Домашнее задание 1 Sorokin Evgeniy

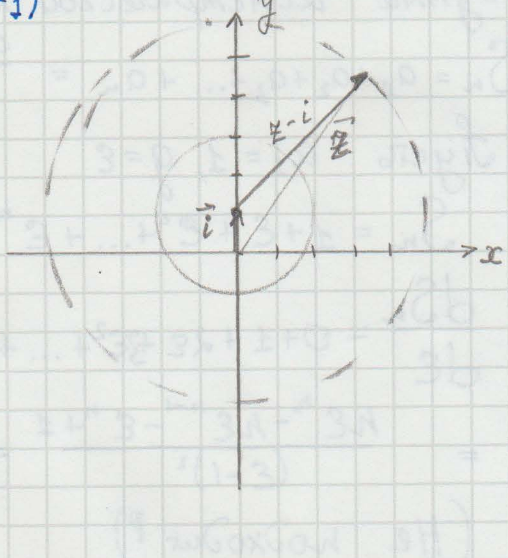
Задача №1

1) $2 \le |z-i| \le 4$

$|z-i| = |x+iy-i| = |x+i(y-1)| = \sqrt{x^2+(y-1)^2}$

$C(0,1)$

$z_c = 0\cdot x + 1\cdot i = i$



$$\left.\begin{aligned} S_R &= \pi R^2 = 16\pi \\ S_r &= \pi r^2 = 4\pi \end{aligned}\right\} \Rightarrow S_{An} = S_R - S_r = 12\pi$$

2) $|z-4i| + |z+4i| = 10$

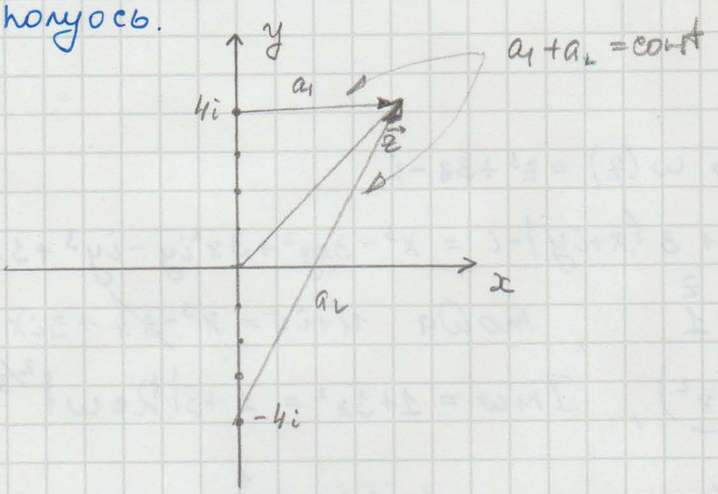
$\sqrt{x^2+(y-4)^2} + \sqrt{x^2+(y+4)^2} = 10$

Аналитическое уравнение эллипса:

$\sqrt{x^2+(y-c)^2} + \sqrt{x^2+(y+c)^2} = 2a$, где $2a$ - большая ось эллипса

$a = 5$ - большая полуось.

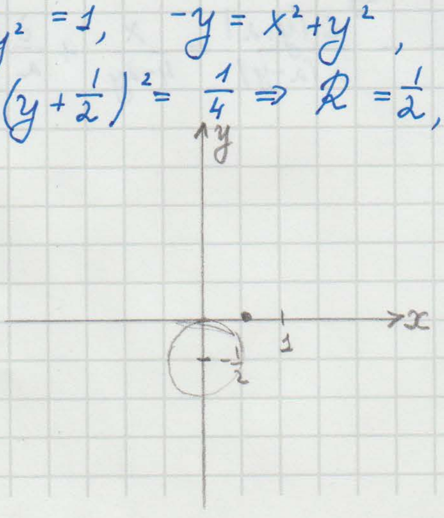
$C_{el}(0,0)$



3) $Im\,\frac{1}{z} = 1$, $\quad \frac{1}{z} = \frac{\bar{z}}{z\bar{z}} = \frac{\bar{z}}{|z|^2} = \frac{x-iy}{x^2+y^2} \Rightarrow Im\,\frac{1}{z} = -\frac{y}{x^2+y^2}$

