> #Лабораторная работа 2
Ряды Фурье
#Жгутов Е.Д., гр. 253504, вариант 7

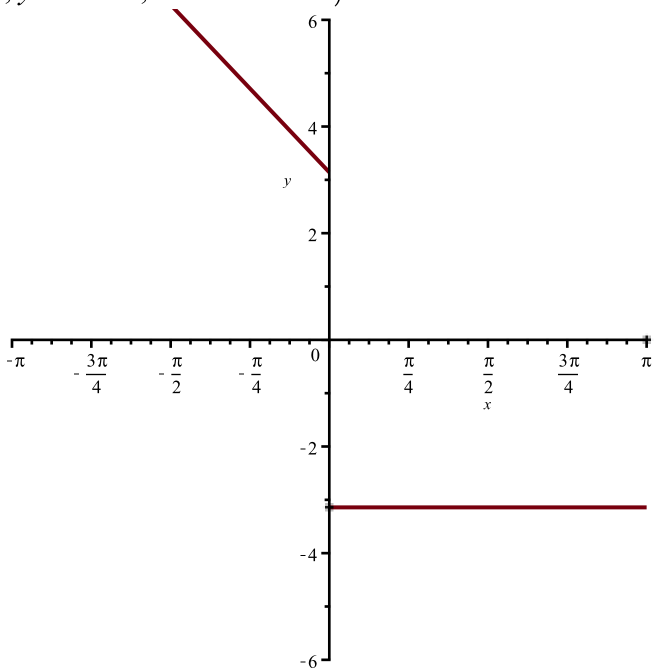
> ***#Задание1 (получить разложение в тригонометрический ряд Фурье для 2Pi-периодической функции, построить на промежутке [-3*Pi, 3*Pi] графики S1(x), S2(x), S7(x) и S(x))***

>

> $f := x \rightarrow piecewise(-\mathrm{Pi} \le x < 0, -2 \cdot x + \mathrm{Pi}, 0 \le x < \mathrm{Pi}, -\mathrm{Pi})$

$$f := x \mapsto \begin{cases} -2\cdot x + \pi & -\pi \le x < 0 \\ -\pi & 0 \le x < \pi \end{cases} \tag{1}$$

> $plot(f(x), x = -\mathrm{Pi} .. \mathrm{Pi}, y = -6 .. 6, discont = true)$



> ***#Коэффициенты***

> $a0 := simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x), x = -\mathrm{Pi} .. \mathrm{Pi})\right)$

$$a0 := \pi \tag{2}$$

> $an := simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x) \cdot \cos(n \cdot x), x = -\mathrm{Pi} .. \mathrm{Pi})\right)$ assuming $n :: posint$

$$an := \frac{2\,(-1)^n - 2}{\pi\, n^2} \tag{3}$$

> $bn := simplify\left(\frac{1}{\mathrm{Pi}} \cdot int(f(x) \cdot \sin(n \cdot x), x = -\mathrm{Pi} .. \mathrm{Pi})\right)$ assuming $n :: posint$

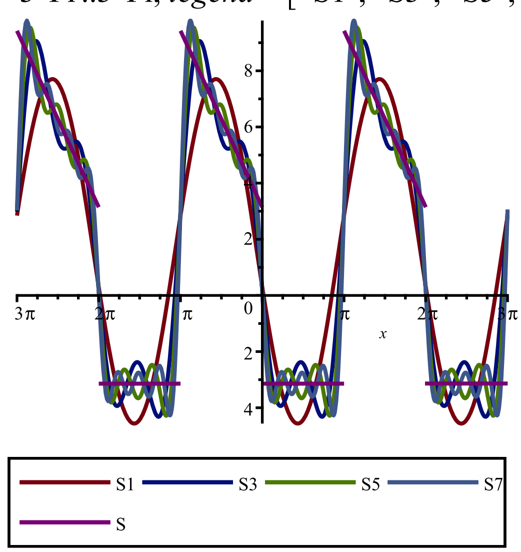
$$bn := \frac{4\,(-1)^n - 2}{n} \tag{4}$$

> $MyFurieSum :=$ **proc**(f, k)
local $a0, an, bn, n;$

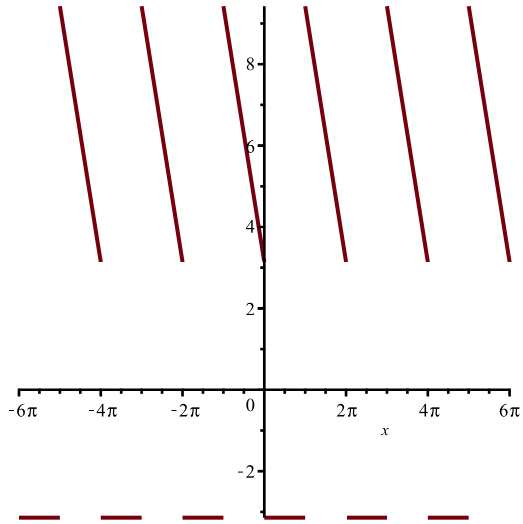
```
a0 := simplify(1/Pi · int(f(x), x =-Pi ..Pi));
assume(n :: posint);
an := simplify(1/Pi · int(f(x) · cos(n·x), x =-Pi ..Pi));
bn := simplify(1/Pi · int(f(x) · sin(n·x), x = -Pi .. Pi));
 return 1/2 · a0 + sum(an·cos(n·x) + bn·sin(n·x), n = 1 ..k)
end proc
```

```
MyFurieSum := proc(f, k)                                        (5)
    local a0, an, bn, n;
    a0 := simplify(int(f(x), x = − π..π) / π);
    assume(n::posint);
    an := simplify(int(f(x) * cos(n * x), x = − π..π) / π);
    bn := simplify(int(f(x) * sin(n * x), x = − π..π) / π);
    return 1 / 2 * a0 + sum(an * cos(n * x) + bn * sin(n * x), n = 1 ..k)
end proc
```

```
> #Найдем частичные функции
> S1 := MyFurieSum(f, 1) :
> S3 := MyFurieSum(f, 3) :
> S5 := MyFurieSum(f, 5) :
> S7 := MyFurieSum(f, 7) :
> S := MyFurieSum(f, infinity) :
> plot([S1, S3, S5, S7, S], x =-3·Pi ..3·Pi, legend = ["S1", "S3", "S5", "S7", "S"], discont = true)
```

