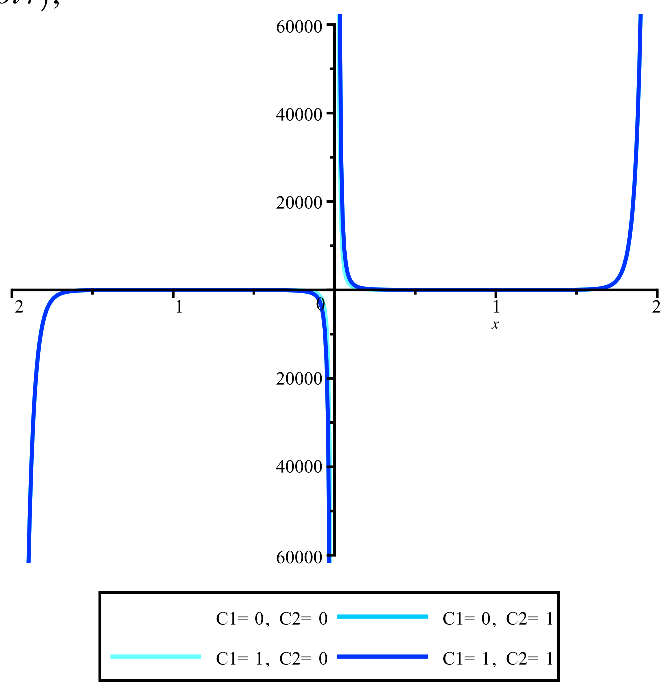
> *y4_ := dsolve(de4) :*
y4_finish := simplify(y4_) :
y4_finish;

$$y(x) = \frac{\_C1 \ln\left(\mathrm{e}^{x^4}\right) + \mathrm{e}^{x^4} + \_C2}{x^3} \quad \mathbf{(12)}$$

>

> *res_plot4 := seq(seq(plot(rhs(y4_finish), x = −2 ..2, colour = COLOUR(RGB, 1 -0.6·(2·_C2 + _C1), 2 − 0.6·(2 ·_C2 + _C1), 3 − 0.6·(2·_C2 + _C1)), legend = typeset("_C1= ", _C1, ",_C2= ", _C2), discont = true, thickness = 1), _C2 = 0 ..1), _C1 = 0 ..1) :*

> *plots[display](res_plot4);*



> **#Задание2(найти общее решение уравнения)**

> **restart;**

> *de5* := $y'''(x) \cdot \cot(2 \cdot x) + 2 \cdot y''(x) = 0$

$$de5 := \left(\frac{d^3}{dx^3}\, y(x)\right) \cot(2\,x) + 2\, \frac{d^2}{dx^2}\, y(x) = 0 \quad \mathbf{(13)}$$

> *y5_ := dsolve(de5, y(x))*

$$y5\_ := y(x) = \frac{\_C1 \sin(4\,x)}{4\sqrt{\frac{2}{\cos(4\,x) \quad 1}}\,\left(\cos(4\,x) \quad 1\right)} + \_C2\, x + \_C3 \quad \mathbf{(14)}$$

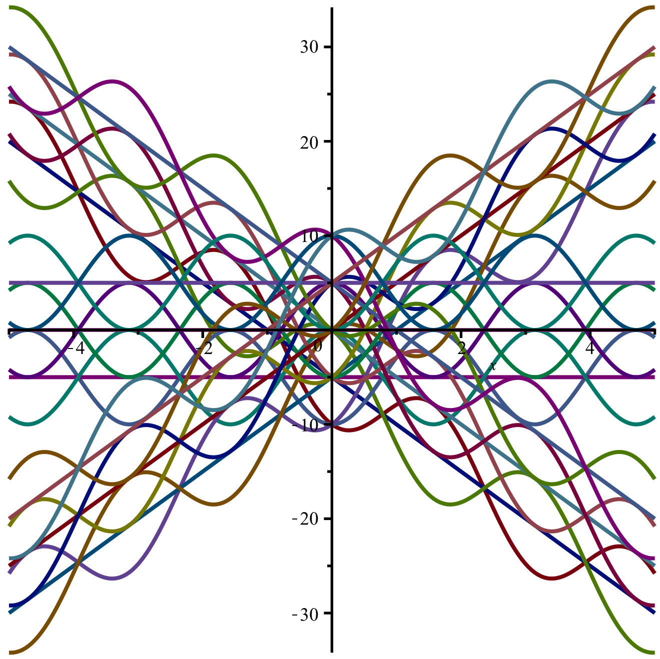
> *y5_ := simplify(convert(simplify(y5_), tan)) assuming x :: real*