$-\frac{y}{x^2+y^2} = 1$, $\quad -y = x^2+y^2$, $\quad x^2+y^2+y=0$, $\quad x^2+y^2+\frac{1}{2}\cdot 2\cdot y + \frac{1}{4} = \frac{1}{4}$

$x^2+\left(y+\frac{1}{2}\right)^2 = \frac{1}{4} \Rightarrow R = \frac{1}{2}$, $\quad C\left(0, -\frac{1}{2}\right) \Rightarrow z = -\frac{1}{2}i$



Задача №2

Геометрическая прогрессия:

$a_1, a_2, \dots, a_n$, где $q = \frac{a_1}{a_0}$ и т.д.

Сумма геометрической прогрессии:

$$S_n = a_1 + a_2 + a_3 + \dots + a_n = \frac{a_1(q^n - 1)}{q - 1}$$

Пусть $a_1 = 1$, $q = \varepsilon$

$$S_n = 1 + \varepsilon + \varepsilon^2 + \dots + \varepsilon^n = \frac{1(\varepsilon^n - 1)}{\varepsilon - 1} = \frac{\varepsilon^n - 1}{\varepsilon - 1}$$

$$\frac{dS_n}{d\varepsilon} = 0 + 1 + 2\varepsilon + 3\varepsilon^2 + \dots + n\varepsilon^{n-1} = \frac{d}{d\varepsilon}\left(\frac{\varepsilon^n - 1}{\varepsilon - 1}\right) = \frac{n\varepsilon^{n-1}(\varepsilon - 1) - 1(\varepsilon^n - 1)}{(\varepsilon - 1)^2} =$$

$$= \frac{n\varepsilon^n - n\varepsilon^{n-1} - \varepsilon^n + 1}{(\varepsilon - 1)^2} = \frac{\varepsilon^{n-1}(n\varepsilon - n - 1) + 1}{(\varepsilon - 1)^2} = \frac{[n(\varepsilon - 1) - 1]\varepsilon^{n-1} + 1}{(\varepsilon - 1)^2}$$

(Не подходит!)

Если $S_n = \varepsilon + \varepsilon^2 + \dots + \varepsilon^n = \frac{\varepsilon^{n+1} - \varepsilon}{\varepsilon - 1}$, то $\frac{dS_n}{d\varepsilon} = \frac{\varepsilon^n(n\varepsilon - n - 1) + 1}{(\varepsilon - 1)^2} =$

$$= \frac{\varepsilon^n(n(\varepsilon - 1) - 1) + 1}{(\varepsilon - 1)^2}$$

Задача №3

1) $\operatorname{Im} z = 1$, $z \longmapsto \omega(z) = z^3 + 3z - i$

$$u + iv = (x + iy)^3 + 3(x + iy) - i = x^3 - 3xy^2 + 3x^2 iy - iy^3 + 3x + 3iy - i$$

$\operatorname{Im} z = 1$, $y = 1$, тогда $u + iv = x^3 - \cancel{3x} + 3ix^2 - i + \cancel{3x} + 3i - i = x^3 + 3ix^2 + i =$

$$= \underbrace{x^3 + i(1 + 3x^2)}_{\omega(z)}, \quad \operatorname{Im}\omega = 1 + 3x^2 = 1 + 3|\operatorname{Re}\omega|^{2/3}$$

2) $\sqrt{x^2 + (y-1)^2} = 1$, $z \longmapsto \omega(z) = \frac{1}{z - 2i}$

$$\omega(z) = \frac{1}{x + iy - 2i} = \frac{1}{x + i(y-2)} = \frac{x - i(y-2)}{x^2 + (y-2)^2} = \frac{x - i(y-2)}{\underbrace{x^2 + y^2}_{2y} - 4y + 4} = \left| \begin{matrix} x^2 + y^2 - 2y + 1 = 1 \\ 2y = x^2 + y^2 \end{matrix} \right| =$$

$$= \frac{x - i(y-2)}{4 - 2y} = \frac{x - i(y-2)}{2(2-y)} = \frac{x}{4 - 2y} - \frac{i(y-2)}{2(2-y)} = \frac{x}{4-2y} + \frac{i}{2}$$