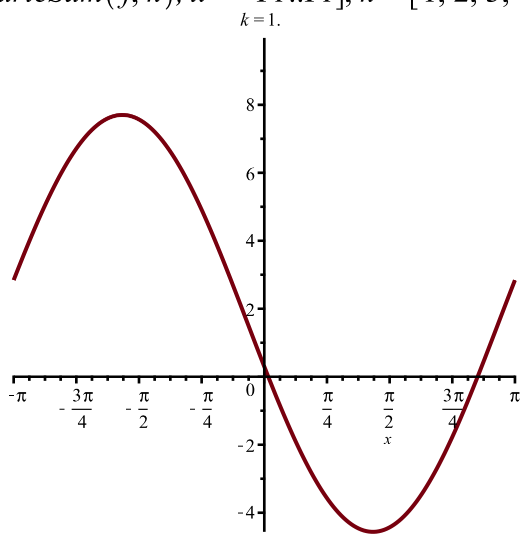



```
> #Отдельно для S
> plot(S, x =   6·Pi ..6·Pi, discont = true)
```



> **#Анимация**

> $plots[animate](plot, [MyFurieSum(f, k), x = -\mathrm{Pi}..\mathrm{Pi}], k = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10])$

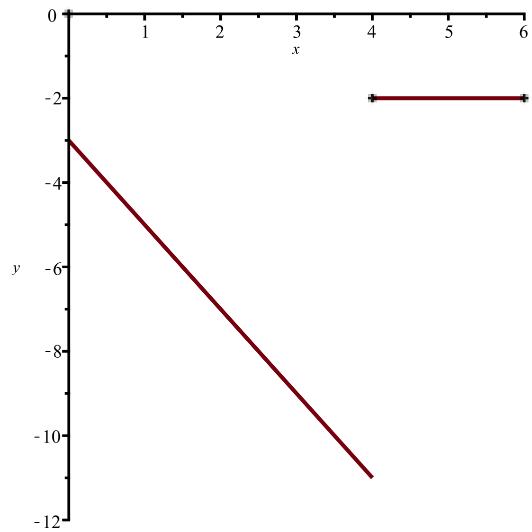


> ***#Задание2 (получить разложение тригонометрический в ряд Фурье для x2-периодической функции, заданной на промежутке (0, x1) формулой y=ax+b, а на промежутке [x1, x2] — y = c. Построить на промежутке [-2*x2, 2*x2] графики S1(x), S3(x), S7(x) и S(x))***

> $f := x \rightarrow piecewise(0 < x < 4, -2\cdot x - 3, 4 \leq x \leq 6, 2)$

$$f := x \mapsto \begin{cases} -2\cdot x - 3 & 0 < x < 4 \\ 2 & 4 \leq x \leq 6 \end{cases} \tag{6}$$

> $plot(f(x), x = 0..6, y = 0..-12, discont = true)$



> ***#Найдем полупериод равен***

> $l := \frac{6}{2}$

$$l := 3 \tag{7}$$

> $a0 := simplify\left(\frac{1}{l} \cdot int(f(x), x = 0..6)\right)$

$$a0 := -8 \tag{8}$$

> $an := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), x = 0..6\right)\right)$ assuming $n :: posint$

$$an := \frac{-13\,\pi\,n \sin\left(\frac{4\,\pi\,n}{3}\right) - 6\cos\left(\frac{4\,\pi\,n}{3}\right) + 6}{n^2\,\pi^2} \tag{9}$$

> $bn := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), x = 0..6\right)\right)$ assuming $n :: posint$

$$bn := \frac{13\,\pi\,n \cos\left(\frac{4\,\pi\,n}{3}\right) - 5\,\pi\,n - 6\sin\left(\frac{4\,\pi\,n}{3}\right)}{n^2\,\pi^2} \tag{10}$$

> $SumWithRanges := $ **proc**$(f, k, x1, x2)$

local $a0, an, bn, n, l;$

$1 := \frac{x2 - x1}{2}$; $a0 := \mathrm{simplify}\left(\frac{1}{l} \cdot \mathrm{int}(f(x), x = 0..2 \cdot 1)\right)$;

assume(n :: posint);

$an := \mathrm{simplify}\left(\frac{1}{l} \cdot \mathrm{int}\left(f(x) \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), x = 0..2 \cdot 1\right)\right)$;

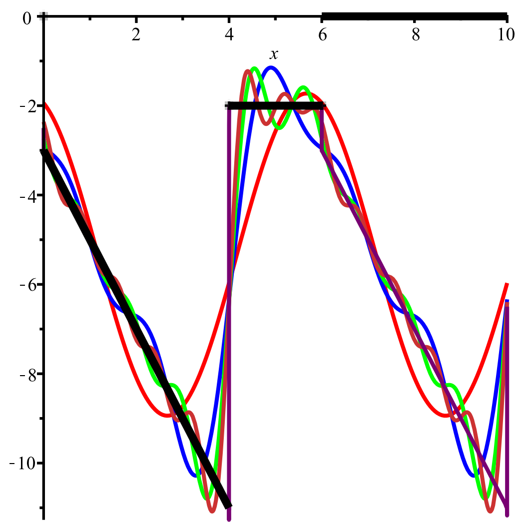
$bn := \mathrm{simplify}\left(\frac{1}{l} \cdot \mathrm{int}\left(f(x) \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), x = 0..2 \cdot 1\right)\right)$;

return $\frac{1}{2} \cdot a0 + \mathrm{sum}\left(an \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right) + bn \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), n = 1..k\right)$

