Self-check problems on Quantum information processing

Part 2: Measurements and evolution

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Problem 1

Consider a qubit in arbitrary pure state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, and a "probe", which is also a qubit, in the initial state $|0\rangle$. Let the first qubit and the probe interact according to Hamiltonian

$$V = \frac{1}{2}\sigma_z \otimes \sigma_y \tag{1}$$

for time t. Let then the probe be measured in $\{|+\rangle, |-\rangle\}$ basis (projection on x-axis).

1. Find the probabilities of obtaining outcomes +1, and -1, corresponding "collapsed" states of the first qubit.

2. Find the state of the first qubit without information about the measurement outcome of the first probe.

3. Write down a POVM that corresponds to this such indirect measurement. At which values of t the measurement becomes projective von Neauman measurement?

Problem 2

Consider a qubit POVM

$$M = \left\{\frac{1}{2}|0\rangle\langle 0|, \frac{1}{2}|1\rangle\langle 1|, \frac{1}{2}\mathbf{1}\right\} \tag{2}$$

(here $\mathbf{1}$ is a 2×2 identity matrix).

1. Check that M is a valid POVM.

2. Design a projective measurement, which corresponds to the given POVM, in an extended space obtained by considering qubit states as states of some 4-level system.

3. Design a projective measurement, which corresponds to the given POVM, on a two qubit state $\rho \otimes |0\rangle\langle 0|$ (here ρ is a states measured with POVM).