Тогда $\operatorname{Im}\omega = \frac{1}{2}$

Задача №4

Условия Коши – Римана: $f(z) = u + iv$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

1) $\omega(z) = x^2 + y^2 = u + iv, \quad u = x^2 + y^2, \; v = 0$

$$\frac{\partial u}{\partial x} = 2x, \quad \frac{\partial v}{\partial y} = 0, \quad \frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = 2, \quad \frac{\partial v}{\partial x} = 0, \quad \frac{\partial u}{\partial y} \neq -\frac{\partial v}{\partial x}$$

Условия К-Р не выполняются!

2) $\omega(z) = x^2 - y^2 + 2ixy, \quad u = x^2 - y^2, \; v = 2xy$

$$\frac{\partial u}{\partial x} = 2x = \frac{\partial v}{\partial y} = 2x$$

$$\frac{\partial u}{\partial y} = -2y = -\frac{\partial v}{\partial x} = -2y$$

Условия К-Р выполняются!

3) $\omega(z) = \frac{1}{x+iy} = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}, \quad u = \frac{x}{x^2+y^2}, \; v = -\frac{y}{x^2+y^2}$

$$\frac{\partial u}{\partial x} = -\frac{x^2-y^2}{(x^2+y^2)^2}, \quad \frac{\partial v}{\partial y} = -\frac{x^2-y^2}{(x^2+y^2)^2}, \quad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = -\frac{2xy}{(x^2+y^2)^2}, \quad \frac{\partial v}{\partial x} = \frac{2xy}{(x^2+y^2)^2}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Условия К-Р выполняются!

Задача №5

1) $|f| = e^{r^2\cos 2\varphi} = e^{r^2(\cos^2\varphi - \sin^2\varphi)} = e^{x^2-y^2} = e^{u+iv}$

$$\omega = \ln f = \ln |f| e^{i\arg f} = \ln|f| + i\arg f \Rightarrow \operatorname{Re}\omega = \ln|f|, \; \operatorname{Im}\omega = \arg f$$

$$\frac{\partial u}{\partial x} = 2x = \frac{\partial v}{\partial y} \Rightarrow v = 2xy + \varphi(x)$$

$$\frac{\partial v}{\partial x} = \cancel{2y} + \varphi'(x) = -\frac{\partial u}{\partial y} = +\cancel{2y} \Rightarrow \varphi(x) = C$$

$v = 2xy + C$, тогда: $u + iv = x^2 - y^2 + 2ixy + iC = \underbrace{(x+iy)}_{z}{}^2 + iC = z^2 + iC$

$$\omega = \ln f \Rightarrow f = e^{\omega} = e^{z^2 + iC} = e^{z^2}e^{iC}$$

2) $\operatorname{Arg} f = xy$

$$w = \ln f = \ln|f|e^{i\,\mathrm{Arg} f} = \ln|f| + i\,\mathrm{Arg} f \Rightarrow \mathrm{Im}\,w = \mathrm{Arg} f,\ \mathrm{Re}\,w = \ln|f|$$

$$w = u + iv \Rightarrow \mathrm{Im}\,w = v = xy$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y};\quad \frac{\partial u}{\partial x} = x \Rightarrow u = \frac{x^2}{2} + \varphi(y)$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x};\quad \frac{\partial u}{\partial y} = \varphi'(y) = -y \Rightarrow \varphi(y) = -\frac{y^2}{2}$$

$$u = \frac{x^2-y^2}{2}$$

$$w = u + iv = \frac{x^2-y^2}{2} + ixy,\quad 2w = x^2 - y^2 + 2ixy = z^2 \Rightarrow w = \frac{z^2}{2}$$

$$w = \ln f \Rightarrow f = e^{\frac{z^2}{2}}$$

Задача №6

$$f(z) = u + iv$$

1) $u = \varphi(x^2 - y^2)$

Условие К-Р:

$$\begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y} \\ \dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x} \end{cases};\quad + \begin{cases} \dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 v}{\partial x \partial y} \\ \dfrac{\partial^2 u}{\partial y^2} = -\dfrac{\partial^2 v}{\partial x \partial y} \end{cases},\quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\quad \Delta u = 0$$

$$\frac{\partial u}{\partial x} = \frac{\partial \varphi(x^2-y^2)}{\partial x} = 2x\varphi'(x^2-y^2)$$

$$\frac{\partial^2 u}{\partial x^2} = 2\varphi'(x^2-y^2) + 4x^2\varphi''(x^2-y^2)$$

$$\frac{\partial u}{\partial y} = -2y\varphi'(x^2-y^2)$$

$$\frac{\partial^2 u}{\partial y^2} = 4y^2\varphi''(x^2-y^2) - 2\varphi'(x^2-y^2)$$

Тогда: $\cancel{2\varphi'(x^2-y^2)} + 4x^2\varphi''(x^2-y^2) - \cancel{2\varphi'(x^2-y^2)} + 4y^2\varphi''(x^2-y^2) =$
$= 0,\quad \varphi''(x^2-y^2)(x^2+y^2) = 0$ (Условие $x = y = 0$, $x = \pm iy$)

$$\varphi''(x^2-y^2) = \varphi''(t) = 0,\quad \varphi'(t) = C_1,\quad \varphi(t) = C_2 + C_1 t$$

Так как $u = \varphi(x^2-y^2)$, то $u = C_2 + C_1(x^2-y^2)$

$$\frac{\partial u}{\partial x} = 2xC_1 = \frac{\partial v}{\partial y} \Rightarrow v = 2xyC_1$$

$$f(z) = u + iv = C_2 + C_1(x^2-y^2) + i2xyC_1 = C_2 + C_1(x^2+2ixy-y^2) = C_2 + C_1 z^2$$

Таким образом, получим:

$f(z) = b + az^2$

2) $u = \varphi\left(\frac{y}{x}\right)$

$\Delta u = 0$

$\frac{\partial u}{\partial x} = \frac{\partial \varphi\left(\frac{y}{x}\right)}{\partial x} = -\frac{y}{x^2}\varphi'\left(\frac{y}{x}\right),$

$\frac{\partial^2 u}{\partial x^2} = \frac{2y}{x^3}\varphi'\left(\frac{y}{x}\right) + \frac{y^2}{x^4}\varphi''\left(\frac{y}{x}\right)$

$\frac{\partial u}{\partial y} = \frac{1}{x}\varphi'\left(\frac{y}{x}\right)$

$\frac{\partial^2 u}{\partial y^2} = 0 + \frac{1}{x^2}\varphi''\left(\frac{y}{x}\right)$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{2y}{x^3}\varphi'\left(\frac{y}{x}\right) + \varphi''\left(\frac{y}{x}\right)\left(\frac{y^2}{x^4} + \frac{1}{x^2}\right) = 0$

$x \neq 0$ (по усл.) $\Rightarrow$ умножим обе части на x^2

$2\frac{y}{x}\varphi'\left(\frac{y}{x}\right) + \varphi''\left(\frac{y}{x}\right)\left(\frac{y^2}{x^2} + 1\right) = 0$

Пусть $\frac{y}{x} = t$, тогда:

$2t\varphi'(t) + \varphi''(t)(t^2+1) = 0$

$\begin{cases} \varphi'(t) = k(t) \\ 2tk + k'(t^2+1) = 0 \end{cases}$, $-2tk = k'(t^2+1)$, $\frac{-2t}{t^2+1} = \frac{dk}{dt}\cdot\frac{1}{k}$

$\int -2\frac{t}{t^2+1}\,dt = \int\frac{dk}{k}$

$-\ln(t^2+1) + \underset{\underset{c_1}{\ln e^c}}{C} = \ln k$, $\frac{C_1}{t^2+1} = k$