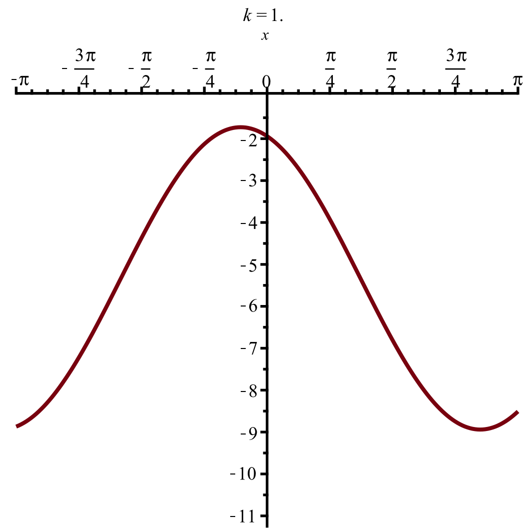
end proc

```
SumWithRanges := proc(f, k, x1, x2)                                                    (11)
    local a0, an, bn, n, l;
    l := 1/2*x2 - 1/2*x1;
    a0 := simplify(int(f(x), x = 0 ..2*l)/l);
    assume(n::posint);
    an := simplify(int(f(x)*cos(π*n*x/l), x = 0 ..2*l)/l);
    bn := simplify(int(f(x)*sin(π*n*x/l), x = 0 ..2*l)/l);
    return 1/2*a0 + sum(an*cos(π*n*x/l) + bn*sin(π*n*x/l), n = 1 ..k)
end proc
```

```
> #Найдем частичные суммы а также опишем порождающую функцию
> partSums := plot([SumWithRanges(f, 1, 0, 6), SumWithRanges(f, 3, 0, 6), SumWithRanges(f,
    5, 0, 6), SumWithRanges(f, 7, 0, 6), SumWithRanges(f, 10000, 0, 6)], x = 0 ..10, discont
    = true, color = [red, blue, green, orange, purple]) :
> origin := plot(f(x), x = 0 ..10, discont = true, color = black, thickness = 3) :
> plots[display](partSums, origin)
```




```
> #Анимация
> plots[animate](plot, [SumWithRanges(f, k, 0, 6), x = -Pi ..Pi], k = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10])
```



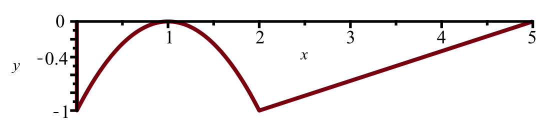
> ***#Задание3 Для графически заданной функции построить три разложения в тригонометриче-***
#ский ряд Фурье, считая, что функция определена:
 # — на полном периоде.
#— на полупериоде (является четной).
#— на полупериоде (является нечетной).
#Постройте графики сумм полученных рядов на промежутке, превышающем длину заданного в 3 раза

>

> $f := x \rightarrow piecewise\left(0 < x \leq 2, -(x-1)^2, 2 < x < 5, \frac{(x-5)\cdot 1}{3}\right)$

$$f := x \mapsto \begin{cases} -(x-1)^2 & 0 < x \leq 2 \\ \frac{x}{3} - \frac{5}{3} & 2 < x < 5 \end{cases} \quad \textbf{(12)}$$

> $plot(f(x), x = 0..5, y = 0..-1, scaling = constrained)$



> *#Полупериод равен*

> $1 := \frac{5}{2};$

$$l := \frac{5}{2} \quad (13)$$

> $a0 := simplify\left(\frac{1}{l} \cdot int(f(x), x = 0\,..\,2 * l)\right);$

$$a0 := -\frac{13}{15} \quad (14)$$

> $an := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \cos\left(\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right)\right), x = 0\,..2 \cdot l\right)\right)$ assuming $n :: posint$

$$an := -\frac{5}{6}\,\frac{7\,\pi\,n\cos\left(\frac{4}{5}\,\pi\,n\right) + 5\,\pi\,n - 15\sin\left(\frac{4}{5}\,\pi\,n\right)}{\pi^3\,n^3} \quad (15)$$

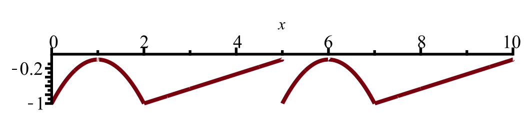
> $bn := simplify\left(\frac{1}{l} \cdot int\left(f(x) \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), x = 0\,..2 \cdot l\right)\right)$ assuming $n :: posint$

$$bn := -\frac{1}{6}\,\frac{6\,\pi^2\,n\sim^2 + 35\,\pi\,n\sim\sin\left(\frac{4}{5}\,\pi\,n\sim\right) + 75\cos\left(\frac{4}{5}\,\pi\,n\sim\right) - 75}{\pi^3\,n\sim^3} \quad (16)$$

> $S := k \rightarrow \frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right) + bn \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), n = 1\,..k\right)$

$$S := k \mapsto \frac{a0}{2} + \sum_{n=1}^{k}\left(an \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right) + bn \cdot \sin\left(\frac{\pi \cdot n \cdot x}{l}\right)\right) \quad (17)$$

> $plot(S(infinity), x = -0\,..10, discont = true, scaling = constrained)$

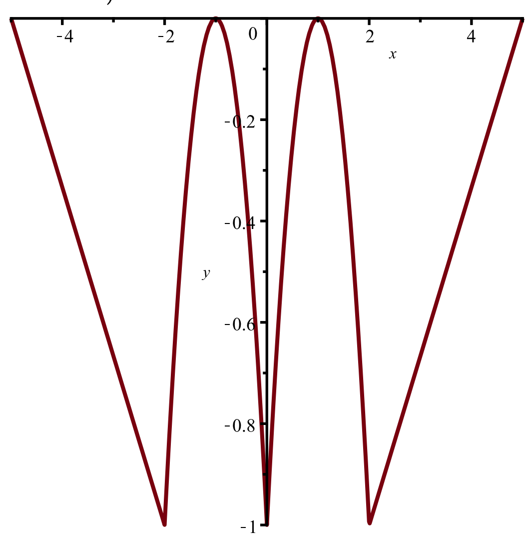


> **#На полупериоде является четной**

$> fEven := x \to piecewise\left(-5 < x < -2, -\frac{(x+5)\cdot 1}{3}, -2 \le x < 0, -(x+1)^2, 0 < x \le 2, -(x-1)^2, 2 < x < 5, \frac{(x-5)\cdot 1}{3}\right)$

$$fEven := x \mapsto \begin{cases} -\frac{x}{3} - \frac{5}{3} & -5 < x < -2 \\ -(x+1)^2 & -2 \le x < 0 \\ -(x-1)^2 & 0 < x \le 2 \\ \frac{x}{3} - \frac{5}{3} & 2 < x < 5 \end{cases} \quad (18)$$