$$y5\_ := y(x) = -\frac{\cos(2x)\,\mathrm{signum}(\sin(2x))\,\_C1}{4} + \_C2\,x + \_C3 \tag{15}$$

> $y5\_ := subs\left(\_C1 = -\frac{\_C1 \cdot 4}{\mathrm{signum}(\sin(2x))}, y5\_\right)$

$$y5\_ := y(x) = \cos(2x)\,\_C1 + \_C2\,x + \_C3 \tag{16}$$

> $plot([seq(seq(seq(rhs(y5\_), \_C1 = -5..5, 5), \_C2 = -5..5, 5), \_C3 = -5..5, 5)], x = -5..5)$



> **#Задание3(найти общее решение ДУ)**

> ***restart;***

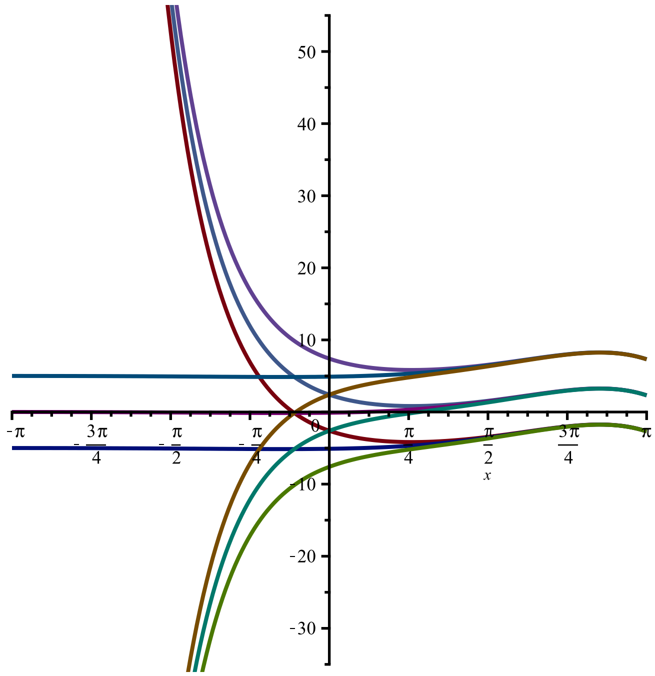
> $de6 := y'' + 2 \cdot y' = e^{x} \cdot (\sin(x) + \cos(x))$

$$de6 := \frac{d^2}{dx^2}\,y(x) + 2\,\frac{d}{dx}\,y(x) = e^{x}\,(\sin(x) + \cos(x)) \tag{17}$$

> $y6\_finish := dsolve(de6)$

$$y6\_finish := y(x) = -\frac{e^{x}\cos(x)}{10} + \frac{3\,e^{x}\sin(x)}{10} - \frac{\_C1}{2\,(e^{x})^{2}} + \_C2 \tag{18}$$

> $plot([seq(seq(rhs(y6\_finish), \_C1 = -5..5, 5), \_C2 = -5..5, 5)], x = -\pi..\pi)$


50
40
30
20
10
-10
-20
-30
$-\pi$
$-\frac{3\pi}{4}$
$-\frac{\pi}{2}$
$-\frac{\pi}{4}$
0
$\frac{\pi}{4}$
$\frac{\pi}{2}$
$\frac{3\pi}{4}$
π
x

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