$\frac{d\varphi}{dt} = \frac{C_1}{t^2+1} = k$, $\int d\varphi = \int\frac{C_1}{t^2+1}\,dt$, $\varphi(t) = C_2 + C_1\,\mathrm{arctg}\, t$

Так как $u = \varphi(t)$, то $u = C_2 + C_1\,\mathrm{arctg}\,t = C_2 + C_1\,\mathrm{arctg}\,\frac{y}{x}$

$\frac{\partial u}{\partial x} = -\frac{C_1 y}{x^2+y^2} = \frac{\partial v}{\partial y} \Rightarrow v = -C_1\frac{\ln(x^2+y^2)}{2}$

$f(z) = u + iv = C_2 + C_1\,\mathrm{arctg}\,\frac{y}{x} - C_1 i\frac{\ln(x^2+y^2)}{2}$

$\frac{y}{x} = \mathrm{tg}\,\varphi$, $\varphi = \mathrm{arctg}\,\frac{y}{x}$, $-i\ln e^{i\varphi} = \varphi = \mathrm{arctg}\,\frac{y}{x}$

$f(z) = C_2 - iC_1\ln e^{i\varphi} - C_1\ln|z| = C_2 - iC_1\ln z$

Задача №7

1) $\oint_C z\,dz = \left| \begin{matrix} z = re^{i\varphi} \\ dz = rie^{i\varphi}d\varphi \end{matrix} \right| = \oint_C ir^2 e^{2i\varphi} d\varphi = ir^2 \oint_C e^{2i\varphi} d\varphi = ir^2 \int_0^{2\pi} e^{2i\varphi} d\varphi =$

$= -\frac{1}{2} r^2 e^{2i\varphi} \Big|_0^{2\pi} = -\frac{1}{2} r^2 \big(\underbrace{e^{4i\pi}}_{1} - \underbrace{e^0}_{1} \big) = 0$

2) $\oint \bar{z}\,dz = \left| \begin{matrix} z = x+iy, \ \bar{z} = x-iy \\ x = r\cos\varphi, \ y = r\sin\varphi \end{matrix} \right| = \oint r(\cos\varphi - i\sin\varphi)\,dz = \left| \begin{matrix} z = r(\cos\varphi + i\sin\varphi) \\ dz = -r\sin\varphi d\varphi + ri\cos\varphi d\varphi = \\ = +r(-\sin\varphi + i\cos\varphi)\,d\varphi \end{matrix} \right| =$

$= \oint_C r(\cos\varphi - i\sin\varphi)\, r(-\sin\varphi + i\cos\varphi)\,d\varphi = r^2 \int_0^{2\pi} (\cancel{-\sin\varphi\cos\varphi} + i\cos^2\varphi + i\sin^2\varphi + \cancel{\sin\varphi\cos\varphi})\,d\varphi =$

$= ir^2 \int_0^{2\pi} d\varphi = 2\pi i r^2$, а т.к $r = 1$, то $\oint \bar{z}\,dz = 2\pi i$

Задача №8

1) $C_{circ}(0,0)$

$\int_C \frac{-(y\,dx - x\,dy)}{x^2+y^2} = \left| \begin{matrix} x = r\cos\varphi & dx = -r\sin\varphi d\varphi \\ y = r\sin\varphi & dy = r\cos\varphi d\varphi \end{matrix} \right| = \int_C \frac{-(r\sin\varphi\, r\sin\varphi d\varphi + r\cos\varphi\, r\cos\varphi d\varphi)}{r^2} =$

$= \int_0^{2\pi} \frac{-r^2(\sin^2\varphi + \cos^2\varphi)}{r^2} d\varphi = -\int_0^{2\pi} d\varphi = -2\pi.$

2) $C_{circ}(2,0)$

$\int_C \frac{y\,dx - x\,dy}{x^2+y^2} = \left| P = \frac{y}{x^2+y^2},\ Q = -\frac{x}{x^2+y^2} \right| = \iint \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\,dy = \left| \frac{\partial Q}{\partial x} = \frac{x^2-y^2}{(x^2+y^2)^2}, \right.$