$> plot(fEven(x), x = -5..5, y = 0..-1)$



$>$ *#Полупериод равен*

$> l := \frac{10}{2};$

$> a0 := simplify\left(\frac{1}{l} \cdot int(fEven(x), x = -l..l)\right);$

$$a0 := -\frac{13}{15} \quad (19)$$

$> an := simplify\left(\frac{1}{l} \cdot int\left(fEven(x) \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), x = -l..l\right)\right)$ assuming $n :: posint$

$$an := \frac{10}{3} \frac{(-1)^n \pi n - 7 \pi n \cos\left(\frac{2}{5} \pi n\right) - 6 \pi n + 30 \sin\left(\frac{2}{5} \pi n\right)}{\pi^3 n^3} \quad (20)$$

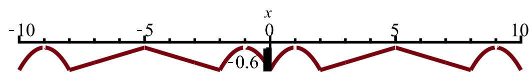
$> bn := simplify\left(\frac{1}{l} \cdot int\left(fEven(x) \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), x = -l..l\right)\right)$ assuming $n :: posint$

$$bn := 0 \quad (21)$$

$> S := k \to \frac{a0}{2} + sum\left(an \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), n = 1..k\right)$

$$S := k \mapsto \frac{a0}{2} + \left(\sum_{n=1}^{k} an \cdot \cos\left(\frac{\pi \cdot n \cdot x}{l}\right)\right) \tag{22}$$

> $plot(S(infinity), x = -10..10, discont = true, scaling = constrained)$

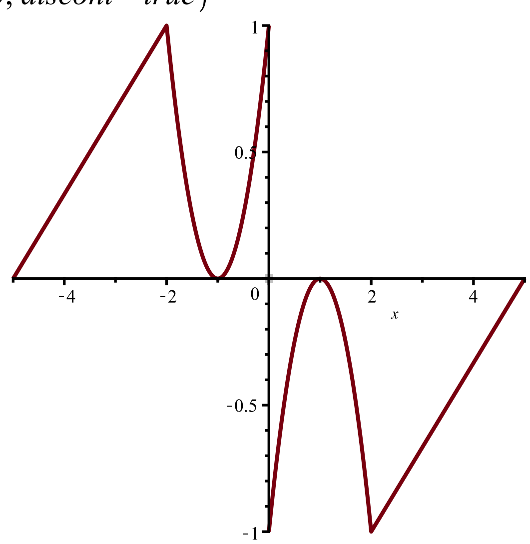


> **#На полупериоде является нечетной**

> $fNoEven := x \rightarrow piecewise\left(-5 < x < -2, \frac{(x+5)\cdot 1}{3}, -2 \leq x < 0, (x+1)^2, 0 < x \leq 2, -(x-1)^2, 2 < x < 5, \frac{(x-5)\cdot 1}{3}\right)$

$$fNoEven := x \mapsto \begin{cases} \frac{x}{3} + \frac{5}{3} & -5 < x < -2 \\ (x+1)^2 & -2 \leq x < 0 \\ -(x-1)^2 & 0 < x \leq 2 \\ \frac{x}{3} - \frac{5}{3} & 2 < x < 5 \end{cases} \tag{23}$$

> $plot(fNoEven(x), x = -5..5, discont = true)$



> $a0 := simplify\left(\frac{1}{l} \cdot int(fNoEven(x), x = -l..l)\right);$

$$a0 := 0 \tag{24}$$

> $an := simplify\left(\frac{1}{l} \cdot int\left(fNoEven(x) \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), x = -l..l\right)\right)$ assuming $n :: posint$

$$an := 0 \tag{25}$$

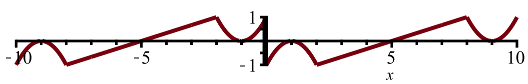
> $bn := simplify\left(\frac{1}{l} \cdot int\left(fNoEven(x) \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), x = -l..l\right)\right)$ assuming $n :: posint$

$$bn := -\frac{2}{3} \frac{3 \pi^2 n^2 + 35 \pi n \sin\left(\frac{2}{5} \pi n\right) + 150 \cos\left(\frac{2}{5} \pi n\right) - 150}{\pi^3 n^3} \tag{26}$$

> $S := k \rightarrow sum\left(bn \cdot \sin\left(\frac{\mathrm{Pi} \cdot n \cdot x}{l}\right), n = 1..k\right)$

$$S := k \rightarrow \sum_{n=1}^{k} bn \sin\left(\frac{\pi n x}{l}\right) \tag{27}$$

> $plot(S(infinity), x = -10..10, discont = true, scaling = constrained)$

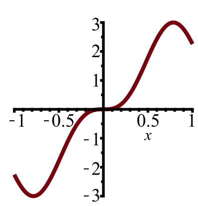


> ***#Задание4 (разложить функцию в ряд Фурье по многочленам Лежандра и Чебышёва на промежутке [-1,1])***

> $f := 3 \cdot \sin^3(2 \cdot x); funcPlot := plot(f, x = -1..1)$

$$f := 3 \sin(2 x)^3$$

$funcPlot :=$



> $with(orthopoly)$

$$[G, H, L, P, T, U] \tag{28}$$

> ***#Найдем коэффициенты многочлена Лежандра***

> **for** n **from** 0 **to** 7 **do** $c[n] := \dfrac{int(f \cdot P(n, x), x = -1 ..1)}{int((P(n, x))^2, x = -1 ..1)}$;**end do**

$$c_0 := 0$$

$$c_1 := -\frac{3}{2}\sin(2)^2\cos(2) - 3\cos(2) + \frac{1}{4}\sin(2)^3 + \frac{3}{2}\sin(2)$$

$$c_2 := 0$$

$$c_3 := -\frac{49}{24}\sin(2)^2\cos(2) + \frac{133}{6}\cos(2) + \frac{469}{144}\sin(2)^3 + \frac{77}{12}\sin(2)$$

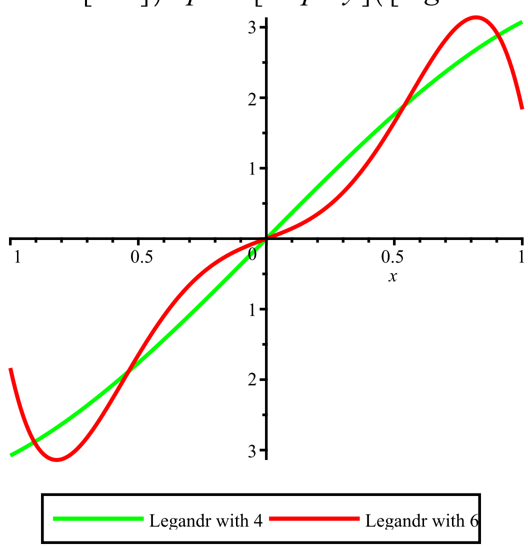
$$c_4 := 0$$

$$c_5 := -\frac{6215}{32}\sin(2) - \frac{6721}{16}\cos(2) + \frac{715}{192}\sin(2)^3 + \frac{209}{32}\sin(2)^2\cos(2)$$

$$c_6 := 0$$

$$c_7 := -\frac{123305}{6912}\sin(2)^3 + \frac{2499805}{288}\sin(2) + \frac{681785}{36}\cos(2) - \frac{8395}{1152}\sin(2)^2\cos(2) \quad \textbf{(29)}$$

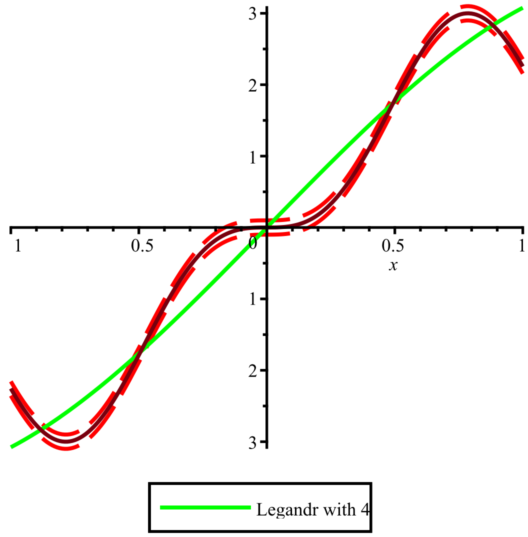
> $legandrsGraph4 := plot(add(c[n] \cdot P(n, x), n = 0 ..4), x = -1 ..1, legend = [\text{"Legandr with 4"}], color = [green]) : legandrsGraph7 := plot(add(c[n] \cdot P(n, x), n = 0 ..7), x = -1 ..1, legend = [\text{"Legandr with 7"}], color = [red]) : plots[display]([legandrsGraph4, legandrsGraph6])$



> $f1 := plot(f + 0.1, x = \ \ 1 ..1, linestyle = dash, color = red)$:

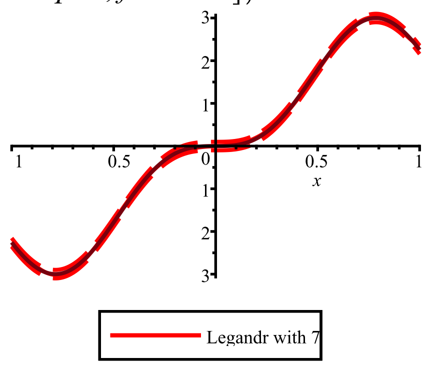
> $f2 := plot(f \quad 0.1, x = \ \ 1 ..1, linestyle = dash, color = red)$:

> $plots[display]([f1, f2, legandrsGraph4, funcPlot])$



>

> $plots[display]([f1, f2, legandrsGraph7, funcPlot])$



> #Эксперементально выяснили, что наименьший порядок частичной суммы,

> #равномерно аппроксимирующей на всем промежутке заданную функцию с точностью 0,1 это 6

>

> ***#Найдем коэффициенты многочлена Чебышева***

> **for** n **from** 0 **to** 7 **do** $c[n] := \frac{2}{\mathrm{Pi}} \cdot int\left(\frac{f \cdot T(n, x)}{\sqrt{1 \quad x^2}}, x = \ 1..1\right)$;**end do**

$$c_0 := 0$$

$$c_1 := \frac{2 \left(\int_{1}^{1} \frac{3 \sin(2\,x)^3\, x}{\sqrt{\quad x^2 + 1}}\, \mathrm{d}x\right)}{\pi}$$

$$c_2 := 0$$

$$c_3 := \frac{2 \left(\int_{1}^{1} \frac{3 \sin(2\,x)^3\, (4\,x^3 \quad 3\,x)}{\sqrt{\quad x^2 + 1}}\, \mathrm{d}x\right)}{\pi}$$