$\left. \frac{\partial P}{\partial y} = \frac{x^2-y^2}{(y^2+x^2)^2} \right| = \iint \left(\frac{x^2-y^2}{(x^2+y^2)^2} - \frac{x^2-y^2}{(y^2+x^2)^2} \right) dx\,dy = 0$

Задача №9

$p(n) = \frac{1}{2\pi i} \int_C dz\, z^{-1-n} \prod_{k=1}^{\infty} \frac{1}{1-z^k}$

Заметим, что:

$\int_C z^n dz = 0$, $n \in \mathbb{N}$

$\int_C z^{-1} dz = 2\pi i$

1) $p(1) = \frac{1}{2\pi i} \int_C \frac{dz}{z^2} \frac{1}{1-z} \frac{1}{1-z^2} \cdots = \frac{1}{2\pi i} \int_C \frac{dz}{z^2} (1+z+z^2+\ldots)(1+z^2+z^4+\ldots)(1+z^3+\ldots) =$

$= \frac{1}{2\pi i} \int_C \frac{dz}{z^2} \big(1 + z + 2z^2 + \ldots\big) = \frac{1}{2\pi i} \int dz \left(\frac{1}{z^2} + \frac{1}{z} + 2 + \ldots \right) = \frac{1}{2\pi i} (J_1 + J_2 + J_3)$

т.к $z < 1$ то раскл. в ряд Тейлора

Далее я получаю степени $n \geq 0 \Rightarrow J = 0$

$J_1 = \int_0^{2\pi} \frac{dz}{z^2} = \int_0^{2\pi} \frac{rie^{i\varphi} d\varphi}{r^2 e^{2i\varphi}} = \frac{i}{r} \int_0^{2\pi} e^{-i\varphi} d\varphi = 0$

$J_2 = \int \frac{dz}{z} = 2\pi i$

$J_3 = \int 2\,dz = \int_0^{2\pi} 2 r i e^{i\varphi} d\varphi = 2ir\int_0^{2\pi} e^{i\varphi} d\varphi = 0 \Rightarrow J_\Sigma = p(1) = \frac{1}{2\pi i} 2\pi i = 1$

2) $p(4) = \frac{1}{2\pi i}\int_C \frac{dz}{z^5}(1+z+z^2+\dots)(1+z^2+z^4+\dots)(1+\dots)\dots = \Big|$ интересуют все комбинации, образующие степени до 5 $\Big| =$

$= \frac{1}{2\pi i}\int_C \frac{dz}{z^5}\left(1+z+2z^2+3z^3+5z^4+\dots\right) = \frac{1}{2\pi i}\sum_{i=1}^{5} J_i$

$J_1 = \int \frac{dz}{z^5} = 0$

$J_2 = \int \frac{dz}{z^4} = 0$

$J_3 = \int \frac{dz}{z^3} = 0$

$J_4 = \int \frac{dz}{z^2} = 0$

$J_5 = \int \frac{dz}{z} = 2\pi i \Rightarrow J_\Sigma = p(4) = \frac{1}{2\pi i} 5\cdot 2\pi i = 5.$

Задача №10

$y(1) = 0, \quad y'(z) = \frac{1}{2z}$

применить вдоль контура

$y(z) = \frac{1}{2}\ln|z| + \frac{1}{2}i\varphi + C \Rightarrow y(1) = \frac{1}{2}\ln 1 + \frac{1}{2}i\cdot 0 + C = 0 \Rightarrow C = 0$

$y(z) = \frac{1}{2}\int_C \frac{dz}{z} = \frac{1}{2}\ln z\Big|_C = \frac{1}{2}\left(\ln|z| + i\arg z\right)\Big|_C = \frac{1}{2}\left(\ln|z| + i\varphi\right)\Big|_C$