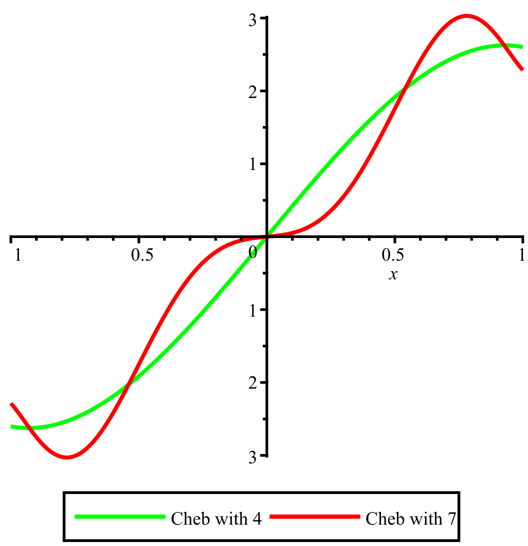
$$c_4 := 0$$

$$c_5 := \frac{2\left(\int_{-1}^{1} \frac{3\sin(2x)^3\left(16x^5-20x^3+5x\right)}{\sqrt{-x^2+1}}\,dx\right)}{\pi}$$

$$c_6 := 0$$

$$c_7 := \frac{2\left(\int_{-1}^{1} \frac{3\sin(2x)^3\left(64x^7-112x^5+56x^3-7x\right)}{\sqrt{-x^2+1}}\,dx\right)}{\pi} \tag{30}$$

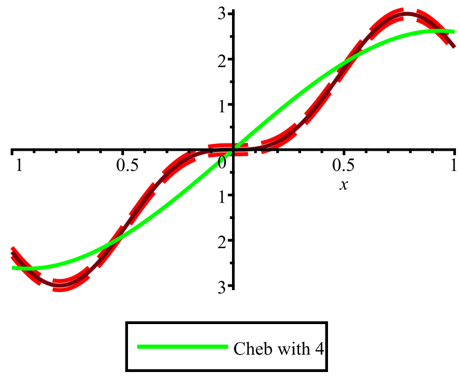
> $chebGraph4 := plot\left(\frac{c[0]}{2} + add(c[n]\cdot T(n,x), n=1..4), x=-1..1, legend = [\text{"Cheb with 4"}], color=[green]\right) : chebGraph7 := plot\left(\frac{c[0]}{2} + add(c[n]\cdot T(n,x), n=1..7), x=-1..1, legend=[\text{"Cheb with 7"}], color=[red]\right) :$
> $plots[display]([chebGraph4, chebGraph7])$



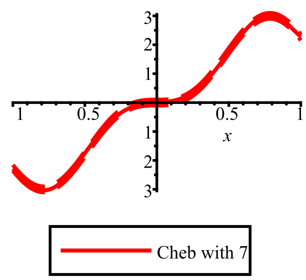
> $cf1 := plot(f + 0.1, x = \ 1..1, linestyle=dash, color=red) :$

> $cf2 := plot(f \quad 0.1, x = \ 1..1, linestyle=dash, color=red) :$

> $plots[display]([cf1, cf2, funcPlot, chebGraph4])$



>
plots[display](cf1, cf2, chebGraph7)



> **#Так как функция нечетная, то нам надо найти только bn**

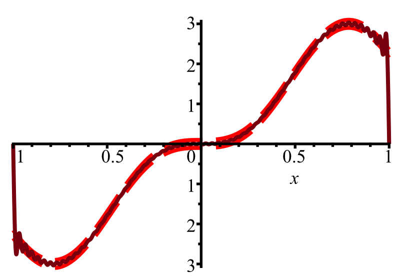
> *bn* := *simplify*(*int*(*f* · sin(Pi·*k*·*x*), *x* = 1 ..1)) assuming *k* :: *posint*

$$bn := \frac{3}{2}\ \frac{(\ \ 1)^{1+k}\,\pi\,k\left(3\,\pi^2\,k^2\sin(2)\quad \pi^2\,k^2\sin(6)\quad 108\sin(2)+4\sin(6)\right)}{\pi^4\,k^4\quad 40\,\pi^2\,k^2+144} \tag{31}$$

> *S* := *mk*→*sum*(*bn*·sin(Pi·*k*·*x*), *k* = 1 ..*mk*)

$$S := mk \rightarrow \sum_{k=1}^{mk} bn\,\sin(\pi\,k\,x) \tag{32}$$

> *furGraph* := *plot*(*S*(60), *x* = 1 .. 1, *discont* = *true*);
plots[*display*]([*f1*, *f2*, *furGraph*])
#При n = 60, видим, что ф-ция не аппроксимирует на [-0.1,0.1]

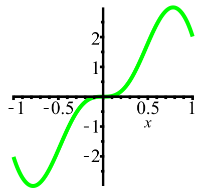


> *St* := *convert*(*taylor*(*f*, *x* = 0, 14), *polynom*)

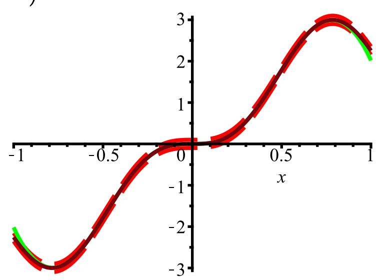
$$St := 24\,x^3\quad 48\,x^5+\frac{208}{5}\,x^7\quad \frac{1312}{63}\,x^9+\frac{10736}{1575}\,x^{11}\quad \frac{2336}{1485}\,x^{13} \tag{33}$$

> *StF* := *plot*(*St*, *x* = 1 .. 1, *color* = *green*)

#При n = 14, видим, что ф-ция не аппроксимирует на [-0.1,0.1]

$StF :=$ 

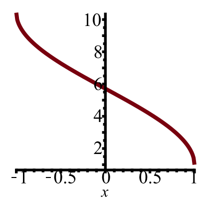
> $plots[display](f1, f2, StF, funcPlot)$