перейти к виду $z = e^{i\varphi}$

1) $y(-1), \ \varphi \in [0, \pi]$

$y(-1) = \frac{1}{2}\left(\ln|-1| + i\pi\right) = \frac{i\pi}{2}$

2) $y(-1), \ \varphi \in [0, -\pi]$

$y(-1) = \frac{1}{2}\left(\ln|-1| - i\pi\right) = -\frac{i\pi}{2}$

Задача №11

$\frac{1+2z^2}{z^3+z^5} = \frac{1}{z^3}\cdot\frac{1+2z^2}{1+z^2} = \frac{1}{z^3}\left[1 + \frac{z^2}{1+z^2}\right] = \frac{1}{z^3} + \frac{1}{z^3}\cdot\frac{z^2}{1+z^2} =$

$= \frac{1}{z^3} + \frac{1}{z}\sum_{n=0}^{\infty} z^{2n} = \underbrace{\frac{1}{z^3} + \frac{1}{z}} + z + \dots$

The principal part of the Laurent series: $\frac{1}{z^3} + \frac{1}{z}$

Задание №12

$f(z) = \frac{1}{z(e^z-1)} = \frac{1}{z}\cdot\frac{1}{e^z-1} = \frac{1}{z}\cdot\frac{1}{\cancel{1}+\sum_{n=1}^{\infty}\frac{z^n}{n!}\cancel{-1}} = \frac{1}{z}\cdot\left(\sum_{n=1}^{\infty}\frac{z^n}{n!}\right)^{-1} =$

$\frac{1}{z+\frac{z^2}{2}+\frac{z^3}{6}+\dots}$

$= \frac{1}{z}\,\frac{1}{z+\frac{z^2}{2}+\frac{z^3}{6}+\dots} \Rightarrow \frac{1}{z}\,\frac{1}{z+\frac{z^2}{2}} = \frac{1}{z^2}\,\frac{1}{1+\frac{z}{2}} = \frac{1}{z^2}\left(1-\frac{z}{2}\right) = \frac{1}{z^2} - \frac{1}{2z}$

Полюс второго порядка, т.к в разложении $(-n)_{max} = 2$

Задача №13

$$f(z)=\frac{1}{z(z-1)}$$

1) $|z|\in(0,1)$

$$\frac{1}{z(z-1)}=\frac{1}{z}\cdot\frac{1}{z-1}=-\frac{1}{z}\cdot\underset{\text{т.к. } |z|<1}{\frac{1}{1-z}}\approx-\frac{1}{z}\cdot\left(\sum_{n=0}^{\infty}z^n\right)=-\frac{1}{z}\left(1+z+z^2+\dots\right)=$$

$$=-\frac{1}{z}-\sum_{k=0}^{\infty}z^k$$

2) $|z|\in(1,+\infty)$

$$\frac{1}{z(z-1)}=\frac{1}{z}\cdot\frac{1}{z(1-\frac{1}{z})}=\frac{1}{z^2}\cdot\underset{\text{т.к. } |z|>1, \text{ то } \frac{1}{|z|}<1}{\frac{1}{1-\frac{1}{z}}}=\frac{1}{z^2}\left(\sum_{n=0}^{\infty}\frac{1}{z^n}\right)=\frac{1}{z^2}\left(\frac{1}{1}+\frac{1}{z}+\frac{1}{z^2}+\dots\right)=$$

$$=\sum_{n=2}^{\infty}\frac{1}{z^n}=\sum_{n=2}^{\infty}z^{-n}$$

Задача №14

$$f(z)=\frac{z}{z^2+1}$$

$$\frac{z}{z^2+1}=\frac{1}{2}\left(\frac{1}{z-i}+\frac{1}{z+i}\right)=\frac{1}{2(z-i)}+\frac{1}{2}\cdot\frac{1}{i+(z-i)+i}=\frac{1}{2(z-i)}+\frac{1}{4i}\left(\frac{1}{1+\frac{z-i}{2i}}\right)=$$

$$=\frac{1}{2}\cdot\frac{1}{z-i}+\frac{1}{4i}\sum_{n=0}^{\infty}(-1)^n\frac{(z-i)^n}{(2i)^n}=\frac{1}{2}\cdot\frac{1}{z-i}-\frac{i}{4}\sum_{n=0}^{\infty}\left(+\frac{i}{2}\right)^n(z-i)^n$$

При условии: $\left|\frac{z-i}{2i}\right|<1$, $|z-i|<2$.