> *restart*

> $f := 3 \cdot \arccos(x) + 1; funcPlot := plot(f, x = -1..1)$

$$f := 3 \arccos(x) + 1$$

$funcPlot :=$ 

> $with(orthopoly)$

$$[G, H, L, P, T, U] \tag{34}$$

> ***#Найдем коэффициенты многочлена Лежандра***

> **for** n **from** 0 **to** 7 **do** $c[n] := \dfrac{int(f \cdot P(n, x), x = -1..1)}{int\left((P(n, x))^2, x = -1..1\right)}$;**end do**

$$c_0 := \frac{3}{2}\pi + 1$$

$$c_1 := -\frac{9}{8}\pi$$

$$c_2 := 0$$

$$c_3 := -\frac{21}{128}\pi$$

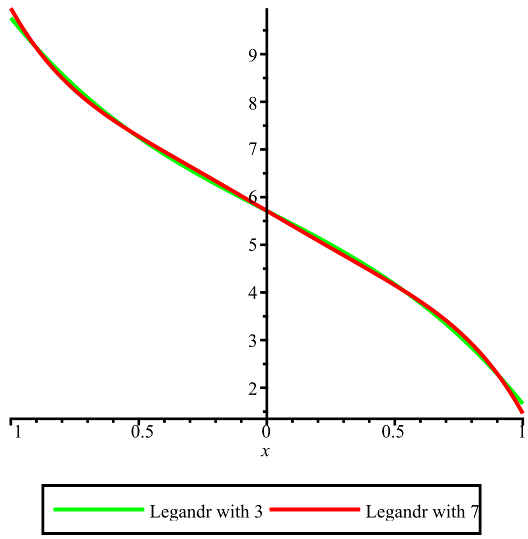
$$c_4 := 0$$

$$c_5 := -\frac{33}{512}\pi$$

$$c_6 := 0$$

$$c_7 := -\frac{1125}{32768}\pi \tag{35}$$

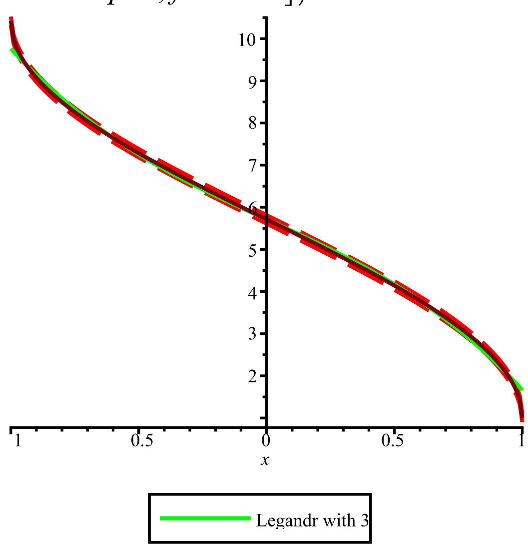
> *legandrsGraph3* := *plot*(*add*(*c*[*n*]·*P*(*n*, *x*), *n* = 0 ..3), *x* = -1 ..1, *legend* = ["Legandr with 3"], *color* = [*green*]) : *legandrsGraph7* := *plot*(*add*(*c*[*n*]·*P*(*n*, *x*), *n* = 0 ..6), *x* = -1 ..1, *legend* = ["Legandr with 7"], *color* = [*red*]) : *plots*[*display*]([*legandrsGraph3*, *legandrsGraph7*])



> *f1* := *plot*(*f* + 0.1, *x* = 1 ..1, *linestyle* = *dash*, *color* = *red*) :

> *f2* := *plot*(*f*   0.1, *x* = 1 ..1, *linestyle* = *dash*, *color* = *red*) :

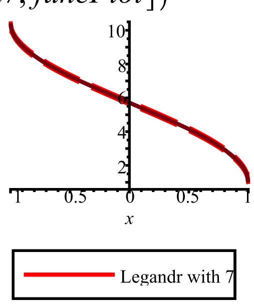
> *plots*[*display*]([*f1*, *f2*, *legandrsGraph3*, *funcPlot*])



> *f1* := *plot*(*f* + 0.1, *x* = 1 ..1, *linestyle* = *dash*, *color* = *red*) :

> *f2* := *plot*(*f*   0.1, *x* = 1 ..1, *linestyle* = *dash*, *color* = *red*) :

> *plots*[*display*]([*f1*, *f2*, *legandrsGraph7*, *funcPlot*])



> #Эксперементально выяснили, что наименьший порядок частичной суммы,

> #равномерно аппроксимирующей на всем промежутке заданную функцию с точностью 0,1

это 7

>

> ***#Найдем коэффициенты многочлена Чебышева***

> **for** n **from** 0 **to** 7**do** $c[n] := \frac{2}{\text{Pi}} \cdot int\left(\frac{f \cdot T(n, x)}{\sqrt{1 - x^2}}, x = -1 .. 1\right)$;**end do**

$$c_0 := \frac{2\left(\frac{3}{2}\pi^2 + \pi\right)}{\pi}$$

$$c_1 := -\frac{12}{\pi}$$

$$c_2 := 0$$

$$c_3 := -\frac{4}{3\pi}$$

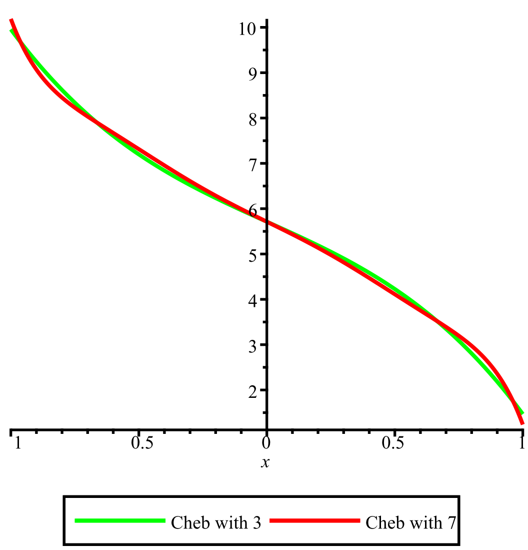
$$c_4 := 0$$

$$c_5 := -\frac{12}{25\pi}$$

$$c_6 := 0$$

$$c_7 := -\frac{12}{49\pi} \qquad \textbf{(36)}$$

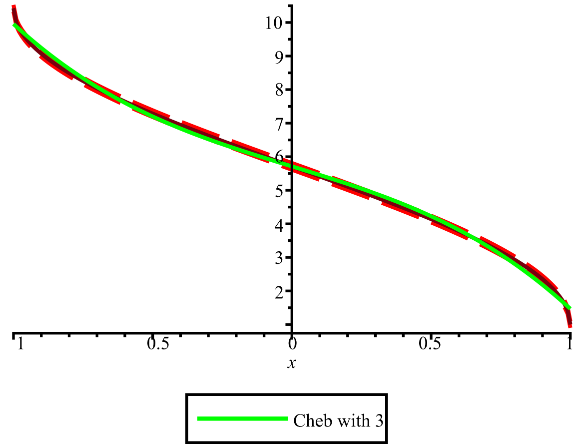
> $chebGraph3 := plot\left(\frac{c[0]}{2} + add(c[n] \cdot T(n, x), n = 1 .. 3), x = -1 .. 1, legend = [\text{"Cheb with 3"}], color = [green]\right) : chebGraph7 := plot\left(\frac{c[0]}{2} + add(c[n] \cdot T(n, x), n = 1 .. 7), x = -1 .. 1, legend = [\text{"Cheb with 7"}], color = [red]\right) :$
$plots[display]([chebGraph3, chebGraph7])$



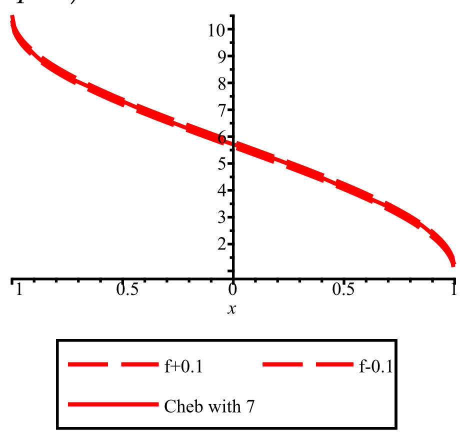
> $cf1 := plot(f + 0.1, x = -1..1, linestyle = dash, color = red)$:

> $cf2 := plot(f - 0.1, x = -1..1, linestyle = dash, color = red)$:

> $plots[display]([cf1, cf2, funcPlot, chebGraph3])$



> $cf1 := plot(f + 0.1, x = \quad 1..1, linestyle = dash, color = red, legend = ["f+0.1"])$:
$cf2 := plot(f \quad 0.1, x = \quad 1..1, linestyle = dash, color = red, legend = ["f-0.1"])$: $chebGraph6$:
$plots[display](cf1, cf2, chebGraph7)$



> ***#Найдем коэффициенты Фурье***

> $a0 := simplify(int(f, x = \quad 1..1))$

$$a0 := 3\pi + 2 \tag{37}$$

> $an := simplify(int(f \cdot \cos(\mathrm{Pi} \cdot through \cdot x), x = \quad 1..1))$ assuming $through :: posint$; $bn :=$ $simplify(int(f \cdot \sin(\mathrm{Pi} \cdot through \cdot x), x = \quad 1..1))$ assuming $through :: posint$

$$an := 0$$

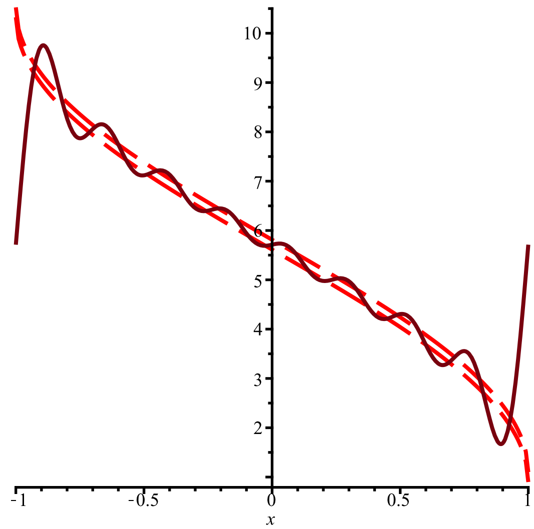
$$bn := \int_{1}^{1} (3 \arccos(x) + 1) \sin(\pi \, through \, x) \, dx \tag{38}$$

> $S := k \rightarrow \frac{a0}{2} + sum(bn \cdot \sin(\mathrm{Pi} \cdot through \cdot x), through = 1..k)$

$$S := k \rightarrow \frac{1}{2} a0 + \sum_{through = 1}^{k} bn \sin(\pi \, through \, x) \tag{39}$$

> $furGraph := plot(S(8), x = \quad 1..1, discont = true)$:

plots[*display*]([*f1*, *f2*, *furGraph*]);

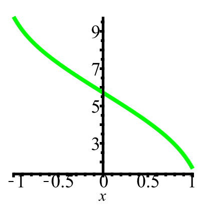


> $St := \mathrm{convert}(\mathrm{taylor}(f, x = 0, 14), \mathrm{polynom})$

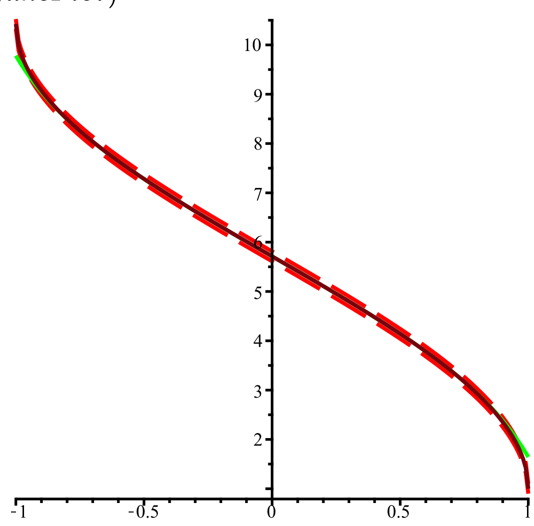
$$St := \frac{3}{2}\pi + 1 - 3x - \frac{1}{2}x^3 - \frac{9}{40}x^5 - \frac{15}{112}x^7 - \frac{35}{384}x^9 - \frac{189}{2816}x^{11} - \frac{693}{13312}x^{13} \quad \textbf{(40)}$$

> $StF := \mathrm{plot}(St, x = -1\,..1, color = green)$

$StF :=$



> *plots*[*display*](*f1*, *f2*, *StF*, *funcPlot*)



>